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Published in:
Monthly Notices of the Royal Astronomical Society

DOI:
10.1111/j.1745-3933.2010.00841.x

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY LETTERS

Mon. Not. R. Astron. Soc. **404,** L89–L93 (2010) doi:10.1111/j.1745-3933.2010.00841.x

Unfolding the hierarchy of voids

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Accepted 2010 February 25. Received 2010 February 7

ABSTRACT

We present a framework for the hierarchical identification and characterization of voids based on the Watershed Void Finder. The Hierarchical Void Finder is based on a generalization of the scale space of a density field invoked in order to trace the hierarchical nature and structure of cosmological voids. At each level of the hierarchy, the watershed transform is used to identify the voids at that particular scale. By identifying the overlapping regions between watershed basins in adjacent levels, the hierarchical void tree is constructed. Applications on a hierarchical Voronoi model and on a set of cosmological simulations illustrate its potential.

Key words: methods: *N*-body simulations – methods: data analysis – techniques: image processing – large-scale structure of Universe.

1 INTRODUCTION

The large-scale distribution of matter observed in galaxy surveys and *N*-body computer simulations features a complex system of cell-like empty regions defined by a dense network of clusters, filaments and walls (Kirshner et al. 1981; Colless et al. 2003; Gott et al. 2005; Huchra et al. 2005). The Cosmic Web is the result of the tidally induced anisotropic nature of the gravitational collapse of density perturbations (Bond, Kofman & Pogosyan 1996; Zel'dovich 1970).

Within this context, voids are the low density depressions from which matter is continuously draining (Icke 1984). Forming a key component of the Cosmic Web, voids emerge out of the density troughs in the primordial Gaussian field of density fluctuations (see van de Weygaert & Platen 2009, for a recent review). As a result of their underdensity, voids represent a region of weaker gravity, resulting in an effective repulsive peculiar gravitational influence. Initially underdense regions expand faster than the Hubble flow and while they expand, matter is squeezed in between them, resulting in void boundaries consisting of sheets and filaments.

1.1 A hierarchy of voids

In addition to its anisotropic nature, the Cosmic Web is also characterized by an evident *hierarchical* structure. As a result of the multi-scale nature of the primordial perturbations, structure builds up via small-scale objects into ever-larger structures. High-resolution *N*-body experiments (Springel 2005) display a complex and tenuous network of substructures within the interior of voids, resembling the prominent Cosmic Web delineated by large haloes. The relation between different levels in the hierarchy of the Cosmic Web can be defined by the voids as, at any given level of the hierarchy, they are the cells within which we observe the web-like infrastructure at the next level.

Because of their relatively simple structure and evolution, we may better understand the gradual hierarchical buildup of the Cosmic Web on the basis of its void population. Two processes dictate the evolution of voids: their *merging* into ever-larger voids as well as the *collapse* and disappearance of small ones embedded in overdense regions. When adjacent voids meet up and merge, the matter in between is squeezed in thin walls and filaments, which subsequently drain towards the outer boundary of the voids (Dubinski et al. 1993). By identifying and assigning critical density values to the two evolutionary void processes of merging and collapse, Sheth & van de Weygaert (2004) managed to describe this hierarchical evolution of the void population in terms of a two-barrier *excursion set* formulation (Bond et al. 1991). The context of this unfolding void hierarchy within the Cosmic Web can be clearly understood within the Lagrangian adhesion description (Sahni, Sathyaprakash & Shandarin 1994).

1.2 Reconstructing the hierarchy of voids

In this study, we describe our formalism for explicitly analysing the hierarchy of voids in the cosmic matter or galaxy distribution. Based on the watershed transform (see e.g. Beucher 1982; Platen, van de Weygaert & Jones 2007), it combines the Watershed Void Finder (WVF) with a formalism to establish the hierarchical structure and relationship of the detected voids.

The grid-based WVF method introduced by Platen et al. (2007) is able to detect voids without restriction on their size and shape. A

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related Voronoi tessellation-based implementation is the ZOBOV void finder (Neyrinck 2008). Both methods are based on the idea following the slope lines connecting a given point in space to the local minima of the valley containing that point. More details on the performance of a variety of other void finders can be found in Colberg et al. (2008) (also see Lavaux & Wandelt 2010).

In Section 2, we describe the basis of the hierarchical void tree formalism. The details of the technique are outlined in Section 3. We then present an illustrative test of its performance on a heuristic hierarchical Voronoi model in Section 4. Its cosmological potential is outlined in Section 5, followed by a short discussion in Section 6.

2 THE HIERARCHICAL VOID TREE

In the void hierarchy framework, we identify voids independently at all levels of the hierarchical space and establish the cross-scale relations between voids at different levels. For establishing the multiscale and nesting properties of the void network, we follow the natural path of multiscale techniques (Aragón-Calvo et al. 2007). Within this context, we evaluate the structure of a scalar field in N dimensions in an $(N + 1)$-dimensional hierarchical space of the original field where the extra dimension represents a scale usually defined by a smoothing function (Iijima 1962; Witkin 1983).

Subsequently, voids between adjacent levels in the hierarchy are linked as a function of well-defined characteristics. A given *parent* void at the hierarchy level i is defined by smaller *children* voids at the next level $i+1$ in the hierarchy. We assign parent–child relations between voids in adjacent levels of the hierarchy by identifying overlapping volumes between the voids. A given child usually shares volume with several parent voids higher in the hierarchy. We enforce a *non-loop* property in the hierarchical tree by assigning each child void exclusively to the one parent void to which the child contributes most of its volume. This constraint assures that all children voids have only one single parent in the *void tree hierarchy*.

3 RECONSTRUCTING THE VOID HIERARCHY

Having established the general scheme for the void hierarchy tree, we need to detail its key ingredients. The first issue is that of the definition of the scale space from which we extract the void hierarchy. The most essential element is the void identification at each level, which is based on the watershed segmentation of the scalar density – or related – fields.

3.1 Scale and hierarchical spaces

Proper scale spaces must have the following set of properties: (1) linearity, (2) spatial shift invariance, (3) isotropy and (4) causality. The Gaussian filter addresses each of these constraints (Florack 1993). However, while the Gaussian function is an optimal scale-space operator, it is not necessarily the only – or the best-suited – option for the study of the hierarchical character of the Cosmic Web.

The spatial filtering approach assumes that the levels of the hierarchy are defined purely and *only* on the basis of their corresponding spatial scale. However, it would be better if our definition of a characteristic hierarchy level was based on the nature of the complex physical processes that give rise to the dark matter and galaxy distribution. Intrinsic hierarchical properties of the Cosmic Web such as halo mass functions, galaxy luminosities, galaxy morphology, etc., are suggestive examples. In the following, the term Hierarchical Spaces is used to indicate a broader class of spaces defined by one or more specific properties which are manifestations of the hierarchical nature of the Cosmic Web. This means that they do not necessarily satisfy the requirements of a proper scale space.

In the case of N-body simulations, we have access to the full evolution of the Cosmic Web. This allows us to control the relation between scales in the primordial density field. By using the information from the power spectrum, we can select those scales in the initial conditions which will grow and evolve faster or, alternatively, those that will not evolve at all. The most straightforward example would be the definition of a linear-regime smoothing procedure that will allow large-scale linear fluctuations to grow while small-scale linear fluctuations will be suppressed. This filter will act on the linear-regime matter distribution where all Fourier modes are independent and grow independently, and allow us to target specific hierarchy levels for further evolution towards collapse, ultimately producing the present-time structures. This low-pass filtered density field will evolve into a universe with all the large-scale structures in place, with their shapes moulded by anisotropic gravitational collapse, but lacking the small-scale details.

This approach is fundamentally different from the usual a posteriori smoothing operation in that it avoids the non-linear effects resulting from cross-talk between Fourier modes. It has the advantage of transparently exposing the hierarchy of structures imprinted in the initial density field.

3.2 Watershed segmentation

The watershed transform segments an image into regions following its intrinsic substructure (see Platen et al. 2007, for a detailed description of the method). The word *watershed* finds its origin in the analogy of the procedure with that of a landscape being flooded by a rising level of water: as the water level rises, the watershed basins around the minima will ultimately meet at the ridges defined by *saddle points* and *maxima* in the density field. The final result of the completely immersed landscape is a division of the landscape into individual cells, separated by *ridge dams*. The cosmological analogy with the landscape is suggestive: the basins represent the underdense void regions, while their boundaries of sheets and ridges form the network of walls, filaments and clusters that defines the Cosmic Web (Aragón-Calvo et al. 2008).

3.2.1 Oversegmentation

One of the practical complications of watershed segmentation is its sensitivity to any structure, whether it is real or an artefact. As a result, it easily partitions a given region into several smaller subregions. This ‘oversegmentation’ is commonly assumed to be the result of ‘noisy’ structures superimposed on top of the more prominent – and usually ‘real’ – features. In reality, the oversegmentation is set not only by the noise level of the image, but also by the presence of intrinsic and significant substructure in the field.

The limitations of the watershed transform due to oversegmentation can be alleviated by the use of hierarchical techniques such as the hierarchical watershed (Olsen 1996, 1997; Gauch 1999). In this approach, the watershed transform is computed on the image after smoothing at several scales or thresholding at several intensity levels. The large-scale images will delineate large regions while smoothing their boundaries. The small-scale images will reveal the small features in the image, as well as the noisy structures, while keeping the original boundaries. In a final step, the scale images are merged following a specific prescription. Often this involves the merging of small regions contained within a common parent region.

In most watershed-based hierarchical reconstruction schemes, the small-scale images in the hierarchy are merely considered as an intermediate step in the reconstruction of the features of interest. The oversegmentation is considered an undesirable effect due to the noise in the image. Here we will use a different approach. Assuming that we may ignore the noise-induced oversegmentation (see Platen et al. 2007), we focus exclusively on the oversegmentation due to intrinsic structures. Instead of using only the largest scale of the hierarchy, we will therefore consider all scales simultaneously.

3.2.2 Hierarchical watershed

We perform the void merging across adjacent levels in the hierarchy by computing only the flooding procedure on the watershed, i.e. without identifying the watershed boundaries. This procedure segments the density field into watershed basins but does not explicitly provide the boundaries between adjacent watershed regions. This 'incomplete watershed' focuses only on the space partitioning aspect of the watershed transform. This makes it straightforward to merge voxels between children voids on the basis of this incomplete watershed. This leads directly to the complete hierarchical void tree. After the merging procedure for completeness, we compute the full watershed transform (i.e. watershed basins and boundaries) by performing a local flooding watershed transform restricted to the boundary voxels as described in Aragón-Calvo et al. (2008).

Once the void hierarchy is stored in a tree structure it is straightforward to define functions to transverse the tree and extract useful information of the properties of the voids, their connectivity and their hierarchical relations.

4 TEST: HIERARCHICAL VORONOI MODELS

We tested our method with a hierarchical implementation of a Voronoi clustering model of the Cosmic Web (van de Weygaert & Icke 1989; Okabe et al. 2000). This model shares similar spatial and hierarchical properties as the observed distribution of matter while making it possible to objectively compare the recovered void hierarchy with the original one. Hierarchical Voronoi models used have the two main properties we seek to study: (1) a clear multiscale nature and (2) a hierarchy of nested structures.

4.1 Implementation

The hierarchical Voronoi model is constructed as follows: the top level of the void hierarchy is generated from a set of sparsely sampled points which define a periodic Voronoi tessellation. Inside each Voronoi cell, we define a new set of points and compute the Voronoi tessellation *locally* on the points inside the cell. This local Voronoi cell is non-periodic and has its parent Voronoi edges as boundaries. This procedure can be repeated iteratively until the desired number of nested levels in the hierarchy is reached. We regularize the size and shape of the Voronoi cells by performing a Voronoi centroid regularization on the seed points. By coupling the scalar and hierarchical aspects of the image, we can study it via the canonical Gaussian scale space.

4.2 The Voronoi test

From the hierarchical Voronoi model, we compute the *normalized distance field* for each point in a regular grid (see Aragón-Calvo et al. 2008). This field is defined as the ratio of the distance to the *closest* and *second closest* Voronoi seeds. It yields a distance field with values of 1 at the cell boundaries and decreasing towards the centre of the cell. We do this for each level in the hierarchy. Finally, all levels are integrated into a single distance field $\mathcal{I}(\boldsymbol{x}) = \sum_{i=0}^{n}[\mathcal{I}^i(\boldsymbol{x})/(i+1)^2]$, where n is the number of levels in the hierarchy. This scale integration scheme is similar to the one used in other synthetic image generation algorithms such as Perlin noise (Perlin 1985). By tuning the denominator of the above equation, it is possible to define different intensity scaling relations between levels in the hierarchy. In our case, the most prominent features in the image will be the largest voids.

Next, we construct the Gaussian scale space of the image and identify voids at each scale independently. Since our image was constructed with two characteristic scales, it makes no sense to use more than two smoothing scales. The scale space then consists of two scales: one with no smoothing and one with a width between the size of the small and large Voronoi cells.

The hierarchical merging of voids is illustrated in Fig. 1. The top and bottom left-hand panels show the original field and its smoothed version, respectively. The centre panels show the corresponding watershed transform. Note that the smoothed field produces a distorted watershed transform. Both the general shape and the boundaries of the voids are affected by the smoothing procedure. On the other hand, the watershed transform of the original field reproduces the original boundaries between voids but it does not differentiate between levels in the hierarchy. The hierarchical merging of voids is shown in the top right-hand panel. One individual void is highlighted in order to illustrate the void merging procedure. The parent void's area overlaps with several children, and one can see that the children that are mostly covered by the parent void are the ones originally inside it. The final result of the merging procedure is shown in the bottom right-hand panel where we emphasize the large voids in the top level of the hierarchy (thick lines) containing smaller voids at the bottom of the hierarchy (thin lines).

The hierarchical reconstruction of the voids has two important advantages over the single-scale WVF: (1) it is not affected by smoothing procedures and (2) it explicitly gives the inner substructure of the voids. The reconstructed hierarchical voids contain both their original shape and their original level of substructure.

5 COSMOLOGICAL APPLICATION

We applied our algorithm to three cosmological simulations that are variants of the cold dark matter scenario. The simulations cover the three possible geometries of the universe: flat, open and closed, with a cosmological parameter of $(\Omega_\mathrm{m}, \Omega_\Lambda) = (0.3, 0.7)$ for the flat Λ cold dark matter (ΛCDM) model, (0.1, 0.7) for the open ΛCDM and (0.5, 07) for the closed ΛCDM universe. Each simulation consists of 256^3 dark matter particles in a $200\,h^{-1}$ Mpc box. All simulations share the same Hubble parameter, $h = 0.7$ and $\sigma_8 = 0.8$ (see Araya-Melo 2008, for a detailed description).

We perform the linear-regime smoothing procedure by generating lower resolution versions of 128^3 and 64^3 particles from the same initial conditions. The 64^3 resolution corresponds to a cut-off scale of $\sim 3\,h^{-1}$ Mpc, enough to trace voids without significant substructure. We followed the evolution of the box from $z = 49$ until the present time $z = 0$ using the GADGET-2 *N*-body code (Springel 2005). From the final particle distribution, we compute the density field inside a cubic grid of 512 voxels per dimension using a recent implementation of the DTFE method (Schaap & van de Weygaert 2000; van de Weygaert & Schaap 2009).

The size distribution of voids in different cosmologies and levels of the hierarchy are shown in Fig. 2. The mean void sizes of voids in all cosmologies are 11, 13 and 18 Mpc h^{-1} for levels 0, 1 and 2,

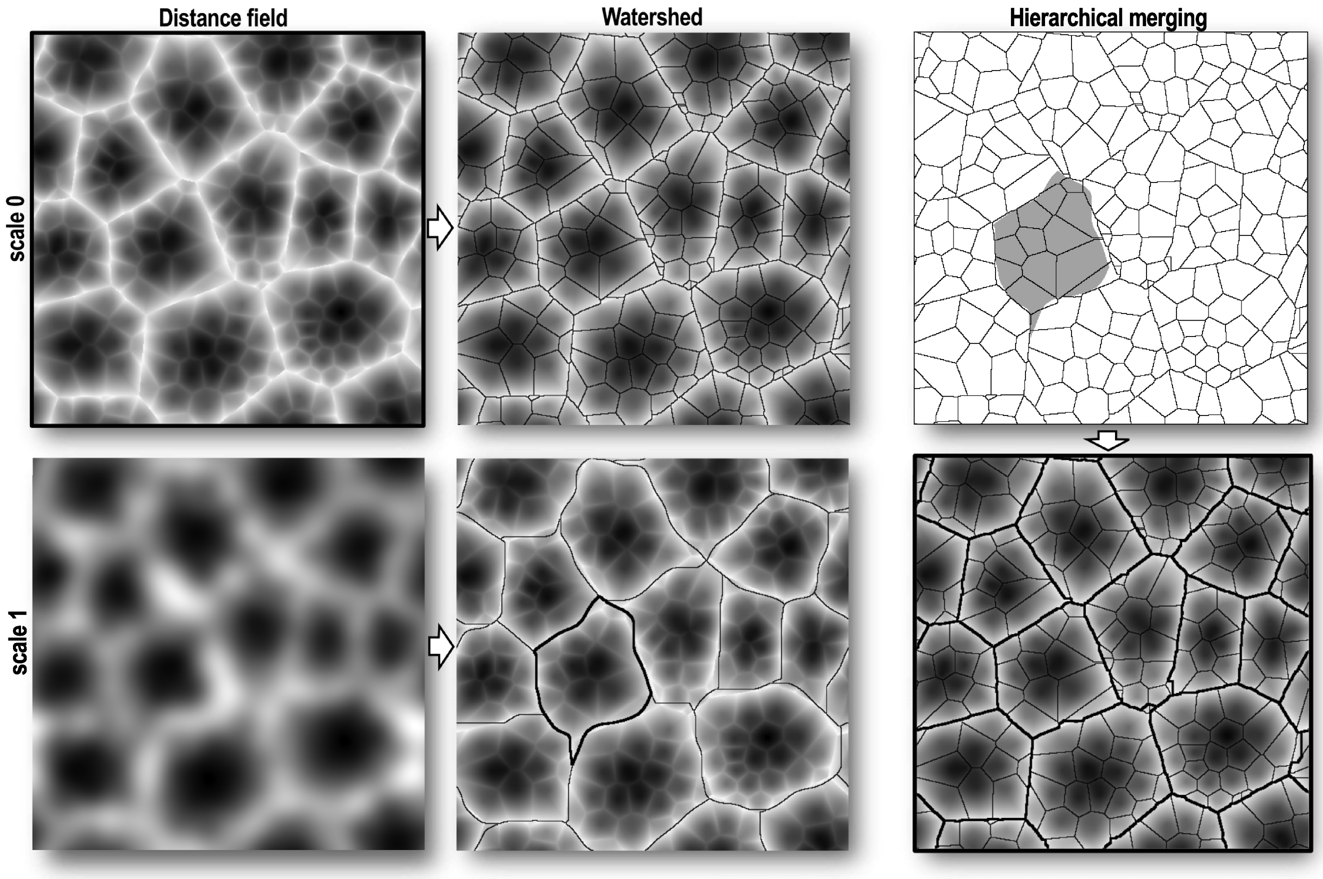

Figure 1. Hierarchical reconstruction of voids in a hierarchical Voronoi model. The original distance field is shown in the top left-hand panel (scale 0) and its smoothed version (scale 1) in the bottom left-hand panel. The centre panels show their corresponding watershed transform. An individual void is depicted at the largest scale in the bottom centre panel. The hierarchical merging of the void with its children subvoids is shown in the top right-hand panel. The final hierarchical reconstruction is shown in the bottom right-hand panel. The original shape of the large voids is reconstructed as well as their inner hierarchy of substructures.

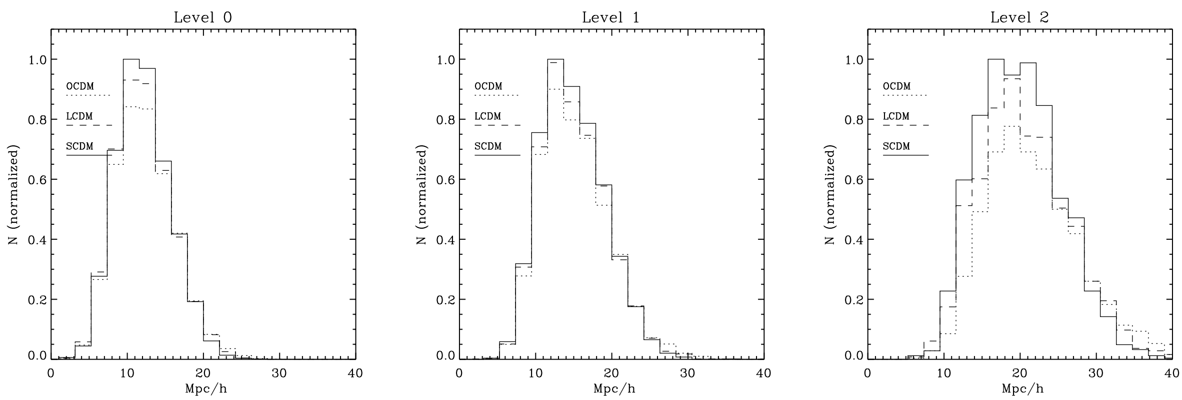

Figure 2. Distribution of void sizes for three cosmologies: SCDM (solid) ΛCDM (dashed) and OCDM (dotted) computed at three different levels of the hierarchy going from the (left) bottom of the hierarchy (smallest scale) to (right) the top of the hierarchy.

respectively. While the voids at the top of the hierarchy (level 2) are clearly the largest, the mean size and distribution of voids in levels 0 and 1 are very similar. All distributions in the three cosmologies have similar peaks at a given level in the hierarchy. However, there are differences in the overall shape of the distributions. Compared to the LCDM and SCDM, the OCDM universe has a higher tail towards large voids. The fact that the order OCDM–LCDM–SCDM is observed in all the distributions gives us a good indication of the ability of our method to discriminate between cosmologies.

6 CONCLUSION AND FUTURE WORK

We introduced a framework for the identification of voids and their hierarchical properties. The hierarchical nature of the void network makes our method a powerful tool for its description and characterization. The Hierarchical Void Finder shares the advantages of the WVF, while addressing some of its limitations such as the oversegmentation and the reconstruction of the void boundaries after strong smoothing of the density field.

In order to test our method, we introduced a hierarchical implementation of Voronoi models. These heuristic models share the multiscale, hierarchical and topological properties of the Cosmic Web. As such, the hierarchical Voronoi models represent a valuable tool for testing algorithms for LSS analysis.

We extend the idea of scale space in order to account for non-linearities and physical processes. We discuss a Gaussian smoothing in the initial conditions. By applying a Gaussian smoothing in the linear regime before there is cross-talk between Fourier modes, we are able to cleanly expose the hierarchy of structures in the evolved non-linear matter distribution.

In a following paper, we will describe the basis of the hierarchical space and explore in more detail the properties of the void network and its potential for constraining cosmological parameters.

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