Topologically massive gauge theory with 32 supercharges

Bergshoeff, Eric; Hohm, Olaf

Published in:
Physical Review D

DOI:
10.1103/PhysRevD.78.125017

Publication date:
2008

PHYSICAL REVIEW D **78,** 125017 (2008)

Topologically massive gauge theory with 32 supercharges

Eric A. Bergshoeff* and Olaf Hohm†

Centre for Theoretical Physics, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

(Received 20 October 2008; published 15 December 2008)

We construct a novel topologically massive Abelian Chern-Simons gauge theory with 32 global supersymmetries in three space-time dimensions. In spite of the 32 supercharges, the theory exhibits massive excitations only up to spin 1. The possibility of such a multiplet shortening is due to the presence of noncentral R-symmetry generators in the Poincaré superalgebra, whose supermultiplets are determined.

DOI: 10.1103/PhysRevD.78.125017 PACS numbers: 04.65.+e, 04.60.Kz, 12.60.Jv

I. INTRODUCTION

According to standard folklore supersymmetric field theories are restricted to 16 supercharges in the case of global supersymmetry or to 32 supercharges in the case of local supersymmetry. This ''no-go theorem'' follows from the requirement that the states of a supermultiplet should not exceed spin 1 (without gravity) or spin 2 (with gravity). This conclusion applies to four dimensions, where a notion of spin can be readily defined, but also to all higher dimensions $4 < D \leq 11$, which are related to the four-dimensional case via dimensional reduction.

However, in three dimensions the situation is more subtle. First of all, in the massless case there is no notion of spin or helicity, as the massless little group degenerates to the trivial SO(1). Moreover, scalars are dual to vectors—obscuring the difference between massless scalar and vector multiplets—, while states of ''higher spin'' are topological. Therefore, supermultiplets might exist for any number $\mathcal{N}$ of supersymmetries. Indeed, free globally supersymmetric theories possessing only massless scalars and Majorana spinor fields can be written for any $\mathcal{N}$ [1]. These theories seem, however, not to be extendable to nonlinear theories, at least not in the form of nonlinear σ models [2].

Notwithstanding the degenerate massless case, a notion of spin does exist in the massive case, where the little group becomes SO(2). Thus, here one expects *a priori* the same bounds as in the massless case in $D = 4$. However, it turns out that the three-dimensional Poincaré superalgebra allows an extension by noncentral R-symmetry generators, which does not have an analogue in higher dimensions [3]. For $\mathcal{N} = 8$ this nonstandard superalgebra appears as the superisometry algebra of the IIB plane wave background [4] and has recently reappeared in the study of mass-deformed multiple M2-branes [5,6]. In somewhat different manifestations the same algebra also occurs in the context of intersecting five-branes [7] and in certain sectors of the AdS_5/CFT_4 correspondence [8–10]. In this paper we will study the supermultiplets and Poincaré invariant field theories based on these unconventional superalgebras. One finds an unexpected type of multiplet shortening, which allows to increase the number of supercharges beyond the barrier mentioned above. As the main result of this paper, we derive a globally supersymmetric massive $\mathcal{N} = 16$ theory, which exhibits 32 supercharges in spite of the fact that the maximum spin is 1. Specifically, this is an Abelian gauge theory, in which the vectors become topologically massive due to the presence of a Chern-Simons term [11–14]. To derive this model we will follow a method which has recently [15–17] been pursued in order to derive the $\mathcal{N} = 8$ membrane actions of [18] from the corresponding supergravity theories [19,20]. Applying the same technique to maximal $\mathcal{N} = 16$ supergravity, one finds, surprisingly, that in contrast to higher dimensions, the topological supergravity fields can be decoupled, leaving a nontrivial $\mathcal{N} = 16$ matter theory. By starting from ungauged supergravity we recover the free massless theories of [1], while the massive theory is obtained by starting from gauged supergravity [21,22].

The organization of the paper is as follows. In Sec. II we determine the massive supermultiplets in the presence of the noncentral R-symmetry charges. In Sec. III we review the Poincaré invariant field theories for massive scalar multiplets with ordinary central charges ($\mathcal{N} = 2$) and with noncentral charges ($\mathcal{N} = 4$ and $\mathcal{N} = 8$). In Sec. IV we consider vector multiplets and determine the $\mathcal{N} = 16$ topologically massive gauge theory. We conclude with an outlook in Sec. V. Our conventions are summarized in an appendix.

II. MASSLESS AND MASSIVE SUPERMULTIPLETS

In this section we determine the massive supermultiplets for the Poincaré superalgebra with noncentral charges. For completeness we first review the standard massless and massive multiplets as well as ordinary Bogomol'nyi-Prasad-Sommerfield (BPS) multiplets. The reader only interested in the multiplet shortening due to the noncentral

*E.A.Bergshoeff@rug.nl
†O. Hohm@rug.nl

charges might skip this part and proceed directly to Sec. II C

A. Standard Poincaré superalgebra

The standard $\mathcal{N}$-extended Poincaré superalgebra for Majorana supercharges Q^i_α reads

$$\{Q^i_\alpha, Q^j_\beta\} = 2(\gamma^\mu C)_{\alpha\beta} P_\mu \delta^{ij}, \tag{2.1}$$

where $i, j, \ldots = 1, \ldots, \mathcal{N}$ and $C_{\alpha\beta}$ is the charge-conjugation matrix. For our SO(1, 2) spinor conventions we refer to Appendix A 1. The other commutation relations are standard, expressing the Poincaré algebra and the transformation properties of the supercharges under the Lorentz group.

We start with the supermultiplets of (2.1) in the massless case $P^2 = 0$, which has been analyzed in [2]. For $P_\mu = (\omega, 0, \omega)$ the superalgebra reads

$$\{Q^i_\alpha, Q^j_\beta\} = 4\omega \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_{\alpha\beta} \delta^{ij}. \tag{2.2}$$

As usual, half of the supercharges disappear, leaving a Clifford algebra for SO($\mathcal{N}$), spanned by Q^i_2. In addition, there is a fermion number operator F, which anticommutes with the supercharges and satisfies $F^2 = \mathbf{1}$. This extends the algebra to the Clifford algebra of SO($\mathcal{N} + 1$). Since in the massless case there is no notion of spin, the only thing we can consider when analyzing the multiplets is the number of bosonic and fermionic states, respectively. These are given by (half of) the dimension of the Clifford algebra, which are known for all values of $\mathcal{N} + 1$. The result is summarized in Table I.

We now turn to the massive case $P^2 = M^2$. We boost into the rest frame, $P_\mu = (M, 0, 0)$, and redefine the supercharges according to [23]

$$(a^i)^\dagger = \tfrac{1}{2}(Q^i_1 + iQ^i_2), \qquad a^i = \tfrac{1}{2}(Q^i_1 - iQ^i_2). \tag{2.3}$$

The superalgebra (2.1) reads in this basis

$$\{a^i, (a^j)^\dagger\} = M\delta^{ij}, \qquad \{a^i, a^j\} = \{(a^i)^\dagger, (a^j)^\dagger\} = 0. \tag{2.4}$$

This redefinition is such that the supercharges and their conjugates can be interpreted as lowering and raising operators. Moreover, they increase and decrease the space-time helicity with respect to the little group SO(2) [23]. In the massive case the supermultiplets are therefore standard, carrying $2^{\mathcal{N}}$ states with helicities ranging from j to $j + \mathcal{N}/2$. In particular, for $\mathcal{N} = 8$ the helicities should go up to 2 and therefore there can be no massive scalar multiplets based on the ordinary superalgebra. This is in contrast to the noncentral charges to be analyzed below.

TABLE I. Number of bosonic states d_n for massless $\mathcal{N}$-extended supermultiplets.

$\mathcal{N}$	1	2	3	4	5	6	7	8	$n+8$
d_n	1	2	4	4	8	8	8	8	$16d_n$

B. Centrally extended Poincaré superalgebra

As an example of a centrally extended superalgebra we consider the $\mathcal{N} = 2$ super-Poincaré algebra

$$\{Q^i_\alpha, Q^j_\beta\} = 2(\gamma^\mu C)_{\alpha\beta} P_\mu \delta^{ij} + 2mi\varepsilon^{ij} C_{\alpha\beta}(Z_1 - Z_2). \tag{2.5}$$

Note that here we have introduced two U(1) generators, Z_1 and Z_2, whose commutation relations with the supercharges read

$$[Z_1, Q^i_\alpha] = \varepsilon^{ij} Q^j_\alpha, \qquad [Z_2, Q^i_\alpha] = \varepsilon^{ij} Q^j_\alpha. \tag{2.6}$$

This implies that the combination $Z \equiv Z_1 - Z_2$ appearing on the right-hand side of the superalgebra (2.5) commutes with the supercharges and therefore represents a central extension. This is also required by consistency with the super-Jacobi identities. The combination $R \equiv Z_1 + Z_2$, on the other hand, rotates the supercharges according to their SO(2) indices and thus represents the SO(2) $\cong$ U(1) R symmetry. Because of the central charges, BPS multiplets are possible in the massive case. First, the oscillator algebra (2.4) gets replaced by

$$\{a^i, (a^j)^\dagger\} = M\delta^{ij} + im\varepsilon^{ij} Z. \tag{2.7}$$

The eigenvalues of the matrix appearing on the right-hand side are given by $M \pm m|Z|$. Unitarity implies therefore the following bound:

$$M \geq m|Z|. \tag{2.8}$$

In case this bound is saturated, one of the oscillators trivializes and thus the multiplets are as for $\mathcal{N} = 1$. For instance, a scalar multiplet contains spins $(0, \frac{1}{2})$. Since this carries only real degrees of freedom, it cannot transform under the required U(1) R symmetry. Thus, we have to complexify, leading to the $\mathcal{N} = 2$ multiplet $(0, \frac{1}{2}) \oplus (0, \frac{1}{2})$. We observe that parity-odd multiplets are natural in the presence of central charges.

C. Noncentrally extended Poincaré superalgebra

The possibility of a noncentrally extended superalgebra arises for $\mathcal{N} \geq 4$. For $\mathcal{N} = 4$, the super-Poincaré algebra can be extended by the following noncentral charges:

$$\{Q^i_\alpha, Q^j_\beta\} = 2(\gamma^\mu C)_{\alpha\beta} P_\mu \delta^{ij} + 2mC_{\alpha\beta}\varepsilon^{ijkl} M_{kl}, \tag{2.9}$$

where M_{ij} denotes the SO(4) R-symmetry generators. In particular, they do not commute but instead satisfy the standard relations

$$\begin{aligned} [M_{ij}, M_{kl}] &= -2(\delta_{k[i} M_{j]l} - \delta_{l[i} M_{j]k}), \\ [M^{ij}, Q^k_\alpha] &= 2\delta^{k[i} Q^{j]}_\alpha. \end{aligned} \tag{2.10}$$

Algebras of this type also appear in the context of AdS supergroups, where the supercharges generically close into the R-symmetry group. The peculiar property here, however, is that this represents a consistent algebra for Poincaré supersymmetry, i.e., despite the commuting translations, the particular choice (2.9) containing an SO(4) Levi-Civita symbol satisfies the super-Jacobi identities [8]. This noncentral extension is also possible for $\mathcal{N} > 4$. We will mainly consider the case of $\mathcal{N}$ being k multiples of 4, for which one has k copies of the algebra (2.9).[1] In this case, the SO($\mathcal{N}$) R-symmetry group will be broken to $SO(4)^k$.[2]

Let us now turn to the supermultiplets of (2.9). We note that (2.9) is related to a central extension of the superalgebra $\hat{\mathfrak{su}}(2|2)$, which appeared in the AdS_5/CFT_4 correspondence. More precisely, the central charges in that algebra can be reinterpreted as a $2+1$ dimensional energy-momentum operator P_μ, while the Lorentz generators do not appear, but rather represent outer automorphisms [9,10]. The representation theory of the latter algebra has been developed in [26]. Here, we are going to apply the standard little group technique to (2.9) and determine the supermultiplets via introducing oscillators.

In the case of $m \neq 0$ there are no massless representations of the superalgebra (2.9), which can be easily seen as follows [8]. For $P_\mu = (\omega, 0, \omega)$ the algebra reads

$$\{Q^i_\alpha, Q^j_\beta\} = 4\omega\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_{\alpha\beta}\delta^{ij} - 2im\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{\alpha\beta}\varepsilon^{ijkl}M_{kl}. \tag{2.11}$$

Thus, like in Eq. (2.2), $\{Q^i_1, Q^j_1\}$ vanishes, and so in a positive-definite Hilbert space Q^i_1 has to act trivially. On the other hand, the off-diagonal bracket $\{Q^1_1, Q^2_2\}$, for instance, is proportional to M^{34}, and therefore also this SO(4) generator has to act trivially. However, this is in conflict with the fact that according to (2.10) the supercharges change the SO(4) quantum numbers, and so the states are generically not singlets. Thus, massless representations can only exist for $m = 0$.

We next consider the massive case. The oscillator algebra now reads

$$\{a^i, (a^j)^\dagger\} = M\delta^{ij} + m\varepsilon^{ijkl}M_{kl}. \tag{2.12}$$

It turns out to be convenient to construct the representations using SU(2) spinor indices via the isomorphism $SO(4) \cong SU(2)_L \times SU(2)_R$. Specifically, the oscillators are bispinors

$$a_{a\dot{a}} = \Gamma^i_{a\dot{a}}a^i, \tag{2.13}$$

where $\Gamma^i_{a\dot{a}}$ are SO(4) gamma matrices and we use undotted and dotted indices for $SU(2)_L$ and $SU(2)_R$, respectively. The SO(4) generators decompose accordingly into the symmetric $SU(2)_{L,R}$ generators M^{ab} and $M^{\dot{a}\dot{b}}$. For further details on this notation we refer to Appendix A 2. Using this notation the algebra (2.12) reads

$$\begin{aligned} &\{a_{a\dot{a}}, a^\dagger_{b\dot{b}}\} = -2M\varepsilon_{ab}\varepsilon_{\dot{a}\dot{b}} - 4m(\varepsilon_{\dot{a}\dot{b}}M_{ab} - \varepsilon_{ab}M_{\dot{a}\dot{b}}), \\ &\{a_{a\dot{a}}, a_{b\dot{b}}\} = \{a^\dagger_{a\dot{a}}, a^\dagger_{b\dot{b}}\} = 0. \end{aligned} \tag{2.14}$$

We note that the two SU(2) factors enter with a relative minus sign, which is due to their respective self-duality and anti-self-duality, cf. Eq. (A15) in the appendix. In addition, the supercharges satisfy the commutation relations (A10), indicating that they act as raising and lowering operators for the SU(2) quantum numbers. To be more precise, if one writes the spinor indices as $a = (+, -)$ and $\dot{a} = (\dot{+}, \dot{-})$, then an undotted or dotted "+" index indicates that the $SU(2)_{L,R}$ spin quantum number is increased by $\frac{1}{2}$, while a "$-$" index indicates that it is decreased by $\frac{1}{2}$. Moreover, M_{+-} corresponds to the J_3 operator and thus measures the spin quantum number ℓ, while M_{++} and M_{--} are $SU(2)$ raising and lowering operators.

In order to construct shortened supermultiplets we must impose a generalized BPS condition. To see how this works, let us consider the bracket

$$\{a_{+\dot{+}}, (a_{+\dot{+}})^\dagger\} = -\{a_{+\dot{+}}, a^\dagger_{-\dot{-}}\} = 2M - 4m(J^L_3 - J^R_3), \tag{2.15}$$

where we used $(a_{+\dot{+}})^\dagger = -\varepsilon^{+-}\varepsilon^{\dot{+}\dot{-}}a^\dagger_{-\dot{-}}$. In case the BPS-like condition $M = 2m(\ell_L - \ell_R)$ is satisfied, positivity of the Hilbert space implies that $a^\dagger_{-\dot{-}}$ is deactivated. Similarly, one derives from (2.14) that each of the four possible raising operators is deactivated provided the corresponding BPS condition is satisfied:

$$\begin{aligned} a^\dagger_{+\dot{+}}&: M = -2m(\ell_L - \ell_R), \\ a^\dagger_{+\dot{-}}&: M = -2m(\ell_L + \ell_R), \\ a^\dagger_{-\dot{+}}&: M = 2m(\ell_L + \ell_R), \\ a^\dagger_{-\dot{-}}&: M = 2m(\ell_L - \ell_R). \end{aligned} \tag{2.16}$$

Note that, in contrast to ordinary BPS multiplets, different sets of supercharges become trivial, depending on which states they act.

Let us now turn to the construction of the supermultiplets for $\mathcal{N} = 4$, which is the first nontrivial case. We label the states $|j; \ell_L, \ell_R\rangle$ by the space-time helicity j and, in the second and third entry, by spin quantum numbers ℓ_L and ℓ_R of $SU(2)_L$ and $SU(2)_R$, respectively. As usual, we start from a "Clifford vacuum" as the lowest state. For the smallest multiplets we choose

$$|\Omega\rangle = |j_0; 0, -\tfrac{1}{2}\rangle, \tag{2.17}$$

which is annihilated by all $a_{a\dot{b}}$. Assuming $M = m$, (2.16)

[1] We should note, however, that this is not a direct sum, since there is only a single energy-momentum operator P_μ.

[2] Theories with $\mathcal{N} = 6$, breaking the R symmetry to $SO(4) \times U(1)$, and $\mathcal{N} = 5$ were considered in [24,25].

implies that only $a^\dagger_{-\dot{+}}$ and $a^\dagger_{+\dot{+}}$ are active. Thus we obtain two states with helicity $j_0 + \frac{1}{2}$: $|j_0 + \frac{1}{2}; \frac{1}{2}, 0\rangle$ and $|j_0 + \frac{1}{2}; -\frac{1}{2}, 0\rangle$. It is not possible to act a second time with the creation operators, which is the main reason for the multiplet shortening. To see this we note that due to (2.16) and the anticommutativity of the oscillators on, say, $|j_0 + \frac{1}{2}; \frac{1}{2}, 0\rangle$ only $a^\dagger_{+\dot{-}}$ can be potentially nonzero. However, one finds

$$a^\dagger_{+\dot{-}}|j_0 + \tfrac{1}{2}; \tfrac{1}{2}, 0\rangle = a^\dagger_{+\dot{-}} a^\dagger_{+\dot{+}}|\Omega\rangle = -a^\dagger_{+\dot{+}} a^\dagger_{+\dot{-}}|\Omega\rangle = 0, \tag{2.18}$$

where we used in the last equation that $a^\dagger_{+\dot{-}}$ is inactive on the vacuum $|\Omega\rangle$. Similarly, one derives that there are no other states of helicity higher than $j_0 + \frac{1}{2}$. Finally, by acting with the SU(2) raising and lowering operators M_{++}, etc., the states combine into complete SU(2) representations. If we choose $j_0 = -\frac{1}{2}$, this $\mathcal{N} = 4$ multiplet consists of two complex scalars ϕ_a and two Dirac fermions $\chi_{\dot{a}}$, corresponding to the following states:

$$\begin{tikzcd} \chi_{\dot{-}} = |-\tfrac{1}{2}; 0, -\tfrac{1}{2}\rangle \arrow[r, "a^\dagger_{+\dot{+}}"] \arrow[d, "M_{\dot{+}\dot{+}}"'] & \phi_+ = |0; \tfrac{1}{2}, 0\rangle \arrow[d, "M_{--}"] \\ \chi_{\dot{+}} = |-\tfrac{1}{2}; 0, \tfrac{1}{2}\rangle \arrow[r, "a^\dagger_{-\dot{-}}"'] & \phi_- = |0; -\tfrac{1}{2}, 0\rangle \end{tikzcd} \tag{2.19}$$

The action of those operators not indicated in (2.19) is either trivial or equivalent to the consecutive action of the given ones. For instance, $a^\dagger_{-\dot{+}}$ acting on the vacuum leads to the same state as $M_{\dot{+}\dot{+}}$ and $a^\dagger_{-\dot{-}}$, which follows from the commutation relations (A10),

$$a^\dagger_{-\dot{-}} M_{\dot{+}\dot{+}}|\Omega\rangle = -[M_{\dot{+}\dot{+}}, a^\dagger_{-\dot{-}}]|\Omega\rangle = -a^\dagger_{-\dot{+}}|\Omega\rangle. \tag{2.20}$$

We note that there is no state with space-time helicity $+\frac{1}{2}$ and therefore the multiplets are parity odd. Summarizing, the action of the creation operators on the vacuum raises the helicity from j_0 to $j_0 + \frac{1}{2}$ and converts the complex $SU(2)_R$ representation $2_{\mathbb{C}} = (0, \frac{1}{2})$ of the ground state into a $SU(2)_L$ representation $\bar{2}_{\mathbb{C}} = (\frac{1}{2}, 0)$ of the next $j_0 + \frac{1}{2}$ state. This pattern will repeat itself in the $\mathcal{N} > 4$ cases which we will discuss now.

First, we discuss $\mathcal{N} = 8$, for which the noncentral charges break the R-symmetry group to

$$\begin{aligned} SO(8) &\to SO(4)^+ \times SO(4)^- \\ &\cong SU(2)^+_L \times SU(2)^+_R \times SU(2)^-_L \times SU(2)^-_R. \end{aligned} \tag{2.21}$$

In this case one has two copies of (2.14), say, with raising operators $a^\dagger_{a\dot{a}}$ and $b^\dagger_{a\dot{a}}$, where we do not distinguish between SU(2) indices on the different operators, but simply understand that $a^\dagger$ acts on the SU(2) factors of $SO(4)^+$ and $b^\dagger$ on those of $SO(4)^-$. Analogous to $\mathcal{N} = 4$ we take the vacuum to be

$$|\Omega\rangle = |j_0; 0, -\tfrac{1}{2}, 0, -\tfrac{1}{2}\rangle, \tag{2.22}$$

where the labels $(\ell^+_L, \ell^+_R, \ell^-_L, \ell^-_R)$ refer to the helicity quantum numbers of (2.21). In the following we will assume that the factors m are the same for both SO(4) sectors, such that the BPS condition $M = m$ gives rise to the maximal possible multiplet shortening. Other cases of less multiplet shortening can be analyzed along similar lines. On (2.22) one can then act either with the $a^\dagger$ or the $b^\dagger$ operators, which in analogy to $\mathcal{N} = 4$ gives four types of states, with space-time helicity $j_0 + \frac{1}{2}$. In contrast to $\mathcal{N} = 4$ it is now possible to act a second time with raising operators, giving states of the type $a^\dagger b^\dagger|\Omega\rangle$. Thus, the multiplet contains also states with space-time helicity $j_0 + 1$. In particular, in the case of $j_0 = -\frac{1}{2}$, which we will consider in the following, the multiplet is parity even, consisting of 8 bosons and 8 fermions. Using the following decomposition of the SO(8) representations $\mathbf{8}_V$, $\mathbf{8}_S$, and $\mathbf{8}_C$ under (2.21)

$$\begin{aligned} \mathbf{8}_V &\to (\tfrac{1}{2}, \tfrac{1}{2}, 0, 0) + (0, 0, \tfrac{1}{2}, \tfrac{1}{2}), \\ \mathbf{8}_S &\to (\tfrac{1}{2}, 0, 0, \tfrac{1}{2}) + (0, \tfrac{1}{2}, \tfrac{1}{2}, 0), \\ \mathbf{8}_C &\to (0, \tfrac{1}{2}, 0, \tfrac{1}{2}) + (\tfrac{1}{2}, 0, \tfrac{1}{2}, 0), \end{aligned} \tag{2.23}$$

one finds that the representations of the supermultiplet are such that the scalars can be combined into $\mathbf{8}_S$ (ϕ^A) and the Majorana spinors into $\mathbf{8}_C$ ($\chi^{\dot{A}}$).[3]

The construction of the multiplets for arbitrary $\mathcal{N} = 4k$ proceeds in exact analogy, using k sets of oscillators. For instance, in the case of $\mathcal{N} = 16$, which will be of relevance below, the R-symmetry group is broken according to

$$SO(16) \to SO(8) \times SO(8), \tag{2.24}$$

and then each SO(8) further according to (2.21). The basic (real) representations decompose under (2.24) into

$$\begin{aligned} \mathbf{16}_V &\to (\mathbf{8}_V, \mathbf{1}) + (\mathbf{1}, \mathbf{8}_V), \\ \mathbf{128}_S &\to (\mathbf{8}_S, \mathbf{8}_S) + (\mathbf{8}_C, \mathbf{8}_C), \\ \mathbf{128}_C &\to (\mathbf{8}_S, \mathbf{8}_C) + (\mathbf{8}_C, \mathbf{8}_S). \end{aligned} \tag{2.25}$$

The $\mathcal{N} = 16$ superalgebra is spanned by four types of oscillators, say, $a^\dagger$, $b^\dagger$, $c^\dagger$, and $d^\dagger$. Starting from a vacuum $|\Omega\rangle$ with space-time helicity j_0, one can now create a state $a^\dagger b^\dagger c^\dagger d^\dagger|\Omega\rangle$, which has helicity $j_0 + 2$. Thus, starting from helicity -1, one obtains helicity $+1$ and so the multiplet can be parity even. However, in spite of the fact that we are dealing with 32 supercharges, spin-2 states are not required. We finally note that according to (2.25) the bosonic degrees of freedom can be combined into the SO(16) representation $\mathbf{128}_S$ (ϕ^A) and the fermionic de-

[3]Here $I, J, \ldots = 1, \ldots, 8$, $A, B, \ldots = 1, \ldots, 8$ and $\dot{A}, \dot{B}, \ldots = 1, \ldots, 8$ denote vector, spinor, and conjugate spinor indices of SO(8).

TABLE II. Multiplet structure for different values of $\mathcal{N}$, containing the space-time helicity j, the (real and complex) representations of the broken R-symmetry group, and the total number of degrees of freedom (d.o.f.).

Helicity	$\mathcal{N}=4$	$\mathcal{N}=8$	$\mathcal{N}=12$	$\mathcal{N}=16$
j_0	$2_{\mathbb{C}}$	$(2, 2)$	$(2, 2, 2)_{\mathbb{C}}$	$(2, 2, 2, 2)$
$j_0+\frac{1}{2}$	$\bar{2}_{\mathbb{C}}$	$(\bar{2}, 2)+(2, \bar{2})$	$(\bar{2}, 2, 2)_{\mathbb{C}}$ + 2 more	$(\bar{2}, 2, 2, 2)$ + 3 more
j_0+1		$(\bar{2}, \bar{2})$	$(\bar{2}, \bar{2}, 2)_{\mathbb{C}}$ + 2 more	$(\bar{2}, \bar{2}, 2, 2)$ + 5 more
$j_0+\frac{3}{2}$			$(\bar{2}, \bar{2}, \bar{2})_{\mathbb{C}}$	$(\bar{2}, \bar{2}, \bar{2}, 2)$ + 3 more
j_0+2				$(\bar{2}, \bar{2}, \bar{2}, \bar{2})$
d.o.f	4_B+4_F	8_B+8_F	64_B+64_F	128_B+128_F

grees of freedom into $\mathbf{128}_C$ ($\chi^{\dot{A}}$), in which, however, 32 of the scalars correspond to Stückelberg fields, that will be eaten by the vectors.

To conclude our discussion of the massive representations, we note that for each $\mathcal{N}=4k$ the structure is rather similar. One starts from a ground state $|\Omega\rangle$ which is in the $(2, 2, \ldots, 2)$ (k factors of 2) representation, being real for k even and complex for k odd. Here, we use the shorthand notation $2_{\mathbb{C}}=(0, \frac{1}{2})$, $\bar{2}_{\mathbb{C}}=(\frac{1}{2}, 0)$, indicating the spin-$\frac{1}{2}$ representations of $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$, respectively. The first excited state, with helicity $j_0+\frac{1}{2}$, is obtained by replacing one of the two representations by a $\bar{2}$. This can be done in k different ways. The next excited state is obtained by replacing in $|\Omega\rangle$ two representations by $\bar{2}$, which can be done in

$$\binom{k}{2}$$

different ways, etc. We have summarized the structure of the short multiplets for $\mathcal{N}=4, 8, 12, 16$ in Table II. The first column indicates the space-time helicities (with j_0 the helicity of the ground state), while the other columns contain the representations under $(\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R)^{\mathcal{N}/4}$. We note that scalar multiplets are possible up to $\mathcal{N}=8$ and spin-1 multiplets up to $\mathcal{N}=16$.

III. MASSIVE SCALAR MULTIPLETS

In this section we discuss massive supersymmetric field theories for $\mathcal{N}=2$, 4, and 8. As the latter is the largest amount of supersymmetry consistent with maximal spin $\frac{1}{2}$, we will focus on massive scalar multiplets. Before doing that, we first briefly illustrate the notion of parity and helicity, which in three dimensions is rather different than in four dimensions. Consider a (real) Majorana fermion χ of mass m. It carries one physical (propagating) degree of freedom. This means that it cannot carry both the two helicities $+\frac{1}{2}$ and $-\frac{1}{2}$, but only one. Thus, an action containing only one Majorana fermion necessarily breaks parity. Consider, for instance, the $\mathcal{N}=1$ Lagrangian

$$\mathcal{L}=\tfrac{1}{2}\partial^\mu\phi\partial_\mu\phi-i\bar{\chi}\gamma^\mu\partial_\mu\chi-\tfrac{1}{2}m^2\phi^2+m\bar{\chi}\chi. \qquad (3.26)$$

In order to decide whether this is parity invariant, we have to define what we mean by parity in three dimensions. In four dimensions one defines a parity transformation as $\vec{x}\to-\vec{x}$, which has determinant -1 and thus is a reflection. In contrast, in three dimensions this is a rotation and therefore we should define a parity transformation rather as inversion $x^i\to-x^i$ of one fixed spatial direction. On spinors this acts as $\chi\to i\gamma^i\chi$, see e.g. [27]. The fermionic kinetic term in (3.26) is invariant under this transformation, but the mass term $m\bar{\chi}\chi$ switches sign. Thus, (3.26) is parity breaking. We note that the sign of the mass term determines the helicity to be, say, $+\frac{1}{2}$, and therefore the corresponding $\mathcal{N}=1$ supermultiplet has the spin content $(0, +\frac{1}{2})$.

A. $\mathcal{N}=2$

In terms of $\mathcal{N}=1$ multiplets *a priori* there are two possibilities to build $\mathcal{N}=2$ multiplets: the parity even $(-\frac{1}{2}, 0, 0, \frac{1}{2})=(-\frac{1}{2}, 0)\oplus(0, \frac{1}{2})$ or the parity odd $(0, \frac{1}{2})\oplus(0, \frac{1}{2})$. As we discussed, the standard Poincaré superalgebra admits the former, while the latter is possible in the presence of central charges. The field content is given by a complex scalar ϕ and a complex (Dirac) fermion χ. For the $(-\frac{1}{2}, 2\times 0, \frac{1}{2})$ multiplet the Lagrangian is given by

$$\mathcal{L}=\tfrac{1}{2}\partial^\mu\phi^\star\partial_\mu\phi-i\bar{\chi}^\star\gamma^\mu\partial_\mu\chi+\tfrac{1}{2}m(\bar{\chi}\chi+\bar{\chi}^\star\chi^\star)-\tfrac{1}{2}m^2\phi^\star\phi, \qquad (3.27)$$

which is invariant under the $\mathcal{N}=2$ supersymmetry transformations

$$\delta_\epsilon\phi=\bar{\epsilon}\chi, \qquad \delta_\epsilon\chi=\tfrac{i}{2}\gamma^\mu\partial_\mu\phi\epsilon^\star+\tfrac{1}{2}m\phi^\star\epsilon, \qquad (3.28)$$

parametrized by the complex spinor ϵ. This can also be obtained through dimensional reduction of the standard $\mathcal{N}=1$ chiral multiplet in four dimensions. In order to see that (3.27) has an equal number of positive and negative helicities and is thus parity invariant, we split the Dirac spinor into real and imaginary parts, $\chi=\chi_1+i\chi_2$. In terms of these two Majorana fermions $\chi_{1,2}$, the mass term reads $m(\bar{\chi}_1\chi_1-\bar{\chi}_2\chi_2)$, i.e., χ_1 and χ_2 have opposite helicity. The parity transformation leaving invariant (3.27) is given by $\chi\to\gamma^i\chi^\star$. Since it involves a relative factor of i as compared to the rule employed for the real case, parity exchanges in addition the real spinors χ_1 and χ_2. This cures the noninvariance of the $\mathcal{N}=1$ action under parity by virtue of the relative sign between the mass terms.

We consider now the parity-odd multiplet. Its action and supersymmetry rules are given by

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\phi^\star\partial_\mu\phi - i\bar{\chi}^\star\gamma^\mu\partial_\mu\chi + m\bar{\chi}^\star\chi - \tfrac{1}{2}m^2\phi^\star\phi,$$
$$\delta\phi = \bar{\epsilon}\chi, \qquad \delta\chi = \tfrac{i}{2}\gamma^\mu\partial_\mu\phi\epsilon^\star + \tfrac{1}{2}m\phi\epsilon^\star. \tag{3.29}$$

Because of the relative complex conjugation in the mass term, the two real fermionic fields enter with the same sign for the mass and, consequently, the action is parity odd. Let us compute the closure of the supersymmetry algebra,

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\phi = \xi^\mu\partial_\mu\phi + m\delta_\Lambda\phi. \tag{3.30}$$

The algebra closes not only into translations with

$$\xi^\mu = \tfrac{i}{2}(\bar{\epsilon}_2\gamma^\mu\epsilon_1^\star - \bar{\epsilon}_1\gamma^\mu\epsilon_2^\star), \tag{3.31}$$

but also into a U(1) rotation, $\delta_\Lambda\phi = i\Lambda\phi$, with real parameter

$$\Lambda = -\tfrac{i}{2}(\bar{\epsilon}_2\epsilon_1^\star - \bar{\epsilon}_1\epsilon_2^\star). \tag{3.32}$$

This is in contrast to the $\mathcal{N} = 1$ and the parity-even $\mathcal{N} = 2$ theory, where a similar term proportional to m drops out of the commutator. Note that this U(1) rotation is not the R symmetry, as this would rotate the supercharges, violating the Jacobi identities. Rather, the action actually has a $U(1) \times U(1)$ symmetry, corresponding to the generators Z_1 and Z_2 in Sec. II B, in which the first U(1) acts only on the scalar, and the second U(1) acts only on the spinor. One linear combination of the U(1)'s corresponds to a central charge—appearing on the right-hand side of (3.30)—, while the other linear combination corresponds to the R symmetry and does not enter the commutator (3.30). The parity-odd multiplet described by (3.29) is in agreement with the findings for standard BPS multiplets discussed in Sec. II B.

B. $\mathcal{N} = 4$

According to the general form of the supermultiplets, for $\mathcal{N} = 4$ we have four bosonic and four fermionic degrees of freedom. Since the SO(4) R-symmetry group is isomorphic to $SU(2)_L \times SU(2)_R$ it is convenient to use complex notation. The $\mathcal{N} = 4$ multiplets found before consist of two complex scalars ϕ^a, transforming under $SU(2)_L$, and two complex spinors $\chi_{\dot a}$, transforming under $SU(2)_R$. This is analogous to the $U(1) \times U(1)$ for $\mathcal{N} = 2$. We use the standard notation that lowering and raising indices corresponds to complex conjugation, $(\phi^a)^* = \phi_a$, etc. The massive theory is given by the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\phi^a\partial_\mu\phi_a - i\bar{\chi}^{\dot a}\gamma^\mu\partial_\mu\chi_{\dot a} - \tfrac{1}{2}m^2\phi^a\phi_a + m\bar{\chi}^{\dot a}\chi_{\dot a}. \tag{3.33}$$

This is invariant under $\mathcal{N} = 4$ supersymmetry,

$$\delta_\epsilon\phi^a = \bar{\epsilon}^{a\dot a}\chi_{\dot a}, \qquad \delta_\epsilon\chi_{\dot a} = \tfrac{i}{2}\gamma^\mu\partial_\mu\phi^a\epsilon_{a\dot a} + \tfrac{1}{2}m\phi^a\epsilon_{a\dot a}, \tag{3.34}$$

where the transformation parameters $\epsilon_{a\dot a}$ satisfy the reality constraint

$$\epsilon^{a\dot a} \equiv (\epsilon_{a\dot a})^* = -\varepsilon^{ab}\varepsilon^{\dot a\dot b}\epsilon_{b\dot b}. \tag{3.35}$$

Though this theory looks quite conventional and manifestly preserves the R symmetry, the latter actually acts as noncentral charges in order to prevent states beyond spin-1/2. To verify this, we compute the commutator of the supersymmetry transformations. On the scalars one finds

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\phi^a = \xi^\mu\partial_\mu\phi^a + m\delta_\Lambda^L\phi^a, \tag{3.36}$$

where apart from the usual translations parametrized by

$$\xi^\mu = \tfrac{i}{4}(\bar{\epsilon}_{1a\dot a}\gamma^\mu\epsilon_2^{a\dot a} - \bar{\epsilon}_{2a\dot a}\gamma^\mu\epsilon_1^{a\dot a}), \tag{3.37}$$

the δ_Λ^L denotes an $SU(2)_L$ R-symmetry transformation with parameters

$$\Lambda^{ab} = \tfrac{1}{2}\varepsilon^{bc}(\bar{\epsilon}_2^{a\dot a}\epsilon_{1c\dot a} - \bar{\epsilon}_1^{a\dot a}\epsilon_{2c\dot a}). \tag{3.38}$$

Note that the symmetry of Λ^{ab} is ensured by the reality condition (3.35). Similarly, one derives for the fermions closure into translations and noncentral terms corresponding to $SU(2)_R$, up to the fermionic equations of motion.

C. $\mathcal{N} = 8$

Let us now discuss the scalar multiplets with $\mathcal{N} = 8$ supersymmetry. The massive multiplet consists of eight real scalars and eight real Majorana spinors, $(\phi^A, \chi^{\dot A})$, the former being in the spinor representation of SO(8) and the latter in the conjugate spinor representation. (Because of SO(8) triality this assignment of representations is rather arbitrary.) The simplest case of a free massive Lagrangian is given by

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\phi^A\partial_\mu\phi^A - i\bar{\chi}^{\dot A}\gamma^\mu\partial_\mu\chi^{\dot A} - \tfrac{1}{2}m^2\phi^A\phi^A - m\bar{\Gamma}^{1234}_{\dot A\dot B}\bar{\chi}^{\dot A}\chi^{\dot B}. \tag{3.39}$$

Here we have restricted to one multiplet (otherwise ϕ and χ would carry an additional SO(N) index labeling the multiplets), and ignored possible gauge couplings as in the massive deformation of multiple M2-branes [5,6]. The supersymmetry parameter transforms as a vector under SO(8), and we have the following supersymmetry rules:

$$\delta\phi^A = \Gamma^I_{A\dot A}\bar{\epsilon}^I\chi^{\dot A},$$
$$\delta\chi^{\dot A} = \tfrac{i}{2}\gamma^\mu\partial_\mu\phi^A\Gamma^I_{A\dot A}\epsilon^I - \tfrac{1}{2}m\bar{\Gamma}^{1234}_{\dot A\dot B}\Gamma^I_{A\dot B}\phi^A\epsilon^I. \tag{3.40}$$

The SO(8) symmetry is explicitly broken to $SO(4) \times SO(4)$ due to the presence of the $\bar{\Gamma}^{1234}$ matrix in the mass term. This matrix has an equal number of positive and negative eigenvalues and hence the theory is parity even.

Let us mention that this theory can be derived from gauged supergravity (see the discussion below, [16]), in which the embedding tensor satisfies a self-duality constraint in agreement with the fact that the multiplets above require a fixed factor between the two SO(4) contributions.

We finally should comment on the following peculiarity. As far as invariance of the action is concerned, the mass matrix $\bar{\Gamma}^{1234}_{\dot{A}\dot{B}}$ can equally be replaced by the SO(8) invariant $\delta_{\dot{A}\dot{B}}$ in (3.39). The analogue of the supersymmetry transformations (3.40) then closes into SO(8) rotations. In fact, this free action has an SO(8) $\times$ SO(8) symmetry, with the first factor acting on the bosons and the second factor acting on the fermions. However, the presence of two independent SO(8) groups violates covariance of the supersymmetry variations, due to the fact that $\Gamma^I_{A\dot{A}}$ is an invariant tensor only with respect to a single SO(8). Consequently, these supersymmetry transformations will not close with the SO(8) generators. Rather, they will close into a sort of generalized supersymmetry, in which instead of the combination $\Gamma^I_{A\dot{A}}\epsilon^I$ a set of 64 independent parameters $\epsilon_{A\dot{A}}$ appear. This is indeed a symmetry which, however, is clearly an artefact of the free theory. Moreover, these "supersymmetry" transformations will not close into ordinary translations. This is not what we want for a supersymmetric theory, in particular, it will not be extendable to an interacting theory—in contrast to (3.39). Thus we will not consider this possibility any further.

IV. THE $\mathcal{N} = 16$ MASSIVE GAUGE THEORY

In this section we construct the topologically massive gauge theory announced in the introduction. We construct the theory by taking the limit of gauged $\mathcal{N} = 16$ supergravity to global supersymmetry by decoupling gravity, following [16,17]. In order to illustrate the procedure we will first in Sec. IV A perform the limit of ungauged supergravity, which results in a massless conformally invariant theory, and then explain the limit for gauged supergravity in Sec. IV B. The final result for the topologically massive deformation is presented in Sec. IV C.

A. The $\mathcal{N} = 16$ massless theory

Ungauged $\mathcal{N} = 16$ supergravity has been constructed in [1], to which we refer the reader for further details. The field content consists of 128 scalar fields ϕ^A, parametrizing the coset space $E_{8(8)}/SO(16)$, and 128 Majorana fermions $\chi^{\dot{A}}$.[4] The metric $e_\mu{}^a$ and the 16 gravitini ψ^I_μ are purely topological in three dimensions and thus do not add any propagating degrees of freedom. The Lagrangian is given by [1]

[4]The indices $I, J = 1, \ldots, 16$, $A = 1, \ldots, 128$ and $\dot{A} = 1, \ldots, 128$ refer now to the vector, spinor, and conjugate spinor representation of SO(16).

$$\mathcal{L} = -\frac{1}{4\kappa^2} eR + \frac{1}{2}\varepsilon^{\mu\nu\rho}\bar{\psi}^I_\mu D_\nu \psi^I_\rho + \frac{1}{4\kappa^2} e g^{\mu\nu} P_\mu{}^A P_\nu{}^A - \frac{i}{2} e \bar{\chi}^{\dot{A}} \gamma^\mu D_\mu \chi^{\dot{A}} + \cdots, \tag{4.41}$$

where we ignored higher-order terms, as these will drop out upon taking the limit to global supersymmetry. Here, $P_\mu{}^A$ is the noncompact part of the Maurer-Cartan forms defined in terms of the $E_{8(8)}$-valued group element $\mathcal{V}(x)$ as

$$\mathcal{V}^{-1}\partial_\mu \mathcal{V} = P_\mu{}^A Y^A + \tfrac{1}{2} \mathcal{Q}_\mu{}^{IJ} X^{IJ}, \tag{4.42}$$

where $t^{\mathcal{M}} \equiv (X^{IJ}, Y^A)$ with adjoint indices $\mathcal{M}, \mathcal{N}, \ldots = 1, \ldots, 248$ are the $\mathfrak{e}_{8(8)}$ generators in the SO(16) decomposition $\mathbf{248} \rightarrow \mathbf{120} + \mathbf{128}$. Upon gauge fixing the local SO(16) symmetry, the group-valued $\mathcal{V}$ can be parametrized in terms of the scalar fields as $\mathcal{V}(x) = \exp(\phi^A(x) Y^A)$, which implies

$$P_\mu{}^A = \partial_\mu \phi^A + \mathcal{O}(\phi^2). \tag{4.43}$$

We finally note that we have kept the explicit dependence on Newton's constant κ, which is of mass dimension $-\frac{1}{2}$. The dimensions of the fields are $(h_{\mu\nu}, \psi^A_\mu, \chi^{\dot{A}}, \phi^A) = (\frac{1}{2}, 1, 1, 0)$, with $h_{\mu\nu}$ denoting the fluctuations of the metric around Minkowski space, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$.

Let us now decouple gravity by sending $\kappa \rightarrow 0$. In order for this limit to be nonsingular, we need to rescale the scalar fields as $\phi^A \rightarrow \kappa\phi^A$ [16]. After setting the topological supergravity multiplet $(h_{\mu\nu}, \psi^I_\mu)$ to zero, the resulting action describes the free theory

$$\mathcal{L}_0 = \frac{1}{4}\partial^\mu \phi^A \partial_\mu \phi^A - \frac{i}{2}\bar{\chi}^{\dot{A}}\gamma^\mu \partial_\mu \chi^{\dot{A}}, \tag{4.44}$$

while the supersymmetry transformations of [1] reduce to

$$\delta_\epsilon \phi^A = \Gamma^I_{A\dot{A}} \bar{\chi}^{\dot{A}} \epsilon^I, \qquad \delta_\epsilon \chi^{\dot{A}} = \frac{i}{2}\Gamma^I_{A\dot{A}}\gamma^\mu \partial_\mu \phi^A \epsilon^I. \tag{4.45}$$

One may easily convince oneself that (4.45) leaves (4.44) invariant, i.e., in spite of the fact that $\mathcal{N} = 16$ represents 32 real supercharges, it is a symmetry of the globally supersymmetric action (4.44). As we noted in the introduction, the existence of this theory is not in conflict with the "higher-spin barrier," which in dimensions $D \geq 4$ excludes globally supersymmetric theories with more than 16 supercharges. In fact, it has already been noticed in [1] that free supersymmetric theories in $D = 3$ can be written for any $\mathcal{N} = 8k$. One simply uses the fact that for multiples of 8, SO($\mathcal{N}$) possesses two inequivalent real spinor representations of the same dimension, with invariant tensor $\Gamma^I_{A\dot{A}}$, such that (4.45) immediately extends to $\mathcal{N} = 8k$.

B. The $\mathcal{N} = 16$ massive theory

We now turn to the global limit of gauged supergravity, which will lead to a massive deformation of (4.44), featuring in addition to massive scalars and spinors topologically massive gauge vectors. The latter is in agreement with the general structure of BPS multiplets discussed in the previous section.

The gauged $\mathcal{N} = 16$ supergravity as constructed in [21,22] is completely determined by means of the so-called embedding tensor $\Theta_{\mathcal{M}\mathcal{N}} = \Theta_{\mathcal{N}\mathcal{M}}$. The latter encodes the subgroup of the rigid invariance group $E_{8(8)}$ that is gauged by determining the covariant derivatives

$$D_\mu = \partial_\mu + \Theta_{\mathcal{M}\mathcal{N}} A_\mu^{\mathcal{M}} t^{\mathcal{N}}. \tag{4.46}$$

More precisely, one introduces 248 vector fields in order to perform the gauging which, however, will only enter through a topological Chern-Simons term and as such do not alter the counting of degrees of freedom. The action is given by

$$\begin{aligned}\mathcal{L} = \mathcal{L}_0 &- \frac{1}{4}\varepsilon^{\mu\nu\rho} A_\mu^{\mathcal{M}} \Theta_{\mathcal{M}\mathcal{N}}\Big(\partial_\nu A_\rho^{\mathcal{N}} \\ &+ \frac{1}{3}\Theta_{\mathcal{K}\mathcal{S}} f^{\mathcal{N}\mathcal{S}}{}_{\mathcal{L}} A_\nu{}^{\mathcal{K}} A_\rho^{\mathcal{L}}\Big) + \frac{1}{2\kappa^2} e A_1^{IJ} \bar{\psi}_\mu^I \gamma^{\mu\nu}\psi_\nu^J \\ &+ \frac{i}{\kappa^2} e A_2^{I\dot{A}} \bar{\chi}^{\dot{A}} \gamma^\mu \psi_\mu^I + \frac{1}{2\kappa^2} e A_3^{\dot{A}\dot{B}} \bar{\chi}^{\dot{A}}\chi^{\dot{B}} - \frac{1}{\kappa^6}\ \mathrm{eV}.\end{aligned} \tag{4.47}$$

Here, $\mathcal{L}_0$ denotes the ungauged Lagrangian (4.41), in which all derivatives have been replaced by the covariant derivatives (4.46). The scalar-dependent Yukawa couplings parametrized by $A_{1,2,3}$ and the scalar potential V, which can be written as a square in A_1 and A_2, are completely determined by the embedding tensor. Their expressions can be found in [21,22]. The action (4.47) is invariant under local supersymmetry, provided the fermionic variations acquire shift terms proportional to $\Theta_{\mathcal{M}\mathcal{N}}$,

$$\delta\psi_\mu^I = \delta_0\psi_\mu^I + iA_1^{IJ}\gamma_\mu\epsilon^J, \qquad \delta\chi^{\dot{A}} = \delta_0\chi^{\dot{A}} + A_2^{I\dot{A}}\epsilon^I, \tag{4.48}$$

and provided the embedding tensor satisfies a linear and quadratic constraint. The explicit form of the linear constraint is given by Eq. (4.6) of Ref. [22]. The quadratic constraint follows by requiring gauge invariance of (4.47) and, consequently, invariance of the embedding tensor. It reads

$$\Theta_{\mathcal{P}\mathcal{K}}\Theta_{\mathcal{L}(\mathcal{M}} f^{\mathcal{K}\mathcal{L}}_{\mathcal{N})} = 0, \tag{4.49}$$

where f denotes the $E_{8(8)}$ structure constants.

Let us now discuss the decoupling limit $\kappa \to 0$. Splitting the $E_{8(8)}$ indices under SO(16), $\mathcal{M} = ([IJ], A)$, we obtain three components of the embedding tensor, $\Theta_{IJ,KL}$, $\Theta_{IJ,A}$, Θ_{AB}, and correspondingly two types of gauge fields, A_μ^{IJ} and A_μ^A. As was shown in [16], this limit is only nonsingular and admits nontrivial supersymmetry transformations for the gauge vectors, provided one performs first certain rescalings with κ. More precisely, the components of the embedding tensor need to be rescaled with κ^2 and the gauge vectors by κ^{-1}. Afterwards, the SO(16) gauge vectors have to be set to zero, as these belong to the supergravity multiplet. This is in accordance with the fact that in globally supersymmetric theories the R-symmetry group cannot be gauged. Instead, the components of Θ in the SO(16) direction will give rise to massive deformations, as we will see below.

The condition of a nonsingular limit requires moreover that certain components of the embedding tensor are set to zero, or in other words, that there are additional linear constraints. These can be determined by expanding the tensors $A_{1,2,3}$ in powers of the scalar fields and Θ and inspecting their scaling behavior with κ, as has been shown in [16]. Rather than repeating these steps in detail here, we will just state the results and refer the reader to [16,22] for explicit formulae. In total, one finds that the available components of the embedding tensor are $\Theta_{IJ,KL}$, satisfying $\Theta_{IK,JK} = 0$ and Θ_{AB}. Together with the linear constraints of [22] this in turn implies

$$\begin{aligned}\Theta_{IJ,KL} = f_{IJKL} \equiv f_{[IJKL]}, \qquad \Theta_{IJ,A} = 0, \\ \Theta_{AB} = \tfrac{1}{96}\Gamma^{IJKL}_{AB} f_{IJKL},\end{aligned} \tag{4.50}$$

i.e., the embedding tensor is parametrized in terms of a totally antisymmetric 4-rank tensor f_{IJKL}. Without referring further to the supergravity limit we will present the Lagrangian and supersymmetry rules of the $\mathcal{N} = 16$ massive gauge theory in the following subsection.

C. $\mathcal{N} = 16$ action and supersymmetry transformations

We find that the Lagrangian corresponding to the action of the $\mathcal{N} = 16$ massive gauge theory is given by

$$\begin{aligned}\mathcal{L} = &\frac{1}{4} D^\mu\phi^A D_\mu\phi^A - \frac{i}{2}\bar{\chi}^{\dot{A}}\gamma^\mu\partial_\mu\chi^{\dot{A}} \\ &- \frac{1}{4}\varepsilon^{\mu\nu\rho}\Theta_{AB} A_\mu{}^A \partial_\nu A_\rho{}^B + \frac{1}{96}\Gamma^{IJKL}_{\dot{A}\dot{B}}\Theta_{IJ,KL}\bar{\chi}^{\dot{A}}\chi^{\dot{B}} \\ &- \frac{1}{16} A_2^{I\dot{A}} A_2^{I\dot{A}},\end{aligned} \tag{4.51}$$

where we defined

$$\begin{aligned}D_\mu\phi^A &= \partial_\mu\phi^A + \Theta_{AB}A_\mu{}^B, \\ A_2^{I\dot{A}} &= \tfrac{1}{16}(\Gamma^{JKL}_{A\dot{A}}\Theta_{IJ,KL} + \tfrac{1}{12}\Gamma^{IJKLM}_{A\dot{A}}\Theta_{JK,LM})\phi^A.\end{aligned} \tag{4.52}$$

The $\mathcal{N} = 16$ supersymmetry transformations (corresponding to 32 real supercharges) read

$$\begin{aligned}\delta_\epsilon\phi^A &= \Gamma^I_{A\dot{A}}\bar{\chi}^{\dot{A}}\epsilon^I, \\ \delta_\epsilon\chi^{\dot{A}} &= \frac{i}{2}\Gamma^I_{A\dot{A}}\gamma^\mu D_\mu\phi^A\epsilon^I + A_2^{I\dot{A}}\epsilon^I, \\ \delta_\epsilon A_\mu{}^A &= i\Gamma^I_{A\dot{A}}\bar{\epsilon}^I\gamma_\mu\chi^{\dot{A}}.\end{aligned} \tag{4.53}$$

In the global limit there is a remnant of the quadratic constraint (4.49), which reads

$$\Gamma^{IJ}_{CD}\Theta_{AC}\Theta_{BD} + \tfrac{1}{2}\Gamma^{KL}_{AC}\Theta_{CB}\Theta_{IJ,KL} = 0. \tag{4.54}$$

To summarize, the action corresponding to (4.51) is invariant under the $\mathcal{N} = 16$ supersymmetry variations (4.53), provided the components of the embedding tensor are given by (4.50), satisfying the quadratic constraint (4.54).

Let us now determine the physical content of (4.51). The scalar potential quadratic in A_2 reduces to pure mass terms for ϕ^A. Similarly, the Yukawa couplings involving $\chi^{\dot{A}}$ lead to mass terms for the spinors. To determine the number of massive spin-0 and spin-1 degrees of freedom, we note that by virtue of (4.54) the Lagrangian (4.51) is invariant under the local shift symmetry

$$\delta_\Lambda \phi^A = \Theta_{AB}\Lambda^B, \qquad \delta_\Lambda A_\mu{}^A = -\partial_\mu \Lambda^A. \tag{4.55}$$

Therefore, the scalar potential does not depend on all scalar fields, but only on a subset determined by the choice of embedding tensor, which are precisely those that become massive due to the presence of A_2. The complementary scalar fields can in turn be gauged to zero by virtue of (4.55). The field equations for the corresponding vector fields then take the form of massive self-duality equations,

$$\Theta_{AB}(F_{\mu\nu}{}^B - \Theta_{BC}\varepsilon_{\mu\nu\rho}A^{\rho C}) = 0. \tag{4.56}$$

After acting with ∂^μ, one obtains the standard massive Yang-Mills equation with mass matrix Θ_{AB} [13]. In other words, the vectors corresponding to a zero eigenvalue of Θ_{AB} disappear from the Lagrangian, leaving a massive scalar, while a nonzero eigenvalue indicates a massive spin-1 field in a Stückelberg formulation.

According to the results summarized in Table II, the 128 bosonic degrees of freedom should be distributed, for any choice of embedding tensor, among 96 massive spin-0 scalars and 32 massive spin-1 vectors. In fact, the possible solutions of (4.54) are quite restricted. It turns out that a solution is given by various copies of the SO(4) Levi-Civita symbol. Thus the R-symmetry group SO(16) is broken into $\mathrm{SO}(4) \times \cdots \times \mathrm{SO}(4)$. Splitting the SO(16) indices into four blocks of SO(4) vector indices $i, j, \ldots = 1, \ldots, 4$, the solution is given by

$$\Theta_{ij,kl} = f_{ijkl} = m\varepsilon_{ijkl}, \tag{4.57}$$

etc. Moreover, the parameter m is restricted by (4.54) to be the same for all four copies of SO(4).[5] This is in agreement with the analysis of the foregoing section, since in the commutator algebra of the supersymmetry transformations (4.53), these parameters multiply the noncentral SO(4) generators, which on the other hand were required to be equal in order to have the maximal multiplet shortening. For the same reason we do not expect the existence of any other solutions of (4.54). We verified with `Mathematica` that inserting (4.57) into (4.50) gives rise to the correct number of zero eigenvalues of Θ_{AB}, in agreement with the expected number of massive spin-0 and spin-1 degrees of freedom (including negative and positive helicities). Moreover, also the scalar mass matrix determined by $A_2^{I\dot{A}}$ and the fermionic mass matrix give rise to the expected eigenvalues.

Let us finally comment on the full gauged supergravity, which gives rise to the given Poincaré invariant theory upon decoupling gravity.[6] This has to be the $\mathrm{SO}(4,4) \times \mathrm{SO}(4,4)$ gauging analyzed in [28], since it has a unique Minkowski ground state, whose mass spectrum coincides with the spectrum above. It would be instructive to study the precise embedding in more detail, but we will leave this for future work.

D. Interacting theories beyond $\mathcal{N} = 8$?

One may wonder whether the limit of gauged supergravity allows the construction of interacting globally supersymmetric theories beyond $\mathcal{N} = 8$. First of all, the free massive deformations as for the $\mathcal{N} = 16$ case just described will also exist for $\mathcal{N} = 9, 10, 12$, simply by taking an embedding tensor in the R-symmetry direction. Concerning the problem of a limit which leaves an interacting theory, $\mathcal{N} = 12$ seems to be a promising candidate, since in this case the coset space in supergravity is $\mathrm{E}_{7(-5)}/(\mathrm{SO}(12) \times \mathrm{SU}(2))$. In particular, the local subgroup H consists not only of the R-symmetry group SO(12), but also of the non-Abelian complement SU(2). If a gauging only of this SU(2) is possible, this would give rise in the limit to a conformally invariant SU(2) Chern-Simons theory. Unfortunately, the general solution of the constraints for compact gauge groups given in [20] [see their Eq. (5.17)] does not allow to consistently switch off the gaugings in the R-symmetry direction. Since, as shown in [16], the components of the embedding tensor in the R symmetry (SO(12)) and global symmetry (SU(2)) directions scale differently with Newton's constant, it follows that these gaugings do not allow a consistent flat space limit. Thus we conclude that $\mathcal{N} = 12$ supergravity does not give rise to a non-Abelian, interacting theory.

V. DISCUSSION AND OUTLOOK

In this paper we analyzed an extension of Poincaré supersymmetry in three dimensions by noncentral R-symmetry generators, both at the level of the supermultip-

[5] To be precise, there is slightly more freedom in that certain relative signs between the four sectors are not fixed. However, one may check that these choices lead to the same mass matrices in (4.51) and so do not represent physically different theories.

[6] We would like to thank Henning Samtleben for discussions on this point and for bringing Ref. [28] to our attention.

lets and at the level of field theoretical realizations. We found an unconventional type of multiplet shortening, which goes beyond the standard one known from central charges and BPS multiplets. In particular, the usual bounds for supersymmetry are stretched by a factor of 2 in that scalar multiplets with maximum spin $\frac{1}{2}$ are possible up to 16 supercharges and vector multiplets with spin 1 up to 32 supercharges. For the latter we determined a field theoretical realization with topologically massive gauge fields by decoupling gravity from gauged $\mathcal{N} = 16$ supergravity.

This unexpected phenomenon suggests interesting further research. First of all, the massive $\mathcal{N} = 16$ theory we constructed in this paper is a free theory. Since the discussed mechanism of multiplet shortening happens also for interacting theories (as the massive deformations of the Bagger-Lambert theory), the question arises whether interacting theories with $\mathcal{N} > 8$ exist. One approach to derive more general theories might be to take the limit of supergravity to nonflat backgrounds. Furthermore, it would be interesting to find out whether the given model or extensions thereof has a direct physical interpretation, say in the context of brane dynamics. Perhaps supersymmetry enhancement as in [7] plays a role here.

Finally, let us note that requiring maximal spin 2 allows supersymmetry up to $\mathcal{N} = 32$ corresponding to 64 supercharges, and so one may hope to construct supergravity theories with this amount of supersymmetry. Actions for *massive* propagating spin-2 fields do exist in three dimensions. Here, the usual (topological) Einstein-Hilbert action is extended by a gravitational Chern-Simons term, quite analogous to the topological mechanism for spin-1 fields encountered above [11]. In fact, these can even be made supersymmetric [29]. However, the supermultiplets discussed in Sec. II require that the spin-2 states transform nontrivially under the R-symmetry group, or in other words, this would require a multigraviton theory. While theories of this type are usually considered to be consistent only in the case of an infinite number of spin-2 fields [30], it might be worth investigating whether this unconventional framework allows for new possibilities.

ACKNOWLEDGMENTS

We thank B. de Wit, J. Engquist, and H. Lin for useful discussions and especially H. Nicolai for asking the right question. This work was partially supported by the European Commission FP6 Program No. MRTN-CT-2004-005104EU and by the INTAS Project No. 1000008-7928.

APPENDIX A: CONVENTIONS AND USEFUL RELATIONS

1. Spinor conventions in $D = 3$

For the SO(1, 2) gamma matrices we choose the purely imaginary basis

$$\gamma^0 = \sigma_2, \qquad \gamma^1 = i\sigma^3, \qquad \gamma^2 = i\sigma^1, \tag{A1}$$

which satisfies the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ for $\eta^{\mu\nu} = (+ - -)$. The charge-conjugation matrix is defined as

$$C = \gamma^0, \tag{A2}$$

satisfying

$$C(\gamma^\mu)^T C^{-1} = -\gamma^\mu. \tag{A3}$$

Consequently, the matrices $(\gamma^\mu C)_{\alpha\beta}$ are symmetric in the spinor indices α, β. We define the bilinear $\bar\psi\psi$ through

$$\bar\psi = \psi^T\gamma^0, \tag{A4}$$

which is invariant under the real three-dimensional Lorentz group $SL(2, \mathbb{R})$. In particular, it can be defined without complex conjugation, even if the spinors are not real. The identities

$$\bar\psi\chi = \bar\chi\psi, \qquad \bar\psi\gamma^\mu\chi = -\bar\chi\gamma^\mu\psi, \tag{A5}$$

etc. readily follow. One can impose a Majorana condition on a spinor ψ, which reads $\bar\psi^* = \psi^T C$. In the given basis, this means that ψ is real. For Majorana spinors ψ, the bilinear $\bar\psi\psi$ is real by virtue of the convention $(\psi_1\psi_2)^* = \psi_2^*\psi_1^*$. We note that due to the definition (A4), there are two different real Lorentz invariant bilinears for complex Dirac spinors $\chi_{\dot a}$ (as, e.g., used for $\mathcal{N} = 4$ in the main text), namely $\bar\chi^{\dot a}\chi_{\dot a}$ and $\bar\chi_{\dot a}\chi_{\dot b}$ + H.c..

2. SO(4) conventions

The SO(4) generators are given by $M_{ij} = -M_{ji}$, $i, j, \ldots = 1, \ldots, 4$, satisfying the algebra (2.10). In order to exhibit the isomorphism $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ it is convenient to introduce spinor indices $a, b, \ldots = 1, 2$, $\dot a, \dot b, \ldots = 1, 2$ and to relate SO(4) vector indices to bispinors via $\Gamma^i_{a\dot a} \equiv (i\mathbf{1}, \sigma^1, \sigma^2, \sigma^3)$. This allows to introduce generators

$$M_{ab} = -\tfrac{1}{4}\Gamma^{ij}_{ab}M_{ij}, \qquad M_{\dot a\dot b} = -\tfrac{1}{4}\bar\Gamma^{ij}_{\dot a\dot b}M_{ij}, \tag{A6}$$

or, inversely,

$$M^{ij} = \tfrac{1}{2}(\Gamma^{ij}_{ab}M^{ab} + \bar\Gamma^{ij}_{\dot a\dot b}M^{\dot a\dot b}), \tag{A7}$$

which are both symmetric in their respective spinor indices. Here we have defined

$$\begin{aligned}\Gamma^{ij}_{ab} &= \tfrac{1}{2}\varepsilon^{\dot a\dot b}(\Gamma^i_{a\dot a}\Gamma^j_{b\dot b} - \Gamma^j_{a\dot a}\Gamma^i_{b\dot b}),\\ \bar\Gamma^{ij}_{\dot a\dot b} &= \tfrac{1}{2}\varepsilon^{ab}(\Gamma^i_{a\dot a}\Gamma^j_{b\dot b} - \Gamma^j_{a\dot a}\Gamma^i_{b\dot b}),\end{aligned} \tag{A8}$$

where we introduced the SU(2) invariant Levi-Civita symbol ε^{ab} (with $\varepsilon^{12} = \varepsilon_{12} = +1$), which allows to raise and lower indices and which satisfies $\varepsilon_{ac}\varepsilon^{cb} = -\delta_a{}^b$. In terms of M_{ab} and $M_{\dot a\dot b}$ the SO(4) algebra (2.10) takes explicitly the direct product form $SU(2)_L \times SU(2)_R$,

$$\begin{aligned}
[M_{ab}, M_{cd}] &= \tfrac{1}{2}(\varepsilon_{ac}M_{bd} + \varepsilon_{bc}M_{ad} + \varepsilon_{ad}M_{bc} + \varepsilon_{bd}M_{ac}),\\
[M_{\dot a\dot b}, M_{\dot c\dot d}] &= \tfrac{1}{2}(\varepsilon_{\dot a\dot c}M_{\dot b\dot d} + \varepsilon_{\dot b\dot c}M_{\dot a\dot d} + \varepsilon_{\dot a\dot d}M_{\dot b\dot c} + \varepsilon_{\dot b\dot d}M_{\dot a\dot c}),\\
[M_{ab}, M_{\dot a\dot b}] &= 0.
\end{aligned} \tag{A9}$$

Moreover, in this language the raising and lowering operators introduced in the main text are bispinors $a^\dagger_{a\dot a}$, etc., and satisfy

$$\begin{aligned}
[M_{ab}, a^\dagger_{c\dot c}] &= \tfrac{1}{2}(\varepsilon_{ac}a^\dagger_{b\dot c} + \varepsilon_{bc}a^\dagger_{a\dot c}),\\
[M_{\dot a\dot b}, a^\dagger_{c\dot c}] &= \tfrac{1}{2}(\varepsilon_{\dot a\dot c}a^\dagger_{c\dot b} + \varepsilon_{\dot b\dot c}a^\dagger_{c\dot a}),
\end{aligned} \tag{A10}$$

and similarly for lowering operators.

The given basis is convenient in order to develop the representation theory, since the generators immediately represent lowering and raising operators for SU(2). To see this, we split the indices according to $a = (+, -)$ and $\dot a = (\dot +, \dot -)$. Then one can identify the $SU(2)_L$ generators,

$$J^L_+ = M_{++}, \qquad J^L_- = M_{--}, \qquad J^L_3 = -M_{+-}, \tag{A11}$$

satisfying the standard algebra

$$\begin{aligned}
[J^L_3, J^L_+] = J^L_+, \qquad [J^L_3, J^L_-] = -J^L_-,\\
[J^L_+, J^L_-] = -2J_3,
\end{aligned} \tag{A12}$$

and analogously for $SU(2)_R$. This notation is chosen such that the index structure of the raising and lowering operators directly indicates how it increases (+) or decreases (−) the SU(2) quantum numbers of a given state. For instance, from (A10) one infers

$$[J^L_3, a^\dagger_{+\dot -}] = \tfrac{1}{2}a^\dagger_{+\dot -}, \qquad [J^R_3, a^\dagger_{+\dot -}] = -\tfrac{1}{2}a^\dagger_{+\dot -}. \tag{A13}$$

Consequently, $a^\dagger_{+\dot -}$ increases the $SU(2)_L$ quantum number and decreases the $SU(2)_R$ quantum number by $\frac{1}{2}$.

We finally give some identities, which we found useful for relating SO(4) to SU(2) quantities. The $\Gamma^i_{a\dot a}$ satisfy

$$\begin{aligned}
\varepsilon^{ab}\varepsilon^{\dot a\dot b}\Gamma^i_{a\dot a}\Gamma^j_{b\dot b} &= -2\delta^{ij}, \qquad \Gamma^i_{a\dot a}\Gamma^i_{b\dot b} = -2\varepsilon_{ab}\varepsilon_{\dot a\dot b},\\
\Gamma^{ij}_{ab}\Gamma^j_{c\dot c} &= -2\varepsilon_{c(a}\Gamma^i_{b)\dot c}, \qquad \bar\Gamma^{ij}_{\dot a\dot b}\Gamma^j_{c\dot c} = -2\varepsilon_{\dot c(\dot a}\bar\Gamma^i_{b)c},
\end{aligned} \tag{A14}$$

while the Γ^{ij} and $\bar\Gamma^{ij}$ obey the (anti)self-duality relations

$$\Gamma^{ij}_{ab} = \tfrac{1}{2}\varepsilon^{ijkl}\Gamma^{kl}_{ab}, \qquad \bar\Gamma^{ij}_{\dot a\dot b} = -\tfrac{1}{2}\varepsilon^{ijkl}\bar\Gamma^{kl}_{\dot a\dot b}. \tag{A15}$$

Finally, we have the reality constraints

$$(\Gamma^\dagger_i)^{a\dot a} = -\varepsilon^{ab}\varepsilon^{\dot a\dot b}(\Gamma_i)_{b\dot b} \qquad (\Gamma^\dagger_{ij})^{ab} = \varepsilon^{ac}\varepsilon^{bd}(\Gamma_{ij})_{cd}, \tag{A16}$$

such that the anti-Hermiticity $(M^{ij})^\dagger = -M^{ij}$ implies for the $SU(2)_L$ generators

$$M^{ab} = (M_{ab})^\dagger = -\varepsilon^{ac}\varepsilon^{bd}M_{cd}, \tag{A17}$$

and analogously for $SU(2)_R$.

[1] N. Marcus and J.H. Schwarz, Nucl. Phys. **B228**, 145 (1983).
[2] B. de Wit, A.K. Tollsten, and H. Nicolai, Nucl. Phys. **B392**, 3 (1993).
[3] W. Nahm, Nucl. Phys. **B135**, 149 (1978).
[4] M. Blau, J.M. Figueroa-O'Farrill, C. Hull, and G. Papadopoulos, J. High Energy Phys. 01 (2002) 047
[5] J. Gomis, A.J. Salim, and F. Passerini, J. High Energy Phys. 08 (2008) 002.
[6] K. Hosomichi, K.M. Lee, and S. Lee, Phys. Rev. D **78**, 066015 (2008).
[7] N. Itzhaki, D. Kutasov, and N. Seiberg, J. High Energy Phys. 01 (2006) 119.
[8] H. Lin and J.M. Maldacena, Phys. Rev. D **74**, 084014 (2006).
[9] D.M. Hofman and J.M. Maldacena, J. Phys. A **39**, 13095 (2006).
[10] N. Beisert, Adv. Theor. Math. Phys. **12**, 945 (2008).
[11] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982); **185**, 406(E) (1988); **281**, 409 2000).
[12] J.F. Schonfeld, Nucl. Phys. **B185**, 157 (1981).
[13] P.K. Townsend, K. Pilch, and P. van Nieuwenhuizen, Phys. Lett. B **136**, 38 (1984); **137**, 443(A) (1984).
[14] S. Deser and R. Jackiw, Phys. Lett. B **139**, 371 (1984).
[15] E.A. Bergshoeff, M. de Roo, and O. Hohm, Classical Quantum Gravity **25**, 142001 (2008).
[16] E.A. Bergshoeff, M. de Roo, O. Hohm, and D. Roest, J. High Energy Phys. 08 (2008) 091.
[17] E.A. Bergshoeff, O. Hohm, D. Roest, H. Samtleben, and E. Sezgin, J. High Energy Phys. 09 (2008) 101.
[18] J. Bagger and N. Lambert, Phys. Rev. D **77**, 065008 (2008).
[19] H. Nicolai and H. Samtleben, Phys. Lett. B **514**, 165 (2001).

[20] B. de Wit, I. Herger, and H. Samtleben, Nucl. Phys. **B671**, 175 (2003).
[21] H. Nicolai and H. Samtleben, Phys. Rev. Lett. **86**, 1686 (2001).
[22] H. Nicolai and H. Samtleben, J. High Energy Phys. 04 (2001) 022.
[23] O. Hohm and J. Louis, Classical Quantum Gravity **21**, 4607 (2004).
[24] K. Hosomichi, K. M. Lee, S. Lee, S. Lee, and J. Park, J. High Energy Phys. 09 (2008) 002.
[25] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk, and H. Verlinde, J. High Energy Phys. 09 (2008) 113.
[26] N. Beisert, J. Stat. Mech. (2007) P017.
[27] M. A. Bandres, A. E. Lipstein, and J. H. Schwarz, J. High Energy Phys. 05 (2008) 025.
[28] T. Fischbacher, H. Nicolai, and H. Samtleben, Classical Quantum Gravity **19**, 5297 (2002).
[29] S. Deser and J. H. Kay, Phys. Lett. B **120**, 97 (1983).
[30] O. Hohm, Phys. Rev. D **73**, 044003 (2006).