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E_{11} and the embedding tensor

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Abstract: We show how, using different decompositions of E_{11}, one can calculate the representations under the duality group of the so-called "de-form" potentials. Evidence is presented that these potentials are in one-to-one correspondence to the embedding tensors that classify the gaugings of all maximal gauged supergravities. We supply the computer program underlying our calculations.

Keywords: Supergravity Models, Global Symmetries, Extended Supersymmetry, Gauge Symmetry.

Contents

1. Introduction **1**

2. Physical states **2**

3. De-forms and top-forms **4**

4. Conclusions **5**

A. SimpLie: a simple program for Lie algebras **6**
- A.1 Level decomposition 6
- A.2 Root system construction 7
- A.3 Outer multiplicities 8

B. Relevant low level results **9**

1. Introduction

For some time now it has been conjectured that the infinite-dimensional algebra of E_{11} is the underlying symmetry of eleven-dimensional supergravity or M-theory [1 − 3]. One piece of evidence for this was provided by the proof [4, 5] that the top-forms[1] consistent with ten-dimensional IIA and IIB supergravity are precisely the ones predicted by E_{11} [3, 6]. The existence of these top-forms do not follow from the representation theory of the supersymmetry algebra since they do not describe physical degrees of freedom. Their existence can be proven by showing that the ten-dimensional supersymmetry algebra can be realized on these fields. A prime example of a top-form is the Ramond-Ramond 10-form that couples to the D9-brane of Type IIB string theory. It turns out that this ten-form is part of a quadruplet of ten-forms transforming according to the **4** representation of the $\mathrm{SL}(2,\mathbb{R})$ duality group [4]. The nine-branes of Type IIB string theory form a non-linear doublet that is embedded into this quadruplet [7].

Another set of fields predicted by E_{11} are the so-called "de-forms".[2] Like the top-forms they do not follow from the representation theory of the supersymmetry algebra.

[1] By a top-form in D dimensions we mean a gauge field with D anti-symmetric indices. Such gauge fields couple to space-filling branes such as the D9-brane in 10 dimensions. Note that, due to the gauge symmetries, top-forms are *inequivalent* to (the product of a Levi-Civita tensor and) scalars.

[2] By a "de-form" in D dimensions we mean a gauge field with D-1 anti-symmetric indices. Every de-form corresponds to a deformation of the corresponding supergravity theory with a masslike parameter. Hence the name (we thank Diederik Roest for suggesting this terminology to us). The masslike parameter occurs as an integration constant to the equation of motion of the de-form.

JHEP09(2007)047

The prime example of a de-form is the ten-dimensional nine-form [8] that is related to the masslike parameter m of massive IIA supergravity [9]. A priori not every deformation of a supergravity theory with a masslike parameter corresponds to a gauged supergravity. In particular, massive IIA supergravity cannot be obtained as the gauging of the $\mathbb{R}^+$ duality group. The Ramond-Ramond 1-form cannot play the role of the candidate gauge field since it is not invariant under the $\mathbb{R}^+$ scaling symmetry. It is only after a torus reduction to nine dimensions that massive IIA supergravity becomes a nine-dimensional maximal gauged supergravity with gauge group $\mathbb{R}^+$ and with the Kaluza-Klein vector as the correct gauge field.

The class of maximal gauged supergravities has been investigated in [10 − 12]. It was shown that a consistent gauging requires that the so-called embedding tensor transforms according to a specific representation of the duality group. Possible gaugings can be explored by verifying which gauge groups lead to an embedding tensor in this particular representation. The embedding tensor may be viewed as a collection of integration constants that transforms according to a representation of the duality group. Equivalently, the integration constants may be described by corresponding de-form potentials. Such de-forms also occur in different decompositions of E_{11}. Therefore, an important piece of evidence in favor of an underlying E_{11} symmetry would be to show that the de-forms predicted by E_{11} transform precisely in the representations required by imposing a consistent supersymmetric gauging following [10 − 12]. It is the purpose of this paper to show that this is indeed the case.

In the final stages of this project we received a paper by Riccioni and West [13] that contains overlap with this work. They derive the same de-forms and top-forms in various dimensions via a different technique and show that the resulting de-forms are in agreement with the literature. There is also an interesting connection with work in progress by de Wit, Nicolai, and Samtleben[3] [14].

This paper is organized as follows. In section 2 we review how the physical states of all $D \leq 11$ maximal supergravities occur as representations of regular subalgebras of E_{11}. In section 3 we derive which de-forms and top-forms in maximal supergravity are predicted by E_{11}. Finally, we comment on our results in the conclusions. We have included two appendices. Appendix A explains how we calculated the different decompositions of E_{11} using a computer program. Appendix B contains the relevant low level results of the spectrum.

2. Physical states

It is well known that maximal supergravities are characterized by hidden symmetries. This means that the $(11-n)$-dimensional supergravities exhibit a duality symmetry group G of rank $0 \leq n \leq 11$ larger than the $\mathrm{SL}(n,\mathbb{R}) \times \mathbb{R}^+$ symmetry group expected to follow from the reduction of $D = 11$ supergravity over an n-torus. In particular, the scalars transform non-linearly under the duality group G and parameterize a coset $G/K(G)$, where K is the maximal compact subgroup of G. We have given G and $K(G)$ for the different dimensions

[3] B. de Wit, private communication.

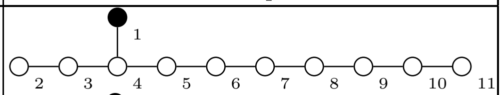
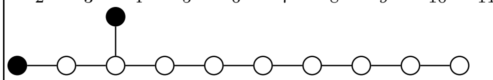
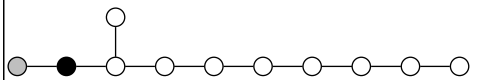
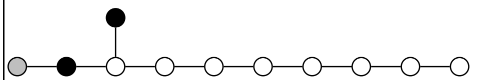
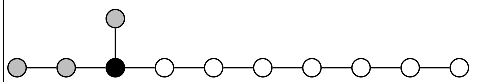
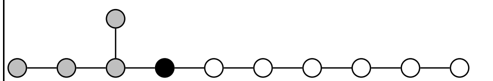
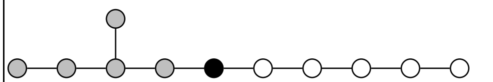
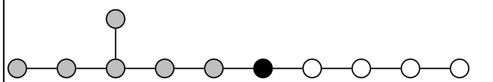
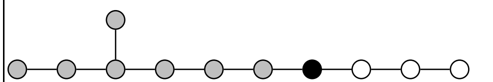
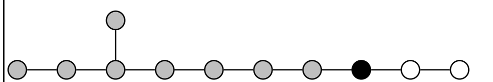
D	G	$K(G)$	$\dim(G/K)$	E_{11} decomposition
11	1	1	0	
IIA	$\mathbb{R}^+$	1	1	
IIB	$\mathrm{SL}(2,\mathbb{R})$	SO(2)	2	
9	$\mathrm{SL}(2,\mathbb{R})\times\mathbb{R}^+$	SO(2)	3	
8	$\mathrm{SL}(3,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$	$\mathrm{SO}(3)\times\mathrm{SO}(2)$	7	
7	$\mathrm{SL}(5,\mathbb{R})$	SO(5)	14	
6	$\mathrm{SO}(5,5)$	$\mathrm{SO}(5)\times\mathrm{SO}(5)$	25	
5	$E_{6(+6)}$	$USp(8)$	42	
4	$E_{7(+7)}$	SU(8)	70	
3	$E_{8(+8)}$	SO(16)	128	

Table 1: The hidden symmetries of all $3 \leq D \leq 11$ maximal supergravities. The duality groups G can be read of from the decomposition of the Dynkin diagram of E_{11}; they correspond to the grey nodes. The white nodes form the gravity line $A_{D-1} = \mathrm{SL}(D)$. In each case the scalars parameterize the coset G/K such that the number of scalars is equal to the dimension of the coset.

in table 1 alongside the corresponding decomposition of E_{11} from which G follows. Note that each G is maximal non-compact.

At low levels the decomposition of E_{11} with respect to different regular subalgebras should contain the physical states of maximal supergravity in $3 \leq D \leq 11$ dimensions. Applying the level decomposition described in appendix A.1 we indeed obtain the physical states of D-dimensional maximal supergravity, see table 2. Each supergravity field transforms as a representation of $A_{D-1} \times G$ where A_{D-1} refers to the spacetime symmetries and G is one of the duality groups given in table 1.

It is straightforward to search for these specific supergravity fields in the different decompositions of E_{11} using the level decomposition rules explained in appendix A. To perform the relevant calculations we developed the computer program SimpLie.

All fields occur in representations of G except the scalars. Dividing out by the maximal compact subgroup of E_{11} means that we restrict the spectrum to the positive levels and therefore we keep only the axions associated to the positive root generators of G and the

D	$g_{\mu\nu}$	$p=0$	$p=1$	$p=2$	$p=3$	$(p=4)^+$
11	$\mathbf{44}\times\mathbf{1}$				$\mathbf{84}\times\mathbf{1}$	
IIA	$\mathbf{35}\times\mathbf{1}$	$\mathbf{1}\times 1$	$\mathbf{8}\times\mathbf{1}$	$\mathbf{28}\times\mathbf{1}$	$\mathbf{56}\times\mathbf{1}$	
IIB	$\mathbf{35}\times\mathbf{1}$	$\mathbf{1}\times 2$		$\mathbf{28}\times\mathbf{2}$		$\mathbf{35}\times\mathbf{1}$
9	$\mathbf{27}\times\mathbf{1}$	$\mathbf{1}\times 3$	$\mathbf{7}\times(\mathbf{1}+\mathbf{2})$	$\mathbf{21}\times\mathbf{2}$	$\mathbf{35}\times\mathbf{1}$	
8	$\mathbf{20}\times(\mathbf{1{,}1})$	$\mathbf{1}\times 7$	$\mathbf{6}\times(\mathbf{3{,}2})$	$\mathbf{15}\times(\mathbf{3{,}1})$	$\mathbf{20}\times\frac{1}{2}\,(\mathbf{1{,}2})$	
7	$\mathbf{14}\times\mathbf{1}$	$\mathbf{1}\times 14$	$\mathbf{5}\times\mathbf{10}$	$\mathbf{10}\times\mathbf{5}$		
6	$\mathbf{9}\times\mathbf{1}$	$\mathbf{1}\times 25$	$\mathbf{4}\times\mathbf{16}$	$\mathbf{6}\times\frac{1}{2}\,\mathbf{10}$		
5	$\mathbf{5}\times\mathbf{1}$	$\mathbf{1}\times 42$	$\mathbf{3}\times\mathbf{27}$			
4	$\mathbf{2}\times\mathbf{1}$	$\mathbf{1}\times 70$	$\mathbf{2}\times\frac{1}{2}\,\mathbf{28}^c$			
3	$-\times\mathbf{1}$	$\mathbf{1}\times 128$				

Table 2: The occurrence of the physical states of all $3 \leq D \leq 11$ maximal supergravities in the level decomposition of E_{11}, which are also listed in appendix B. The p-columns indicate which p-form potentials are present. All entries apart from $p = 0$ are of the form "physical d.o.f. $\times$ G representation," where G is the duality group. For $p = 0$ the entries read "physical d.o.f. $\times$ number of scalars."

dilatons associated to the Cartan generators of G. Together, these scalars parameterize the coset G/K. Note that the curl of the 4-form potential in IIB is self-dual, we therefore count this potential as 35 degrees of freedom. Furthermore, in $D = 8$ the 3-form potential and its dual together form a doublet $(\mathbf{1{,}2})$. Thus the potential is counted as $1/2 \times (\mathbf{1{,}2})$ as indicated in the table. The same applies to the 2-forms in $D = 6$ and the 1-forms in $D = 4$. In each dimension the total number of states adds up to 128.

3. De-forms and top-forms

We now extend the analysis of the previous section to include the de-forms and top-forms in our calculations. The results ensue from appendix B and are summarized in table 3. The top-forms are identified using the observations made in [15].

A few remarks are in order. First of all, there are no de-forms or top-forms in $D = 11$ dimensions. Furthermore, table 3 reproduces the known de-forms and top-forms in $D = 10$ dimensions [4, 5, 8]. More importantly, the de-form representations coincide precisely with the embedding tensor calculations given in [10 − 12]. This is non-trivial given the fact that the de-form calculation is based upon E_{11} whereas the calculations of [10 − 12] involve supergravity.

It is interesting to compare the de-form calculation with some of the known results on gauged and/or massive supergravities in dimensions $D \leq 10$. As explained in the introduction, the IIA theory has a singlet massive deformation which is the massive supergravity of [9]. Note that there is a single maximal gauged supergravity in $D = 10$ [16] but this theory can only be defined at the level of the equations of motion. Apparently, E_{11} does not give rise to theories without an action.

Maximal gauged supergravities in $D = 9$ have been considered in [17]. As observed in [13] the $D = 9$ de-forms agree perfectly with the triplet and doublet deformations of [17].

JHEP09(2007)047

D	IIA	IIB	9	8	7	6	5	4	3
de-forms	**1**		**2**	**6**	**15**	**144**	**351**	**912**	**1**
			3	**12**	**40**				**3875**
top-forms	$2\times\mathbf{1}$	**2**	$2\times\mathbf{2}$	$2\times\mathbf{3}$	**5**	**10**	**27**	**133**	**248**
		4	**4**	**9**	**45**	**126**	**1728**	**8645**	**3875**
				15	**70**	**320**			**147250**

Table 3: E_{11} predictions for de-forms and top-forms in all $3 \leq D \leq 10$ maximal supergravities. These are representations of the respective duality groups G given in table 1.

Both deformations are gauged supergravities with a corresponding action. The analysis of [17] contains more $D = 9$ maximal gauged supergravities but they do not have an action.

Maximal gauged supergravities in $D = 8$ have been studied (but not exhaustively classified) in [18]. Only the ones that follow from a group manifold reduction of $D = 11$ supergravity have been given. They are contracted and non-compact versions of the SO(3) gauged supergravity of [19]. These results have been compared with the de-form calculation [20, 13]. It seems there are more maximal gauged supergravities than the ones constructed in [18]. Finally, we note that extensive studies of the possible gauge groups, using the embedding tensor, have been performed in $D = 7$ [21] and $D = 5$ [22].

4. Conclusions

In this paper we have given evidence that the de-forms that follow from E_{11} and the embedding tensors that follow from supersymmetric gauging are in one-to-one correspondence to each other. In particular, we have shown that they occur in precisely the same representations of the duality group. This is to be expected assuming that the embedding tensor plays the role of the integration constants of the de-form equations of motion. This requires a duality relation of the following schematic form:

$$^\star G_{(D)} \sim \text{embedding tensor}\,, \tag{4.1}$$

where $G_{(D)}$ is the curl of the de-form potential. The fact that the representations coincide is a good hint that such a duality relation indeed exists. It would be interesting to more precisely analyze this relation.

Finally, from the IIB case we know that not all top-forms couple to 1/2 BPS branes. This is the reason that there is no quadruplet (but instead a non-linear doublet) of IIB nine-branes given the fact that the D9-brane and its SL(2, $\mathbb{R}$) rotations couple to a quadruplet of 10-form potentials. It would be interesting to see what the situation is for the top-forms in $D < 10$ dimensions. This would teach us something about the possible space-filling branes in lower dimensions. The same applies to the de-forms and domain walls in $D < 10$ dimensions.

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A. SimpLie: a simple program for Lie algebras

In this appendix we show how the root system of a Lie algebra $\mathfrak{g}$ can be ordered in representations of any of its subalgebras. We shall mainly be interested in infinite-dimensional Kac-Moody algebras and their regular subalgebras. The analysis follows the lines of [3, 23–25] but keep in mind it is only valid for algebras whose Cartan matrix is symmetric. As we are dealing with infinite-dimensional algebras the resulting calculations are rather cumbersome to work out by hand.

To automate the process we have written "SimpLie", which is a Java computer program. The following sections explain the math behind it. SimpLie and its source are available from [27].

A.1 Level decomposition

Given a Lie algebra $\mathfrak{g}$ with associated Cartan matrix A, we can form a regular subalgebra $\mathfrak{s}$ by deleting rows and columns from A. The resulting matrix is then the Cartan matrix of the regular subalgebra of $\mathfrak{g}$. This procedure corresponds to deleting nodes from the associated Dynkin diagram of $\mathfrak{g}$.

Consequently, a root α can be decomposed into contributions from the deleted part and the regular subalgebra (repeated indices are summed over):

$$\alpha = m^i \alpha_i = l^a \alpha_a + m^s \alpha_s. \tag{A.1}$$

The values of the vector l^a are more commonly called the levels of α, and m^i the root labels. We have introduced the following indices:

$$i = \{1, \ldots, r\}, \qquad \text{full algebra} \tag{A.2}$$

$$a = \{1, \ldots, n\}, \qquad \text{deleted nodes} \tag{A.3}$$

$$s = \{n+1, \ldots, r\}, \qquad \text{regular subalgebra} \tag{A.4}$$

where r is the rank of $\mathfrak{g}$ and n is the number of deleted nodes.

Because the root system of $\mathfrak{g}$ is graded with respect to l^a and all the roots of $\mathfrak{s}$ have levels equal to zero, all roots of $\mathfrak{g}$ with a particular l^a form highest weight modules of $\mathfrak{s}$

under the adjoint action:

$$[\mathfrak{s}, \mathfrak{g}_{l^a}] \subset \mathfrak{g}_{l^a}. \tag{A.5}$$

These modules are characterized by the Dynkin labels of their highest weight state, which are given by

$$p_s = A_{si} m^i = A_{st} m^t + A_{sa} l^a. \tag{A.6}$$

For given levels l^a it is easy to scan for possible valid highest weight modules. They have to satisfy three conditions:

(i) Their Dynkin labels all have to be integer and non-negative.

(ii) The m^s have to be integers.

(iii) The length squared of the root must not exceed the maximum value.

In the finite-dimensional case, the maximum value is given by the length squared of the highest root. However, for Kac-Moody algebras there is no such thing as a highest root. Fortunately, for the simply-laced cases the length squared of the roots is bounded by $\alpha^2 \leq 2$ [26]. Thus we have

$$\alpha^2 = A_{ij} m^i m^j = S^{st} p_s p_t + \left(A_{ab} - S^{st} A_{sa} A_{tb}\right) l^a l^b \leq 2, \tag{A.7}$$

where S is the inverse of the Cartan matrix of $\mathfrak{s}$. When S has no negative entries, the above formula is a monotonically increasing function of p_s at fixed levels. This is the case when $\mathfrak{s}$ is finite.

To check condition (ii) we can invert (A.6) to obtain

$$m^s = S^{st}(p_t - A_{ta} l^a). \tag{A.8}$$

When equations (A.8) and (A.7) respectively satisfy conditions (ii) and (iii) we have found a possible highest weight representation of $\mathfrak{s}$. Yet it remains to be seen whether or not this representation actually occurs within $\mathfrak{g}$. To that end, the root system of $\mathfrak{g}$ must be constructed up to the levels we are interested in. Furthermore, one has to check if a particular representation occurs as a weight in another representation. These are the subjects of the next sections.

Note that, instead of scanning for the highest weight, we can also look for the lowest weight of the representation. It has Dynkin labels equal to minus the Dynkin labels of the highest weight state of the conjugate representation. An easy way to do this is to replace p_s with $-p_s$ in the above analysis. The advantage of this particular approach is that lowest weight states have a lower height (given by $h = \sum_i m^i$), so that it suffices to construct the root system of $\mathfrak{g}$ to lower heights.

A.2 Root system construction

Assuming the root system of $\mathfrak{g}$ has been constructed up to height h, there is a simple procedure to determine all the roots of height $h+1$. Specifically one considers, for all roots β of height h, the string $s_{\alpha_i;\beta}$ of simple roots α_i given by

$$s_{\alpha_i;\beta} = \{\beta + k\alpha_i \mid k = -p, -p+1, \ldots, q-1, q\}, \tag{A.9}$$

where p and q satisfy

$$\frac{2(\beta|\alpha_i)}{(\alpha_i|\alpha_i)} = p - q. \tag{A.10}$$

In order for $\beta + \alpha_i$ to be a root, q has to be positive. This is the case when the inner product between β and α_i is negative. If it is positive, we have to perform a search through the previously generated roots in the string to determine the value of p. Following this procedure, it is possible to construct the root system to an arbitrary height starting from just the simple roots.

Knowing if a particular root α occurs in the root system of $\mathfrak{g}$ is not enough: one would also like to know its multiplicity $\mathrm{mult}(\alpha)$. The most straightforward method to calculate these multiplicities, albeit not the most elegant one, is perhaps the Peterson recursion formula. It reads

$$\Big((\alpha|\alpha) - 2h(\alpha)\Big)c_\alpha = \sum_{\substack{\alpha=\beta+\gamma\\ \beta,\gamma>0}} (\beta|\gamma)c_\beta c_\gamma, \tag{A.11}$$

where the co-multiplicity c_α is given by

$$c_\alpha = \sum_{k\geq 1} \frac{1}{k}\mathrm{mult}\left(\frac{\alpha}{k}\right). \tag{A.12}$$

Factors for which α/k is not a root do not contribute to the sum. The β and γ in (A.11) do not have to be roots but can also be non-negative linear combinations of the simple roots. If they are not roots, they have to be integer multiples of roots. Otherwise their co-multiplicity would be zero and they would not contribute to the sum (A.11).

A.3 Outer multiplicities

The number of times a representation of $\mathfrak{s}$ actually occurs within the root system of $\mathfrak{g}$ is called its outer multiplicity μ. In order to determine μ we need to know the multiplicity of the representation as a weight in other representations at the same level. Furthermore the multiplicity of the root α in $\mathfrak{g}$ associated to its highest weight state is needed. The outer multiplicity μ then follows from [23]

$$\mathrm{mult}(\alpha) = \sum_i \mu(R_i)\mathrm{mult}_{R_i}(\alpha), \tag{A.13}$$

where i runs over the number of representations at a fixed level, and R_i is the i-th representation. The only unknowns remaining are the weight multiplicities mult_{R_i} of the representations of $\mathfrak{s}$. These can be calculated with the Freudenthal recursion formula, which reads

$$\Big((\Lambda|\Lambda){-}2h(\Lambda){-}(\lambda|\lambda){-}2h(\lambda)\Big)\mathrm{mult}_{R(\Lambda)}(\lambda) = 2\sum_{\alpha>0}\sum_{k\geq 1}(\lambda{+}k\alpha|\alpha)\mathrm{mult}_{R(\Lambda)}(\lambda{+}k\alpha). \tag{A.14}$$

The first sum is over all positive roots of $\mathfrak{s}$. The second terminates when $\lambda + k\alpha$ leaves the weight system of $R(\Lambda)$, i.e. when the height of the corresponding root exceeds that of Λ. In our case, the highest weight Λ is given by its Dynkin labels p_s.

l	p_{grav}	m	α^2	d_{reg}	mult(α)	μ	fields
0	1 0 0 0 0 0 0 0 0 0 1	0 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	2	120	1	1	$\bar{g}_{\mu\nu}$
0	0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0	0	1	11	1	$\hat{g}_{\mu\nu}$
1	0 0 1 0 0 0 0 0 0 0 0	1 0 0 0 0 0 0 0 0 0 0	2	165	1	1	$p = 3$
2	0 0 0 0 0 1 0 0 0 0 0	2 1 2 3 2 1 0 0 0 0 0	2	462	1	1	$\star\,(p = 3)$
3	1 0 0 0 0 0 0 0 1 0 0	3 1 3 5 4 3 2 1 0 0 0	2	1760	1	1	$\star\, g_{\mu\nu}$

Table 4: A_{10} representations in E_{11} ($D = 11$)

l		p_{grav}	m	α^2	d_{reg}	mult(α)	μ	fields
0	0	1 0 0 0 0 0 0 0 0 1	0 0 -1 -1 -1 -1 -1 -1 -1 -1 -1	2	99	1	1	$\bar{g}_{\mu\nu}$
0	0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0	0	1	11	2	$p = 0, \hat{g}_{\mu\nu}$
1	0	0 1 0 0 0 0 0 0 0 0	1 0 0 0 0 0 0 0 0 0 0	2	45	1	1	$p = 2$
0	1	1 0 0 0 0 0 0 0 0 0	0 1 0 0 0 0 0 0 0 0 0	2	10	1	1	$p = 1$
1	1	0 0 1 0 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0 0	2	120	1	1	$p = 3$
2	1	0 0 0 0 1 0 0 0 0 0	2 1 2 3 2 1 0 0 0 0 0	2	252	1	1	$\star\,(p = 3)$
3	1	0 0 0 0 0 0 0 1 0 0	3 1 3 5 4 3 2 1 0 0 0	2	120	1	1	$\star\,(p = 1)$
4	1	0 0 0 0 0 0 0 0 0 1	4 1 4 7 6 5 4 3 2 1 0	2	10	1	1	de
2	2	0 0 0 0 0 1 0 0 0 0	2 2 3 4 3 2 1 0 0 0 0	2	210	1	1	$\star\,(p = 2)$
3	2	1 0 0 0 0 0 0 1 0 0	3 2 3 5 4 3 2 1 0 0 0	2	1155	1	1	$\star\, g_{\mu\nu}$
3	2	0 0 0 0 0 0 0 0 1 0	3 2 4 6 5 4 3 2 1 0 0	0	45	8	1	$\star\,(p = 0)$
4	2	0 1 0 0 0 0 0 0 1 0	4 2 4 6 5 4 3 2 1 0 0	2	1925	1	1	
4	2	1 0 0 0 0 0 0 0 0 1	4 2 4 7 6 5 4 3 2 1 0	0	99	8	1	
4	2	0 0 0 0 0 0 0 0 0 0	4 2 5 8 7 6 5 4 3 2 1	-2	1	46	2	top
3	3	1 0 0 0 0 0 0 0 1 0	3 3 4 6 5 4 3 2 1 0 0	2	440	1	1	

Table 5: A_9 representations in E_{11} (IIA)

The procedure in SimpLie is the same at each level: first all possible highest weight representations R_i of $\mathfrak{s}$ found by the scanning procedure described in section A.1 are gathered. Next we calculate mult(α) by the brute force method of section A.2. Then all the relevant weights and their multiplicities of every R_i are calculated using, amongst others, the Freudenthal recursion formula. Finally, the outer multiplicity of R_i is determined using an iterative implementation of (A.13).

B. Relevant low level results

Here we list the output of SimpLie at low levels, using the various decompositions of E_{11} listed in table 1. The regular subalgebra splits into a part belonging to the gravity line A_n (the white nodes) and a part belonging to the internal duality group G (the grey nodes).

In the following tables we respectively list the levels, the Dynkin labels of A_n and G, the root labels, the root length, the dimension of the representations of A_n and G, the multiplicity of the root, the outer multiplicity, and the interpretation as a physical field. These physical fields are also listed in table 2. The de-forms and top-forms are indicated by 'de' and 'top', respectively. When the internal group does not exist, we do not list the corresponding columns. In all cases the Dynkin labels of the lowest weights of the representations are given. The order of the Dynkin labels and the root labels is determined

JHEP09(2007)047

l	p_{grav}	p_{int}	m	α^2	d_{reg}	d_{int}	mult(α)	μ	fields
0	1 0 0 0 0 0 0 0 0 1	0	-1 0 0 -1 -1 -1 -1 -1 -1 -1 -1	2	99	1	1	1	$\bar{g}_{\mu\nu}$
0	0 0 0 0 0 0 0 0 0 0	2	0 -1 0 0 0 0 0 0 0 0 0	2	1	3	1	1	$p=0$
0	0 0 0 0 0 0 0 0 0 0	0	0 0 0 0 0 0 0 0 0 0 0	0	1	1	11	1	$\hat{g}_{\mu\nu}$
1	0 1 0 0 0 0 0 0 0 0	1	0 0 1 0 0 0 0 0 0 0 0	2	45	2	1	1	$p=2$
2	0 0 0 1 0 0 0 0 0 0	0	1 1 2 2 1 0 0 0 0 0 0	2	210	1	1	1	$p=4$
3	0 0 0 0 0 1 0 0 0 0	1	2 1 3 4 3 2 1 0 0 0 0	2	210	2	1	1	$^\star\,(p=2)$
4	1 0 0 0 0 0 0 1 0 0	0	2 2 4 5 4 3 2 1 0 0 0	2	1155	1	1	1	$^\star\,g_{\mu\nu}$
4	0 0 0 0 0 0 0 0 1 0	2	3 1 4 6 5 4 3 2 1 0 0	2	45	3	1	1	$^\star\,(p=0)$
5	0 1 0 0 0 0 0 0 1 0	1	3 2 5 6 5 4 3 2 1 0 0	2	1925	2	1	1	
5	1 0 0 0 0 0 0 0 0 1	1	3 2 5 7 6 5 4 3 2 1 0	0	99	2	8	1	
5	0 0 0 0 0 0 0 0 0 0	3	4 1 5 8 7 6 5 4 3 2 1	2	1	4	1	1	top
5	0 0 0 0 0 0 0 0 0 0	1	4 2 5 8 7 6 5 4 3 2 1	-2	1	2	46	1	top

Table 6: $A_9 \times A_1$ representations in E_{11} (IIB)

l	p_{grav}	p_{int}	m	α^2	d_{reg}	d_{int}	mult(α)	μ	fields
0 0	1 0 0 0 0 0 0 0 1	0	0 0 0 -1 -1 -1 -1 -1 -1 -1 -1	2	80	1	1	1	$\bar{g}_{\mu\nu}$
0 0	0 0 0 0 0 0 0 0 0	2	0 -1 0 0 0 0 0 0 0 0 0	2	1	3	1	1	$p=0$
0 0	0 0 0 0 0 0 0 0 0	0	0 0 0 0 0 0 0 0 0 0 0	0	1	1	11	2	$p=0, \hat{g}_{\mu\nu}$
1 0	1 0 0 0 0 0 0 0 0	0	1 0 0 0 0 0 0 0 0 0 0	2	9	1	1	1	$p=1$
0 1	1 0 0 0 0 0 0 0 0	1	0 0 1 0 0 0 0 0 0 0 0	2	9	2	1	1	$p=1$
1 1	0 1 0 0 0 0 0 0 0	1	1 0 1 1 0 0 0 0 0 0 0	2	36	2	1	1	$p=2$
1 2	0 0 1 0 0 0 0 0 0	0	1 1 2 2 1 0 0 0 0 0 0	2	84	1	1	1	$p=3$
2 2	0 0 0 1 0 0 0 0 0	0	2 1 2 3 2 1 0 0 0 0 0	2	126	1	1	1	$^\star\,(p=3)$
2 3	0 0 0 0 1 0 0 0 0	1	2 1 3 4 3 2 1 0 0 0 0	2	126	2	1	1	$^\star\,(p=2)$
3 3	0 0 0 0 0 1 0 0 0	1	3 1 3 5 4 3 2 1 0 0 0	2	84	2	1	1	$^\star\,(p=1)$
2 4	0 0 0 0 0 1 0 0 0	0	2 2 4 5 4 3 2 1 0 0 0	2	84	1	1	1	$^\star\,(p=1)$
3 4	1 0 0 0 0 0 1 0 0	0	3 2 4 5 4 3 2 1 0 0 0	2	720	1	1	1	$^\star\,g_{\mu\nu}$
3 4	0 0 0 0 0 0 0 1 0	2	3 1 4 6 5 4 3 2 1 0 0	2	36	3	1	1	$^\star\,(p=0)$
3 4	0 0 0 0 0 0 0 1 0	0	3 2 4 6 5 4 3 2 1 0 0	0	36	1	8	1	$^\star\,(p=0)$
4 4	1 0 0 0 0 0 0 1 0	0	4 2 4 6 5 4 3 2 1 0 0	2	315	1	1	1	
4 4	0 0 0 0 0 0 0 0 1	2	4 1 4 7 6 5 4 3 2 1 0	2	9	3	1	1	de
3 5	1 0 0 0 0 0 0 1 0	1	3 2 5 6 5 4 3 2 1 0 0	2	315	2	1	1	
3 5	0 0 0 0 0 0 0 0 1	1	3 2 5 7 6 5 4 3 2 1 0	0	9	2	8	1	de
4 5	0 1 0 0 0 0 0 1 0	1	4 2 5 7 5 4 3 2 1 0 0	2	1215	2	1	1	
4 5	1 0 0 0 0 0 0 0 1	1	4 2 5 7 6 5 4 3 2 1 0	0	80	2	8	2	
4 5	0 0 0 0 0 0 0 0 0	3	4 1 5 8 7 6 5 4 3 2 1	2	1	4	1	1	top
4 5	0 0 0 0 0 0 0 0 0	1	4 2 5 8 7 6 5 4 3 2 1	-2	1	2	46	2	top
3 6	1 0 0 0 0 0 0 0 1	0	3 3 6 7 6 5 4 3 2 1 0	2	80	1	1	1	

Table 7: $A_8 \times A_1$ representations in E_{11} ($D=9$)

by the numbering of the nodes in table 1. All tables are truncated at the point when the number of indices of the gravity subalgebra representations exceed the dimension.

The interpretation of the representations at level zero as the graviton is, unlike the p-forms at higher levels, not quite straightforward. The graviton emerges when one combines the adjoint representation of A_n with a scalar coming from one of the deleted nodes, see [24, 3]. We have indicated these parts of the graviton by $\bar{g}_{\mu\nu}$ and $\hat{g}_{\mu\nu}$, respectively.

l	$p_{\rm grav}$	$p_{\rm int}$	m	α^2	$d_{\rm reg}$	$d_{\rm int}$	mult(α)	μ	fields
0	1 0 0 0 0 0 0 1	0 0 0	0 0 0 0 -1 -1 -1 -1 -1 -1 -1	2	63	1	1	1	$\bar{g}_{\mu\nu}$
0	0 0 0 0 0 0 0 0	0 1 1	0 -1 -1 0 0 0 0 0 0 0 0	2	1	8	1	1	$p = 0$
0	0 0 0 0 0 0 0 0	2 0 0	-1 0 0 0 0 0 0 0 0 0 0	2	1	3	1	1	$p = 0$
0	0 0 0 0 0 0 0 0	0 0 0	0 0 0 0 0 0 0 0 0 0 0	0	1	1	11	1	$\hat{g}_{\mu\nu}$
1	1 0 0 0 0 0 0 0	1 0 1	0 0 0 1 0 0 0 0 0 0 0	2	8	6	1	1	$p = 1$
2	0 1 0 0 0 0 0 0	0 1 0	1 0 1 2 1 0 0 0 0 0 0	2	28	3	1	1	$p = 2$
3	0 0 1 0 0 0 0 0	1 0 0	1 1 2 3 2 1 0 0 0 0 0	2	56	2	1	1	$p = 3$, $\star\ (p = 3)$
4	0 0 0 1 0 0 0 0	0 0 1	2 1 2 4 3 2 1 0 0 0 0	2	70	3	1	1	$\star\ (p = 2)$
5	0 0 0 0 1 0 0 0	1 1 0	2 1 3 5 4 3 2 1 0 0 0	2	56	6	1	1	$\star\ (p = 1)$
6	1 0 0 0 1 0 0 0	0 0 0	3 2 4 6 4 3 2 1 0 0 0	2	420	1	1	1	$\star\ g_{\mu\nu}$
6	0 0 0 0 0 1 0 0	0 1 1	3 1 3 6 5 4 3 2 1 0 0	2	28	8	1	1	$\star\ (p = 0)$
6	0 0 0 0 0 1 0 0	2 0 0	2 2 4 6 5 4 3 2 1 0 0	2	28	3	1	1	$\star\ (p = 0)$
7	1 0 0 0 0 1 0 0	1 0 1	3 2 4 7 5 4 3 2 1 0 0	2	216	6	1	1	
7	0 0 0 0 0 0 0 1	1 2 0	3 1 4 7 6 5 4 3 2 1 0	2	8	12	1	1	de
7	0 0 0 0 0 0 0 1	1 0 1	3 2 4 7 6 5 4 3 2 1 0	0	8	6	8	1	de
8	0 1 0 0 0 1 0 0	0 1 0	4 2 5 8 6 4 3 2 1 0 0	2	720	3	1	1	
8	1 0 0 0 0 0 0 1	0 0 2	4 2 4 8 6 5 4 3 2 1 0	2	63	6	1	1	
8	1 0 0 0 0 0 0 1	2 1 0	3 2 5 8 6 5 4 3 2 1 0	2	63	9	1	1	
8	1 0 0 0 0 0 0 1	0 1 0	4 2 5 8 6 5 4 3 2 1 0	0	63	3	8	1	
8	0 0 0 0 0 0 0 0	0 2 1	4 1 4 8 7 6 5 4 3 2 1	2	1	15	1	1	top
8	0 0 0 0 0 0 0 0	2 1 0	3 2 5 8 7 6 5 4 3 2 1	0	1	9	8	1	top
8	0 0 0 0 0 0 0 0	0 1 0	4 2 5 8 7 6 5 4 3 2 1	-2	1	3	46	2	top

Table 8: $A_7 \times (A_2 \times A_1)$ representations in E_{11} $(D = 8)$

l	$p_{\rm grav}$	$p_{\rm int}$	m	α^2	$d_{\rm reg}$	$d_{\rm int}$	mult(α)	μ	fields
0	1 0 0 0 0 1	0 0 0 0	0 0 0 0 0 -1 -1 -1 -1 -1 -1	2	48	1	1	1	$\bar{g}_{\mu\nu}$
0	0 0 0 0 0 0	1 1 0 0	-1 -1 -1 -1 0 0 0 0 0 0 0	2	1	24	1	1	$p = 0$
0	0 0 0 0 0 0	0 0 0 0	0 0 0 0 0 0 0 0 0 0 0	0	1	1	11	1	$\hat{g}_{\mu\nu}$
1	1 0 0 0 0 0	0 0 0 1	0 0 0 0 1 0 0 0 0 0 0	2	7	10	1	1	$p = 1$
2	0 1 0 0 0 0	0 1 0 0	1 0 1 2 2 1 0 0 0 0 0	2	21	5	1	1	$p = 2$
3	0 0 1 0 0 0	1 0 0 0	1 1 2 3 3 2 1 0 0 0 0	2	35	5	1	1	$\star\ (p = 2)$
4	0 0 0 1 0 0	0 0 1 0	2 1 2 4 4 3 2 1 0 0 0	2	35	10	1	1	$\star\ (p = 1)$
5	1 0 0 1 0 0	0 0 0 0	3 2 4 6 5 3 2 1 0 0 0	2	224	1	1	1	$\star\ g_{\mu\nu}$
5	0 0 0 0 1 0	1 1 0 0	2 1 3 5 5 4 3 2 1 0 0	2	21	24	1	1	$\star\ (p = 0)$
6	1 0 0 0 1 0	0 0 0 1	3 2 4 6 6 4 3 2 1 0 0	2	140	10	1	1	
6	0 0 0 0 0 1	0 1 1 0	3 1 3 6 6 5 4 3 2 1 0	2	7	40	1	1	de
6	0 0 0 0 0 1	2 0 0 0	2 2 4 6 6 5 4 3 2 1 0	2	7	15	1	1	de
7	0 1 0 0 1 0	0 1 0 0	4 2 5 8 7 5 3 2 1 0 0	2	392	5	1	1	
7	1 0 0 0 0 1	1 0 1 0	3 2 4 7 7 5 4 3 2 1 0	2	48	45	1	1	
7	1 0 0 0 0 1	0 1 0 0	4 2 5 8 7 5 4 3 2 1 0	0	48	5	8	1	
7	0 0 0 0 0 0	1 2 0 0	3 1 4 7 7 6 5 4 3 2 1	2	1	70	1	1	top
7	0 0 0 0 0 0	1 0 1 0	3 2 4 7 7 6 5 4 3 2 1	0	1	45	8	1	top
7	0 0 0 0 0 0	0 1 0 0	4 2 5 8 7 6 5 4 3 2 1	-2	1	5	46	1	top

Table 9: $A_6 \times A_4$ representations in E_{11} $(D = 7)$

l	$p_{\rm grav}$	$p_{\rm int}$	m	α^2	$d_{\rm reg}$	$d_{\rm int}$	mult(α)	μ	fields
0	0 0 0 0 0	0 0 1 0 0	-1 -1 -2 -2 -1 0 0 0 0 0 0	2	1	45	1	1	$p=0$
0	1 0 0 0 1	0 0 0 0 0	0 0 0 0 0 0 -1 -1 -1 -1 -1	2	35	1	1	1	$\bar{g}_{\mu\nu}$
0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0	0	1	1	11	1	$\hat{g}_{\mu\nu}$
1	1 0 0 0 0	0 0 0 0 1	0 0 0 0 0 1 0 0 0 0 0	2	6	16	1	1	$p=1$
2	0 1 0 0 0	0 1 0 0 0	1 0 1 2 2 2 1 0 0 0 0	2	15	10	1	1	$p=2$, $^\star\,(p=2)$
3	0 0 1 0 0	1 0 0 0 0	1 1 2 3 3 3 2 1 0 0 0	2	20	16	1	1	$^\star\,(p=1)$
4	0 0 0 1 0	0 0 1 0 0	2 1 2 4 4 4 3 2 1 0 0	2	15	45	1	1	$^\star\,(p=0)$
4	1 0 1 0 0	0 0 0 0 0	3 2 4 6 5 4 2 1 0 0 0	2	105	1	1	1	$^\star\,g_{\mu\nu}$
5	1 0 0 1 0	0 0 0 0 1	3 2 4 6 5 5 3 2 1 0 0	2	84	16	1	1	
5	0 0 0 0 1	1 1 0 0 0	2 1 3 5 5 5 4 3 2 1 0	2	6	144	1	1	de
6	1 0 0 0 1	0 0 0 1 0	3 2 4 6 6 6 4 3 2 1 0	2	35	120	1	1	
6	0 1 0 1 0	0 1 0 0 0	4 2 5 8 7 6 4 2 1 0 0	2	189	10	1	1	
6	0 0 0 0 0	0 1 1 0 0	3 1 3 6 6 6 5 4 3 2 1	2	1	320	1	1	top
6	0 0 0 0 0	2 0 0 0 0	2 2 4 6 6 6 5 4 3 2 1	2	1	126	1	1	top
6	1 0 0 0 1	0 1 0 0 0	4 2 5 8 7 6 4 3 2 1 0	0	35	10	8	1	
6	0 0 0 0 0	0 1 0 0 0	4 2 5 8 7 6 5 4 3 2 1	-2	1	10	46	1	top

Table 10: $A_5 \times E_5$ representations in E_{11} $(D=6)$

l	$p_{\rm grav}$	$p_{\rm int}$	m	α^2	$d_{\rm reg}$	$d_{\rm int}$	mult(α)	μ	fields
0	0 0 0 0	1 0 0 0 0 0	-2 -1 -2 -3 -2 -1 0 0 0 0 0	2	1	78	1	1	$p=0$
0	1 0 0 1	0 0 0 0 0 0	0 0 0 0 0 0 0 -1 -1 -1 -1	2	24	1	1	1	$\bar{g}_{\mu\nu}$
0	0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0	0	1	1	11	1	$\hat{g}_{\mu\nu}$
1	1 0 0 0	0 0 0 0 0 1	0 0 0 0 0 0 1 0 0 0 0	2	5	27	1	1	$p=1$
2	0 1 0 0	0 1 0 0 0 0	1 0 1 2 2 2 2 1 0 0 0	2	10	27	1	1	$^\star\,(p=1)$
3	0 0 1 0	1 0 0 0 0 0	1 1 2 3 3 3 3 2 1 0 0	2	10	78	1	1	$^\star\,(p=0)$
3	1 1 0 0	0 0 0 0 0 0	3 2 4 6 5 4 3 1 0 0 0	2	40	1	1	1	$^\star\,g_{\mu\nu}$
4	0 0 0 1	0 0 1 0 0 0	2 1 2 4 4 4 4 3 2 1 0	2	5	351	1	1	de
4	1 0 1 0	0 0 0 0 0 1	3 2 4 6 5 4 4 2 1 0 0	2	45	27	1	1	
5	1 0 0 1	0 0 0 0 1 0	3 2 4 6 5 5 5 3 2 1 0	2	24	351	1	1	
5	0 0 0 0	1 1 0 0 0 0	2 1 3 5 5 5 5 4 3 2 1	2	1	1728	1	1	top
5	0 1 1 0	0 1 0 0 0 0	4 2 5 8 7 6 5 3 1 0 0	2	75	27	1	1	
5	1 0 0 1	0 1 0 0 0 0	4 2 5 8 7 6 5 3 2 1 0	0	24	27	8	1	
5	0 0 0 0	0 1 0 0 0 0	4 2 5 8 7 6 5 4 3 2 1	-2	1	27	46	1	top

Table 11: $A_4 \times E_6$ representations in E_{11} $(D=5)$

l	p_{grav}	p_{int}	m	α^2	d_{reg}	d_{int}	mult(α)	μ	fields
0	0 0 0	0 1 0 0 0 0 0	-2 -2 -3 -4 -3 -2 -1 0 0 0 0	2	1	133	1	1	$p=0$
0	1 0 1	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 -1 -1 -1	2	15	1	1	1	$\bar{g}_{\mu\nu}$
0	0 0 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0	0	1	1	11	1	$\hat{g}_{\mu\nu}$
1	1 0 0	0 0 0 0 0 0 1	0 0 0 0 0 0 0 1 0 0 0	2	4	56	1	1	$p=1$, $^\star\,(p=1)$
2	0 1 0	0 1 0 0 0 0 0	1 0 1 2 2 2 2 2 1 0 0	2	6	133	1	1	$^\star\,(p=0)$
2	2 0 0	0 0 0 0 0 0 0	3 2 4 6 5 4 3 2 0 0 0	2	10	1	1	1	$^\star\,g_{\mu\nu}$
3	0 0 1	1 0 0 0 0 0 0	1 1 2 3 3 3 3 3 2 1 0	2	4	912	1	1	de
3	1 1 0	0 0 0 0 0 0 1	3 2 4 6 5 4 3 3 1 0 0	2	20	56	1	1	
4	0 0 0	0 0 1 0 0 0 0	2 1 2 4 4 4 4 4 3 2 1	2	1	8645	1	1	top
4	1 0 1	0 0 0 0 0 1 0	3 2 4 6 5 4 4 4 2 1 0	2	15	1539	1	1	
4	0 2 0	0 1 0 0 0 0 0	4 2 5 8 7 6 5 4 2 0 0	2	20	133	1	1	
4	1 0 1	0 1 0 0 0 0 0	4 2 5 8 7 6 5 4 2 1 0	0	15	133	8	1	
4	0 0 0	0 1 0 0 0 0 0	4 2 5 8 7 6 5 4 3 2 1	-2	1	133	46	1	top
4	2 1 0	0 0 0 0 0 0 0	6 4 8 12 10 8 6 4 1 0 0	2	45	1	1	1	
4	1 0 1	0 0 0 0 0 0 0	6 4 8 12 10 8 6 4 2 1 0	-2	15	1	44	1	

Table 12: $A_3 \times E_7$ representations in E_{11} $(D=4)$

l	p_{grav}	p_{int}	m	α^2	d_{reg}	d_{int}	mult(α)	μ	fields
0	0 0	0 0 0 0 0 0 0 1	-3 -2 -4 -6 -5 -4 -3 -2 0 0 0	2	1	248	1	1	$p=0$
0	1 1	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 -1 -1	2	8	1	1	1	$\bar{g}_{\mu\nu}$
0	0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0	0	1	1	11	1	$\hat{g}_{\mu\nu}$
1	1 0	0 0 0 0 0 0 0 1	0 0 0 0 0 0 0 0 1 0 0	2	3	248	1	1	$^\star\,(p=0)$
2	0 1	0 1 0 0 0 0 0 0	1 0 1 2 2 2 2 2 2 1 0	2	3	3875	1	1	de
2	2 0	0 0 0 0 0 0 0 1	3 2 4 6 5 4 3 2 2 0 0	2	6	248	1	1	
2	0 1	0 0 0 0 0 0 0 0	6 4 8 12 10 8 6 4 2 1 0	-2	3	1	44	1	de
3	0 0	1 0 0 0 0 0 0 0	1 1 2 3 3 3 3 3 3 2 1	2	1	147250	1	1	top
3	1 1	0 0 0 0 0 0 1 0	3 2 4 6 5 4 3 3 3 1 0	2	8	30380	1	1	
3	1 1	0 1 0 0 0 0 0 0	4 2 5 8 7 6 5 4 3 1 0	0	8	3875	8	1	
3	0 0	0 1 0 0 0 0 0 0	4 2 5 8 7 6 5 4 3 2 1	-2	1	3875	46	1	top
3	3 0	0 0 0 0 0 0 0 1	6 4 8 12 10 8 6 4 3 0 0	2	10	248	1	1	
3	1 1	0 0 0 0 0 0 0 1	6 4 8 12 10 8 6 4 3 1 0	-2	8	248	44	1	
3	0 0	0 0 0 0 0 0 0 1	6 4 8 12 10 8 6 4 3 2 1	-4	1	248	206	1	top
3	1 1	0 0 0 0 0 0 0 0	9 6 12 18 15 12 9 6 3 1 0	-4	8	1	192	1	

Table 13: $A_2 \times E_8$ representations in E_{11} $(D=3)$

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