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AdS_3 OM theory and the self-dual string or membranes ending on the five-brane

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Abstract

We describe properties of the M-theory five-brane containing Q coincident self-dual strings on its worldvolume. This is the five-brane description of Q membranes ending on the five-brane. In particular, we consider a Maldacena-like low energy limit in the six-dimensional worldvolume which yields a near 'horizon' description of the self-dual string using light open membranes, i.e., OM theory, in an $AdS_3 \times S^3$ geometry.

1. Introduction

The world volume description of the M-theory five-brane has been the subject of much study, see [1–3]. In some sense the five-brane is the M-theory analogue of a D-brane in that open membranes may end on five-branes [4,5]. Just as for D-branes there exists an effective description that describes the low energy worldvolume dynamics of the brane. For the five-brane this has the field content of a $(2, 0)$ tensor multiplet consisting of a self-dual two form, five scalars and symplectic Majorana spinors. (Throughout this Letter we will restrict ourselves to a single M5-brane; the correct worldvolume description of coincident M5-branes is not known.) The covariant equations of motion were found using various methods in [3]; the remarkable feature of these equations

is the non-linear self-duality constraint. This gives a non-linear, interacting theory on the five-brane worldvolume. The non-linearities are governed by the eleven-dimensional Planck length, ℓ_p, which is associated to the tension of the M-theory branes.

There are string-like solutions to these equations [6] that are charged with respect to the self-dual two-form. These solutions, known as self-dual strings, have the interpretation of the ending of a membrane on the five-brane. As such it is the M-theory analogue of the BIon [7].

Recently, there have been attempts to investigate the five-brane in a particular limit where the Planck length is taken to zero and at the same time the background field strength on the five-brane becomes near-critical. This leads to a new effective scale, ℓ_6, on the five-brane in the limit, while decoupling the brane from the background eleven-dimensional supergravity. This has been called open membrane (OM) theory, [8–10]. One way of describing this limit,

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given in detail in [9], is through a conjectured effective metric on the five brane worldvolume called the *open membrane metric*, $G_{\mu\nu}^{\mathrm{OM}}$. This metric it was argued, is the metric as seen by open membranes ending on the five-brane and is the natural analogue of the open string metrics on the D-branes. The crucial property of the OM limit is that the open membrane metric scales in such away that the effective open membrane tension given by $\ell_{\mathrm{p}}^{-2} G_{\mu\nu}^{\mathrm{OM}}$ is fixed. This limit is the M-theory analog of the noncommutative open string limit, [11]. Thus the OM theory has been conjectured to be the UV-completion of the $(2,0)$ theory with constant background three form. As such it would subsume all the decoupled noncommutative theories on D-branes and NS5-branes [8,9,12].

It is not certain how one should view the open membrane metric given that it is not derived from a fundamental theory of membranes. Being pessimistic one might simply say that it is just a convenient way to encode the OM limit. It seems very natural however that given there is an open string metric such an object will lift to some sort of open membrane metric in M-theory. (We will discuss the quantitative evidence for this metric when we introduce its precise form in the following section.)

A conceptual drawback of the above OM limit is that the near-critical field must be switched on by hand by introducing an an external source of delocalized (smeared) membranes, which breaks the five-brane $ISO(5,1)$ symmetry. This raises the natural question whether one could also create an environment with critical field by going close to a single localized open membrane, that is the above-mentioned self-dual string soliton.

In what follows we wish to describe the five-brane worldvolume region near the core of the self-dual string in a low energy, 'near horizon' limit a la Maldacena [15]. In the geometric picture where the five-brane is embedded in a flat eleven-dimensional target space this region is a tube with the topology $\mathsf{R}^2 \times \mathsf{R}^+ \times S^3$ that extends away from the five-brane such that the radius of the transverse three-sphere goes to zero far from the five-brane. We shall refer to this tube region as the near horizon region. There is three-form field strength trapped inside the tube, which turns out to be near-critical (see below (9)). By considering the effective open membrane tension in this limit, it turns out that the dynamics in the tube region is that of

OM theory expanded around $AdS_3 \times S^3$. The germ of this idea was suggested already in [16].

We of course expect similar results to hold for many other worldvolume solitons, such as a q-brane soliton in a p-brane will give rise to open q-brane theory [8,12] in a geometry which is conformal to $AdS_{q+1} \times S^{p-q}$ (with conformal factor playing the role of a running brane coupling), where the anti-de Sitter space corresponds to the directions of the worldvolume electric field strength and the sphere to the directions of the worldvolume dual magnetic field strength. Related ideas concerning the fuzzy geometry of BIons have been explored extensively in [13]. We wish to point out that here we are moving beyond the low energy theory given by the $(2,0)$ tensor multiplet on the brane which becomes invalid deep in the throat as pointed out in [13] for the case of the BIon.

Of course, there is a complementary picture of this near horizon region given by the theory of Q coincident membranes with a boundary [14].

By examining the absorption behavior of the self-dual string we find that the near horizon region of the self-dual string decouples from the asymptotic region in the low energy limit.

In summary, the AdS_3 OM theory provides a near horizon description of the self-dual string that decouples from the rest of the five-brane in the prescribed limit.

The outline of the Letter is as follows. In Section 2 we review the five-brane equations of motion and their self-dual string solution. We then describe in detail the low energy, near horizon limit. In Section 3 we compute the self-dual string absorption cross section and show its vanishing in the limit. We conclude in Section 4.

2. Self-dual strings in the M5-brane and a Maldacena style limit

The bosonic equations of motion of the five-brane in flat eleven-dimensional spacetime are given by the scalar equation ($\mu = 0, \ldots, 5; i = 6, \ldots, 11$):

$$G^{\mu\nu} \nabla_\mu^{(g)} \partial_\nu \phi^i = 0 \tag{1}$$

and the following nonlinear self-duality condition:

$$\frac{\sqrt{-\det g}}{6} \epsilon_{\mu\nu\rho\sigma\lambda\tau} \mathcal{H}^{\sigma\lambda\tau} = \frac{1+K}{2} \left(G^{-1}\right)_\mu{}^\lambda \mathcal{H}_{\nu\rho\lambda}. \tag{2}$$

Here ϕ^i are the five transverse scalars in a static gauge, $\mathcal{H} = db$ the three-form worldvolume field strength, $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \phi^i \partial_\nu \phi^i$ the induced metric, $\nabla_\mu^{(g)}$ the corresponding covariant derivative and, finally, the scalar K and the tensor $G_{\mu\nu}$ are given by

$$K = \sqrt{1 + \frac{\ell_p^6}{24} \mathcal{H}^2},$$ (3)

$$G_{\mu\nu} = \frac{1+K}{2K}\left(g_{\mu\nu} + \frac{\ell_p^6}{4}\mathcal{H}_{\mu\nu}^2\right).$$ (4)

The constant ℓ_p is the eleven-dimensional Planck length, which is the only parameter of the theory. Thus, the five-brane field equations are reliable as an effective description of M-theory dynamics only for energies smaller than ℓ_p^{-1}.

To describe the self-dual string solution we decompose the five-brane worldvolume coordinates x^μ into the x^0, x^1, which are the coordinates of the string worldsheet, and the remaining four coordinates, denoted by $y^m, m = 1, \ldots, 4$, which are coordinates of the space transverse to the string. Going to radial coordinates we introduce $r^2 = y^m y^m$. For the self-dual string solution, the fields are functions of r only.

As shown in [6] the equations of motion (1) and (2) can be solved by the following:

$$\phi^6 = \ell_p f, \qquad \mathcal{H}_{01p} = \frac{1}{4}\ell_p^{-2}\partial_p f,$$

$$\mathcal{H}_{mnp} = \frac{1}{4}\ell_p^{-2}\epsilon_{mnpq}\partial_q f,$$ (5)

where f is a harmonic function on the transverse space:

$$f = 1 + \frac{Q\ell_p^2}{r^2}.$$ (6)

The parameter Q is essentially the charge of the string as evaluated by integrating the $\mathcal{H}$-flux over the S^3 surrounding the string. Since the field strength is self-dual one can show that the electric and magnetic charges are equal. The fermions and remaining overall transverse scalars $\phi^{7,\ldots,11}$ are zero. This solution preserves 8 of the 16 five-brane worldvolume supercharges [6].

This solution has the interpretation of a semi infinite membrane ending on the five-brane. The energy of this configuration is therefore infinite corresponding to the fact that the tension of the string will be given by the tension of the membrane times the membrane extension out of the five-brane. In order for the string to have finite tension one regulates the solution by demanding that the membrane is not infinite but ends on another five-brane some finite distance, L, away (measured in the flat target space metric). The solution is still good for sufficiently large L and r. The string tension becomes:

$$T_{\text{string}} = \ell_p^{-3} L.$$ (7)

We will now examine the near horizon of this solution using the conjectured *open membrane* metric. The tensor $G_{\mu\nu}$ is conformally equivalent to the open membrane metric, which is given by [9,17,19]:

$$G_{\mu\nu}^{\text{OM}} = \left(\frac{1 - \sqrt{1 - K^{-2}}}{K^2}\right)^{1/3}\left(g_{\mu\nu} + \frac{\ell_p^6}{4}\mathcal{H}_{\mu\nu}^2\right).$$ (8)

This was first determined in the near-critical limit in [9] by analyzing the scaling properties of the OM limit, and also by invoking IIB/M-theory duality. Away from the critical limit, the conformal factor can be fixed by demanding that $G_{\mu\nu}^{\text{OM}}$ evaluated on a probe five-brane in the background of the SUGRA dual to OM theory [10] is independent of the three form deformation of the solution. This is discussed at length in [17] where the idea is based on generalising the known properties of open string metrics [18]. Perhaps most naturally, one may also consider its dimensional reduction and map it to the open string metric for the D4-brane as was also done in [17, 19]. These three different approaches are all consistent which encourages us to believe in the form of this open membrane metric.

We note, however, that the near horizon geometry only requires the near-critical behavior of the conformal factor; in fact the near horizon geometry of the self-dual string was identified up to the conformal scale already in [16], though the precise definition the limit below was not appreciated at the time. A conformally related metric was introduced in [20] that had advantages in simplifying the equations of motion and was used in studying a different strong coupling limit.

The scaling limit we wish to consider is a low energy limit, where we keep fixed the energy on the self-dual string fixed as we send ℓ_p to zero. In two dimensions scalar fields have dimension zero so a vacuum expectation value of such a scalar, describing the separation of a string from the stack, is $rT_s^{1/2}$,

where T_s is the tension of the self dual string as given by (7). Thus, as one requires $r/\ell_p^{3/2}$ to be fixed (L is obviously fixed).

We wish to compare this limit to the near horizon limit for a stack of membranes in M-theory. The fixed energy on a stack of membranes associated with separating off a single membrane from the stack by a distance r is given also by $r/\ell_p^{3/2}$. This may be computed either by considering the Higgsing of D2-branes and then going to strong coupling and invoking $SO(8)$ invariance, or by directly Higgsing the scalar fields on the $SO(8)$ invariant membrane.[1] Moreover, from (5) we see that this limit implies that the tube region close to the self-dual string, which is the region we are interested in examining, is cut out and kept fixed in the fixed eleven-dimensional background. This is obvious given the interpretation of the self-dual string as arising from the membrane. Thus we are led to the following low energy and near horizon limit:

$$\ell_p \to 0, \qquad \frac{r^2}{\ell_p^3} = u \text{ fixed},$$

$$Q = \text{fixed}. \tag{9}$$

After inserting the solution, (5) into the open membrane metric (8) and then taking the above limit we obtain:

$$ds^2(G^{OM}) = \ell_p^2\big(Q^{-2/3}v^2\big(-dt^2 + dx^2\big) \\ + Q^{2/3}v^{-2}\,dv^2 + Q^{2/3}\,d\Omega_3^2\big), \tag{10}$$

where we have performed the trivial coordinate transformation, $v = Q^{-1/3}u$ so as to put the line element in the canonical form for $AdS_3 \times S^3$. Here the radii of these two spaces are equal and given by

$$R_{AdS} = Q^{1/3}\ell_p = R_{S^3}. \tag{11}$$

We wish to analyze the energy $E(r)$ of an object located at distance r corresponding to an open membrane excitation scaling as $1/\ell_p$. It is, however, the energy ω at infinity that we wish to associate with the energy on the self-dual string and keep fixed. Taking into account

the red-shift these two energies are related by:

$$\omega = E(r)\sqrt{-G_{tt}^{OM}(r)} \\ = \ell_p^{-2}E(r)Q^{-2/3}r^2 \\ = \ell_p E(r)Q^{2/3}u = \text{fixed}. \tag{12}$$

Thus such energies as measured at infinity are fixed as the limit is taken even though the energies locally at r diverge.

Examining the OM metric (10), naively, one would think the whole metric is becoming of zero size since we have ℓ_p^2 multiplying the whole expression. However, the important property of this limit is that the open membrane theory on the 5-brane is kept finite as can be seen by looking at the effective open membrane tension given by

$$\ell_p^{-2}G_{\mu\nu}^{OM} = \text{fixed}, \tag{13}$$

which is kept fixed in the limit but is a function of u. This is just as for the usual OM theory only there the background field, and thus the OM metric, is a constant function [9]. Here we are performing a similar limit, sending the tension to infinity but by going to the near horizon region of the self-dual string as described, the field strength increases appropriately so that the scale of open membrane excitations remain finite (though a function of u). Of course we only expect to be able to believe this description when the fields are slowly varying which would indicate having a large R_{AdS} (in plank units). This implies the description is only valid when:

$$Q \gg 1. \tag{14}$$

After the dust has settled, we see that in the limit described above we are left with the OM theory on $(AdS_3 \times S^3)_Q$. The subscript Q denotes that the radii in plank units are proportional to $Q^{1/3}$.

3. Absorption by the self-dual string

In this section we wish to calculate the cross section for the absorption of massless scalars by the self-dual string in the world volume of the M-theory five-brane. We will adopt an entirely world volume approach similar to that of [21–23]. We begin by writing the equation satisfied by the s-wave with

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[1] A physical (fixed) scalar in three dimensions has scaling dimension half. Thus the vacuum expectation value of such a scalar corresponding to the separation of the membrane is given by $r/\ell_p^{3/2}$.

energy ω, $\phi(r,t) = \phi(r)e^{i\omega t}$, of the linear fluctuations of the four overall transverse scalars about the self-dual string, (it is known that there are problems when one considers higher angular momentum modes [23], one must take care with the validity of the linearized approximation, this is discussed in [13]):

$$\left(\rho^{-3}\frac{d}{d\rho}\rho^3\frac{d}{d\rho} + 1 + \frac{R^6\omega^6}{\rho^6}\right)\phi(\rho) = 0, \qquad (15)$$

where $\rho = r\omega$, $R = Q^{1/3}\ell_p$. Note, as pointed out by [11] world volume solitons have a much sharper potential than the Coulomb type potential typical of brane solutions in supergravity; thus this scattering is different to that of the string in six-dimensional supergravity. Nevertheless, for small ωR one may solve this problem by matching an approximate solution in the inner region to an approximate solution in the outer region; this follows closely the supergravity calculation [24].

To approximate the inner region we change variables, $z = (R\omega)^3\rho^{-2}$. Then (15) becomes:

$$\left(4\frac{d^2}{dz^2} + \frac{(R\omega)^3}{z^3} + 1\right)\phi(z) = 0. \qquad (16)$$

This equation may be solved when $z \gg R\omega$. This implies $\rho \ll R\omega$. In this region the solution becomes simply:

$$\phi(\rho) = A\cos\left(\frac{(R\omega)^3}{4\rho^2}\right) + B\sin\left(\frac{(R\omega)^3}{4\rho^2}\right). \qquad (17)$$

The fact the fluctuation equation becomes a linear wave equation with the coordinate change given by the excited scalar of the background solution was noted for the D-brane analogue in [21]. A and B are undetermined constants.

Now, consider the substitution, $\phi(\rho) = \rho^{-3/4}\psi(\rho)$. Eq. (15) becomes:

$$\left(\frac{d^2}{d\rho^2} - \frac{3}{4\rho^2} + 1 + \frac{(R\omega)^6}{\rho^6}\right)\psi = 0. \qquad (18)$$

One can neglect the potential term when $(R\omega)^6/\rho^4 \ll 1$. This implies $\rho \gg (R\omega)^{3/2}$. In this region we may solve the equation by Bessel functions, $J(\rho)$, and Neumann functions, $N(\rho)$, as follows:

$$\phi(\rho) = \rho^{-1}\left(A'J_1(\rho) + B'N_1(\rho)\right). \qquad (19)$$

Here A' and B' are undetermined constants.

We wish to calculate the ratio of fluxes between the exterior and interior and so we need to find an overlap region that allows us to fix A' and B' in terms of A and B. There is an overlap region, $R\omega \gg \rho \gg (R\omega)^{3/2}$, when $R\omega \ll 1$. In this region we can match the solutions provided:

$$A = A'/2, \qquad B = \frac{2}{\pi}\frac{1}{(R\omega)^3}B'. \qquad (20)$$

For small $R\omega$ the ratio of fluxes of interior to asymptotic regions is (neglecting numerical constants)

$$\mathcal{P} \sim (R\omega)^6. \qquad (21)$$

The formula for the absorption cross section in d spatial dimensions is:

$$\sigma = \frac{(2\pi)^{d-1}\mathcal{P}}{\omega^{d-1}\Omega_{d-1}}, \qquad (22)$$

where Ω_D is the volume of the unit D-sphere. For our case, $d = 4$ giving the following (up to a numerical constant):

$$\sigma \sim R^6\omega^3. \qquad (23)$$

If we return to the limit described previously, where now we can identify $R = \ell_p Q^{1/3}$ and ω is kept fixed, we see that σ vanishes and the asymptotic region indeed decouples.

4. Discussion

In summary, we have defined a Maldacena-like low energy limit on the M-theory five-brane such that the near horizon region of a self-dual string becomes near-critical and gives rise to a AdS_3 phase of the OM theory. This describes the emerging stack of Q membranes from the five-brane perspective.

Now, in the above discussion we have neglected the ambient gravity. To decouple this brane system from gravity, one takes ℓ_p to zero with v^2/ℓ_p^3 fixed where v is now the eleven-dimensional distance from membranes. A strictly rigorous analysis is not possible since there is no supergravity solution, however, for large Q, one may ignore the back reaction of the single M5-brane and we have a decoupled $AdS_4 \times S^7$ spacetime with radius of curvature (using the naive estimate) going like $Q^{1/6}$ with the $AdS_3 \times S^3$ is embedded inside whose radius of curvature goes like $Q^{1/3}$. A similar set up is described in [14].

By compactifying the whole system, one introduces a plethora of various decoupled open brane theories [8, 9,12] with coupling moduli. We expect all these to give rise to similar behavior as the one found here, though presumably the conformal invariance is broken in most of these cases. This of course requires the identification of the appropriate open brane metrics and coupling; as done in [17].

One might imagine that we are describing simply the usual stack of Q coincident membranes in the limit. This is not the case however because crucially the membranes have a boundary which is of course the five-brane. They must end on the five-brane in a smooth way that also conserves flux. It is this that essentially alters the geometry of the membranes.

Finally, we wish to entertain the following possibility. One may imagine that the low energy effective action of the self-dual string may be determined by writing down the action corresponding to goldstone modes of the self-dual string, or using the more powerful superembedding formalism of [25] which was applied to the self-dual string in [5]. As such it will be an $N = (4, 4)$ SCFT in $1 + 1$ dimensions. Actually knowing what the precise SCFT is problematic because of how one determines the Q dependence. The approaches described above actually only gives the action when $Q = 1$ but allows one to determine the supersymmetry. This is similar to the problem of knowing what the conformal theories are for any of the M-theory branes, at the moment the only thing known about them is the supersymmetry and properties determined by the assumed duality with eleven-dimensional supergravity. It would be interesting to see if the work of [26] could be useful in describing this *non-Abelian* self-dual string theory. One significant objection to this proposal is that the solution is singular and has infinite tension unless regulated by the arbitrary cut off L as indicated here. Nevertheless, if such a low energy effective action exists and could be trusted; it would indicate a low energy duality between the two descriptions of the decoupled region.

As an aside we wish to point out that the six-dimensional five-brane theory should not be thought of as the $Q \to \infty$ limit of the AdS_3 version of the OM theory that we have found here, just as one should not think of the ten-dimensional type IIB string theory as the large N limit of the five-dimensional anti-de Sitter string theory. The flat limit instead consists of uplifting to the maximally symmetric theory in the higher dimension, where the brane charge can be set equal to zero. In fact, in analogy with the type IIB string theory, we expect the AdS_3 OM theory to be given in terms of a curvature expansion for large R with an essential singularity at $1/R = 0$. In type IIB this pole origins from the singularity in the string tension in the curvature expansion of string theory about the ten-dimensional flat vacuum [27]. When expanded about the five-dimensional anti-de Sitter vacuum the string tension is traded for the cosmological constant by letting it absorb powers of the string coupling (dilaton). As a result the string tension can be sent to zero at fixed Planck length and radius (which corresponds to sending the string coupling to zero). In the resulting tensionless phase the original singularity thus has translated into the singularity at $1/R = 0$.

One also might be worried that in fact the OM theory does not really decouple from the background due to its thermal properties; as was suggested for NCOS theories in [28]. It was not clear there what such a thermal analysis revealed about OM theory as such it remains an open question. The relation if any to the usual $AdS_3 \times S^3$, CFT corresspondence [29], also remains undetermined.

Acknowledgements

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