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Structure of the $11/2^-$ analog state in ^{91}Nb populated by the ^{90}Zr(α, t) reaction

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Abstract

Decay via proton emission of isobaric analog states (IAS's) in ^{91}Nb was studied using the ^{90}Zr(α, t) reaction at $E_\alpha = 180$ MeV. This study provides information about the damping mechanism of these states. Decay to the ground state and low-lying phonon states in ^{90}Zr was observed. The experimental data are compared with theoretical predictions wherein the IAS 'single-particle' proton escape widths are calculated in a continuum RPA approach. The branching ratios for decay to the phonon states are explained using a simple model. © 2001 Published by Elsevier Science B.V.

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1. Introduction

The study of the decay of single-particle (s.p.) or single-hole states gives insight into their damping mechanisms. To expound on this, consider the width of a s.p. state embedded in the continuum. This is usu-

ally described by a width $\Gamma = \Gamma^\uparrow + \Gamma^\downarrow$, where $\Gamma^\uparrow$ is the escape width and $\Gamma^\downarrow$ is the spreading width. The escape width is related to the direct decay by particle emission of a s.p. state. This width is small for unbound states near the particle-decay threshold, but increases rapidly as a function of excitation energy. The spreading/fragmentation width is related to the coupling of the state to more complex many-particle-many-hole states, where the complexity increases up

to statistical equilibrium. It is expected that the first step in this damping mechanism is mediated via coupling to a special class of particle–hole states, i.e., the surface vibrations (phonon states) [1,2]. This mechanism plays an important role at low excitation energies. From these phonon states, acting as a doorway, coupling will take place to more complex states until statistical equilibrium is reached. Valuable information about the damping process and, therefore, on the structure of the excited s.p. states can be obtained by the experimental determination of the direct escape width and the width for semi-direct decay to these phonon states. Since isobaric analogs of s.p. states have similar structure as the s.p. states in the parent nucleus, the study of their decay will reflect the damping mechanisms of the parent s.p. state, i.e., their coupling to the continuum and its spreading/fragmentation.

In the seventies and eighties proton-stripping reactions on medium-weight nuclei were used to study IAS's; see among others Finkel et al. [3,4]. Assume for simplicity that the excess neutrons in a core nucleus (Z, N) with $N > Z$ fill exactly one complete neutron subshell with spin j. An extra neutron (indicated with ν) can be added in the next higher shell with spin j_ν. Then the IAS of this ground state (g.s.) has a simple structure. If one applies the isospin-lowering operator to the g.s. wave function, one gets two components: one part comes from the transition of the valence neutron into the corresponding high-lying proton orbit and the second part comes from the transition of a neutron from the filled subshell into the corresponding open proton orbit; thus forming a two-particle-one-hole state. Note, the second part will only be possible for a core where $N > Z$. It should be emphasized here that in the proton-stripping reaction on the core nucleus (Z, N) populating the IAS in the nucleus $(Z + 1, N)$, only the first part will contribute to the cross section.

The proton decay of the analog of a single-neutron state of a 'core-plus-valence-neutron' in a parent nucleus can be described in terms of the IAS s.p. proton decay populating the g.s. of the core nucleus. Considerable efforts have been undertaken in the last three decades to describe in a quantitative way the s.p. proton widths of the IAS's (see, e.g., Refs. [5, 6] and references therein). However, only in the last decade the corresponding self-consistent continuum random-phase approximation (CRPA) or continuum

Tamm–Dancoff approximation (CTDA) approaches have been developed and applied to the closed-shell [7–9] and to the 'closed-(neutron)shell + valence-neutron' parent nuclei [7]. In Ref. [7], the CRPA approach taken in the form developed by Muraviev and Urin [10] was used together with a phenomenological mean field and the Landau–Migdal particle–hole interaction. The partial self-consistency condition, which is a result of the approximate isospin-symmetry conservation in nuclei (see, e.g., Ref. [11]), was used by Rumyantsev and Urin [7] who explained a number of experimental data satisfactorily.

In the present work, the one-proton stripping (α, t) reaction was studied to investigate the damping mechanism of IAS's in a medium-weight nucleus. Because the (α, t) reaction is Q-value mismatched, predominantly high-spin states were populated. First results of this work have been reported elsewhere [12]. In addition to the population and decay of IAS's, data were also obtained for high-lying s.p. states in the same experiment. Those data and a full description of the experimental procedure and data analysis will be published elsewhere [13].

2. Experimental procedure and results

The measurement of the ^{90}Zr(α, t) proton-stripping reaction was performed at the Research Center for Nuclear Physics (RCNP). The beam energy used for the reaction was 180 MeV and the triton ejectiles were momentum analyzed using the Grand Raiden spectrometer [14]. The ^{90}Zr target used in the experiment was a metallic foil with a thickness of 1 mg/cm^2 (98% enrichment). For calibration purposes, a natural carbon target was used. Since the cross sections for exciting high-spin states in this reaction peak at a scattering angle of $0°$, the spectrometer was set at $0.3°$ and a solid angle of 1.6 msr was used. The direct beam from the cyclotron was stopped inside the spectrometer using a well-shielded beam dump. The protons emitted from the excited states were detected in a 37-element solid-state detector array, covering the backward hemisphere in the angular range between $100°$ and $160°$. Each of these detectors was mounted at a distance of 10 cm from the target, subtending a solid angle of 15.4 msr. The excitation-energy region covered by the setting of the spectrometer ranged from 5

to 20 MeV in ^{91}Nb. This region was chosen, because it covers the range of known IAS's [3,4,15] and because the threshold for proton emission starts at an excitation energy of 5.16 MeV. The energy calibration of the tritons was made using the ^{12}C(α, t) reaction at the same bombarding energy. In this reaction, coincidences with protons emitted from excited ^{13}N nuclei were measured. In Fig. 1, the energy of protons emitted from the residual nuclei is plotted versus the excitation energy in the relevant nuclei (upper panel for the ^{90}Zr target, lower panel for the natural carbon target). In the lower panel of this figure, one clearly recognizes the decay of states in ^{13}N to the g.s. in ^{12}C. Strong states in ^{13}N are the $J^\pi = 5/2^+$ state at $E_x = 3.55$ MeV and the $J^\pi = 5/2^-, 7/2^-$ state at $E_x = 10.36$ MeV. Both in the carbon target and in the ^{90}Zr target, contaminations of ^{16}O were present. In the region of interest, these contaminations lead to the population of the $J^\pi = 3/2^+$ state at $E_x = 5.82$ MeV in ^{17}F. With the use of these calibration spectra obtained from the ^{12}C(α, t) reaction, it was possible to identify in the singles and coincidence spectra, the IAS's in ^{91}Nb. In the upper panel of Fig. 1, the loci corresponding to different final states in ^{90}Zr are easily recognized and indicated. Furthermore, the same figure shows the calculated loci for the $\alpha \rightarrow t + p$ reaction induced on the contaminants ^{12}C and ^{16}O.

Because of kinematics, the decay of excited states in ^{91}Nb to the ^{90}Zr g.s., and in the excitation-energy region beyond 10 MeV in ^{91}Nb also to the $2_1^+/5_1^-$ doublet in ^{90}Zr, can be separated clearly from the reactions induced on the contaminants in the target. A singles and a coincidence spectrum for the ^{90}Zr(α, t) reaction are shown in Fig. 2. The triton energy resolution was determined from the measured, but here not shown, excitation spectrum for the population of low-lying states in ^{91}Nb to be 150 keV (fwhm) which is much larger than the intrinsic width of the observed IAS's.

Comparing the singles and coincidence spectra, it is clear that the large physical background observed in the singles spectrum is reduced substantially because of the detection of a coincident proton emitted in the backward direction, i.e., the region where the breakup process yields very small cross sections. Furthermore, it is clear from the coincidence spectrum, that above the neutron-emission threshold at an excitation energy
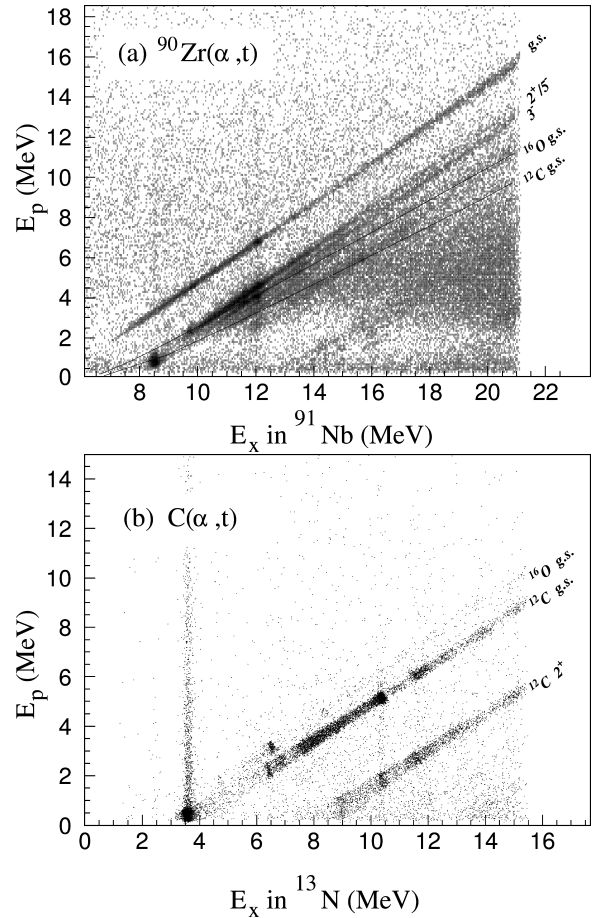

Fig. 1. (a) Measured proton energy versus the excitation energy in ^{91}Nb for the ^{90}Zr(α, t) reaction at $E_\alpha = 180$ MeV and $\theta_t = 0.3°$. The solid lines indicate the calculated loci for the $\alpha \rightarrow t + p$ reaction induced on the oxygen and carbon contaminants leading to the decay to their respective ground states. (b) Measured proton energy versus the excitation energy in ^{13}N for the C(α, t) reaction at $E_\alpha = 180$ MeV and $\theta_t = 0.3°$.

of 12.05 MeV, the yields for coincidences with protons drops significantly.

As the momenta for a triton ejectile and a proton emitted from a ^{91}Nb nucleus were measured, it is possible to determine the final-state energy $E_{fs} = E_x(^{91}\text{Nb}) - E_p - S_p - E_r$, where S_p and E_r are the proton separation energy and the energy of the recoiling ^{90}Zr nucleus, respectively. The energy resolution achieved for the final-state spectra was 300 keV (fwhm) which is not sufficient to resolve the 2_1^+ and 5_1^- states in ^{90}Zr, which are located at $E_x = 2.19$ and
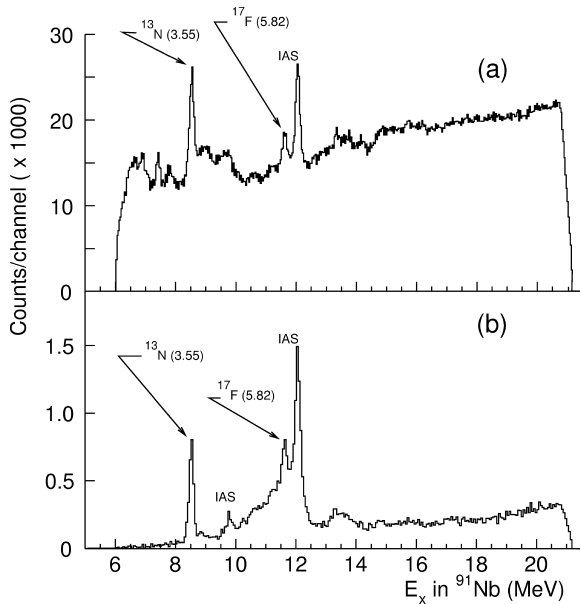
Fig. 2. (a) The singles triton spectrum for the ^{90}Zr(α, t) reaction at $E_\alpha = 180$ MeV and $\theta_t = 0.3°$. (b) The same spectrum as given in (a), but requiring in addition a coincidence with a proton detected at backward angles. Indicated are the IAS's and peaks due to contamination of the target with oxygen and carbon leading to states in ^{17}F at $E_x = 5.82$ MeV and ^{13}N at $E_x = 3.55$ MeV, respectively.

2.32 MeV, respectively. As an example, Fig. 3 shows the final-state energy spectrum for the decay of states of ^{91}Nb in the excitation-energy region between 11.85 and 12.20 MeV. Because the 37 proton detectors were mounted in the backward hemisphere, in the angular range between 100° and 160°, it was possible to measure the angular correlation between the direction of the protons emitted and the recoil axis of the excited ^{91}Nb nuclei; see Fig. 4 for the decay of the peak at 12 MeV shown in Fig. 2(b).

The analog state of the $J^\pi = 5/2^+$ ^{91}Zr g.s. is located at $E_x = 9.86$ MeV in ^{91}Nb [3]. Because of the relatively small cross section, this IAS cannot be identified in the present singles spectrum. But in the coincidence data the nuclear continuum caused by other processes is reduced substantially and, therefore, this IAS is clearly seen in the coincidence spectrum; see Fig. 1. Our data are consistent with earlier measurements done by Finkel et al. [4] who find that the decay of this state proceeds to the g.s. of ^{90}Zr only. Since we cannot determine the singles cross section for the population of this state, the absolute branching ratio and,
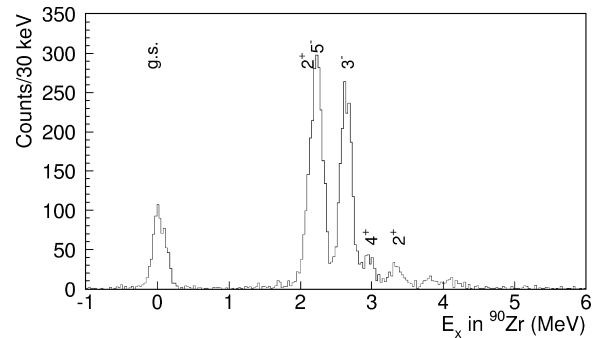

Fig. 3. Final-state spectrum after decay by proton emission from the excitation-energy region in ^{91}Nb between 11.85 and 12.2 MeV.
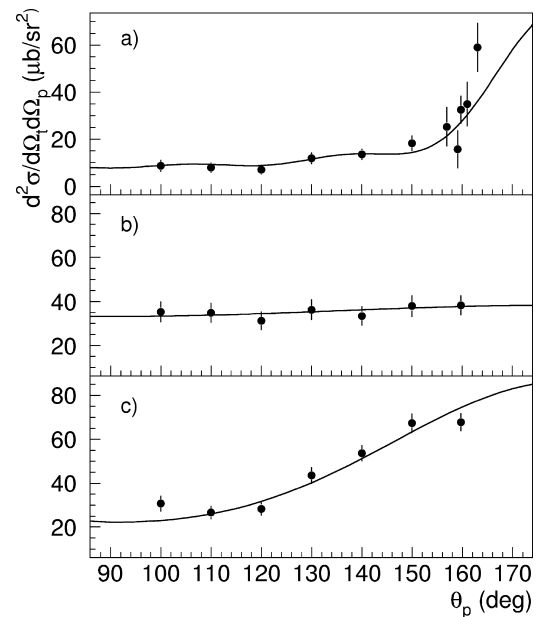
Fig. 4. Experimental (dots) and calculated (lines) angular correlations for the decay from the 12 MeV IAS to different final states in ^{90}Zr; (a) to the g.s., (b) to the $2^+/5^-$ states and (c) the 3^- state. The dominant l-values used for the calculated angular correlations are listed in Table 1. Other allowed l-values for decay to the 3^- and 5^- states are included in the fit presented, but these have minor contributions and no effect on the extracted branching ratio.

therefore, the escape width cannot be deduced from the present data.

For the higher lying IAS at $E_x = 12$ MeV, both the singles cross section and the cross sections for decay to the lowest states in ^{90}Zr have been extracted. Using the calculated angular correlations shown in

Table 1
The weights $|a_c|^2$ of different components of the $11/2^-$ parent state(s) in ^{91}Zr deduced from the experimental data and calculated within the coupled-channel approach; see Eqs. (3) and (4)

c decay channel	$\epsilon_c^{\exp}$ [MeV]	l_c [$\hbar$]	b_c	Γ_c^{calc}(s.p.)[a] [keV]	$|a_c|^2$			β_L
					Experiment	Theory		
						$\Psi^{(1)}$	$\Psi^{(2)}$	
g.s.	6.90	5	0.12 ± 0.01	0.42	0.45 ± 0.03	0.26	0.01	–
5^-	4.56	0	0.27 ± 0.05	2.90	0.14 ± 0.03	0.12	0.88	0.08
3^-	4.16	2	0.33 ± 0.03	1.25	0.41 ± 0.10	0.62	0.11	0.20

[a] Obtained from Eqs. (1) and (3).

Fig. 4, the absolute branching ratios b_c for the decay by proton emission are obtained by integrating them over 4π of the proton solid angle and dividing this by the singles cross sections. The branching ratios are listed in Table 1 together with the values for the angular momentum l_c transferred in the decay channel. Note that the 'isotropic' angular correlation for the decay to the $2^+/5^-$ doublet clearly indicates that the predominant decay goes to the 5^- state of this doublet. Since the intrinsic width of this IAS cannot be determined experimentally, the partial decay widths Γ_c cannot be deduced from the present data. In view of the present analysis, this peak at an excitation energy of 12 MeV is assumed to be the $J^\pi = 11/2^-$ state, located at 12.07 MeV in ^{91}Nb [15] and, therefore, to be the analog of the $E_x = 2.17$ MeV state in ^{91}Zr. Although other high-spin states at this excitation energy in ^{91}Nb might exist, e.g., a $J^\pi = 13/2^-$ state at $E_x = 12.084$ MeV [15], the 'isotropic' decay of the peak at 12 MeV shown in Fig. 2 to the $J^\pi = 5^-$ state in ^{90}Zr and the assumed $5\hbar$ momentum transfer for the decay to the ^{90}Zr g.s., substantiate this assumption which is further supported by Finkel et al. [3,4] and by the conclusions of Blok et al. [16] for the study of the analog nucleus ^{91}Zr.

3. Comparison with calculations

In the present work, we use the CRPA approach developed by Moukhai et al. [17] which has been applied recently [10] for the description of the IAS s.p. proton decay in medium-heavy nuclei [18]. This approach is equivalent to earlier work [7] but it is simpler in practice. In agreement with Ref. [18], we find that the proton pairing can be neglected in the description of the decay of the IAS's in ^{91}Nb by proton emission, provided that the proton-decay energy ϵ and the spectroscopic factor S_ν for the valence-neutron states are taken from experiment.[1] This assumption means that the calculated width $\Gamma_c^{\text{calc}} = S_\nu^{\exp} \cdot \Gamma_c^{\text{calc}}$(s.p.) with

$$\Gamma_c^{\text{calc}}(\text{s.p.}) = \Gamma_c^{\text{RPA}} \cdot \frac{1}{S_\nu} \cdot \frac{P_{l_\pi}(\epsilon^{\exp})}{P_{l_\pi}(\epsilon)} \tag{1}$$

can be compared with the corresponding experimental one. Here, $c = \{l_\pi, j_\pi\}$ represents the quantum numbers of the emitted proton with $l_\pi = l_\nu$ and $j_\pi = j_\nu$, and Γ_c^{RPA} is the IAS s.p. width calculated within the CRPA, taken at the calculated escape-energy ϵ of the proton. In Eq. (1), $P_l(\epsilon)$ is the proton potential-barrier penetrability (see, e.g., Ref. [19]) and $S_\nu^{\exp}$ is the experimental spectroscopic factor, while within the CRPA $S_\nu = 1$. In case of the ^{91}Zr parent nucleus we take within the BCS model (see, e.g., Ref. [20]) the proton pairing into account only to describe the proton separation energy.

The results of the calculations given below have been obtained with the use of the isoscalar part of the nuclear mean field taken from Ref. [10] and it contains four phenomenological parameters (two strength parameters and two geometrical ones). The solution of the BCS equations for the proton subsystem is found

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[1] Note that we do not use the convention C^2S for the s.p. spectroscopic factor, but rather S_ν. For the IAS's of ^{91}Nb the isospin Clebsch–Gordon coefficient is $C^2 = 1/11$.

Table 2
Calculated and experimental separation energies

Nucleus	S_n [MeV]		S_p [MeV]	
	Experiment	Calculated	Experiment	Calculated
^{91}Zr	7.19	7.11	8.70	8.55
^{91}Nb	12.04	12.04	5.15	5.15

using only the proton s.p. bound states. The intensity of the pairing forces for protons, $G_p = 27/A$ MeV, is chosen to reproduce the experimental proton-pairing energy $P^{\exp}(^{90}\text{Zr}) = 1.15$ MeV [21]. This intensity is close to the one mentioned by Soloviev [20]. The experimental neutron and proton separation energies are reproduced in our calculations provided that $f' = 0.96$ is chosen as the dimensionless amplitude of the isovector part of the Landau–Migdal particle–hole interaction. This value of f' is close to the one used in earlier work [7,17]. Thus, all model parameters are fixed. In Table 2, the calculated and experimental separation energies are listed for ^{91}Zr and ^{91}Nb.

Consider now the calculation of the parameters for the IAS($5/2^+$), which is the analog of the g.s. of ^{91}Zr. The Fermi strength function calculated for the ^{91}Zr parent nucleus exhibits a narrow state with a peak energy $E_{\text{IAS}} = 11.52$ MeV with respect to the g.s. of ^{91}Zr. The state corresponding to the mentioned IAS($5/2^+$) in ^{91}Nb exhausts 96.5% of the Fermi sum rule $N - Z = 11$. The calculated excitation energy of the state $E_x = E_{\text{IAS}} - S_n(^{91}\text{Zr}) + S_p(^{91}\text{Nb})$ is equal to 9.47 MeV. This value is in good agreement with the experimental one, $E_x^{\exp} = 9.86$ MeV, deduced from the ^{90}Zr(^{3}He, d) reaction [3]. The s.p. proton escape width of the IAS($5/2^+$) calculated with Eq. (1) is equal to 3.3 keV. In this calculation we use the spectroscopic factor $S_\nu^{\exp}(d_{5/2}) = 0.95$ [22] and the experimental escape energy $\epsilon^{\exp} = E_x^{\exp}(^{91}\text{Nb}) - S_p^{\exp}(^{91}\text{Nb}) = 4.71$ MeV; ($\epsilon^{\text{calc}} = 4.32$ MeV, $\Gamma_c^{\text{RPA}} = 1.7$ keV). The calculated value agrees well with the width $\Gamma^{\exp}(2d_{5/2}) = 4.0 \pm 0.5$ keV deduced from the ^{90}Zr(p, p_0) reaction [23].

The present analysis of the s.p. proton width of the IAS($5/2^+$) in ^{91}Nb is refined as compared with the one given in Ref. [7]. In that work, the Coulomb mean field was calculated in a non-consistent way and the spectroscopic factor $S_\nu = 0.86$ was used. Nevertheless, the values of $\Gamma(2d_{5/2})$ calculated presently and

in Ref. [7] differ little due to the partial cancellation of the contributions to the width of the two mentioned effects.

The proton decay of the IAS($5/2^+$) to the 2^+ phonon state in ^{90}Zr is preferred as compared to the decay to the 5^- and 3^- states because the escaping proton has a higher energy and the lowest angular momentum ($l_c = 0$). The preliminary calculations in an extended CRPA yield that the partial width for the decay to the 2^+ state has a value of several eV. This is in qualitative agreement with the experimental result that the decay of the IAS($5/2^+$) to the g.s. of ^{90}Zr exhausts about 100% of the total proton width of this IAS [4].

The IAS($11/2^-$) has an excitation energy $E_x^{\exp} = 12.07$ MeV [3]. The excitation energy of the corresponding $11/2^-$ parent state in ^{91}Zr is 2.17 MeV which is very close to the energy difference between the IAS($11/2^-$) and the IAS($5/2^+$) in ^{91}Nb. The spectroscopic factor of this parent state is $S_\nu^{(1)} = 0.37$ ($S_\nu^{(1)} = 0.45$) in Ref. [22] ([24]). Another $11/2^-$ state in ^{91}Zr is located at an excitation energy of 2.32 MeV and has a small spectroscopic factor $S_\nu^{(2)}(11/2^-) = 0.05$ [22]. The IAS of this state will, therefore, be hardly excited in the present experiment.

We propose simple wave functions to describe some of the measured properties of the IAS($11/2^-$) state and the possible parent states in ^{91}Zr:

$$\Psi^{(i)}(11/2^-) = a_1^i |1h_{11/2}\rangle + a_2^i |5^- \otimes 2d_{5/2}\rangle$$
$$+ a_3^i |3^- \otimes 2d_{5/2}\rangle, \quad i = 1, 2. \quad (2)$$

The first term is due to the non-zero spectroscopic factors of the $11/2^-$ parent states, i.e., $S_\nu^{(i)} = |a_1^i|^2$. In addition, one has to account for the measured relative branching ratios for the proton decay of the IAS($11/2^-$) to the 5_1^- and 3_1^- phonon states in ^{90}Zr at $E_x = 2.32$ and 2.75 MeV, respectively. For the parent states, this is achieved by coupling a $d_{5/2}$ neutron to these negative-parity phonon states in ^{90}Zr close in excitation energy to that of the $11/2^-$ states in ^{91}Zr. Using the wave function given by Eq. (2) and taking only the IAS s.p. proton decay into account, we obtain the following expressions for the partial proton widths of the IAS($11/2^-$) for the decay to the g.s., and to the 5^- and 3^- states of the core nucleus:

$$\Gamma(\text{g.s.}) = |a_1|^2 \Gamma_1^{\text{calc}}(\text{s.p.}), \quad \text{subscript 1: } 1h_{11/2}, \epsilon_1^{\exp},$$

$$\Gamma(5^-) = |a_2|^2 \Gamma_2^{\text{calc}}(\text{s.p.}), \quad 2: 2d_{5/2}, \epsilon_2^{\text{exp}},$$

$$\Gamma(3^-) = |a_3|^2 \Gamma_3^{\text{calc}}(\text{s.p.}), \quad 3: 2d_{5/2}, \epsilon_3^{\text{exp}}, \quad (3)$$

where the subscript c for the calculated s.p. width $\Gamma_c^{\text{calc}}(\text{s.p.})$ means that this width is calculated using the indicated s.p. quantum numbers and the experimental escape-energy ϵ^{exp}. The results of the calculations are given in Table 1. The calculated s.p. widths $\Gamma_c^{\text{calc}}(\text{s.p.})$ are used to find the experimental probabilities $|a_c|^2$ by comparing the relative experimental branching ratios $\Gamma(c)/\Gamma(c')$ and by using the condition that $\sum_{c=1}^3 |a_c|^2 = 1.0$. The resulting values for $|a_c|^2$, which are obtained from the present experiment are listed in Table 1. The value of the quantity $|a_1|^2 = 0.45 \pm 0.03$ is in good agreement with the experimental spectroscopic factor of the $11/2^-$ parent state $S_\nu(1h_{11/2}) = 0.37$ [22], $S_\nu(1h_{11/2}) = 0.45$ [24]. The analysis of the experimental data on proton decay of the analog of the rather complicated $11/2^-$ parent state in ^{91}Zr is found to be reasonable.

Theoretical evaluation of probabilities $|a_c|^2$ could be an additional argument to judge the validity of the analysis. Such an evaluation can be done in a coupled-channel approach using phenomenological one-phonon transition potentials, which are expressed via the dynamic-deformation parameter β_L and the nuclear mean field for neutrons $U^n(r)$ (see, e.g., Ref. [25]). Let ν_0 and ν_1 be the set of quantum numbers for the $2d_{5/2}$ and $1h_{11/2}$ states, respectively; let L^π and $\pi = (-1)^L$ be the phonon angular momentum and parity ($L^\pi = 5^-, 3^-$). The coupling matrix elements are equal to:

$$V_{\nu_1\nu_0}^L = -\frac{\beta_L R}{\sqrt{2L+1}} \frac{\langle(\nu_1)||Y_L||(\nu_0)\rangle}{\sqrt{2j_{\nu_1}+1}}$$

$$\times \int \chi_{\nu_1}(r) \frac{\partial U^n}{\partial r} \chi_{\nu_0}(r)\, dr. \quad (4)$$

Here, R is the nuclear radius, $\langle(\nu_1)||Y_L||(\nu_0)\rangle$ is the reduced matrix element and $r^{-1}\chi_\nu(r)$ are the neutron bound-state radial wave functions. Using the model parameters fixed previously and values for β_L from Ref. [26] listed in Table 1, we find: $V_{\nu_1\nu_0}^{L=5} = 0.37$ MeV and $V_{\nu_1\nu_0}^{L=3} = 1.51$ MeV. The last matrix element is comparable to the energy difference between the $1h_{11/2}$ s.p. state ($E_x^{\text{calc}} = 4.40$ MeV) and the configuration $(3^- \otimes 2d_{5/2})_{11/2^-}$ ($E_x^{\text{exp}} = 2.75$ MeV). This means that at least one of the two states of

Eq. (2) can have a rather large admixture of the $h_{11/2}$ s.p. state (i.e., a rather large spectroscopic factor). In fact, we have a three-level problem with known basis states having the mentioned energies and with only two non-zero coupling matrix elements. The solution of this problem gives three $11/2^-$ states with energies $E_x^{\text{calc}} = 1.78$, 2.36 and 5.32 MeV. The calculated squared amplitudes $|a_c|^2$ for the wave functions $\Psi^{(1,2)}$ of the two states with the lowest excitation energies are given in Table 1. The $11/2^-$ state at $E_x^{\text{calc}} = 1.78$ MeV should be assigned to the $11/2^-$ parent state at $E_x = 2.17$ MeV, because the calculated and experimental spectroscopic factors for this state are at least in qualitative agreement. For the same reason, the second $11/2^-$ state at $E_x^{\text{calc}} = 2.36$ should be assigned to the $11/2^-$ state in ^{91}Zr at $E_x = 2.32$ MeV. It should be remarked that the above analysis of the structure of the $11/2^-$ states in ^{91}Zr at $E_x \sim 2$ MeV can be considered only as qualitative because rather schematic one-phonon transition potentials have been used. A more detailed investigation can be made, for instance, in the quasiparticle-phonon model [27]. Nevertheless, the performed analysis unravels the main structure of these states. Increasing the number of basis states by taking into consideration the $1g_{7/2}$ and $3s_{1/2}$ single-neutron states in addition to the $2d_{5/2}$ state changes the calculated probabilities $|a_c|^2$ and partial proton widths of the lowest two IAS($11/2^-$) only slightly. However, such an increase could lead to a strong fragmentation of the strength of the third IAS($11/2^-$) at higher excitation energies, which is presumably the reason why no third $11/2^-$ state was observed in the experiment.

4. Conclusion

The one-neutron pickup spectroscopic factor S_ν of the $11/2^-$ parent state in ^{91}Zr found in the present analysis is in agreement with the values found in Refs. [22,24]. The assumed structure of the $11/2^-$ parent state is qualitatively confirmed by the calculations within the coupled-channel approach. The abilities of the continuum-RPA approach are also checked by the description of the partial proton width of the IAS($5/2^+$) in ^{91}Nb for the decay to the g.s. of ^{90}Zr. The present analysis shows that the character of the IAS($5/2^+$) and its parent state, i.e., the g.s. of ^{91}Zr, is a pure $2d_{5/2}$ s.p. state coupled to the core nucleus

^{90}Zr. The IAS($11/2^-$) in ^{91}Nb, however, is of a more complex character; it consists of the combination of a $1h_{11/2}$ s.p. state and a coupling of a $1d_{5/2}$ state to low-lying negative-parity phonon states in ^{90}Zr.

References

[1] S. Galès, Ch. Stoyanov, A.I. Vdovin, Phys. Rep. 166 (1988) 125.
[2] N. Van Giai, Ch. Stoyanov, Phys. Lett. B 272 (1991) 178.
[3] G. Finkel, D. Ashery, A.I. Yavin, G. Bruge, A. Chaumeaux, Nucl. Phys. A 217 (1973) 197.
[4] G. Finkel, D. Ashery, A.I. Yavin, A. Boudard, G. Bruge, A. Chaumeaux, M. Rouger, Phys. Rev. C 19 (1979) 1782.
[5] N. Auerbach, Phys. Rep. 98 (1983) 273.
[6] V.G. Guba, M.G. Urin, Nucl. Phys. A 460 (1986) 222.
[7] O.A. Rumyantsev, M.H. Urin, Phys. Rev. C 49 (1994) 537.
[8] G. Colò, N. Van Giai, P.F. Bortignon, R.A. Broglia, Phys. Rev. C 50 (1994) 1496.
[9] D.P. Knobles, S.A. Stotts, T. Udagawa, Phys. Rev. C 52 (1995) 2257.
[10] S.E. Muraviev, M.H. Urin, Nucl. Phys. A 572 (1994) 267.
[11] B.L. Birbrair, V.A. Sadovnikova, Sov. J. Nucl. Phys. 20 (1974) 1010.
[12] H.K.T. van der Molen, Ph.D. thesis Rijksuniversiteit Groningen, Groningen, The Netherlands, 1999, unpublished.
[13] H.K.T. van der Molen et al., to be published.
[14] M. Fujiwara et al., Nucl. Instrum. Methods A 422 (1998) 484.
[15] C.M. Baglin, Nucl. Data Sheets 86 (1999) 1.
[16] H.P. Blok, L. Hulstman, E.J. Kaptein, J. Blok, Nucl. Phys. A 273 (1976) 142.
[17] E.A. Moukhai, V.A. Rodin, M.H. Urin, Phys. Lett. B 447 (1999) 8.
[18] M.L. Gorelik, A.G. Zvenigorodsky, O.A. Rumyantsev, M.H. Urin, Bull. Rus. Acad. Sci. Phys. 63 (1999) 703;
M.L. Gorelik, M.H. Urin, to be published.
[19] A. Bohr, B. Mottelson, Nuclear Structure, Benjamin, New-York, 1969.
[20] V.G. Soloviev, Theory of Complex Nuclei, Pergamon, Oxford, 1976.
[21] A.H. Wapstra, G. Audi, Nucl. Phys. A 432 (1985) 1.
[22] A. Graue, L.H. Herland, K.J. Lervik, J.T. Nesse, E.R. Cosman, Nucl. Phys. A 187 (1972) 141.
[23] H.L. Scott, C.P. Swann, F. Rauch, Nucl. Phys. A 134 (1969) 667.
[24] W. Booth, S.M. Dalgliesh, K.C. McLean, R.N. Glover, F.R. Hudson, Phys. Lett. B 30 (1969) 335.
[25] V.V. Samoilov, M.H. Urin, Nucl. Phys. A 567 (1994) 237.
[26] W.S. Gray, R.A. Kenefick, J.J. Kraushaar, G.R. Satchler, Phys. Rev. 142 (1966) 735.
[27] Ch. Stoyanov, V.V. Voronov, N. Van Giai, Nucl. Phys. A 672 (2000) 141.