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Space-time-filling branes and strings with sixteen supercharges

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Abstract

We discuss branes whose world-volume dimension equals the target space-time dimension, i.e. ‘‘space-time-filling branes’’. In addition to the D9-branes, there are 9-branes in the NS–NS sectors of both the IIA and IIB strings. The world-volume actions of these branes are constructed, via duality, from the known actions of branes with codimension larger than zero. Each of these types of branes is used in the construction of a string theory with sixteen supercharges by modding out a Type II string by an appropriate discrete symmetry and adding 32 9-branes. These constructions are related by a web of dualities and each arises as a different limit of the Hořava–Witten construction. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction and overview

This paper is concerned with space-time-filling branes and their role in the construction of string theories with 16 supersymmetries. The most familiar case is that of the D9-brane of the IIB theory. It is usually not consistent to consider a background with an arbitrary number of D9-branes, but if 32 D9-branes are included and an orientifold is taken of the IIB theory, then the Type I string theory results. In [1], a number of other 9-branes were shown to arise. Applying an *S*-duality to the D9-brane gives a new 9-brane of the IIB theory which carries a NS–NS charge and which we will refer to as the NS–9B brane, and a *T*-duality takes this to a similar NS–NS 9-brane of the IIA theory, the NS–9A brane. At finite string coupling, the IIA string should become M-theory compactified on a circle, and the NS–9A brane and the D8-brane have a common M-theory origin, which was postulated to be an M9-brane (which is of course

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not a space-time-filling brane). However, the D8-brane arises in the version of the IIA theory with a mass parameter, and the lifting of this to 11 dimensions involves a number of complications, as we will discuss below. The charges for these 9-branes necessarily arise in the superalgebras [1], as we will review in Section 2, giving further evidence that they occur in the theory.

The low-energy effective action for a D9-brane is a Born–Infeld-type action for the ten-dimensional super-Yang–Mills theory. A space-time-filling brane cannot move, of course, and this is reflected in the absence of any scalars in the effective action that could represent translation zero-modes. The action gives the gauge-field part of the Type I effective action. In Section 4, we will construct the effective actions for the other space-time-filling branes, which will give the gauge sectors of the various string theories with 16 supercharges; for example, the NS–9B brane action gives a Born–Infeld-type action with unusual dilaton dependence for the Yang–Mills sector of the SO(32) heterotic string. These actions also encode some other useful information, such as the dependence of the brane tension on the moduli of the theory, as discussed in Section 3.

In Section 5, we discuss how each of the space-time-filling branes, together with some of the domain-wall branes, can be used in the construction of superstrings with 16 supersymmetries, in the same way that D9-branes are used in the construction of the Type I string. In each case, a Type II string theory is modded out by a $\mathbb{Z}_2$ discrete symmetry in the presence of 32 space-time-filling branes to give a new theory with 16 supercharges. The space-time-filling branes give rise to the gauge symmetry in each case, as branes that end on the space-time-filling branes carry gauge charges. For example, the SO(32) heterotic string is constructed by modding out the IIB string by the *S*-dual of the world-sheet parity symmetry in the presence of 32 NS–9B branes [1,2]. This puts the new space-time-filling branes on a similar footing to the D9-branes, and gives a unified picture of the various string theories with 16 supersymmetries. These constructions are lifted to M-theory in Section 6, and it is shown that they arise as particular limits of the Hořava–Witten construction [3], giving an M-theoretic justification for some of the assumptions made in Section 5 and Ref. [1]. Our conclusions are given in Section 7.

We now return to the discussion of the IIA mass parameter and the M9-brane. The D8-branes [4,5] are domain walls that divide regions with different values of the mass parameter of the *massive* IIA string theory whose low-energy limit is the *massive* IIA supergravity of Romans [6]. In the massless case, the IIA string at finite coupling is M-theory compactified on a circle whose radius depends on the string coupling [7], and it is natural to seek a similar interpretation of the massive theory. In [8] it was shown that if the Romans supergravity is lifted to 11 dimensions, it must be to a theory with explicit dependence on the Killing vector in the compact direction. Then 11-dimensional covariance does not emerge at strong coupling if the mass is non-zero, and it is not clear if this limit exists. This non-covariant theory does indeed have M9-brane solutions that give the D8-brane on double-dimensional reduction [8,9] and these give the NS–9A brane on vertical reduction.

Although the massive IIA supergravity cannot be obtained from conventional 11-dimensional supergravity, it was recently shown that the massive IIA string theory can be obtained from M-theory [10] by compactifying M-theory on a T^2 bundle over a circle and taking a limit in which all three 1-cycles shrink to zero size. The quantized mass parameter arises as the topological twist of the bundle. It seems plausible that the supergravity of [8] could emerge as a limit of this construction, with the explicit dependence on the Killing vector emerging from the M-theory background, not from any intrinsic modification of M-theory. We will not discuss this further here, but some of our results give support to the picture given by the effective theory of [8].

2. Branes and charges

Due to recent developments in string theory, it is by now well understood that there is an intricate relationship between the 'central' charge structure of the space-time supersymmetry algebra and the spectrum of BPS states that are described by supersymmetric brane solutions [11]. The generic rule is that a p-form charge in D dimensions contains the charges for a p-brane and a $(D-p)$-brane [1,12][1]. This gives rise to the well-known BPS spectrum of Type II string theory and M-theory, together with the extra 9-branes discussed in Section 1 we now review this, following Ref. [1].

The ten-dimensional IIA supersymmetry algebra with central charges is given by $(\alpha = 1,\ldots,32;\ M = 0,\ldots,9)$

$$\{Q_\alpha, Q_\beta\} = (\Gamma^M C)_{\alpha\beta} P_M + (\Gamma_{11} C)_{\alpha\beta} Z + \left(\Gamma^M \Gamma_{11} C\right)_{\alpha\beta} Z_M + \frac{1}{2!} (\Gamma^{MN} C)_{\alpha\beta} Z_{MN}$$
$$+ \frac{1}{4!} \left(\Gamma^{MNPQ} \Gamma_{11} C\right)_{\alpha\beta} Z_{MNPQ} + \frac{1}{5!} \left(\Gamma^{MNPQR} \Gamma_{11} C\right)_{\alpha\beta} Z_{MNPQR} \,. \tag{2.1}$$

Note that the right-hand-side contains the maximum number of allowed central charges:

$$\frac{1}{2} \times 32 \times 33 = 1 + 10 + 10 + 45 + 210 + 252 \,. \tag{2.2}$$

Scanning the known IIA branes we find the following correspondences between charges and BPS states:

$$\begin{aligned} &P_M \to \text{W–A}\,, \qquad Z \to \text{D0}\,, \qquad Z_M \to \text{NS–1A and NS–9A}\,,\\ &Z_{MN} \to \text{D2 and D8}\,, \qquad Z_{MNPQ} \to \text{D4 and D6}\,,\\ &Z_{MNPQR} \to \text{NS–5A and KK–A}\,. \end{aligned} \tag{2.3}$$

We find a gravitational wave (W–A), a fundamental string (NS–1A), Dp-branes $(p = 0,2,4,6,8)$, a solitonic five-brane (NS–5A), a Kaluza–Klein monopole (KK–A)

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[1] The spatial components $Z_{i_1\ldots i_p}$ of a p-form charge $Z_{M_1\ldots M_p}$ give the charge carried by a p-brane, whereas the $Z_{0i_1\ldots i_{p-1}}$ components can be dualized to give a spatial $(D-p)$-form charge:

$$\tilde{Z}_{i_p\ldots i_D} = \frac{1}{p!} \epsilon^{0i_1\ldots i_{p-1}}_{i_p\ldots i_D} Z_{0i_1\ldots i_{p-1}}\,,$$

which is the charge carried by a $(D-p)$-brane. The exceptions are a 0-form central charge, a self-dual central charge and the translation generator. The BPS solutions carrying these are a 0-brane, a $D/2$-brane and a gravitational wave, respectively. These cases are special because the 0-form central charge has no time component, the self-dual central charge has space components that are not independent from the time components, while the time component of the translation generator is identified as the Hamiltonian.

and a nine-brane (NS–9A). All cases are well understood except for the NS–9A brane, which would be a space-time-filling brane in IIA string theory [1].

Another example is the ten-dimensional IIB supersymmetry algebra with central charges. In this case there are two Majorana–Weyl charges Q^i_α ($i = 1,2$) with the same chirality. The algebra is given by

$$\{Q^i_\alpha, Q^j_\beta\} = \delta^{ij}(\mathscr{P}\Gamma^M C)_{\alpha\beta} P_M + (\mathscr{P}\Gamma^M C)_{\alpha\beta} Z^{ij}_M + \frac{1}{3!}\epsilon^{ij}(\mathscr{P}\Gamma^{MNP}C)_{\alpha\beta} Z_{MNP} + \frac{1}{5!}\delta^{ij}(\mathscr{P}\Gamma^{MNPQR}C)_{\alpha\beta} Z^+_{MNPQR} + \frac{1}{5!}(\mathscr{P}\Gamma^{MNPQR}C)_{\alpha\beta} Z^{+,ij}_{MNPQR}. \tag{2.4}$$

Here $\mathscr{P}$ is a chiral projection operator and $Z^{ij}_M, Z^{+,ij}_{MNPQR}$ are doublets of SO(2) (symmetric traceless representations). The upper index $+$ indicates that the charge is a self-dual 5-form. As in the IIA case we have the maximum number of p-form charges:

$$\frac{1}{2} \times 32 \times 33 = 10 + 20 + 120 + 126 + 252. \tag{2.5}$$

Scanning the known IIB branes we find the following correspondences[2]:

$$P_M \to \text{W–B}, \qquad Z^{ij}_M \to \text{D1 and NS–1B}, \quad \text{D9 and NS–9B},$$
$$Z_{MNP} \to \text{D3 and D7}, \qquad Z^{+,ij}_{MNPQR} \to \text{D5 and NS–5B}, \qquad Z^+_{MNPQR} \to \text{KK–B}. \tag{2.6}$$

In this case we find a gravitational wave (W–B), a fundamental string (NS–1B), Dp-branes ($p = 1,3,5,7,9$), a solitonic five-brane (NS–5B), a Kaluza–Klein monopole (KK–B) and a further nine-brane (NS–9B). We see that the IIB central charges suggest the existence of two space-time-filling branes: the D9-brane and the NS–9B brane. The first one has been discussed in the context of D-branes (see e.g. Ref. [16]), and the second occurs in the work of [1]. All cases are well understood except for the NS–9B brane.

The last example to consider is the eleven-dimensional supersymmetry algebra ($\alpha = 1,\ldots,32; M = 0,\ldots 10$):

$$\{Q_\alpha, Q_\beta\} = (\Gamma^M C)_{\alpha\beta} P_M + \frac{1}{2!}(\Gamma^{MN}C)_{\alpha\beta} Z_{MN} + \frac{1}{5!}(\Gamma^{MNPQR}C)_{\alpha\beta} Z_{MNPQR}. \tag{2.7}$$

Again, the algebra contains the maximum number of allowed central charges:

$$\frac{1}{2} \times 32 \times 33 = 11 + 55 + 462. \tag{2.8}$$

[2] The D7-brane is not characterized uniquely by its 7-form charge, but depends also on its $SL(2,\mathbb{Z})$ monodromy, and one encounters various types of 7-branes in the literature (see e.g. Refs. [13–15]). This is related to the fact that for objects of co-dimension 2 an $SL(2,\mathbb{R})$-transformation keeps the charge invariant but changes the monodromy and the couplings to strings and 5-branes [1].

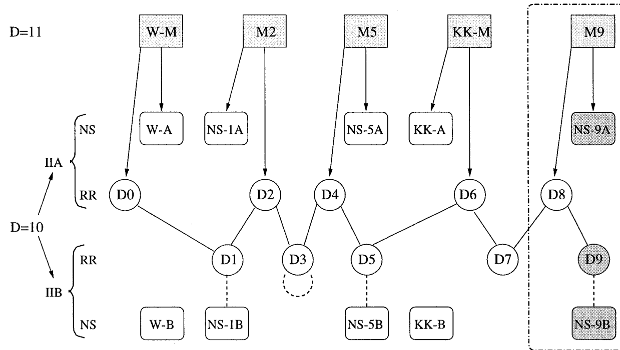

Fig. 1. Branes in ten and eleven dimensions. This figure depicts the states of IIA, IIB and M-theories preserving half the supersymmetry. Each carries a charge occurring in the supersymmetry algebras (see Eqs. (2.3), (2.6) and (2.9)). Arrows indicate dimensional reduction. In particular, in this figure vertical (diagonal) arrows indicate direct (double) dimensional reduction. The solid (dashed) lines indicate *T*-duality (*S*-duality) relations (the figure does not indicate all duality relations). The five branes in the box at the right are the focus of this work.

These central charges are related to the following M-branes:

$$P_M \to \mathrm{W}-M\,, \qquad Z_{MN} \to \mathrm{M2\,and\,M9}\,, \qquad Z_{MNPQR} \to \mathrm{M5\,and\,KK}-M\,. \tag{2.9}$$

We find a gravitational wave (W–M), a membrane (M2), a five-brane (M5), a Kaluza–Klein monopole (KK–M) and a nine-brane (M9). For a recent discussion of the M9-brane, see Ref. [9]. Note that in this case we do not find any space-time-filling branes.

All branes mentioned above can be related to each other via *T*-duality, *S*-duality and/or dimensional reduction (see Fig. 1). Formally, the D9-brane is obtained from other D-branes by *T*-duality, the NS–9B brane is obtained from the D9-brane by *S*-duality, and the NS–9A brane of the IIA theory is obtained from the NS–9B brane by *T*-duality. Finally, the M9-brane provides the 11-dimensional origin of both the D8-brane and the NS–9A brane. This is formal, as it is usually not consistent to have a single D9-brane, and the same will apply to the other 9-branes related to this by duality. However, it is consistent to have 32 D9-branes together with an orientifold plane in the construction of the Type I string, and one of the purposes of this paper is to give the analogous constructions involving the other 9-branes to give the various superstrings with sixteen supercharges, and show that they are related to the Type I construction by the appropriate S and *T*-dualities. This we will do in Sections 5 and 6, but first we will discuss the effective actions and tensions for these 9-branes.

3. Effective tensions

In Section 4 we will construct the world-volume actions of the different space-time-filling branes. It is instructive to first consider the *p*-brane effective tensions, defined as the mass per unit *p*-volume, which can be read off from the coefficient of the leading Nambu–Goto term in the action.

In general the world-volume actions of branes with p spacelike dimensions take the form:

$$S=\frac{1}{l^{p+1}}\int d^{p+1}\xi\, e^{\alpha\phi}|k|^{\beta}\sqrt{|\det g|}+\dots, \tag{3.1}$$

where the leading term is the Nambu–Goto term and the dots stand for other world-volume fields and the Wess–Zumino term. Here l is the fundamental length scale, which is the (eleven-dimensional) Planck length $l=l_p$ in M-theory and is the string length $l=l_s=\sqrt{\alpha'}$ in string theory, in which case we use a (dimensionless) string-frame metric[3]. For M2- and M5-branes, $\alpha=\beta=0$, while for conventional Type II branes $\beta=0$ and $\alpha=-1$ for D-branes, $\alpha=0$ for the fundamental string and $\alpha=-2$ for NS-5 branes.

The KK monopole in D dimensions it the product of a self-dual Taub-NUT space with $D-4$ dimensional Minkowski space and can be considered as an extended object with $D-5$ spatial dimensions and one extra isometry direction, the Taub-NUT fiber direction, which is transverse to the world-volume. In order to get the right counting of degrees of freedom this isometry is gauged, such that the effective number of embedding scalars is 3, fitting in a $D-4$ dimensional vector multiplet [1]. The effective action of the monopole is that of a $D-4$ dimensional gauged sigma model, with Killing vector k^{μ} [17]. For the M-theory KK monopole, the world-volume action is a gauged sigma-model, with a target space isometry generated by a (spacelike) Killing vector k^{μ} being gauged [17], and this leads to a term $|k|^{\beta}$ in the world-volume action [17], where $|k|^2=-k^{\mu}k^{\nu}g_{\mu\nu}$ and $\beta=2$ (and $\alpha=0$). This particular dependence with $|k|^2$ in front of the action is needed in order to get, for $D=11$, the correct $e^{-\phi}$ dilaton coupling of the D6-brane after dimensional reduction. The actions for the Type II KK monopoles have $\alpha=-2,\beta=2$ [18].

Given the world-volume action (3.1) one can read off the effective tension τ:

$$\tau=\frac{(g_s)^{\alpha}R^{\beta}}{l^{p+\beta+1}}, \tag{3.2}$$

where R is the size of the Killing direction, $\langle|k|^2\rangle\equiv(R/l)^2$, and $g_s=e^{\langle\phi\rangle}$ is the string coupling constant. Note that in string theory we have $\tau=\tau(l_s,g_s,R)$ while in M-theory we have $\tau=\tau(l_p,R)$, since $\alpha=0$. Conventional branes have $\beta=0$, while the KK monopole has non-trivial R-dependence, and more general objects with R-dependence have been found in [9,19–22]. For example, the *T*-dual of a D6-brane is a circularly symmetric D7-brane [5], i.e. one 'smeared' over a circle, and the *S*-dual of this is a 7-brane with $\tau\propto 1/g^3$.

In particular, the M9-brane has $\beta=3$ [9,19], and the effective tensions of the other 9-branes then follow from this via dualities, or can be read off from the actions of Section 4. In Fig. 2 we have given the effective tensions of the three ten-dimensional space-time-filling branes. The same figure also gives the effective tension of the ten-dimensional (eleven-dimensional) domain wall D8 (M9).

[3] We use here the same conventions as [11]. In the following sections we will use the convention of, e.g., Ref. [4] where one takes $l_s=1$ and works with the (string-frame) metric $g_{\mu\nu}$.

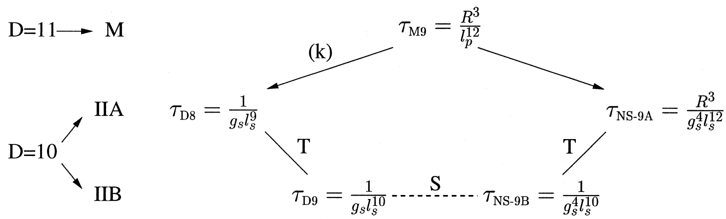

Fig. 2. Effective tensions. The figure gives the effective tensions of the three ten-dimensional space-time-filling branes (NS–9A, D9, NS–9B) and the ten-dimensional (eleven-dimensional) domain wall D8 (M9). It also indicates the different duality and reduction relations, using the same conventions as Fig. 1.

It is instructive to verify the *S*- and *T*-duality transformations of the effective tensions of the branes indicated in Fig. 2. The *T*-duality rules for R, g_s and l_s are

$$R' = \frac{l_s^2}{R}\,, \qquad g_s' = \frac{g_s l_s}{R}\,, \qquad l_s' = l_s\,, \tag{3.3}$$

and the *S*-duality rules are given by

$$R' = R\,, \qquad g_s' = \frac{1}{g_s}\,, \qquad l_s' = (g_s)^{1/2} l_s\,. \tag{3.4}$$

Finally, the reduction rules from $D = 11$ to $D = 10$ are given by

$$l_p = g_s^{1/3} l_s\,, \qquad R = \begin{cases} l_s g_s & \text{reduction in the } R \text{ direction} \\ R & \text{reduction in other direction} \end{cases} \tag{3.5}$$

Using the rules (3.3), (3.4) and (3.5) one can check that under *T*-, *S*-duality and reduction the different effective tensions given in Fig. 2 correctly transform into each other. It is understood that the *T*-duality relations are given by $(R\tau_{\text{D9}})' = \tau_{\text{D8}}$ and $(R\tau_{\text{NS}-9\text{B}})' = \tau_{\text{NS}-9\text{A}}$, respectively.

From Fig. 2 we see that the NS–9A brane and the M9-brane are special in the sense that their effective tensions are proportional to R^3 and it is interesting to compare these with KK monopoles which have $\tau \propto R^2$. The KK monopole in D dimensions has $D-5$ flat directions (which can be taken to be $\mathbb{R}^{D-5}$) and a Killing direction of radius R, and so can be thought of as a $D-5$-brane with 3 transverse dimensions. The NS–9A brane and the M9-brane each have 8 flat directions (which can be taken to be $\mathbb{R}^8$) and a Killing direction of radius R and so should be thought of as 8-branes. In particular, we take $p = 8$ in Eq. (3.1). The NS–9A brane is space-filling while the M9-brane is a wall with one transverse direction.

The KK-monopole cannot move in the Taub-NUT isometry direction which, in order to give a charge to the monopole is compact with finite radius R. Taking the limit $R \to \infty$ of a multi-monopole metric gives an ALE space. For the NS-9A and M9 eight-branes, the extra compact R direction can be decompactified to give nine-branes only for a certain number of coinciding eight-branes (in combination with an orientifold plane) but not for a single brane.

The KK monopole is a solution of the usual supergravity equations of motion, but the M9-brane only occurs as a solution of supergravity equations modified by explicit dependence on a mass parameter and a Killing vector; for more details, see Refs. [8,9].

One of the results of this work is that we will clarify the special role of the Killing vector direction in the case of the NS–9A brane and M9-brane.

The KK monopole with $D-5$ flat spatial dimensions and a circular fiber direction carries a $D-5$ form charge [1] while the NS-9A and M9-branes both have 8 flat spatial dimensions and a circular direction and both carry a 9-form charge. Thus in the KK monopole case, the circular fiber is transverse to the brane while in the NS–9A and M9-branes the circular direction is longitudinal, so that the brane should be thought of as wrapping the circle.

4. World-volume actions

In this section we construct the world-volume effective actions of the different space-time-filling branes. Our starting point is the action for the D9-brane. By applying *S*-duality we will obtain the action for the NS–9B brane, and from it we will derive the action of the NS–9A brane through a *T*-duality transformation. The dimensional reduction of the D9-brane and NS–9B brane leads to two space-time-filling 8-branes in nine dimensions: the RR-8 brane and the NS-8 brane (see Fig. 3). All these branes will play a role when we discuss in Section 5 the relation between space-time-filling branes and strings with sixteen supercharges.

Space-time-filling branes are special in the sense that they have zero codimension: there is no transversal direction. At first sight this leads to two problems:

1. The lines of force cannot escape to infinity.
2. The ‘‘naive’’ space-time solution is flat Minkowski space-time since the harmonic function characterizing the brane solution only depends on the transverse directions. Therefore the space-time solution does not appear to break any supersymmetry.

The D9-brane of the Type IIB string theory [23] is a brane carrying charge with respect to a non-dynamical RR 10-form potential, and N of these lead to an open string sector with $U(N)$ Chan–Paton factors. This is inconsistent due to gauge anomalies and charge non-conservation, but if there are precisely 32 coincident D9-branes, then orientifolding by the world-sheet parity reversal symmetry Ω gives a consistent theory,

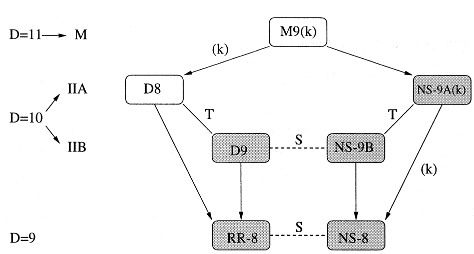

Fig. 3. Space-time-filling branes in ten and nine dimensions. The grey areas indicate the (ten- and nine-dimensional) space-time-filling branes whose actions are constructed in this section. To indicate the different reduction and duality relations, we use the same notation as in Fig. 1. Reduction over a Killing isometry direction is indicated by the label (k). The figure contains two more branes: the D8-brane or domain wall in ten dimensions and the M9-brane in eleven dimensions. We have indicated that the M9-brane and NS–9A brane require the existence of a Killing vector k. (See below).

which is the Type I SO(32) string theory. The orientifolding introduces a space-filling O9 orientifold fixed plane which has negative charge with respect to the RR 10-form potential, and the role of the 32 D9-branes is to cancel this charge.

Similar issues arise for any p-brane in a space-time in which the transverse space is compact, as the lines of force again cannot escape to infinity. In general it is not consistent to put branes in such a space-time, but it can be consistent if the charge of the branes is cancelled by that of the fixed planes of some discrete symmetry, or in some other way. In general, then, there are consistency conditions restricting which branes can be placed in which space-times, and the case of the D9-brane can be thought of as an example of this.

Nonetheless, it is sometimes convenient to think of a brane in an arbitrary background. In the actions for Dp-branes in arbitrary Type II supergravity backgrounds, the coupling to the NS-NS fields is through a Born–Infeld kinetic term and to the RR n-form gauge fields $C^{(n)}$ is through a Wess–Zumino term. In particular, this gives an action for the D9-brane coupling to an arbitrary IIB supergravity background with fields[4]:

$$\{g\,,\phi\,,B^{(2)}\,,B^{(10)}\,,C^{(0)}\,,C^{(2)}\,,C^{(4)}\,,C^{(10)}\}\,. \tag{4.1}$$

Acting with S-duality then gives the action for the NS–9B brane coupling to the same set of fields, and acting with T-duality using the rules of [24] then gives the action for the NS–9A brane coupling to the IIA supergravity fields

$$\{g\,,\phi\,,B^{(2)}\,,B^{(10)}\,,C^{(1)}\,,C^{(3)}\,,C^{(9)}\}\,. \tag{4.2}$$

In the orientifold construction, the IIB massless supergravity multiplet is truncated to the $N=1$ supergravity multiplet, with the following bosonic fields set to zero:

$$B^{(2)}=C^{(0)}=C^{(4)}=B^{(10)}=0\,, \tag{4.3}$$

and it is to this truncated background to which the D9-brane of the Type I theory couples. The Yang–Mills sector arises from the world-volume gauge theory. Below, we will give the action for the Type I D9-brane coupling to this truncated set of fields. We will give the abelian form of the action, as the formulation of a non-abelian Born–Infeld action describing a number of coincident D-branes is still an open problem; one candidate for such an action is given in [25]. Acting with S and T-dualities then gives the $N=1$ truncations of the IIA and IIB backgrounds to which the other space-time-filling branes will couple.

Since S-duality acts within the massless sector, after applying S-duality, the NS–9B brane will couple to the S-dual of the truncation (4.3), given by

$$C^{(2)}=C^{(0)}=C^{(4)}=C^{(10)}=0\,. \tag{4.4}$$

The IIA supergravity background (4.2) is truncated by

$$C^{(1)}=C^{(3)}=C^{(9)}=0\,, \tag{4.5}$$

i.e. the T-dual of the projection (4.4). The NS–9A brane couples to this $N=1$ truncation.

[4] We include the SL(2) doublet of non-dynamical 10-form potentials: $B^{(10)}$ and $C^{(10)}$, to which the NS–9B brane and D9-brane couple, and the dual potentials.

Space-time-filling branes in nine dimensions couple to $N=1$ truncations of the $D=9$ $N=2$ supergravity background. These truncations follow via reduction from ten dimensions. Let us first introduce our notation for the $D=9$ $N=2$ supergravity background. We can consider either its IIA or IIB origin, but it will be useful to consider both.

From IIB one gets the following $D=9$ $N=2$ background fields[5]:

$$g_{\mu\nu} \to \left\{ g_{\mu\nu}\,,\, A^{(1)}\,,\, \kappa \right\}, \qquad \phi \to \phi, \qquad B^{(2)} \to \{ B^{(2)}\,,\, B^{(1)} \}\,, \qquad B^{(10)} \to B^{(9)}\,,$$
$$C^{(0)} \to C^{(0)}\,, \qquad C^{(2)} \to \{ C^{(2)}, C^{(1)} \}\,, \qquad C^{(4)} \to C^{(3)}\,, \qquad C^{(10)} \to C^{(9)}\,. \qquad (4.6)$$

The RR-8 brane couples to the following $N=1$ truncation:

$$B^{(2)} = B^{(1)} = C^{(0)} = C^{(3)} = B^{(9)} = 0\,, \qquad (4.7)$$

since this is the reduction of the Ω projection (4.3) to 9 dimensions. The NS-8 brane couples to the reduction of (4.4):

$$C^{(2)} = C^{(1)} = C^{(0)} = C^{(3)} = C^{(9)} = 0\,. \qquad (4.8)$$

We now indicate the IIA origin of the $D=9$ $N=2$ background fields since it will be useful for the construction of the nine-dimensional space-time-filling branes in Subsection 4.2:

$$g_{\mu\nu} \to \left\{ g_{\mu\nu}\,,\, B^{(1)}\,,\, \kappa \right\}, \qquad \phi \to \phi\,, \qquad B^{(2)} \to \{ B^{(2)}\,,\, A^{(1)} \}\,, \qquad B^{(10)} \to B^{(9)}\,,$$
$$C^{(1)} \to \{ C^{(1)}, C^{(0)} \}\,, \qquad C^{(3)} \to \{ C^{(3)}, C^{(2)} \}\,, \qquad C^{(9)} \to \{ C^{(9)}, C^{(8)} \}\,. \qquad (4.9)$$

Since space-time-filling branes have no transverse directions their corresponding world-volume actions do not contain physical transverse embedding scalars. Indeed, all background fields depend on the world-volume embedding scalars only. For instance, the dependence of the dilaton background field ϕ is given by

$$\phi = \phi(X^{\mu}(\xi)) \quad (\mu = 0,\ldots,9)\,, \qquad (4.10)$$

where X^{μ} are the ten world-volume embedding scalars and ξ are the ten world-volume coordinates. In the physical gauge

$$\boldsymbol{X} = \boldsymbol{\xi} \qquad (4.11)$$

we can identify the world-volume of the space-time-filling brane with the target space and we obtain

$$\phi = \phi(\boldsymbol{\xi})\,. \qquad (4.12)$$

We thus end up with a ten-dimensional field theory consisting of a vector multiplet coupled to $N=1$ supergravity background fields.

In Subsection 4.1 we will first construct the (bosonic part of) the world-volume actions of the three ten-dimensional space-time-filling branes: the D9-brane, the NS–9B brane and the NS–9A brane. In the following subsection we will construct the world-volume actions of the two nine-dimensional space-time-filling branes given in

[5] The notation in [24] can be obtained after the replacements: $A^{(1)}_{\mu} \to B_{\mu}$, $B^{(2)}_{\mu\nu} \to B^{(1)}_{\mu\nu}$, $B^{(1)}_{\mu} \to A^{(2)}_{\mu}$, $C^{(2)}_{\mu\nu} \to B^{(2)}_{\mu\nu}$, $C^{(1)}_{\mu} \to A^{(1)}_{\mu}$ and $C^{(0)} \to \ell$.

Fig. 3: the RR-8 brane and the NS-8 brane, and show that they are related by an *S*-duality transformation in nine dimensions.

4.1. Actions for D = 10 space-time-filling branes

4.1.1. The D9-brane action

The world-volume theory for the space-time-filling D9-brane of the Type II theory is a ten-dimensional supersymmetric gauge theory with gauge field b coupling to the background fields (4.1), while that of the Type I D9-brane couples to the remaining fields after the truncation (4.3). The (bosonic part of the) world-volume action of the Type I D9-brane, in the physical gauge (4.11), is given by

$$S^{(\mathrm{D9})} = S^{(\mathrm{D9})}_{\mathrm{DBI}} + S^{(\mathrm{D9})}_{\mathrm{WZ}}\,, \tag{4.13}$$

with Dirac–Born–Infeld (DBI) action

$$\begin{aligned} S^{(\mathrm{D9})}_{\mathrm{DBI}} &= -T_9 \int d^{10}\xi\, e^{-\phi}\sqrt{|\det(g + 2\pi\alpha' F)|} \\ &= -T_9 \int d^{10}\xi\, e^{-\phi}\sqrt{|g|}\left\{1 - \frac{1}{4}(2\pi\alpha')^2 \operatorname{tr} F^2 \right.\\ &\qquad \left. + + \frac{1}{8}(2\pi\alpha')^4\left[\frac{1}{4}(\operatorname{tr} F^2)^2 - \operatorname{tr} F^4\right] + \ldots\right\}, \end{aligned} \tag{4.14}$$

where the DBI field strength is

$$F = 2\partial b\,, \tag{4.15}$$

and the trace is taken over the space-time components, so that e.g. $\operatorname{tr} F^2 = F_m{}^n F_n{}^m$. The Wess–Zumino term is

$$\begin{aligned} S^{(\mathrm{D9})}_{\mathrm{WZ}} = -T_9 \int \left\{ C^{(10)} + \frac{1}{2!}(2\pi\alpha')^2 C^{(6)} \wedge F \wedge F + \right.\\ \left. + \frac{1}{4!}(2\pi\alpha')^4 C^{(2)} \wedge F \wedge F \wedge F \wedge F + \ldots \right\}. \end{aligned} \tag{4.16}$$

Here $C^{(10)}$ is the RR 10-form, $C^{(6)}$ is the 6-form dual of $C^{(2)}$, the dots indicate gravitational contributions from the ‘‘roof genus’’ [26,27] and we have used differential form notation[6].

The cosmological term in the expansion of the DBI action and the $C^{(10)}$ term in the Wess–Zumino part are cancelled when one introduces the Orientifold O9 plane. The O9 plane does not have any world-volume dynamics but contributes, in the tadpole cancellation, with an energy density and charge opposite to that of 32 D9 branes, and so corresponds to the following action:

$$S^{(\mathrm{RR-O9})} = 32 \times T_9 \int d^{10}\xi \left\{ e^{-\phi}\sqrt{|g|} + C^{(10)} + \ldots \right\}. \tag{4.17}$$

The dots represent other coupling terms, some of which have been discussed in [28,29].

---

[6] Our convention for a rank r form is: $A_{(r)} = \frac{1}{r!} A_{(r)\mu_1\ldots\mu_r} dX^{\mu_1} \wedge \ldots \wedge dX^{\mu_r}$.

4.1.2. The NS–9B action

In order to obtain the world-volume action of the NS–9B brane, we apply an *S*-duality transformation to the D9-brane action. The *S*-duality rules for the fields present in this action are

$$\phi \to -\phi\,, \qquad g_{\mu\nu} \to e^{-\phi} g_{\mu\nu}\,, \qquad C^{(10)} \to B^{(10)}\,, \tag{4.18}$$

$$C^{(2)} \to B^{(2)}\,, \qquad C^{(6)} \to B^{(6)}\,, \tag{4.19}$$

where $B^{(6)}$ is the 6-form dual to $B^{(2)}$. The BI world-volume 1-form b transforms into a 1-form $c^{(1)}$ as follows:

$$b \to -c^{(1)}\,, \qquad c^{(1)} \to b\,. \tag{4.20}$$

Applying these rules to the action of the D9-brane we obtain the following world-volume action for the NS–9B brane:

$$S^{(\mathrm{NS-9B})} = S^{(\mathrm{NS-9B})}_{\mathrm{DBI}} + S^{(\mathrm{NS-9B})}_{\mathrm{WZ}}\,, \tag{4.21}$$

with DBI term

$$\begin{aligned} S^{(\mathrm{NS-9B})}_{\mathrm{DBI}} &= -T_9 \int d^{10}\xi\, e^{-4\phi} \sqrt{\left|\det\big(g - (2\pi\alpha')\, e^{\phi} F\big)\right|} \\ &= -T_9 \int d^{10}x \sqrt{|g|} \left\{ e^{-4\phi} - \frac{1}{4} (2\pi\alpha')^2 e^{-2\phi} \operatorname{tr} F^2 \right. \\ &\qquad \left. + \frac{1}{8} (2\pi\alpha')^4 \left[\frac{1}{4} (\operatorname{tr} F^2)^2 - \operatorname{tr} F^4 \right] + \dots \right\}, \end{aligned} \tag{4.22}$$

and Wess–Zumino term

$$\begin{aligned} S^{(\mathrm{NS-9B})}_{\mathrm{WZ}} &= -T_9 \int \left\{ B^{(10)} + \frac{1}{2!} (2\pi\alpha')^2 B^{(6)} \wedge F \wedge F \right. \\ &\qquad \left. + \frac{1}{4!} (2\pi\alpha')^4 B^{(2)} \wedge F \wedge F \wedge F \wedge F + \dots \right\}. \end{aligned} \tag{4.23}$$

Here F is the curvature of $c^{(1)}$:

$$F = 2\,\partial c^{(1)}\,. \tag{4.24}$$

As in the case of the D9-brane, we expect that the cosmological term and the $B^{(10)}$ dependence of the WZ term will disappear when we consider 32 coincident NS-9B branes and mod out by the *S*-dual of Ω, so that there is a contribution from the *S*-dual of the RR O9 Orientifold (see Subsection 5.3), which will carry $B^{(10)}$ charge and has action

$$S^{(\mathrm{NS-O9})} = 32 \times T_9 \int d^{10}\xi \left\{ e^{-4\phi} \sqrt{|g|} + B^{(10)} + \dots \right\}. \tag{4.25}$$

4.1.3. The heterotic string effective action

As we shall discuss in the next section, the heterotic SO(32) string is constructed by modding out the IIB string by a discrete symmetry, introducing the *S*-dual of the RR O9 orientifold, and adding 32 NS-9B branes. This then gives the Yang–Mills effective action of the heterotic string to be the sum of (the non-abelian form of) the DBI action

(4.22), the Wess–Zumino term (4.23) and the 9-plane contribution (4.25), to give the action

$$
\begin{aligned}
S_{\text{het}} = & -T_9\int d^{10}\xi\, e^{-4\phi}\Big[\sqrt{|\det\big(g-(2\pi\alpha')e^{\phi}F\big)|}-\sqrt{|\det g|}\,\Big] \\
& -T_9\int\left\{\frac{1}{2!}(2\pi\alpha')^2 B^{(6)}\wedge F^2+\frac{1}{4!}(2\pi\alpha')^4 B^{(2)}\wedge F^4+\ldots\right\} \\
= & -T_9\int d^{10}x\sqrt{|g|}\left\{-\frac{1}{4}(2\pi\alpha')^2 e^{-2\phi}\operatorname{tr}F^2\right. \\
& \left.+\frac{1}{8}(2\pi\alpha')^4\left[\frac{1}{4}(\operatorname{tr}F^2)^2-\operatorname{tr}F^4\right]+\ldots\right\} \\
& -T_9\int\left\{\frac{1}{2!}(2\pi\alpha')^2 B^{(6)}\wedge F^2+\frac{1}{4!}(2\pi\alpha')^4 B^{(2)}\wedge F^4+\ldots\right\}.
\end{aligned}
\tag{4.26}
$$

It is interesting to compare with known results for the heterotic action. A similar comparison between the D9-brane and the Type I effective action has been made in [30]. The WZ term involving $B^{(6)}$ gives the Chern–Simons coupling, the WZ term involving $B^{(2)}$ gives the Green–Schwarz anomaly canceling term and the first few terms of the expansion of the DBI action agree with the results of [31,32].

4.1.4. The NS–9A action

We next apply a T-duality transformation on the NS–9B brane (a T-duality on the D9-brane leads to the D8-brane). As stressed in the caption of Fig. 3, we will see that this case is special in the sense that the world-volume action we obtain contains a Killing vector, which effectively means that the action can only be defined in nine uncompactified space-time dimensions. Nevertheless, as will become clear later, it is useful to consider this case on its own.

The NS–9A brane is in some ways like an 8-brane; it has $8+1$ non-compact directions, and there is one which is compactified. It has an $8+1$ dimensional world-volume, like an 8-brane, but the target space has 10 scalars X^{μ}, $8+1$ of which are identified with the world-volume coordinates in physical gauge, and one of which is gauged. T-duality in this case can be understood through a double-dimensional reduction. The target space is taken to have a circular direction z, say, with $k=\partial/\partial z$ a Killing vector, and so (in physical gauge) the world-volume indices can be divided into (i,σ) where σ is the NS–9B brane world-volume direction wrapping the circle and i runs over the remaining 9 dimensions. For the target space fields we apply the standard T-duality rules [24]. The T-duality takes the NS–9B brane world-volume vector field $c^{(1)}$ to a nine-dimensional vector $d_i^{(1)}$ and a scalar $c^{(0)}$, which are the bosonic world-volume fields of the NS–9A brane. The T-duality rules for the world-volume fields are given by

$$
c_i^{(1)'}=-d_i^{(1)},\qquad c_\sigma^{(1)'}=-c^{(0)}. \tag{4.27}
$$

In order to explicitly calculate the T-dual of the NS–9B brane action (4.22) it is convenient to use the basic matrix identity ($\hat{\imath}=(i,\sigma)$):

$$
\det A_{\hat{\imath}\hat{\jmath}}=A_{\sigma\sigma}\det\left(A_{ij}-\frac{1}{A_{\sigma\sigma}}A_{i\sigma}A_{\sigma j}\right), \tag{4.28}
$$

where the determinant on the right-hand-side is of one dimension lower than the determinant on the left-hand-side. Applying this identity to the world-volume action (4.22) we obtain for the determinant

$$|g_{zz}|\left|\det\left(g_{ij} - \frac{1}{g_{zz}} g_{iz} g_{jz} - (2\pi\alpha') e^{\phi}\left(2\partial_{[i c^{(1)}{}_{j]}} - 2\partial_{[i c^{(1)}{}_{\sigma g j]z}}/g_{zz}\right)\right.\right.$$
$$\left.\left. + (2\pi\alpha')^2 \frac{e^{2\phi}}{g_{zz}} \partial_i c_\sigma^{(1)} \partial_j c_\sigma^{(1)}\right)\right| \,. \tag{4.29}$$

Using this intermediate result, we find that the (bosonic part of the) NS–9A brane has world-volume fields (in the physical gauge (4.11))

$$\left\{d_i^{(1)}, c^{(0)}\right\}, \tag{4.30}$$

in terms of which the world-volume action is given by

$$S^{(\mathrm{NS-9A})} = S_{\mathrm{DBI}}^{(\mathrm{NS-9A})} + S_{\mathrm{WZ}}^{(\mathrm{NS-9A})}, \tag{4.31}$$

with DBI term

$$S_{\mathrm{DBI}}^{(\mathrm{NS-9A})}$$
$$= -T_8 \int d^9\xi\, e^{-4\phi} |k|^3 \sqrt{\left|\det\left(\Pi - (2\pi\alpha')^2 e^{2\phi}\, \partial c^{(0)} \partial c^{(0)} + 2\pi\alpha' |k|^{-1} e^{\phi} \mathscr{H}^{(2)}\right)\right|}\,, \tag{4.32}$$

where $T_8 = \int d\sigma\, T_9$ and Π_{ij} is the reduced metric

$$\Pi_{ij} = \partial_i X^\mu \partial_j X^\nu \left(g_{\mu\nu} + |k|^{-2} k_\mu k_\nu\right). \tag{4.33}$$

This action is invariant under translations generated by the Killing vector k^μ. We can define covariant derivatives $D_i X^\mu = \partial_i X^\mu + A_i k^\mu$, with A_i a dependent gauge field, $A_i = |k|^{-2} \partial_i X^\mu k_\mu$, in such a way that $\Pi_{ij} = D_i X^\mu D_j X^\nu g_{\mu\nu}$. The field strength $\mathscr{H}^{(2)}$ is given by

$$\mathscr{H}^{(2)} = H - 2\left(i_k B^{(2)}\right) \partial c^{(0)}, \tag{4.34}$$

where $H = 2\partial d^{(1)}$.

The Wess–Zumino term reads

$$\begin{aligned} S_{\mathrm{WZ}}^{(\mathrm{NS-9A})} = -T_8 \int \Big\{ & i_k B^{(10)} + \frac{1}{2!} (2\pi\alpha')^2 \left(i_k B^{(6)}\right) \wedge H \wedge H \\ & - (2\pi\alpha')^2 \left(i_k N^{(7)}\right) \wedge H \wedge dc^{(0)} \\ & - (2\pi\alpha')^2 \left(i_k B^{(6)}\right) \wedge \left(i_k B^{(2)}\right) \wedge H \wedge dc^{(0)} \\ & + \frac{1}{3!} (2\pi\alpha')^4 \left(B^{(2)} - A \wedge \left(i_k B^{(2)}\right)\right) \wedge H \wedge H \wedge H \wedge dc^{(0)} \\ & + \frac{1}{4!} (2\pi\alpha')^4 A \wedge H \wedge H \wedge H \wedge H \Big\}. \end{aligned} \tag{4.35}$$

The field $N^{(7)}$ is the Poincaré dual of the Killing vector k_μ considered as a 1-form, and it was first encountered in the action of the Kaluza–Klein monopole [33]. The T-duality transformation rules for $N^{(7)}$ and $B^{(6)}$ can be found in [18]. The field $B^{(10)}$ is the NS 10-form potential T-dual to the NS 10-form potential of the Type IIB theory. Again, a cancellation of the cosmological constant in the expansion of the DBI term and the $i_k B^{(10)}$ term is supposed to take place when we consider 32 coincident NS-9A branes and we include the contribution of the plane related by an S- and a T-duality to the O9 orientifold plane of the Type IIB theory.

The world-volume theory for the NS–9A brane is a nine-dimensional gauge theory with a non-linear action for the super-Yang–Mills multiplet with bosonic fields $\{d_i^{(1)}, c^{(0)}\}$, coupled to 10 scalars X^μ, nine of which are eliminated in the physical gauge, and one of which drops out due to the invariance under the isometry symmetry $X^\mu \to X^\mu + \alpha k^\mu$, where α is an arbitrary local function on the world-volume. We see that the action of the NS–9A brane is a gauge theory coupled to a gauged sigma-model and contains a Killing vector. The isometry is an essential ingredient, and the gauge symmetry means that the collective coordinate for translations in the Killing direction is absent, so that such translations are not physical. It therefore requires an isometry direction and is effectively defined in nine non-compact space-time dimensions only. Dimensional reduction over the Killing direction gives the action of the NS-8 brane (see Subsection 4.2.2). We will see in Section 6 the reason why the 9-form central charge of the IIA supersymmetry algebra cannot be realized by a proper ten-dimensional space-time-filling brane whereas we have seen that this is possible for the two 9-form central charges of the IIB supersymmetry algebra.

4.2. Actions for $D = 9$ space-time-filling branes

In the previous subsection we have seen that the NS–9A brane has an $8+1$-dimensional world-volume and couples to background fields which break ten-dimensional general covariance. The dimensional reduction of the NS–9A brane over the Killing isometry direction leads to a space-time-filling brane, which we will refer to as the NS-8 brane, in nine dimensions. Furthermore, by considering the direct reduction of the D8-brane we obtain a second space-time-filling brane, referred to as the RR-8 brane, in nine dimensions. These two nine-dimensional space-time-filling branes will play a role when we discuss the relation between space-time-filling branes and strings with sixteen supercharges. In this subsection we will construct their world-volume actions and show that they are related by an S-duality transformation.

4.2.1. The RR-8 action

To obtain the action of the RR-8 brane we perform a direct reduction of the D8-brane. Note that the ten-dimensional D8-brane is not a space-time-filling brane but a domain wall. It therefore couples to a IIA supergravity background. After direct reduction of the D8-brane, when the brane has become a space-time-filling one, we perform the truncation (4.7).

Our starting point is the D8-brane action

$$S^{(\mathrm{D8})} = S^{(\mathrm{D8})}_{\mathrm{DBI}} + S^{(\mathrm{D8})}_{\mathrm{WZ}}\,, \tag{4.36}$$

with DBI term given by

$$S_{\rm DBI}^{\rm (D8)} = -T_8 \int d^9\xi\, e^{-\phi} \sqrt{|\det(g + 2\pi\alpha'\mathcal{F})|}\,, \tag{4.37}$$

with

$$\mathcal{F} = 2\partial b + \frac{1}{2\pi\alpha'} B^{(2)}, \tag{4.38}$$

and Wess–Zumino term given by

$$\begin{aligned} S_{\rm WZ}^{\rm (D8)} = T_8 \int \bigg\{ & C^{(9)} - (2\pi\alpha')C^{(7)} \wedge \mathcal{F} + \frac{1}{2!}(2\pi\alpha')^2 C^{(5)} \wedge \mathcal{F} \wedge \mathcal{F} \\ & - \frac{1}{3!}(2\pi\alpha')^3 C^{(3)} \wedge \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \\ & + \frac{1}{4!}(2\pi\alpha')^4 C^{(1)} \wedge \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \bigg\}. \end{aligned} \tag{4.39}$$

The direct reduction of this action along the single transversal coordinate Z gives, after performing the truncation (4.7), the action of the RR-8 brane, which contains the following world-volume fields:

$$\{b_i, Z\}\,. \tag{4.40}$$

Explicitly[7],

$$S_{\rm DBI}^{\rm (RR-8)} = -T_8 \int d^9\xi\, e^{-\phi}\kappa^{-1/2} \sqrt{\left|\det\!\left(g_{ij} - \kappa^2\partial_i Z\partial_j Z + 2\pi\alpha'\mathcal{F}_{ij}\right)\right|}\,, \tag{4.41}$$

where the field strength $\mathcal{F}$ is defined as

$$\mathcal{F} = 2\partial b - \frac{2}{(2\pi\alpha')} A^{(1)}\partial Z. \tag{4.42}$$

The WZ term reads

$$\begin{aligned} S_{\rm WZ}^{\rm (RR-8)} = T_8 \int \bigg\{ & C^{(9)} - (2\pi\alpha')C^{(6)} \wedge dZ \wedge \mathcal{F} + + \frac{1}{2!}(2\pi\alpha')^2 C^{(5)} \wedge \mathcal{F} \wedge \mathcal{F} \\ & - \frac{1}{3!}(2\pi\alpha')^3 \big(C^{(2)} - C^{(1)} \wedge A^{(1)}\big) \wedge dZ \wedge \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \\ & + \frac{1}{4!}(2\pi\alpha')^4 C^{(1)} \wedge \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \bigg\}. \end{aligned} \tag{4.43}$$

Here $C^{(9)}$ is the 9-form potential obtained after reducing the D8-brane's 9-form $C^{(9)}$ and $C^{(6)}$ ($C^{(5)}$) denotes the 6-form (5-form) dual to $C^{(1)}$ ($C^{(2)}$) in nine dimensions.

4.2.2. The NS-8 brane action

Similarly, the dimensional reduction of the NS–9A brane along the Killing isometry direction gives the action of the NS-8 brane, with world-volume fields

$$\{d_i^{(1)}, c^{(0)}\}\,. \tag{4.44}$$

[7] Now all target-space fields are nine-dimensional.

The action is given by

$$S^{(\mathrm{NS}-8)} = S^{(\mathrm{NS}-8)}_{\mathrm{DBI}} + S^{(\mathrm{NS}-8)}_{\mathrm{WZ}}\,, \tag{4.45}$$

with DBI term[8]:

$$\begin{aligned} S^{(\mathrm{NS}-8)}_{\mathrm{DBI}} = &-T_8\int d^9\xi\, e^{-4\phi} \\ &\times \kappa\sqrt{\left|\det\left(g_{ij} - (2\pi\alpha')^2 e^{2\phi}\kappa\partial_i c^{(0)}\partial_j c^{(0)} + (2\pi\alpha')\,\kappa^{-1/2}e^{\phi}\mathcal{H}_{ij}\right)\right|}\,, \end{aligned} \tag{4.46}$$

with $\mathcal{H}$ given by

$$\mathcal{H} = 2\,\partial d^{(1)} + 2\,A^{(1)}\partial c^{(0)} \equiv H + 2\,A^{(1)}\partial c^{(0)}\,. \tag{4.47}$$

The Wess–Zumino term reads

$$\begin{aligned} S^{(\mathrm{NS}-8)}_{\mathrm{WZ}} = &-T_8\int\Big\{B^{(9)} - (2\pi\alpha')^2 B^{(6)}\wedge H\wedge dc^{(0)} \\ &+\frac{1}{2!}(2\pi\alpha')^2 B^{(5)}\wedge H\wedge(H + 2\,A^{(1)}\wedge dc^{(0)}) \\ &-\frac{1}{3!}(2\pi\alpha')^4 B^{(2)}\wedge H\wedge H\wedge H\wedge dc^{(0)} \\ &-\frac{1}{4!}(2\pi\alpha')^4 B^{(1)}\wedge H\wedge H\wedge H\wedge H\Big\}\,. \end{aligned} \tag{4.48}$$

Here $B^{(9)}$ is the 9-form potential obtained after the reduction of the 10-form potential $B^{(10)}$ of the NS–9A brane, and $B^{(6)}$ ($B^{(5)}$) is the 6-form (5-form) dual to $B^{(1)}$ ($B^{(2)}$) in nine dimensions.

Note that both the RR-8 and the NS-8 branes contain one extra world-volume scalar: Z and $c^{(0)}$, respectively. However, it is only in the case of the RR-8 brane that this extra scalar can be used to oxidize the brane to a conventional brane in ten dimensions (the D8-brane). In the case of the NS-8 brane it is not possible to do so in a $D=10$ covariant way. Instead one gets a $D=10$ action with a Killing vector (the NS–9A brane).

Finally, we will show that the actions of the RR-8 brane and NS-8 brane are S-dual (in nine dimensions) to each other. The reduction of the S-duality rules (4.18) leads to the following S-duality rules for the nine-dimensional target space fields [24]:

$$g_{\mu\nu}\to\sqrt{\kappa}\,e^{-\phi}g_{\mu\nu}\,,\qquad A^{(1)}\to A^{(1)}\,,\qquad \kappa\to\kappa^{3/4}e^{\phi/2}\,,\qquad e^{\phi}\to\kappa^{7/8}e^{-3\phi/4}\,, \tag{4.49}$$

$$C^{(2)}\to -B^{(2)}\,,\qquad C^{(1)}\to B^{(1)}\,, \tag{4.50}$$

and for their duals

$$C^{(5)}\to -B^{(5)}\,,\qquad C^{(6)}\to B^{(6)}\,. \tag{4.51}$$

[8] The scalar κ arises here from $|k|^2=\kappa^2$.

The 9-form potentials transform as

$$C^{(9)} \rightarrow -B^{(9)}. \tag{4.52}$$

Furthermore, the S-duality rules for the world-volume fields are given by

$$Z \rightarrow -(2\pi\alpha')c^{(0)}, \qquad b \rightarrow d^{(1)}. \tag{4.53}$$

Applying these rules one may verify that the world-volume action of the RR-8 brane is S-dual (in nine dimensions) to the world-volume action of the NS-8 brane.

5. String theories, orbifolds and orientifolds

In this section we will discuss how the space-time-filling branes are used in the construction of string theories with 16 supercharges, and the way these are related by dualities. In the previous section we constructed the world-volume actions of three ten-dimensional (D9, NS–9B and NS–9A) and two nine-dimensional (RR–8 and NS–8) space-time-filling branes and all actions are related to each other via duality and/or reduction. Orientifolding the Type IIB string theory by the world-sheet parity reversal operator Ω of the fundamental string (NS–1B brane) requires the addition of 32 D9-branes in order to cancel the anomalies and tadpole introduced by the O9-orientifold fixed plane and gives the Type I SO(32) string theory. However, the Type I SO(32) string theory is related, via duality and/or reduction, to other string theories with sixteen supercharges and this suggests that one might also be able to describe these other $N = 1$ superstring theories by dividing out Type IIA or IIB string theory by a discrete symmetry, with the addition of a set of space-time-filling branes in order to cancel the anomalies introduced by the projection. In [1,2], it was argued that the SO(32) heterotic string can be obtained by modding out the IIB string with a (non-perturbative generalization of) the operator $(-1)^{F_L}$ (where F_L is the left-moving fermion number) in the presence of 32 NS-9B branes. It is the purpose of this section to generalize this to other cases and identify the perturbative symmetries of Type IIA/IIB string theories responsible for the projections onto the various string theories with sixteen supercharges. In

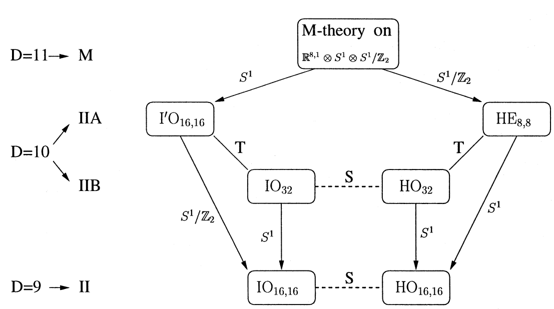

Fig. 4. String theories with 16 supercharges. The duality and/or reduction relations between theories that result from $\mathbb{Z}_2$ projections of theories with 32 supercharges (which are indicated on the left-hand side of the figure). The M-theory origin of two of the theories with 16 supercharges is also indicated. The abbreviations used in the Figure are explained in the text.

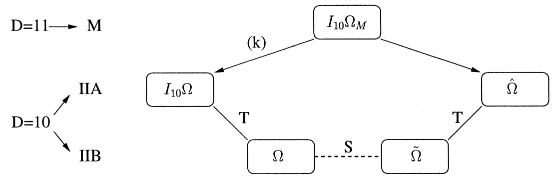

Fig. 5. $\mathbb{Z}_2$-symmetries. The $\mathbb{Z}_2$-symmetries of Type II and M theories and their relations under duality. These are used in the constructions of the corresponding theory with 16 supersymmetries. These symmetries are discussed further in Subsections 5.3, 5.4, 5.5 and 6.1.

Section 6, we will discuss the (compactified) M-theory origin of the Type II string theory discrete symmetries and show how the constructions of this section all arise as limits of the Hořava–Witten construction.

A unified description of the theories with 16 supercharges emerges. The string theories with 16 supercharges are related by the dualities displayed in Fig. 4 and can be obtained from M-theory compactifications. We will refer to any duality between a theory and its strong coupling dual as an *S*-duality, so that in this sense the *S*-dual of the Type I theory is the SO(32) heterotic string.

This *defines* the discrete symmetries

$$\tilde{\Omega} = S\Omega S^{-1}, \qquad \hat{\Omega} = T\tilde{\Omega}T^{-1} \tag{5.1}$$

while $I_{10}\,\Omega = T\Omega T^{-1}$ where I_{10} is reflection in the x^{10} direction. Each of these arise from a theory with 32 supercharges on modding out by a $\mathbb{Z}_2$ symmetry, in a background with 16 9-branes or 8-branes (plus their 16 mirror images). The Type I theory arises from modding out the IIB string by the world-sheet parity Ω with 32 D9-branes, and then the others arise from acting on these constructions with T and *S*-dualities. The set of discrete symmetries obtained in this way and used in the construction are shown in Fig. 5.

5.1. Orientifolds and orbifolds

An important ingredient in the discussion below is provided by the orientifold and orbifold constructions. For an extensive review on both types of construction and related issues, see Ref. [34][9].

Modding out a theory with a group of the form $G = G_1 \cup G_2\,\Omega$ (G_1, G_2 are target-space symmetries and Ω involves an orientation–reversal on the world-sheet) gives either an orbifold (if G_2 is empty) or an orientifold (if G_2 is non-empty). For consistency one also has to add a sector with twisted boundary conditions and perform a projection on the twisted spectrum as well. In the orientifold-construction this usually gives an open-string sector, i.e. strings which are closed modulo Ω. Here we will only consider groups $G = \mathbb{Z}_2$.

For orientifolds, the p-planes that are invariant under the orientation reversal Ω are called (perturbative) orientifold-fixed Op-planes. They couple to a RR–potential and are negatively charged. When the Op-planes have no non-compact transverse direction their

[9] For early references on orientifolds, see Refs. [35–41].

charge must be cancelled. This is achieved through the addition of a fixed number of Dp-branes, with opposite charge with respect to the same RR–potential, on which the open strings can end. They provide the Chan–Paton factors for the open strings. For other (non-orientifold) groups G, anomalies are cancelled and charges conserved if other types of branes are added, such as the NS-9 branes, as we will see. When such 'background' D-branes or other branes are added, p-branes that end on these background branes carry generalized Chan–Paton indices and the action of G is extended to act on these indices.

If G is a symmetry of a perturbative string theory X, this construction leads to a new perturbative string theory X/G, provided the correct set of anomaly canceling branes is added. Acting with T-duality then relates this to other perturbative constructions. The question arises as to whether the construction of X/G from X can be extended to the non-perturbative string theory. First of all, this requires that G should be a symmetry of the full non-perturbative generalization of X. If it is, then we can consider modding out the full theory by G (adding a twisted sector and projecting onto invariant states) and this should give a non-perturbative generalization of X/G. In particular, we can then take the strong coupling limit in which X is replaced by a dual theory $\tilde{X}$ which is a perturbation theory in $\tilde{g}=1/g$, where g is the coupling of X. (For example, if X is the IIA string, $\tilde{X}$ is M-theory on a circle of radius R, with coupling constant $\tilde{g}=l_s/R$.) Then the original symmetry extrapolates to a symmetry $\tilde{G}$ of the dual theory $\tilde{X}$, and modding out $\tilde{X}$ by $\tilde{G}$ to give $\tilde{X}/\tilde{G}$, which should be the strong coupling limit of X/G, this will be a perturbative construction (in $\tilde{g}$) if $\tilde{G}$ is a perturbative symmetry of $\tilde{X}$; this will be the case for the examples considered here. If the original construction of X/G required the addition of background branes, then the construction of $\tilde{X}/\tilde{G}$ will require the addition of the branes that are the strong coupling extrapolation of these. If X/G is an orientifold construction, then its strong coupling dual $\tilde{X}/\tilde{G}$ often turns out to be an orbifold construction, so that non-perturbatively it does not make sense to differentiate between orientifolds and orbifolds, and we shall simply refer to modding out by a discrete symmetry. Note that in some cases, different results arise depending on whether or not background branes are added.

For example, if X is the IIB string and $G=\Omega$, then X/G with 32 D9-branes gives X/G as the Type I string. The strong coupling limit is given by a dual perturbative IIB string theory in which the symmetry acts through $\tilde{G}=(-1)^{F_L}$. Then $\tilde{X}/\tilde{G}$ with 32 NS-9B branes gives the SO(32) heterotic string, which is indeed the strong coupling limit of the Type I string. However, in this case one can also consider modding out the IIB theory by $(-1)^{F_L}$ without adding any 9-branes, and it has been argued that this gives the IIA string [34], at least perturbatively. In what follows, we will discuss these and other examples in more detail.

5.2. D9 / Type I SO(32)

This case is the one that is best understood (for a review, see e.g. Ref. [16]) and we have discussed it already in different parts of the paper. The operator Ω acts on the perturbative IIB theory through the world-sheet parity reversal of the fundamental string:

$$\Omega: \sigma \to \pi - \sigma\,. \tag{5.2}$$

In orientifolding by Ω, it is necessary to add 32 coincident D9-branes to cancel the negative RR-charge of the O9 orientifold fixed plane. The invariant sector is the unoriented Type I closed string theory and there is a ‘twisted sector’ of open strings (i.e. strings that are closed modulo an Ω transformation) whose Chan–Paton factors are provided by the 32 D9-branes. The SO(32) gauge symmetry arises after the projection of the U(32) world-volume gauge group of the 32 D9-branes. For the massless bosonic fields, restricting to the invariant sector gives the truncation (4.3) of the IIB supergravity theory, while the open strings give the SO(32) gauge fields.

5.3. D8 / Type I’

This case has also been discussed in the literature (see e.g. Ref. [16]). It can be obtained, via T-duality, from the previous case compactified on a circle. The T-duality takes the IIB theory to the IIA string theory and the operator Ω to

$$T\,\Omega\,T^{-1} = I_{10}\,\Omega\,, \tag{5.3}$$

where I_{10} is the space-time reversal of the compact x^{10} direction and Ω is the world-sheet parity reversal operator acting on the fundamental string of the IIA or IIB theory. Note that Ω is not a symmetry of the IIA string theory, but $I_{10}\,\Omega$ is, acting as

$$I_{10}\,\Omega\colon x^{10} \to -x^{10}\,, \quad \sigma \to \pi - \sigma\,. \tag{5.4}$$

The Type I′ theory is defined by orientifolding the IIA string by $I_{10}\,\Omega$. As the compact direction is divided out by the I_{10} operator, the Type I′ theory is defined over the manifold $\mathbb{R}^{8,1} \times S^1/\mathbb{Z}_2$. There are two O8-planes (each with RR charge -16) sitting at the two fixed points of the $\mathbb{Z}_2$, $x^{10} = 0,\pi$. To cancel the RR charge, it is necessary to add 16 D8-branes (and their 16 mirror images), and these can be located anywhere on the circle. The space $S^1/\mathbb{Z}_2$ can be viewed as the interval $0 \leqslant x^{10} \leqslant \pi$, so the construction gives the IIA string compactified on the interval with 16 D8-branes that can be located anywhere on the interval. This orientifold gives the truncation (4.7) of the massless bosonic sector of the Type IIA string theory.

The Type I string compactified on a circle gives a nine-dimensional theory with gauge group $\mathrm{SO}(32) \times \mathrm{U}(1) \times \mathrm{U}(1)$ (or $\mathrm{SO}(32) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ at the self-dual radius) and this is T-dual to the Type I′ theory with the same gauge group. However, Wilson lines can be introduced in the compactification of the Type I string and by choosing these the gauge group can be made to be $G_{17} \times \mathrm{U}(1)$ where G_{17} is any simply laced rank-17 gauge group, and for any choice there is a Type I′ dual with the same gauge group. The 16 parameters determining the Wilson lines in the Type I picture correspond to the 16 positions of the D8-branes in the Type I′ theory. The Type I momentum p^9 and D-string winding number in the x^{10} direction correspond in the Type I′ picture to the fundamental string winding number in the x^{10} direction and the D0-brane charge.

The relation between the positions of the D8-branes and the gauge symmetry has been discussed in [42] (an alternative description of the symmetry enhancement is given in [43]). If there are 8 coincident D8-branes (and their mirrors) at each O8-plane then the gauge group is $\mathrm{SO}(16) \times \mathrm{SO}(16) \times \mathrm{U}(1) \times \mathrm{U}(1)$. If one of the D8 branes is displaced from each of the O8-planes by a distance d, the generic configuration has $\mathrm{SO}(14) \times \mathrm{SO}(14) \times \mathrm{U}(1)^4$ symmetry, but if the displacement d takes a certain critical value, this is enhanced to $E_8 \times E_8 \times \mathrm{U}(1) \times \mathrm{U}(1)$, and then if the size of the $S^1/\mathbb{Z}_2$ is tuned to be twice this displacement, so that the two displaced branes are coincident, the group is enhanced to $E_8 \times E_8 \times SU(2) \times U(1)$. If all the D8-branes are at the same O8-plane, then the symmetry is generically $SO(32) \times U(1)^2$, and if the coupling constant is tuned to a critical value, this is enhanced to $SO(34) \times U(1)$. Bound states of D0-branes with O8-planes play a crucial role in these symmetry enhancements [42,44,45].

The D8-branes are domain walls that divide regions with different values of the mass parameter of the *massive* IIA string theory whose low-energy limit is the *massive* IIA supergravity of Romans [6].[10] In the $SO(16) \times SO(16)$ configuration the 8 D8-branes at each orientifold plane cancel the RR charge of the O8-plane locally and the bulk theory is the usual massless IIA string theory, while in all other cases the charges are only cancelled globally and the massive IIA string is the bulk theory in at least some of the regions between branes. At strong coupling the Type I′ string theory can be described in terms of (compactified) M-theory where an eleventh direction (with coordinate x^{11}) has been developed. The coupling constant $g_{I'}$ of the Type I′ theory is given by

$$g_{I'} = R_{11}^{3/2}, \tag{5.5}$$

where R_{11} is the compactification radius of the x^{11} coordinate. Only for the $SO(16) \times SO(16)$ configuration can the limit $R_{11} \to \infty$ be defined [4]; this will be discussed further in Section 6.

Note that although a D8-brane in $\mathbb{R}^{9,1}$ is a domain wall, when it is embedded in a manifold $\mathbb{R}^{8,1} \times S^1/\mathbb{Z}_2$ it has no non-compact transverse direction, and therefore effectively behaves like a space-time-filling brane. Reduction along the $S^1/\mathbb{Z}_2$-direction gives an RR–8 brane, filling the whole $\mathbb{R}^{8,1}$ space-time. These space-time-filling branes couple to the truncation (4.7) of the massless $D=9$ Type II background fields, and provide Chan–Paton factors to the open strings of the $D=9$ Type I string theory. For example, for the $SO(16) \times SO(16)$ configuration, the branes provide $SO(16) \times SO(16)$ Chan–Paton factors and give rise to the nine-dimensional theory with gauge group $SO(16) \times SO(16) \times U(1) \times U(1)$, which we will refer to as the $IO_{16,16}$ theory. The connection between this theory and the other $N=1$ theories is indicated in Fig. 4.

5.4. NS–9B / heterotic SO(32)

This case has been discussed in [1], and more extensively in [2], and arises from taking the strong coupling limit of the orientifold construction of the Type I theory. Acting with the $SL(2,\mathbb{Z})$ transformation S that takes weak to strong coupling takes the IIB theory to itself[11], and takes Ω to a symmetry

$$S\Omega S^{-1} = \tilde{\Omega}. \tag{5.6}$$

This acts as the perturbative symmetry

$$\tilde{\Omega} = (-1)^{F_L} \tag{5.7}$$

[10] The explicit D8-brane solution has been given in [4,5].

[11] The *S*-duality transformation takes the full non-perturbative IIB theory to itself, but takes the perturbative IIB string theory defined by perturbation theory in g to the dual string theory defined by perturbation theory in $\tilde{g} = 1/g$; the two dual string theories are equivalent, but arise as different ‘slices’ of the full non-perturbative theory.

on perturbative IIB string states (where F_L is the left-moving fermion number operator), and reverses the parity of the D-string world-sheet. The S-duality takes the 32 D9-branes to 32 NS-9B branes and as the S-dual (i.e. strong-coupling limit) of the Type I string is the SO(32) heterotic string, modding out the IIB string by $\tilde{\Omega}$ in the presence of 32 NS-9B branes should give the SO(32) heterotic string [1,2]. Projecting onto the $\tilde{\Omega}$-invariant sector projects the IIB supergravity multiplet onto the $N=1$ supergravity multiplet with the truncation (4.4) of the massless bosonic fields. In [2] it is shown that the gauge structure of the heterotic string emerges as follows. There are open 'D-strings' (arising from the IIB D-strings) which end on the 32 NS-9B branes and so carry SO(32) Chan–Paton factors. There are also open D-strings which have one end on the fundamental string and the other on the NS-9B brane, so that they tether an SO(32) charge to the fundamental string. Some of the segments of the 'fundamental' string will in fact be $(1,p)$ strings for various values of p, as follows from charge conservation. For example, attaching a D-string and an anti D-string to a fundamental string leads to a 'fundamental' string split into two segments by the junctions and which is a (1,0) string for one segment and a (1,1) string for the other segment. In the weak coupling limit, the tension of these D-strings becomes infinite, so that the SO(32) charges are pulled onto the heterotic string world-sheet give rise to the SO(32) current algebra and the gauge sector of the theory, and at the same time the $(1,p)$ segments of the string also contract, leaving a fundamental string; see Ref. [2] for further details.

Thus, the heterotic SO(32) string theory arises by modding out the Type IIB string theory by $\tilde{\Omega}$.

This can be generalized to the case of (p,q) 9-branes. The (p,q) 9-brane is obtained from the D9-brane via an $SL(2,\mathbb{Z})$ transformation $S_{p,q}$ and this takes Ω to

$$S_{p,q}\Omega S_{p,q}^{-1}=\tilde{\Omega}_{p,q} \tag{5.8}$$

and leads to a construction in which the IIB theory is modded out by $\tilde{\Omega}_{p,q}$ with 32 (p,q) 9-branes.

5.5. NS–9A / heterotic

This case has not been discussed in the literature, although it is related to known cases. Our starting point is the construction in Subsection 5.4 of the SO(32) heterotic string by modding out the IIB string with $\tilde{\Omega}$ in the presence of 32 NS–9B branes. Compactifying on a circle and performing a T-duality (without Wilson lines), the IIB string becomes the IIA string, the SO(32) heterotic string is mapped to itself, the NS-9B branes are mapped to NS–9A branes and $\tilde{\Omega}$ is mapped to

$$T\tilde{\Omega}T^{-1}=\hat{\Omega}, \tag{5.9}$$

where $\hat{\Omega}$ is the symmetry which acts on perturbative IIA string states through the perturbative symmetry

$$T(-1)^{F_L}T^{-1}=(-1)^{F_L^c}, \tag{5.10}$$

where F_L^c is the left-handed fermion number of the IIA string. This then gives a construction of the SO(32) heterotic string compactified on a circle from the Type IIA string compactified on a circle modded out by $\hat{\Omega}$ with 32 NS–9A branes. In particular, it yields the truncation (4.5) of the massless bosonic sector of the IIA string. The open D-strings of the IIB construction that give rise to the gauge sector of the heterotic strings become, after the *T*-duality, D0-branes and open D2-branes.

For the heterotic string compactified on a circle, one can obtain the gauge group $G_{17} \times \mathrm{U}(1)$ for any simply laced rank-17 group G_{17} by adding Wilson lines, and the question arises as to what these Wilson lines correspond to in the dual picture; they cannot correspond to positions of the NS–9A branes as they are space-filling (and in the perturbative IIA string, there is no extra dimension in which to displace them). In the IIB picture, the NS–9B brane has world-volume vector fields $c^{(1)}$ taking values in the SO(32) Lie algebra, and the Wilson lines correspond to giving expectation values to the component $c^{(1)}_\sigma$ in the compact direction which take values in a Cartan sub-algebra of SO(32). The NS-9A brane world-volume theory is, after fixing the Killing symmetry $\delta X^\mu = \alpha k^\mu$, a nine-dimensional super-Yang–Mills multiplet (with a vector field and Lie algebra-valued scalar $c^{(0)}$) coupled to a supergravity background, with a non-linear DBI action. After the *T*-duality transformation (4.27), the Wilson lines correspond to giving expectation values to the scalars $c^{(0)}$ that arise in the NS–9A brane world-volume theory, where the expectation values are in a Cartan subalgebra.

Dimensionally reducing to 9 dimensions, this gives a construction of the nine-dimensional heterotic string theory from the nine-dimensional Type II string with 32 of the space-time-filling NS-8 branes discussed in Subsection 4.2.2. These branes couple to the truncation (4.8) of the $D = 9$ Type II massless background fields.

In Section 6 we will lift this construction to M-theory, and show that the expectation values of the scalars $c^{(0)}$ correspond to displacing the M9-branes in an 11th dimension which is an $S^1/\mathbb{Z}_2$.

5.6. Orbifolding with $(-1)^{F_L}$

Orbifolding the perturbative IIB string with $(-1)^{F_L}$, in the absence of any 9-branes, gives the IIA string [34]. In the untwisted sector, all R–R and R–NS states of the IIB string are projected out, and the twisted sector introduces states that make up the R–R and R–NS sectors of the IIA string. In the massless sector, the IIB supergravity can be decomposed into an $N = 1$ supergravity multiplet and a right-handed $N = 1$ gravitino multiplet, and in the untwisted sector this gravitino multiplet is projected out. The twisted sector introduces a left-handed $N = 1$ gravitino multiplet, which combines with the $N = 1$ supergravity multiplet to give the IIA supergravity multiplet. The orbifolding of the IIB string to obtain the IIA string is a conformal field theory construction, and it is interesting to ask whether it can be extended to a non-perturbative construction that works for finite coupling or strong coupling.

In Subsection 5.4 we considered modding out the IIB string by the same symmetry, $(-1)^{F_L}$, but in a background with 32 NS-9B branes, and this gave the SO(32) heterotic string. Thus it appears that modding out by the same symmetry can give different results, depending on whether or not 9-branes are added. The perturbative symmetry $(-1)^{F_L}$ extends to the symmetry $\tilde{\Omega}$ of the full IIB theory and at strong coupling becomes the world-sheet parity operator Ω of the dual weakly coupled IIB string theory, while the NS–9B branes become the D9-branes. Modding out by $\tilde{\Omega}$ with 32 NS-9B branes to give the heterotic string is then *S*-dual to the modding out by Ω with 32 D9-branes to give the Type I string. Suppose that the construction without 9-branes of the IIA from the IIB extends to the full non-perturbative theory. Then applying an S-duality would give the modding out of the IIB string by Ω, without any D9-branes, to give the strong-coupling limit of the IIA string, which is M-theory. The untwisted sector gives an $N=1$ ten-dimensional supergravity multiplet, and the absence of D9-branes means that there are no open strings. The twisted sector must provide the extra states needed to give M-theory.

Thus modding out the IIB string by $\tilde{\Omega}$ gives different results, depending on whether or not NS–9B branes are added. With 32 NS–9B branes, this gives the SO(32) heterotic string and, as we shall see in more detail in Section 6, extends to the full non-perturbative theory. Without any 9-branes, no gauge group is introduced and this takes the perturbative IIB string to the perturbative IIA string. Either this does not extend to the full non-perturbative theory, in which case it would be interesting to understand what goes wrong, or it does extend, in which case it would have an S-dual construction of M-theory on a large circle of radius R from orientifolding the IIB string with Ω at small coupling $g_B = l_s/R$, and this would be interesting to understand directly.

Similarly, the IIA string can be orbifolded by $(-1)^{F_L^c}$ to give the IIB string. If this can be extrapolated to strong coupling, the S-dual construction would be as follows. The IIA string becomes M-theory compactified on a large circle of radius $R = l_s g_A$ at strong string coupling g_A, and the symmetry $(-1)^{F_L^c}$ extends to the M-theory symmetry used in the Hořava–Witten construction [3] (we refer to this symmetry as $\Omega_M I$ in Section6). Then modding out M-theory with this symmetry would give a Hořava–Witten-type construction, but without any gauge sectors introduced at the fixed points of the symmetry, and would give the weakly coupled IIB string with coupling $g_B = l_s/R$. Again, it would be interesting to investigate this further.

6. Relation with M-theory

6.1. The Hořava— Witten picture

In this section, we will show how the constructions of string-theories with 16 supersymmetries of the last section can all be lifted to M-theory, and that they all arise as particular limits of the Hořava–Witten picture of M-theory compactified on $\mathbb{R}^{8,1} \times S^1 \times S^1/\mathbb{Z}_2$ [3]. This will give an M-theoretic justification of some of the assumptions made in Section 5, allow us to discuss the non-perturbative generalizations of the constructions, and give insights into the M-theoretic origin of D8-branes.

Consider first M-theory compactified on a 2-torus with radii R_{10}, R_{11}. When one of the radii is large and the other small, the theory is described by a weakly coupled IIA string theory; for example, if R_{11} is small, this is a IIA string theory with coupling constant $g_{\mathrm{IIA}} = (R_{11}/l_p)^{3/2}$ compactified on a circle of radius R_{10}. If both radii are small, then the theory is IIB string theory with coupling constant $g_{\mathrm{IIB}} = R_{11}/R_{10}$ compactified on a circle of radius $R_{\mathrm{IIB}} = l_p^3/R_{10}R_{11}$. The limit in which $R_{10}, R_{11} \to 0$ gives the IIB string in 10 dimensions [46,47]. The moduli space is depicted in Fig. 6, and should be identified under the reflection that interchanges R_{10} with R_{11}.

The conjectured $\mathbb{Z}_2$ symmetry of M-theory used in the Hořava–Witten construction is $I_{10}\,\Omega_M$ where I_{10} takes $x^{10} \to -x^{10}$ and Ω_M reverses the orientation of the M2-brane

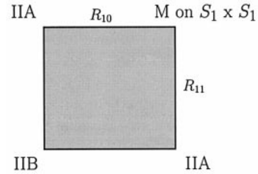

Fig. 6. M theory compactified on a T^2 and the various string theory limits.

and the M5-brane and acts in the supergravity as $C \to -C$ where C is the 3-form potential. For M-theory compactified on a 2-torus with radii R_{10}, R_{11}, this symmetry reduces to the various string theory symmetries considered in the last section in the string theory limits. If R_{11} is large and R_{10} is small, $I_{10}\,\Omega_M$ acts as the symmetry $\hat{\Omega}$ of the IIA string, acting as $(-1)^{F_L^c}$ in the perturbative theory. If R_{10} is large and R_{11} is small, on the other hand, $I_{10}\,\Omega_M$ acts as the symmetry $I_{10}\,\Omega$ of the IIA string compactified on a circle of radius R_{10}, where Ω is the IIA string world-sheet parity operator. If both radii are small and the IIB string is weakly coupled, so that $g_{\mathrm{IIB}} = R_{11}/R_{10}$ is small, $I_{10}\,\Omega_M$ acts as Ω, the IIB string world-sheet parity operator, while for strong coupling, the theory is the dual IIB string theory with coupling $\tilde{g}_{\mathrm{IIB}} = R_{10}/R_{11}$ and $I_{10}\,\Omega_M$ is the IIB string symmetry $\tilde{\Omega}$, which acts as $(-1)^{F_L}$ in the perturbative theory. This is depicted in Fig. 7.

The M-theory construction gives a non-perturbative picture of both the IIA and IIB theories, and shows that the symmetries discussed in the last section all extend to the same symmetry of the non-perturbative theory, namely the Hořava–Witten symmetry. In particular, we see that $\tilde{\Omega}$ is indeed the strong coupling limit of Ω.

Consider M-theory on T^2 modded out by $I_{10}\,\Omega_M$. The circle in the x^{10} direction becomes the interval $S^1/\mathbb{Z}_2$ and the torus is replaced by a cylinder. It was argued in [3] that in the limit $R_{11} \to \infty$ and $R_{10} \to 0$ this gives the $E_8 \times E_8$ heterotic string, in the limit $R_{10} \to \infty$ and $R_{11} \to 0$ this gives the Type I′ string and in the limit in which $R_{11} \to 0$ and $R_{10} \to 0$, this gives the Type I string with coupling $g_I = R_{11}/R_{10}$ if this is small, and the SO(32) heterotic string with coupling $\tilde{g}_{\mathrm{het}} = R_{10}/R_{11}$ if this is small. These are depicted in Fig. 8.

Comparing with the above, the Hořava–Witten construction reduces to the string theory constructions of the last section in each of the string theory corners of the moduli space. In the corner in which R_{10} is large and R_{11} is small, the Hořava–Witten construction reduces to orientifolding the IIA string with $I_{10}\,\Omega$ to obtain the Type I′ string, while in the corner in which R_{10} is large and R_{11} is small, the Hořava–Witten construction reduces to modding out the IIA string by $\hat{\Omega} \sim (-1)^{F_L^c}$ to obtain the

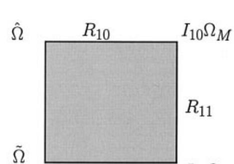

Fig. 7. The Hořava–Witten square, with the limiting forms of the Hořava–Witten symmetry arising in the various string theory limits marked.

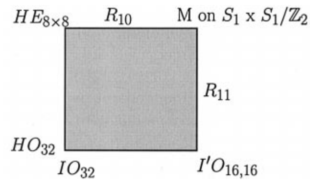

Fig. 8. M-theory on a T^2 modded out with $I_{10}\,\Omega_M$ and the various string theory limits.

heterotic string. If both R_{10} and R_{11} are small, then if R_{11}/R_{10} is small the construction gives the orientifold of the IIB string with Ω to get the Type I string, while if it is large it gives the IIB string modded out by $\tilde{\Omega} \sim (-1)^{F_L}$ to give the SO(32) heterotic string. However, in each of these constructions the 9-branes play a vital role and we will now examine the individual cases in more detail.

6.2. R_{10} and R_{11} both small

For M-theory on T^2, the limit in which both radii tend to zero gives the IIB string with coupling constant given by the limiting value of $g_{\mathrm{IIB}} = R_{11}/R_{10}$ [46,47]. Modding out by $\Omega_M I_{10}$ gives M-theory on a cylinder and we shall consider the limits in which both R_{10} and R_{11} tend to zero; if R_{11}/R_{10} is small, this gives the Type I string with coupling $g_I = R_{11}/R_{10}$, while if it is large it gives the SO(32) heterotic string with $g_h = R_{10}/R_{11}$ [3].

For M-theory on T^2, the symmetry $\Omega_M I_{10}$ reduces to perturbative symmetries of the IIA or IIB theories in the corners of the moduli space square in Fig. 6. In particular, taking the limit in which $R_{10} \to 0, R_{11} \to 0$ with R_{11}/R_{10} small, $\Omega_M I_{10}$ becomes the symmetry Ω of the IIB theory with coupling constant $g_{\mathrm{IIB}} = R_{11}/R_{10}$ that is the fundamental string world-sheet parity symmetry, while taking the limit in which $R_{10} \to 0, R_{11} \to 0$ with R_{11}/R_{10} large, $\Omega_M I_{10}$ becomes the symmetry $\tilde{\Omega}$ of the dual IIB theory with coupling constant $\tilde{g}_{\mathrm{IIB}} = R_{10}/R_{11}$ which acts in the perturbative theory as $(-1)^{F_L}$. Thus if $\Omega_M I_{10}$ is a symmetry of M-theory, then the IIB string theory has a non-perturbative symmetry which is the limiting form of $\Omega_M I_{10}$ in the IIB limit, and which acts as Ω at weak IIB coupling and as $\tilde{\Omega}$ in the S-dual IIB string. Then M-theory modded out by $\Omega_M I_{10}$ becomes, in the limit, the (non-perturbative) IIB theory modded out by the $\tilde{\Omega}/\Omega$ symmetry to give the SO(32) heterotic/Type I theory. If R_{11}/R_{10} is small, this is the orientifold construction of the Type I theory and if it is large, this is the construction of the SO(32) heterotic string of Refs. [1,2].

6.3. R_{10} large, R_{11} small

This limit gives the Type I′ theory, with gauge group determined by the positions of the D8-branes, as discussed in Subsection 5.3, and coupling constant given by (5.5). For general D8-brane configurations, the bulk supergravity theory between the D8-branes is given by the *massive* IIA supergravity, and the limit $R_{10} \to \infty$ cannot be taken, as the dilaton becomes infinite at some value(s) of x^{10} as R_{10} is increased to a finite critical value [4].

However, in the special case in which 8 D8-branes positioned on top of one orientifold and 8 on top of the other, corresponding to the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ vacuum, there is no dilaton gradient and the limit $R_{10} \to \infty$ can be taken [4]. In this limit, bound states of D0-branes with the O8-planes become massless and lead to an enhancement of the gauge symmetry to $E_8 \times E_8$ [42,44,45]. This can be seen by first separating one D8-brane a critical distance from each O8-plane to get the Type I′ theory with $E_8 \times E_8 \times \mathrm{U}(1) \times \mathrm{U}(1)$ symmetry [42] and then taking the limit in which $R_{10} \to \infty$, in which the critical separation tends to zero so that all D8-branes coincide with the O8-planes in the limit. At the orientifold planes, each D8-brane contributes one unit to the cosmological constant and the total contribution is cancelled by the contribution of the O8-plane. The bulk theory in the region between the O8-planes is given by the *massless* IIA superstring, which has an M-theory origin and so the decompactification limit can be understood conventionally in terms of M-theory.

For finite R_{10} we know that the theory has 16 moduli corresponding to the positions of the D8-branes, at least in the weak-coupling limit $R_{11} \to 0$, and there should be an interpretation for them at finite R_{11}. If there is such an interpretation, it should correspond to the positions (at particular values of x^{10}) of the M-theory branes that give rise to the D8-branes; these are the M9-branes [1,9], wrapped around the x^{11} circle. However, the bulk theory between the D8-branes is the massive string theory and so the M9-branes should arise in the M-theory origin of the massive string theory. The massive deformation of 11-dimensional supergravity proposed in [8] involves an explicit Killing vector (which is here $\partial/\partial x^{11}$), and the IIA configurations considered here can be lifted to a solution of this theory consisting of a system of 16 M9-branes at various positions between two planes at the end points of the interval, and the positions can be arbitrary provided R_{10}, R_{11} are finite. We will refer to the fixed point planes that arise from oxidizing the O8-planes as M-theory O9-planes. The fundamental strings ending on D8-branes are oxidized to M2-branes ending on the M9-branes.

6.4. R_{11} large, R_{10} small

This is the construction of the heterotic string, compactified on a circle of radius R_{11}, from the IIA string (on a circle) with 32 NS-9A branes by modding out by $\hat{\Omega} \sim (-1)^{F_L^c}$. The gauge group is determined by the heterotic string Wilson lines, which arise here as the 16 moduli given by the expectation values of the scalars $c^{(0)}$ in a Cartan subalgebra of SO(32). This is related by *T*-duality in the x^{11} direction to the SO(32) heterotic string constructed from the IIB string, where the gauge sector arose from open D-strings with one end on the fundamental heterotic string and the other on a D9-brane, which collapse to zero length at weak coupling. After the *T*-duality, the open D-strings become D0-branes and open D2-branes ending on the NS-9A branes. The D2-branes are cylindrical, wrapping the circular x^{11} direction and with one end on the fundamental string in the nine-dimensional space orthogonal to x^{11} and the other on an NS-9A brane, and the length of the cylinder tends to zero at weak coupling. It is these D0- and D2-branes that are responsible for the gauge sector at weak coupling.

At finite heterotic coupling, the non-perturbative theory is M-theory on a cylinder, with an eleventh dimension which is a finite interval of length $R_{10}\pi$. The NS-9A branes oxidize to 16 M9-branes, moving between two O9-planes with the 16 moduli now giving the positions of the M9-branes on the interval (or to 16 M9-branes plus their mirror images on the circle). The non-BPS D2-branes ending on NS-9A branes become cylindrical M2-branes ending on M9-branes and orthogonal to the x^{10} direction and the fundamental IIA strings become M2-branes that are tangential to the x^{10} direction, and again end on M9-branes. The heterotic string in 10 dimensions results from the decompactification limit $R_{11} \to \infty$ with all the M9-branes at one O9-plane, in which case the SO(32) heterotic string arises, or with 8 M9-branes at each of the O9-planes, giving the $E_8 \times E_8$ heterotic string.

In the limit $R_{11} \to \infty$ with 8 M9-branes at each O9-plane, the M9-branes can no longer move. This is in accord with the fact that the tension of a single M9-brane is proportional to $(R_{11})^3$ (see Fig. 2) so that they become infinitely massive in the limit, and that their world-volume theory has no scalar and so no collective coordinate for translations in the transverse direction.

6.5. M9-branes and the Hořava–Witten construction

M-theory compactified on a line interval in the x^{10} direction of length $R_{10}\pi$ gives a non-perturbative formulation of the $E_8 \times E_8$ heterotic string and requires an E_8 super-Yang–Mills theory on each end-of-the-world plane [3]. If this is further compactified on a circle in the x^{11} direction, then we have learned that each end-of-the-world plane consists of an O9-plane and 8 M9-branes which can move away from the O9-plane if R_{11} is finite. This is because we know that as $R_{11} \to 0$ this gives the IIA theory on an interval, and the two end-of-the-world planes are each O8-planes with 8 D8-branes that can move away from the O8-plane, and for finite R_{11} this must oxidize to M9-branes wrapped on the x^{11} circle moving between O9-planes.

The world-volume theory of the 8 M9-branes at an O9-plane has the E_8 super-Yang–Mills theory on $R^{8,1} \times S^1$ as its low-energy limit and moving the M9-branes in the x^{10} direction breaks the gauge group, as the M9-brane positions are moduli that correspond to Wilson lines in the x^{11} direction. The compact x^{11} direction plays a special role in the theory, which has explicit dependence on the Killing vector $k = \partial/\partial x^{11}$. The theory between the M9-branes is the massive version of M-theory depending explicitly on k [8], and the M9-brane world-volume effective action also depends explicitly on k. Dependence on k and on the mass parameter drops out if 8 M9-branes are placed at each O9-plane, and it is only in this configuration that the limit $R_{11} \to \infty$ can be taken, to give the Hořava–Witten configuration.

7. Conclusions

In this paper we have discussed the space-time-filling branes of Type IIA and Type IIB string theories, which are connected by a web of duality transformations and/or reductions (see Fig. 3). In the first part of this work we have constructed their world-volume effective actions, together with their reductions to nine dimensions.

In the second part of this work we investigated the roles of D9, NS-9A and NS-9B branes in string theory. Each of these lead to inconsistencies when introduced in a Type II string theory, but in each case 16 of these branes plus 16 mirrors can be added to the Type II string theory modded out by the appropriate $\mathbb{Z}_2$ symmetry to obtain a construction of string theories with 16 supercharges: the IIB theory modded out by Ω with 32 D9-branes gives the Type I string, the IIA string modded out by $I_{10}\Omega$ with

16(+16) D8 branes gives the Type I′ string, the IIB string modded out by $\tilde{\Omega}$ with 32 NS-9B branes gives the SO(32) heterotic string and the IIA string compactified on a circle and modded out by $\hat{\Omega}$ with 32 NS-9A branes gives the heterotic string compactified on a circle. In the latter case, the large radius limit can be taken only in the $O(16)\times O(16)$ vacuum, to obtain the $E_8\times E_8$ heterotic string from the IIA string. These constructions are related to one another by dualities (see Figs. 4 and 5).

Moreover, these four constructions each arise as a different limit of the Hořava–Witten construction, and so are unified in M-theory. This gives the origin of the dualities relating these constructions. These connections also show that, when compactified on a circle of radius R, the end-of-the-world-branes of Hořava and Witten each become a combination of an O9-plane and 8 M9-branes, all wrapped on the circle, and the 8 M9-branes can be moved to arbitrary points on the interval $S^1/\mathbb{Z}_2$. In the small radius limit $R\to 0$, these become the D8-branes and O8-planes of the Type I′ string theory. However, the large radius limit $R\to\infty$ does not exist unless 8 M9-branes are at each O9-plane, in which case the Hořava–Witten picture is recovered.

Our work implies that the group structure of the $E_8\times E_8$ string theory can be described by D0-branes and open non-BPS D2-branes which in the strong coupling limit oxidize to open M2-branes ending on the M9-branes. These are the analogs of the D1-branes that provide the group structure in the case of the SO(32) heterotic string theory.

Finally, we have found that if the orbifolding of the IIB (IIA) string by $(-1)^{F_L}$ to obtain the IIA (IIB) string were to extend to the full non-perturbative theory, it would have some peculiar features: for example, the orientifolding of the IIB string *without* introducing D9-branes would lead to M-theory. It would be of considerable interest to investigate whether or not this construction does extend to the non-perturbative theory, and if so, how its peculiar features can be understood directly.

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