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Multiple intersections of D-branes and M-branes

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Abstract

We give a classification of all multiple intersections of D-branes in ten dimensions and M-branes in eleven dimensions that correspond to threshold BPS bound states. The residual supersymmetry of these composite branes is determined. By dimensional reduction composite p-branes in lower dimensions can be constructed. We emphasize in dimensions $D \geqslant 2$ those solutions which involve a single scalar and depend on a single harmonic function. For these extremal branes we obtain the strength of the coupling between the scalar and the gauge field. In particular, we give a D-brane and M-brane interpretation of extreme p-branes in two, three and four dimensions.

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1. Introduction

Classical solutions of the low-energy effective supergravity action have played a crucial role in the unification of string theories. This unification, in terms of the conjectured M-theory [1,2], has led to a renewed interest in $D = 11$ supergravity. Since $D = 11$ supergravity is the low-energy limit of M-theory, its classical solutions, and their descendants which can be obtained in lower dimensions by dimensional reduction, play a particularly important role. The basic eleven-dimensional extended objects (Mp-branes or, shortly, M-branes) are the M2-brane [3] and the M5-brane [4].

In ten dimensions a class of solutions of Type IIA/IIB string theory, satisfying Dirichlet boundary conditions in certain directions, has received much attention [5,6]. These solutions are called Dirichlet p-branes or shortly Dp-branes (or just D-branes). The charge of the Dp-branes is carried by a Ramond–Ramond gauge field. Dp-branes exist for all values of $0 \leqslant p \leqslant 9$ and are all related by T-duality [6–10].

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Both the M-brane and the D-brane solutions are characterized by a function H which depends only on the coordinates transverse to the brane, and is harmonic on this transverse space. This suggests that the M-brane and D-brane solutions are related, and indeed one finds that direct dimensional reduction of the M2-brane and double-dimensional reduction of the M5-brane in $D = 11$ leads to the D2- and D4-branes in IIA supergravity.

It is natural to consider solutions that correspond to intersections of D-branes and M-branes. This was first done in [11] where certain solutions occurring in [4] were interpreted as intersections of M-branes. Soon after, these results were generalized both for M-branes [12–17] as well as D-branes [12,18,14,20]. In the latter case the solutions correspond to bound states of D-branes [6,19].

Under certain conditions the force between two M-branes or D-branes vanishes [21], and composite configurations of branes can be static solutions to the equations of motion. The conditions for the existence of such configurations have been formulated. In particular, the so-called harmonic function rule [12] prescribes how products of powers of the harmonic functions H_i of the N intersecting branes must occur in the composite solution. In particular, it implies that if one of the H_i is set equal to one, a solution with $N - 1$ intersecting branes is obtained.

For a more detailed analysis of configurations of two branes, we assume that the powers of harmonic functions are as stated by the harmonic function rule, and then split the coordinates in three parts: the overall world-volume coordinates (d), which are common to the two branes, the overall transverse coordinates (t), and the remainder, which are called relative transverse (n), and are transverse to only one of the two branes. Three kinds of intersections of a p_1- and a p_2-brane are possible, with the following conditions, valid for D-branes in $D = 10$ and the basic M-branes in $D = 11$, on H_1 and H_2 [6,12,14,21,18]:

(1) Both H_i depend only on the t overall transverse coordinates. Then in $D = 10$ we must have $n = 4$ (i.e. 4 relative transverse directions), while in $D = 11$ the only possibilities are $(0|2,2)$ and $(3|5,5)$ $(n = 4)$, and $(1|2,5)$ $(n = 5)$. [1]

(2) One H_i depends on the overall transverse coordinates, the other on the relative coordinates. In this case the conditions on n are as in case (1).

(3) Both H_i depend on the relative coordinates. Then in $D = 10$ and $D = 11$ we must have $n = 8$, which in $D = 11$ can only be realized as $(1|5,5)$ [14].

In case more than two branes intersect, the above rules must apply for each pair of branes in the composite system. As we will see, this basic requirement enormously restricts the number of allowed multiple intersections.

The aim of this paper is to give a systematic and complete classification of intersecting branes in ten and eleven dimensions *satisfying the above conditions*. This paper will mainly concentrate on the intersecting branes that satisfy condition (1). The ones that satisfy conditions (2) or (3) are separately discussed in an Appendix, since their status

[1] We denote the intersection of a p_1- and a p_2-brane over a common q-brane with $d = q + 1$ by $(q|p_1, p_2)$. We do not include the case $n = 0$ $(p_1 = p_2)$, for which the intersection is described by the sum of H_1 and H_2 (multiple branes of the same type with different locations).

in string theory is less clear.

Note that the harmonic function rule implies restrictions on the form of the metric of the solution, and that therefore our analysis does not exclude the existence of other (static) multiple brane configurations. In particular, we only consider intersections where each participating brane corresponds to an independent harmonic function in the solution. Such solutions correspond to threshold BPS bound states, i.e. they satisfy the no force condition. To summarize, in this paper we only consider solutions corresponding to threshold BPS bound states. We do *not* consider the following solutions (see, however, Section 5):

- $D = 10$ intersections that involve NS–NS strings, five-branes and/or their T-duals. Neither do we consider in D=11 intersections that involve a gravitational wave (boosted M-branes [22]) and/or its magnetic partner.
- Solutions corresponding to non-threshold BPS bound states such as the $D = 10$ D-brane bound states with $n = 2$ or 6 [6], the $D = 10$ (q_1, q_2) string solutions of [23] and the $D = 10$ ($D = 11$) solution given in Ref. [24] (Ref. [25]) that interpolates between a 2-brane and a 5-brane.

Our main conclusions are that under condition (1) there are three inequivalent ways of eight intersecting p-branes, both in $D = 10$ and $D = 11$. If intersections with $n = 8$ are allowed as well, a ninth brane can be added to these configurations.

A single M- or D-brane preserves half the supersymmetry of the corresponding supergravity theory (which has 32 real supersymmetry generators). As a general rule, a configuration of N intersecting branes preserves *at least* $1/2^N$ of the maximal supersymmetry [6,12]. In our analysis we will see when and how the "at least" becomes relevant: in some cases an additional brane can be added to a composite system without additional breaking of supersymmetry ([19,13,14]). Our maximal intersecting configurations with eight branes preserve $1/32$ or $1/16$ of the maximal supersymmetry, for the intersection with nine branes this is $1/32$.

The conditions (1)–(3) follow from the gauge-field equation of motion, and so are *a priori* necessary conditions. The Einstein equation, and, in $D = 10$, the dilaton equation of motion, need to be checked. In all cases considered in this paper we find that the full equations of motion are satisfied by multiple intersections satisfying the above conditions. It is an interesting fact that all multiple configurations based on (1)–(3) preserve at least $1/32$ of the maximal supersymmetry. The reason must be that the condition of vanishing force between branes is implied by supersymmetry. If this is true, then on the one hand it should be possible to derive the conditions (1)–(3) from the requirement of supersymmetry, while on the other hand preservation of supersymmetry should imply the complete equations of motion. However, we have not proved this.

The organisation of this paper is as follows. In the body of the paper we will extensively discuss case (1) of the above conditions: $n = 4, 5$ and dependence on the overall transverse coordinates. We will do this for $D = 10$ in Section 2, and for $D = 11$ in Section 3. In Section 4 we discuss some aspects of the reduction of our composite solutions to lower dimensions, with emphasis on extreme p-branes in two, three and four dimensions. Further remarks, in particular on the inclusion of NS–NS branes and/or non-threshold BPS bound states, are made in Section 5. Explicit representations of the solutions with maximum numbers of intersecting branes are given in Appendices A and B for $D = 10$ and $D = 11$, respectively. In Appendix C we discuss the additional possibilities which arise from the cases (2), (3) of the above conditions.

2. Intersections of D-branes in ten dimensions

The single elementary Dirichlet p-brane solution in the string frame in ten dimensions is given by the following metric, dilaton and gauge field: [2]

$$\begin{aligned} ds^2 &= H_p^{-1/2}\, ds^2_{p+1} - H_p^{1/2}\, ds^2_{9-p}, \\ e^{2\phi} &= (H_p)^{-\frac{1}{2}(p-3)}, \qquad F_{01\dots pi} = \partial_i H_p^{-1}, \end{aligned} \tag{1}$$

where H_p is a harmonic function which depends on the $9-p$ transverse coordinates. [3] The line element ds^2_{p+1} contains the time coordinate $t = x^0$.

T-duality transforms a p-brane into a $(p+1)$-brane if H_p is independent of one of the transverse coordinates, say x. The duality rule for the metric is given by

$$\tilde{g}_{xx} = 1/g_{xx}, \tag{2}$$

so that a transverse direction gets dualized to a world-volume direction. The other rules of T-duality can be found in [27,10]. It is in principle possible to perform T-duality in the opposite direction and change a world-volume coordinate into a transverse one. However, this is a "dangerous" T-duality, since we have to suppose that the harmonic function after dualization depends on the direction in which we have dualized and it is not guaranteed that the result is still a solution of the equations of motion. In this paper we will perform only "safe" T-duality transformations.

It is convenient to represent every coordinate that corresponds to a world-volume direction by $\times$ and every direction transverse to the brane by $-$. We thus obtain the following representation of the metric of a D-brane solution:

$$ds^2 = \underbrace{\times\ \times\ \dots\ \times}_{p+1}\ \overbrace{-\ -\ \dots\ -}^{9-p}. \tag{3}$$

It is easy to see that acting with a "safe" T-duality on this metric, a $-$ changes into a $\times$.

[2] We use here the basis of RR gauge fields that occur naturally in the Wess–Zumino terms of the D-brane actions [26]. In this basis the Chern–Simons terms in the curvatures for the RR gauge fields always contain an NS–NS 3-form curvature. These CS terms vanish for the class of solutions we are considering in this paper.

[3] So far we used a notation where the subscript i on the harmonic function indicated the ith intersecting D-brane. Sometimes, however, like here, it is more convenient to use a notation where the subscript i on the harmonic function indicates the number p of the corresponding p-brane. It should be clear from the context which of the notations is used.

The ansatz we use to describe intersecting D-branes follows from the harmonic function rule: the metric is diagonal and every dx^2 gets multiplied by a factor which is a product of the powers of the harmonic functions involved in the intersection. D-branes that have the coordinate x as a world-volume direction contribute a factor[4] $H^{-1/2}$ and the ones that have x as a transverse direction contribute $H^{1/2}$. The dilaton is given by the product of the dilaton expressions for each separate brane and we have a gauge field of the form given in (1) for every D-brane in the intersection.

As an example we give the expression for the metric and dilaton of a $(p+r)$-brane intersecting with a $(p+s)$-brane over a p-brane, i.e. a $(p|p+r, p+s)$ configuration:

$$\begin{aligned} ds^2 &= \left(H_{p+r}H_{p+s}\right)^{-1/2} ds^2_{p+1} - \left(\frac{H_{p+r}}{H_{p+s}}\right)^{1/2} ds^2_s \\ &\quad - \left(\frac{H_{p+s}}{H_{p+r}}\right)^{1/2} ds^2_r - \left(H_{p+r}H_{p+s}\right)^{1/2} ds^2_{9-p-r-s}\,, \\ e^{2\phi} &= e^{-\frac{1}{2}(p+r-3)} e^{-\frac{1}{2}(p+s-3)}\,. \end{aligned} \tag{4}$$

In this case $d = p+1$, $n = r+s$ and $t = 9-p-r-s$. In this section we will only consider intersections that satisfy condition (1) of the Introduction, the other two cases will be discussed in Appendix C. For such intersections, the two harmonic functions depend on the t overall transverse coordinates, and we must have $n = 4$ in order that the intersecting configuration is a solution to the equations of motion. The expressions for the gauge fields for this case are given by

$$F_{0\ldots p1\ldots ri} = \partial_i H^{-1}_{p+r}\,, \qquad F_{0\ldots p1\ldots si} = \partial_i H^{-1}_{p+s}\,. \tag{5}$$

Using the notation of (3) we can rewrite the general $N = 2$ intersection given in (4). For example, a possible two-intersection is given by

$$ds^2 = \begin{cases} \times \;\times\;\times\;\times\;\times\; -\;-\;-\;-\;- & : H_1 \\ \times\; \underbrace{-\;-\;-\;-}_{x_n}\; \underbrace{-\;-\;-\;-\;-}_{x_t} & : H_2. \end{cases} \tag{6}$$

This is a $(0|0,4)$-solution, i.e. a 0-brane lying in a 4-brane, with $d = 1$, $n = 4$, $t = 5$. The harmonic functions H_i both depend on the coordinates x_t.

In an intersection such as (6) T-duality acts on a column, changing every $\times$ in a $-$ and vice versa. If we act with T-duality on the relative transverse directions and on the overall transverse direction we recover the other T-dual solutions with $n = 4$ relative transverse directions given in [18,14]. Clearly n, the number of relative transverse directions, is a T-invariant quantity. It is also clear that (6) represents the complete $N = 2$ $n = 4$ family, since all other members can be obtained from it by "safe" T-duality transformations. For the same reason we can limit ourselves, for arbitrary N, to those intersections with $d = 1$ (intersections over a 0-brane).

[4] These factors are given for the string frame.

When adding further D-branes to (6), in the form of horizontal lines with ×'s and −'s, we need to have $n = 4$ (four different entries of × or −) for every pair in the intersection. To streamline the construction, it is useful to characterize an intersection by the contents of the columns (components of the metric) corresponding to the relative transverse coordinates. For an N-intersection we can, by using T-duality, bring each column in a form such that no more than $[N/2]$ ×'s are present. Such columns are the "building blocks" of the intersection. Given N, there are $\binom{N}{k}$ building blocks with k ×'s.

In an N-intersection ($N \geqslant 2$) there are $\binom{N}{2}$ intersecting pairs. The total number of differences between × and − in the N-intersection is therefore equal to $4\binom{N}{2}$. A column with k ×'s contributes $k(N-k)$ differences. Let n_k be the number of building blocks with k ×'s. Then we must have

$$\sum_{k=1}^{[N/2]} k(N-k)n_k = 4\binom{N}{2}, \tag{7}$$

with $\sum_k n_k < 9$. Given N, this is an equation for the n_k.

Let us give a few examples. For $N = 2$ there is only one type of building block with $k = 1$. Eq. (7) for this case reduces to the equation $n_1 = 4$ which is condition (1) of the Introduction. For $N = 3$ there is again only one type of building block with $k = 1$ and we find $n_1 = 6$. For $N = 4$, there are two types of building blocks, with $k = 1$ and with $k = 2$. Eq. (7) reduces to $3n_1 + 4n_2 = 24$ which has three solutions namely $(n_1, n_2) = (8,0), (4,3)$ and $(0,6)$. Finally, for $N = 5$ there are again two types of building blocks with $k = 1,2$ and we find $4n_1 + 6n_2 = 40$ leading to two solutions given by $(n_1, n_2) = (4,4)$ and $(1,6)$. From now on, we will use the numbers n_k to label solutions. Note that the remaining T-duality and the interchange of columns and/or rows in the representation of the metric (corresponding to a relabeling of the space-time coordinates or the intersecting branes), do not change the n_k.

Clearly, (7) is only a necessary condition for the existence of a solution. Given a set of n_k allowed by (7), it is not clear that one can actually realize such a solution in terms of the available building blocks and consistent with condition (1) of the Introduction. In practice, we have found that such a realization is possible only in a small number of cases. In the actual construction it is not always useful to use only building blocks with $k \leqslant [N/2]$. Instead, it is convenient to start the N-intersection with a 0-brane. Since $n = 4$, all other branes in the intersection must then be 4-branes. We find that for $N = 2, \ldots, 5$ all configurations that satisfy the consistency condition (7) actually satisfy the stronger condition (1) of the Introduction for each intersection. However, for $N = 6$, the $(n_1, n_2, n_3) = (3,0,5)$ configuration does not survive. We have repeated this analysis until we reach $N = 8$ with three different configurations. At this point, our procedure stops. Although (7) has solutions for $N = 9$, it turns out to be impossible to add a ninth brane in such a way that it has $n = 4$ relative transverse directions with all other eight D-branes. An overview of the different intersections and their relations is given in Fig. 1, the explicit form of the three 8-intersections is given in Appendix A.

A crucial role in the classification given in Fig. 1 is played by the observation that up to T-duality and interchanges of rows and/or columns there is a *unique* D-brane configuration that realizes the numbers $(n_1, n_2, \ldots)$ obtained from (7) and given in Fig. 1. So far, we have not been able to give a general proof of this fact. Instead, we checked it by brute force in a case by case analysis.

As we mentioned in the Introduction, at this stage one should still check the Einstein equation and the dilaton equation of motion. We have checked these for the three $N=8$ configurations using the computer. This implies that the intersections with $N \geqslant 5$ are also solutions. For lower N the number t of overall transverse coordinates increases, so that the harmonic functions can depend on more coordinates. We checked that the equations of motion indeed allow this.

Let us now consider supersymmetry. A single D-brane has half of its supersymmetry unbroken, since the supersymmetry transformations for the Killing spinor in the string frame

$$\begin{aligned} \delta\psi_\mu &= \partial_\mu\epsilon - \tfrac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon + \frac{(-)^p}{8(p+2)!}\, e^\phi\, F_{\mu_1\ldots\mu_{p+2}}\gamma^{\mu_1\ldots\mu_{p+2}}\gamma_\mu\epsilon'_{(p)} = 0\,, \\ \delta\lambda &= \gamma^\mu(\partial_\mu\phi)\epsilon + \frac{3-p}{4(p+2)!}\, e^\phi\, F_{\mu_1\ldots\mu_{p+2}}\gamma^{\mu_1\ldots\mu_{p+2}}\,\epsilon'_{(p)} = 0\,, \end{aligned} \tag{8}$$

for the D-brane yield

$$\epsilon + \gamma_{01\ldots p}\,\epsilon'_{(p)} = 0\,. \tag{9}$$

Eq. (9) defines a projection operator on ϵ that breaks half of the supersymmetry. For the IIA cases (p even) we have $\epsilon'_{(p)} = \epsilon$ for $p = 0,4,8$, $\epsilon'_{(p)} = \gamma_{11}\epsilon$ for $p = 2,6$; for IIB (p odd) $\epsilon'_{(p)} = i\epsilon$ for $p = -1,3,7$, $\epsilon'_{(p)} = i\epsilon^\star$ for $p = 1,5$, while always $\epsilon = H^{-1/8}\epsilon_0$, for constant ϵ_0. It is known [6,18] that a configuration of two intersecting D-branes can only be supersymmetric (keeping 1/4 of maximal supersymmetry) if they intersect in such a way that there are $n = 4$ or eight relative transverse directions, exactly the condition necessary to be also a solution of the equations of motion.

Adding more D-branes to the composite system, implies that more projection operators are introduced. Each time we add a new projection operator, half of the remaining supersymmetry gets broken. However, sometimes it is possible to add a D-brane in such a way that its projection operator is not independent, but given by a product of previous operators[5] [19,13,14]. In that case no additional supersymmetry generator is broken. In Fig. 1 we see this happens for example in the $N=4$ intersection. For $N=3$ we have one 0-brane and two 4-branes which preserve 1/8 of the supersymmetry because of the three independent projection operators

$$\begin{aligned} (1+\gamma_0)\epsilon &= 0\,, \\ (1+\gamma_{01234})\epsilon &= 0\,, \end{aligned} \tag{10}$$

[5] A similar mechanism has been observed in lower dimensions, where p-brane solutions with different numbers of participating field strengths preserve the same amount of supersymmetry [28].

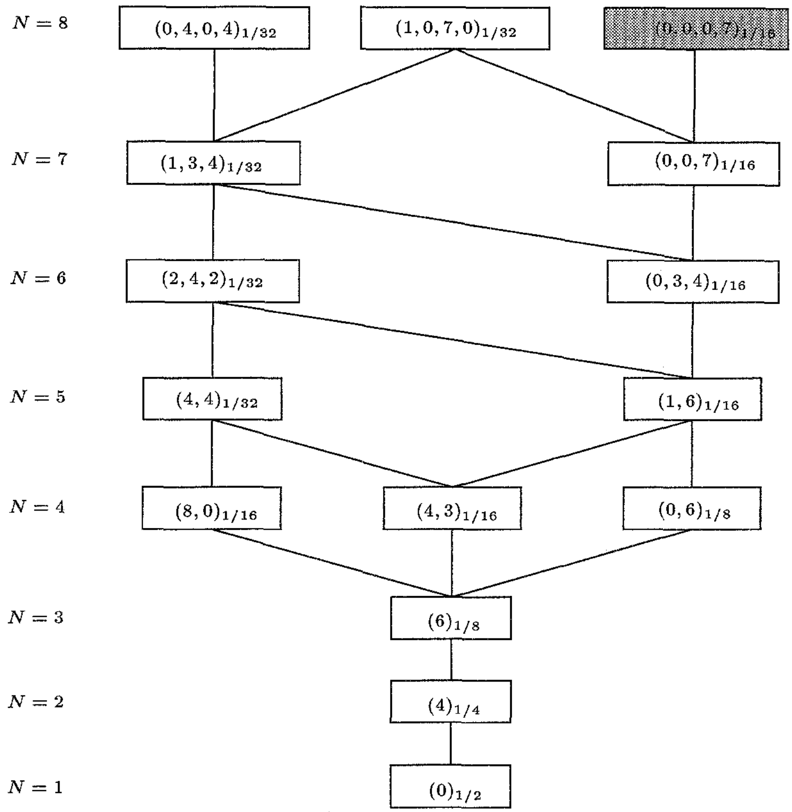

Fig. 1. D-brane intersections with $n = 4$ in ten dimensions: the numbers $(n_1, n_2, \ldots)$ label the number of times a building block with $(1,2,\ldots)$ world-volume directions is used. The subscript in the figure indicates the amount of supersymmetry preserved in each solution. The number N indicates the number of independent harmonics. The lines between solutions indicate how one configuration follows from another by adding (or deleting) a harmonic function. The configuration (0,0,0,7) cannot be extended to eleven dimensions in terms of (non-boosted) 2- and 5-branes only.

$$(1+\gamma_{01256})\epsilon = 0\,.$$

From Fig. 1 we see that there are three different ways to add a fourth brane. Two of them break an extra half of the remaining supersymmetry (configurations (8,0) and (4,3)), since in these cases the new brane introduces an independent projection operator. The third way (corresponding to configuration (0,6)) is by adding a 4-brane oriented in such a way that its projection operator

$$(1+\gamma_{03456})\epsilon = 0 \tag{11}$$

is exactly the product of the previous three operators (10). In this way no extra conditions on the Killing spinor arise.

The construction of projection operators for supersymmetry is another way of building up Fig. 1. Apparently supersymmetry and the equations of motion go hand in hand:

supersymmetry protects the stability of a configuration and vice versa, all stable solutions are supersymmetric. [6] The amount of unbroken supersymmetry of each configuration can be found in Fig. 1.

One $N = 8$ solution in Fig. 1 is special. By using T-duality it cannot be expressed in terms of 2- and 4-branes only, and therefore it cannot be lifted to eleven dimensions as an intersection of (non-boosted) M-branes. It is the solution $(0,0,0,7)$, indicated by a grey box in Fig. 1. We will discuss this solution in the next section.

3. Intersections of M-branes in eleven dimensions

The basic solutions in $D = 11$ are the M2-brane solution [3]

$$\begin{aligned} ds^2_{E,11} &= H_2^{-2/3}\, ds_3^2 - H_2^{1/3}\, ds_8^2, \\ F_{012i} &= \partial_i H_2^{-1}, \end{aligned} \tag{12}$$

and the M5-brane solution [7] [4]

$$\begin{aligned} ds^2_{E,11} &= H_5^{-1/3}\, ds_6^2 - H_5^{2/3}\, ds_5^2, \\ F_{012345i} &= \partial_i H_5^{-1}, \end{aligned} \tag{13}$$

where H_2 and H_5 are harmonic functions on the eight- and five-dimensional space transverse to the brane, respectively. As in the previous section, we will construct all multiple intersections satisfying condition (1) in the Introduction, and obtain their residual supersymmetry. As stated in the Introduction, each pair of M-branes in a composite configuration must be $(0|2,2)$, $(3|5,5)$ $(n=4)$ or $(1|2,5)$ $(n=5)$ [11,12].

We use the same representation as in the previous chapter: $\times$ for the world-volume coordinates and $-$ for the transverse coordinates. The three allowed $N = 2$ intersections of M-branes can therefore be represented as: [8]

$$(0|2,2):\quad \left\{ \begin{array}{c|cc|cc|cc|cc|cc|} \times & \times & \times & - & - & - & - & - & - & - & - \\ \times & - & - & \times & \times & - & - & - & - & - & - \end{array} \right. \tag{14}$$

$$(1|2,5):\quad \left\{ \begin{array}{c|cc|cc|cc|cc|cc|} \times & \times & \times & - & - & - & - & - & - & - & - \\ \times & \times & - & \times & \times & \times & \times & - & - & - & - \end{array} \right. \tag{15}$$

$$(3|5,5):\quad \left\{ \begin{array}{c|cc|cc|cc|cc|cc|} \times & \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & \times & \times & \times & - & - & \times & \times & - & - & - \end{array} \right. \tag{16}$$

[6] The procedure for constructing all independent projection operators for ten-dimensional supersymmetry resembles the procedure for constructing the independent central charges of a supersymmetry algebra in lower than ten dimensions (K. Stelle, private communication).

[7] Note that we represent the fivebrane in terms of a six-form gauge field, the field strength $F_{012345i}$ being the dual of F_{jklm}. This will be done for all $D = 11$ solutions presented in the paper. The $D = 11$ Chern–Simons term does not contribute to the solutions considered in this paper. This can be easily seen by noting that all non-zero gauge-field curvatures have a time component.

[8] Note that we cannot apply T-duality in eleven dimensions to relate these three intersections. Therefore we must also consider intersections of M-branes that intersect over a p-brane with $p > 0$.

Next, we add further M2-branes and/or M5-branes, always satisfying condition (1) for each pair. Like in $D = 10$, we find that this procedure stops at $N = 8$. We will not present the details of our constructive procedure but instead present the results below. It might be thought that the $D = 11$ result can be immediately obtained from the results of Section 2, but this is not quite true. One can go from M-branes in $D = 11$ to D-branes in $D = 10$ only if there is a direction such that all M2-branes are reduced to D2-branes, and all M5-branes to D4-branes. This will not be true in general, some configurations (which have $N \geqslant 4$) in $D = 11$ will only reduce to $D = 10$ intersections that involve NS–NS branes.

To characterize the configurations, we use again the contents of the columns (the components of the metric corresponding to the spacelike directions except the overall transverse coordinates) in the representation of the metric. For an N-intersection each column can have $1, \dots, N$ $\times$'s, indicating world-volume directions. The numbers of columns with k world-volume directions label the solutions, in the notation $\{n_1, \dots, n_N\}$ (using curly brackets). It is convenient to classify, in a first stage, the eleven-dimensional intersections up to T-duality. T-duality works as follows in $D = 11$ [27]. Two $D = 11$ solutions are called T-dual if, upon reduction to $D = 10$ dimensions, they lead to T-dual D-brane configurations. These T-dual $D = 11$ solutions can be represented by the labels $(n_1, \dots, n_{[N/2]})$ (using round brackets) which were used in the previous section to label T-dual D-brane configurations. Such a classification in terms of $D = 10$ solutions was also used by [20] for $N = 2$. Of course, this notation can only be used for $D = 11$ intersections that can be reduced to D-branes only.

The results we find in $D = 11$ can be represented in three different ways. First of all, in Fig. 2 we present the solutions up to T-duality in $D = 11$. For those M-brane intersections that reduce to one of the D-brane intersections given in Fig. 1, we use the same notation $(n_1, \dots, n_{[N/2]})$ as in the previous section. The gray rectangles indicate the solutions which necessarily contain NS–NS branes in $D = 10$, and for those the $D = 11$ notation $\{n_1, \dots, n_N\}$ is used. As in $D = 10$, we can have at most eight intersecting branes. Secondly, in Table 1 we provide more details about the contents of Fig. 2 by showing all $D = 11$ solutions that correspond to the same $D = 10$ D-brane intersection. Finally, in Appendix B the $N = 8$ intersections are given explicitly. We have checked that these intersections indeed solve the equations of motion.

As in $D = 10$, the complete structure of the $D = 11$ intersections can be recovered by the requirement of partially unbroken supersymmetry. Since the procedure is identical to the one used in $D = 10$ we will not give the details. The amount of unbroken supersymmetry for the different solutions is indicated in Fig. 2.

As an example consider the intersection of seven M5-branes: $\{0,0,7,0,0,0,2\}$. This solution has recently been considered in [30]. This configuration cannot be extended to $N = 8$ by adding another M-brane but is equivalent, via T-duality in $D = 10$, to a second $N = 7$ configuration $\{0,0,6,2,0,0,0\}$ (see Table 1). This T-dual $N = 7$ configuration can be extended to $N = 8$ as indicated in Fig. 2. Note that the third $N = 7$ configuration $\{0,0,0,7,0,0,1\}$, belonging to the same $(0,0,7)$ class, and its extension to $N = 8$ $\{1,0,0,7,0,0,0,1\}$ were given in [14].

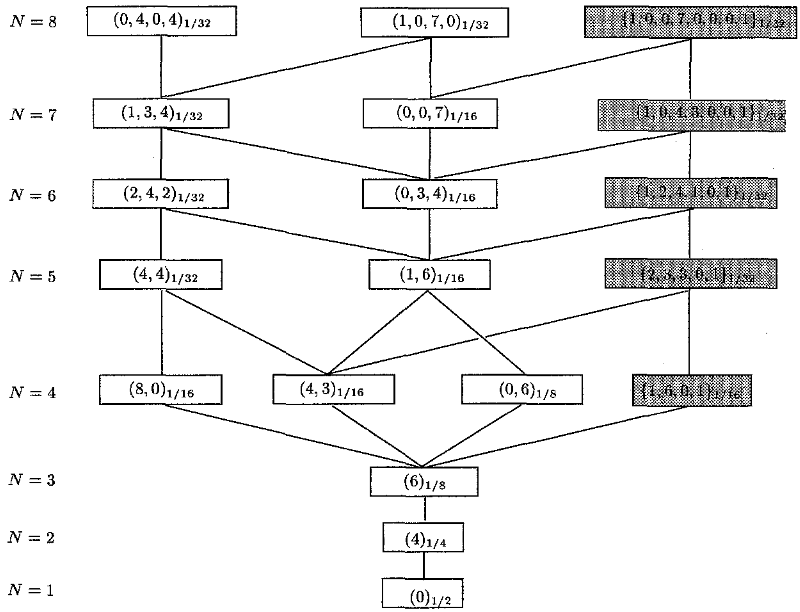

Fig. 2. M-brane intersections with $n = 4, 5$ in eleven dimensions: the numbers $(n_1, \ldots, n_{[N/2]})$ are the same labels used in $D = 10$, and indicate to which D-brane intersection the $D = 11$ solution reduces. The configurations in gray rectangles only reduce to $D = 10$ intersections involving NS–NS branes. For these configurations we use the eleven-dimensional notation $\{n_1, \ldots, n_N\}$ explained in the text. The subscripts indicate the amount of residual supersymmetry.

Finally, we consider the eleven-dimensional origin of the $N = 8$ D-brane intersection $(n_1, n_2, n_3, n_4) = (0, 0, 0, 7)$. In Section 2 we found that this intersection does not follow from the dimensional reduction of a $D = 11$ intersection consisting of (non-boosted) 2- and 5-branes only. Instead we find that it corresponds to an $N = 7$ intersection $\{0, 0, 0, 7, 0, 0, 1\}$ boosted along the common string direction. This configuration can be viewed as an intersection of seven M5-branes and a $D = 11$ gravitational wave and has a non-diagonal metric:

$$\begin{aligned} ds_{11}^2 = {} & (H_2H_3H_4H_5H_6H_7H_8)^{-\frac{1}{3}}[(2 - H_1)dt^2 - H_1dx_{10}^2 + 2(1 - H_1)dtdx_{10} \\ & -(H_2H_5H_8)dx_1^2 - (H_2H_6H_7)dx_2^2 - (H_3H_6H_8)dx_3^2 - (H_3H_5H_7)dx_4^2 \\ & -(H_4H_7H_8)dx_5^2 - (H_4H_5H_6)dx_6^2 - (H_2H_3H_4)dx_7^2 \\ & -(H_2H_3H_4H_5H_6H_7H_8)(dx_8^2 + dx_9^2)] \, . \end{aligned} \tag{17}$$

The solution has two overall transverse directions, and all H_i are harmonic on this two-dimensional space.

Table 1
Table of M-brane intersections in $D = 11$: The number N indicates the number of independent harmonics. The boldface labels $(n_1, \ldots, n_{[N/2]})$ correspond to the $D = 10$ D-brane intersection to which the $D = 11$ solutions reduce (when applicable). The numbers between square brackets indicate the number of M2-branes and M5-branes involved in the intersection. The labels $\{n_1, \ldots, n_N\}$ specify the structure of the $D = 11$ metric as explained in the text

N = 8	**(0,4,0,4)**	**(1,0,7,0)**	**{1,0,0,7,0,0,0,1}**	
	$[2^4,5^4]\{0,4,0,5,0,0,0,0\}$	$[2^4,5^4]\{1,0,6,1,1,0,0,0\}$	$[2^1,5^7]\{1,0,0,7,0,0,0,1\}$	
N = 7	**(1,3,4)**	**(0,0,7)**	**{1,0,4,3,0,0,1}** 7	
	$[5^7]\{1,0,4,0,3,0,1\}$ $[5^7]\ \{0,3,0,4,0,1,1\}$ $[2^3,5^4]\{1,2,4,1,1,0,0\}$ $[2^3,5^4]\{1,3,1,4,0,0,0\}$ $[2^4,5^3]\{1,3,4,1,0,0,0\}$	$[5^7]\{0,0,7,0,0,0,2\}$ $[5^7]\{0,0,0,7,0,0,1\}$ $[2^3,5^4]\{0,0,6,2,0,0,0\}$	$[2^1,5^6]\{1,0,4,3,0,0,1\}$	
N = 6	**(2,4,2)**	**(0,3,4)**	**{1,2,4,1,0,1}**	
	$[5^6]\{1,2,2,2,1,1\}$ $[2^2,5^4]\{1,4,2,1,1,0\}$ $[2^2,5^4]\{2,2,2,3,0,0\}$ $[2^3,5^3]\{2,3,3,1,0,0\}$ $[2^4,5^2]\{2,5,2,0,0,0\}$	$[5^6]\{0,0,4,3,0,1\}$ $[5^6]\{0,3,4,0,0,2\}$ $[2^2,5^4]\{0,2,4,2,0,0\}$ $[2^3,5^3]\{0,3,5,0,0,0\}$	$[2^1,5^5]\{1,2,4,1,0,1\}$	
N = 5	**(4,4)**	**(1,6)**	**{2,3,3,0,1}**	
	$[5^5]\{2,2,2,2,1\}$ $[2^1,5^4]\{3,1,3,2,0\}$ $[2^2,5^3]\{3,3,2,1,0\}$ $[2^3,5^2]\{4,3,2,0,0\}$ $[2^4,5^1]\{5,4,0,0,0\}$	$[5^5]\{1,4,2,0,2\}$ $[5^5]\{0,2,4,1,1\}$ $[2^1,5^4]\{0,4,2,2,0\}$ $[2^1,5^4]\{1,6,0,1,1\}$ $[2^2,5^3]\{1,3,4,0,0\}$ $[2^3,5^2]\{1,6,1,0,0\}$	$[2^1,5^4]\{2,3,3,0,1\}$	
N = 4	**(8,0)**	**(4,3)**	**(0,6)**	**{1,6,0,1}**
	$[2^2,5^2]\{6,1,2,0\}$ $[2^4]\{8,0,0,0\}$ $[5^4]\{4,0,4,1\}$	$[5^4]\{3,3,1,2\}$ $[5^4]\{1,3,3,1\}$ $[2^1,5^3]\{4,3,1,1\}$ $[2^1,5^3]\{2,3,3,0\}$ $[2^2,5^2]\{3,4,1,0\}$ $[2^3,5^1]\{5,3,0,0\}$	$[2^2,5^2]\{0,7,0,0\}$ $[5^4]\{0,6,0,2\}$	$[2^1,5^3]\{1,6,0,1\}$
N = 3	**(6)**			
	$[5^3]\{6,0,3\}$ $[5^3]\{0,6,1\}$	$[5^3]\{3,3,2\}$ $[2^1,5^2]\{5,2,1\}$	$[2^1,5^2]\{2,5,0\}$ $[2^2,5^1]\{5,2,0\}$ $[2^3]\{6,0,0\}$	
N = 2	**(4)**			
	$[5^2]\{4,3\}$	$[2^1,5^1]\{5,1\}$	$[2^2]\{4,0\}$	

4. Reduction to lower dimensions

A natural application of our results is the reduction of the M-brane and D-brane intersections we found in the previous two sections to dilatonic p-branes in lower dimensions. This will lead to dilatonic p-brane solutions which can be understood as D- and/or M-brane bound states in $D = 10, 11$. The interpretation of lower-dimensional solutions in terms of bound states of D- and/or M-branes in $D = 10, 11$ is a useful tool for understanding the properties of these lower-dimensional solutions, especially in the case of (extremal) black holes where it has opened up the possibility for a microscopic explanation of the Bekenstein–Hawking entropy [31]. It was recently discovered that the $D = 4$ (extremal) dilaton black holes preserving $\frac{1}{2}$ of the supersymmetry can be interpreted as bound states of D-branes (M-branes) compactified on a six-torus (seven-torus) [11–14,32,33]. It was shown, using the $N = 4$ (0,6) intersection in $D = 10$ (see Fig. 1), that the four values of the dilaton coupling a^2 in $D = 4$ could be reproduced by identifying the harmonic functions (equal charges) and truncating to intersections with smaller N (by setting some of the harmonic functions equal to one).

Using the D- and M-intersections constructed in this paper we find many other intersections which can be reduced to p-branes in lower dimensions. The general (Einstein frame) form of our reduced action (upon identifying some of the harmonic functions and setting the others equal to one) for $D > 2$ will always be in the class of Lagrangians of the form

$$\mathcal{L}_{E,D} = \sqrt{g}\left[R + \tfrac{1}{2}(\partial\phi)^2 + \frac{(-1)^{p+1}}{2(p+2)!}e^{a\phi}F^2_{(p+2)}\right]. \tag{18}$$

Using the ansatz

$$\begin{aligned} ds^2_{E,D} &= H^{\alpha} ds^2_{p+1} - H^{\beta} ds^2_{d-p-1}, \\ e^{2\phi} &= H^{\gamma}, \\ F_{0\dots pi} &= \delta\ \partial_i H^{-1}, \end{aligned} \tag{19}$$

we know that the general p-brane solution ($D > 2$) is given by [9]

$$\begin{aligned} \alpha &= -\frac{4(D-p-3)}{\Delta(D-2)}, \qquad \beta = \frac{4(p+1)}{\Delta(D-2)}, \\ \gamma &= \frac{4a}{\Delta}, \qquad \delta^2 = \frac{4}{\Delta}, \end{aligned} \tag{20}$$

with

$$\Delta = a^2 + 2\frac{(p+1)(D-p-3)}{D-2}. \tag{21}$$

The lower-dimensional p-brane solutions which follow from the reduced D-brane and M-intersections (now containing only one independent harmonic function) must fall inside this class of solutions. For supersymmetric solutions one must have [34,28]

[9] We use here a form of the solution as given in [34].

$$\Delta = 4/N, \tag{22}$$

where N is an integer labeling the number of participating field strengths, in our case this is the number of intersecting branes.

Any toroidal Kaluza–Klein reduction of the $D = 10, 11$ intersections will be a supersymmetry preserving p-brane solution in a lower dimension. Because the number of participating field strengths is equal to the number of intersecting branes we can immediately read off our dilatonic p-brane solution from (20) and (21)

As an illustration, consider the $N = 8$ D-brane intersections (see Fig. 1). We see that one of them, labeled by (0,0,0,7), can be naturally reduced to 0-branes in $D = 3$ by reducing over all relative transverse directions. Every truncation of this solution can of course also be reduced to $D = 3$ 0-branes, giving rise to eight different supersymmetry preserving solutions in $D = 3$. Doing the explicit Kaluza–Klein reduction we find that the different values of a^2 representing the different solutions (the explicit solution can be determined using (20)) are given by

$$a^2 = 4/N, \tag{23}$$

which is just (21) with $p = 0$, $D = 3$ and N running from 1 to 8. So we find eight supersymmetry preserving 0-branes in $D = 3$ (in contrast to the four 0-branes in $D = 4$) with the dilaton coupling given by (23) [35].

The general rule is to find the highest N intersection in the D- and/or M-intersections that can be reduced to a single p-brane in a lower dimension. The p-brane solutions in that lower dimension are given by (20) and (21) with $\Delta = 4/N$. Note that N is the only parameter, and that therefore different configurations of intersecting D- and/or M-branes with the same N, will all reduce to the same p-brane in lower dimensions upon identification of the harmonic functions (even if the $D = 10, 11$ intersecting solutions preserve different amounts of supersymmetry).

We will now discuss the various p-branes in lower dimensions obtained after reduction of D- and/or M-brane intersections.

• $D = 3$: So far we discussed the reduction of D-brane intersections to 0-branes in $D = 3$. Because the $N = 8$ configuration (0,0,0,7) cannot be oxidated to a (non-boosted) $D = 11$ M-brane intersection, the reduction from $D = 11$ to 0-branes in $D = 3$ will give only seven different solutions. To be precise, the $N = 7$ M-brane intersections labeled by $\{0, 0, 6, 2, 0, 0, 0\}$ and $\{0, 0, 0, 7, 0, 0, 1\}$ can be reduced to $D = 3$ 0-branes (giving the same solutions as reduction of the $N = 7$ (0,0,7) D-brane intersection). Of course, if we extend our ansatz and use the boosted $D = 11$ $N = 7$ $\{0, 0, 0, 7, 0, 0, 1\}$ solution (17) we do obtain the eighth 0-brane in $D = 3$.

In $D = 3$ the other possibility is to consider string (domain wall[10]) solutions. Because we have eight 0-branes in $D = 3$ and we can always do T-duality in one of the overall transverse directions on the $N = 8$ (0,0,0,7) D-brane intersections we find eight domain wall solutions. These same domain wall solutions also have an M-brane

[10] We use the name domain wall to indicate a $(D - 2)$-brane solution in D dimensions.

bound state interpretation because the $\{1,0,0,7,0,0,0,1\}$ M-brane intersection can be reduced to domain walls in $D = 3$ (note that this intersection cannot be reduced to $D = 10$ D-branes).

This means that in $D = 3$ we find eight 0-branes and eight domain walls, both series of solutions can be described as bound states of D-branes. One of the 0-branes cannot be interpreted as a bound state of (non-boosted) M-branes, but can be obtained from $D = 11$ using a boosted $N = 7$ solution.

• $D = 4$: Having a closer look at the $\{0,0,0,7,0,0,1\}$ M-brane intersection we see that this intersection can be reduced to strings (1-branes) in $D = 4$, which implies that there are seven supersymmetry preserving string solutions in $D = 4$ [28] with the solutions given by (20) and (21). From the reduction of the D-intersections we can obtain four different string solutions in $D = 4$ by using T-duality in one of the overall transverse directions in $D = 10$ on the (0,6) intersection. So we get three extra string solutions in $D = 4$ from the M-brane intersections.

By performing T-duality on an overall transverse direction we obtain four domain walls coming from the (0,6) D-brane intersection [29]. Surprisingly, we can get three extra domain wall solutions in $D = 4$ from the M-intersections. The $\{0,0,7,0,0,0,2\}$ $N = 7$ M-brane intersection can be reduced to domain walls in $D = 4$ and will give three extra domain wall solutions [30].

This completes the $D = 4$ case, which has four 0-branes, seven strings and seven domain walls (four coming from D-branes, seven coming from M-branes).

• $D > 4$: In dimensions higher than four there are fewer possibilities. In $D = 5$ we find three particle, three string, three membrane and three domain wall solutions coming from the $\{6,0,0\}$, $\{0,6,1\}$, $\{3,3,2\}$ and $\{6,0,3\}$ M-brane intersections respectively [12]. Only the $N = 1,2$ have a D-brane origin, all the solutions have an M-brane origin.

In $D = 6$ p-branes come in pairs and have a D- and M-brane origin. In $D = 7$ there exist two supersymmetry preserving 0-branes, both having an M-brane interpretation, only one having a D-brane interpretation. The basic D-branes in $D = 10$ and/or the basic M-branes in $D = 11$ (M2- and M5-brane) can be reduced to supersymmetry preserving p-branes in $D > 7$. Because the D8-brane in $D = 10$ has no (known) $D = 11$ origin there will be no domain wall solution in $D = 9$ with an M-brane origin.

• $D = 2$: So far we did not discuss $D = 2$. We see that in principle all $N = 8$ intersections in $D = 10, 11$ can be reduced to $D = 2$ 0-branes. In this case, however, we must work in the string frame and (21) is no longer valid. Therefore we redo the Kaluza–Klein reduction of the $D = 10$ intersections, keeping the string-frame metric. The reduction to $D = 2$ will always fall in the following class of Lagrangians (only 0-branes)

$$\mathcal{L}_{S,2} = \sqrt{|g|}e^{-2\phi}[R - 4(\partial\phi)^2] - \tfrac{1}{4}\sqrt{|g|}e^{a\phi}F_{(2)}^2. \tag{24}$$

Weyl invariance in $D = 2$ ensures that the reduced Lagrangian can always be written in the above way. The general 0-brane solution of this Lagrangian is

Table 2
D-brane and/or M-brane interpretation of dilatonic p-branes in $D \leqslant 6$ dimensions: The numbers $r[D^s, M^t]$ indicate that there are r solutions, for given D, p, of which s have a D-brane interpretation and t have an (non-boosted) M-brane interpretation. Note that one of the eight $D = 3$ $p = 0$ branes has a $D = 11$ interpretation only as a boosted $N = 7$ 5-brane solution

D	$p = 0$	$p = 1$	$p = 2$	$p = 3$	$p = 4$
6	$2[D^2, M^2]$	$2[D^2, M^2]$	$2[D^2, M^2]$	$2[D^2, M^2]$	$2[D^2, M^2]$
5	$3[D^2, M^3]$	$3[D^2, M^3]$	$3[D^2, M^3]$	$3[D^2, M^3]$	–
4	$4[D^4, M^4]$	$7[D^4, M^7]$	$7[D^4, M^7]$	–	–
3	$8[D^8, M^7]$	$8[D^8, M^8]$	–	–	–
2	$8[D^8, M^8]$	–	–	–	–

$$\alpha = \frac{2}{a} - 1, \qquad \beta = -\frac{2}{a} - 1, \tag{25}$$

$$\gamma = \frac{2}{a}, \qquad \delta^2 = -\frac{4}{a}, \tag{26}$$

where it is understood that the same ansatz (19) is used as in the Einstein frame case.

We find that for every number N of intersecting D-branes there is only one dilaton coupling constant representing the 0-brane solution. For example, all three $N = 8$ intersections give rise to the same two-dimensional 0-brane, thus confirming that for every N there is only one 0-brane solution, just like in $D > 2$ [34]. The dilaton couplings representing the supersymmetric solutions are given by

$$a = -4/N, \tag{27}$$

with $N = 1, \ldots, 8$. Note that we now give the string-frame dilaton coupling (defined by the Lagrangian in (24)) with a definite sign. This is done because in the $D = 2$ string frame there is no symmetry that flips the sign. Because the different solutions are labeled by N and most of the M-brane intersections can be reduced to D-branes in $D = 10$ we are convinced that the reduction from $D = 11$ will give the same results. So there are eight supersymmetry preserving 0-brane solutions in $D = 2$ with dilaton coupling given in (27), all of them having a D- and M-brane interpretation. It would be interesting to see whether the D-brane interpretation could shed any new light on the structure of black holes in two dimensions [36].[11]

Finally, we mention that all p-brane solutions in lower dimensions preserve half of the maximal supersymmetry in contrast to the intersecting D- and/or M-intersections in $D = 10, 11$. This gain in supersymmetry is a result of the identification of the different harmonics (equal charges). For an overview of the number of dilatonic p-brane solutions in lower dimensions ($D \leqslant 6$) with a D- and/or (non-boosted) M-brane bound state interpretation we refer to Table 2.

[11] In fact, O.A. Soloviev informed us that he is studying this connection.

5. Conclusions

In this paper we have given a classification of all multiple intersections of D-branes in ten dimensions and M-branes in eleven dimensions that correspond to threshold BPS bound states. In both cases we found that the maximum number of participating branes is eight. Allowing one $n = 8$ pair we can extend the number of intersecting branes to the maximum of nine. Furthermore, we found that not all D-brane intersections can be lifted up to non-boosted M-brane intersections in eleven dimensions. Conversely, not all M-brane intersections can be reduced to a ten-dimensional configuration of intersecting D-branes only. We also investigated the supersymmetry of the intersections, both in ten and eleven dimensions, and found that for all configurations at least $1/32$ of the supersymmetry is preserved.

There are several ways in which the classification presented in this work can be extended. First of all, we may consider boosted M-brane intersections [11–16]. The rule seems to be that, in case the intersection has a common string isometry direction x, one can add a Brinkmann wave with non-trivial (non-diagonal) metric components in the (x^0, x) direction. The Brinkmann wave is the eleven-dimensional origin of the D0-brane. A similar mechanism should exist where the wave is replaced by its magnetically charged partner (being the eleven-dimensional origin of the D6-brane). We thus obtain intersections with more than eight independent harmonics. It is expected that this wider class of intersecting M-branes gives rise, upon dimensional reduction to ten dimensions, to the class of intersections that contains not only D-branes but also NS–NS strings, five-branes and/or their T-duals. Of course, these intersections do not involve 8-branes whose eleven-dimensional origin so far has been a mystery. This concludes the classification of solutions that correspond to threshold BPS bound states.

One may also extend the solutions to the ones that correspond to non-threshold BPS bound states. For instance, by considering M-branes finitely boosted in a transverse direction [22] one obtains $D = 11$ solutions that reduce to D-brane bound states with $n = 2$ or 6. Furthermore, by considering a wave propagating along a generic cycle of a 2-torus [22] one obtains $D = 11$ solutions that reduce to the $D = 10(q_1, q_2)$ string solutions of [23]. There are also non-threshold BPS bound states in eleven dimensions, like the one given in [25]. It would be interesting to see how they fit in the general classification scheme.

Finally, we note that all knowledge about intersecting configurations is contained in the $D = 11$ solution with the maximum number of independent harmonics. The other ones can be obtained from these basic solutions via truncation and/or dimensional reduction. We have seen that there are very few of these basic configurations. Even including intersections with NS–NS branes and/or non-threshold BPS bound states, we expect the number of basic solutions to be limited. It would be of interest to construct these basic M-brane configurations explicitly.

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Appendix A. $N = 8$ D-brane intersections

In this appendix we give the explicit form of the metric for eight intersecting D-branes. There are three inequivalent $N = 8$ intersections. Note that the first two can be written via T-duality (in the x_2 and x_3 direction) as an intersection of four 2-branes and four 4-branes, and therefore can be lifted up to intersecting (non-boosted) M-brane solutions in eleven dimensions. The third solution can not be written as intersecting 2- and 4-branes and requires a non-diagonal form of the metric in eleven dimensions (see Eq. (17)). Using the notation explained in Section 2, the metric for the three $N = 8$ intersections are given by

$$(0,4,0,4): \left\{ \begin{array}{c|cc|cc|cc|cc|c} \times & - & - & - & - & - & - & - & - & - \\ \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & \times & \times & - & - & \times & \times & - & - & - \\ \times & - & - & \times & \times & \times & \times & - & - & - \\ \times & \times & - & \times & - & \times & - & \times & - & - \\ \times & - & \times & \times & - & \times & - & - & \times & - \\ \times & \times & - & \times & - & - & \times & - & \times & - \\ \times & - & \times & \times & - & - & \times & \times & - & - \end{array} \right. \tag{A.1}$$

$$(1,0,7,0): \left\{ \begin{array}{c|cc|cc|cc|cc|c} \times & - & - & - & - & - & - & - & - & - \\ \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & \times & \times & - & - & \times & \times & - & - & - \\ \times & - & - & \times & \times & \times & \times & - & - & - \\ \times & \times & - & \times & - & \times & - & \times & - & - \\ \times & - & \times & \times & - & \times & - & - & \times & - \\ \times & - & \times & - & \times & \times & - & \times & - & - \\ \times & - & \times & \times & - & - & \times & \times & - & - \end{array} \right. \tag{A.2}$$

$$(0,0,0,7): \left\{ \begin{array}{c|cc|cc|cc|cc|c} \times & - & - & - & - & - & - & - & - & - \\ \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & \times & \times & - & - & \times & \times & - & - & - \\ \times & - & - & \times & \times & \times & \times & - & - & - \\ \times & \times & - & \times & - & \times & - & \times & - & - \\ \times & \times & - & - & \times & - & \times & \times & - & - \\ \times & - & \times & \times & - & - & \times & \times & - & - \\ \times & - & \times & - & \times & \times & - & \times & - & - \end{array} \right. \tag{A.3}$$

All intersections with $N < 8$ can be obtained from these via different truncations. There are, of course, many different ways of truncating to lower intersections. However, as we can see in Fig. 1, many of them will lead to the same class. To recognize the class one has to determine the n_k's representing the class, this means counting the times a particular building block occurs in the configuration (see Section 2).

Appendix B. $N = 8$ M-brane intersections

In this appendix we give the explicit form of the metric for the configurations with $N = 8$ $n = 4, 5$ M-branes. As explained in Section 3 we can label classes of M-brane intersections by the D-brane intersection classes they reduce to. Some possible M-brane intersections cannot be reduced to D-brane intersections and then we are forced to use the particular $D = 11$ building block numbers. The configurations below are given with the $D = 11$ building block numbers (and their preserved supersymmetry). The first one can also be labelled with $(0,4,0,4)_{1/32}$ representing the $N = 8$ D-brane intersection it reduces to. The second one reduces to the $(1,0,7,0)_{1/32}$ $N = 8$ D-brane intersection and the third one cannot be reduced to a D-brane intersection.

$$\{0,4,0,5,0,0,0,0\}_{1/32}: \quad \left\{ \begin{array}{c|cc|cc|cc|cc|cc}
\times & \times & \times & - & - & - & - & - & - & - & - \\
\times & - & - & \times & \times & - & - & - & - & - & - \\
\times & - & - & - & - & \times & \times & - & - & - & - \\
\times & - & - & - & - & - & - & \times & \times & - & - \\
\times & - & \times & \times & - & \times & - & \times & - & \times & - \\
\times & \times & - & - & \times & \times & - & \times & - & \times & - \\
\times & \times & - & \times & - & - & \times & \times & - & \times & - \\
\times & \times & - & \times & - & \times & - & - & \times & \times & -
\end{array} \right. \tag{B.1}$$

$$\{1,0,6,1,1,0,0,0\}_{1/32}: \quad \left\{ \begin{array}{c|cc|cc|cc|cc|cc}
\times & \times & \times & - & - & - & - & - & - & - & - \\
\times & - & - & \times & \times & - & - & - & - & - & - \\
\times & - & - & - & - & \times & \times & - & - & - & - \\
\times & - & - & - & - & - & - & \times & \times & - & - \\
\times & - & \times & - & \times & \times & - & - & \times & \times & - \\
\times & \times & - & - & \times & \times & - & \times & - & \times & - \\
\times & - & \times & \times & - & \times & - & \times & - & \times & - \\
\times & \times & - & \times & - & \times & - & - & \times & \times & -
\end{array} \right. \tag{B.2}$$

$$\{1,0,0,7,0,0,0,1\}_{1/32}: \quad \left\{ \begin{array}{c|cc|cc|cc|cc|cc}
\times & \times & \times & - & - & - & - & - & - & - & - \\
\times & \times & - & \times & \times & \times & \times & - & - & - & - \\
\times & \times & - & \times & \times & - & - & \times & \times & - & - \\
\times & \times & - & - & - & \times & \times & \times & \times & - & - \\
\times & \times & - & \times & - & \times & - & \times & - & \times & - \\
\times & \times & - & - & \times & - & \times & \times & - & \times & - \\
\times & \times & - & - & \times & \times & - & - & \times & \times & - \\
\times & \times & - & \times & - & - & \times & - & \times & \times & -
\end{array} \right. \tag{B.3}$$

As in the case of D-branes we can obtain lower intersections through truncation. In order to obtain all configurations in Table 1 we have to make use of T-duality in $D = 10$. This T-duality should be carried out such that we keep $D2$ and/or $D4$-branes in $D = 10$, so we can lift up the solution back to intersecting M-branes in $D = 11$. The $D = 11$ building block numbers (n_k) can then be read off in a straightforward manner.

Appendix C. D-brane and M-brane intersections with $n = 4, 5, 8$ relative transverse directions

In this appendix we discuss intersections with $n = 4, 8$ $(D = 10)$ or $n = 4, 5, 8$ $(D = 11)$, and the dependence on relative transverse coordinates, i.e. corresponding to the conditions (2) and (3) in the Introduction.

It is easy to see that it is impossible to construct a configuration with three intersecting D-branes such that all pairs have $n = 8$ with a non-trivial dependence of the harmonic functions on the relatively transverse coordinates.

Let us work out in some detail how the dependence on relative coordinates can be brought in, since we will have to be careful about the allowed dependence on these coordinates. Consider any $n = 4$ configuration with dependence on overall transverse coordinates only, e.g. the $D = 10$ solution given in (A.1), which we copy below in (C.1). Now, suppose H_2, the harmonic function corresponding to the second line in (C.1), does not depend on x_9 but instead on the relative coordinates $x_5, \ldots, x_8$. Then we have realized a configuration satisfying condition (2). However, in verifying the equations of motion we find that the dependence on $x_5, \ldots, x_8$ has to be further restricted: the metric components g_{ii} have to be the same for each of the relative coordinates x_i on which the brane depends. Only then do the equations of motion lead to a harmonic equation for H_2. In this case that means that H_2 can depend on only one of the coordinates x_5, x_6, x_7 or x_8. Note that we can do this only for one harmonic function at the time, since any pair which both depend on relative coordinates must have $n = 8$.

$$N = 8,\ (0,4,0,4):\quad \left\{ \begin{array}{c|cc|cc|cc|cc|c} \times & - & - & - & - & - & - & - & - & - \\ \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & \times & \times & - & - & \times & \times & - & - & - \\ \times & - & - & \times & \times & \times & \times & - & - & - \\ \times & \times & - & \times & - & \times & - & \times & - & - \\ \times & - & \times & \times & - & \times & - & - & \times & - \\ \times & \times & - & \times & - & - & \times & - & \times & - \\ \times & - & \times & \times & - & - & \times & \times & - & - \end{array} \right. \qquad (C.1)$$

Since we have a different dependence on one harmonic function, we also have to change our ansatz for the gauge field: the gauge field of the D-brane represented by H_2 is now

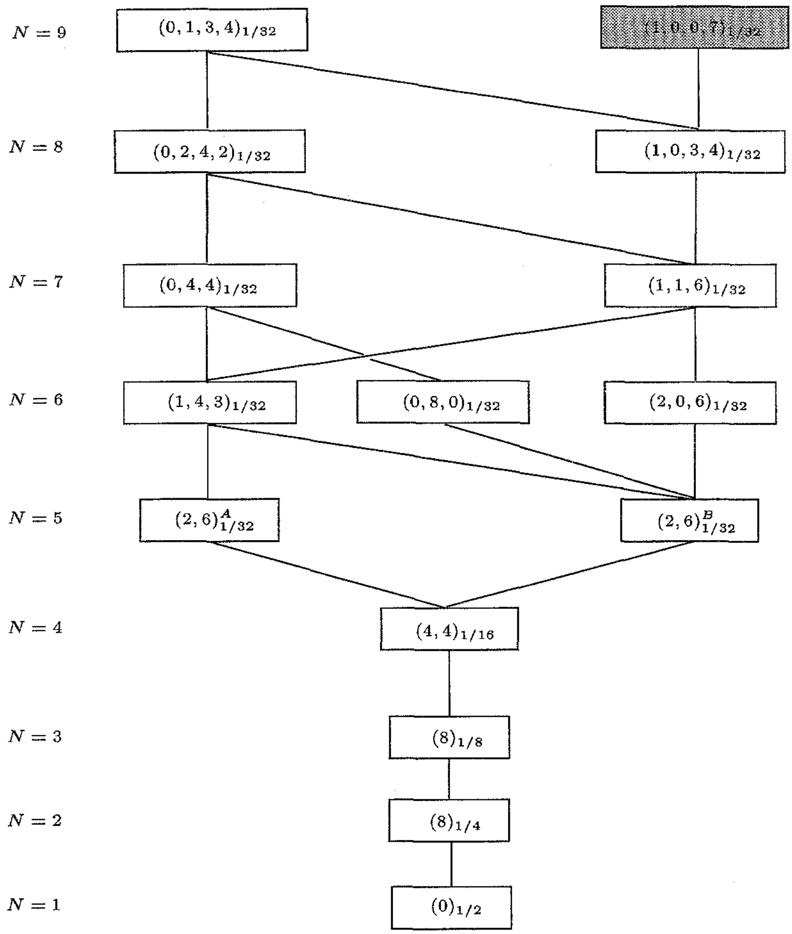

Fig. C.1. D-brane intersections with $n = 4, 8$ in ten dimensions: The solutions are labelled by $(n_1, \ldots n_{[N/2]})$, as explained in Section 2. For $N = 5$ an extra superscript is added to distinguish between the two sets of labels. Subscripts indicate the supersymmetry of the configurations. The $(1, 0, 0, 7)$ configuration given in the grey rectangle cannot be extended to eleven dimensions in terms of (non-boosted) 2- and 5-branes.

given by

$$F_{01234r} = \partial_r H_2^{-1}, \tag{C.2}$$

where x_r can be either x_5, x_6, x_7 or x_8.

It turns out that in this way all configurations satisfying condition (2) of the Introduction can be obtained from the ones satisfying condition (1) of the Introduction. Therefore, the classification of intersections satisfying condition (2) is the same as the one satisfying condition (1) and is therefore given by Fig. 1 as well. This concludes our classification of the configurations satisfying condition (2).

We next consider the configurations satisfying condition (3) of the Introduction. Consider the "mirror" configuration of H_2 in the above configuration, i.e. the brane

$$\times|-\ -|-\ -|\times\ \times|\times\ \times|-, \tag{C.3}$$

in which all $\times$'s in the relative coordinates have been replaced by $-$'s and vice versa. This 4-brane has $n = 8$ with H_2, and $n = 4$ with the other seven branes included in (C.1). Since it has $n = 8$ with H_2, its harmonic function, H_9 must depend on (some of the) coordinates $x_1, \ldots, x_4$, to satisfy the conditions (3) of the Introduction. An investigation of the equations of motion reveals that only dependence on one of the coordinates x_1, x_2, x_3 or x_4 is allowed: the metric must again be of the same form in the relative transverse coordinates.

This simple mechanism makes it possible to introduce an additional brane into any $n = 4$ configuration, by constructing an $n = 8$ pair with one of the constituents. In the present case this leads to the $N = 9$ configuration:

$$N=9,\ (0,1,2,5):\quad \left\{\begin{array}{c|cc|cc|cc|cc|c}
\times & - & - & - & - & - & - & - & - & - \\
\times & \times & \times & \times & \times & - & - & - & - & - \\
\times & \times & \times & - & - & \times & \times & - & - & - \\
\times & - & - & \times & \times & \times & \times & - & - & - \\
\times & \times & - & \times & - & \times & - & \times & - & - \\
\times & - & \times & \times & - & \times & - & - & \times & - \\
\times & \times & - & \times & - & - & \times & - & \times & - \\
\times & - & \times & \times & - & - & \times & \times & - & - \\
\times & - & - & - & - & \times & \times & \times & \times & -
\end{array}\right. \tag{C.4}$$

In Fig. C.1 we give the extension to $n = 4, 8$ of Fig. 1. Note that, in contrast to the $n = 4$ case, the labels $(n_1, \ldots, n_{[N/2]})$ do not uniquely specify the configuration: the two configurations with $N = 5$ have the same building block numbers (n_k's), although they are inequivalent. To distinguish between them we have added a superscript A or B. For the sake of completeness we also give the form of the gauge fields of the D-branes that depend on the relative coordinates x_r and x_s [18]

$$\begin{aligned}
F^{(2)}_{01234r} &= H_9 \partial_r H_2^{-1}, \\
F^{(9)}_{05678s} &= H_2 \partial_s H_9^{-1}.
\end{aligned} \tag{C.5}$$

Note that these curvatures indeed satisfy the Bianchi identity, and that we obtain the correct truncation by setting either H_2 or H_9 equal to one.

We next repeat this analysis for M-branes in $D = 11$. The only possibility for two M-branes to have $n = 8$ is two 5-branes intersecting over a string [14]:

$$n=8,\ (1|5,5):\quad \left\{\begin{array}{c|cc|cc|cc|cc|cc}
\times & \times & \times & \times & \times & \times & - & - & - & - & - \\
\times & - & - & - & - & \times & \times & \times & \times & \times & -
\end{array}\right. \tag{C.6}$$

This solves the equations of motion with H_1, H_2 depending on the relative transverse directions. As in $D = 10$, we will only consider the possibility of having a single $n = 8$

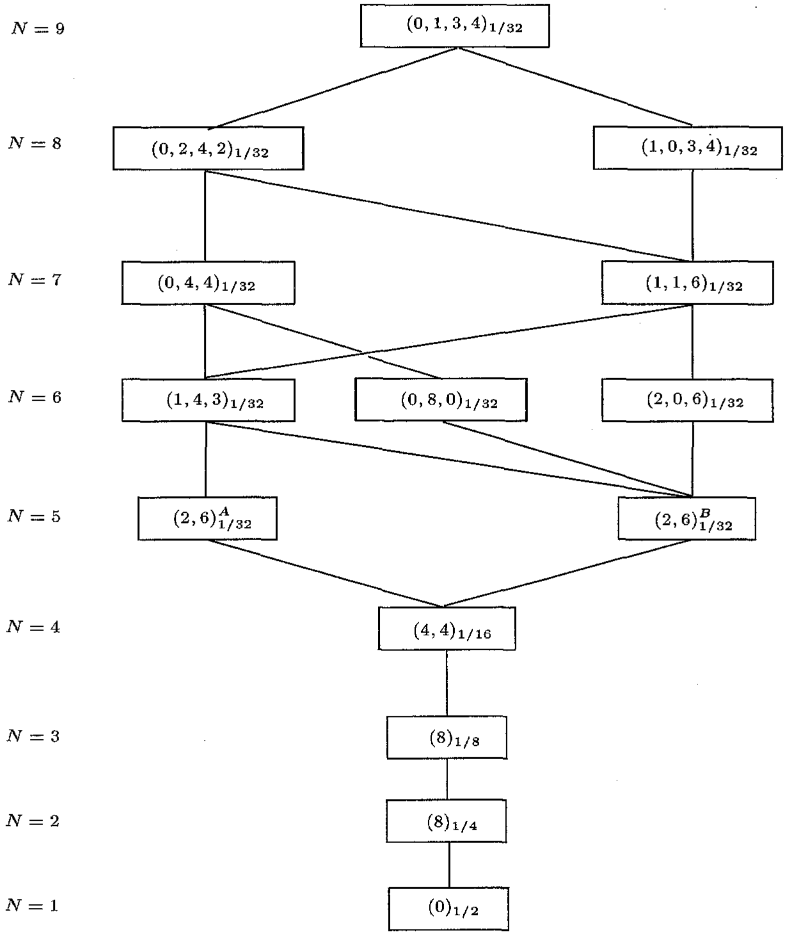

Fig. C.2. M-brane intersections with $n = 4, 5, 8$ in eleven dimensions. Since all configurations reduce to D-branes in $D = 10$ with $n = 4, 8$ we use $D = 10$ labels to classify the solutions. For $N = 5$ an extra superscript is added to distinguish between the two sets of labels. Subscripts indicate the unbroken supersymmetry.

pair.[12] Then there are no essential differences between the $D = 10$ and the $D = 11$ construction and we will skip the details. Fig. C.2 represents the result.

Finally we give an example of an $N = 9$ configuration in $D = 11$. Note that H_5 and H_6 (the lines 5 and 6 in (C.7)) are the $n = 8$ pair. H_5 may depend on one of the coordinates x_2, x_4, x_5 or x_8, H_6 on x_1, x_3, x_6 or x_7:

[12] However, in $D = 11$ configurations with several $n = 8$ pairs exist. While this paper was in press, this was pointed out in [37].

$$
(0,1,3,4)_{1/32}: \quad \left\{ \begin{array}{c|cc|cc|cc|cc|cc}
\times & \times & \times & - & - & - & - & - & - & - & - \\
\times & - & - & \times & \times & - & - & - & - & - & - \\
\times & - & - & - & - & \times & \times & - & - & - & - \\
\times & - & - & - & - & - & - & \times & \times & - & - \\
\times & \times & - & \times & - & - & \times & \times & - & \times & - \\
\times & - & \times & - & \times & \times & - & - & \times & \times & - \\
\times & \times & - & - & \times & \times & - & \times & - & \times & - \\
\times & - & \times & \times & - & \times & - & \times & - & \times & - \\
\times & \times & - & \times & - & \times & - & - & \times & \times & -
\end{array} \right. \tag{C.7}
$$

As a final remark, we mention that in the D-intersections for $n = 4, 8$ we find a configuration, $(1,0,0,7)$, which cannot be obtained through dimensional reduction of an intersection of (non-boosted) M-branes with $n = 4, 5, 8$. As in Section 3, we see that instead the result in $D = 11$ has a non-diagonal metric and involves a $[5^8]$ configuration and the Brinkmann wave. This can be interpreted as a boosted eight $[5^8]$ intersection in eleven dimensions. More explicitly, the configuration $(1,0,0,7)$ $(n = 4, 8)$ in $D = 10$ can be written as

$$
\begin{aligned}
ds_{10}^2 = {} & (H_1H_2H_3H_4H_5H_6H_7H_8H_9)^{-1/2}\{dt^2 - (H_1H_2H_4H_5)dx_1^2 \\
& -(H_1H_2H_4H_5H_6H_7H_8H_9)dx_2^2 \\
& -(H_1H_2H_7H_8)dx_3^2 - (H_1H_2H_6H_9)dx_4^2 \\
& -(H_1H_3H_5H_6H_7)dx_5^2 - (H_1H_3H_5H_8H_9)dx_6^2 \\
& -(H_1H_3H_4H_7H_9)dx_7^2 - (H_1H_3H_4H_6H_8)dx_8^2 \\
& -(H_1H_2H_3H_4H_5H_6H_7H_8H_9)dx_9^2\}, \\
e^{-2\phi} = {} & H_1^{-3/2}(H_2H_3H_4H_5H_6H_7H_8H_9)^{1/2}, \\
A_0 = {} & 1 - H_1^{-1}.
\end{aligned} \tag{C.8}
$$

Lifted up to eleven dimensions it has the form

$$
\begin{aligned}
ds_{11}^2 = {} & (H_2H_3H_4H_5H_6H_7H_8H_9)^{-1/3}\{(2 - H_1)dt^2 - H_1dx_{10}^2 + 2(1 - H_1)dtdx_{10} \\
& -(H_2H_4H_5)dx_1^2 - (H_2H_4H_5H_6H_7H_8H_9)dx_2^2 - (H_2H_7H_8)dx_3^2 \\
& -(H_2H_6H_9)dx_4^2 - (H_3H_5H_6H_7)dx_5^2 - (H_3H_5H_8H_9)dx_6^2 \\
& -(H_3H_4H_7H_9)dx_7^2 - (H_3H_4H_6H_8)dx_8^2 \\
& -(H_2H_3H_4H_5H_6H_7H_8H_9)dx_9^2\}.
\end{aligned} \tag{C.9}
$$

It represents an intersection boosted in the direction x_{10} where H_1 parametrizes the boost. If we set $H_1 = 1$ we recover the $[5^8]$ M-brane intersection. If instead we set all $H = 1$ except H_1, we get the Brinkmann wave in eleven dimensions.

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