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The massive Kaluza-Klein monopole

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Abstract

We construct the (bosonic) effective worldvolume action of an M-theory Kaluza–Klein monopole in a background given by the bosonic sector of eleven-dimensional massive supergravity, i.e. a “massive Kaluza–Klein monopole”. As a consistency check we show that the direct dimensional reduction along the isometry direction of the Taub–NUT space leads to the massive D-6-brane. We furthermore perform a double dimensional reduction in the massless case and obtain the effective worldvolume action of a Type IIA Kaluza–Klein monopole. © 1998 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

It is well-known that a classification of worldvolume (scalar, vector and tensor) multiplets leads to the brane scans for p-branes, D-branes and M-branes. On the other hand, requiring that branes in different dimensions are related to each other via (direct or double) dimensional reduction leads one to include gravitational waves and Kaluza–Klein (KK) monopoles [1] as well. These objects are missing in the standard brane scans which are based upon the so-called Bose–Fermi matching conditions. For the gravitational waves the reason is simply that their effective action is given by a one-dimensional field theory [2] and for such low-dimensional systems the Bose–Fermi matching conditions do not apply.

It turns out that the worldvolume action of a d -dimensional KK-monopole corresponds to a $(d - 5)$ -dimensional field theory [3] and for such case the Bose–Fermi matching conditions do apply. A natural question to ask is then why is the KK-monopole missing in the standard brane scans? As discussed in [3], a special feature of the KK-monopole is that one of its four transverse directions corresponds to the isometry direction in

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the Taub-NUT space of the monopole⁴. Since the monopole cannot move in this direction one should not associate a physical worldvolume scalar to it. On the other hand, given the fact that a monopole is a $(d - 5)$ -brane one cannot use the worldvolume diffeomorphisms to gauge away this unphysical scalar. In order to write down a Lorentz-covariant action one therefore needs a new mechanism to eliminate unphysical worldvolume scalars. We proposed in [2] that the effective action of the Heterotic KK-monopole corresponds to a *gauged sigma model* [4] with a gauging in the isometry direction of the Taub–NUT space. The effect of this gauging is to eliminate the unwanted scalar.

The purpose of this letter is to extend the results of [2] (for other partial results on the KK-monopole see [5]) to the Type IIA and M-theory KK-monopoles. For these cases the additional feature arises that the KK-monopole may move in a background with a non-zero cosmological constant proportional to m^2 with m a mass parameter. Branes moving in such a background are called “massive branes”. Their properties have been investigated in [6] (for earlier results on massive branes, see [7]). It turns out that eleven-dimensional massive branes are described by a *gauged sigma model* with the gauge coupling constant proportional to m . This letter extends the work of [6] to the case of the massive M-theory KK-monopole⁵.

At this point an obvious question arises. On the one hand, it follows from [2] that the massless M-theory KK-monopole is described by a gauged sigma model. On the other hand, the work of [6] suggests that in order to put the M-theory KK-monopole in a massive background we again have to use a gauged sigma model. In order to reproduce the massive D-6-brane it is clear that in both cases the gauging must be done in the isometry direction of the Taub–NUT space. However, we can only gauge a given isometry direction once! The resolution of this apparent puzzle is as follows. In a first stage, in order to describe the massless M-theory KK-monopole we gauge the isometry direction using a *dependent*⁶ worldvolume gauge field $\hat{A}^{(1)}$ with gauged isometry parameter $\hat{\sigma}^{(0)}$, see Eqs. (2.5) and (2.8). We remind that in order to put the M-wave, M-2-brane and M-5-brane in a massive background we used a gauging with an *independent* gauge field \hat{b} [6]. In the case of the M-theory KK-monopole, it turns out that the dependent worldvolume field $\hat{A}^{(1)}$, when put in a massive background, has exactly the same massive transformation as the independent worldvolume field \hat{b} we introduced for the other branes. Therefore the dependent field $\hat{A}^{(1)}$ takes over the role of the independent field \hat{b} .

Although in this sense the gauge field \hat{b} is not needed to put the KK-monopole in a massive background, it turns out that the worldvolume action of the monopole already contains this field, even in a massless background. Whereas for the other branes it is an auxiliary field needed to put the brane in a massive background, in the case of the M-theory KK-monopole it plays the role of a propagating Born–Infeld (BI) field.

Usually, a propagating worldvolume field describes the flux of another brane. For instance, the BI 1-form in the D-brane worldvolume action describes the flux of a fundamental string and the self-dual 2-form in the M-5-brane worldvolume action describes the flux of an M-2-brane⁷. At this point a puzzle arises because the 1-form field \hat{b} appearing in the KK-monopole action describes the flux of a 1-brane and no such brane is known to exist in M-theory. The resolution to this puzzle is that in fact the 1-form \hat{b} describes the flux of a *wrapped* M-2-brane. One way to see this is to observe that an M-2-brane can only intersect with an M-theory

⁴ For $d = 11$, a direct dimensional reduction over this isometry direction leads to the D-6-brane. This does not imply that the M-theory KK-monopole is equivalent to the ten-dimensional D-6-brane. The difference is that the D-6-brane is a *singular* object moving in ten dimensions whereas the M-theory KK-monopole is a *nonsingular* object defined in eleven dimensions (of which one is compact). We thank Paul Townsend for pointing this out to us.

⁵ For some recent work on KK-monopoles, see e.g. [8].

⁶ In case the Wess-Zumino (WZ) term is ignored there is an alternative formulation of the gauged sigma model using an *independent* worldvolume gauge field [2]. However, in the presence of a WZ term the two formulations are not equivalent and one must use the formulation with the dependent gauge field.

⁷ The same worldvolume fields also play a role in the construction of worldvolume solitons. For instance, the BI 1-form corresponds to a 0-brane soliton and the self-dual 2-form is used in constructing the self-dual string on the M-5-brane worldvolume. Worldvolume solitons for the KK-monopole action have been considered in [9] recently.

Table 1
Worldvolume fields

Worldvolume Field	Gauge Parameter	Field Strength	# of d.o.f
$\hat{X}^{\hat{\mu}}$			$11 - 7 - (1) = 3$
$\hat{b}_{\hat{t}}$	$\hat{\rho}^{(0)}$	$\hat{\mathcal{F}}_{\hat{t}\hat{j}}$	$7 - 2 = 5$
$\hat{\omega}_{\hat{t}_1 \dots \hat{t}_6}^{(6)}$	$\hat{\rho}^{(5)}$	$\hat{\mathcal{H}}_{\hat{t}_1 \dots \hat{t}_7}^{(7)}$	–

In this table we give the worldvolume fields, together with their gauge parameters, field strengths and number of degrees of freedom, that occur in the worldvolume action of the massive M-theory KK-monopole. The worldvolume scalars $\hat{X}^{\hat{\mu}}$ are the embedding coordinates, \hat{b} is a BI 1-form and $\hat{\omega}^{(6)}$ is a non propagating 6-form that describes the tension of the monopole. Due to the gauging the embedding scalars describe 3 and not 4 degrees of freedom as indicated in the table.

KK-monopole over a 0-brane such that one of the worldvolume directions of the M-2-brane coincides with the isometry direction z of the Taub-NUT space [10]⁸:

$$(0|M2, \mathcal{H}\mathcal{H}) = \left\{ \begin{array}{c|cccccccc} \times & - & - & \times & - & - & - & - & - & - \\ \times & - & - & z & \times & \times & \times & \times & \times & \times \end{array} \right.$$

A compactification of this intersection along the z -direction gives the $(0|F1, D6)$ configuration. The reduction of \hat{b} leads in this case to the BI field of the D-6-brane describing the tension of the F1 fundamental string.

The organization of this letter is as follows. In Section 2 we present the worldvolume action of the massive M-theory KK-monopole. As a consistency check we verify in Section 3 that the direct reduction of this action along z gives the action of the massive D-6-brane. Next, in Section 4 we perform a double dimensional reduction of the M-theory KK-monopole and obtain the action of the Type IIA KK-monopole. We perform this double dimensional reduction only for the massless case leaving the (more involved) massive case for a future publication. Finally, in Section 5 we present our conclusions.

2. The action

Before presenting the (bosonic) worldvolume action of the massive M-theory KK-monopole, it is useful to first summarize the worldvolume and target space fields that are involved in the construction of this action. We first discuss the worldvolume fields. Since the monopole in 11 dimensions behaves like a 6-brane and breaks half of the supersymmetry, it must correspond, after gauge fixing, to a 7-dimensional multiplet [3]. The natural candidate is the vector multiplet involving 3 scalars and 1 vector. As discussed in the introduction, the embedding coordinates describe $11 - 7 = 4$ degrees of freedom and one must eliminate a further scalar by gauging an isometry of the background. Besides scalars and a vector, we also introduce a non-propagating worldvolume 6-form that describes the tension of the monopole⁹. The resulting worldvolume field content of the KK-monopole is summarized in Table 1.

We next discuss the target space fields. The target space background corresponds to massive 11-dimensional supergravity [6], which has the same field content as the massless 11-dimensional supergravity theory (see

⁸ We use here a notation where a $\times(-)$ indicates a worldvolume (transverse) direction. The z -direction corresponds to the isometry direction of the Taub-NUT space and the first \times in a row indicates the time direction.

⁹ The same 6-form field occurs in the massive M-5-brane action. In that case it leads to the spontaneous creation of a KK-monopole whenever an M-5-brane crosses an M-9-brane [6].

Table 2
Target space fields

Target space Field	Gauge Parameter
$\hat{g}_{\hat{\mu}\hat{\nu}}$	
$\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}$	$\hat{\chi}_{\hat{\mu}\hat{\nu}}$
$(i_{\hat{k}}\hat{C})_{\hat{\mu}\hat{\nu}}$	$(i_{\hat{k}}\hat{\chi})_{\hat{\mu}}$
$\hat{C}_{\hat{\mu}_1\dots\hat{\mu}_6}$	$\hat{\chi}_{\hat{\mu}_1\dots\hat{\mu}_5}$
$(i_{\hat{k}}\hat{C})_{\hat{\mu}_1\dots\hat{\mu}_5}$	$(i_{\hat{k}}\hat{\chi})_{\hat{\mu}_1\dots\hat{\mu}_4}$
$\hat{N}_{\hat{\mu}_1\dots\hat{\mu}_8}$	$\hat{\Sigma}_{\hat{\mu}_1\dots\hat{\mu}_7}$
$(i_{\hat{k}}\hat{N})_{\hat{\mu}_1\dots\hat{\mu}_7}$	$(i_{\hat{k}}\hat{\Sigma})_{\hat{\mu}_1\dots\hat{\mu}_6}$

This table shows the 11-dimensional target space fields together with their gauge parameters. We also include the contractions with the Killing vector $\hat{k}^{\hat{\mu}}$, denoted by $(i_{\hat{k}}\hat{S})$ for a given field \hat{S} . The field \hat{C} is the Poincaré dual of \hat{C} and \hat{N} is the Poincaré dual of the Killing vector, considered as a 1-form $\hat{k}_{\hat{\mu}}$.

Table 2). The massive background has an isometry generated by a Killing vector $\hat{k}^{\hat{\mu}}$, such that the Lie derivative of all target space fields and gauge parameters with respect to \hat{k} vanish¹⁰:

$$\mathcal{L}_{\hat{k}}\hat{C} = \mathcal{L}_{\hat{k}}\hat{\chi} = \dots = 0. \quad (2.1)$$

Notice that in the massive case the gauge transformation rules of the target space fields contain extra terms proportional to a mass parameter m .

We proceed by presenting the (bosonic) action for the massive M-theory KK-monopole:

$$\hat{S}[\hat{X}^{\hat{\mu}}, \hat{b}_{\hat{\nu}}] = -\hat{T} \int d^7\xi \hat{k}^2 \sqrt{|\det(D_{\hat{\nu}}\hat{X}^{\hat{\mu}} D_{\hat{\nu}}\hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}} + (2\pi\alpha')\hat{k}^{-1}\hat{\mathcal{F}}_{\hat{\nu}\hat{\rho}})|} + \frac{1}{7!} (2\pi\alpha')\hat{T} \int d^7\xi \epsilon^{\hat{\nu}_1\dots\hat{\nu}_7} \hat{\mathcal{H}}_{\hat{\nu}_1\dots\hat{\nu}_7}^{(7)}. \quad (2.2)$$

We use the following notation. First of all, $\hat{k}^2 = -\hat{k}^{\hat{\mu}}\hat{k}^{\hat{\nu}}\hat{g}_{\hat{\mu}\hat{\nu}}$. The field-strength $\hat{\mathcal{F}}$ of the BI 1-form \hat{b} is defined by¹¹

$$\hat{\mathcal{F}} = 2\partial\hat{b} + \frac{1}{2\pi\alpha'} \partial\hat{X}^{\hat{\mu}}\partial\hat{X}^{\hat{\nu}}(i_{\hat{k}}\hat{C})_{\hat{\mu}\hat{\nu}}, \quad (2.3)$$

where $(i_{\hat{k}}\hat{C})$ is the contraction of the field \hat{C} with \hat{k} . The covariant derivative is given by

$$D_{\hat{\nu}}\hat{X}^{\hat{\mu}} = \partial_{\hat{\nu}}\hat{X}^{\hat{\mu}} + \hat{A}_{\hat{\nu}}^{(1)}\hat{k}^{\hat{\mu}}, \quad (2.4)$$

where the gauge field $\hat{A}^{(1)}$ is a dependent field given by

$$\hat{A}_{\hat{\nu}}^{(1)} = \hat{k}^{-2}\partial_{\hat{\nu}}\hat{X}^{\hat{\mu}}\hat{k}_{\hat{\mu}}. \quad (2.5)$$

¹⁰ We use the following notation for the hats. Hats on target space fields and indices indicate they are 11-dimensional. Absence of hats indicates they are 10-dimensional. Furthermore, we use hats for the worldvolume fields of branes in 11 dimensions and no hats for the worldvolume fields of branes in 10 dimensions. Finally, we use hats for worldvolume indices $\hat{\nu}$ if the dimension of the worldvolume is 6+1 and no hats for a 5+1 dimensional worldvolume.

¹¹ We omit the worldvolume indices. It is understood that the expression is completely antisymmetrized in these indices. We use this notation throughout the paper.

Finally, $\hat{\mathcal{H}}^{(7)}$ is the field strength of the non propagating worldvolume 6-form $\hat{\omega}^{(6)}$:

$$\begin{aligned} \hat{\mathcal{H}}^{(7)} = & 7 \left\{ \partial \hat{\omega}^{(6)} - \frac{1}{7(2\pi\alpha')} (i_{\hat{k}} \hat{N}) + 3(i_{\hat{k}} \hat{C}) \hat{\mathcal{F}} - \frac{5}{2\pi\alpha'} D\hat{X}^{\hat{\mu}} D\hat{X}^{\hat{\nu}} D\hat{X}^{\hat{\rho}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} (i_{\hat{k}} \hat{C}) (i_{\hat{k}} \hat{C}) \right. \\ & - 30 D\hat{X}^{\hat{\mu}} D\hat{X}^{\hat{\nu}} D\hat{X}^{\hat{\rho}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} (i_{\hat{k}} \hat{C}) \partial \hat{b} - 60(2\pi\alpha') D\hat{X}^{\hat{\mu}} D\hat{X}^{\hat{\nu}} D\hat{X}^{\hat{\rho}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} \partial \hat{b} \partial \hat{b} \\ & \left. - 120(2\pi\alpha')^2 \hat{A}^{(1)} \partial \hat{b} \partial \hat{b} \partial \hat{b} - 15m(2\pi\alpha')^3 \hat{b} \partial \hat{b} \partial \hat{b} \partial \hat{b} \right\}. \end{aligned} \quad (2.6)$$

Notice that we write explicitly the pullbacks only when covariant derivatives are used.

The full action is invariant under worldvolume gauge transformations (see Table 1), target space gauge transformations (see Table 2) and local isometry transformations (with parameter $\hat{\sigma}^{(0)}$) as given below. The gauge transformations of the worldvolume fields are given by:

$$\begin{aligned} \delta \hat{X}^{\hat{\mu}} &= -\hat{\sigma}^{(0)} \hat{k}^{\hat{\mu}}, \quad \delta \hat{b} = \partial \hat{\rho}^{(0)} - \frac{1}{2\pi\alpha'} (i_{\hat{k}} \hat{\chi}), \\ \delta \hat{\omega}^{(6)} &= 6\partial \hat{\rho}^{(5)} + \frac{1}{2\pi\alpha'} (i_{\hat{k}} \hat{\Sigma}) - 30\hat{b} \partial (i_{\hat{k}} \hat{\chi}) - 180(2\pi\alpha') \partial \hat{\chi} \hat{b} \partial \hat{b} \\ &\quad - 120(2\pi\alpha')^2 \partial \hat{\sigma}^{(0)} \hat{b} \partial \hat{b} \partial \hat{b} - 45m(2\pi\alpha')^2 (i_{\hat{k}} \hat{\chi}) \hat{b} \partial \hat{b} \partial \hat{b} - 15m(2\pi\alpha')^3 \partial \hat{\rho}^{(0)} \hat{b} \partial \hat{b} \partial \hat{b}. \end{aligned} \quad (2.7)$$

The dependent gauge field $\hat{A}^{(1)}$ transforms as

$$\delta \hat{A}^{(1)} = \partial \hat{\sigma}^{(0)} + \frac{m}{2} (i_{\hat{k}} \hat{\chi}). \quad (2.8)$$

Note that $\hat{\omega}^{(6)}$ transforms both under $\hat{\sigma}^{(0)}$ and $\hat{\rho}^{(0)}$. Only if we identify the two parameters

$$\hat{\sigma}^{(0)} = -\frac{m}{2} (2\pi\alpha') \hat{\rho}^{(0)}, \quad (2.9)$$

the transformation rule of $\hat{\omega}^{(6)}$ coincides with the one given in [6] where it occurred in the coupling to a massive M-5-brane. We deduce that the $\hat{\omega}^{(6)}$ describing the monopole tension actually has more symmetries than when it is coupled to a massive M-5-brane. The reason for this is that in the case of the M-5-brane the gauge transformation of \hat{b} is identified with the gauged isometry parameter.

Finally, the gauge transformations of the target space fields are given by:

$$\begin{aligned} \delta \hat{g}_{\hat{\mu}\hat{\nu}} &= -m(i_{\hat{k}} \hat{\chi})_{(\hat{\mu}} (i_{\hat{k}} \hat{g})_{\hat{\nu})} - \hat{\sigma}^{(0)} \hat{k}^{\hat{\lambda}} \hat{\chi}_{\hat{\lambda}} \hat{g}_{\hat{\mu}\hat{\nu}}, \quad \delta \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 3\partial_{[\hat{\mu}} \hat{\chi}_{\hat{\nu}\hat{\rho}]} - \frac{3}{2} m(i_{\hat{k}} \hat{\chi})_{[\hat{\mu}} (i_{\hat{k}} \hat{C})_{\hat{\nu}\hat{\rho}]} - \hat{\sigma}^{(0)} \hat{k}^{\hat{\lambda}} \hat{\chi}_{\hat{\lambda}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}, \\ \delta (i_{\hat{k}} \hat{C})_{\hat{\mu}\hat{\nu}} &= 2\partial_{[\hat{\mu}} (i_{\hat{k}} \hat{\chi})_{\hat{\nu}]} - \hat{\sigma}^{(0)} \hat{k}^{\hat{\lambda}} \hat{\chi}_{\hat{\lambda}} (i_{\hat{k}} \hat{C})_{\hat{\mu}\hat{\nu}}, \\ \delta (i_{\hat{k}} \hat{C})_{\hat{\mu}_1 \dots \hat{\mu}_5} &= 5\partial_{[\hat{\mu}_1} (i_{\hat{k}} \hat{\chi})_{\hat{\mu}_2 \dots \hat{\mu}_5]} + 15\partial_{[\hat{\mu}_1} \hat{\chi}_{\hat{\mu}_2 \hat{\mu}_3} (i_{\hat{k}} \hat{C})_{\hat{\mu}_4 \hat{\mu}_5]} - 10\hat{C}_{[\hat{\mu}_1 \hat{\mu}_2 \hat{\mu}_3} \partial_{\hat{\mu}_4} (i_{\hat{k}} \hat{\chi})_{\hat{\mu}_5]} \\ &\quad - \hat{\sigma}^{(0)} \hat{k}^{\hat{\lambda}} \hat{\chi}_{\hat{\lambda}} (i_{\hat{k}} \hat{C})_{\hat{\mu}_1 \dots \hat{\mu}_5}, \\ \delta (i_{\hat{k}} \hat{N})_{\hat{\mu}_1 \dots \hat{\mu}_7} &= 7 \left\{ \partial_{[\hat{\mu}_1} (i_{\hat{k}} \hat{\Sigma})_{\hat{\mu}_2 \dots \hat{\mu}_7]} + 15\partial_{[\hat{\mu}_1} (i_{\hat{k}} \hat{\chi})_{\hat{\mu}_2 \dots \hat{\mu}_5} (i_{\hat{k}} \hat{C})_{\hat{\mu}_6 \hat{\mu}_7]} + 30\partial_{[\hat{\mu}_1} \hat{\chi}_{\hat{\mu}_2 \hat{\mu}_3} (i_{\hat{k}} \hat{C})_{\hat{\mu}_4 \hat{\mu}_5} \right. \\ &\quad \left. \times (i_{\hat{k}} \hat{C})_{\hat{\mu}_6 \hat{\mu}_7]} - 20\hat{C}_{[\hat{\mu}_1 \hat{\mu}_2 \hat{\mu}_3} (i_{\hat{k}} \hat{C})_{\hat{\mu}_4 \hat{\mu}_5} \partial_{\hat{\mu}_6} (i_{\hat{k}} \hat{\chi})_{\hat{\mu}_7]} \right\} - \hat{\sigma}^{(0)} \hat{k}^{\hat{\lambda}} \hat{\chi}_{\hat{\lambda}} (i_{\hat{k}} \hat{N})_{\hat{\mu}_1 \dots \hat{\mu}_7}. \end{aligned}$$

3. Direct dimensional reduction

As a consistency check on the monopole action presented in the previous Section we perform a direct dimensional reduction along the direction associated to the isometry of the Taub-NUT space. It is convenient to use adapted coordinates $\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}z}$. The embedding scalars then split

$$\hat{X}^{\hat{\mu}} = X^{\mu}, \quad \hat{X}^z = Z, \quad (3.1)$$

and the background fields reduce as follows¹²:

$$\begin{aligned} \hat{g}_{\hat{\mu}\hat{\nu}} &= e^{-\frac{2}{3}\phi} g_{\mu\nu} - e^{\frac{4}{3}\phi} C_{\mu}^{(1)} C_{\nu}^{(1)}, \quad \hat{g}_{\hat{\mu}z} = -e^{\frac{4}{3}\phi} C_{\mu}^{(1)}, \quad \hat{g}_{zz} = -e^{\frac{4}{3}\phi}, \\ (i_{\hat{k}} \hat{C})_{\mu_1 \dots \mu_5} &= C_{\mu_1 \dots \mu_5}^{(5)} - 5 C_{[\mu_1 \mu_2 \mu_3}^{(3)} B_{\mu_4 \mu_5]}, \quad \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = C_{\mu\nu\rho}^{(3)}, \\ (i_{\hat{k}} \hat{N})_{\mu_1 \dots \mu_7} &= C_{\mu_1 \dots \mu_7}^{(7)} - 5 \cdot 7 C_{[\mu_1 \mu_2 \mu_3}^{(3)} B_{\mu_4 \mu_5} B_{\mu_6 \mu_7]}, \quad (i_{\hat{k}} \hat{C})_{\mu\nu} = B_{\mu\nu}. \end{aligned}$$

Similarly, the reduction rules of the gauge parameters are given by

$$(i_{\hat{k}} \hat{\chi})_{\mu_1 \dots \mu_4} = \Lambda_{\mu_1 \dots \mu_4}^{(4)}, \quad (i_{\hat{k}} \hat{\chi})_{\mu} = \Lambda_{\mu}, \quad \hat{\chi}_{\hat{\mu}\hat{\nu}} = \Lambda_{\mu\nu}^{(2)}, \quad (i_{\hat{k}} \hat{\Sigma})_{\mu_1 \dots \mu_6} = \Lambda_{\mu_1 \dots \mu_6}^{(6)}. \quad (3.2)$$

Besides these gauge parameters, we include as well the gauge parameter $\Lambda^{(0)}$ associated to the KK vector $C^{(1)}$.

Concerning the worldvolume fields, the BI field \hat{b} reduces as

$$\hat{b}_{\hat{i}} = b_{\hat{i}}. \quad (3.3)$$

Furthermore, we have

$$\hat{A}_{\hat{i}}^{(1)} = -C_{\hat{i}}^{(1)} - \partial_{\hat{i}} Z, \quad D_{\hat{i}} \hat{X}^z = -C_{\hat{i}}^{(1)}, \quad D_{\hat{i}} \hat{X}^{\mu} = \partial_{\hat{i}} X^{\mu}. \quad (3.4)$$

Substituting the above reduction rules into the kinetic term of the monopole action one gets the usual kinetic term for D -branes

$$\hat{k}^2 \sqrt{|\det(D_{\hat{i}} \hat{X}^{\hat{\mu}} D_{\hat{j}} \hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}} + (2\pi\alpha') \hat{k}^{-1} \hat{\mathcal{F}}_{\hat{i}\hat{j}})|} = e^{-\phi} \sqrt{|\det(\partial_{\hat{i}} X^{\mu} \partial_{\hat{j}} X^{\nu} g_{\mu\nu} + (2\pi\alpha') \mathcal{F}_{\hat{i}\hat{j}})|}, \quad (3.5)$$

where

$$\mathcal{F} = 2\partial b + \frac{1}{2\pi\alpha'} B \quad (3.6)$$

is the BI curvature.

We next discuss the reduction of $\hat{\omega}^{(6)}$. Since the KK-monopole reduces to the D-6-brane it is expected that $\hat{\omega}^{(6)}$ will reduce to the field $c^{(6)}$ describing the tension of the D-6-brane. The gauge transformations of $c^{(6)}$ are given by [6]:

$$\begin{aligned} \delta c^{(6)} &= 6\partial\kappa^{(5)} - \frac{1}{2\pi\alpha'} \Lambda^{(6)} + 30\Lambda^{(4)}\partial b - 180(2\pi\alpha') \Lambda^{(2)}\partial b\partial b + 120(2\pi\alpha')^2 \Lambda^{(0)}\partial b\partial b\partial b \\ &\quad - 15m(2\pi\alpha')^3 \rho^{(0)}\partial b\partial b\partial b + 45m(2\pi\alpha')^2 \Lambda b\partial b\partial b. \end{aligned} \quad (3.7)$$

We find the following reduction rule for $\hat{\omega}^{(6)}$ ¹³

$$\hat{\omega}^{(6)} = -c^{(6)} + 120(2\pi\alpha')^2 \partial Z b \partial b \partial b. \quad (3.8)$$

¹² We use the notation and conventions of [6].

¹³ This reduction rule coincides with the one given in [6] provided we make the identification (2.9) and rename $Z = (2\pi\alpha')c^{(0)}$.

Making use of the above reduction rules we find that, after direct reduction, the coordinate Z disappears from the action and we obtain the action of a massive $D6$ -brane:

$$S[X^\mu, b_i] = -\hat{T} \int d^7\xi e^{-\phi} \sqrt{|\det(g + (2\pi\alpha')\mathcal{F})|} - \frac{1}{7!} (2\pi\alpha') \hat{T} \int d^7\xi \epsilon^{\hat{i}_1 \dots \hat{i}_7} \mathcal{G}_{\hat{i}_1 \dots \hat{i}_7}^{(7)}. \quad (3.9)$$

Here $\mathcal{G}^{(7)}$ is the curvature of the worldvolume field $c^{(6)}$:

$$\mathcal{G}^{(7)} = 7 \left\{ \partial c^{(6)} + \frac{1}{7(2\pi\alpha')} C^{(7)} - 3C^{(5)}\mathcal{F} + 15(2\pi\alpha')C^{(3)}\mathcal{F}\mathcal{F} - 15(2\pi\alpha')^2 C^{(1)}\mathcal{F}\mathcal{F}\mathcal{F} + 15m(2\pi\alpha')^3 b\partial b\partial b\partial b \right\}. \quad (3.10)$$

The gauge transformations of the worldvolume and target space fields coupled to the D -6-brane can be found, for instance, in [6].

4. Double dimensional reduction

In this Section we perform the double dimensional reduction of the M-theory KK-monopole given in Section 2. As a result we will obtain the action of the Type IIA KK-monopole. We consider only the massless case¹⁴, i.e. our starting point is the action (2.2) with $m = 0$. In order to perform the double dimensional reduction, we must introduce an extra isometry for the background, generated by a Killing vector \hat{h} , and then wrap one direction of the monopole, $\hat{\xi}^6$, around this new compact direction:

$$\partial_6 \hat{X}^{\hat{\mu}} = \hat{h}^{\hat{\mu}}. \quad (4.1)$$

We have now two different isometries, one is given by \hat{h} , which is in a direction tangent to the worldvolume, and the other one is given by \hat{k} , in a direction transverse to the worldvolume. For the action (2.2) to be invariant under both isometries we must require that $\mathcal{L}_{\hat{h}}\hat{k} = 0$. Moreover, this implies that we can find a coordinate system adapted to both isometries¹⁵: $\hat{h}^{\hat{\mu}} = \delta^{\hat{\mu}y}$, $\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}z}$. Splitting the embedding coordinates as $\hat{X}^{\hat{\mu}} = (X^\mu, Y)$ we find the gauge-fixing condition:

$$Y = \hat{\xi}^6. \quad (4.2)$$

In this double dimensional reduction the Killing vector \hat{k} becomes, after reduction, the Killing vector associated to the isometry of the Taub-NUT space of the IIA KK-monopole:

$$\hat{k}^y = 0, \quad \hat{k}^\mu = k^\mu. \quad (4.3)$$

We use the following reduction rules for the background fields:

$$\begin{aligned} \hat{g}_{\mu\nu} &= e^{-\frac{2}{3}\phi} g_{\mu\nu} - e^{\frac{4}{3}\phi} C_\mu^{(1)} C_\nu^{(1)}, \quad \hat{g}_{\mu y} = -e^{\frac{4}{3}\phi} C_\mu^{(1)}, \quad \hat{C}_{\mu\nu\rho} = C_{\mu\nu\rho}^{(3)}, \quad \hat{g}_{yy} = -e^{\frac{4}{3}\phi}, \quad \hat{C}_{\mu\nu y} = B_{\mu\nu}, \\ \left(i_{\hat{k}} \hat{N}\right)_{\mu_1 \dots \mu_6 y} &= -\left(i_k \tilde{N}\right)_{\mu_1 \dots \mu_6}, \quad i_{\hat{k}} \hat{C}_{\mu y} = -\left(i_k B\right)_\mu, \\ \hat{C}_{\mu_1 \dots \mu_5 y} &= C_{\mu_1 \dots \mu_5}^{(5)} - 5C_{[\mu_1 \mu_2 \mu_3}^{(3)} B_{\mu_4 \mu_5]}, \quad \hat{C}_{\mu_1 \dots \mu_6} = -\tilde{B}_{\mu_1 \dots \mu_6}, \\ \left(i_{\hat{k}} \hat{C}\right)_{\mu_1 \dots \mu_4 y} &= -\left(i_k C^{(5)}\right)_{\mu_1 \dots \mu_4} + 3\left(i_k C^{(3)}\right)_{[\mu_1 \mu_2} B_{\mu_3 \mu_4]} + 2C_{[\mu_1 \mu_2 \mu_3}^{(3)} \left(i_k B\right)_{\mu_4]}, \end{aligned}$$

¹⁴ For a discussion of the massive case, see the Conclusions.

¹⁵ Notice, that the usage of the adapted coordinates does not imply that $\hat{k}^{\hat{\mu}} \hat{h}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}} = 0$.

where \hat{N} reduces to \tilde{N} , the Poincaré dual of the Killing vector, considered as a 1-form k_μ . Similarly, the reduction rules of the gauge parameters are given by

$$\begin{aligned}\hat{\chi}_{\mu_1 \dots \mu_5} &= -\tilde{\Lambda}_{\mu_1 \dots \mu_5}, \quad \hat{\chi}_{\mu y} = \Lambda_\mu, \quad \hat{\chi}_{\mu_1 \dots \mu_4 y} = \Lambda_{\mu_1 \dots \mu_4}^{(4)}, \quad (i_{\hat{k}} \hat{\chi})_y = (i_k \Lambda), \\ (i_{\hat{k}} \hat{\chi})_{\mu_1 \dots \mu_3 y} &= -(i_k \Lambda^{(4)})_{\mu_1 \dots \mu_3}, \quad (i_{\hat{k}} \hat{\chi})_\mu = (i_k \Lambda^{(2)})_\mu, \quad (i_{\hat{k}} \hat{\Sigma})_{\mu_1 \dots \mu_5 y} = -(i_k \Sigma^{(6)})_{\mu_1 \dots \mu_5}.\end{aligned}\quad (4.4)$$

We now turn to the reduction of the gauge field $\hat{A}^{(1)}$. Using the reduction rules given above one obtains:

$$\begin{aligned}\hat{A}_i^{(1)} &= \left(1 + e^{2\phi} k^{-2} (i_k C^{(1)})^2\right)^{-1} \left(A_i^{(1)} - e^{2\phi} k^{-2} C_i^{(1)} (i_k C^{(1)})\right), \\ \hat{A}_6^{(1)} &= -\left(1 + e^{2\phi} k^{-2} (i_k C^{(1)})^2\right)^{-1} e^{2\phi} k^{-2} (i_k C^{(1)}),\end{aligned}\quad (4.5)$$

where $k^2 = -k^\mu k^\nu g_{\mu\nu}$ and the field $A^{(1)}$ has been defined as the dependent gauge field associated to the gauged isometry of the Type IIA KK-monopole:

$$A_i^{(1)} = k^{-2} \partial_i X^\mu k_\mu. \quad (4.6)$$

The covariant derivative is defined by

$$DX^\mu = \partial X^\mu + A^{(1)} k^\mu. \quad (4.7)$$

Concerning the worldvolume fields, the 1-form \hat{b} gives rise to a scalar and a 1-form, and the 6-form $\hat{\omega}^{(6)}$ reduces to a 5-form $\omega^{(5)}$ describing the tension of the IIA KK-monopole:

$$\hat{b}_i = \omega_i^{(1)}, \quad \hat{b}_6 = \omega^{(0)}, \quad \hat{\omega}_{i_1 \dots i_5 6}^{(6)} = \omega_{i_1 \dots i_5}^{(5)}. \quad (4.8)$$

Using the above reduction rules in the action of the massless M-theory KK-monopole, we find the following action of a massless Type IIA KK-monopole:

$$\begin{aligned}S[X^\mu, \omega^{(0)}, \omega^{(1)}] &= -T \int d^6 \xi \, e^{-2\phi} k^2 \sqrt{1 + e^{2\phi} k^{-2} (i_k C^{(1)})^2} \\ &\times \sqrt{\left| \det \left(D_i X^\mu D_j X^\nu g_{\mu\nu} - (2\pi\alpha')^2 k^{-2} \mathcal{Z}_i^{(1)} \mathcal{Z}_j^{(1)} + \left[\frac{(2\pi\alpha') k^{-1} e^\phi}{\sqrt{1 + e^{2\phi} k^{-2} (i_k C^{(1)})^2}} \mathcal{Z}_{ij}^{(2)} \right] \right| \right.} \\ &\left. + \frac{1}{6!} (2\pi\alpha') T \int d^6 \xi \, \epsilon^{i_1 \dots i_6} \mathcal{Z}_{i_1 \dots i_6}^{(6)} \right).\end{aligned}\quad (4.9)$$

Here T is the tension of the Type IIA KK-monopole, which is related to the tension of the M-theory KK-monopole by:

$$T = \hat{T} \int_{S^1} d\hat{\xi}^6. \quad (4.10)$$

The 1-form $\mathcal{Z}^{(1)}$ is the field strength of the scalar $\omega^{(0)}$

$$\mathcal{Z}^{(1)} = \partial\omega^{(0)} - \frac{1}{2\pi\alpha'} (i_k B), \quad (4.11)$$

and the field strength $\mathcal{Z}^{(2)}$ of the 1-form $\omega^{(1)}$ is given by

$$\mathcal{Z}^{(2)} = 2\partial\omega^{(1)} + \frac{1}{2\pi\alpha'} (i_k C^{(3)}) - 2\mathcal{Z}^{(1)} (C^{(1)} + (i_k C^{(1)}) A^{(1)}). \quad (4.12)$$

Finally, $\mathcal{F}^{(6)}$ is the field strength of the non propagating worldvolume 5-form $\omega^{(5)}$:

$$\begin{aligned} \mathcal{F}^{(6)} = & \left\{ 6\partial\omega^{(5)} + \frac{1}{2\pi\alpha'}(i_k\tilde{N}) - 30(i_k C^{(5)})\partial\omega^{(1)} - \frac{15}{2\pi\alpha'}(i_k C^{(5)})(i_k C^{(3)}) - 6(i_k\tilde{B})\mathcal{F}^{(1)} \right. \\ & - 120(2\pi\alpha')DX^\mu DX^\nu DX^\rho C_{\mu\nu\rho}^{(3)}\mathcal{F}^{(1)}\partial\omega^{(1)} + \frac{30}{2\pi\alpha'}DX^\mu DX^\nu B_{\mu\nu}(i_k C^{(3)})(i_k C^{(3)}) \\ & + \frac{50}{2\pi\alpha'}DX^\mu DX^\nu DX^\rho C_{\mu\nu\rho}^{(3)}(i_k B)(i_k C^{(3)}) - 30DX^\mu DX^\nu DX^\rho C_{\mu\nu\rho}^{(3)}(i_k C^{(3)})\partial\omega^{(0)} \\ & - 180(2\pi\alpha')DX^\mu DX^\nu B_{\mu\nu}\partial\omega^{(1)}\partial\omega^{(1)} - 360(2\pi\alpha')^2 A^{(1)}\partial\omega^{(1)}\partial\omega^{(1)}\partial\omega^{(0)} \\ & \left. + 15(2\pi\alpha')^2 \frac{e^{2\phi}k^{-2}(i_k C^{(1)})}{1 + e^{2\phi}k^{-2}(i_k C^{(1)})^2}\mathcal{F}^{(2)}\mathcal{F}^{(2)}\mathcal{F}^{(2)} \right\}, \end{aligned} \quad (4.13)$$

where again, we are writing the pullbacks explicitly only when covariant derivatives are used.

The action is invariant under (massless) gauge transformations and the gauged isometry. The transformations for the worldvolume fields are:

$$\begin{aligned} \delta X^\mu = & -\sigma^{(0)}k^\mu, \quad \delta\omega^{(0)} = \frac{1}{2\pi\alpha'}(i_k\Lambda), \quad \delta\omega^{(1)} = \partial\rho^{(0)} - \frac{1}{2\pi\alpha'}(i_k\Lambda^{(2)}) + \partial\Lambda^{(0)}\omega^{(0)}, \\ \delta\omega^{(5)} = & 5\partial\rho^{(4)} - \frac{1}{2\pi\alpha'}(i_k\Sigma^{(6)}) - 5\partial(i_k\tilde{\Lambda})\omega^{(0)} + 20\omega^{(1)}\partial(i_k\Lambda^{(4)}) \\ & - 30(2\pi\alpha')\partial\Lambda^{(2)}(\omega^{(1)}\partial\omega^{(0)} + \partial\omega^{(1)}\omega^{(0)}) + 60(2\pi\alpha')\omega^{(1)}\partial\omega^{(1)}\partial\Lambda \\ & + 60(2\pi\alpha')^2\sigma^{(0)}\partial\omega^{(1)}\partial\omega^{(1)}\partial\omega^{(0)}, \end{aligned} \quad (4.14)$$

whereas that of the target space field $(i_k\tilde{N})$ is given by:

$$\begin{aligned} \delta(i_k\tilde{N}) = & 6\partial(i_k\Sigma^{(6)}) + 60(i_k C^{(3)})\partial(i_k\Lambda^{(4)}) - 30\partial(i_k\tilde{\Lambda})(i_k B) - 60(i_k C^{(3)})(i_k C^{(3)})\partial\Lambda \\ & + 120\partial\Lambda^{(2)}(i_k C^{(3)})(i_k B) + 60(i_k C^{(3)})\partial(i_k\Lambda^{(2)})B - 40C^{(3)}\partial(i_k\Lambda^{(2)})(i_k B) \\ & - 20C^{(3)}(i_k C^{(3)})\partial(i_k\Lambda) - \sigma^{(0)}k^\lambda\lambda_\lambda(i_k\tilde{N}). \end{aligned} \quad (4.15)$$

The transformations of the other target space fields can be found in [6].

Notice that the truncation of the action (4.9) by setting the RR fields and $\omega^{(1)}$ to zero leads to the action of the Heterotic KK-monopole given in [11].

5. Conclusions

In this paper we have presented the worldvolume action of the massive M-theory KK-monopole. We find that the action is given by a gauged sigma-model. This agrees with the results of [6] where the effective actions of massive branes were generally shown to be described by gauged sigma-models.

A new feature that arises is that the M-theory KK-monopole action is already given by a gauged sigma-model in the massless case. This gauging is needed to effectively eliminate the Taub-NUT isometry coordinate [2]. We find that the dependent gauge field that is needed for the gauging in the massless case transforms under the massive transformations such that the massless gauged action can be made invariant under the massive transformations without the need to introduce an independent auxiliary gauge field. Besides a dependent gauge field, the monopole action contains an independent BI gauge field, which already occurs in the

massless case. It turns out that in the massive case extra terms proportional to the mass parameter m need to be added to the WZ term. As a check on the action we have presented in this work, we have performed a direct dimensional reduction along the Taub–NUT isometry direction and have obtained the massive D-6-brane.

We have as well performed a double dimensional reduction and obtained the action of the Type IIA KK-monopole. We have performed this double dimensional reduction only for the massless case. In a forthcoming publication we will perform the double dimensional reduction in the massive case and present the massive Type IIA KK-monopole [12]. A new feature that arises in the double dimensional reduction of the massive monopole is that we first need to construct a massive monopole in which the isometry associated to the mass is realized along a coordinate *different* from the Taub–NUT isometry direction. We thus end up with an M-theory KK-monopole with two gauged isometries. One isometry is the z -direction in the Taub–NUT space. This isometry is already gauged in the massless case. The other isometry is only gauged when the monopole is put in a massive background.

Finally, our results for the Type IIA monopole also lead, via T-duality, to the worldvolume action of a Type IIB NS5-brane, which coincides with that of a D-5-brane in a $SL(2, Z)$ -transformed background, see [12]. In order to work out this duality transformation it is necessary to first derive the set of T-duality rules which apply to the different world-volume fields that need to be introduced.

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