Algorithms in behavioral systems theory

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Foreword

If I had to summarize in one sentence what this thesis is about, I guess the best way of putting it would be that it is trying to formulate and constructively answer a number of questions that naturally arise when modeling physical systems.

In the next few pages, with the help of figure 1, I wish to give a feeling of what kind of questions are being asked and what kind of solutions are being proposed. The tone for the whole work is set in Chapter 1, where

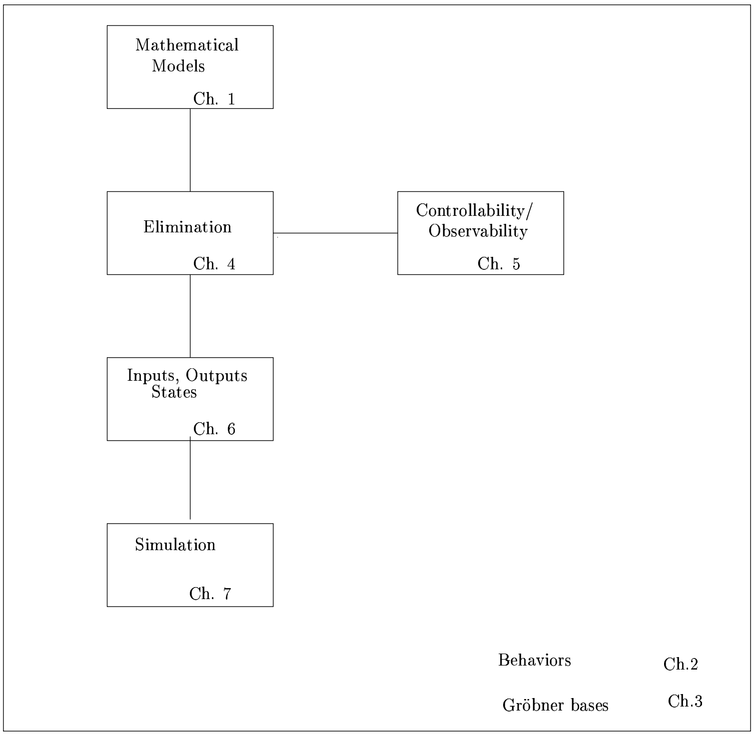

Figure 1: Outline of this work

I formalize a method for describing complex physical systems viewed as interconnection of simpler subsystems. Using a number of examples from the electrical, mechanical and fluid domain I try to show that the method proposed is extremely systematic as well as flexible, and equally suited for describing models of a very different nature.

Of the many kind of models that can arise when following the approach described in chapter 1, I concentrate on the specific case of systems which can be described by linear constant coefficient partial differential equations. Chapter 2 provides the basic notions on how this class of systems can be described using the language of behavioral systems theory. In this way an elegant mathematical framework is provided, that perfectly matches the more intuitive approach to modeling introduced in the first chapter.

A first consequence of the systematic approach for describing systems presented in the first chapter is that the models one obtains in first instance include both the variables the model aims at (called manifest variables) and some auxiliary variables (called latent variables). The latter typically arise in the description of the component subsystems and are very useful in the phase of constructing a model. Latent variables, however, become redundant once it comes to analyzing the system which has been built, because the goal of modeling is to describr how the manifest variables alone behave. The question arises whether one can start with equations that use both latent and manifest variables, and derive equations that describe the evolution of the manifest variables, only containing the manifest variables themselves. This is, in a nutshell, the elimination problem, which is addressed in detail in Chapter 4.

Once a model of the behavior of the manifest variables is obtained, I proceed in two directions. On the one hand I wish to analyze constitutive system properties. This is done in Chapter 5, where definitions of controllability and observability are given and algorithms to asses these properties in terms of the equations describing the model are given. On the other hand I want to be able to numerically reproduce (i.e. simulate) trajectories of the variables which are being modeled. Chapter 7 of this work is dedicated to showing how this can be done for systems described by ordinary differential equations.

Before addressing simulation issues, however, I dedicate a chapter to discussing the concepts of inputs, outputs and state. A crucial consequence of the way in which I build models in chapter 1, in fact, is that all the variables need to be considered on an equal footing, without separation between inputs and outputs, and without state variables being used. Many examples in the first chapter show why it is desirable not to use i/s/o models as the a priori general framework for describing physical systems. On the other hand, the possibility of a posteriori partitioning variables into inputs and outputs and of constructing state variables reveals a lot about the structure of the model and turns out to be extremely useful, for example, when it comes to simulation. In chapter 6, therefore, I show how an input/(state)/ output description can be obtained starting from the more general descriptions used in the earlier chapters.

So much for *what* I am planning to do, for the questions I intend to address. In the beginning I said I wanted to answer them constructively, so I briefly need to explain *how* I am going to deal with them. The answer is given in chapter 3, where the basic concepts of the theory of Gröbner bases are outlined. All along, in fact, I am going to present algorithms that allow to compute the answers to the questions being asked in each chapter. For the class of systems I work with (i.e. linear constant coefficient PDE's) the theory of Gröbner bases provides a unified framework to describe elegantly the operations I need to perform on the system equations in order to extract the necessary information. For the special case of ordinary differential systems, I establish connections to more traditional tools such as canonical forms of one variable polynomial matrices.

Having discussed *what* I intend to do and *how* I plan to do it, I wish to close this section with a few words on *why* I find the problems addressed in the next pages interesting. The answer lies in the fact that I consider myself an hybrid between an engineer, a mathematician and a computer scientist (maybe this is the best definition of a system theorist !) and I wanted these three sides to be equally represented in my work.

As an engineer, I was extremely interested in working on problems related to modeling of physical systems. As a mathematician, I was looking for an elegant framework in which to describe physical phenomena. As a computer scientist, finally, I wanted to develop something as close as possible to implementation as a software package. In all these respects, I found behavioral systems theory, as developed over the last ten years, a very appealing framework to work in. What I find especially attractive in this theory was the possibility of having a firm mathematical basis for ideas such as modularity and interconnection of systems that are, at a more intuitive level, very much part of engineering thinking. At the same time, I was appealed by the possibility of relating to constructive algebraic methods, such as Gröbner bases, enabling me to really do computations on models.

As computer power grows, we are able to perform automatically computations and manipulations of growing complexity. In my opinion, a direction where Systems Theory as a science can still achieve a lot is that of providing tools for describing physical systems which are, on the one hand, easy and intuitive to handle for the human modeler sitting behind the screen and on the other hand computationally tractable for the automatic modeler sitting inside the CPU. This thesis was written in this spirit and I hope the next pages will convey at least part of it.

I wish you all a pleasant reading!

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