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The Timing of Initial Public Offerings

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ABSTRACT

In this paper, we study the dynamics of initial public offerings (IPOs) by examining the tradeoff between an entrepreneur's private benefits, which are lost whenever the firm is publicly traded, versus the advantages from diversification. We characterize the timing dimension of the decision to go public, derive a function for firm value, and describe their effect on the evolution of firm risk over time. Our model, which endogenizes the timing of the decision to stay private or to go public, is able explain two puzzling phenomena: the clustering of IPOs and buyouts in time and the long-run underperformance of recently issued stock relative to the shares of longer-listed companies.

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I. Introduction

Several reasons have been proposed for why entrepreneurs sell shares of their firms to the public. Trivially, companies may issue stock to finance future investments. Yet, this in itself does not justify initial public offerings (IPOs) as bank loans or private equity placements may equally well finance a need for funds. Moreover, Pagano, Panetta, and Zingales [1998] find that investments of firms actually *decline* after an IPO. Holmström and Tirole [1993] and Bolton and Von Thadden [1998] suggest that by selling stock on the public market, companies subject themselves to monitoring by outsiders (e.g., investment banks, auditors, analysts, investors); activities that may enhance the value of the firm. They also suggest, like Amihud and Mendelson [1988], that IPOs make the firms' shares more liquid, which raises firm value even more. Benveniste and Spindt [1989], Dow and Gorton [1997], Habib and Ljungqvist [1998], Subrahmanyam and Titman [1999], and Maug [2000] argue that IPOs allow entrepreneurs to use the share price to infer investors' valuations of their firm; this information can be used in post-IPO investment decisions and for management's incentive compensation. Along similar lines, Chemmanur and Fulghieri [1999] argue that both public and private ownership entail information advantages and that the optimal decision on this structure minimizes the related costs.

All the suggested reasons for going public exhibit some tradeoff between the benefits of being publicly traded and its associated costs. Consequently, as the conditions under which the firm operates change, the incentives to be public or private may also change. Yet, most of the papers above model the decision to go public as a single shot: entrepreneurs have but one chance to decide whether to go public or to stay private. Clearly, this way of modeling ignores the ability of entrepreneurs to *time* their IPO, since the decision to remain private *today* does not eliminate the possibility of going public at some future date. Furthermore, such an analysis also ignores any opportunities to take the firm private again, either directly (e.g., in a management buyout

(MBO) or a leveraged buyout (LBO)) or indirectly (i.e., by being purchased by another company).

This paper complements the current literature on the decision to go public by explicitly considering the timing dimension of IPOs as well as the timing of going-private transactions. Specifically, we analyze the optimal conditions to take a company public and the circumstances to reverse this decision (to become a private firm again). In our model, the investments of the firm have been made. The owner takes the company public because outside investors, being more diversified, are willing to pay a higher price for the risky cash flows of the firm than the entrepreneur's own valuation of these flows.¹ Going public, however, means that the owner gives up her "private benefits of control" (for example, because of more intense monitoring of a public firm as in Bolton and Von Thadden [1998]).² If entrepreneurs have but one chance to go public, they would simply trade off the benefits and costs of an IPO and choose their best course of action accordingly. In our model, however, the decision to remain private in any given period may be reversed at later dates (and vice versa). Therefore, the decision to go public entails more than a straightforward comparison of *immediate* costs and benefits. In this paper we analyze the optimal timing of an IPO explicitly considering the dynamics of a firm's cash flows while also allowing for reversibility of today's decisions in future periods.³

¹ The higher price outside investors are willing to pay for the firm's cash flows can also be viewed as capturing the added value of monitoring public companies or as the value which is contributed by the increased post-IPO liquidity to the "private" cash flow value of the shares.

² The "private benefits" can also be regarded as the agency costs saved by a firm that is not traded publicly. These costs (as considered in detail by Jensen [1986]) include any costs of separating ownership from control, but may also refer to administrative costs (e.g., filing requirements, audited financial statements, etc.), the costs of information gathered by outside investors, and increased disclosure of inside information that may reduce the competitive advantages of the firm.

³ The dynamics of similar flexibilities have been studied by Ikenberry and Vermaelen [1996] and Fluck [1999].

Some empirical regularities suggest that entrepreneurs indeed time their decisions to go public. For example, there are waves in IPOs, a phenomenon called “hot issue markets” (Ritter [1984]). Moreover, these waves are often disproportionately populated with firms in a particular industry. One possible reason for the “hot” markets in IPOs is that firms, especially in certain industries, face better investment opportunities during some periods than in other times, so that IPOs merely allow for increased fund raising. However, Loughran, Ritter, and Rydqvist [1994] find that hot issue markets do not coincide with an increase in subsequent investments. Rather, IPOs appear to cluster during periods in which investors place relatively high values on the cash flows of the firms that go public; a result we derive in the context of our model. Specifically, we show that entrepreneurs issue shares when the “public” cash flow value of their firm is relatively high. Conversely, our model indicates that firms are taken private when the market valuation of the expected cash flows is low (relative to the private benefits). This is consistent with evidence of Halpern, Kieschnick, and Rotenberg [1999].

A puzzling aspect of IPOs, which still remains unexplained, is that the returns on recently issued stock over the five years following the IPO are substantially below the market returns (see Ritter [1991], Loughran and Ritter [1995], Nelson [1999], and Baker and Burglar [1999]).⁴ Our model can explain such “underperformance”: its source stems from the possibility to re-privatize publicly traded companies. In case the firm’s risky cash flows have fallen sufficiently, so that the gains from diversification have been diminished and there is no justification to incur the costs of being public (i.e., the associated loss in private benefits), the firm will be taken private. On average, the value of the option to re-privatize represents a larger proportion of total firm value for a company that has been listed recently than for a firm that has been traded for a longer

⁴ The empirical significance of these findings has been the subject of recent debate (see Brav and Gompers [1997] and Brav, Geczy, and Gompers [1999]).

period of time. Accordingly, the risk of recently issued “young” firms (for which this “put option” is a relatively large fraction of firm value) is smaller than the risk of “older” companies (with a relatively low “option” value). Hence, the returns on recently issued stock should be smaller than the returns on longer-listed shares.

Empirically, the option to re-privatize recently issued firms is not trivial. To see this, in Table I we present numbers (derived from Welch [1999]) that show the fraction of firms dropped from the exchange, liquidated, merged, or acquired within five years after their IPO. The striking empirical regularity documented by Welch [1999] is that almost half the firms that go public are de-listed, one way or another, within five years after the IPO. Admittedly, firms may be de-listed or absorbed into another firm not only to save the costs of running a public company. Yet, this large fraction of de-listings suggests that the option to re-privatize is important in understanding the public-private decision and its impact on firm value and firm risk.

The remainder of the paper is structured as follows. Section II presents the framework within which the entrepreneur’s decision to go public is analyzed. In Section III we derive the value function for the firm and characterize its properties. Section IV discusses the empirical implications of our model. In Section V we present two extensions of our framework. Finally, Section VI offers some concluding remarks.

II. Setup of the model

We consider a firm that is currently owned by an entrepreneur who may decide, *at the start of each of the coming periods*, whether to take the firm public or to keep it private. We assume that the decision to go public or to stay private is reversible: at any point in time the firm may be taken public (if it is private) or can be privatized again (in case it is public). Thus, effectively at any date the firm faces the question whether it should be private or public during the next period.

We separate the investment decisions from the question of ownership, taking the capital budgeting decisions of the firm as given by assuming that the firm has made all its investments. These investments generate a stream of uncertain cash flows to the firm's owners. (In periods in which the firm is private, the technology also returns a flow of "private benefits", which are specified below). We model the evolution of the cash flows (which are taken to be net of any necessary investments) in a binomial framework. Specifically, if at time t the cash flow is CF , then the cash flow at time $t+1$ will be either $u CF$ or $d CF$, where $u > 1 > d$. The states of nature attached to u or d are called the "up state" and the "down state", respectively. We consider a model with an infinite horizon in which u and d are time- and state-independent.

For every period in which the firm is private, the entrepreneur derives some "private benefits of control".⁵ These private benefits, denoted by PB , capture the private value of control as well as any savings of monitoring, bonding, and agency costs a public firm incurs due to the separation between ownership and control. PB can be viewed more generally as capturing any difference between the public and private value of a firm (see footnote 2). For now we assume that PB is some positive constant (an assumption we relax in Section V). Hence, at each pair $\{t,s\}$ of time and state the total stream of benefits from the firm is its cash flow $CF_0 u^s d^{t-s}$ if it is public (where CF_0 is the initial cash flow), and $CF_0 u^s d^{t-s} + PB$ in case the firm is private.

Next we specify the valuation in our model. We assume that the risk-free rate of return is $r > 0$ in all periods. We assume that risk-free investments are equally available to all agents so that the same risk-free rate r is used by both the entrepreneur as well as outside investors to discount risk-free cash flows. To value risky cash flows, we employ

⁵ We do not require that it is the *same* agent who owns the firm during every period in which it is privately held. However, to simplify the language, we refer to *the* entrepreneur throughout the paper, as if she were one.

the up state and another for the down state). These prices depend on whether the firm is private or public. The difference between the private and the public pairs of state prices captures the typical situation in which the entrepreneur is less diversified than the investors who own the firm when it is publicly traded.

If the firm is private, its flows are valued by the entrepreneur at her *private* state prices, which we denote by p_u for the up state and p_d for the down state. If the firm is publicly traded, the *public* state prices are given by q_u for the up state and q_d for the down state. Since both the entrepreneur and outside investors can invest in the risk-free asset, the sum of the private state prices and of the public state prices has to be the same:

$$(1) \quad p_u + p_d = q_u + q_d = \frac{1}{1+r} \equiv \frac{1}{R}$$

To capture the incomplete diversification of the entrepreneur's "portfolio", which makes her more averse to the firm's unique risks, we assume that $p_u < q_u$ and $p_d > q_d$. To see intuitively, why this *spread* between these prices captures the higher tolerance to the firm's risk of the well-diversified investor than that of the incompletely diversified owner, note that in the up state the entrepreneur has "too much" consumption relative to diversified investors. Hence, the private state price, p_u , is *smaller* than the public one, q_u . Similarly, because in the cash flow's down state the entrepreneur has "too little" consumption relative to a well-diversified investor, the private state price, p_d , is *greater* than the public one, q_d .⁶ Thus, these state prices capture the idea that diversification allows the entrepreneur to hedge against the down state by selling part of her consumption in the up state (at the market state prices q_u and q_d).

⁶ This intuition can be expressed formally by considering the state prices as the probability-adjusted marginal rates of substitution of an investor. If the utility function is concave, the assumed spread in these state prices will result from the lack of diversification.

Another intuitive way to interpret our assumption is based on the relative valuations both pairs of state prices imply for the firm's cash flows. Since well-diversified investors are "less averse" to the unique risk of the firm, we expect their valuation of the uncertain cash flows to be higher than the value a non-diversified entrepreneur attaches to the same stream. The following lemma shows that, given our assumptions, the public value of the firm's cash flows is higher than their private value:

Lemma: If $u > 1 > d$, $p_u < q_u$ and $p_d > q_d$, the private value of the uncertain cash flow stream is lower than its public value. That is:

$$(2) \quad CE^{Private} \equiv p_u \cdot u + p_d \cdot d < q_u \cdot u + q_d \cdot d \equiv CE^{Public}$$

where $CE^{Private}$ and CE^{Public} denote the private and public certainty equivalents, respectively, of the uncertain cash flow over the next period expressed per unit of current cash flow.

Proof: Since we assume that $p_u + p_d = q_u + q_d$, it follows that $q_u - p_u = p_d - q_d > 0$. Hence, as $u > d$, we have $(q_u - p_u) \cdot u > (p_d - q_d) \cdot d$, which can be rewritten to the result desired. \parallel

Lastly, we assume that $CE^{Public} < 1$, which guarantees that the value of the firm is always finite.⁷

⁷ The value of an always-public firm is given by:

$$\begin{aligned} V(\text{always public}) &= \sum_{t=1}^{\infty} \sum_{i=0}^t CF_0 \cdot \binom{i}{t} \cdot u^i \cdot d^{t-1} \cdot q_u^i \cdot q_d^{t-1} \\ &= \sum_{t=1}^{\infty} CF_0 \cdot (u \cdot q_u + d \cdot q_d)^t = CF_0 \cdot \sum_{t=1}^{\infty} (CE^{Public})^t \end{aligned}$$

This will be finite if and only if $CE^{Public} < 1$.

III. Value of the firm

In this section we define the value function of the firm and derive its properties. This value function is an option-like function which takes into account that, at any future date, the firm can either be taken public or bought out to become private again.

Consider some time t with an associated cash flow CF . If at the beginning of this date the firm is private, the entrepreneur receives the firm's cash flow, CF , plus the private benefits, PB . On the other hand, in case the firm is public at the beginning of date t , its shareholders only get the cash flow CF .

After receiving the cash flow and the private benefits (if the firm is currently private at t), the entrepreneur can choose whether the firm will be public or private in the *next* period. Since our model is stationary and has an infinite horizon, the value of the firm is a time-independent function of its cash flow, $V(CF)$. Now consider the case in which the firm has decided to stay private at time t . Then the next period, the entrepreneur's payoff will be $u \cdot CF + PB + V(uCF)$ in the up state and $d \cdot CF + PB + V(dCF)$ in the down state.

The value of the firm to the entrepreneur in this case is:

$$\begin{aligned}
 (3) \quad V^{Private}(CF) &= p_u * (u \cdot CF + PB + V(uCF)) + p_d * (d \cdot CF + PB + V(dCF)) \\
 &= CE^{Private} \cdot CF + \frac{PB}{R} + p_u \cdot V(uCF) + p_d \cdot V(dCF)
 \end{aligned}$$

Thus, the firm value is the sum of the value of the immediate cash flows, the immediate private benefits, and the future value of the firm all discounted at the private state prices. Analogously, the value of the firm in case the entrepreneur chooses to go public at date t is:

$$\begin{aligned}
 (4) \quad V^{Public}(CF) &= q_u * (u \cdot CF + V(uCF)) + q_d * (d \cdot CF + V(dCF)) \\
 &= CE^{Public} \cdot CF + q_u \cdot V(uCF) + q_d \cdot V(dCF)
 \end{aligned}$$

The decision at time t to be public or private during the next period given the firm's current cash flow CF depends on whether $V^{Public}(CF) \leq V^{Private}(CF)$. This gives the following (recursive) value function:⁸

$$(5) \quad \begin{aligned} V(CF) &= \text{Max}\{V^{Public}(CF), V^{Private}(CF)\} \\ &= \text{Max}\left\{ \begin{array}{l} CE^{Public} \cdot CF + q_u V(uCF) + q_d V(dCF), \\ CE^{Private} \cdot CF + \frac{PB}{R} + p_u V(uCF) + p_d V(dCF) \end{array} \right\} \end{aligned}$$

Note that the definition of the value function V implicitly assumes that upon re-privatization the entrepreneur will have to pay the full private value of the firm (i.e., including a premium above its public value). This is motivated by free-riding and holdup problems (e.g., see Grossman and Hart [1980]). Allowing for some other, say negotiation-driven, split of the difference between these private and public values of the firm will not change our results.

We divide our discussion of the properties of the value function into two parts. The next proposition derives some relatively obvious, asymptotic properties of the function so its proof is suppressed. The second proposition derives some deeper properties of the value function.

Proposition 1: The following are the asymptotic properties of the value function $V(CF)$:

- If the firm is always public its value is equal to:

$$V^{Public}(CF) = \left(\frac{CE^{Public}}{1 - CE^{Public}} \right) \cdot CF$$

⁸ As CE^{Public} , q_u , q_d , $CE^{Private}$, $1/R$, p_u and p_d are all less than one, the value function equations define a contraction, which means that a unique value function exists.

- If the firm is always private its value equals:

$$V^{Private}(CF) = \left(\frac{CE^{Private}}{1 - CE^{Private}} \right) \cdot CF + \frac{PB}{r}$$

- The Lemma implies that the slope of the “always public” function is greater than the slope of the “always private” value function: $\frac{CE^{Public}}{1 - CE^{Public}} > \frac{CE^{Private}}{1 - CE^{Private}}$
- For cash flows sufficiently large (or private benefits sufficiently small), the firm is public for many periods and its value can be approximated by:

$$V(CF) \approx \left(\frac{CE^{Pi}}{1 - CE^{Pi}} \right) \cdot CF$$

- If the current cash flow is equal to zero, the firm is always private and its value will be: $V(0) = \frac{PB}{r}$

The next proposition gives the monotonicity and convexity properties of the value function.

Proposition 2: The value function $V(CF)$ is continuous, increasing, and convex in CF .

Proof: Define the continuous, increasing, and convex function $W(CF, t, n)$ for a fixed horizon n by the following recursive relation:

$$(6) \quad W(CF, t, n) = \begin{cases} 0 & , t \geq n \\ \text{Max} \left\{ CE^{Private} \cdot CF + \frac{PB}{R}, CE^{Public} \cdot CF \right\} & , t = n - 1 \\ \text{Max} \left\{ \begin{array}{l} CE^{Private} \cdot CF + \frac{PB}{R} + p_u W(uCF, t + 1, n) \\ \quad + p_d W(dCF, t + 1, n), \\ CE^{Public} \cdot CF + q_u W(uCF, t + 1, n) \\ \quad + q_d W(dCF, t + 1, n), \end{array} \right\} & , t < n - 1 \end{cases}$$

V is the limit of $W(\cdot, \cdot, n)$ as n grows to infinity and inherits the properties of W . This means that continuity and the positive slope of V are immediate. It also means that as $CF \rightarrow 0$, the slope of $V \rightarrow \frac{CE^{Private}}{1 - CE^{Private}}$, and as $CF \rightarrow \infty$, the slope of $V \rightarrow \frac{CE^{Public}}{1 - CE^{Public}}$.

To prove convexity of $V(CF)$ we proceed by induction. The function $W(CF, n-1, n)$ is convex since it is the maximum of two linear functions and such a maximum is always convex. Now suppose that $W(CF, n-k, n)$ is convex. Then $W(CF, n-k-1, n)$ is convex as it is the maximum of two convex functions. This proves that $W(CF, t, n)$ is convex for any fixed horizon n . Since, keeping t fixed, $V(CF) = \lim_{n \rightarrow \infty} W(CF, t, n)$, this proves the convexity of V . \parallel

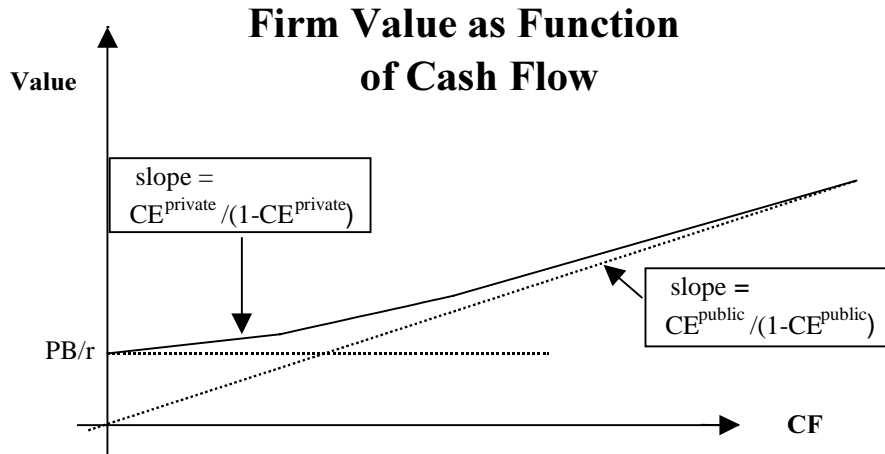


Figure 1.

Propositions 1 and 2 show that the value function looks like the graph shown in Figure 1. Note that the function looks like the value of a “call option” on the public value of the firm’s cash flows shifted upwards by PB/r , the present value of all future private benefits. Alternatively, the value function can be viewed as the sum of:

- The value of the risky cash flows as if the company will always be public, and;
- The value of a “put option” allowing entrepreneurs to reclaim (the present value of) the flow of private benefits (in addition to the firm’s stream of uncertain cash flows).

The functional form of the value function suggests that at low cash flow levels the firm is private while at higher levels it is public. This is indeed the case as the next two propositions, which begin our characterization of the optimal timing of IPOs, show.

Proposition 3: Suppose that, at time t with current cash flow CF , it is optimal to keep the firm private for the next period, meaning that $V^{private}(CF) > V^{public}(CF)$. Thus:

$$(7) \quad V(CF) = CE^{Private} \cdot CF + \frac{PB}{R} + p_u \cdot V(uCF) + p_d \cdot V(dCF)$$

(We call this “ V is private at CF ”). Then V is also private for any cash flow $X < CF$.

Proof: $V(CF)$ is private means that:

$$(8) \quad CE^{Private} \cdot CF + \frac{PB}{R} + p_u V(uCF) + p_d V(dCF) > CE^{Public} \cdot CF + q_u V(uCF) + q_d V(dCF)$$

Defining $D \equiv q_u - p_u = p_d - q_d > 0$ and rewriting (8), we get that $V(CF)$ is private if:

$$(9) \quad (CE^{Public} - CE^{Private}) \cdot CF + D \cdot [V(uCF) - V(dCF)] < \frac{PB}{R}$$

Intuitively, $V(CF)$ is private if the gains from diversification, both on the immediate cash flows of the current date and on the expected value at the end of the next period, are smaller than the loss of this period’s private benefits. To prove that $V(X)$ is private for all $X < CF$, we have to show that the condition holds for all $X < CF$:

$$(10) \quad (CE^{Public} - CE^{Private}) \cdot X + D \cdot [V(uX) - V(dX)] < \frac{PB}{R}$$

This is true since $CE^{Public} > CE^{Private}$, which implies that:

$$(11) \quad (CE^{Public} - CE^{Private}) \cdot X < (CE^{Public} - CE^{Private}) \cdot CF$$

and as $D > 0$, $X < CF$, and $V(CF)$ is convex in CF , which means that:

$$(12) \quad D \cdot [V(uX) - V(dX)] < D \cdot [V(uCF) - V(dCF)] \quad \parallel$$

Proposition 4: Similarly, suppose that, at time t with current cash flow CF , it is optimal for the firm to be public in the next period, meaning that $V^{Public}(CF) > V^{Private}(CF)$. Thus:

$$(13) \quad V(CF) = CE^{Public} \cdot CF + q_u V(uCF) + q_d V(dCF)$$

(We call this “ V is public at CF ”). Then $V(Y)$ is also public at any $Y > CF$.⁹

⁹ The proof of Proposition 4 is similar to that of Proposition 3 above.

Propositions 3 and 4 imply that there is some critical cash flow level, CF^* , such that for all cash flows greater than or equal to CF^* the firm is public and for all cash flows below CF^* the firm is private. CF^* is that cash flow at which the firm's value as a public company for the next period just equals the value of the firm as a privately held entity. The firm goes public when its cash flow rises above CF^* and it is re-privatized when the cash flow falls below CF^* .¹⁰ The following figure plots this *public-private* decision rule:

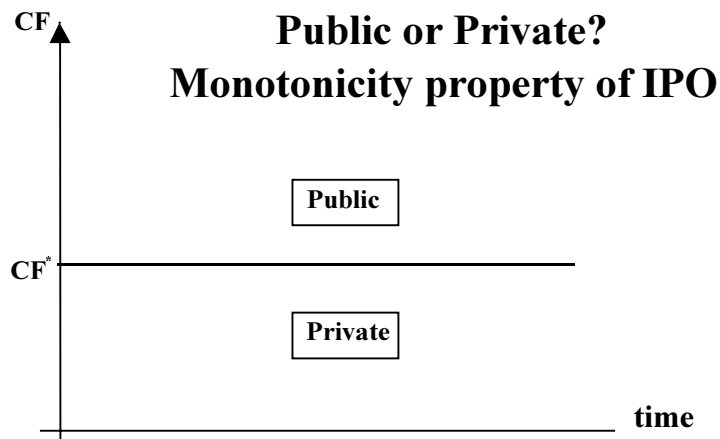


Figure 2.

Based on the “monotonicity property” derived from Propositions 3 and 4 and on the stationarity of all the parameters in the model it is trivial to prove the following “triangular” properties:

¹⁰ So far we have ignored any switching costs of going public or private. We consider such costs in Section V.

Proposition 5: If V is public (private) at uCF and at dCF , then V is also public (private) at CF .

Proposition 6: If V is public (private) today at CF , then V is public (private) in at least one state of the world tomorrow.

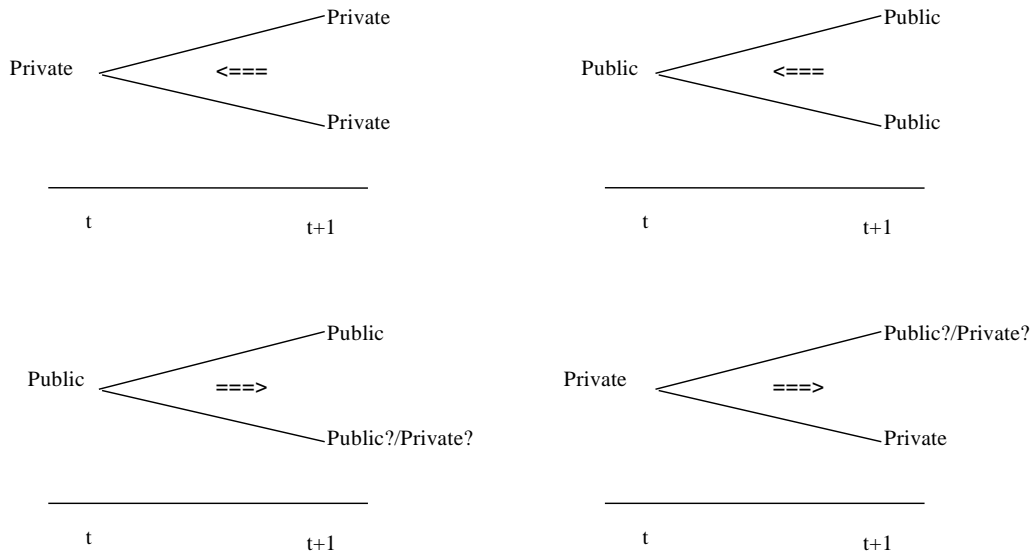


Figure 3.

These key properties of our value function and of the optimal timing of IPOs can be derived without specifying the probabilities of the up and down state (i.e., without any specification of the expected return of the firm's activities). Next, we characterize the evolution of the risk of the firm over time and provide a sufficient condition for the value of the firm to rise, on average, over time (i.e., for the expected return to be positive in terms of capital gains).

Proposition 7: Denote the probability of the up state by π and the rate of return of the firm by $r_s \equiv \frac{V(s \cdot CF) - V(CF)}{V(CF)}$, where $s \in \{u, d\}$. Then the variance of this return,

$Var(r_s)$, is increasing in the firm's cash flow.

Proof: The variance of the firm's rate of return can be expressed by:

$$(14) \quad Var(r_s) = \pi \cdot (1 - \pi) \cdot \left[\frac{V(uCF) - V(dCF)}{V(CF)} \right]^2$$

Since $0 < \pi < 1$, we have to prove that for all $Y > X$:

$$(15) \quad \frac{V(uY) - V(dY)}{V(Y)} > \frac{V(uX) - V(dX)}{V(X)}$$

As $V(CF)$ is convex, we know that for $Y > X$:

$$(16) \quad \frac{V(uY) - V(dY)}{(u - d) \cdot Y} > \frac{V(uX) - V(dX)}{(u - d) \cdot X}$$

Sufficient for the inequality in equation (15) to hold is that $\frac{V(Y)}{Y} < \frac{V(X)}{X}$ for all $Y > X$

(i.e., the function $V(X)/X$ declines monotonically in X). This condition follows from Propositions 1 and 2, which jointly imply that:

$$(17) \quad V'(X) \leq \frac{CE^{Public}}{1 - CE^{Public}} < \frac{V(X)}{X} \quad \parallel$$

Proposition 7 means that if the firm's cash flow grows over time, the variance of its rate of return will increase as well. This is an outcome of the convexity of the value function, which reflects the option to re-privatize publicly traded firms. Note that the impact of this convexity of $V(CF)$ on the variance of the return is more pronounced at lower levels of the current cash flow since this "put option" has little impact on firm value at high cash flow levels.

Proposition 8: If $\pi \cdot u + (1-\pi) \cdot d \geq 1$, the expected value of the firm at the end of the next period will be higher than its current value: $\pi \cdot V(uCF) + (1-\pi) \cdot V(dCF) > V(CF)$ (i.e., the expected rate of return in terms of capital gains, $E(r_s)$, is positive).

Proof: Let π^* be such that $\pi^* \cdot u + (1-\pi^*) \cdot d = 1$. Then, by the convexity of $V(CF)$:

$$(18) \quad \pi^* \cdot V(uCF) + (1-\pi^*) \cdot V(dCF) \geq V(CF)$$

with a strict inequality if $V(CF)$ is strictly convex. As $u > d$, $\pi \cdot u + (1-\pi) \cdot d > 1$ for all $\pi > \pi^*$. Since $V(CF)$ is monotone, for all $\pi > \pi^*$ it holds that:

$$(19) \quad \pi \cdot V(uCF) + (1-\pi) \cdot V(dCF) > \pi^* \cdot V(uCF) + (1-\pi^*) \cdot V(dCF) \geq V(CF). \quad \parallel$$

The intuition underlying Proposition 8 is based on the fact that (in case the expected cash flow is non-decreasing over time) the dispersion of the cash flows the firm can possibly generate strictly increases over time. Since the shareholders have the option to re-privatize the firm, which makes firm value a convex function of the cash flows, this increase in the cash flow dispersion causes the expected value of the firm to rise over time. This means that the expected rate of return on the firm's activities (in terms of capital gains) is positive even if the cash flows are expected to remain constant over time and, even more so, in case the current cash flow is expected to grow.

IV. Empirical Implications

In the preceding sections we characterized the timing of the decision to go public (to re-privatize) based on the tradeoff between private benefits of control and better diversification of the firm's risk. We showed that the optimal timing of an IPO occurs when the firm's cash flow rises above a certain critical level, which we denoted by CF^* . At this cash flow level, the value of the firm as a privately held entity is equal to its value as a publicly traded company. The reverse is true for the decision to take the company private again: it is optimal to buy out the firm (e.g., by an MBO, an LBO, an acquisition

by private parties, etc.) when its cash flow falls below CF^* . Because the company can be re-privatized, firm value is a convex function of its cash flows. Intuitively, the value function looks like the present value that well-diversified investors attach to the cash flows the firm is expected to generate plus the value of a “put option” allowing entrepreneurs to reclaim (the present value of) the private benefits. The characterization of the optimal timing of IPOs and the resulting value function have several empirical implications, which we discuss in this section.

First, consider the phenomenon of “hot issue markets”: the observation that many firms go public at about the same time, typically when the values of already-traded firms are high. Our results are consistent with this phenomenon and explain it via the cross-sectional correlation in the profitability of firms. Since changes in macro-economic conditions (e.g., due to cyclicalities) simultaneously affect multiple industries and companies, firm profitability tends to be positively correlated. In particular, good economic circumstances positively impact the cash flows of many firms. Our model predicts that firms go public when their cash flows are high, which means that when one firm finds it optimal to issue stock so do other firms. Therefore, our model predicts that IPOs will come in waves. Furthermore, since the correlation between the cash flows of firms within the same industry is likely to be greater than the cross-sectional correlation at large, our results are consistent with the industry concentration that characterizes waves in IPOs. Finally, good economic conditions impact the cash flows of both publicly traded and privately held firms. Hence, the waves in IPOs, which happen when the cash flows of the issuing firms are high, occur when the cash flows of publicly traded firms are high as well. Thus, IPO waves coincide with times of relatively high share prices in general (i.e., a boom on the stock market).

The discussion above shows that the model predicts some of the stylized facts regarding IPOs well-documented in the literature. Based on our model, we can also derive implications for the patterns we expect to observe in the reverse, mirror

transaction: the re-privatization of firms. Specifically, our model predicts that going-private transactions (by an MBO, LBO, or otherwise) will occur in waves as well and that these waves will coincide with times of relatively low stock prices. The model also predicts that these waves in buyouts will be concentrated in specific industries for which cash flows are especially low.

There are several papers supporting our conclusions on the clustering of going-private transactions. Kaplan and Stein [1993], for example, document the “hot privatization market” of the 1980s. Lehn and Poulsen [1989] as well as Opler and Titman [1993] have studied about the same time frame and conclude that the buyouts they examined were mainly caused by the great amount of financial slack and large free cash flows within the firms which were privatized. This is perfectly consistent with our story that when the private benefits, such as the agency costs associated with free cash flows (see Jensen [1986]), are large relative to the firm’s cash flow, the company will be privatized again to reclaim these opportunity costs. Kaplan [1991] has studied such firms in the period after their LBO and discovered that most of these re-privatizations were not permanent, but went public again after some form of reorganization (the firms were private for a median period of 6,8 years). This finding is also consistent with the tradeoff underlying our main story. If the private benefits (in the form of agency costs associated to free cash flows) are large, the company goes private. While it is private, the firm will be reorganized to recapture this free cash flow and internalize it into the “regular” cash flow of the firm, *CF*. When this is done and the private benefits (agency costs) have diminished, the advantages from diversification are too large to justify staying private any longer, and the firm goes public again. In this respect, the model is able to describe the sequence of transactions as documented in the literature.

There is another way in which our model is able to explain the clustering of IPOs over time in “hot issue markets”. This explanation is related to learning by outside investors and takes a somewhat different angle for interpreting the nature of the private

benefits. These benefits can also be regarded to represent any costs of information gathered by outsiders to learn more about the activities of the company. Now, consider a particular industry with a number of firms which are still privately held. These firms probably all face the same level of private benefits as viewed to be the costs of gathering information. The companies can be ordered according to the level of their current cash flow, and now, let's suppose that the firm with the highest cash flow within the industry finds it worthwhile to go public. During and after this IPO outside investors learn more about the company and its activities. But, they also learn about the whole industry and implicitly this learning by outsiders affects the level of private benefits of the remaining (private) firms (i.e., it lowers the costs of gathering information about their activities). Due to this externality, the firm with the next highest cash flow now may also find it worthwhile to go public, since its private benefits have diminished while the advantages from diversification are still the same and appealing. This way an iterative process starts which stops until of all private firms within the industry the company with the next highest cash flow will want to remain private based on its own tradeoff between being private and becoming public. A direct consequence of our reasoning here is that in case outsiders are able to learn very quickly about a new industry and its activities, that there will be a waves in IPOs of firms active in that business. In such a way, the externality of learning in general affects not only the company that goes public, but also other private firms within the same industry or related sectors of the economy.

One of the more puzzling observations regarding IPOs is the “underperformance” of recently issued shares relative to the return of the market over several years following the initial offering.¹¹ The benchmark for the market return has been measured by the return of several indices; common to all of them is that they are

¹¹ Recall from our introduction that several papers have questioned the statistical support for this finding.

largely comprised of relatively “old” firms which have been publicly traded for a long time. Thus, the underperformance of recently offered stock is measured relative to the performance of longer-listed shares.

The results of our model suggest a possible explanation for such underperformance. This explanation is based on our characterization of the value of the firm as the sum of two values: the “public” value of the risky cash flows and the value of the option to re-privatize the firm for its stream of private benefits. The decomposition of firm value into this cash flow value and that of the “put option” implies a similar breakdown of firm risk. Specifically, the risk of the company is the weighted average of the risk of the cash flows and of the risk of the option to re-privatize. The weights of these two risk components are equal to the fractions of their values relative to total firm value. Typically, cash flows have more systematic (i.e., priced) risk than put options (which may even exhibit *negative* risk as their value usually moves in the opposite direction to the economy at large). Moreover, the option to re-privatize has a proportionally higher value for firms with borderline cash flows (i.e., close to CF^*) than for companies with high levels of cash flow. Thus, the “put option” represents a larger proportion of firm value for recently listed shares than for longer-traded stock, since the cash flows of these “young” firms, by definition, are close to CF^* , while the “older” (public) companies have higher cash flows. Hence, it follows that the average risk of recently issued shares is lower than that of longer-listed stock.

Looking at this point from a different angle, consider the cross section of “old” firms, which have been traded for quite some time. The average cash flow of these companies will be considerably higher than the critical cash flow CF^* . Therefore, given the results of our model, the risk of those longer-listed stocks is greater than the risk of recently offered firms, which have cash flows close to CF^* . Consequently, the returns the shareholders of these “older” companies demand will have to be higher and thus justifiably “outperform” the rates of return on recently issued shares.

V. Two extensions of the model

In this section we consider two extensions of our model: allowing for switching costs between being private and being public, and for private benefits to increase in the firm's cash flow. We show that these extensions do not invalidate our main results about the optimal timing of IPOs and the resulting properties of firm value and firm risk.

Extension 1: Switching costs

In our basic model, we do not explicitly consider any costs of switching from being private to being public or vice versa. Here, we show how such costs, which may include IPO discounts due to underpricing and premiums paid in going-private transactions, can be incorporated into the model without affecting our main intuition and results. The only difference between the models with and without switching costs is that in case there are such costs we have two value functions: one for the value of the firm if it is private and the other for its value when the company is public.

To see this, we consider a finite horizon of n and switching costs equal to X . Suppose we are one period before the horizon (i.e., $t = n-1$). Furthermore, suppose that the firm is currently private and its cash flow is equal to CF . Then the firm's value is:

$$(20) \quad W(CF, t, n, \text{private}) \equiv W(CF, t, n, 0) = \text{Max} \left\{ CE^{\text{private}} \cdot CF + \frac{PB}{R}, CE^{\text{public}} \cdot CF - X \right\} \quad t = n-1$$

where the "state of being private" is denoted by "0" (we will denote the "state of being public" by "1"). Note that the value function for the private firm incorporates a cost X of switching from private to public. Similarly if the firm is public at $t = n-1$:

$$(21) \quad W(CF, t, n, \text{public}) \equiv W(CF, t, n, 1) = \text{Max} \left\{ CE^{\text{private}} \cdot CF + \frac{PB}{R} - X, CE^{\text{public}} \cdot CF \right\} \quad t = n-1$$

Now suppose the firm is at $t = n-2$. Then if the firm is private (i.e., the state is "0"), we have:

$$\begin{aligned}
& W(CF, t, n, 0) \\
(22) \quad & = \text{Max} \left\{ \begin{array}{l} CE^{\text{Private}} \cdot CF + \frac{PB}{R} + p_u W(uCF, t, n, 0) + p_d W(dCF, t, n, 0), \\ CE^{\text{Public}} \cdot CF - X + q_u W(uCF, t, n, 1) + q_d W(dCF, t, n, 1) \end{array} \right\}, \quad t = n-2
\end{aligned}$$

Similarly, if at $t = n-2$ the firm is public, we get:

$$\begin{aligned}
& W(CF, t, n, 1) \\
(23) \quad & = \text{Max} \left\{ \begin{array}{l} CE^{\text{Private}} \cdot CF + \frac{PB}{R} - X + p_u W(uCF, t, n, 0) + p_d W(dCF, t, n, 0), \\ CE^{\text{Public}} \cdot CF + q_u W(uCF, t, n, 1) + q_d W(dCF, t, n, 1) \end{array} \right\}, \quad t = n-2
\end{aligned}$$

Continuing this recursively, we can derive:

$$\begin{aligned}
& W(CF, t, n, \text{state} = 0) \\
& = \begin{cases} 0 & , t \geq n \\ \text{Max} \left\{ CE_p \cdot CF + \frac{PB}{R}, CE_q \cdot CF - X \right\} & , t = n-1 \\ \text{Max} \left\{ \begin{array}{l} CE_p \cdot CF + \frac{PB}{R} + p_u W(uCF, t+1, 0) + p_d W(dCF, t+1, 0), \\ CE_q \cdot CF + q_u W(uCF, t+1, 1) + q_d W(dCF, t+1, 1) - X, \end{array} \right\} & , t < n-1 \end{cases} \\
& W(CF, t, n, \text{state} = 1) \\
(24) \quad & = \begin{cases} 0 & , t \geq n \\ \text{Max} \left\{ CE_p \cdot CF + \frac{PB}{R} - X, CE_q \cdot CF \right\} & , t = n-1 \\ \text{Max} \left\{ \begin{array}{l} CE_p \cdot CF + \frac{PB}{R} + p_u W(uCF, t+1, 0) + p_d W(dCF, t+1, 0) - X, \\ CE_q \cdot CF + q_u W(uCF, t+1, 1) + q_d W(dCF, t+1, 1), \end{array} \right\} & , t < n-1 \end{cases}
\end{aligned}$$

Taking the limit of these finite-horizon functions such that $n \rightarrow \infty$, we get the value functions for an infinite horizon:

$$V(CF,0) = \lim_{n \rightarrow \infty} W(CF,t,n,0), V(CF,1) = \lim_{n \rightarrow \infty} W(CF,t,n,1).$$

Now, we can establish the following properties of the functions $V(CF,h)$ for $h = 0$ and 1 separately, in the same way as we did for the case without any switching costs.

Proposition 9 $V(CF,h)$, for $h = 0$ and 1, has the following properties:

- Both value functions are increasing in CF .
- Both value functions are convex.
- Both value functions exhibit the “triangular” properties.

It is important to remember, however, that now there are *two* distinct value functions for the firm. In the graph below we plot the difference between $V(CF,0) - V(CF,1)$ for a particular set of numerical values.

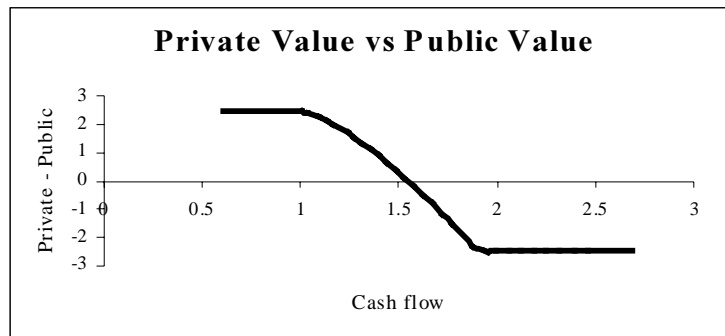


Figure 4.

At very low cash flows ($CF < 0.95$ in the graph), the firm will stay private if it is already private or will become private if it is currently public. Therefore, the difference between the two value functions is equal to the switching cost X . Similarly if the cash flows are

high ($CF > 1.94$ in the graph), the firm will remain public if it is already public or become public if it happens to be private. Therefore, the difference between the value functions for high cash flows is $-X$. For intermediate cash flows (between 0.95 and 1.94), the costs X are greater than the potential gain from any switch. Thus, for this range of cash flows, the firm will remain private (public) for one more period if it is currently private (public). The difference between the two functions moves from X to $-X$ as the probability of a switch from being private to being public increases.

Extension 2: Linear private benefits

In the basic model we assumed that the private benefits are fixed at some constant level PB . In this part we replace that assumption and take the private benefits to be increasing linearly in the cash flows of the firm: $PB = a + b \cdot CF$. This means that if the firm is currently private, the entrepreneur's payoff in the up state becomes:

$$(25) \quad u \cdot CF + (a + b \cdot u \cdot CF) + V(uCF) = (1 + b)uCF + a + V(uCF)$$

and in the down state her payoff becomes:

$$(26) \quad d \cdot CF + (a + b \cdot d \cdot CF) + V(dCF) = (1 + b)dCF + a + V(dCF).$$

Thus the value function $V^{private}(CF)$, as it was given in equation (3), changes to:

$$(27) \quad V^{private}(CF) = (1 + b) \cdot CE^{private} \cdot CF + \frac{a}{R} + p_u \cdot V(uCF) + p_d \cdot V(dCF)$$

while $V^{public}(CF)$ remains as defined in Section III. Thus, by simply redefining the cash flow CF in case the firm is private to include a multiplication by $(1+b)$ we can keep all of our analysis intact. For our problem to remain interesting, however, we need to ensure that the newly defined private benefits do not dominate the diversification gains of going public. Assuming that the parameter a is positive, this means that the slope of the value function in case the firm is always private has to be smaller than the slope of the function for the case in which the firm will be public forever. Formally we can express this by:

$$(28) \quad \left(\frac{CE^{Private}}{1 - CE^{Private}} \right) \cdot (1 + b) < \left(\frac{CE^{Public}}{1 - CE^{Public}} \right)$$

which can be rewritten to:

$$(29) \quad b < \left(\frac{CE^{Public} - CE^{Private}}{(1 - CE^{Public}) \cdot CE^{Private}} \right)$$

This inequality has to hold since otherwise entrepreneurs will never take their firms public. In addition, we can derive a sufficient condition for our “triangular” properties to hold, which is:

$$(30) \quad CE^{Private} \cdot (1 + b) < CE^{Public}$$

Since $CE^{Public} < 1$, this condition is stricter than the inequality given in equation (29). Thus, by assuming parameter b is small enough for the condition in equation (30) to hold, we can keep our model’s results intact, including the characterization of the optimal timing of IPOs and buyouts, and the resulting properties for firm value and risk.

VI. Conclusions

This paper studies the timing dimension of the decision to go public. The current literature on IPOs has considered this decision as a single shot: entrepreneurs have but one chance to take their firm public. We complement this literature by examining the ability of entrepreneurs to *time* their IPO and also investigate the possibility to *re-privatize* publicly traded firms.

In our model, the entrepreneur trades off the gains of diversification against the benefits of being private. During times in which cash flows are sufficiently high, the potential advantages from diversification outweigh these private benefits and the firm goes public. As entrepreneurs can choose to take their firm public at any date and reverse this choice later on, the decision to go public reflects more than the *immediate* costs and benefits.

We characterize the optimal timing of IPOs and derive implications for firm value and firm risk. Our results are consistent with several empirical regularities:

- The documented clustering of IPOs over time in “hot issue markets”, which are often disproportionately populated with firms in a particular industry. Moreover, we derive mirror implications regarding the timing of going-private transactions.
- The stylized fact that waves in IPOs coincide with times of relatively high stock prices. Again, the mirror prediction of our model regarding waves in buyouts is that these transactions coincide with periods of relatively low stock prices.
- The puzzling below-market returns recently issued shares have earned, over several years following the IPO, relative to the stock returns of companies that have been listed a long time ago.

We also show that the model’s results and its properties for firm value and firm risk given our characterization of the timing of IPOs are robust to two extensions of the basic model. The first extension allows for switching costs: transaction costs of going public or costs attached to buyouts. The second extension allows for private benefits increasing in the firm’s cash flow.

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Table I: The Fraction of Firms De-listed within Five Years after their IPO

The table below shows the number of firms that initially sold their shares to the public by year and the number and fraction of the IPOs that were dropped from the exchange, liquidated (“deleted”), or merged / acquired (“M&A”) within five years after the IPO. The source of these data is Welch [1999].

| Year | # of IPOs in year | # of IPOs deleted | % of IPOs deleted | # M&A | % of IPOs M&A | Total # of IPOs de-listed | % of IPOs de-listed |
|-------------|------------------------------|------------------------------|------------------------------|------------------|------------------------------|--|--------------------------------|
| 1980 | 30 | 2 | 7% | 15 | 50% | 17 | 57% |
| 1981 | 21 | 6 | 29% | 9 | 43% | 15 | 71% |
| 1982 | 17 | 8 | 47% | 4 | 24% | 12 | 71% |
| 1983 | 60 | 8 | 13% | 18 | 30% | 26 | 43% |
| 1984 | 64 | 9 | 14% | 25 | 39% | 34 | 53% |
| 1985 | 109 | 24 | 22% | 41 | 38% | 65 | 60% |
| 1986 | 327 | 68 | 21% | 114 | 35% | 182 | 56% |
| 1987 | 412 | 74 | 18% | 120 | 29% | 194 | 47% |
| 1988 | 242 | 33 | 14% | 60 | 25% | 93 | 38% |
| 1989 | 197 | 22 | 11% | 44 | 22% | 66 | 34% |
| 1990 | 209 | 22 | 11% | 44 | 21% | 66 | 32% |
| 1991 | 394 | 30 | 8% | 108 | 27% | 138 | 35% |
| 1992 | 684 | 76 | 11% | 143 | 21% | 219 | 32% |
| 1993 | 1034 | 106 | 10% | 176 | 17% | 282 | 27% |
| 1994 | 839 | 79 | 9% | 102 | 12% | 181 | 22% |
| | Average | | 16% | | 29% | | 45% |