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Brane Solutions of Gravity–Dilaton–Axion Systems

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Abstract. We consider general properties of brane solutions of gravity-dilaton-axion systems. We focus on the case of 7-branes and instantons. In both cases we show that besides the standard solutions there are new deformed solutions whose charges take value in any of the three conjugacy classes of $SL(2, \mathbb{R})$. In the case of 7-branes we find that for each conjugacy class the 7-brane solutions are 1/2 BPS. Next, we discuss the relation of the 7-branes with the DW/QFT correspondence. In particular, we show that the two (inequivalent) 7-brane solutions in the $SO(2)$ conjugacy class have a nice interpretation as a distribution of (the so-called near horizon limit of) branes. This suggests a way to define the near-horizon limit of a 7-brane.

In the case of instantons only the solutions corresponding to the \mathbb{R} conjugacy class are 1/2 BPS. The solutions corresponding to the other two conjugacy classes correspond to non-extremal deformations. We first discuss an alternative description of these solutions as the geodesic motion of a particle in a two-dimensional AdS_2 space. Next, we discuss the instanton-soliton correspondence. In particular, we show that for two of the conjugacy classes the instanton action in D dimensions is given by the mass of the corresponding soliton which is a (non-extremal) black hole solution in $D+1$ dimension. We speculate on the role of the non-extremal instantons in calculating higher-derivative corrections to the string effective action and, after a generalization from a flat to a curved AdS_5 background, on their role in the AdS/CFT correspondence.

INTRODUCTION

Gravity coupled to the two scalars (dilaton and axion) that parametrise an $SL(2, \mathbb{R})/SO(2)$ coset space is an important subsector of the low-energy limit of type IIB superstring theory. Among the different solutions of this system are seven-brane solutions that carry magnetic charges with respect to the three generators of $SL(2, \mathbb{R})$. These magnetic charges combine into a traceless 2×2 charge matrix Q which transforms in the adjoint representation of $SL(2, \mathbb{R})$. The combination $\det(Q)$, being invariant under these transformations, labels the three different conjugacy classes of $SL(2, \mathbb{R})$. Each pair of solutions in the same conjugacy class is related via $SL(2, \mathbb{R})$. On the other hand, two solutions that belong to two different conjugacy classes can not be related via $SL(2, \mathbb{R})$. The “circular” 1/2 BPS D7^c-brane of [1] is represented by the $\det Q = 0$ conjugacy class.

It is well-known that the electric-magnetic dual of the D7-brane is the D-instanton [2]. The D-instanton is a half-supersymmetric solution of the Euclidean gravity-dilaton-

axion system, and carries electric charge with respect to the Euclidean $SL(2, \mathbb{R})$ symmetry. In complete analogy to the case of seven-branes, the three Euclidean $SL(2, \mathbb{R})$ charges combine into a 2×2 charge matrix Q that transforms in the adjoint representation of the Euclidean $SL(2, \mathbb{R})$. The D-instanton is represented by the same conjugacy class that represents the circular $D7^c$ -brane, i.e. the one with $\det(Q) = 0$.

It is natural to ask whether there exist 7-branes and instantons with $SL(2, \mathbb{R})$ charges corresponding to the other two conjugacy classes of $SL(2, \mathbb{R})$. It is the aim of this work to construct and investigate such solutions. This talk is a summary of [3, 4, 5, 6, 7].

GRAVITY-DILATON-AXION SYSTEMS

In this section we briefly review some basic properties of the gravity-dilaton axion system. The basic fields are the metric $g_{\mu\nu}$, the dilaton ϕ and the axion χ . An axion, as opposed to a dilaton, only occurs via its spacetime derivative in the Lagrangian. In D-dimensional Minkowski spacetime this Lagrangian is given by

$$\mathcal{L}_M = \frac{1}{2} \sqrt{|g|} \left[R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} e^{b\phi} (\partial\chi)^2 \right], \quad (1)$$

where $b \neq 0$ is an arbitrary dilaton coupling parameter. The Lagrangian (1) is invariant under a nonlinear $SL(2, \mathbb{R})$ symmetry. For $b = 0$ this symmetry reduces to

$$SL(2, \mathbb{R}) \rightarrow ISO(2). \quad (2)$$

From now on we will assume that $b \neq 0$. The case $b = 0$ should be treated separately and has been discussed in [8].

The two scalars ϕ and χ parametrize the coset

$$\frac{SL(2, \mathbb{R})}{SO(2)}. \quad (3)$$

This can be made manifest by combining ϕ and χ into the following $SL(2, \mathbb{R})$ matrix \mathcal{M} :

$$\mathcal{M} = e^{b\phi/2} \begin{pmatrix} \frac{1}{4} b^2 \chi^2 + e^{-b\phi} & \frac{1}{2} b \chi \\ \frac{1}{2} b \chi & 1 \end{pmatrix}. \quad (4)$$

In terms of \mathcal{M} the $SL(2, \mathbb{R})$ symmetry is given by

$$\mathcal{M} \rightarrow \Omega \mathcal{M} \Omega^T \quad \text{with} \quad \Omega \in SL(2, \mathbb{R}). \quad (5)$$

Note that we defined the gravity-dilaton-axion system in Minkowski spacetime. One can show that similar formulae hold in Euclidean space. In the Euclidean case the axion and dilaton parametrise the coset

$$\frac{SL(2, \mathbb{R})}{SO(1, 1)}. \quad (6)$$

TABLE 1. The three conjugacy classes of $SL(2, \mathbb{R})$

$\det Q < 0$	$\det Q = 0$	$\det Q > 0$
SO(1,1)	\mathbb{R}	SO(2)

For more details, see [7].

In the following discussion we will keep the dimension D and the dilaton coupling parameter $b \neq 0$ arbitrary. The standard example is $D = 10$ and $b = 2$ corresponding to IIB supergravity. Other cases may correspond to (truncations of) compactifications of N=2 supergravity. For instance, the Euclidean Lagrangian for the universal hypermultiplet that arises from a Calabi-Yau compactification of type II strings [9, 10] can be truncated to a D=4 Euclidean gravity-dilaton-axion system with $b = 1$ or $b = 2$.

Given the $SL(2, \mathbb{R})$ symmetry one can define corresponding Noether currents

$$J_\mu \sim (\partial_\mu \mathcal{M}) \mathcal{M}^{-1}. \quad (7)$$

For the 7-brane and instanton solutions that we study here one can define corresponding Noether charges¹

$$\begin{aligned} 7\text{-branes} : Q &\sim \int_{S^1} J, \\ \text{instantons} : Q &\sim \int_{S^{D-1}} J. \end{aligned} \quad (8)$$

These charges transform under $SL(2, \mathbb{R})$ as follows:

$$Q \rightarrow \Omega Q \Omega^{-1}. \quad (9)$$

We now come to an important point that will be crucial for the remaining part of this work. From the above transformation rule we see that the determinant $\det Q$ is *invariant* under the $SL(2, \mathbb{R})$ transformations. This means that we have a family of distinct conjugacy classes which are labelled by the value of $\det Q$. It is natural to distinguish between the three cases indicated in table 1. In this table we have also indicated the one-dimensional subgroup of $SL(2, \mathbb{R})$ associated to each of the three types of conjugacy classes. This association means that each element g of the given conjugacy class can be written as an element h of the corresponding subgroup conjugated with an arbitrary $SL(2, \mathbb{R})$ group element Ω , i.e.

$$g = \Omega h \Omega^{-1}. \quad (10)$$

¹ For nonlinear symmetries, like $SL(2, \mathbb{R})$, it is *a priori* not guaranteed that the integrals of the currents are finite. In our case, where the solutions depend on only one coordinate, the integrals are finite. We thank E. Ivanov for a discussion on this point.

Any 7-brane or instanton solution of the gravity-dilaton-instanton system will carry $SL(2, \mathbb{R})$ charges that fall into one of the three types of conjugacy classes of table 1. Let us first consider the standard D7-brane solution of IIB string theory. In general we wish to consider branes with *two* transverse directions, i.e. $(D - 3)$ -branes in D dimensions. For simplicity we will often call these branes just 7-branes instead of $(D - 3)$ -branes. The position of the D7-brane is often indicated by the diagram

$$\text{D7} : \times | \times \times \times \times \times \times \times \times \text{---} \quad (11)$$

where $\times(-)$ indicates a worldvolume (transverse) direction. The first \times before the $|$ indicates the worldvolume time direction. A $(D - 3)$ -brane naturally couples to a C_{D-2} -form which is just the dual of the axion:

$$\partial\chi \sim (\partial C_{D-2})^* \quad (12)$$

The special thing about a brane with *two* transverse directions is that the harmonic function over this space is given by a logarithm

$$H(r) \sim \ln r, \quad (13)$$

where r indicates the radial transverse direction. This solution is half-supersymmetric. It turns out that it is not possible to reduce this brane solution over one of the two transverse directions to a half-supersymmetric domain wall solution in one dimension lower. Neither can one define the near-horizon limit of the standard D7-brane solution. Nevertheless, as we will see, the analysis below will suggest a way to define the near-horizon limit which brings the 7-branes on the same footing with the other branes in the so-called DW/QFT correspondence [11, 12].

In order to dualize the 7-brane to, for instance, the D8 brane solution of type IIA string theory it is necessary to introduce the so-called *circular* $D7^c$ -brane [1] which has an extra isometry in one of the two transverse directions. The harmonic in the remaining single direction r is given by

$$H(r) \sim r. \quad (14)$$

Indeed, one can show that this $D7^c$ -brane solution is T-dual to the D8-brane of type IIA string theory. It turns out that both the D7-brane and the circular $D7^c$ -brane have charges that are in the $\det Q = 0$ conjugacy class of $SL(2, \mathbb{R})$.

We next consider the Euclidean D-instanton solution of [2]. The D-instanton can be viewed as the extreme case of a $D(-1)$ brane in the family of T-dual D_p -brane solutions with only transverse and no worldvolume directions. The corresponding diagram is given by

$$\text{D-1} : \text{-----} \quad (15)$$

The solution can be given in terms of a harmonic over the D -dimensional Euclidean transverse space:

$$H(r) \sim 1/r^{D-2}. \quad (16)$$

One can define Euclidean $SL(2, \mathbb{R})$ charges for the D-instanton. Like the D7-brane and the D7^c-brane these charges take value in the $\det Q = 0$ conjugacy class.

Clearly, by performing arbitrary $SL(2, \mathbb{R})$ rotations on a $\det Q = 0$ solution one only obtains solutions that fall into the same conjugacy class. The following obvious question arises that will be central theme of this investigation:

What about 7-branes and instantons with $\det Q > 0$ and $\det Q < 0$?

We will show that both for the 7-branes and the D-instantons deformations can be found which carry charges that belong to the $\det Q > 0$ or $\det Q < 0$ conjugacy classes. Apart from this there are also major differences between the 7-brane and instanton case. We first discuss the 7-brane solutions.

7-BRANES AND THE DW/QFT CORRESPONDENCE

It is indeed possible to find generalizations of the D7^c-brane solution. This extended class of solutions is characterized by two arbitrary holomorphic functions $f(z)$ and $g(z)$. Here $z = x + iy$ is the complex coordinate parametrizing the two-dimensional transverse space. We have found the following class of 1/2 BPS 7-brane solutions [3, 13]:

$$\boxed{\begin{aligned} ds^2 &= ds_{D-2}^2 + \Im f(z) e^{-\Re g(z)} dz d\bar{z}, \\ \tau &\equiv \chi + ie^{-\phi} = f(z). \end{aligned}} \quad (17)$$

All solutions are half-supersymmetric with the Killing spinor given by

$$\varepsilon = e^{i\Im g} \varepsilon_0 \quad \text{with} \quad \Gamma_z \varepsilon = 0. \quad (18)$$

These solutions have been obtained by first considering the 1/2 BPS domain wall solutions of the maximally gauged supergravities in D=9 dimensions. These gauged supergravities can be obtained from IIB supergravity by a so-called *twisted* reduction over a circle with radius R . Assuming that x parametrizes the circle this means that the fields at x and $x + 2\pi R$ are related to each other via an $SL(2, \mathbb{R})$ matrix Ω that takes value in the one-dimensional subgroup corresponding to one of the three conjugacy classes, i.e.

$$\Phi(x) = \Omega \Phi(x + 2\pi R) \quad \text{with} \quad \Omega \in SO(1, 1), \mathbb{R} \text{ or } SO(2) \quad (19)$$

for any field Φ . These twisted reductions of IIB supergravity lead to maximally-supersymmetric $SO(1, 1), \mathbb{R}$ or $SO(2)$ gauged supergravities in D=9 dimensions. Note that this construction method implies that we only find 1/2 BPS 7-brane solutions in

D=10 dimensions that can be reduced to 1/2 BPS domain wall solutions in D=9 dimensions. For instance, this method will not lead to the D7-brane solution since that solution can not be reduced to a 1/2 supersymmetric domain wall solution.

For branes with two transverse directions one usually calculates the monodromy matrix Λ instead of the charge matrix Q . We assume that the two-dimensional space has the topology of a cylinder and that x, y are cylindrical coordinates with x the circle direction parametrizing a circle of radius R . The two matrices Λ and Q are related via [13]

$$\Lambda = \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{\text{monodromy}} = e^{2\pi R \overset{\text{charge}}{\downarrow} Q}. \quad (20)$$

For a given set of holomorphic functions $f(z), g(z)$ it is straightforward to calculate the monodromy matrix and hence the charge matrix. For example, the circular D7^c-brane solution is specified by

$$f(z) = mz, \quad g(z) = 0. \quad (21)$$

This leads to the monodromy matrix

$$\Lambda = \begin{pmatrix} 1 & 2\pi mR \\ 0 & 1 \end{pmatrix}. \quad (22)$$

Following (20) the corresponding charge matrix is given by

$$Q = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix}, \quad (23)$$

which belongs to the $\det Q = 0$ conjugacy class.

It is not too difficult to find choices of holomorphic functions that lead to 1/2 BPS 7-branes with charges corresponding to the other two conjugacy classes. We found these solutions by first constructing the domain wall solutions they give rise to after a twisted reduction over the circular x direction. The result is given in table 2 [3]. Note that in the $SO(2)$ conjugacy class we find two inequivalent solutions. The second one is a locally flat spacetime which in the transverse directions has a cone-like structure, i.e. there is a non-trivial deficit angle.

Sofar we did not consider any quantization conditions. String theory requires that we must impose the following condition on the monodromy matrix Λ :

$$\Lambda \in SL(2, \mathbb{Z}). \quad (24)$$

This is in general a diophantine equation which in the case of the $\det Q = 0$ conjugacy class has the following simple general solution:

$$\Lambda = 2\pi R \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}, \quad n \in \mathbb{Z}. \quad (25)$$

TABLE 2. The choices of $f(z), g(z)$ corresponding to the three conjugacy classes of $SL(2, \mathbb{R})$. The last column indicates the one-dimensional subgroup characterizing each conjugacy class.

class	$f(z)$	$g(z)$	group
$\det Q = 0$	mz	0	\mathbb{R}
$\det Q < 0$	ie^{mz}	mz	$SO(1,1)$
$\det Q > 0$	$\tan \frac{1}{2}mz$	$\ln \cos \frac{1}{2}mz$	$SO(2)$
	i	imz	$SO(2)$

The integer n specifies how many circular 7-branes we have. For the other two conjugacy class the quantization conditions are much more difficult to work out. The general solution can be found in [14, 15].

The two solutions given in table 2 corresponding to the $SO(2)$ conjugacy class are especially interesting in the context of the AdS/CFT correspondence since after reduction over x they lead to 1/2 supersymmetric domain wall solutions in $SO(2)$ gauged supergravity theories. The general scheme in the DW/QFT correspondence is that the near horizon limit of a brane with n transverse directions leads, after spherical reduction, to domain wall solutions of $SO(n)$ gauged supergravities. We will show below that 7-branes naturally fit this picture in the special case that $n = 2$.

The DW/QFT correspondence is a non-conformal generalization of the AdS/CFT correspondence [16]. The standard example of the AdS/CFT correspondence is the D3-brane, see table 3. The near-horizon geometry of the D3-brane is the $AdS_5 \times S^5$ vacuum configuration. After compactification over the spherical part this leads to a $SO(6)$ gauged supergravity in $D=5$ dimensions. This supergravity allows a maximally supersymmetric AdS_5 vacuum configuration. At the boundary of this AdS_5 spacetime lives the dual conformal $N=4$ supersymmetric Yang-Mills theory.

There are two ways to break the conformal symmetry. In each of these two cases the maximally supersymmetric AdS_5 space is replaced by a 1/2 BPS domain wall solution. Such a solution requires the coupling of a dilaton to the cosmological constant. We distinguish between two kinds of dilatons. First we have the volume dilaton ϕ which occurs as an overall factor in the potential of the gauged supergravity. This dilaton is absent in the potential, and hence does not couple to the cosmological constant, if the corresponding brane is conformal, i.e. D3, M2 and M5. The activation of this dilaton is required if we consider one of the other non-conformal branes. The maximally supersymmetric AdS space is in these cases replaced by a 1/2 BPS domain wall solution with a non-trivial profile for the volume dilaton. The field theory living at the boundary of this domain wall spacetime is not a conformal field theory but a non-conformal quantum field theory. Second, we have the remaining so-called distribution dilatons. These always couple to the cosmological constant. Each of these dilatons specifies whether the corresponding brane, be it a conformal brane or not, is distributed in a given transverse direction. It turns out that a brane with n transverse directions can be distributed in maximal $n - 1$ directions. In the conformal case, the distribution of the branes means that in the dual conformal field theory some scalars have obtained a non-

TABLE 3. The standard example of the AdS/CFT correspondence: the D3-brane

brane	vacuum configuration	gauged supergravity
D3	$AdS_5 \times S^5$	D=5 SO(6)

TABLE 4. Branes and the DW/QFT correspondence. The third column indicates whether the volume dilaton occurs in the potential yes or no.

D	n	volume dilaton ϕ	supergravity	Brane
8	3	✓	IIA on S^2	D6
7	5	-	11D on S^4	M5
6	5	✓	IIA on S^4	D4
5	6	-	IIB on S^5	D3
4	8	-	11D on S^7	M2
3	8	✓	IIA on S^7	F1A
2	9	✓	IIA on S^8	D0

vanishing expectation value, i.e. we are in the (non-conformal) Coulomb branch of the gauge theory [17, 18, 19].

It is natural to extend the AdS/CFT correspondence of the conformal branes to a DW/QFT correspondence for the non-conformal branes [11, 12]. In both cases the branes can be distributed depending on whether some of the distribution dilatons are activated. This leads to the relations of table 4 which is a generalization of table 3 to the general brane case²

The point we want to make is that 7-branes can naturally be added to the top of tabel 4 if, instead of performing an ordinary circle reduction we perform a $SO(2)$ *twisted* reduction. This leads to the extension given in table 5, where from now on we assume that $b = 2$ for the remaining part of this section. The two solutions of table 2 are now naturally interpreted as distributions of 7-branes. To see this it is instructive to first consider the case of D5-branes and D6-branes. D5-branes have a 4-dimensional transverse space and can be distributed into at most 3 independent transverse directions.

TABLE 5. The D7-brane and the DW/QFT correspondence

D	n	volume dilaton ϕ	supergravity	Brane
9	2	✓	IIB with $SO(2)$ twist	D7

² We have not indicated the branes of string theory that follow from M-branes by so-called direct or double dimensional reduction. In D=3 there is furthermore an independent possibility with the fundamental string F1B of type IIB string theory. This leads to an inequivalent maximally supersymmetric $SO(8)$ gauged supergravity in D=3 dimensions, compare to [20].

This happens when all three distribution dilatons are activated in the domain wall solution of the D=7 SO(5) gauged supergravity. The branes are distributed as branes with positive (+) charge on the surface of a 3-dimensional ellipsoid and as branes with negative (-) charge inside this ellipsoid, see the first picture in figure 1. Note that the volume dilaton is always activated since the D5-brane is not a conformal brane.

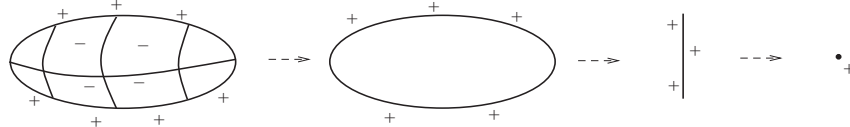


FIGURE 1. Distribution of D5-branes in 3, 2, 1 and 0 directions

Setting one of the distribution dilatons equal to zero corresponds to undoing the distribution in one of the three directions. Due to a cancellation of + branes on the boundary, except the ones at the equator, with - branes in the bulk one is left with a two-dimensional distribution with + branes only positioned at the boundary of a two-dimensional ellipsoid, see the second picture in figure 1. This undoing of the distribution in a given direction can be done two more times, see the third and fourth picture in figure 1. At the end one ends up with a set of stacked branes with no distribution at all.

We next consider the case of D6-branes, see figure 2. Since D6-branes have a 3-dimensional transverse space, we start with a maximal distribution in 2 directions, see the first picture in figure 2. The + branes are positioned at the boundary of a two-dimensional ellipsoid while the - branes are distributed inside this ellipsoid. Undoing the distribution in one of the 2 directions we end up with a two-centred solution. The uplifting of this solution to M-theory leads to the Eguchi-Hanson metric which is the near-horizon limit of the two-centred Kaluza-Klein monopole solution. Undoing the distribution in the second direction we end up with (the near-horizon limit of) a set of stacked D6-branes.

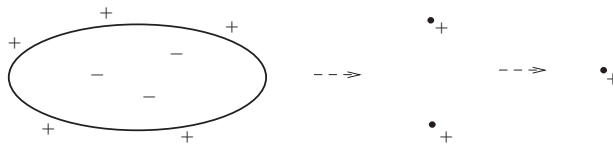


FIGURE 2. Distribution of D6-branes in 2, 1 and 0 directions

Finally, we consider the case of interest, i.e. 7-branes, see figure 3. 7-branes have two transverse directions and therefore we would like to interpret the first picture of figure 3, representing the first SO(2) conjugacy class solution of table 2, as a distribution of branes

in one direction with + branes positioned at the end of the line and - branes distributed along the line. Somewhat to a surprise, this is indeed possible provided we view the first picture in figure 3 as a distribution of D7-branes, which themselves belong to the \mathbb{R} conjugacy class and cannot be reduced to 1/2 supersymmetric domain walls! The reason that this is possible is related to the fact that the product of two monodromy matrices in the \mathbb{R} conjugacy class can yield a monodromy matrix in the $SO(2)$ conjugacy class, i.e. one can view the $SO(2)$ conjugacy solution as a bound state of \mathbb{R} conjugacy class solutions. Undoing the distribution in the single direction leads to the second picture in figure 3 which represents the second $SO(2)$ solution of table 2. Due to a cancellation of charges one is left with no charge at all! Indeed, this is consistent with the fact that the second solution is a locally flat spacetime where the two transverse directions have a cone-like structure with quantized deficit angle. It is very suggestive to define this configuration as the near-horizon limit of the D7-brane solution.

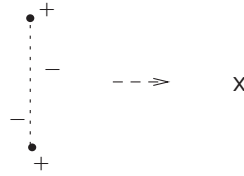


FIGURE 3. Distribution of D7-branes in 1 and 0 directions.

This concludes our discussion of the relation between 7-branes and the DW/QFT correspondence. In the remaining part of this work we will discuss the instanton solutions and their relation to the AdS/CFT correspondence.

INSTANTONS AND THE ADS/CFT CORRESPONDENCE

To construct generalizations of the D-instanton we allow in the Ansatz for *conformally flat* metrics. Allowing a non-zero conformal factor in the Euclidean spacetime metric leads to the following class of solutions:

$$\begin{aligned}
 ds^2 &= \left(1 - \frac{\vec{q}^2}{r^{\frac{2}{D-2}}}\right)^{\frac{2}{D-2}} (dr^2 + r^2 d\Omega_{D-1}^2), \\
 e^{b\phi(r)} &= \left(\frac{q_-}{q} \sinh(H(r) + C)\right)^2, \\
 \chi(r) &= \frac{2}{b q_-} (q \coth(H(r) + C) - q_3).
 \end{aligned}
 \tag{26}$$

Here $H(r)$ is the harmonic over the conformally Euclidean metric in (26) which is given by

$$H(r) = \frac{bc}{2} \log(f_+(r)/f_-(r)), \quad f_{\pm}(r) = 1 \pm \frac{q}{r^{D-2}}. \quad (27)$$

Remember that b is the dilaton coupling parameter. The numerical constant c is given by

$$c = \sqrt{2(D-1)/(D-2)}. \quad (28)$$

The solutions (26) contain 4 integration constants. The integration constant C is related to the fact that one can perform special $SL(2, \mathbb{R})$ transformations

$$\Omega = e^{\lambda Q} \quad (29)$$

that commutes with the charge matrix Q . The other 3 integration constants \bar{q}^2, q_3, q_- are related to the $SL(2, \mathbb{R})$ charge matrix Q in the following way:

$$Q = \begin{pmatrix} q_3 & iq_+ \\ iq_- & -q_3 \end{pmatrix}, \quad \det Q = -\bar{q}^2, \quad q \equiv \sqrt{\bar{q}^2}. \quad (30)$$

Similar solutions have been discussed in [21]. Note that $q \equiv \sqrt{\bar{q}^2}$ can be imaginary for negative \bar{q}^2 . In that case a real solution can be obtained by analytic continuation in which the hyperbolic functions in (26) get replaced by goniometric ones. It is easiest to discuss the three cases $\bar{q}^2 > 0$, $\bar{q}^2 = 0$ and $\bar{q}^2 < 0$ separately. We do this below.

- $\bar{q}^2 > 0$: *Black Holes*

In this case there is a curvature singularity for $r_c \sim \bar{q}^2$. There are two ways in which the singularity might be resolved. It could be that string effects will soften the singularity since $e^{\phi} \rightarrow \infty$ at $r \rightarrow r_c$. For certain values of the dilaton coupling parameter b the singularity is resolved by uplifting the solution to a p-brane solution in $D + p + 1$ dimensions, see later in this section. Note that for $\bar{q}^2 > 0$ the limit $q_- \rightarrow 0$ is well-defined. It leads to a special case in which the axion decouples.

- $\bar{q}^2 = 0$: *Extremal Instantons*

This is the case of the 1/2 BPS D-instanton [2]. In this limit the solution is given by

$$\boxed{\begin{aligned} ds^2 &= dr^2 + r^2 d\Omega_{D-1}^2, \\ e^{b\phi(r)/2} &= g_s^{b/2} + \frac{bcq_-}{r^{D-2}}, \\ \chi(r) &= \frac{2}{b} \left(e^{-b\phi(r)/2} - \frac{q_3}{q_-} \right). \end{aligned}} \quad (31)$$

This limiting case can be obtained either by starting from $\tilde{q}^2 > 0$ or $\tilde{q}^2 < 0$ and taking the limit $\tilde{q}^2 \rightarrow 0$. This limit is facilitated by first making the redefinition

$$C \rightarrow \frac{q}{q_-} C. \quad (32)$$

• $\tilde{q}^2 < 0$: *Wormholes*

To obtain a real solution we redefine $q \rightarrow i\tilde{q}$ such that $\tilde{q}^2 > 0$. After an analytic continuation the solution for this case is given by

$$\begin{aligned} ds^2 &= \left(1 + \frac{\tilde{q}^2}{r^{2(D-2)}}\right)^{\frac{2}{(D-2)}} (dr^2 + r^2 d\Omega_{D-1}^2), \\ e^{b\phi(r)} &= \left(\frac{q_-}{\tilde{q}} \sin(bc \arctan(\frac{\tilde{q}}{r^{D-2}}) + C)\right)^2, \\ \chi(r) &= \frac{2}{bq_-} \tilde{q} \cot(bc \arctan(\frac{\tilde{q}}{r^{D-2}}) + C) - q_3. \end{aligned}$$

The metric is regular for $0 < r < \infty$ and the scalars are regular for $bc < 2$. It turns out that the singularity in the scalars for $bc \geq 2$ can be understood from the fact that, after the Wick rotation from Minkowski spacetime to Euclidean space the dilaton and axion do not define a global coordinate system for the AdS_2 scalar sigma manifold. This is due to the fact that the Poincare coordinates they define do cover the *Euclidean AdS₂* space but only half of the *Minkowskian AdS₂* space. For more details, see [22].

We mention that, unlike the case of 7-branes only the D-instanton, belonging to the $\det Q = 0$ conjugacy class, is 1/2 BPS. The instantons with charges in the other two conjugacy classes, i.e. the ones with $\det Q > 0$ and $\det Q < 0$, are *not* supersymmetric. For this reason, and others, see below, we call these instantons *non-extremal*.

It is well-known that the D-instanton has a wormhole geometry in the string frame metric [2]. This wormhole is asymptotically flat with a neck of physical radius $\rho = \rho_{sd}$ positioned at the fixed point $r = r_{sd}$, see figure 4.

It turns out that the non-extremal $\tilde{q}^2 < 0$ instantons also have a wormhole geometry in the Einstein frame. This can be deduced from the fact that the metric given in (33) has a \mathbb{Z}_2 isometry corresponding to the reflection

$$r^{D-2} \rightarrow \tilde{q} r^{2-D} \quad (33)$$

which interchanges the two asymptotically flat regions. This reflection has a fixed point, corresponding to the selfdual radius

$$r_{sd}^{D-2} = \tilde{q}. \quad (34)$$

The thickness of the neck was computed to be [7]

$$\rho_{sd}^{D-2} = 2\tilde{q}. \quad (35)$$

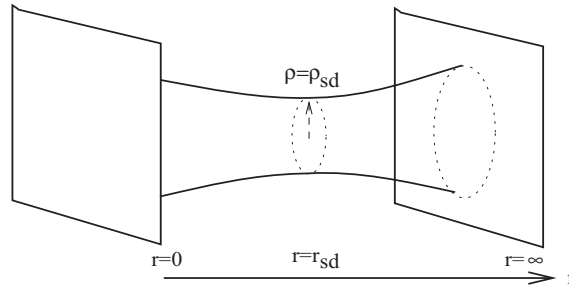


Figure 4. The geometry of a wormhole. The two asymptotically flat regions at $r = 0$ and $r = \infty$ are connected via a neck with a minimal physical radius ρ_{sd} at the self-dual radius r_{sd} .

The $\tilde{q}^2 > 0$ instantons, on the other hand, have a wormhole geometry in the so-called dual frame only when the dilaton coupling parameter b is given by $bc = 2$. In summary, wormhole geometries occur in the following frames:

- $\tilde{q}^2 > 0$: dual frame (only if $bc = 2$)
- $\tilde{q}^2 = 0$: string frame
- $\tilde{q}^2 < 0$: Einstein frame

Similar wormhole geometries have been studied in the eighties, see e.g. [23]. The new thing about the situation here is that we have been able to construct *regular* wormhole solutions for $\tilde{q}^2 < 0$ and $bc < 2$. In type IIB string theory in ten dimensions this is not satisfied. Toroidal compactifications of string theory only lead to values of b for which $bc \geq 2$, so no wormholes exist for these cases. However, for the universal hypermultiplet, which descends from a Calabi-Yau compactification of type II strings, one can have the value $b = 1$ in $D = 4$, and so $bc = \sqrt{3} < 2$. The solution is then characterized by the dilaton and the RR scalar that descends from the RR three-form gauge potential in type IIA string theory in $D = 10$ dimensions. Since the extremal case $\tilde{q}^2 = 0$ corresponds to a wrapped type IIA Euclidean membrane over a (supersymmetric) three-cycle, it is natural to suggest that the regular wormhole, with $\tilde{q}^2 < 0$, corresponds to a wrapped non-extremal Euclidean D2-brane [24].

It turns out that there is an interesting alternative way of describing the instantons as the geodesic motion of a particle in a Minkowskian AdS_2 spacetime. The technique described below is taken from a similar particle description in the case of accelerating cosmologies [25]. In fact, the same technique can be applied to domain walls as well, see [22].

Our starting point is the following Lagrangian and Ansatz:

$$\begin{aligned}
\hat{\mathcal{L}} &= \hat{R} + (\partial \hat{\phi})^2 + e^{b\hat{\phi}} (\partial \hat{\chi})^2, \\
d\hat{s}^2 &= e^{c\varphi(r)} e(r)^2 dr^2 + e^{c\varphi(r)/(D-1)} dS_{D-1}^2, \\
\hat{\phi}(r) &= \phi(r), \quad \hat{\chi}(r) = \chi(r).
\end{aligned} \tag{36}$$

Remember that the numerical constant c is defined in (28). The above Ansatz is a standard Kaluza-Klein Ansatz for a spherical reduction to 1 dimension. The metric contains two functions: The function $\varphi(r)$ is the would-be conformal factor of the instanton solution and the function $e(r)$ is the einbein in 1 dimension with r playing the role of Euclidean time. After reduction the Lagrangian is given by

$$\mathcal{L} \sim e^{-1} (\dot{\phi}^2 + e^{b\phi} \dot{\chi}^2) + \underbrace{e^{-1} \dot{\phi}^2 + e(D-1)(D-2)e^{2\varphi/c}}_{\text{Liouville equation}}. \tag{37}$$

As indicated in this equation the Kaluza-Klein scalar decouples and its equation of motion is given by the Liouville equation

$$\ddot{\phi} - 2(D-1)(D-2)/c e^{2\varphi/c} = 0. \tag{38}$$

After a coordinate transformation

$$d\tilde{r} = e dr \tag{39}$$

the general solution for the conformal factor is given by

$$e^{2\varphi/c} = \frac{-8q^2}{(D-2) \left[1 + \cosh\left(\frac{2}{c} \sqrt{8(D-1)q^2 \tilde{r}}\right) \right]}. \tag{40}$$

After substituting this solution back into the action we obtain

$$\mathcal{L} \sim e^{-1} \underbrace{(\dot{\phi}^2 + e^{b\phi} \dot{\chi}^2)}_{AdS_2} + e \bar{q}^2. \tag{41}$$

We deduce that the dilaton coupling parameter b can be identified with the radius R of the AdS_2 space:

$$b = 2/R. \tag{42}$$

Moreover the nature of the geodesic (spacelike, timelike or null) or, equivalently, the mass of the particle (tachyonic, massless or massive) is determined by the value of \bar{q}^2 , i.e. by the conjugacy class. In summary, we obtain the following relations:

- $\bar{q}^2 > 0$: Black Holes \leftrightarrow Tachyonic Particle

TABLE 6. The D-instanton and the DW/QFT correspondence

D	n	volume dilaton ϕ	supergravity	Brane
1	10	$\sqrt{\quad}$	IIA on S^9	D-instanton

- $\bar{q}^2 = 0$: Extremal Instantons \leftrightarrow Massless Particle
- $\bar{q}^2 < 0$: Wormholes \leftrightarrow Massive Particle

The spherical reduction from D to 1 dimension naturally triggers the question whether the D-instanton plays a similar role in the AdS/CFT and DW/QFT correspondence like the other branes do. In other words, it is natural to extend table 4 not only with 7-branes, as indicated in table 5 but also with instantons, see table 6. Whether or not this analogy with the other branes can be made remains to be seen. For an earlier reference to this suggestion, see [26]. Below we will investigate the relation between (extremal and non-extremal) D-instantons and the AdS/CFT correspondence from a quite different point of view.

The non-extremal instanton solutions we have constructed fall naturally in the class of non-extremal Dp-brane ($0 \leq p \leq 6$) solutions which have been constructed in the literature. In fact, there are two classes of non-extremal branes available in the literature. In the first class the isometries of the worldvolume and transverse space are broken. These solutions are given by [27]

$$ds^2 = e^{\alpha H} (-e^{2f} dt^2 + dx_p^2) + e^{\beta H} (e^{-2f} dr^2 + r^2 d\Omega^2) \quad (43)$$

for some constants α and β . The function H is harmonic and f is the non-extremality function. In the second class the isometries remain unbroken [28]

$$ds^2 = e^A (-dt^2 + dx_p^2) + e^B (dr^2 + r^2 d\Omega^2) \quad (44)$$

Here $A \sim B$ is harmonic in the extremal case but $A \neq B$ and not harmonic in the non-extremal case. The above formulae are only valid for $0 \leq p \leq 6$. However, they can be analytically continued to $p = -1$ as well. It turns out that the non-extremal instantons we constructed just fits in as the $p = -1$ member of the second class of non-extremal Dp-branes given in (44). In fact in all cases there are *two* non-extremal deformations, one corresponding to $\bar{q}^2 > 0$ and one corresponding to $\bar{q}^2 < 0$. Only when $p = -1$ has \bar{q}^2 an interpretation in terms of $SL(2, \mathbb{R})$ charges.

At the other side of the chain, it is not clear how to define non-extremal 7-branes. Note also that the double Wick rotation of the $\bar{q}^2 > 0$ non-extremal Dp-branes leads to Sp-branes. Recently this exercise has been repeated, via a *single* Wick rotation, to the non-extremal $\bar{q}^2 > 0$ D-instanton leading to a ‘‘S-1-brane’’ solution [29].

We now wish to discuss another aspect of the instanton solutions. It is well-known that there is an intricate relation between instantons and solitons in one dimension higher. The uplifting of the gravity-dilaton-axion system leads to the following Lagrangian in D+1 dimensions containing a metric, a dilaton and a vector:

$$\mathcal{L}_{D+1} \sim \hat{R} + (\partial\hat{\phi})^2 + e^{a\hat{\phi}} \hat{F}^2 \quad (45)$$

The constant a defines the dilaton coupling in D+1 dimensions. The reduction of this Lagrangian over time leads to the gravity-dilaton-axion system (1) with $bc \geq 2$. The case $bc = 2$ corresponds to zero dilaton coupling in D+1 dimensions, i.e. $a = 0$.

It is instructive to consider the instanton-soliton correspondence for the case $bc = 2$. It turns out that for that case the uplift of the instantons are given by the (non-extremal) Reissner-Nordström black hole with mass M and charge Q . We find the following relations between the different cases:

$$\begin{aligned} \text{extremal BH :} & \quad M^2 = Q^2 \Leftrightarrow \bar{q}^2 = 0, \\ \text{non-extremal BH :} & \quad M^2 > Q^2 \Leftrightarrow \bar{q}^2 > 0, \\ \text{singular BH :} & \quad M^2 < Q^2 \Leftrightarrow \bar{q}^2 < 0. \end{aligned} \quad (46)$$

The relation between M, Q and the $SL(2, \mathbb{R})$ charges is as follows:

$$M \sim \sqrt{q_-^2 + \bar{q}^2}, \quad Q \sim q_- . \quad (47)$$

Inverting these relations we find that

$$\bar{q}^2 \sim M^2 - Q^2 \quad (48)$$

This relation shows that \bar{q}^2 acts as the non-extremality parameter.

The above uplifting can easily be extended to the uplift of the instantons to p-branes with $p > 0$. The condition for uplifting to a p-brane, i.e. the analogue of $bc = 2$, becomes

$$(bc)^2 = \frac{4(p+1)(D-1)}{D+p-1}. \quad (49)$$

We are thus led to the Instanton Scan given in table 7.

The value of the action, evaluated on the instanton solution, is a key ingredient in the semiclassical approximation of the euclidean path integral. Using the relations between instantons and black holes we have obtained an elegant answer for the $\bar{q}^2 \geq 0$ instanton action: the action is just the mass of the black hole! More precisely, we have found that (for more details, see [7])

TABLE 7. The Instanton Scan

bc	Dimension	Regular Solutions
<2	D	wormholes with $\vec{q}^2 < 0$
=2	D+1	RN black hole with $\vec{q}^2 \geq 0$
>2	D+1	dilatonic BH with $\vec{q}^2 = 0$
= (49)	D+p+1	non-dilatonic p-branes with $\vec{q}^2 \geq 0$
> (49)	D+p+1	dilatonic p-branes with $\vec{q}^2 = 0$

$$S_{inst} = \frac{4}{b^2} (D-2) Vol(S^{D-1}) bc \sqrt{\frac{q_-^2}{g_s^b} + \vec{q}^2} \quad (50)$$

There are two interesting limits to consider. First, the extremal limit is given by $\vec{q}^2 = 0$ in which case the action reaches its lowest value and reduces to that of the D-instanton:

$$S_{inst} \sim \frac{1}{g_s^{b/2}} |q_-| = \frac{1}{g_s^{b/2}} Q. \quad (51)$$

The D-instanton of ten-dimensional IIB string theory corresponds to taking $b = 2$. Other cases in D=4 dimensions include the membrane instanton ($b=1$) and the NS-fivebrane instanton ($b=2$) [9, 10]. The other limit is the Schwarzschild limit $q_- \rightarrow 0$. In that case the action is given by

$$S_{inst} \sim |q|. \quad (52)$$

A natural question to ask is whether the non-extremal instantons we have been constructing give rise to extra corrections to the string effective action like the D-instanton case does. It is well-known that the D-instantons, together with contributions from tree-level graviton scattering and one-loop contributions, give rise to terms of the form [30, 31]

$$f(\tau, \bar{\tau}) R^4. \quad (53)$$

Based on a field theory analysis one can argue that the non-extremal instantons constructed in this work give rise to terms of the form

$$f(\tau, \bar{\tau}) R^8 \quad (54)$$

for some $SL(2, \mathbb{Z})$ modular forms $f(\tau, \bar{\tau})$. This remains to be investigated.

Finally, we would like to make some comments about the dual picture of the non-extremal instantons in the AdS/CFT correspondence. It is well-known that the D-instanton with a flat metric can be generalized to an instanton in a $AdS_5 \times S^5$ background. One can view this configuration as the near-horizon limit of a D3-D(-1) bound

state configuration. Such $AdS_5 \times S^5$ instantons exactly correspond to the standard self-dual Yang-Mills instanton in $N=4$, $D=4$ supersymmetric Yang-Mills theory. To obtain such instanton solutions one should consider the following deformation of the gravity-dilaton-axion system ³

$$\mathcal{L}_M \sim R + (\partial\phi)^2 + e^{b\phi}(\partial\chi)^2 + \Lambda \quad (55)$$

The extremal instantons of this system, i.e. for $\vec{q}^2 = 0$, have been constructed sometime ago and are given by [32]

$$ds^2 = d\rho^2 + l^2 \sinh^2(\rho/l) d\Omega_4^2, \quad (56)$$

$$e^{b\phi/2} = H(\rho), \quad (57)$$

$$\chi(r) = 2/b(H^{-1}(\rho) - q_3/q_-), \quad (58)$$

with $l = \sqrt{12/|\Lambda|}$.

We have now instantons with $\vec{q}^2 \neq 0, \Lambda = 0$ and instantons with $\vec{q}^2 = 0, \Lambda \neq 0$. The natural question to ask is: are there instantons with $\Lambda \neq 0$ and $\vec{q}^2 \neq 0$? Indeed they do exist and are given by [8, 23, 33]

$$ds^2 = (1 + r^2/l^2 + \vec{q}^2/r^{2(D-2)})^{-1} + r^2 d\Omega_4^2, \quad (59)$$

$$e^{b\phi(r)} = (q_-/q \sinh(H(r) + C))^2, \quad (60)$$

$$\chi(r) = 2/bq_-(q \coth(H(r) + C) - q_3). \quad (61)$$

It would be interesting to see whether such bulk solutions correspond to non-selfdual instantons of $N=4$, $D=4$ supersymmetric Yang-Mills theory [34].

CONCLUSIONS

In this work we discussed the properties of brane solutions to the gravity-dilaton-axion system in string theory. A central theme was played by the underlying $SL(2, \mathbb{R})$ duality symmetries underlying this system. In particular we exploited the fact that the group $SL(2, \mathbb{R})$ has three conjugacy classes. Our results and conclusions can be summarized as follows.

- 7-branes

Each conjugacy class is represented by a 1/2 BPS 7-brane solution. Using the monodromy relations one can view the $\det Q > 0$ and $\det Q < 0$ 7-branes as bound states

³ We assume that a spherical reduction has been performed. The flux of the selfdual 5-form curvature leads to the cosmological constant Λ in (55).

of 7-brane solutions belonging to the $\det Q = 0$ conjugacy class. Our work shows that 7-branes fit nicely with the other branes in a general DW/QFT correspondence as far as the supergravity description goes. It led us to make a conjecture for a sensible definition of the near-horizon limit of a D7-brane. Whether all these nice features survive in a more rigorous string theory approach to the AdS/CFT correspondence remains to be seen. It would be interesting to see whether there is some better understanding of our results from a F-theory perspective.

- Instantons

Only the $\det Q = 0$ conjugacy class is represented by a 1/2 BPS D-instanton. The instantons in the $\det Q > 0$ and $\det Q < 0$ conjugacy classes are represented by non-supersymmetric non-extremal instantons. Whether or not these non-extremal instantons play an important role in string theory remains to be seen. Apart from the fact whether or not they give rise to corrections in the string effective action, it is also of interest to investigate what their picture is in the dual gauge theory. For the extremal D-instanton there is a well-established relation with the selfdual YM instanton. For the non-extremal instantons one is tempted to consider the idea of non-selfdual instantons. These and other issues are presently under investigation [34].

- S – branes

A third class of solutions that we did not consider in this work are the so-called S-branes. They are related, via a double Wick rotation to the non-extremal Dp-branes with $\vec{q}^2 > 0$ and $0 \leq p \leq 6$. Recently, it has been shown that the single Wick rotation of the $\vec{q}^2 > 0$ D-instanton leads to a S-1-brane [29]. Another issue is whether S7-branes exist for each of the three conjugacy classes. Their existence is related to the fact whether non-extremal 7-branes for $\vec{q}^2 > 0$ can be defined. We expect that each 1/2 BPS 7-brane solution indeed has such a non-extremal deformation. We hope to come back to this point in [22].

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