

University of Groningen

## Implementation of new regulatory rules in a multistage ALM model for Dutch pension funds

Klein Haneveld, Wim; Streutker, Matthijs; van der Vlerk, Maarten

**IMPORTANT NOTE:** You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2007

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Klein Haneveld, W., Streutker, M., & van der Vlerk, M. (2007). Implementation of new regulatory rules in a multistage ALM model for Dutch pension funds. s.n.

**Copyright**

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

**Take-down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

# Implementation of New Regulatory Rules in a Multistage ALM Model for Dutch Pension Funds

Willem K. Klein Haneveld   Matthijs H. Streutker \*

Maarten H. van der Vlerk

Department of Econometrics & OR  
University of Groningen  
PO Box 800, 9700 AV Groningen  
The Netherlands

March 2007

## Abstract

This paper discusses the implementation of new regulatory rules in a multistage recourse ALM model for Dutch pension funds. The new regulatory rules, which are called the ‘Financieel Toetsingskader’, are effective as of January 2007 and have deep impact on the issues of valuation of liabilities, solvency, contribution rate, and indexation. Multistage recourse models have proved to be valuable for pension fund ALM. The ability to include the new regulatory rules would increase the practical value of these models.

## 1 Introduction

Dutch pension funds are adapting to far-reaching modifications of regulatory rules. Not only a new valuation foundation is introduced, but also the reserves of pension funds must comply with new risk-based solvency rules. The body of new supervision rules is called the ‘Financieel Toetsingskader’ (FTK, Financial Assessment Framework) and was originally formulated by the Pensioen- en Verzekeringskamer (PVK), which merged into De Nederlandsche Bank (DNB) [7]. The FTK intends to be in line with international developments like market valuation of assets and liabilities, and it can be conceived as a precursor of the upcoming European risk-based regime Solvency II [10].

---

\*Corresponding author: [m.h.streutker@rug.nl](mailto:m.h.streutker@rug.nl)

As a consequence, Asset Liability Management (ALM) models supporting strategic decision making for Dutch pension funds must be adapted to the new FTK rules. An upcoming methodology in ALM for pension funds is multistage recourse modeling. See, for instance, Consigli and Dempster [3], Dert [8], Drijver [9], Kouwenberg [20], and Mulvey et al. [26]. Sodhi [31] points out that, in general, the foundations of stochastic programming are highly suitable for ALM. Multistage recourse modeling offers a flexible way of dynamic optimization modeling. All characteristics of importance for a strategic decision support model for pension funds can be addressed in a multistage recourse framework. In this paper we investigate which current multistage recourse model is most suited for modeling the FTK and explore the possibilities for implementing the FTK in this model in detail. First, we state the specific regulatory rules. Second, we compare eight multistage recourse models on FTK compatibility. Third, we research the implementation of the rules for the model of Drijver [9], which we find most compatible with the FTK rules.

## 1.1 FTK and Multistage Recourse ALM Models

The FTK [28] states that it is intended to improve the insight of both the supervised institution and the supervisory authority into the institution's financial position and its possible development over the short and medium term. Its main goal is to make the ins and outs of pension funds more transparent. As an illustration, the contribution premiums should be cost-effective, and the conditions for indexation of pension rights must be clear. Important is the identification and control of risks. The transparency of the funds' financial position is attained by market valuation of both assets and liabilities, and by establishing sound financial healthiness indicators. The FTK provides standardized methods to determine whether the requirements are met. Alternatively, pension funds may use their own methodology, which is called an internal model, subject to approval by the DNB, see [5].

Besides a number of requirements for every report date, which we will state in this document, pension funds need to do a continuity analysis every 3 years as well. This continuity analysis must give insight in the financial position of the fund for the coming 15 years. The analysis is to be of stochastic nature, where the FTK prescribes the minimum expected inflation and the maximal expected returns on assets allowed, see [22, 23]. A multistage recourse model is well-suited for this kind of analysis.

Notwithstanding the importance of the new regulatory rules, one can still question whether it is recommendable to incorporate them into a multistage recourse ALM model. The major difference is, of course, that ALM models generate policies, while the FTK mainly evaluates the financial position of the fund, i.e., the outcomes of policies. Therefore, under normal circumstances the FTK will never forbid policies. However, in case of financial distress it prescribes actions and hence influences policy making. There are three characteristics of the FTK that

makes it compatible with multistage recourse ALM models for pension funds. First, the intention of both the FTK and multistage recourse models is to better indicate and handle the risks faced by pension funds. Second, like multistage recourse models, the FTK is mainly formulated at the strategic level. Third, both consider yearly periods, either for decision making in the ALM model, or for reporting in the FTK.

The main question is how to implement the FTK in multistage recourse ALM models. This has proved to be no fill-in-the-blanks exercise. The research questions we address in this paper are:

1. *What are the current FTK rules?*

The FTK is implemented on 1 January 2007, and under normal circumstances the current rules will at least hold till 1 January 2010. We will describe the regulations as stated in the Financial Assessment Framework Provisions of December 2006 [22] and Regulation Pension Act and Occupational Pension Scheme (Obligatory Participation) Act of December 2006 [24]. Moreover, the Indexation Matrix of January 2007 [6] is consulted.

2. *Which of the available multistage recourse models is most suitable for implementing the FTK?*

In recent years, a number of multistage recourse ALM models for pension funds have been proposed. We will describe how these models relate to the FTK.

3. *How can the FTK rules be implemented?*

The translation of evaluative rules to policy making rules is not trivial. Too strict implementations will result in forbidding certain choices, which does not occur under the evaluative rules of the FTK. We will give directions for adapting the most suitable model such that it complies with the FTK by either using the standardized method or an internal method.

## 1.2 Outline

The organization of this paper is as follows. The next section describes the FTK rules, subdivided into the valuation of liabilities, solvency of the fund, contribution rate, and indexation. Section 3 discusses multistage recourse ALM models for pension funds. Special attention is paid to the model of Drijver [9]. The possibilities for implementation of the FTK in this model is investigated in section 4. We end with a summary and conclusion. Two appendices are added. The first one concerns the required solvencies for various risk types and is related to the solvency part of section 2. The second one lists the notation used throughout this paper.

## 2 FTK Requirements

In this section, we state the FTK requirements. The continuity analysis part of the FTK is not treated, since it does not yield any specific rules that can be modeled. We distinguish the subjects of valuation of liabilities, solvency, contribution rate, and indexation. Each field is briefly introduced after which the specific rules are given.

### 2.1 Valuation of Liabilities

The *liabilities* of a pension fund are the accrued rights of its participants, which in the future will result in benefit payments. These rights comprise the unconditional parts of the pension contract plus granted indexation (correction for inflation). The provision for the liabilities, which we shall refer to as the value of the liabilities, is calculated by discounting the *expected future benefit payments*. These expected payments are to be determined using prudent underwriting principles, which involve the foreseeable mortality trends and expected developments of a demographic, legal, social, financial, and economic nature.

Let us denote<sup>1</sup>  $B_t(n)$  the expected benefit payments due  $n$  years from time  $t$ , and let  $d_t(n)$  be the *discount factor* for payments  $n$  years from time  $t$ . Then, the value of the liabilities  $L_t$  at time  $t$  is given by

$$L_t = \sum_{n=1}^{\infty} d_t(n)B_t(n). \quad (1)$$

Until recently, pension funds based their discount factors on a single *discount rate*, the so-called actuarial interest rate. Theoretically, this rate could change from year to year, but in practice it was constant (4%) ever since October 1969 [30]. If we let  $r_t^s$  be this single discount rate at time  $t$ , then the discount factors would be given by  $d_t(n) = (1 + r_t^s)^{-n}$ . Presently, it is believed that the use of a single discount rate does not give a fair estimation of the value of the liabilities. Therefore, for each year ahead a different discount rate needs to be specified. The liabilities, as they stem from unconditional parts of the pension contract and granted indexation in the past, are to be considered as guaranteed future cash flow streams, and should be valued based on risk-free financial products. A logical choice is to use an interest rate term structure for discounting.

The DNB [4] derives their term structure of discount rates from inter-banking *swap rates*. Zero-coupon rates are derived from the par European swap rates for 1 to 10-year maturities (yearly intervals) and 12, 15, 20, 25, 30, 40 and 50-year maturities, as they are listed on a daily basis by Bloomberg. The interest rate term structure is published monthly by the DNB. Let  $z_t(n)$  be the zero-coupon rate for maturity  $n$  at time  $t$  calculated as in [4]. The values of the discount

---

<sup>1</sup>For convenience, all notation and definitions are listed in Appendix B.

factors prescribed by DNB are

$$d_t(n) = \frac{1}{(1 + z_t(n))^n}.$$

## 2.2 Solvency of the Fund

The most stringent supervision in the FTK is on the solvency (also called actuarial surplus) of the pension fund. The *solvency*  $S_t$  of a fund at time  $t$  is defined by  $S_t = A_t - L_t$ , where  $A_t$  is the value of the assets at time  $t$  and  $L_t$  is the value of the liabilities at time  $t$ . Constraints on solvency can also be formulated as funding ratio constraints. The *funding ratio* of a fund at time  $t$  is defined by  $F_t = A_t/L_t$ . As the FTK is formulated in solvency terms, while most ALM models in funding ratio terms, we will give both formulations.

The short term solvency condition that should hold at each report date  $t$  is

$$S_t \geq S_t^m \quad (F_t \geq 1 + \frac{S_t^m}{L_t}), \quad (2)$$

with  $S_t^m$  the *minimum required solvency*, which is to be calculated as prescribed by the European Union [11]. This minimum required solvency approximates 5 percent of the value of the liabilities. If the condition is not satisfied, it is said that there is a *funding shortage*. When such a funding shortage occurs, the pension fund has to formulate a recovery plan. The recovery period allowed is 3 years, provided that the chance of recovery increases and that the participants' risk and the chance on future indexation are not negatively influenced due to the plan. In case this is not satisfied, the recovery period allowed is only 1 year.

The medium term solvency condition that should hold at each report date is

$$P(S_{t+1} < 0 | S_t) \leq 0.025 \quad (P(F_{t+1} < 1 | t) \leq 0.025). \quad (3)$$

If the condition is not satisfied, it is said that there is a *reserves shortage*. If such a reserves shortage appears at report date  $t$ , then the pension fund has to formulate a plan which shows that the fund strives to satisfy equation (3) again at the latest at report date  $t + 15$ . The plan should exhibit a steady recovery ex ante. The solvency value such that  $P(S_{t+1} < 0 | S_t^r) = 0.025$  is called the *required solvency* and is denoted by  $S_t^r$ .

The FTK states that the second solvency condition comes down to a desired present funding ratio of 1.30 for an average pension fund, which invests 50% in stocks and 50% in bonds and faces a duration of the bonds and liabilities of, respectively, 5 and 16 years. Thus, for this average pension fund equation (3) boils down to:

$$S_t \geq 0.30L_t \quad (F_t \geq 1.30). \quad (4)$$

### Standardized Method

The DNB standardized method for calculating the required solvency is by using the estimator  $\hat{S}_t^r$ , which is based on desired solvencies for a number of risk types:

$$\hat{S}_t^r = \sqrt{S_{1t}^2 + S_{2t}^2 + 2\rho_{12}S_{1t}S_{2t} + S_{3t}^2 + S_{4t}^2 + S_{5t}^2 + S_{6t}^2}$$

with  $S_{it}$  the required solvency for risk of type  $i$ : interest rate risk ( $S_{1t}$ ), market risk ( $S_{2t}$ ) with respect to equity and real estate, foreign exchange risk ( $S_{3t}$ ), commodities risk ( $S_{4t}$ ), credit risk ( $S_{5t}$ ), and underwriting risk ( $S_{6t}$ ), and  $\rho_{12}$  is the correlation between effects of interest risk and market risk, which equals 0.5. In Appendix A it is shown how the required solvency for each risk type is calculated.

### Internal Method

Rather than testing the medium term solvency by the standardized method, a pension fund may use an internal method. An internal method must correctly model the probability distribution of the solvency at a horizon of one year.

## 2.3 Contribution Rate

First and foremost, the FTK states that the contribution rate must be cost-effective, i.e., cover the costs of the pension plan. That is, the contribution rate is completely determined by the pension plan and the risk attitude of the fund. Nonetheless, in situations of underfunding or overfunding the fund may adjust the ‘normal’ contribution rate. In the second place, the FTK advocates a stable contribution rate. Highly volatile contribution rates might cause social upheaval.

The ‘normal’ contribution rate at pension contract level in the FTK consists of four components. The first component is the actuarially necessary contribution for the purchase of the unconditional parts of the pension agreement. The second component is the solvency surcharge for the unconditional parts of the pension agreement. Its value is based on the risk profile of the fund, and thus on the asset portfolio. The third element is the surcharge for handling costs. The fourth component is the actuarially necessary contribution for the conditional parts of the pension agreement, which are mostly indexation commitments.

In case of underfunding or overfunding two other components come to the scene. The fifth component is the recovery premium, which appears when a funding shortage (condition (2)) or a reserves shortage (condition (3)) is present. The pension fund can set the recovery premium freely. The sixth component is the premium discount, which may be granted if the capital of the fund is such that fulfillment of the unconditional and conditional parts are guaranteed for the foreseeable future.

## 2.4 Indexation

Indexation is the correction of pension rights for wage or price inflation. In general, the unconditional rights of pension agreements are nominal. However, most pension funds strive to grant the participants a real pension. In the past, full indexation of the pensions was common practice. Currently, more and more pension funds only grant partial indexation.

The indexation policy of a pension fund should be in line with the indexation ambition communicated to the participants. The current indexation matrix [6] distinguishes six indexation categories: no indexation commitment, no purposeful indexation policy, conditional indexation commitment not linked to an ex ante defined criterion, conditional indexation commitment linked to an ex ante defined criterion, combination of unconditional and conditional indexation commitment, and unconditional indexation commitment. In the indexation matrix [6] the characteristics of each category are given, e.g., the way the indexation is financed and on which criterion the decision of granting indexation is based.

Whether or not conditional indexation may be given depends on the financial state of the pension fund. In case of a funding or a reserves shortage, the pension fund is still allowed to grant indexation. However, it should not jeopardize the execution of the recovery plan. If the solvency of the fund is higher than the required solvency, the fund has own funds. These own funds are intended to be used for the fulfillment of the current conditional parts, but can also be used for the recovery of indexation in the past.

## 3 Multistage Recourse ALM Modeling for Pension Funds

In this section, we review eight multistage recourse ALM models for pension funds from the perspective of compatibility with the FTK. We found the model of Drijver [9] to be the most suited. Therefore, we describe this model in more detail in the second part of this section.

### 3.1 Literature Review

Asset liability management modeling is an active topic of research in the stochastic programming community. The headway made is described in surveys by Mulvey [25] and Sodhi [31]. We restrict ourselves to multistage recourse ALM models for pension fund management. Furthermore, we only consider defined benefit plans, as this is the standard in the Netherlands. The goal of this review is to investigate which model is most suited for implementing the FTK, i.e., which model is most in line with the outlines of the FTK. As described in the previous section, the main features of the FTK are the market valuation of liabilities, the presence of solvency requirements, cost-effective contribution



rates, and guidelines for indexation.

We will discuss eight recent multistage optimization models given in the literature. Five of them are based on Dutch data: the chance constrained model by Dert [8], the hybrid simulation/optimisation model of Boender [1], a model using CVaR constraints by Bogentoft et al. [2], the model of Kouwenberg [20], which was applied to real data of a Dutch pension fund by Gondzio and Kouwenberg [12], and the model of Drijver [9]. Furthermore, we consider four abroad based models: the Watson pension fund case for the CALM model of Consigli and Dempster [3], the model designed for Towers Perrin-Tillinghast by Mulvey et al. [26], and a model for a Finnish pension company by Hilli et al. [15].

### **Valuation of Liabilities**

In most models, for the valuation of the liabilities a single interest rate is utilized. The development of the liabilities in the models of Boender [1], Bogentoft et al. [2], Dert [8], and Kouwenberg [20] are provided by ORTEC [27] and are based on a fixed actuarial interest. Hilli et al. [15] utilize a so-called technical interest rate, which is a single interest rate that changes from year to year. As opposed to the models mentioned above, Drijver [9] and Mulvey et al. [26] use a term structure of discount rates. Mulvey et al. base it solely on a bond yield curve, while Drijver adds an equity component.

### **Solvency of the Fund**

Solvency constraints are modeled in various ways. Boender [1] works with funding ratio ranges. Whenever the ratio is too low, the contribution rate will be raised. Kouwenberg [20] does not put restrictions on the solvency level, but penalizes deficits in the objective. Dert [8] limits the probability of bankruptcy by a chance constraint on the wealth of the fund. Bogentoft et al. [2] utilize a CVaR constraint on the wealth. For details on CVaR see Rockafellar and Uryasev [29]. Drijver [9] applies the closely related integrated chance constraint [19]. Besides this medium term solvency constraint, Drijver [9] also adds a short term solvency requirement. Most closely related to the FTK short term and medium term requirements are the solvency constraints of Hilli et al. [15]. There is a short term condition demanding the solvency to be at least equal to the so-called solvency capital (similar to the minimum required solvency). The medium term condition concerns the so-called solvency border, which is the solvency level such that the probability of ruin in one year is 2.5%. This solvency border is equivalent to the required solvency under FTK. The calculation method, however, differs. In the model the non-convex solvency border constraint is replaced by a convex approximation, which is still non-linear though.

### **Contribution Rate**

The financing policies are similarly modeled in all models. The contribution rate decision and its fluctuations are usually bounded. Boender [1] and Drijver [9] use

soft bounds for the fluctuations. Dert [8] also bounds the absolute contribution to the fund in each year. Hilli et al. [15] are divergent. In their model, the contribution rate is determined by the Ministry of Social Affairs and Health of Finland, and is thus a parameter instead of a decision variable as in the other models.

### **Indexation**

Looking at indexation, we see that the liabilities scenarios provided by ORTEC [27], which are utilized by Boender [1], Bogentoft et al. [2], Dert [8], and Kouwenberg [20] are based on a fully-indexed final earnings scheme. As a consequence, no (indexation) decisions on the liabilities side are modeled, i.e., the liabilities are purely data. None of the other models, except for Drijver [9], have indexation decisions.

### **Conclusion**

Our conclusion is that the model of Drijver [9] bears the most resemblance to the FTK rules. It scores well on all subjects under consideration. Therefore, we will check suitability of this model for FTK implementation, but first let us have a closer look at this model, which we name model D05.

## **3.2 Model D05**

Model D05 was constructed when the FTK was still in its developing stages. Ideas present on the general direction of the FTK at that moment were used by Drijver to anticipate the FTK as best as possible.

Model D05 is a multistage mixed-integer linear recourse model made for supporting strategic decisions of a Dutch pension fund. This fund utilizes a defined benefit contract based on an average earnings scheme with conditional indexation. Valuation of the liabilities is based upon a term structure of discount rates. The decisions to be supported concern the investment (asset class positions), financing (level of contribution rates, regular and remedial), and indexation (level of indexation) policy. The objective of the model is to minimize the funding costs, while fixed and proportional penalties are given for undesirable events like, e.g., underfunding, remedial contributions by the sponsor, and incomplete indexation.

In the model, a number of accounting, policy, and regulatory constraints appear. The accounting constraints are part of all ALM models. They cover the bookkeeping after the realizations of stochastic parameters are observed and make sure that no money is ‘lost’ in the decision process. Next, we have the three policy areas of investment, financing, and indexation. First, the investment policy constraints assure that no short positions are possible, and that the proportional amount invested in a certain asset class is bounded. Second, the

financing policy constraints mitigate the contribution rate and remedial contribution. The model has some stabilizing constraints on the contribution rate. However, these constraints are soft and do not prohibit big jumps when needed. Whether a remedial contribution by the sponsor is allowed or even compulsory is modeled as well. Also rules for restitutions to the sponsor are specified. Third, the indexation is modeled by making the value of the liabilities a decision variable bounded by the value of the nominal and fully-indexed rights. The benefit payments are adjusted according to the indexation decision. Finally, the regulatory constraints are a rough approximation of the FTK solvency constraints. The constraints included limit next year's expected loss.

## 4 Model D05 in view of FTK

In this section, the implementation of the FTK in model D05 is discussed. We deal with the four subjects of valuation of liabilities, solvency of the fund, contribution rate, and indexation subsequently. First, it is explained how Drijver [9] deals with the field. Next, the confrontation with the FTK is made.

### 4.1 Valuation of Liabilities

As is customary, the value of the liabilities in model D05 equals the discounted expected benefit payments. Drijver bases his discount factors  $d_t(n)$  on the *pension spot curve* of H.A. Klein Haneveld [16], which is derived from the *yield curve* and the *ex-ante equity risk premium*. According to Luenberger [21], a bond's *yield* is the interest rate at which the present value of the stream of payments (consisting of the coupon payments and the final face-value redemption payment) is exactly equal to the current price. Furthermore, the yield curve displays yield as a function of time to maturity.

The pension spot curve is defined as follows:

$$PSC_t(n) = \begin{cases} y_t(n) + \frac{n}{20} earp_t & \text{if } 0 \leq n < 20, \\ y_t(n) + earp_t & \text{if } n \geq 20, \end{cases}$$

with  $y_t(n)$  the yield corresponding to a risk-free zero-coupon bond maturing  $n$  years from time  $t$  and  $earp_t$  the ex-ante equity risk premium at time  $t$ . The yields are computed as (Haugen [14])

$$y_t(n) = (a_1 + a_2 n)e^{-a_3 n} + a_{4t},$$

where the constants  $a_1$ ,  $a_2$ , and  $a_3$  do not depend on  $t$ . The dynamics of the yield curve are thus driven by the return on the bank account.  $a_{4t}$  is the yield on bonds with the longest terms to maturity, and its values are computed in

$$a_{4t} = a_{4,t-1} - y_{t-1}(1) + \mathbb{E}[r_{t+1}^{ba}],$$

where  $r_{t+1}^{ba}$  is the return on the bank account in year  $t+1$ . Based on the Gordon growth model [13], the value of the ex-ante equity risk premium is

$$earp_t = R_{1t} - y_t(1),$$

with  $R_{1t}$  the internal rate of return of the stock portfolio at time  $t$ .

The discount factors<sup>2</sup> are now given by

$$d_t(n) = \frac{1}{(1 + PSC_t(n))^n}.$$

## Discussion

Drijver's model does not satisfy the FTK requirement of market valuation of the liabilities, since the discount rates contain equity-based elements, which is inappropriate for valuation purposes. The pension spot curve, which is used for valuation of the liabilities, is the sum of a yield curve and ex-ante equity risk premiums.

Another important issue is the value of future discount factors. In a multi-stage model one needs to be able to calculate the value of the liabilities in the future. How to implement this is far from trivial. For  $t = 0$ , the term structures based on swap rates or the pension spot curve are deterministic and can be computed, but for future moments in time ( $t > 0$ ) the term structures are based on stochastic variables and are thus stochastic themselves. Note that in the past when a fixed discount rate was used, future discount factors were deterministic.

To implement the DNB term structure method in model D05, we need a stochastic model that captures the swap curve dynamics. An alternative would be to base the discount rates on a yield curve. Since model D05 already contains a yield curve, it seems a logical step to use this curve for valuation purposes. However, it first needs to be investigated whether the yield curves are generated consistently in time. For instance, the development of the yield curves should be arbitrage free.

## 4.2 Solvency of the fund

In Model D05, there are three conditions on the funding ratio, two of which concern the current funding ratio, and one concerns next years funding ratio. In these conditions we encounter the value of the fully-indexed liabilities, in addition to the actual liabilities (unconditional rights + granted indexation). We define  $\bar{L}_t$  to be the value of the fully-indexed liabilities, i.e., when all conditional rights in the past have been guaranteed. The value of the nominal liabilities  $\underline{L}_t$  is the value of the nominal unconditional rights. Hence,  $\underline{L}_t \leq L_t \leq \bar{L}_t$ .

---

<sup>2</sup>Drijver [9] on p. 98 mistakenly states that  $d_t(n) = \frac{1}{\prod_{i=1}^n (1 + PSC_t(i))}$ .

The first condition is that if the funding ratio is below  $\theta$ ,

$$F_t < \theta \quad (S_t < (\theta - 1)L_t), \quad (5)$$

the sponsor of the fund has to make an immediate remedial contribution. The sponsor of the fund will make a payment such that the funding ratio is restored to at least  $\theta$ , where  $\theta$  is specified such that a drop below this level is absolutely unacceptable.

The second condition is that if the funding ratio is below  $\alpha$ ,

$$F_t < \alpha \quad (S_t < (\alpha - 1)L_t), \quad (6)$$

in  $a$  consecutive years, a remedial contribution of the sponsor is needed (with  $\alpha > \theta \geq 0$ ). The amount of this remedial contribution is such that the funding ratio is restored to at least  $\alpha$ . The level  $\alpha$  is specified such that it is undesirable for the funding ratio to be below  $\alpha$ , but if the funding ratio is still above  $\theta$  no direct action has to be taken. It might be that the low funding ratio results from a temporal dip of the financial markets. However, if the funding ratio is in poor health for a longer time, action is needed.

Finally, a one-stage risk condition states that the expected funding shortage at the next report date, which is the expectation of  $\max\{0, \alpha\bar{L}_{t+1} - A_{t+1}\}$ , is not greater than an amount depending on the current value of the liabilities:

$$\mathbb{E}[A_{t+1} - \alpha\bar{L}_{t+1}]^- \leq \varphi L_t \quad (7)$$

with  $\alpha \geq 1$  and  $(x)^- := \max\{0, -x\}$ . Constraint (7) is a so-called *integrated chance constraint* [17, 19]. If one takes  $\alpha = 1$ , i.e.,  $\mathbb{E}[A_{t+1} - \bar{L}_{t+1}]^- \leq \varphi L_t$  then this constraint can be interpreted as the quantitative variant of the qualitative constraint (3) in the FTK requirements. Instead of limiting the probability of underfunding irrespective of the size of the shortage, the expected amount of underfunding is limited.

## Discussion

Both Drijver and the FTK underline the importance of controlling the solvency on the short and medium term. The short term solvency condition of the FTK, which says that if the solvency at a certain report date is below the minimum required solvency, then in either 1 or 3 years it should be above this minimum required solvency again, is similar to equations (5) and (6). These equations prescribe action (remedial contribution) when the funding ratio is below a certain level for a given period of time.

For guaranteeing the existence of the fund on the medium term, the visions of the FTK and Drijver drift apart. The FTK on the one hand, has a qualitative appreciation on long term risk (probability of default), since they put a

limit on the chance that the next years value of the liabilities is higher than that of the assets. On the other hand, Drijver has a qualitative appreciation of risk (expected shortage), since he limits the expected difference between next years value of the liabilities and the assets.

### Implementation of Standardized Method

The short term solvency requirement can be modeled in various ways using the constraints of model D05. For the minimum required solvency we will use the 5% of the liabilities. A very strict interpretation of the minimum required solvency would be to see it as a strict lower bound on the solvency, i.e., use condition (5) with  $\theta = 1.05$ . In this case funding shortages are not allowed. If you do allow funding shortages, and the related recovery periods, this can be modelled with condition (6) with  $\alpha = 1.05$ . A one-year recovery period can be modeled in model D05 by setting  $a = 2$ . This disallows the funding ratio to be below 1.05 at two consecutive report dates. If in a certain year the funding ratio threatens to become below 1.05, and in the last year it was already below 1.05, then the sponsor is obliged to make a remedial contribution that restores the funding ratio at least to 1.05. Similarly, for a three-year recovery period  $a = 4$  would be used. Given the high penalty costs for a remedial contribution, the pension fund board will strive to prevent this situation. The 3-year recovery period conditions cannot be captured by the constraints present in model D05.

For the long term solvency condition, the FTK provides a ‘simple’ and a standardized method. The simple method (for the average pension fund), which tells that the funding ratio should be larger than 1.30 with a recovery period of 15 years, can be implemented by using constraint (6) with  $\alpha = 1.30$  and  $a = 16$ . However, the time horizon of multistage recourse models with yearly decision moments, like Drijver’s, is usually much shorter than 15 years. Horizons of, say, more than 5 year make these complex models intractable. Moreover, the recovery should be steady ex ante. How to incorporate a 15-year recovery period with ex ante steady recovery in a model with a much shorter horizon is an open problem.

It is more complicated to implement the standardized method. Recall that the standardized method breaks up the required solvency into a number of risk types and calculates in that way the required solvency such that the long term requirement is satisfied in approximation. The problem is that the required solvency is a nonlinear function of the risk types. Calculation of the required solvency for the market risk, foreign exchange risk, and commodity risk is straight forward in Drijver’s model. The calculation of the interest rate solvency and the underwriting solvency might also be trouble-free, since most of the calculations can be done in the scenario tree and are thus available as data. Note that we still need to deal with the 15-year recovery period with the ex ante steady recovery.

## Implementation of Internal Method

For the solvency requirements also the FTK allows pension funds to use their own methods. This concerns only the medium term solvency requirement, since the short term requirement does not give much space for other interpretations. The direct implementation of constraint (3) would result in chance constraints. The use of integrated chance constraints is an alternative, see Klein Haneveld et al. [18]. Drijver uses the quantitative constraint (7) as an alternative to the qualitative constraint (3). However, there is no straight forward relation between the induced feasible sets of these two constraints.

### 4.3 Contribution Rate

Drijver and the FTK seem to have a totally different view on contributions. In Drijver’s model, the contribution rate is one of the main decision variables. The FTK sees the contribution rate more as a state variable, and only in extreme situations the fund will directly alter the contribution rate. This difference can be explained by a difference in goals, as model D05 generates policies and the FTK evaluates the results of policies.

Drijver does not distinguish different components of the contribution rate. The contribution rate  $c_t$  can be chosen ‘freely’, and is actually one of the main decision variables. Bounds are given on the contribution rate:

$$\underline{c} \leq c_t \leq \bar{c}.$$

Stabilization of the contribution rate is also one of the goals of Drijver’s model. The constraint

$$-\eta \leq \Delta c_t \leq \rho,$$

which bounds the change in contribution rate  $\Delta c_t := c_t - c_{t-1}$ , is implemented as a soft constraint in the model. For exceeding the bounds penalty costs are associated. In addition, a conditional lower bound on the contribution rate is included:

$$c_t \geq c^*$$

in case of a remedial contribution.

## Discussion

The perception of the contribution rate in the new regulatory rules differs from that of model D05. In Drijver’s model the contribution rate is one of the three major decisions (contribution rate, asset portfolio mix, indexation degree). However, in the FTK in the normal situation the contribution rate is a state variable. Only in cases of shortage or substantial surplus the contribution rate can be influenced directly by the pension fund.

Nevertheless, Model D05 supports a cost-effective contribution rate, since too

high or too low contribution rates lead to extra costs. Furthermore, the need for a stable contribution rate is recognized in Model D05.

#### 4.4 Indexation

The pension fund in Drijver's model has a strong ambition to index, albeit on a conditional basis. The fund strives to index the liabilities every report date with respect to last year's increase in the general wage level. Moreover, if in a certain year the pension rights are not fully compensated in the general wage level, it strives to give this compensation in a later year.

Indexation is modeled by making the value of the liabilities at time  $t$ , which is denoted by  $L_t$ , a decision variable. This value should be between the value of the nominal liabilities  $\underline{L}_t$  and the value of the fully-indexed liabilities  $\bar{L}_t$ . Penalty costs are used to model the strong indexation ambition.

First, the total deviation of full indexation up to and including time  $t$  is penalized by including the following term in the objective function

$$\varsigma_L(L_t - \bar{L}_t)^-,$$

with  $\varsigma_L$  a penalty parameter.

In addition, Drijver penalizes the decision to not fully index last years liabilities even more. He introduces the binary variable  $m_t$  indicating whether the current decision is to fully index last years liabilities ( $m_t = 0$ ) or not ( $m_t = 1$ ). The term included in the objective function is

$$\lambda_m m_t,$$

with  $\lambda_m$  a penalty parameter.

There are no further constraints on the decision on the liabilities. In particular, in case of financial distress it is still possible to grant indexation. On the other hand, it is also possible to turn back indexation that was granted in the past.

#### Discussion

The indexation ambition level is the basis for the FTK. Six indexation categories are distinguished. The pension fund of Drijver's model has a conditional indexation ambition. Changing the ambition of the fund to (partly) unconditional can be achieved by raising  $L_t$  with the expected value of guaranteed indexation rights. To get a no-indexation ambition one could drop the penalty parameters and use bonuses instead in case indexation is granted.

A difficult point, though, is the matter of buffers and categorized contributions,



as in the FTK. Since Drijver's model does not categorize the contribution rate, it is not clear how much is reserved for indexation. A fund with no indexation ambition could have the same contribution policy.

The decisions on the indexation are free in Drijver's model. No matter the financial position of the fund, indexation can be granted. Also indexation granted in the past can be taken back. However, the FTK seems more restrictive. In case of a funding or reserves shortage, the fund should be careful with indexation. Moreover, most pension contracts do not allow indexation to be turned back.

## 5 Summary and Conclusion

In this paper, we researched if and how the new FTK regulatory rules for pension funds can be implemented in multistage recourse ALM models. To this end, we answered the following three questions:

1. What are the current FTK rules?
2. Which of the available multistage recourse models is most suitable for implementing the FTK?
3. How can the FTK rules be implemented?

Now, we will summarize our findings and pose our conclusion.

### 5.1 What are the current FTK rules?

The FTK is involved with the following four subjects:

- valuation of liabilities,
- solvency of the fund,
- contribution rate, and
- indexation.

Below, we will treat these subjects in succession.

The FTK prescribes market valuation of the liabilities, which concerns discounting the expected benefit payments with a term structure of risk-free discount rates.

The FTK has two conditions on the solvency (assets minus liabilities). First, the solvency should be greater or equal to the minimum required solvency, which approximates 5% of the liabilities. If not, a recovery period of either one or three years is allowed depending on the characteristics of the recovery plan. Second, the solvency should be such that the probability of underfunding in the next

year is at most 2.5%. If not, a recovery period of 15 years is allowed. However, the recovery must be steady ex ante.

The contribution rate must be cost-effective. The FTK distinguishes six components comprising the contribution rate. In addition, the FTK advocates a stable contribution rate in order to prevent social upheaval.

The indexation policy must be in line with the indexation ambition communicated to the participants. Depending on this ambition, funds must construct buffers and can ask contribution for indexation. Further, in case of financial distress, of which the solvency level is the indicator, funds must be cautious with indexation.

## **5.2 Which of the available multistage recourse models is most suitable for implementing the FTK?**

We compared eight recently published multistage recourse ALM models on their suitability for adopting the FTK. The valuation of liabilities based upon a term structure of discount rates was found in the models of Drijver [9] and Mulvey et al. [26]. The solvency constraints of Drijver [9], but even more Hillie et al. [15] showed the most resemblance with the FTK. The contribution rate constraints of all models were similar. Drijver's model [9] was the only model that considers indexation. We concluded that the model of Drijver [9], which we named Model D05, is most suited for implementing the FTK.

## **5.3 How can the FTK rules be implemented?**

Here, we will treat the implementation of the above mentioned FTK rules in Model D05. Again we will treat the four subjects successively.

For discounting the expected benefit payments either a swap curve based method (FTK) or a yield curve can be used. However, the yield curve present in Model D05 cannot be used thoughtlessly. First, the time consistency of the curve needs to be investigated.

By and large, the short term solvency condition can be implemented by making use of existing constraints in Model D05. Only for the 3-year recovery period conditions, new constraints need to be modeled. The implementation of the medium term solvency condition can be done in various ways. A direct translation would be chance constraints. A computationally nicer alternative is integrated chance constraints. The FTK provides two methods as well. First, it is said that for the average fund the condition comes down to a solvency of 30% of the liabilities. Second, they give an approximation of the solvency needed by considering various risk types.

The contribution rate regulations are mostly formulated at the individual contract and operational level. It is not desirable to implement these rules exactly in a strategic model. On top of that, it is unclear how the different components should be calculated, specifically the conditional parts. The current constraints on the contribution rate in Model D05 seem to be sufficient for now.

Modeling the indexation ambition in Model D05 can be done by the specification of the penalty costs for incomplete indexation. It is possible to relate the indexation decisions to the solvency level, so that the fund is prudent with indexation in times of financial distress. Model D05 needs to be adapted in that it allows to reverse previously granted indexation, since this is not allowed in most pension contracts.

## 5.4 Conclusion

To conclude, we give our views on the possibilities for adopting the FTK in model D05.

Drijver and the DNB have the same leading concept: the identification and control of risks will better guarantee a good pension for the participants. We will now state the parts of the FTK that can be implemented relatively easily and those that will take some more considerations for implementation.

Valuation of the liabilities according to the FTK rules can be implemented relatively easily. This can be accomplished by using the zero-coupon bond yield curve in Model D05. However, the dynamics of this yield curve need to be researched first. The implementation of the short term solvency requirement should not be a major problem either. The same holds for stabilizing the contribution rate. Furthermore, making the indexation dependent on the financial position of the funds is doable, as well as adding a no-turning-back-indexation condition.

Difficulties occur for the medium term solvency requirement. First of all, we need to deal with the 15-year recovery period with ex-ante a steady recovery in a model with a much shorter horizon. Second, how the probability condition should be implemented is cumbersome. The simple method is just too unrealistic. The standardized method results in nonlinear constraints. A direct translation results in chance constraints. An alternative to chance constraints would be integrated chance constraints.

Another difficult point is the prescribed cost-effective component-wise contribution rate prescribed by the DNB. A switch from the ‘free’ contribution rate in Model D05 to the prescribed FTK contribution rate is immense. One could ask whether this switch is necessary, since the contribution rate in Model D05 will be cost-effective after all. Herewith connected is the matter of the contribution rate component for indexation. Model D05 does not distinguish components.

Moreover, it is difficult to categorize Model D05 in the indexation matrix.

## References

- [1] C.G.E. Boender. A hybrid simulation/optimisation scenario model for asset/liability management. *European Journal of Operational Research*, 99:126–135, 1997.
- [2] E. Bogentoft, H.E. Romeijn, and S. Uryasev. Asset/liability management for pension funds using CVaR constraints. *The Journal of Risk Finance*, 3(1):57–71, 2001.
- [3] G. Consigli and M.A.H. Dempster. Dynamic stochastic programming for asset-liability management. *Annals of Operations Research*, 81:131–161, 1998.
- [4] De Nederlandsche Bank. Adoption of interest rate term structure methodology. [http://www.dnb.nl/dnb/bin/doc/vaststelling%20methode%20rentetermijnstructuur%20FTK%20English\\_tcm12-53179.pdf](http://www.dnb.nl/dnb/bin/doc/vaststelling%20methode%20rentetermijnstructuur%20FTK%20English_tcm12-53179.pdf), March 2005.
- [5] De Nederlandsche Bank. Interne modellen methode: Filosofie en uitgangspunten (Internal model method: Philosophy and basic principles). [http://www.dnb.nl/dnb/bin/doc/interne%20modellen%20methoden%20-%20filosofie%20en%20uitgangspunten\\_tcm12-66385.pdf](http://www.dnb.nl/dnb/bin/doc/interne%20modellen%20methoden%20-%20filosofie%20en%20uitgangspunten_tcm12-66385.pdf), October 2005.
- [6] De Nederlandsche Bank. Consultatie toeslagenmatrix (consultation indexation matrix). [http://www.dnb.nl/dnb/home/file/brief%20consultatie%20beleidsregel%20toeslagenmatrix\\_tcm12-114969.pdf](http://www.dnb.nl/dnb/home/file/brief%20consultatie%20beleidsregel%20toeslagenmatrix_tcm12-114969.pdf), January 2007.
- [7] De Nederlandsche Bank (DNB). <http://www.dnb.nl>.
- [8] C.L. Dert. *Asset Liability Management for Pension Funds: A Multistage Chance Constrained Programming Approach*. PhD thesis, Erasmus University Rotterdam, 1995.
- [9] S.J. Drijver. *Asset Liability Management for Pension Funds Using Multistage Mixed-Integer Stochastic Programming*. PhD thesis, University of Groningen, 2005.
- [10] European Insurance and Occupational Pensions Committee. Solvency accords. [http://ec.europa.eu/internal\\_market/insurance/solvency\\_en.htm](http://ec.europa.eu/internal_market/insurance/solvency_en.htm).
- [11] European Parliament and the Council of the European Union. Directive 2003/41/EC of the European Parliament and of the Council of 3 June 2003 on the activities and supervision of institutions for occupational retirement provision. *Official Journal of the European Union*, L 235:10–21, 2003.

- [12] J. Gondzio and R. Kouwenberg. High performance computing for asset liability management. *Operations Research*, 49(6):879–891, 2001.
- [13] M. Gordon. *The Investment, Financing, and Valuation of the Corporation*. Richard D. Irwin, Inc., Homewood, Ill., 1962.
- [14] R.A. Haugen. *Modern Investment Theory*. Prentice Hall, Upper Saddle River, N.J., 4<sup>th</sup> edition, 1997.
- [15] P. Hilli, M. Koivu, T. Pennanen, and A. Ranne. A stochastic programming model for asset liability management of a Finnish pension company. *Annals of Operations Research*, to appear.
- [16] H.A. Klein Haneveld. *Solvabiliteitscriteria Voor Pensioenfondsen*. PhD thesis, University of Groningen, 1999.
- [17] W.K. Klein Haneveld. *Duality in stochastic linear and dynamic programming*, volume 274 of *Lecture Notes in Economics and Mathematical Systems*. Springer-Verlag, Berlin, 1986.
- [18] W.K. Klein Haneveld, M.H. Streutker, and M.H. van der Vlerk. An ALM model for pension funds using integrated chance constraints. Research Report 05A03, SOM, University of Groningen, <http://irs.ub.rug.nl/ppn/274795531>, 2005.
- [19] W.K. Klein Haneveld and M.H. van der Vlerk. Integrated chance constraints: reduced forms and an algorithm. *Computational Management Science*, 3(4):245–269, 2006.
- [20] R. Kouwenberg. Scenario generation and stochastic programming models for asset liability management. *European Journal of Operational Research*, 134:279–292, 2001.
- [21] D.G. Luenberger. *Investment Science*. Oxford University Press, New York, 1998.
- [22] Ministry of Social Affairs and Employment. Besluit financieel toetsingskader pensioenfondsen (financial assessment framework provisions). *Staatsblad van het Koninkrijk der Nederlanden*, 710, 2006.
- [23] Ministry of Social Affairs and Employment. Regeling parameters pensioenfondsen (regulation parameters pension funds). *Staatscourant*, 250, December 2006.
- [24] Ministry of Social Affairs and Employment. Regeling Pensioenwet en Wet verplichte beroepspensioenregeling (Regulation Pension Act and occupational pension scheme (obligatory participation) act). *Staatscourant*, 253, December 2006.
- [25] J.M. Mulvey. Introduction to financial optimization: Mathematical programming special issue. *Mathematical Programming*, 89:205–216, 2001.

- [26] J.M. Mulvey, G. Gould, and C. Morgan. An asset and liability management system for Towers Perrin-Tillinghast. *Interfaces*, 30(1):96–114, 2000.
- [27] ORTEC. <http://www.ortec.com>.
- [28] Pensioen- & Verzekeringskamer. Financial assessment framework consultation document. [http://www.dnb.nl/dnb/bin/doc/FTK%20Consultation%20Document%20English%20translation\\_tcm13-47968.pdf](http://www.dnb.nl/dnb/bin/doc/FTK%20Consultation%20Document%20English%20translation_tcm13-47968.pdf), October 2004.
- [29] R.T. Rockafellar and S. Uryasev. Optimization of conditional value-at-risk. *Journal of Risk*, 2(3):21–41, 2000.
- [30] M. van Rooij, A. Siegmann, and P. Vlaar. PALMNET: A pension Asset and Liability Model for the Netherlands. <http://ideas.repec.org/p/dnb/wormem/760en.html>, 2004.
- [31] M.S. Sodhi. LP modeling for asset-liability management: A survey of choices and simplifications. *Operations Research*, 53(2):181–196, 2005.

## A Required Solvency for Various Risk Types

This appendix contains the formulas for the required solvency for various risk types, as specified by the FTK, see [24]. They are used in the standardized method of the medium term solvency condition. As before, we will denote the value of the assets at time  $t$  by  $A_t$ , and the liabilities by  $L_t$ . The FTK considers a number of categories of the assets and the liabilities, see Table 1. So, for instance,  $A_t^{dm}$  is the value of the stocks invested in developed stock markets at time  $t$ .

### Interest Rate Risk

The interest rate risk is the risk that the surplus of the pension fund will decrease, due to the interest rate showing an upward or downward shock. Such a shock will influence the value of both the assets as well as the liabilities, since both have components related to the fixed income market. The DNB gives for each duration level two factors that represent a downward or upward shock, with which the market interest rate for the given duration should be multiplied to get the new interest rate. The desired solvency for a shock equals the difference between the effect the shock has on the liabilities and the assets. The required solvency for interest rate risk is the maximum of the upward and downward shock.

Denoting the duration of a financial product  $f$  by  $D_f$ , then  $D_{L_t}$  is the duration of the liabilities, and  $D_{A_t^f}$  is the duration of the fixed-income portfolio. Moreover, we simplify the notation of the interest term structure interest rate

**Table 1:** Asset and liability categories specified by the FTK in the standardized solvency test.

Category	Index
<i>Assets</i>	
developed markets	<i>dm</i>
emerging markets	<i>em</i>
private equity	<i>pe</i>
commodities	<i>cm</i>
fixed income	<i>fi</i>
credit bond	<i>cb</i>
currency exposure	<i>ce</i>
credit bond	<i>cb</i>
<i>Liabilities</i>	
retirement pension	<i>rp</i>
dependant's pension	<i>dp</i>

for the duration of a certain financial product to  $z(f)$  (instead of  $z(D_f)$ ). Introducing  $\delta^-(d)$  as the downward factor for a duration of  $d$ , the required solvency for a downward interest rate shock is

$$S_{1t}^- = \left[ \left( \frac{1 + z(L_t)}{1 + \delta^-(D_{L_t})z(L_t)} \right)^{D_{L_t}} - 1 \right] L_t - \left[ \left( \frac{1 + z(A_t^{fi})}{1 + \delta^-(D_{A_t^{fi}})z(A_t^{fi})} \right)^{D_{A_t^{fi}}} - 1 \right] A_t^{fi}.$$

The value of  $S_{1t}^+$ , the solvency for an upward interest rate shock, is calculated analogously. The maximum of both solvencies is the required solvency for interest rate risk, i.e.,

$$S_{1t} = \max\{S_{1t}^-, S_{1t}^+\}.$$

### Market Risk

The required solvency for market risk accounts for the risk faced in more risky investment classes, which are equity (developed, which includes indirect real estate, and emerging markets), private equity, and (direct) real estate. The required solvency is based on the amount invested in these classes:

$$\begin{aligned} S_{2t}^{dm} &= 0.25A_t^{dm}, \\ S_{2t}^{em} &= 0.30A_t^{em}, \\ S_{2t}^{pe} &= 0.30A_t^{pe}, \\ S_{2t}^{re} &= 0.15A_t^{re}. \end{aligned}$$

The total required solvency for market risk is calculated using mutual correlations of 0.75.

### **Foreign Exchange Risk**

The foreign exchange risk concerns the possible devaluation of foreign currencies, which affects the investments in these countries. The required solvency for this risk is 20% of the value of investments exposed to currency risk:

$$S_{3t} = 0.20A_t^{ce}.$$

### **Commodity Risk**

The risk that the value of the commodity portfolio decreases enormously is called commodity risk. The required solvency for this type of risk equals 30% of the value invested in commodities:

$$S_{4t} = 0.30A_t^{cm}.$$

### **Credit risk**

Credit risk is expressed in the credit spread. This is the difference between the effective yields on a collection of cash flows whose payment depends on the creditworthiness of counter parties and the effective yields on the same collection of cash flows as if they were certain to be paid. The higher the credit spread, the less the creditworthiness of the counter parties involved in the investments. Generally, bonds of a highly credit worthy government are regarded as default free.

The desired solvency for credit risk is 40% of the product of the duration of the credit bonds, the credit spread, and the value of the investment in credit bonds. So if we let  $cs_t$  be the credit spread of the credit bonds at time  $t$ , then the required solvency for credit risk is

$$S_{5t} = 0.40D_{A_t^{cb}} cs_t A_t^{cb}.$$

### **Underwriting Risk**

The required solvency for underwriting risk is to be determined by the pension fund itself. It should account for future mortality trend uncertainty and negative stochastic deviations from the expected value of the benefit payments.

## **B Symbols and Definitions**

In this section, the notation used throughout the paper is listed. The main ingredients are:



$t$	yearly time indicator, either a moment in time, or period of one year.
$A_t$	value of assets at time $t$ .
$L_t$	value of liabilities at time $t$ .

The other notation is grouped by subject.

## B.1 Valuation of Liabilities

$B_t(n)$	expected benefit payments $n$ years from time $t$ .
$d_t(n)$	discount factor for payments $n$ years from time $t$ .
$W_t$	total level of the pensionable wages of the active participants in year $t$ .
$w_t$	change in general wage level in year $t$ .
$r_t^s$	single discount rate at time $t$ .
$z_t(n)$	swap rate for maturity of $n$ years at time $t$ .
$y_t(n)$	yield of a risk-free zero-coupon bond maturing $n$ years from time $t$ .
$earp_t$	ex-ante risk premium at time $t$ .
$PSC_t(n)$	pension spot curve value for a maturity of $n$ years from time $t$ .
$a_1, a_2, a_3$	time independent yield curve constants.
$a_{4t}$	yield on bonds with longest terms to maturity at time $t$ .
$r_t^{ba}$	return on bank account for year $t$ .
$R_{1t}$	internal rate of return of stock portfolio at time $t$ .

## B.2 Solvency of the Fund

$S_t$	solvency at time $t$ .
$F_t$	funding ratio at time $t$ .
$S_t^m$	minimum required solvency at time $t$ .
$S_t^r$	required solvency at time $t$ .
$\hat{S}_t^r$	estimated required solvency at time $t$ .
$i$	risk type index.
$S_{it}$	solvency required for risk type $i$ at time $t$ .
$\bar{L}_t$	value of fully-indexed liabilities at time $t$ .
$\underline{L}_t$	value of nominal liabilities at time $t$ .
$\alpha, \theta$	funding ratio parameters.
$a$	number of consecutive years before remedial contribution.
$\varphi$	integrated chance constraint parameter.

## B.3 Contribution Rate

$c_t$	contribution rate for year $t + 1$ .
$\underline{c}, \bar{c}$	lower and upper bound on contribution rate.
$\Delta c_t$	change in contribution rate.
$-\eta, \rho$	lower and upper bound on contribution rate change.
$c^*$	lower bound on contribution rate in case of remedial contribution.

## B.4 Indexation

$\varsigma_L$	total indexation deviation penalty parameter.
$m_t$	binary indicating full indexation of last years pensions.
$\lambda_m$	last year's partly indexation penalty parameter.

## B.5 Required Solvency for Various Risk Types

$f$	financial product.
$A_t^f$	value of financial product $f$ assets at time $t$ .
$D_f$	duration of financial product $f$ .
$\delta^-(d)$	downward factor interest shock for a duration of $d$ .
$\delta^+(d)$	upward factor interest shock for a duration of $d$ .
$S_{1t}^-$	required solvency for downward interest rate shock.
$S_{1t}^+$	required solvency for upward interest rate shock.
$S_{1t}$	required solvency for interest rate risk.
$cs_t$	credit spread at time $t$ .