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An Experimental Application of Total Energy Shaping Control: Stabilization of the Inverted Pendulum on a Cart in the Presence of Friction

J.C.M. van der Burg, R. Ortega, J.M.A. Scherpen, J.A. Acosta and H.B. Siguerdidjane

Abstract— **In this paper we report the experimental application of a state-feedback controller derived via the principles of Total Energy Shaping Control for the stabilization of underactuated mechanical systems. The particular application concerns the well-known inverted pendulum on a cart. We describe the first steps taken towards** *global* **stabilization of the inverted pendulum with Total Energy Shaping Control. The results show that performance of the nonlinear controller in the neighborhood of the equilibrium position is better compared to a linear H-infinity controller, since the transients are smoother and there is less overshoot. Furthermore, it is shown that the energy shaping controller has a great tuning potential that allows proper functioning of the closed-loop system in the presence of friction. This paper is intended to be the starting point in the development of tools that enable the control engineer to make deliberate choices in tuning the energy-shaping controller, based on performance in the presence of friction and in a later stage also for parameter uncertainties, input constraints and other issues that are relevant in a practical environment.**

Index Terms— **Energy shaping, Hamiltonian systems, nonlinear control, passivity, underactuated mechanical systems with friction.**

I. INTRODUCTION

Total Energy Shaping Control is a controller design methodology that achieves (asymptotic) stabilization of mechanical systems endowing the closed-loop system with a Lagrangian or Hamiltonian structure and a desired energy function that qualifies as a Lyapunov function for the desired equilibrium. Also known as Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) in the Hamiltonian framework and as the Method of Controlled Lagrangians in the Lagrangian framework[1], the methodology was first published in [1]. Inspired by the attractive asset of clear physical interpretability of the controller and the closed-loop system in terms of potential and kinetic energy, researchers have succeeded in applying Total Energy Shaping to a number of simple underactuated mechanical systems. Simulations reported for well-known physical systems such as the Ball and Beam and the Inertia Wheel Pendulum [1], the VTOL Aircraft [6], and the Acrobot [7] show smooth closed-loop behavior and stabilization for a large domain of attraction.

The success of the method is limited by the possibility of solving two partial differential equations (PDEs) which identify the kinetic and potential energy functions that can be assigned to the closed-loop. Therefore, current research focusses on developing methods that make IDA-PBC applicable to a broader class of systems; especially [3] shows encouraging progress on the matter, making it possible to derive state-feedback controllers for the Inverted Pendulum on a Cart and the Furuta Pendulum.

As an obvious next step, these controllers need to be tested experimentally, because first, a successful experiment would prove the functioning of the IDA-PBC controller. And second, it is likely that the questions that are raised by extending the applicability to experimental set-ups will contribute to a better understanding of the methodology while at the same time unexpected or neglected phenomena demand for new analysis.

For example, the effect of friction on the closed-loop behavior needs to be assessed. This issue was first studied in [5], where a design specific condition has been derived, called the dissipation condition, that ensures that the total energy function of the closed-loop still qualifies as a storage function in the presence of friction. For systems that satisfy this condition, two methods are presented that allow the shaping of the closed-loop dissipation terms. Unfortunately, the article [5] only considers the case where a system satisfies the dissipation condition. While in fact a number of systems, including the Inertia Wheel Pendulum and the Inverted Pendulum on a Cart, do *not* satisfy it.

In this paper we are interested in the practical application of Total Energy Shaping to the inverted pendulum on a cart. The main contributions of the paper are:

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[1]In [8] and [9], it has been shown that the PDEs of the controlled Lagrangian method and IDA-PBC are the same.

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- A successful experimental application of the method of Total Energy Shaping, where the pendulum is stabilized from an initial angle in the upper half plane and where the cart is stabilized at a desired location on the rail.
- The proof that there does not exist a controller redesign that makes the dissipation condition hold; for the inverted pendulum on a cart, it is impossible to fulfill the condition deriving from energy shaping and the dissipation condition at the same time.
- The verification that, as indicated in [2] and well-known for PBC designs, there is a robustness margin against friction, depending on the artificial damping injection gain together with the other controller parameters.

II. Background on Total Energy Shaping Control in the Hamiltonian framework

In this section, a brief review is presented of the control method of Total Energy Shaping in the Hamiltonian framework, also known as IDA-PBC. IDA-PBC was first introduced in [1] to regulate the position of frictionless underactuated mechanical systems of the form

$$\Sigma : \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \nabla_q \mathscr{H} \\ \nabla_p \mathscr{H} \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u, \quad (1)$$

where $q \in \mathbb{R}^n$, $p \in \mathbb{R}^n$ are the generalized position and momenta, respectively, $u \in \mathbb{R}^m$ the input and $G \in \mathbb{R}^{n \times m}$ with rank $G = m < n$ accounting for underactuation.

$$\mathscr{H}(q,p) = \frac{1}{2} p^\top M^{-1}(q) p + \mathscr{V}(q) \quad (2)$$

is the total energy with $M = M^\top > 0$ the inertia matrix and $\mathscr{V}$ the potential energy function.

A. Energy Shaping

The main result of [1] is the proof that for all matrices $M_d = M_d^\top \in \mathbb{R}^{n \times n}$ and functions $\mathscr{V}_d(q)$ that satisfy the PDEs

$$G^\perp \{ M_d M^{-1} \nabla_q (p^\top M_d^{-1} p) - 2 J_2 M_d^{-1} p \} = G^\perp \nabla_q (p^\top M^{-1} p) \quad (3)$$

$$G^\perp \{ \nabla_q \mathscr{V} \} = G^\perp \{ M_d M^{-1} \nabla_q \mathscr{V}_d \}, \quad (4)$$

for some $J_2(q,p) = -J_2^\top(q,p) \in \mathbb{R}^{n \times n}$ and a full rank left annihilator $G^\perp(q) \in \mathbb{R}^{m \times n}$ of G, i.e., $G^\perp G = 0$ and rank $(G^\perp) = n - m$, the system (1) in closed-loop with the state-feedback control law $u = \hat{u}(q,p) + v$, where

$$\hat{u}(q,p) = (G^\top G)^{-1} G^\top (\nabla_q \mathscr{H} - M_d M^{-1} \nabla_q \mathscr{H}_d + J_2 M_d^{-1} p), \quad (5)$$

takes the Hamiltonian form Σ_d:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & M^{-1} M_d \\ -M_d M^{-1} & J_2 \end{bmatrix} \begin{bmatrix} \nabla_q \mathscr{H}_d \\ \nabla_p \mathscr{H}_d \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} v, \quad (6)$$

where the new total energy function is

$$\mathscr{H}_d(q,p) = \frac{1}{2} p^\top M_d^{-1}(q) p + \mathscr{V}_d(q). \quad (7)$$

Further, if M_d is positive definite in a neighborhood of $q^\star \in \mathbb{R}^{n \times n}$ and

$$q^\star = \arg\min \mathscr{V}_d(q), \quad (8)$$

then $(q^\star, 0)$ is a stable equilibrium point of (6) with Lyapunov function $\mathscr{H}_d$.

B. Asymptotic Stability

For the closed-loop system to converge to the desired stable equilibrium, a negative feedback of the passive output is applied:

$$v = -K_v G^\top \nabla_p \mathscr{H}_d, \quad (9)$$

where $K_v > 0$. This yields

$$\dot{\mathscr{H}}_d = -\nabla_p \mathscr{H}_d G K_v G^\top \nabla_p \mathscr{H}_d \leq 0. \quad (10)$$

If the system is detectable, then $(q^\star, 0)$ can be proven to be asymptotically stable.

C. Asymptotic Stability in the Presence of Friction

In the design strategy as presented in [1], friction is not taken into consideration, while in reality it occurs in every mechanical system. In [5], the issue is analyzed in detail. Here we state the most important results. First, friction is to be modeled in the open-loop system as a motion-opposing force that is velocity dependent, namely

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -R(q) \end{bmatrix} \begin{bmatrix} \nabla_q \mathscr{H} \\ \nabla_p \mathscr{H} \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u, \quad (11)$$

where the same definitions hold as in (1) and $R(q)$ is the friction matrix, smooth and bounded as a function of q. Then after applying both energy-shaping and damping injection, the closed-loop system with friction becomes

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & A_{12} \\ -A_{12} & A_{22} \end{bmatrix} \begin{bmatrix} \nabla_q \mathscr{H}_d \\ \nabla_p \mathscr{H}_d, \end{bmatrix} \quad (12)$$

with $A_{12} = M^{-1} M_d$, $A_{22} = J_2 - G K_v G^\top - R M^{-1} M_d$. Now

$$\dot{\mathscr{H}}_d = -\nabla_p \mathscr{H}_d \left(G K_v G^\top + R M^{-1} M_d \right) \nabla_p \mathscr{H}_d. \quad (13)$$

Assuming full-rank of the friction matrix R, stability is ensured for $\dot{\mathscr{H}}_d < 0$, hence all closed-loop systems have to satisfy the following necessary condition on the damping of the *unactuated* coordinate(s), see [5],

$$\frac{1}{2} G^\perp (R M^{-1} M_d + M_d M^{-1} R) (G^\perp)^\top > 0. \quad (14)$$

This dissipation condition is system dependent via friction matrix R and inertia matrix M. It is design dependent by the desired inertia matrix M_d. For diagonal matrices R and M, (14) automatically holds for all matrices M_d that satisfy the energy-shaping PDEs. The Ball and Beam, analyzed in [5] is such a case. However, for non-diagonal matrices R and M, it is possible that the matching conditions (3) and (4) stemming from energy-shaping, conflict with the dissipation condition (14).

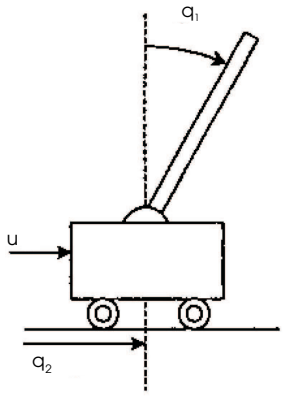

Fig. 1. Pendulum on a Cart

III. CONTROLLER DESIGN FOR THE PENDULUM-CART SYSTEM

A. The System Model

The dynamic equations of the Inverted Pendulum on a Cart, depicted in figure 1, with friction are given by (11), where[2]

$$M(q_1) = d\begin{bmatrix} 1 & b\cos q_1 \\ b\cos q_1 & c \end{bmatrix}, \qquad \mathscr{V}(q_1) = ad\cos q_1,$$

$$G = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ a = \tfrac{g}{l},\ b = \tfrac{1}{l},\ c = \tfrac{M+m}{ml^2},\ d = ml^2. \tag{15}$$

Coordinates q_1 and q_2 denote the pendulum angle relative to the upright vertical and the cart position, respectively. M is the mass of the cart, m and l the mass and length of the pendulum[3] and g is the gravitational acceleration. Friction is likely to occur in both coordinates, hence $R = diag\{r_1, r_2\}$, where the friction terms can be chosen to depend on the system states (q,p) in order to model Coulomb friction, see [5].

B. Controller Design

First, we assume that there is no friction and we follow the procedure of section II-A, starting by solving the two partial differential equations (3) and (4). Since the inertia matrix M of the pendulum system depends on coordinate q_1, the so-called forcing term $G^{\perp}\nabla_q(p^{\top}M^{-1}p)$ is nonzero, seriously complicating the kinetic energy PDE. In [3], a coordinate transformation is proposed such that the PDEs become solvable. Applying the technique developed in [3], we can derive the solution obtained in [6] without partial feedback linearization. Thus, the closed-loop parameterization $\{M_d, J_2, \mathscr{V}_d\}$ is

$$M_d = M\tilde{M}_d M \Rightarrow \tilde{M}_d = \begin{bmatrix} \frac{kb^2}{3}\cos^3 q_1 & -\frac{kb}{2}\cos^2 q_1 \\ -\frac{kb}{2}\cos^2 q_1 & k\cos q_1 + m_{22}^0 \end{bmatrix}, \tag{16}$$

with $k > 0$ and $m_{22}^0 \geq 0$. Furthermore

$$J_2(q,p) = M\tilde{J}_2(q, M^{-1}p)M + S(q,\tilde{p})\tilde{M}_d M - M\tilde{M}_d S^{\top}(q,\tilde{p}) \tag{17}$$

where $\tilde{p} = M^{-1}p$ and

$$\tilde{J}_2 = \tilde{p}^{\top}\tilde{M}_d^{-1}\mathscr{J}W, \qquad W = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

$$\mathscr{J} = \begin{bmatrix} \frac{k^2b^3}{12}\cos^4 q_1 \sin q_1 \\ -\frac{k^2b^2}{12}\cos^3 q_1 \sin q_1 \end{bmatrix}, \qquad S(q,\tilde{p}) = \nabla_q(M(q)\tilde{p}) \tag{18}$$

and finally for a free function $\Phi(.)$ with a minimum in $q^{\star}$

$$\mathscr{V}_d = \tfrac{3a}{kb^2\cos^2 q_1}\Phi(z(q))$$

$$\Rightarrow z(q) = q_2 - q_2^{\star} + \tfrac{3}{b}\ln(\sec q_1 + \tan q_1) + \tfrac{6m_{22}^0}{kb}\tan q_1, \tag{19}$$

where we take a quadratic function $\Phi(z(q)) = \frac{P}{2}[z(q)]^2$, with $P > 0$. Substituting the triplet $\{M_d, J_2, \mathscr{V}_d\}$ into (5) yields the energy-shaping state-feedback controller. This control law together with an artificial damping injection (9) ensures asymptotic stability of the desired equilibrium $(0, q_2^{\star}, 0, 0)$ with a domain of attraction equal to the set $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \mathbb{R}^3$. The fact that the domain of attraction is limited to the upper-half plane, can be seen in (19) where there is a $\cos q_1$ in the denominator, causing the controller to saturate at $q_1 = \frac{\pi}{2}$ and $q_1 = -\frac{\pi}{2}$. Simulations however, show a larger domain of attraction: the pendulum swings up from any position in the lower half plane, including the exact hanging position $(q_1 = \pi)$, note that since this is an unstable equilibrium of the closed-loop, a little initial momentum p_1 is needed to start the swing-up, see figure 2, where the system parameters are fixed to match the experimental set-up, namely $g = 9.81$ m/s^2 and

$$M = 0.60kg, \qquad m = 0.39kg, \qquad l = 0.36m. \tag{20}$$

C. Satisfying the Dissipation Condition

To see whether friction can destabilize the closed-loop system, the dissipation condition (14) is checked for the Inverted Pendulum on a Cart with its shaped mass matrix M_d (16). It equals:

$$-\frac{1}{6}r_1 kb^2\cos^3(q_1) < 0, \qquad \forall q_1 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \tag{21}$$

hence (14) does not hold and the total energy function of the closed-loop does not qualify as a storage function in the presence of friction. The question then comes up whether there exists another matrix M_d that does satisfy (14). First,

[2]Note that in [3], on which the controller derivation is based, use is made of a scaled Hamiltonian. Both the inertia matrix and the potential energy function differ by a factor d from the real ones presented here. This does not influence the solution of the PDEs of (3) and (4), but the resulting controller differs.

[3]Note that the m is a point mass and l the length of a massless rod, hence the inertia of the pendulum equals $d = ml^2$. For a pendulum that can not be modeled in this way, one has to compute an equivalent coefficient d.

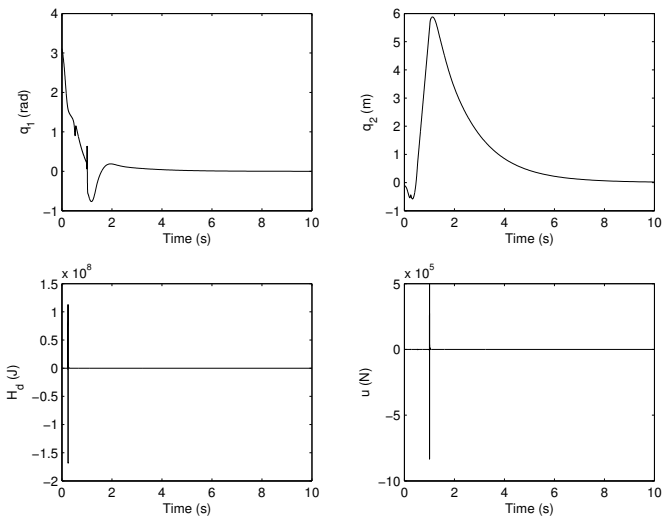

Fig. 2. Trajectories of q_1 and q_2, closed-loop energy $\mathscr{H}_d$ and input signal u with the pendulum swinging up to the upright stabilized position, cart end position $q_2^\star = 0$, $(q(0), p(0)) = (0.99\pi, -0.1, -0.1, 0)$, full state feedback: $(P, k, m_{22}^0, K_v) = (40, 0.007, 0.005, 0.01)$

we substitute the relation $M_d = M\tilde{M}_d M$ from [3] into (14), transforming the condition into

$$\frac{1}{2}G^{\perp}(R\tilde{M}_d M + M\tilde{M}_d R)(G^{\perp})^{\top} > 0. \tag{22}$$

Then for a generic positive definite symmetric matrix

$$\tilde{M}_d = \begin{bmatrix} m_1(q_1) & m_2(q_1) \\ m_2(q_1) & m_3(q_1) \end{bmatrix} > 0, \tag{23}$$

(22) yields the following condition on the elements of $\tilde{M}_d$:

$$dr_1(m_1(q_1) + m_2(q_1)b\cos q_1) > 0. \tag{24}$$

On the other hand, the potential energy PDE (4) yields the following condition on the elements of $\tilde{M}_d$, see [6]

$$m_1(q_1) + m_2(q_1)b\cos q_1 < 0, \tag{25}$$

hence for physically allowable coefficients r_1 and d, there does not exist a matrix $\tilde{M}_d$ (nor a matrix M_d) that satisfies both the energy shaping constraint and the dissipation condition at the same time.

The closed-loop Hamiltonian $\mathscr{H}_d$, which used to serve as a Lyapunov function in the frictionless case, is no longer a Lyapunov function when friction is present. Instead of looking for a new Lyapunov function, a cumbersome task, Lyapunov's linearization method is proposed to assess stability close to the desired equilibrium of the closed-loop system[4]. This analysis shows a robustness margin against friction in both coordinates that depends on all four controller parameters in the control loop (K_v, P, k, m_{22}^0).

This result that can be interpreted in two ways: first, given a set of controller parameters, a set of allowable friction coefficients r_1, r_2 can be found such that the closed-loop equilibrium is still asymptotically stable; this is the analyst's point of view. Second, given an estimated upperbound on the friction coefficients, a set of controller parameters can be found that makes the equilibrium asymptotically stable; this is the designer's point of view.

In most mechanical systems, the type of friction that occurs in rubbing contact is Coulomb friction. The accurate modeling of this type of friction, as proposed in [5] with a state-dependent friction matrix R, yields very high values of the coefficients $r_1(q,p)$, $r_2(q,p)$ in the neighborhood of zero velocity. By exceeding the stability bound imposed by a given controller on r_1 and r_2, this so-called stick-slip phenomenon is likely to cause instability of the equilibrium position. Simulations show limit-cycle behavior of the closed-loop system, which indeed indicates a small unstable region around the equilibrium followed by a stable region further away from the equilibrium position.

In a study on controller design for the Inverted Pendulum on a Cart in the Lagrangian framework, the authors of [10] reached the same conclusions. However, the experimental approach was different. Instead of trying to stabilize the inverted pendulum with the energy-shaping controller alone, a Lyapunov-based switching algorithm was proposed, where a conventional controller was used to stabilize the pendulum in the neighborhood of the desired equilibrium. This was done to remedy for the limit-cycle behavior that remained after the action of the energy-shaping controller. Here, we attempt a control strategy that achieves non-oscillatory stabilization *without* any switching.

IV. EXPERIMENTAL RESULTS

All experiments have been conducted on an experimental device that was at our disposal at the Laboratoire d'Automatique de l'École Supérieure d'Électricite. Figure 3 gives an overview picture of the set-up.

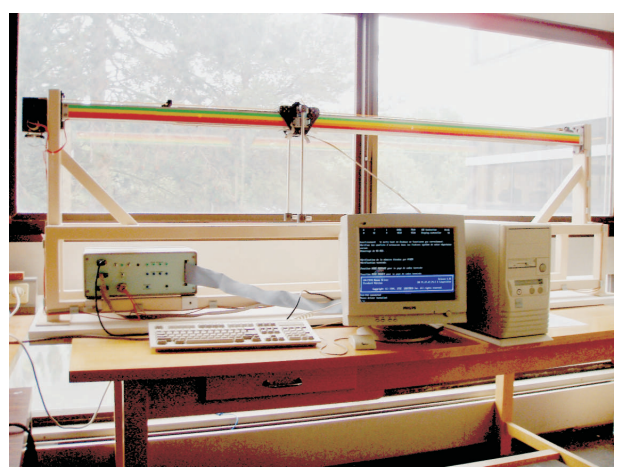

Fig. 3. The experimental apparatus, (Laboratoire d'Automatique, Supélec, France)

A. Description of the Set-up

The pendulum is attached to the cart and the cart is free to slide over a rail, where several bearings support the cart. The cart is attached to a stiff rubber conveyer belt that runs over

[4]Notice that we now lack a theoretical proof of global stability.

a set of pulleys, one attached to a DC motor (type ESCAPE 36L2R12-422P). The maximum cart travel is 1.8 m.

Two incremental sensors monitor the position of the cart and the angle of the pendulum with a resolution of 0.33 mm on the position and 0.0015 rad on the angle. The sensor output is fed through a 16 bits analog-digital convertor implemented in a HP Vectra QS/2 micro-computer that ensures real-time command. A 16 bits digital-analog converter generates a control signal that is amplified before being applied as an input-voltage to the DC-motor. Remember from the model (1) that the controller is defined in terms of a force applied to the cart, where actually a voltage needs to be calculated. However, since there is a PI-loop around the motor, assuming a linear relation between the input-voltage to the output force only introduces a negligible error. Hence the controller signal that is calculated needs to be multiplied by an actuator gain ($3.04\ N/V$). Furthermore, it is noted that the motor has a maximum allowable magnitude of the input voltage of ten volts. This input constraint is programmed via a straightforward if-then-statement. There is a delay of 50 ms in the loop due to the time that is needed to execute the controller routine.

Finally, a linear observer estimates the angular velocity of the pendulum and the speed of the cart. Then, with the relation $p = M\dot{q}$, the corresponding momenta can be calculated and fed into the control law.

B. Friction Compensation

The friction in the pendulum joint is negligible for this particular set-up. The friction between the cart and the rail is substantial and is compensated by a preliminary feedback loop (in other words, cascaded inner loop): to the controller of (5) is added

$$\hat{u}_{comp}(\dot{q}_2) = \frac{\beta \dot{q}_2}{\sqrt{\alpha + \dot{q}_2^2}}, \tag{26}$$

where the pair $\alpha = 10^{-3}$ and $\beta = 0.55$ best cancels the Coulomb friction present in the set-up.

C. Experimental Results

For the well-compensated system, two sets of controller parameters are tested. Figure 4 shows the trajectories for the first set of parameters

$$K_v = 0.01, \qquad P = 20, \qquad k = 0.007, \qquad m_{22}^0 = 0.005, \tag{27}$$

whereas figure 5 shows the trajectories for the second set of controller parameters with a higher damping injection gain

$$K_v = 0.13, \qquad P = 20, \qquad k = 0.07, \qquad m_{22}^0 = 0. \tag{28}$$

Both controllers show smooth convergence to the equilibrium position. Note the excellent performance with parameter set (27); the controller of (28) has a relatively large overshoot compared to (27). An analysis of the linearized closed-loop system shows that for the parameters of (28) the system is more robust to friction, meaning it can handle higher friction between cart and rail before becoming unstable. At the same time, this controller (28) gives less satisfactory performance when friction is well-compensated; the higher value of K_v enhances robustness, but also slows down the transients.

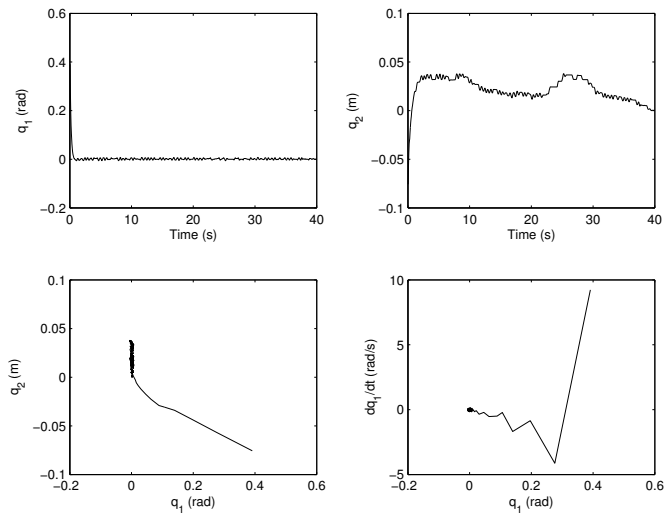

Fig. 4. Trajectories of the experimental set-up with the pendulum starting at about 0.4 rad from its upright stabilized equilibrium position, cart end position $q_2^\star = 0$, full state feedback given by 27, friction compensation parameters: $(\alpha, \beta) = (10^{-3}, 0.55)$.

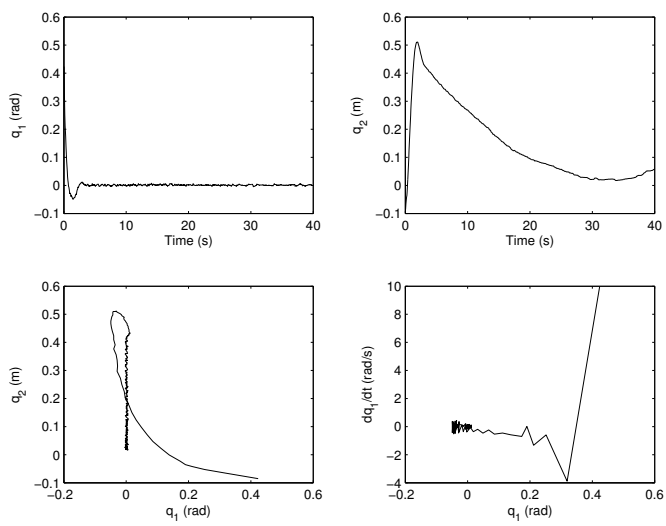

Fig. 5. Trajectories of the experimental set-up; cart end position $q_2^\star = 0$, full state feedback given by 28 with friction compensation parameters: $(\alpha, \beta) = (10^{-3}, 0.55)$.

In order to see the effect of friction on both controllers, the coefficient β of the preliminary feedback is adjusted to 0.20, meaning that only approximately 40 % of the friction is compensated. Experiments show that the parameters of (27) then yield unstable behavior: the pendulum is kept upright while the cart drifts away until it reaches the end of the rail. The parameters of (28), represented in figure 6, on the other hand succeed in keeping the pendulum upright with a pronounced oscillatory motion of the cart.

The fact that parameterset (28) succeeds in keeping the pendulum upright whereas parameterset (27) obviously fails,

confirms the earlier claim that for higher values of K_v robustness against friction increases. This experiment, compared to the previous experiments of figure 4 and figure 5, also confirms the fact that Coulomb friction leads to limit-cycle behavior of the closed-loop system: the better friction is compensated, the smaller the oscillations become.

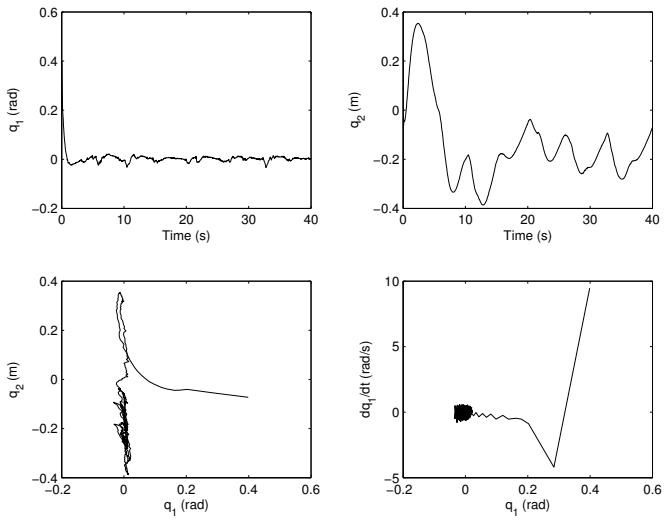

Fig. 6. Trajectories of the experimental set-up; cart end position $q_2^\star = 0$, full state feedback given by 28 with friction compensation parameters: $(\alpha, \beta) = (10^{-3}, 0.20)$.

As a final experiment, an H-infinity controller is tested. This controller is derived based on a linear system model that is valid only in a small domain around the pendulum's upright position. The closed-loop trajectories are presented in figure 7. Compared to the IDA-PBC controller of (27), figure 4, the H-infinity controller shows a larger overshoot. Furthermore, the IDA-PBC controller has smoother trajectories. Note that both controllers show only little oscillatory behavior[5]. Remind that IDA-PBC should have the main advantage of having a larger domain of attraction.

V. CONCLUSIONS AND FUTURE RESEARCH

A. Conclusions

It was already known from [12] that for the stabilization of the Inertia Wheel Pendulum in its upright position, no controller redesign exists that allows the closed-loop energy function $\mathscr{H}_d$ to be used as a Lyapunov function in the presence of friction. In this paper, it has been proven that for the Inverted Pendulum on a Cart, the same holds: no desired inertia matrix can be found that satisfies both the matching conditions and the dissipation condition at the same time.

Knowing that in the presence of friction, Lyapunov analysis is no longer applicable with the closed-loop Hamiltonian as a Lyapunov function, new ways of assessing closed-loop stability need to be adopted. For the inverted pendulum on a cart, Lyapunov's linearization method reveals the stabilizing properties of the controller parameters and the destabilizing

[5]Little oscillatory behavior can be seen, in both cases, as a limit-cycle motion due to uncompensated frictional effects.

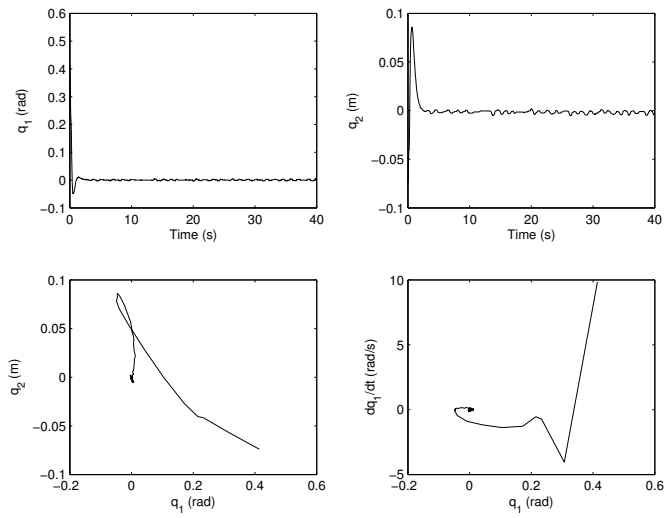

Fig. 7. Trajectories of the experimental set-up. The controller is an H-infinity controller: $q_2^\star = 0$, full state feedback: $u = 27.43q_1 + 7.67(q_2 - q_2^\star) + 5.32\dot{q}_1 + 12.44\dot{q}_2$, (output is a force, not a voltage). No friction compensation.

properties of the friction terms. With the help of this linear analysis, a defendable choice can be made between enlarging the stability margin of the closed-loop system and increasing the gain of the loop.

With a preliminary feedback loop to compensate the mayor friction source in the actuated coordinate, IDA-PBC has been successfully applied to an experimental set-up. Allowing some more friction into the system reveals that tuning, with the help of the linear stability analysis, indeed improves robustness.

B. Future Research

There are still a number of steps to be taken before the pendulum can be stabilized in the upright position starting from any possible angle. This is currently under investigation and the results will be reported elsewhere.

It would not be surprising, although (not yet) theoretically confirmed, to see the pendulum swing up with the same controller from any position in the lower half plane as well, as simulations show, figure 2. However, the swing-up might reveal itself to be very demanding on the system's hardware. Figure 2 for example shows a position overshoot of the cart of almost six meters, whereas most practical set-ups have a limited cart travel distance of at most two meters. Of course, other masses of the pendulum and the cart are likely to yield a smaller overshoot. Therefore, it should be tested beforehand by simulation if a given experimental set-up is appropriate.

Furthermore, note that the problem of input constraints is likely to show up if one wishes to swing up the pendulum in one smooth movement, for a DC motor yields a limited maximum force. This input bound should be taken into account during controller design and controller tuning. For example, there is still some freedom left in assigning the desired potential energy function (19). Instead of choosing a

quadratic function $\Phi(.)$, one could also take logarithmic or saturated functions to account for input constraints or rate-saturations, see [6].

Finally, if this has been done, the controlled Inverted Pendulum on Cart can be used to further develop physically motivated theory to analyze robustness against all types of uncertainties (including the practically relevant parameter uncertainty), or to develop strategies that ease the tuning of the controller. For instance, it could be attempted to investigate the possibilities of compensating friction in the *unactuated* coordinate. For the particular set-up that is used for this paper, friction in this coordinate is negligible, but other set-ups are likely to be different. In [5], it has been shown that there are some possibilities in using the additional freedom of assigning a skew symmetric matrix J_{20} that has its influence on the friction terms in closed-loop.

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