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Robust decentralized output regulation for uncertain heterogeneous systems

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Abstract— We consider the problem in which N heterogeneous uncertain linear systems aim at tracking a reference signal generated by a given exosystem under the restriction that not all the systems are directly connected to the exosystem. To tackle this problem, the reference signal is reconstructed via local interaction of the systems among themselves and the exosystem in accordance with the given communication graph. Then decentralized robust controllers which use the reconstructed reference signals are designed and shown to track the prescribed reference signal.

I. INTRODUCTION

In multi-agent coordination problems one of the possible tasks which the agents may have to carry out is to track an exogenous signal ([1], [8], [12]). The reference signal is not available to all the agents, and strategies to overcome this limitation are put in place ([3], [12], [4]).

In [16] it is shown that output synchronization occurs for N heterogeneous linear systems if and only if an exosystem ([10]) exists which generates the "reference signal" to which all the systems' outputs converge. As a result, controllers which guarantee output synchronization are those which solve an output regulation problem associated to that exosystem.

Motivated by this result, we turn the attention to the problem in which the systems aim at tracking a reference signal generated by an exosystem given in advance. Differently from [16], we do not assume that the systems' models are perfectly known and robust regulators have to be designed ([10]). The problem in [16] for the case of uncertain systems has been studied in [11] as well but, compared with the latter, the problem formulation in our paper is slightly different and the approach taken in this paper seems to lead to a simple analysis.

Other approaches have been proposed very recently in the literature. In [14], the systems which do not have direct access to the exosystem exchange information about the local tracking errors. Compared to our contribution, however, the authors require the communication graph to contain no cycles and the uncertainties to be sufficiently small. The restrictions on the graph in [14] have been weakened in [9] but assuming that the systems have all the same model and no uncertainty is considered.

Internal-model-based velocity tracking in coordination problems for passive systems is dealt with in [2, Chapter 3], while leader-follower problems using the internal model principle have been studied in [15]. In Section III of [12] an internal model approach to (position and) velocity tracking in networks of Euler-Lagrange systems is pursued, but the exosystem is restricted to the trivial one (constant reference velocity). To deal with non-constant reference velocities the authors rely on a discontinuous control law and require information about one-hop and two-hop neighbors. Related work is also available in [4]. Antecedents on the use of ideas from output regulation theory and multi-agent systems can be found in the work [5], later developed in e.g. [6], [7].

In Section II we formulate the problem along with the standing assumptions. In Section III, the main results are stated. The actual design of the controllers is described in IV and then illustrated via a numerical example in Section V. Conclusions are drawn in Section VI. Due to space limitations, proofs could not be included in the paper.

II. PROBLEM STATEMENT AND STANDING ASSUMPTIONS

Consider N heterogeneous uncertain linear dynamical systems

$$S_i : \quad \begin{array}{rcl} \dot{x}_i & = & A_i(\mu_i)\, x_i + B_i(\mu_i)\, u_i \\ y_i & = & C_i(\mu_i)\, x_i \,, \end{array} \quad (1)$$

with state vector $x_i \in \mathbb{R}^{n_i}$, control input $u_i \in \mathbb{R}^{p_i}$, and output vector $y_i \in \mathbb{R}^q$ for $i = 1, \cdots, N$. Each matrix of the system (1) depends on a vector μ_i of uncertain parameters which is assumed to range over a given set $\mathcal{P}_i$.

Consider also another system, which we will refer to as the "leader", whose dynamical behavior is described by the following equation:

$$\dot{w}_0 = S w_0, \quad r = R w_0 \,, \quad (2)$$

where $w_0 \in \mathbb{R}^m$, $r \in \mathbb{R}^q$, and matrices $S \in \mathbb{R}^{m \times m}$, $R \in \mathbb{R}^{q \times m}$. These matrices are assumed to satisfy the following:

Assumption 1 *The real parts of the eigenvalues of S are zero, i.e., $\sigma(S) \subset \mathbb{C}^0$ and (R, S) is detectable.*

The N systems (1) exchange information according to the pattern described by the directed graph $G = (V, E)$. Each system is represented by a node in the set $V = \{1, 2, \ldots, N\}$ and system j sends information to system i if and only if $(j, i) \in E \subseteq V \times V$. Associated to the graph G is the adjacency matrix $A = [a_{ij}]$. The entry $a_{ij} > 0$ if and only if $(j, i) \in E$ and 0 otherwise. If $a_{ij} > 0$, we say that j is a neighbor of i. We set $a_{ii} = 0$ for each $i = 1, 2, \ldots, N$. The Laplacian L is the matrix $L = D - A$, with $D = \mathrm{diag}(d_1, \ldots, d_N)$ and $d_i = \sum_{j \neq i} a_{ij}$. A directed path from i to j is a sequence of edges $(v_0, v_1), (v_1, v_2), \ldots, (v_{k-1}, v_k)$ in E such that $v_0 = i$ and $v_k = j$.

In addition to the graph G, we consider the directed graph $G_0 = (V_0, E_0)$, obtained as follows. Let the system (2) (the

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leader) be associated with the node 0 and set $V_0 = V \cup \{0\}$. Moreover, for $i = 1, 2, \ldots, N$, we set $a_{i0} > 0$ if and only if there is an arc from 0 to i and $a_{i0} = 0$ otherwise. Then we set $E_0 = E \cup \{(0, i) : a_{i0} > 0\}$. Compared with G, the graph G_0 additionally describes which followers have direct access to the information of the leader.

In what follows we exploit the following lemma ([12]), where we refer to the graphs G, G_0 and the Laplacian L introduced above.

Lemma 1 *If in the graph G_0 the node 0 has directed paths to all the nodes $i = 1, 2, \ldots, N$, then all the eigenvalues of the matrix $L + \mathrm{diag}(a_{10}, \ldots, a_{N0})$ have strictly positive real part.*

The objective of the paper is to design the control laws u_i which guarantee

$$\lim_{t \to \infty} ||y_i(t) - R w_0(t)|| = 0, \quad \text{for all } i = 1, \cdots, N,$$

under the following restrictions on the available measurements: (i) Only the systems S_i for which $a_{i0} > 0$ can access the leader and therefore the reference signal r. Hence, only these systems S_i can measure the *tracking error* $\epsilon_i = y_i - R w_0$. (ii) The systems S_i's exchange only relative information. (iii) For all $i = 1, \cdots, N$, the system S_i has access to the relative information with respect to S_j if and only if S_j is a neighbor of S_i.

Other assumptions are needed in order to state our main result in the next section.

Assumption 2 *(i) the μ_i-dependent Francis' equations*

$$\Pi_i(\mu_i)S = A_i(\mu_i)\Pi_i(\mu_i) + B_i(\mu_i)\Gamma_i(\mu_i)$$
$$0 = C_i(\mu_i)\Pi_i(\mu_i) - R \tag{3}$$

have a μ_i-dependent solution $\Pi_i(\mu_i)$, $\Gamma_i(\mu_i)$ for each $i = 1, \cdots, N$.

(ii) there exist matrices $\Phi_i, H_i, \Sigma_i(\mu_i)$, with Φ_i, H_i independent of μ_i, such that

$$\Sigma_i(\mu_i)S = \Phi_i \Sigma_i(\mu_i)$$
$$\Gamma_i(\mu_i) = H_i \Sigma_i(\mu_i) \tag{4}$$

(iii) there exists a matrix G_i independent of μ_i such that the linear system defined by the triplet

$$\begin{pmatrix} A_i(\mu_i) & B_i(\mu_i)H_i \\ 0 & \Phi_i \end{pmatrix} \quad \begin{pmatrix} B_i(\mu_i) \\ G_i \end{pmatrix} \quad \begin{pmatrix} C_i(\mu_i) & 0 \end{pmatrix}$$

is robustly stabilizable by the dynamic output feedback, i.e. there are matrices K_i, L_i, M_i independent of μ_i, such that the matrix

$$\begin{pmatrix} \begin{pmatrix} A_i(\mu_i) & B_i(\mu_i)H_i \\ 0 & \Phi_i \end{pmatrix} & \begin{pmatrix} B_i(\mu_i) \\ G_i \end{pmatrix} M_i \\ K_i \begin{pmatrix} C_i(\mu_i) & 0 \end{pmatrix} & L_i \end{pmatrix} \tag{5}$$

is Hurwitz.

A few comments on Assumption 2 are in order.
– Fix $i \in \{1, 2, \ldots, N\}$. Suppose that there exists a controller of the form

$$\dot{\xi}_i = L_i \xi_i + K_i \epsilon_i$$
$$u_i = H_i \xi_i + M_i \epsilon_i \tag{6}$$

which robustly stabilizes the system S_i. Then, provided that $\sigma(S) \subset \mathbb{C}^0$ (see Assumption 1), equations (3), (4) are well-known ([10, Proposition 1.4.1]) necessary and sufficient conditions for the controller (6) to solve the tracking problem for the system (1) for each $\mu_i \in \mathcal{P}_i$. Recall that the controller (6) is said to solve the tracking problem for the system (1) for each $\mu_i \in \mathcal{P}_i$ if, for each $\mu_i \in \mathcal{P}_i$, (i) the equilibrium $(x_i, \xi_i) = (\mathbf{0}, \mathbf{0})$ of the unforced closed-loop system (1), (6) is asymptotically stable; (ii) the response of the closed-loop system (1), (6) is such that $\lim_{t \to \infty} \epsilon_i(t) = \mathbf{0}$.
– If in addition condition (iii) in Assumption 2 holds, then one can prove that the dynamic feedback control law

$$\begin{aligned} \dot{\eta}_i &= \Phi_i \eta_i + G_i M_i \xi_i \\ \dot{\xi}_i &= L_i \xi_i + K_i \epsilon_i \\ u_i &= H_i \eta_i + M_i \xi_i \end{aligned} \tag{7}$$

solves the tracking problem. Due to the fact that the tracking error ϵ_i may not be available to the controller of system S_i, the previous controller cannot be implemented. In the next section, we overcome this lack of information on ϵ_i with the use of the information collected from the neighbors of system S_i.

III. MAIN RESULTS

The control strategy we propose to solve the decentralized output regulation problem formulated in the previous section comprises two steps. Since not all the systems S_i may have access to the reference signal r, we first design systems which aim at asymptotically reconstructing the reference signal using only locally available relative information. As a second step, we use such an asymptotic estimate of the reference signal to feed the tracking controllers and show that they achieve the prescribed control objective.
Motivated by Lemma 1, we introduce the following:

Assumption 3 *In the graph G_0 the node 0 has directed paths to all the nodes $i = 1, 2, \ldots, N$.*
The assumption implies that there exist N_1 systems, with $1 \leq N_1 \leq N$, which have direct access to the *reference* r generated by the leader. Without loss of generality and for the sake of simplicity, we assume that $N_1 = 1$ and that the system with direct access to the leader is the first one. To reconstruct the reference signal, the systems cooperate to estimate the internal state of the exosystem. For system S_1, the estimation is carried out by

$$\begin{aligned} \dot{\hat{w}}_0 &= S\hat{w}_0 + PR(w_0 - \hat{w}_0) \\ \dot{w}_1 &= Sw_1 + \sum_{j=1}^{N} a_{1j}(w_j - w_1) + a_{10}(\hat{w}_0 - w_1), \end{aligned} \tag{8}$$

where the matrix P is properly chosen in such a way that $\sigma(S - PR) \subset \mathbb{C}^-$ and $\hat{w}_0$ is an asymptotic estimate of the leader's internal state w_0. For system S_i, the system which carries out the asymptotic estimation is given by

$$\dot{w}_i = Sw_i + \sum_{j=1}^{N} a_{ij}(w_j - w_i). \tag{9}$$

For the system (2), (8), (9) we have the following result for the convergence of w_i:

Lemma 2 *Let Assumption 1 and 3 hold. Then $||w_i(t) - w_0(t)|| \to 0$ exponentially $\forall i = 1, \cdots, N$, as $t \to \infty$.*

Remark 1 Clearly the signals $Rw_i(t)$, $i = 1, 2, \ldots, N$, converge exponentially to $r(t)$.

Next, we introduce the controllers for systems (1) as follows. As system S_1 has access to w_0, we design u_1 as

$$
\begin{aligned}
\dot{\hat{w}}_0 &= S\hat{w}_0 + PR(w_0 - \hat{w}_0) \\
\dot{w}_1 &= Sw_1 + \sum\nolimits_{j=1}^{N} a_{1j}(w_j - w_1) + a_{10}(\hat{w}_0 - w_1) \\
\dot{\eta}_1 &= \Phi_1 \eta_1 + G_1 M_1 \xi_1 \\
\dot{\xi}_1 &= L_1 \xi_1 + K_1(y_1 - Rw_1) \\
u_1 &= H_1 \eta_1 + M_1 \xi_1
\end{aligned}
\tag{10}
$$

For agent $i = 2, \cdots, N$, we design u_i as

$$
\begin{aligned}
\dot{w}_i &= Sw_i + \sum\nolimits_{j=1}^{N} a_{ij}(w_j - w_i) \\
\dot{\eta}_i &= \Phi_i \eta_i + G_i M_i \xi_i \\
\dot{\xi}_i &= L_i \xi_i + K_i(y_i - Rw_i) \\
u_i &= H_i \eta_i + M_i \xi_i
\end{aligned}
\tag{11}
$$

The matrices $\Phi_i, G_i, M_i, L_i, K_i, H_i$ are those found in Assumption 2.

Theorem 1 *Consider N heterogeneous linear systems (1) coupled via the dynamic couplings (10) and (11). Suppose Assumptions 1–3 hold. Then, $||y_i(t) - Rw_0(t)||$ exponentially converges to 0 as $t \to \infty$ for all $i = 1, \cdots, N$.*

To explicitly design the regulators in the next section, we need the following corollary which deals with the case in which the dynamics of each system (1) are affected by the signals w_i, namely

$$
S_i^w : \quad
\begin{aligned}
\dot{x}_i &= A_i(\mu_i) x_i + B_i(\mu_i) u_i + P_i(\mu_i) w \\
y_i &= C_i(\mu_i) x_i,
\end{aligned}
\tag{12}
$$

where $w = (w_1^T \ldots w_N^T)^T$ is the vector of signals generated by (8), (9) and $P_i(\mu_i) = (P_{i1}(\mu_i) \ldots P_{iN}(\mu_i))$. The previous theorem can then be easily extended provided that Assumption 2 is modified as follows:

Assumption 4 *(i) the μ_i-dependent Francis' equations*

$$
\begin{aligned}
\Pi_i(\mu_i)S &= A_i(\mu_i)\Pi_i(\mu_i) + B_i(\mu_i)\Gamma_i(\mu_i) + \sum\nolimits_{j=1}^{N} P_{ij}(\mu_i) \\
0 &= C_i(\mu_i)\Pi_i(\mu_i) - R
\end{aligned}
\tag{13}
$$

have a μ_i-dependent solution $\Pi_i(\mu_i)$, $\Gamma_i(\mu_i)$ for each $i = 1, \cdots, N$. (ii) and (iii) are as in Assumption 2.

The result below is used in the next section to design the output regulators.

Corollary 1 *Consider N heterogeneous linear systems (12). Suppose the systems are coupled via the dynamic couplings (10) and (11). Suppose Assumptions 1, 3 and 4 hold. Then, $||y_i(t) - Rw_0(t)||$ exponentially converges to 0 as $t \to \infty$ for all $i = 1, \cdots, N$.*

IV. DESIGN OF THE CONTROLLERS

The actual design of the controllers in the previous section reposes on the fulfillment of the conditions in Assumption 2 or 4. In this section we discuss how this can be achieved. The arguments follow the treatment in [10, Section 1.5]. We start with condition (ii), namely with the fulfillment of the internal model principle. Let Φ, H and $\Sigma_i(\mu_i)$ be the matrices

$$
\Phi_i = \begin{pmatrix}
0 & I_{p_i} & 0 & \ldots & 0 \\
0 & 0 & I_{p_i} & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & I_{p_i} \\
-a_0 I_{p_i} & -a_1 I_{p_i} & -a_2 I_{p_i} & \ldots & -a_d I_{p_i}
\end{pmatrix},
$$

$$
H_i = \begin{pmatrix} I_{p_i} \\ 0 \\ 0 \\ \cdots \\ 0 \end{pmatrix}^T, \quad
\Sigma_i(\mu_i) = \begin{pmatrix}
\Gamma_i(\mu_i) \\
\Gamma_i(\mu_i)S \\
\vdots \\
\Gamma_i(\mu_i)S^{d-2} \\
\Gamma_i(\mu_i)S^{d-1}
\end{pmatrix}
$$

where $\lambda^d + a_{d-1}\lambda^{d-1} + a_1 d + a_0$ is the minimal polynomial of S and $\Gamma_i(\mu_i)$ is the matrix which appears in the regulator equations (3). It is straightforward to check that these matrices satisfy the internal model condition (4).

To the purpose of fulfilling also the robust stability condition (iii), it is convenient to introduce other matrices F_i, G_i, Ψ_i, T_i which also fulfill the internal model principle. These matrices are detailed in the following lemma ([10, Lemma 1.5.6]):

Lemma 3 *Let F_i be any Hurwitz $s \times s$ matrix and let G_i be any $s \times 1$ vector such that the pair (F_i, G_i) is controllable. Let Φ be any $s \times s$ matrix whose eigenvalues are all in $\overline{\mathbb{C}^+}$ and let H be any $1 \times s$ vector such that the pair (H, Φ) is observable.*
Then there exist a $1 \times s$ vector Ψ_i and a nonsingular $s \times s$ matrix T_i such that

$$
\begin{aligned}
(F_i + G_i\Psi_i)T_i &= T_i\Phi_i \\
\Psi_i T_i &= H_i.
\end{aligned}
\tag{14}
$$

It is immediate to see that the matrix $\tilde{\Sigma}_i(\mu_i) = T_i\Sigma_i(\mu_i)$ satisfies

$$
\begin{aligned}
\tilde{\Sigma}_i(\mu_i)S &= (F_i + G_i\Psi_i)\tilde{\Sigma}_i(\mu_i) \\
\Gamma_i(\mu_i) &= \Psi_i\tilde{\Sigma}_i(\mu_i).
\end{aligned}
$$

Hence, the internal model principle property (4) is fulfilled by the matrices $F_i + G_i\Psi_i$, Ψ_i, $\tilde{\Sigma}_i(\mu_i)$.
The controllers introduced in Section III can be rewritten as:

$$
\begin{aligned}
\dot{\hat{w}}_0 &= S\hat{w}_0 + PR(w_0 - \hat{w}_0) \\
\dot{w}_i &= Sw_i + \sum\nolimits_{j=1}^{N} a_{ij}(w_j - w_i) + a_{i0}(\hat{w}_0 - w_i) \\
\dot{\eta}_i &= (F_i + G_i\Psi_i)\eta_i + G_i M_i \xi_i \\
\dot{\xi}_i &= L_i \xi_i + K_i(y_i - Rw_i) \\
u_i &= \Psi_i\eta_i + M_i\xi_i
\end{aligned}
\tag{15}
$$

where $i = 1, \cdots, N$. Note that $a_{10} = 1$ and $a_{i0} = 0$ for $i = 2, \cdots, N$.

To the purpose of stabilizing the overall closed-loop system (requirement (iii) in Assumption 2), it is more convenient to work with these controllers rather than with those in (8), (9). In the rest of the section, we turn now to the problem of determining the stabilizing matrices L_i, K_i, M_i, $i = 1, 2, \ldots, N$.

For each i, consider the system (1) with output $\varepsilon_i = y_i - R w_i$, namely

$$
\begin{aligned}
\dot{x}_i &= A_i(\mu_i) x_i + B_i(\mu_i) u_i \\
\varepsilon_i &= C_i(\mu_i) x_i - R w_i .
\end{aligned} \tag{16}
$$

As in [10], to reduce the notational burden, we focus on the case in which the inputs u_i are scalar, i.e. $p_i = 1$ for $i = 1, 2, \ldots, N$. Further assume that $\mathcal{P}_i$ is a compact set and that for each $\mu_i \in \mathcal{P}_i$, the system (16) has the same relative degree r_i from u_i to ε_i. Namely, there exists an integer $r_i \geq 1$ such that for each $\mu_i \in \mathcal{P}_i$

$$
\begin{aligned}
C_i(\mu_i) A_i^j(\mu_i) B_i(\mu_i) &= 0 , \quad j = 0, 1, \ldots, r_i - 2 \\
C_i(\mu_i) A_i^{r_i - 1}(\mu_i) B_i(\mu_i) &\neq 0 .
\end{aligned}
$$

Then there exists a μ_i-dependent change of coordinates

$$
\begin{pmatrix} z_i \\ e_i \end{pmatrix} = \begin{pmatrix} Z_i(\mu_i) \\ \hline C_i(\mu_i) \\ C_i(\mu_i) A_i(\mu_i) \\ \vdots \\ C_i(\mu_i) A_i(\mu_i)^{r_i - 1} \end{pmatrix} x_i =: \tilde{Z}_i(\mu_i) x_i, \tag{17}
$$

where $Z_i(\mu_i)$ is a suitable matrix such that $\tilde{Z}_i(\mu_i)$ is non-singular, such that the system (16) in the new coordinates becomes

$$
\begin{aligned}
\dot{z}_i &= A_i^{(11)}(\mu_i) z_i + A_i^{(12)}(\mu_i) e_i \\
\dot{e}_{i1} &= e_{i2} \\
&\vdots \\
\dot{e}_{i, r_i - 1} &= e_{i r_i} \\
\dot{e}_{i, r_i} &= A_i^{(21)}(\mu_i) z_i + A_i^{(22)}(\mu_i) e_i + b_i(\mu_i) u_i \\
\varepsilon_i &= e_{i1} - R w_i = \overline{C} e_i - R w_i,
\end{aligned} \tag{18}
$$

where in particular $b_i(\mu_i) = C_i(\mu_i) A_i^{r_i - 1}(\mu_i) B_i(\mu_i) \neq 0$.

Further change the coordinates in the following way:

$$
\tilde{e}_i = e_i + Q_i w
$$

where $Q_i = (Q_{i1}^T \ldots Q_{ir}^T)^T$, $w = (w_1^T \ldots w_N^T)^T$,

$$
\begin{aligned}
Q_{i1} &= (\mathbf{0}_{1 \times m} \ldots \mathbf{0}_{1 \times m} \ -R \ \mathbf{0}_{1 \times m} \ldots \mathbf{0}_{1 \times m}) , \\
Q_{i, j+1} &= Q_{i,j} \tilde{S}, \quad j = 1, 2, \ldots r_i - 1
\end{aligned}
$$

and $\tilde{S} = (I_N \otimes S - L \otimes I_m)$. Then we obtain

$$
\begin{aligned}
\dot{z}_i &= A_i^{(11)}(\mu_i) z_i + A_i^{(12)}(\mu_i) \tilde{e}_i + \overline{Q}_i(\mu_i) w \\
\dot{\tilde{e}}_{i1} &= \tilde{e}_{i2} \\
&\vdots \\
\dot{\tilde{e}}_{i, r_i - 1} &= \tilde{e}_{i r_i} \\
\dot{\tilde{e}}_{i, r_i} &= A_i^{(21)}(\mu_i) z_i + A_i^{(22)}(\mu_i) \tilde{e}_i + \tilde{Q}_i(\mu_i) w \\
&\qquad + b_i(\mu_i) u_i \\
\varepsilon_i &= \tilde{e}_{i1},
\end{aligned} \tag{19}
$$

with $\overline{Q}_i(\mu_i) = -A_i^{(12)}(\mu_i) Q_i$, $\tilde{Q}_i(\mu_i) = -A_i^{(22)}(\mu_i) Q_i$.

Below we use the following partition for the two matrices:

$$
\begin{aligned}
\overline{Q}_i(\mu_i) &= (\overline{Q}_{i1}(\mu_i) \ldots \overline{Q}_{iN}(\mu_i)) \\
\tilde{Q}_i(\mu_i) &= (\tilde{Q}_{i1}(\mu_i) \ldots \tilde{Q}_{iN}(\mu_i)).
\end{aligned}
$$

Observe that due to the latter change of coordinates the signal w affects the dynamics of the systems. Hence, (19) falls in the class of systems considered in (12) and Corollary 1 applies. Before doing this, we need an additional assumption. Let the system (16) be minimum-phase, namely

Assumption 5 *For each $\mu_i \in \mathcal{P}_i$, all the eigenvalues of $A_i^{(11)}(\mu_i)$ have strictly negative real parts.*

As a consequence of this assumption it is promptly verified (see [10], page 27) that the matrices

$$
\begin{aligned}
\Pi_i(\mu_i) &= \begin{pmatrix} \Pi_{i1}(\mu_i)^T & 0 & \ldots & 0 \end{pmatrix}^T \\
\Gamma_i(\mu_i) &= -\frac{1}{b_i(\mu_i)} \left[A_i^{(21)}(\mu_i) \Pi_{i1}(\mu_i) - \sum_{j=1}^{N} \tilde{Q}_{ij}(\mu_i) \right],
\end{aligned} \tag{20}
$$

where $\Pi_{i1}(\mu_i)$ is the unique $r_i \times r_i$ matrix which solves the Sylvester equation

$$
\Pi_{i1}(\mu_i) S = A_i^{(11)}(\mu_i) \Pi_{i1}(\mu_i) + \sum_{j=1}^{N} \overline{Q}_{ij}(\mu_i), \tag{21}
$$

satisfy condition (i) in Assumption 4 with

$$
P_i(\mu_i) = \left(\overline{Q}_i^T(\mu_i), 0, \cdots, 0, \tilde{Q}_i^T(\mu_i) \right)^T .
$$

The design of the matrices K_i, L_i, M_i such that condition (iii) is satisfied can be carried out in two steps.

Consider the system (19) and write it in the compact form

$$
\begin{aligned}
\dot{z}_i &= A_i^{(11)}(\mu_i) z_i + A_i^{(12)}(\mu_i) \tilde{e}_i + \overline{Q}_i(\mu_i) w \\
\dot{\tilde{e}}_i &= \overline{A} \tilde{e}_i + \overline{B} \left[A_i^{(21)}(\mu_i) z_i + A_i^{(22)}(\mu_i) \tilde{e}_i + \right. \\
&\qquad \left. \tilde{Q}_i(\mu_i) w + b_i(\mu_i) u_i \right] \\
\varepsilon_i &= \overline{C} \tilde{e}_i,
\end{aligned} \tag{22}
$$

where $\overline{A}, \overline{B}, \overline{C}$ are understood from the context. Also consider a controller of the form

$$
\dot{\eta}_i = F_i \eta_i + G_i u_i \quad u_i = \Psi_i \eta_i + v_i , \tag{23}
$$

where v_i is an additional control input and obtain the closed-loop system

$$
\begin{aligned}
\dot{\eta}_i &= (F_i + G_i \Psi_i) \eta_i + G_i v_i \\
\dot{z}_i &= A_i^{(11)}(\mu_i) z_i + A_i^{(12)}(\mu_i) \tilde{e}_i + \overline{Q}_i(\mu_i) w \\
\dot{\tilde{e}}_i &= \overline{A} \tilde{e}_i + \overline{B} \left[A_i^{(21)}(\mu_i) z_i + A_i^{(22)}(\mu_i) \tilde{e}_i + \right. \\
&\qquad \left. \tilde{Q}_i(\mu_i) w + b_i(\mu_i)(\Psi_i \eta_i + v_i) \right] \\
\varepsilon_i &= \overline{C} \tilde{e}_i.
\end{aligned} \tag{24}
$$

Consider the dynamic feedback controller

$$
\dot{\xi}_i = L_i \xi_i + K_i \varepsilon_i, \quad v_i = M_i \xi_i, \tag{25}
$$

where

$$L_i = \begin{pmatrix} -g_i c_{i,r_i-1} & 1 & \ldots & 0 \\ -g_i^2 c_{i,r_i-2} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -g_i^{r_i-1} c_{i,1} & 0 & \ldots & 1 \\ -g_i^{r_i} c_{i,0} & 0 & \ldots & 0 \end{pmatrix}, K_i = \begin{pmatrix} g_i c_{i,r_i-1} \\ g_i^2 c_{i,r_i-2} \\ \vdots \\ g_i^{r_i-1} c_{i,1} \\ g_i^{r_i} c_{i,0} \end{pmatrix}$$
$$M_i = -k_i \overline{M}_i, \overline{M}_i = (d_{i0}\ d_{i1}\ \ldots\ d_{i,r_i-2}\ 1) \quad (26)$$

the polynomials $\lambda^{r_i} + c_{i,r_i-1}\lambda^{r_i-1} + \ldots + c_{i,1}\lambda^1 + c_{i,0}$, $\lambda^{r_i-1} + d_{i,r_i-2}\lambda^{r_1-2} + \ldots + d_{i0}$ have both all the roots with strictly negative real part and $k_i, g_i > 0$ are design parameters. Under Assumption 5, if $b_i(\mu_i) \geq b_i > 0$ for all $\mu_i \in \mathcal{P}_i$, it can be shown ([10, Lemma 1.5.4 and 1.5.5]) that there exist a positive gain $k_i^* > 0$ such that, for any fixed $k_i \geq k_i^*$, there exists g_i^* for which the controller (25), with any $g_i \geq g_i^*$, asymptotically stabilizes the system (24) for all $\mu_i \in \mathcal{P}_i$. The latter statement allows us to summarize as follows:

Proposition 1 *Consider the system (22). Let Assumption 5 hold and assume that $b_i(\mu_i) \geq b_i > 0$ for all $\mu_i \in \mathcal{P}_i$, with $\mathcal{P}_i$ a compact set. Then there exist gains $k_i g_i > 0$ for which the matrices L_i, K_i, M_i defined in (26) are such that the dynamic feedback controller (25) globally asymptotically stabilizes (24) for all $\mu_i \in \mathcal{P}_i$.*

Remark 2 The overall controller is given by the interconnection of the internal model (23) and the stabilizer (25). We observe that the design of the two controllers requires local information only. As a matter of fact, the matrices F_i, G_i, Ψ_i of the internal model can be obtained via Lemma 3. On the other hand, the controller (25) is designed to robustly stabilize the system (22). Since the only terms in the system (22) which depend on the Laplacian matrix L are the "disturbance" vectors $\overline{Q}_i(\mu_i), \tilde{Q}_i(\mu_i)$ which play no role in the stability property of the closed-loop system, one infers that the design of L_i, K_i, M_i is independent of the knowledge of the graph topology.

V. NUMERICAL EXAMPLE

In this section we illustrate the design of the robust controllers for decentralized output regulation via a numerical example. The example we consider corresponds to a network of double integrators with different actuator dynamics, namely we consider the case in which the systems (1) are modeled as

$$\begin{aligned} \dot{x}_i &= \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & c_i \\ 0 & -d_i & -a_i \end{pmatrix}}_{A_i(\mu_i)} x_i + \underbrace{\begin{pmatrix} 0 \\ 0 \\ b_i \end{pmatrix}}_{B_i(\mu_i)} u_i \\ y_i &= \underbrace{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}}_{C_i(\mu_i)} x_i, \quad i = 1, 2, \ldots, N, \end{aligned} \quad (27)$$

where $\mu_i = (a_i\, b_i\, c_i\, d_i)^T$ is the vector of uncertain parameters. The example was proposed in [16] where it was

assumed that the parameters appearing in the equations are known and used to design the controllers. Here we consider the case when these parameters are uncertain. Hence the controllers have to be designed differently. We assume that μ_i is not precisely known and ranges over a compact set $\mathcal{P}_i$ which is contained in $\mathbb{R}_{>0}^3 \times \mathbb{R}_{\geq 0}$. Observe that the uncertain parameters a_i, b_i, c_i are bounded away from zero. We consider the problem in which the matrices which define the leader's equation (2) are given by

$$S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 0 \end{pmatrix}. \quad (28)$$

In other words, the position of the systems (27) has to asymptotically evolve as the ramp reference signal set by the leader.

Following the previous section, we first compute the relative degree r_i of each system. It is easily verified that

$$C_i(\mu_i)B_i(\mu_i) = C_i(\mu_i)A_i(\mu_i)B_i(\mu_i) = 0$$
$$C_i(\mu_i)A_i^2(\mu_i)B_i(\mu_i) = b_i c_i .$$

Since $b_i c_i \neq 0$ for each $\mu_i \in \mathcal{P}_i$, the previous equalities show that each system has a relative degree $r_i = 3$. As the relative degree equals the dimension of the systems, the matrix $\tilde{Z}_i(\mu_i)$ in the change of coordinates (17) writes as

$$\tilde{Z}_i(\mu_i) = \begin{pmatrix} C_i(\mu_i) \\ C_i(\mu_i)A_i(\mu_i) \\ C_i(\mu_i)A_i(\mu_i)^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c_i \end{pmatrix}$$

and in the new coordinates the system (18) writes as

$$\begin{aligned} \dot{e}_{i1} &= e_{i2} \\ \dot{e}_{i2} &= e_{i3} \\ \dot{e}_{i3} &= -c_i d_i e_{i2} - a_i e_{i3} + b_i c_i u_i. \end{aligned} \quad (29)$$

When compared with (18), we observe that the system has no zero dynamics and checking Assumption 5 becomes superfluous. Moreover,

$$A_i^{(21)}(\mu_i) = 0, A_i^{(22)}(\mu_i) = -(0\ c_i d_i\ a_i), b_i(\mu_i) = b_i c_i,$$

from which we conclude that $b_i(\mu_i) \geq \bar{b}_i > 0$, for all $\mu_i \in \mathcal{P}_i$, for some $\bar{b}_i$.

Having verified that all the assumptions of Proposition 1 hold, we can determine the controllers. First of all, we determine the matrices F_i, G_i, Ψ_i in (23). This computation is carried out as in the proof of [10, Lemma 1.5.6]. Since the minimal polynomial of S is λ^2, we have

$$\Phi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad H = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

and let (see Lemma 3 above)

$$F_i = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}, \quad G_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

be a pair of matrices with F_i Hurwitz and (F_i, G_i) controllable. Here, for the sake of simplicity, we take F_i, G_i to be the same for each $i = 1, 2, \ldots, N$. Following the proof of

[10, Lemma 1.5.6], one can construct the vector Ψ_i and the nonsingular matrix T_i which satisfy (14) and obtain

$$\Psi_i = \begin{pmatrix} 1 & 2 \end{pmatrix}, \quad T_i = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}.$$

This concludes the computation of the matrices F_i, G_i, Ψ_i which appear in (23).

We turn now to the design of the matrices L_i, K_i, M_i which appear in (7). Since $r_i = 3$, and letting $\lambda^2 + d_{i1}\lambda + d_{i0} = \lambda^2 + 2\lambda + 1$, $\lambda^3 + c_{i2}\lambda^2 + c_{i1}\lambda + c_{i0} = \lambda^3 + 3\lambda^2 + 3\lambda + 1$ two polynomials with all the roots having strictly negative real parts, the matrices L_i, K_i, M_i are given by

$$L_i = \begin{pmatrix} -3g_i & 1 & 0 \\ -3g_i^2 & 0 & 1 \\ -g_i^3 & 0 & 0 \end{pmatrix}, \quad K_i = \begin{pmatrix} 3g_i \\ 3g_i^2 \\ g_i^3 \end{pmatrix},$$
$$M_i = -k_i \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$$

where k_i, g_i are gains to be chosen sufficiently large. Finally, we let $P = (2\ 1)^T$ be such that $S - PR$ is Hurwitz.

We conclude that the controller (15) with the matrices $F_i, G_i, \Psi_i, L_i, K_i, M_i, P$ computed above, solve the decentralized output regulation problem for the systems (27), (28). We have run simulation for $N = 4$ systems with parameters $\{a_i, b_i, c_i, d_i\}$ chosen equal to $\{1 + \mu_1, 1 + \mu_2, 1 + \mu_3, \mu_4\}$, $\{2.5 + \mu_1, 2 + \mu_2, 1 + \mu_3, \mu_4\}$, $\{2 + \mu_1, 1 + \mu_2, 1 + \mu_3, 0.5 + \mu_4\}$, $\{2 + \mu_1, 1 + \mu_2, 1 + \mu_3, 1 + \mu_4\}$ respectively, where $\mu_1, \mu_2, \mu_3, \mu_4$ are $0.3, 0.4, 0.5, 0.7$ respectively. We set the gains $k = 1.1$, $g = 14$. As for the communication graph, we have chosen one with a direct link between the exosystem and the system S_1, i.e. there is a directed link $(0,1)$. The communication graph among the systems S_i, $i = 1, 2, 3, 4$ is set to be the undirected and static graph with edges $\{(1,2), (2,3), (3,4), (4,1)\}$. The initial value for the exosystem w_0 is taken equal to $(2\ 1)^T$ while all the other initial values are randomly chosen in the interval $[0, 10]$.

Fig. 1 shows that the outputs y_i, $i = 1, 2, 3, 4$ of the systems successfully track the exosystem output Rw_0. The simulation result supports the conclusions of Theorem 1 and the controller design method.

VI. CONCLUSION

We have tackled the problem of designing decentralized controllers able to track a prescribed reference signal generated by an exosystem under the restriction that not all the systems can access the information available at the exosystem. Under the assumption that the exosystem has a directed path to all the systems, we have shown that there exist decentralized controllers which achieve the desired regulation task in the presence of arbitrarily large but bounded uncertainties in the systems' models.

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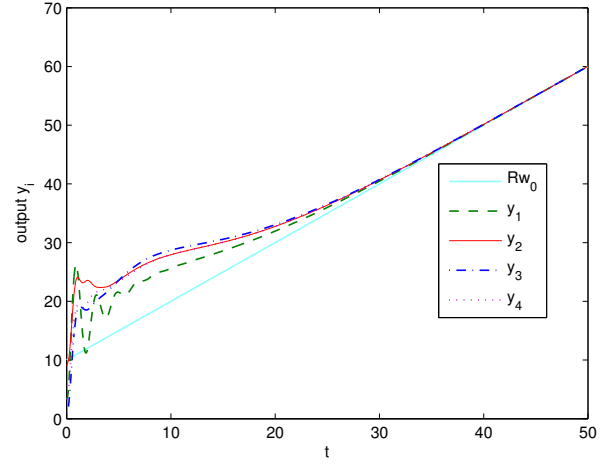
Fig. 1. The outputs of the systems track the signal Rw_0.

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