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Inequalities in network structures

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ABSTRACT

We use a model of continuous attachments in networks to generate propositions concerning inequalities in network structures, and test the propositions on data from organizational settings. Our network model, inspired by that of [Gould, Roger 2002. The origins of status hierarchies: A formal theory and empirical test. *American Journal of Sociology 107*, 1143–1178], is based on a theoretically informed actor model, in which each network member sets attachment strengths based on perceived partner quality, reciprocity, influence from others, attribute homophily, and attachment resistance. A computer algorithm finds the single robust equilibrium configuration of attachment strengths. This allows us to generate six propositions concerning inequalities at the individual, dyadic, triadic, and network levels. We test the propositions on network data for four kinds of attachments over four waves for five organizations, and find that the results generally support the propositions. The results suggest that partner quality, reciprocity, and attachment resistance are the most important elements in the network members' choices.

1. Introduction

Structures of attachments in groups, or network structures, often exhibit inequalities. Examples are networks of communication (Bales, 2001), trust (Buskens and Raub, 2002), and liking or popularity (Coleman, 1961; Feld, 1991; Moody, 2000, 2001). In this study, following Gould (2002), we take a generative approach (see Fararo, 1989; Sawyer, 2005) to understanding these inequalities. We present a model of attachment formation in networks that can generate testable predictions about inequalities, and that can be applied to many different kinds of attachments. The advantage to such a model is the greater understanding that comes with increased generality, with not being tied to a particular empirical network or even a particular kind of attachment.

Specifically, we define an attachment as some kind of cognitive or behavioral link from one person to another person. Typically, many different kinds of attachments exist in a social group. Take, for example, Chris and Pat, members of a work group: Chris may like Pat, hate Pat, trust Pat, envy Pat, sit with Pat at lunch, talk to Pat, ask Pat's advice, direct Pat's work, play practical jokes on Pat, etc. At the group level, a network structure exists for each kind of attachments. Commonly studied attachment structures in groups are the communication network, the authority structure, and the friendship network.

Furthermore, for any given network structure, it is possible and may be interesting to measure and analyze the distribution of the attachment in question, or, in other words, inequalities in the attachment. For example, if the attachment is friendship, we can look at inequalities in popularity. For trust, we can examine the distribution of "being trusted;" for liking, the distribution of "being liked;" and so forth.

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In this study, we use an actor model, based on past research on attachments in networks and in social psychology more generally, to construct a network model of the attachments of network members to each other. By "model" we mean a formalized representation of the production of a particular output, and "network model" and "actor model" refer to models at different levels of aggregation (cf. Whitmeyer, 1994). As is typical for applied mathematical models of empirical phenomena, for example, dynamical systems or partial differential equations models, we cannot derive analytic solutions to the model and have to use numerical methods to find them. We use the solutions of the model, then, to generate propositions concerning inequalities in the attachment patterns of individuals, linked dyads and triads, and the network as a whole. These propositions are interesting in their own right but also can be used for empirical evaluation of the models.

We proceed as follows: We begin with a presentation of the network and actor models: their assumptions and formal representations. We, then, describe how solutions are found and the propositions concerning equality that the solutions generate. Finally, we describe the testing of these propositions with empirical network data for four kinds of attachments over four waves for five organizations. The kinds of attachment are trust, communication, cooperation, and advice receipt, all of which fall within the scope of the model.

2. The actor and network models

A model necessarily embodies a set of assumptions. Thus, the output of the actor model is the attachment behavior of a prototypical individual; we assume this model and use it as an input into the network model. The output of the network model is a characteristic of the network, specifically, an attachment structure. Alternative actor and network models are possible. Indeed, our study differs from Gould's (2002) in that, although we use the same network model, we employ a different actor model.

2.1. The network model

The specific precursor of our network model is that of Gould (2002). Based on an actor model of individual network members' formation of attachments, Gould's network model posits that networks tend toward a Nash equilibrium, meaning that the strengths of attachments between network members tend toward levels such that no member can benefit by changing any attachment strength unilaterally. The existence of this equilibrium point allows prediction of inequalities in attachments at the individual, dyadic, triadic, and network level. Gould conceived of his model as potentially applicable to a variety of attachments, including trust, liking, friendship, communication, and cooperation, and indeed tested it with data on two kinds of attachments, friendship and advice-related communication.

While Gould's model is pathbreaking and presents many valuable features, it is flawed mathematically. When worked out correctly, the model breaks down once group size increases beyond 8 and yields nonsensical results.[1] Yet, in principle such a model should be applicable to groups of medium size, as indeed Gould intended. In addition, most of the parameters and variables in the Gould model are not standardized. While not wrong, this makes understanding the model and applying it to data more difficult than is necessary. One consequence is that there is no explicit or implicit constraint on the strength of attachments, which is partly responsible for the breakdown of the model beyond size 8.

The network approach of our model is the same as Gould's (2002): it assumes that individuals form attachments to enhance their individual welfare, and that collectively they can approach a Nash equilibrium, which allows solution of the network model. This then allows it to generate testable propositions. The principal assumptions of the network model are:

1. For the given kind of attachment, all network members have attachments or potential attachments to all other network members (attachments can be of strength 0 or nearly 0).
2. Over time, the network structure for a given kind of attachment will change through all members' simultaneous adjustment of their attachments. Through this change the network structure will approach a Nash equilibrium, a situation in which no individual can improve her welfare by a unilateral change in one or more of her attachments.
3. Network members are behaviorally homogeneous. That is, the same actor model can be used for all actors. We do allow heterogeneity in the weights of a few components of the actor model.
4. Network members vary in quality, that is, in the value other network members place on forming an attachment to them. We allow variation in the perceptions of quality.
5. Network members are more similar to some and less similar to others in some important, unspecified attributes that for purposes of implementing homophily can be approximated by a single "abstract attribute" variable.

The first assumption implies that the physical setting of the network is such that every attachment is possible. Examples are an organization in a single physical location, students of a single grade in a single school, and a small village. Moreover, cognitive limitations restrict the network to perhaps 150 members at most (cf. Dunbar, 1993; Hill and Dunbar, 2003), and probably considerably less for networks whose members also have attachments outside the network setting.

[1] Many of the results presented in Gould (2002) are in error, including several of the equations and a key graph. We refer here to the corrected elaboration of the model (contact first author for details).

Assumption two means that there must be reasonable continuity in the network membership for a sufficient period of time for attachments to approach the equilibrium. This cannot be expected of a group that is together for too short of a time or whose membership is too fluid.

It is almost certain that the third assumption is not strictly true. If variation among the actors is small enough, however, the model still should produce reasonable outcomes. Assumption four encompasses a variety of differences. First, network members may differ in desirable qualities: some are likely to give better advice than others, some are likely to be more trustworthy than others. Second, unidentified differences between network members may make some better partners for some and others better partners for others. Chris may find Sam's advice more helpful and Pat may find Tracy's more helpful. Third, the actual quality of people may be difficult to estimate and accordingly different people may come up with different estimates.

The intrinsic quality of actors in the network must be distributed in some fashion. We allow for several possibilities. One is a uniform distribution with the range specified by a parameter. Alternatively we permit a normal distribution, given that many human characteristics, including many abilities, seem to be approximately normally distributed in the population. Finally, many groups, including many companies, are selective in granting admission to the group, which may have the effect of truncating the bottom of the distribution. Accordingly, we also allow for a normal distribution with the bottom truncated at a point specified by a parameter. If that truncation point were the mean, for example, it would have the effect of distributing quality according to a monotone decreasing probability distribution function, such that actors of the lowest quality were the most common and those of the highest quality were the least common.

Much evidence suggests that people value homophily in making attachments (see below), so to implement this in the model the fifth assumption is necessary: there must be variation between network members such that some are more similar and some less similar. Possible attributes on which homophily might work include age, social background, gender, race, and cultural tastes. Rather than implement variation in specific attributes, for purpose of operationalizing homophily the model abstracts them all into a single "attribute" variable, which is distributed randomly among network members. A small difference in this variable, then, indicates high similarity overall on attributes that matter, while a large difference indicates dissimilarity.

2.2. The actor model

We bypass the mathematical problems with Gould's model by introducing an alternative actor model, which differs from Gould's in both mathematical form and content. We discuss the content of the model first, then present its mathematical implementation.

Our actor model embodies the following assumptions:

1. For a given kind of attachment, how an individual's attachments affect her welfare can be captured as a function of five preferences connected with those attachments: quality (the value of the partner to her), reciprocity (the strength of the partner's attachment to her), social influence (how strong others' attachments are to the partner), attribute homophily (how similar she is to the partner), and attachment resistance (a declining value to additional attachments the more total attachments she has).
2. The benefit from an attachment increases with the quality of the partner.
3. People vary in their perceptions of a given person's quality (see network model assumption 4, above).
4. People prefer for their attachments to a person to be similar to others' attachments to that person. They may be more strongly influenced by others with whom they have stronger attachments.
5. People prefer equality in their attachments: that a partner is attached to them about as strongly as they are to the partner.
6. People prefer attachments with people similar to themselves in various attributes.
7. People's preference for more attachments will diminish the more total attachments they have. This satiation effect may vary between people.
8. An individual's adjustment of his attachments of a given kind to other network members can be modeled as the incremental improvement or local maximization of his welfare as determined by his attachments of that kind.
9. For the kind of attachment in question, each network member has full and accurate knowledge of all attachments in the network.

It is perhaps worth stating two omissions from the model explicitly as assumptions:

10. An actor's formation of attachments of one kind is unaffected by his other kinds of attachments.
11. Actors do not form attachments with strategic or political ends, or as dictated by institutional rules.

Formally, the first assumption is embodied in a welfare function of the five preferences. Specifically, welfare is given as a weighted sum of five components, corresponding to the preferences, where the welfare in each component is the sum of squared deviations from an ideal point (see Whitmeyer, 1998). Thus, we use a least squares model; alternatives, such as the Cobb–Douglas model (see Coleman, 1990), could be used.

Eq. 1 is the basic model:

$$U_i = -vQ_i - w_iI_i - sR_i - hH_i, -a_iA_i. \tag{1}$$

In Eq. (1), u_i is actor i's welfare and the variables Q, I, R, H, and A are the contributions from the different components or preferences: quality, social influence, reciprocity, homophily, and resistance. See below for how each is computed. The lower-case letters are the weights for the components. The weights are required to be not less than 0 and not greater than 1 and to sum to 1. Larger values for a component indicate a greater distance from the actor's ideal and thus lower welfare; hence, each component is given a minus sign.

The formal expressions for each of the components or preferences, together with brief justifications, are as follows. The quality component for actor i, Q_i, is the sum over all other network members j of the squared deviations of i's attachment to j from j's quality as i perceives it:

$$Q_i = \sum_{j \neq i} \{x_{ij} - [(1-U)q_j + Uf_{ij}]\}^2. \tag{2Q}$$

Here, x_{ij} denotes i's attachment to j, q_j indicates actor j's quality, and f_{ij} denotes random noise added (with weight U) to j's quality (with weight $1-U$) to give i's perception of j's quality.

The importance of the influence of others in making judgments is well established in the social psychological literature (see, e.g., Aronson, 2004; Fiske and Taylor, 1991) and in studies of what is called field dependence (Witkin and Goodenough, 1981). To determine the social influence component for actor i, I_i, we find for each network member j the deviation of i's attachment to j from the mean of other network members' (ks') attachments to j. We square those deviations and sum over the $n-1$ network members (excluding i):

$$I_i = \sum_{j \neq i} (x_{ij} - \frac{1}{n-2} \sum_{k \neq i,j} x_{kj})^2. \tag{2I}$$

Eq. (2I) assumes that all network members are equally influential. To implement the possibility that people's judgments are influenced by others to the extent that they have attachments to those others, we can weight the effect on actor i of each actor k's attachment to actor j by the strength of i's attachment to k:

$$I_i = \sum_{j \neq i} \left(x_{ij} - \frac{\sum_{k \neq i,j} x_{kj} x_{ik}}{\sum_{k \neq i,j} x_{ik}} \right)^2. \tag{2I*}$$

It is likely that people prefer equality in many kinds of attachments (Fiske, 1991; Jasso, 2002), such as cooperation, although they may prefer to be superior for some kinds of attachments, such as advice (Homans, 1974). The reciprocity component for actor i, R_i, expresses actor i's preference for equality in bilateral attachments. It is the sum over all j of the squared differences in reciprocal attachments:

$$R_i = \sum_{j \neq i} (x_{ij} - x_{ji})^2. \tag{2R}$$

There is much evidence of homophily and its effects, that is, that people prefer attachments with people with whom they share attributes and that this preference may influence the attachments they form (e.g., Coleman, 1961; Kandel, 1978; McPherson and Smith-Lovin, 1987; Moody, 2001; Verbrugge, 1977). To calculate the homophily component, we first distribute an abstract homophily attribute randomly across the network members. This is a variable from 0 to 1, to be thought of as a circle of circumference 1, with 0 identical to 1 and a maximum distance on the circumference of .5. Dissimilarity (d_{ij}) is given by twice the absolute value of the difference between actors i's and actor j's values on this circumference, which yields $0 \leqslant d_{ij} \leqslant 1$. Finally, the homophily component is the sum over all j of squared deviations of i's attachment to j from i's similarity to j:

$$H_i = \sum_{j \neq i} [x_{ij} - (1 - d_{ij})]^2. \tag{2H}$$

Lastly, people are likely to reach a limit in how much attachment they want to confer on others and are likely to vary in that limit. For example, some people are more willing to place trust than others (Yamagishi, 2001), some are more eager to cooperate than others (see, e.g., Blau, 1955), and some are more receptive to advice (Harvey and Fischer, 1997) than others. This can be seen as having interests that conflict with attachments: in self-sufficiency, a feeling of invulnerability, or simply alternative uses of time, for example. In the actor model, we collapse all such interests into a component for attachment resistance and allow its weight to vary across network members. We calculate the component as the squared deviation of actor i's mean attachment to others from complete self-reliance, that is, no attachments at all:

$$A_i = \left(\frac{1}{n-1} \sum_{j \neq i} x_{ij} \right)^2. \tag{2A}$$

A last note concerning the formalization of the actor model (assumptions 1–7) is that the weights for quality (v), reciprocity (s), and homophily (h) are identical for all network members. In keeping with the above discussion, however, we allow the option of having the weights for social influence (w_i) and attachment resistance (a_i) vary across actors, drawn randomly from a normal distribution with the mean and standard deviation as parameters. To avoid truncation issues at the boundaries of the 0–1 interval we restrict possible values for the standard deviation such that at least 2.5 standard deviations on either side of the mean weight lie in the interval. If either or both weights vary, then, for a given actor the sum of the weights is not likely to sum to 1. We deal with this by normalizing the weights: for each actor, we create new weights by dividing each of the five weights by the sum of the five weights.

Assumption 8, that all network members have full and accurate knowledge of other members' attachments, only affects the reciprocity and social influence components and so will matter only if they do. Full and accurate knowledge is more likely to be true of some kinds of attachment than others. In particular, people are more likely to be aware of others behaviors toward them, such as communicating with them or giving them advice and less likely to know others' cognitive states toward them, such as how well others like them or trust them.

The assumption of incremental improvement, assumption 9, is that individuals change their attachments gradually, through the accrual of small adjustments rather than by identifying an optimal state and jumping to it. In game theoretic terms, this means that the network structure will move to a Nash equilibrium if one exists, but not necessarily a Pareto optimum (a situation such that no alternative situation exists that is at least as good for everyone and better for at least one). As it turns out, with the least squares actor model and our network model the Nash equilibrium appears to be also a Pareto optimum and the assumption of incremental improvement is not crucial. Under alternative circumstances it might be, however.

Assumption 10, that an actor's attachments of one kind do not affect her attachments of a different kind, is almost certainly not true. For example, there is likely to be mutual pairwise positive correlation between communication, cooperation, and trust. It seems unlikely, however, that there is a dominant attachment structure that causally determines the others. Moreover, it is likely that to the extent that attachment structures are not independent they reinforce each other. For these reasons, modeling attachment structures separately is unlikely to introduce much inaccuracy. In future work, however, it may be desirable to include the connections between attachment structures in the model.

Assumption 11 is also like not to be true. In a given network setting, such as an organization or portion of one, informal or formal rules may obtain concerning the formation of attachments. A company or department, for example, may have written or unwritten rules about cooperation, urging employees to work at some particular point on the continuum between full collaboration and complete independence. Similarly, there may be rules or norms about placing trust in colleagues, dispensing advice (see Blau, 1955), and the extent of communication. If we were interested in the cardinal level of attachments to others that individuals formed, it probably would be necessary to include a component for norms and rules in the actor model. Because we are interested only in patterns in the configuration of attachments, however, the relative strength of attachments is what matters. It is reasonable to assume that, within an organizational setting, the effect of norms and rules is to increase or decrease the strength of attachments fairly uniformly, either additively or multiplicatively. This would have little effect on patterns in the configuration of attachments, and thus we can leave this component out of our actor model.

We also assume that network members do not form attachments strategically, beyond the mechanisms embodied in our actor model. It is possible that people form attachments to others of particular strengths to get attachments of particular strengths in return. This could be in pursuit of relationship goals, for example, due to a romantic interest, or in pursuit of a position in the network hierarchy, for example, from an interest in being the most popular or most powerful person. Individuals in groups may pursue prestige in those groups, through performance but also through social activities or "politicking" (Loch, Huberman, & Stout, 2000; Loch, Huberman, & Ülkü, 2001).

We do not model such strategic behavior for several reasons. First, some strategy pursuit may be embodied already in parts of the actor model, such as the social influence and attribute homophily components. Second, we think it unlikely that strategic goals pertain directly to the particular attachments we investigate empirically here. That is, we doubt that any network member is pursuing the goal of being "most trusted," "most cooperated with," and so forth. High prestige and popularity are more likely to be strategic goals. Third, if there is a strategic goal such as high prestige or popularity, its pursuit is likely to involve a sophisticated combination of several kinds of attachment. Such sophistication, in turn, implies effort, which means it is unlikely to be undertaken by many group members. In short, we think that strategic formation of the kinds of attachments we study here—trust, communication, cooperation, and receipt of advice—beyond what is embodied already in the actor model, is likely to be low-level and thus can be ignored in this initial exploration. In future work, it may be worthwhile to incorporate some strategic behavior into the model, to see if the effects produced warrant the increase in complexity.

2.3. Solution of the network model

We used a computer algorithm to find the Nash equilibrium, the set of $n(n-1)$ attachment strengths for which no network member will have an incentive to change her attachment to any other network member unilaterally. An explicit analytic solution may be found when the actors are homogeneous in all the initial conditions except for quality, that is, when the homophily weight (h), the weight of the noise added to quality perception (U), and the two standard deviations, of the social

Table 1
Parameters in the simulation.

Parameter	Symbol	Range
Group size	n	3–150
Quality weight	v	0–1
Reciprocity weight	s	0–1
Social influence weight	w	0–1
Standard deviation of social influence weight	s.d. w	0–[.4∗(.5–|.5–w|)]
Homophily weight	h	0–1
Attachment resistance weight	a	0–1
Standard deviation of attachment resistance weight	s.d. a	0–[.4∗(.5–|.5–a|)]
Quality uncertainty	U	0–1
Radius of uniformly distributed quality	r	0–.5
Standard deviation of normally distributed quality	r	0–.2
Truncation point for distribution of quality		0–1

influence weight (w) and attachment resistance weight (a), are 0.[2] Otherwise, the actor model includes non-homogeneous variables, which makes analytic solution difficult. We have no formal proof that the equilibrium found is unique, again, except in the situation where h, U, and the two standard deviations are 0. Nevertheless, extensive exploration over a wide range of initial attachment values shows that the algorithm always reaches the same, single equilibrium for a given set of parameter and non-homogeneous variable values.

Another attractive feature of the model solutions is that, for any network size, equilibrium attachment strengths are reasonable, falling in the 0–1 interval with an overall mean almost always between .3 and .6. The effective limits on attachment strengths are suited to empirical testing and application to most data, in which attachment strengths are likely to be measured with a constrained set of values. Even beyond the constraints of measurement, attachment strengths are likely to have actual upper and lower limits.

Table 1 displays the parameters for our model and their ranges. It includes the component weights, with the mean and standard deviation as parameters for the two that are allowed to vary. The ranges for the standard deviations are limited such that only about .6% of cases are expected to fall outside the 0–1 range; the program rounds any such cases to 0 or 1.

The table also includes three additional parameters: the network size (n), uncertainty concerning others' quality (U), and the range within the 0–1 interval over which the network members' quality is distributed. As noted above, we allow either a uniform or truncated normal distribution of quality. If the distribution is uniform, we fix the mean at .5, and allow the maximal deviation on either side, r, to be an input parameter. Thus, for example, if the group size n is 5, the distribution is uniform, and the deviation r is .3, the qualities of the group members would be .8, .65, .5, .35, and .2. If the distribution is truncated normal, we fix the mean to be .5, and both the standard deviation (r again) and the truncation point are input parameters. If the truncation point is 0, the distribution is simply normal.

The specific algorithm we employed to find the Nash equilibrium is as follows: A focal network member considers each of his attachments to other network members. The focal member evaluates his welfare with his current attachment, that attachment plus a small increment, and that attachment minus a small increment. The increment size is an initial input, typically .001. The focal member chooses the single change of attachment that increases his welfare the most, or makes no attachment change if none yields an increase. Then another network member is chosen to be the focal member. The algorithm cycles through the entire network in this way, and then begins another cycle. The process terminates when either no attachments change though an entire cycle of network members, or 500,000 cycles have elapsed. Whether the algorithm varies the order of network members in the cycle or keeps a constant order does not affect the equilibrium found.

We emphasize that our algorithm is *not* a simulation of actual behavioral and social processes. It is a computational method of finding the Nash equilibrium. We will have strong reason to suppose that networks will tend to these Nash equilibria if they are also Pareto optima, that is, configurations of attachments such that there is no other configuration better for at least one network member and worse for none. This would mean that even the changing of many attachment strengths by many network members simultaneously would be likely to lead in the long run to the Nash equilibrium. We cannot prove that the Nash equilibria the computer algorithm finds are Pareto optima, but we suspect they are. For variety of points in the parameter space, we searched extensively for a set of attachments better than the Nash equilibrium for at least one network member and worse for none, but found no such Pareto-superior points. This leads us to believe and predict that naturally occurring networks will tend toward the Nash equilibria and, thus, that the propositions our analysis generates will hold empirically.

[2] When $h = 0$, $U = 0$, s.d. $w = 0$, $a = 0$, and s.d. $a = 0$, the equilibrium attachment x_{ij} directed from actor i to actor j, is:

$$x_{ij} = \frac{(n-2)svQ_i + v[(n-2)v + (n-2)s + (n-1)w]Q_j + sw\sum_{k=1}^{n} Q_k}{(n-2)v^2 + 2(n-2)sv + nsw + (n-1)vw}$$

As a check, the computer algorithm finds the identical equilibrium.

3. Propositions concerning inequalities

It is common to evaluate theoretical models by testing propositions implied by the model outcomes rather than by testing the outcomes directly. Moreover, to generate propositions for models with probabilistic elements, such as ours, the models must be run multiple times with some kind of robustness criterion in place to identify certain outcome relationships as propositions. We use the criterion that a relationship must appear in at least 95% of runs in order to characterize it as an implication of the model, a proposition to be tested.

One more element of indeterminacy is that, like many models, our model includes a number of parameters. In order to fully understand the implications of the model, therefore, it is necessary to explore the parameter space, that is, to examine the outcomes of the model for the entire range of plausible parameter settings. There are three possible results of this. First, a proposition may be generated in nearly the entire parameter space, which means it is a general implication of the model. Second, a proposition may be generated in a substantial portion of the parameter space but not in another substantial portion of the space (where, perhaps the inverse, is generated). This means the proposition is contingent on the parameter settings. This is likely to be interesting, but is testable only if the parameters can be estimated for a particular empirical situation. Third, a proposition may be generated only for very particular parameter settings. In this case, the proposition is unstable and probably will not be seen in empirical situations. We note that these considerations hold generally for parameterized models, for example, models of dynamical systems (see, e.g., Strogatz, 1994), including those without probabilistic elements.

3.1. Propositions 1–4

To generate our propositions, we systematically explored parameter space, using the computer algorithm to find the equilibrium solutions of our network model. The attachments at equilibrium were plausible except when the quality weight (v) was 0. Then, typically, the equilibrium attachments were negative, which is nonsensical.

We found, in addition, that two options and one parameter had only small effects on quantitative outcomes and no effects on qualitative outcomes. One option was transitivity in social influence, which yielded the same outcomes, qualitatively, as allowing all network members equal social influence. The other option was a truncated normal distribution of network member quality. We tried sampling from only the upper half of a normal distribution and from a full normal distribution, but these did not produce results qualitatively different from a uniform distribution. Results reported below, therefore, refer only to the latter. Specifically, we distributed network member quality by fixing the mean at .5 and varying a parameter that gave the range around that mean. Finally, allowing non-zero variance of the weight of social influence (w) made no qualitative difference to outcomes.

From our explorations of the parameter space, we created six propositions concerning inequalities in network structures at different levels of aggregation. Proposition 1 concerns individuals, proposition 2 dyads, propositions 3 and 4 triads, and propositions 5–6 the entire group. Note that the intrinsic quality (q) of network members, which might be problematic to measure in empirical tests, is not an element of the propositions we present.

We generated the first four propositions as follows. We had the parameter settings range over the entire parameter space, and ran the computer algorithm 1000 times for each parameter setting. We used statistical significance of the given measure as an indication of the given relationship. These propositions, similar to propositions Gould (2002) presents, concern the relative amount of attachments received ("choice status," in Gould (2002)).

1. *Those who direct stronger attachments will tend to receive stronger attachments.* Specifically, the correlation between attachments made and attachments received will be positive.
2. *If ego receives more total attachments than alter, then ego's attachment to alter is likely to be less than alter's attachment to ego*. Specifically, the correlation between the difference in a pair of network members' attachments to each other and the difference in their total attachments received will be negative.
3. *If ego receives more total attachments than alter, then ego is likely to have a stronger attachment to any other network member than alter does, and the greater the former difference, the greater the latter difference*. Specifically, the correlation between the difference in a pair of network members' attachments received and the mean of the differences in their attachments to third parties will be positive.
4. *The more similar ego's and alter's total attachments received, the more similar the attachments they direct to other network members will tend to be*. Specifically, the correlation between the absolute value of the difference in a pair of network members' attachments received and the mean of the absolute values of the differences in their attachments to third parties will be positive.

These propositions hold for almost the entire parameter space. Proposition 1 fails to hold only, but not always, when the weights of both reciprocity and attribute homophily are zero. Proposition 2 fails to hold only, but not always, when: quality is constant but not *perceived* uniformly; the weight of quality (v) is fairly low, less than .4; and the mean weight of attachment resistance is not extreme, neither 0 or 1. Proposition 3 almost never fails and then only when proposition 1 also fails. Proposition 4 almost never fails and then only when propositions 1 and 3 also fail.

Results for configurations of the five actor model weights suggest that three components of the actor model are most important in affecting attachment strength: network member quality, reciprocity, and attachment resistance. Their weights can vary the most and can assume the highest values. Two of these, network member quality and attachment resistance, are indispensable. Specifically, the weight of quality must be at least .1 and can be as high as .8 for some parameter configurations. The weight of attachment resistance must be at least .1 and can be as high as .6. The weight of reciprocity can be as low as 0 and as high as .6.

The other two model components, social influence and attribute homophily, are less important. The lack of effect of social influence is in fact consonant with the settings for quality range and uncertainty. If network members are similar in quality but differently suited to different people, then the strengths of others' ties to alter will not be helpful to ego in choosing the strength of her tie to alter.

3.2. Proposition 5–6

The remaining two propositions concern two structural measures we used to examine inequality in the network as a whole further. One was Freeman's (1979) measure of degree centralization, with the denominator adjusted appropriately for continuous rather than binary measurement of attachment strengths. Let m_i denote member i's number of partners, which will be $n-1$ unless data are missing. The measure for out-degrees is:

$$C_{out} = \frac{1}{n-1}\left[n \max_i \left(\frac{1}{m_i}\sum_{j \neq i} x_{ij}\right) - \sum_i \left(\frac{1}{m_i}\sum_{j \neq i} x_{ij}\right)\right]. \tag{3}$$

The measure for in-degrees, C_{in}, may be obtained by replacing x_{ij} with x_{ji} in the formula for C_{out}.

The other measure is a global measure of bilateral asymmetry in attachments, which we call the graph asymmetry index (GAI):

$$\mathrm{GAI} = \frac{\sum_i \sum_{j \neq i} |x_{ij} - x_{ji}|}{\sum_i \sum_{j \neq i} (|x_{ij}| + |x_{ji}|)}. \tag{4}$$

The GAI has the following properties. It ranges from 0 to 1; higher values mean greater asymmetry or inequality in the mutual attachments in pairs. Specifically, when all attachments are identically reciprocated (i.e., $x_{ij} = x_{ji}$, for all $i \neq j$), the GAI is 0. When all mutual attachments are maximally unequal (i.e., here, one of x_{ij} and x_{ji} is 0 and the other is 1, for all $i \neq j$), the GAI is 1.[3]

We generated two propositions concerning these measures by carrying out 100 runs of the computer algorithm for each of a variety of settings of the parameters, and within each run comparing the equilibrium outcome to 1000 random permutations of the equilibrium attachment strengths. The fifth proposition says that bilateral inequality will tend to be low in the networks:

5. Conditional on attachment strengths, the graph asymmetry index (GAI) will be lower than if attachments were randomly distributed.

The sixth proposition says that centralization will tend to be higher in attachments directed than in attachments received:

6. Conditional on attachment strengths, out-degree centralization (C_{out}) will be greater than in-degree centralization (C_{in}).

4. Testing the propositions

Testing the model requires sociometric data in which the members of small to medium-sized groups express their attachments towards each other. Furthermore, in order to show that the model holds for different kinds of attachments, it is necessary to have sociometric choice data on substantially different kinds of attachments. Preferably, the networks should have natural boundaries that determine membership or non-membership in the group. Since work groups in organizations meet these requirements well, we used sociometric information from organizational settings.

4.1. Research design and data

Data were collected in the context of a panel study on social network dynamics in five work organizations in the Netherlands and Germany. Data collection took place between 1995 and 1997, and was preceded or accompanied by extended periods of participant observation by the researchers.

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[3] We thank James Moody for advice on this index.

Table 2
Summary statistics for five organizations.

	Organization					
Department	Hospital Care unit	Hospital Dialysis unit	Computer firm Project team	Housing corp. All five units	Paper factory Manag. team	Total
Size/response	49/45	30/29	31/28	78/74	22/21	210/197
Response (%)	91.8	96.7	90.3	94.8	95.4	93.8
Mean age	38.5	35.2	43.1	38.0	41.2	39.2
Supervisors (%)	8.8	10.3	60.7	18.9	19.0	23.5
Men (%)	24.5	20.0	83.9	56.4	100	56.9

The organizations vary considerably in the task structure and characteristics of employees. Selection of research sites was based on existing contacts. A total of 197 respondents participated in the survey: the management team of a German paper factory, the project team of a computer company, a care and a dialysis department of a hospital, and all five departments of a housing corporation. The panel consisted of four waves with intervals of between 3 and 6 months. Summary statistics for all organizations are presented in Table 2. Mean response rate across organizations was 93.8%. Mean age ranges from 35.2 to 43.1. The organizations studied vary considerably with regard to size and the percentage of women and supervisors among the respondents. The biggest unit of investigation is the housing corporation ($n = 74$), followed by the care unit of the hospital ($n = 45$). The size of the networks of the remaining three organizations is 21 (paper factory), 28 (computer firm) and 29 (dialysis department). The two departments of the hospital have the highest proportion of female employees and the lowest proportion of respondents with a supervisory function. The highest proportion of supervisors can be found in the computer firm, and the paper factory has the highest proportion of men.

In each organization, network size fluctuated due to employees leaving the unit and new employees joining it. This holds particularly for the computer firm, where only 68% of the respondents who collaborated during the first wave were also present at the second wave. A dominant common goal was present only in one of the organizations (the paper factory), during the first of the four measurements, when the managers were absorbed in the completion of a prestigious project (building of a new production hall and transfer of a new paper machine).

Sociometric information was collected in form of written questionnaires that were either distributed to the respondents using the internal mailing system of the organization, or by using computer-aided interviewing (the housing corporation and the computer firm). All sociometric questions were followed by a list with names of the respondent's colleagues in the department or organization of study. Each name was accompanied by five-point response categories that covered the frequency, intensity, or quality of the tie.

4.1.1. Communication

Communication ties of an actor were measured by asking each respondent to indicate on a five-point scale how frequently during office hours he or she had communicated with each colleague in their department or organization during the last 3 months. The question stated that the content and location of the communication did not matter, but that the communicative event should transcend the transmission of a simple message or a greeting. Response categories were "never", "less than once a month", "1–3 times a month", "1–3 times a week", and "daily".

4.1.2. Trust

The respondents were asked for evaluations of their relationships with colleagues in terms of closeness and trust. Trust relationships were defined in the questionnaire as ties to colleagues with whom one discusses personal problems that have their origin either in the work sphere or in one's private life. The question was introduced as follows: "We all feel closer to some people than to others. By 'closeness' we mean how much you trust a particular colleague, e.g., in whom you would confide important personal information. The latter can be either private or firm-related matters." For each colleague in their department or organization respondents then indicated on a five-point scale how close they felt towards that person. The five categories were defined as follows: unknown ("you either do not know this colleague, or you just know his or her name or face"), distant ("you would certainly not take into your confidence"), neutral ("you do not know this colleague well enough to take him or her into your confidence for personal matters"), close ("you take this colleague into your confidence for matters that are relatively important to you"), and very close ("you take this colleague into your confidence for matters that are extremely important to you"). Respondents were then asked to indicate which of these five statements best described their relationship to each colleague.

4.1.3. Cooperation

The measurement of the quality of cooperative relationships was introduced with the remark that "people working in teams often find it easier to cooperate with particular colleagues than with others." Mutual cooperation was then defined as "each situation in which the way a colleague carries out a job is important for your own work." The respondents indicated on a six-point scale to what degree they considered cooperation with each colleague during the last 3 months as "very difficult", "difficult", "neither difficult nor good", "good", and "very good," or whether there was no cooperation necessary with this particular person.

Table 3
Support for propositions by organizational setting. Percentage of networks with measures in predicted directions, and percentage significantly so compared with networks of randomly permuted attachments.

Proposition	Criterion	Data set				
		PF	Hosp-D	CF	Hosp-C	Housing C
1	r positive	100	77	100	38	100
	$p < .05$[a]	69	46	62	8	62
	$p < .05$[b]	0	0	15	0	0
2	r negative	92	92	85	100	85
	$p < .05$[a]	8	31	0	92	62
	$p < .05$[b]	38	38	23	92	62
3	r positive	100	92	100	46	100
	$p < .05$[a]	69	38	69	8	54
	$p < .05$[b]	0	0	15	0	0
4	r positive	92	46	85	54	85
	$p < .05$[a]	77	0	46	8	69
	$p < .05$[b]	62	0	31	0	62
5	GAI low					
	$p < .05$[a]	100	92	100	69	100
6	Centralization out > centralization in	92	77	69	92	100
	$p < .05$[a]	46	8	0	69	23
Network *n*		17–19	20–26	17–21	35–40	59–64

$N = 13$ networks, for each organization (including 4 kinds of attachments, 1–4 waves each).
[a] $p < .05$ tested against random redistribution of attachments.
[b] $p < .05$ tested against random redistribution of attachments with bilateral reciprocity (i.e., fixing pairs of attachments).

4.1.4. Advice

Respondents had to indicate how frequently during the past 3 months they asked each colleague in their department or organization for advice or help if they could not solve a problem on their own or had to make a decision. Response categories were ‘‘never”, ‘‘less than once a month”, ‘‘1–3 times a month”, ‘‘1–3 times a week”, and ‘‘daily.”

4.2. Hypotheses evaluated

We consider all propositions as hypotheses for these data. Following Gould (2002), we analyze the data with a resampling method using Monte Carlo simulation, a mathematically valid alternative to statistical model-based tests (Lunneborg, 2000). Resampling is unfamiliar to many researchers because it has become practicable only with the advent of high-speed computers. The reason we use it is that, in contrast to most statistical model based methods, the validity of resampling does not rest on the assumption that cases are independent. Moreover, known or plausible constraints and dependencies in the data can be built into the resampling process and so into the determination of significance. The advantages of resampling methods are especially useful in analyzing network or small group data such as ours in which the cases are not independent. For example, there will be interdependencies in the amount of attachments that individuals in any of our networks receive.

Specifically, we use the following permutation method (see Good, 2000). We compare each empirical measure to that which would obtain if the same set of attachment strengths were randomly redistributed among attachments, which we call a random model. If the random model is sufficiently unlikely to generate results as extreme in the predicted direction as the empirical results, we reject the null hypothesis that some random process dictates the distribution of attachment strengths. This implies support for the alternative hypothesis that the network model we present determines the distribution.

Thus, for each of the 65 networks, we permute the $n(n-1)$ empirical attachments randomly 1000 times, and take our p-value to be the proportion of those random networks that give results as extreme or further in the predicted direction. For the first five propositions, we also test outcomes against a second null hypothesis of a random model maintaining bilateral reciprocity.[4] This means we control for a simple bias toward reciprocity (cf. Wasserman and Faust, 1994) as follows: we fix reciprocal attachments (x_{ij} with x_{ji}), and then permute the $n(n-1)/2$ pairs of reciprocal attachments. Again, we take our p-value to be the proportion of 1000 random-with-fixed-pairs networks that give results as extreme or further in the predicted direction.

Table 3 shows the support for the propositions by organizational setting. For each hypothesis, Table 3 reports the number of networks for which the measure is in the predicted direction, and the number for which the measure is significantly different in the predicted direction from the random models.

Table 3 shows general support for all six propositions. All measures are in the predicted direction for most networks, in many cases significantly so. Support was weakest for proposition 4, that the more similar network members are in attachments received, the more similar their attachments to others. Organizations also appear to vary in the extent to which they fit the propositions. Support for the propositions was weakest for the two hospital departments, which we discuss further

[4] Tom Snijders suggested this comparison model.

Table 4
Support for propositions by kind of attachment in network. Percentage of networks with measures in predicted directions, and percentage significantly so compared with networks of randomly permuted attachments.

Proposition	Criterion	Kind of attachment			
		Advice	Trust	Communication	Cooperation
1	r positive	55	74	100	83
	$p < .05$[a]	30	32	80	67
	$p < .05$[b]	0	0	10	0
2	r negative	100	100	70	100
	$p < .05$[a]	60	53	15	17
	$p < .05$[b]	60	63	30	50
3	r positive	80	84	100	83
	$p < .05$[a]	25	32	75	67
	$p < .05$[b]	0	0	10	0
	r positive	80	58	80	67
4	$p < .05$[a]	20	42	55	33
	$p < .05$[b]	10	21	50	33
5	GAI low				
	$p < .05$[a]	80	95	100	100
	Centralization out > Centralization in	95	79	85	83
6	$p < .05$[a]	35	26	35	17
No. of networks		20	19	20	6

[a] $p < .05$ tested against random redistribution of attachments.
[b] $p < .05$ tested against random redistribution of attachments with bilateral reciprocity (i.e., fixing pairs of attachments).

below. There was no indication that network size limited the validity of the model within the range of network sizes considered here. Support for the propositions was strong for the largest networks here, those from the housing corporation, with around 60 members.

Propositions 5 and 6 concern the network as a whole. The empirical results consistently support Proposition 5, that the graph asymmetry index (GAI) for the networks generally will be low. Substantively, this means that in bilateral attachments inequality tends to be low. As predicted in proposition 6, in-degree centralization is usually—56 out of 65 networks—less than out-degree centralization. Inequality in in-degree, or attachments received, is more indicative of what we usually mean by inequality than inequality in out-degree.

Table 4 reports the number of networks for which each measure is in the predicted direction and significant measures, this time arranged according to the kind of attachment. Support for the propositions appears to be roughly at the same levels across the different networks, suggesting that the network model is reasonable for all four kinds of attachments in our data.

We can refer these results back to the model parameters. Propositions 1 and 2 especially provide some constraints on the parameters. The implication is that the components that matter most are partner quality, attachment resistance, and reciprocity. The weights for reciprocity and attribute homophily cannot both be zero. On the other hand the weight of attribute homophily, along with that for social influence, can take only zero or very small values. The weight for quality is not likely to be low, especially when attachment resistance has a moderately high weight. Partner quality has a constant distribution with low levels of uncertainty, or is distributed in a narrow range with a high level of uncertainty.

We can state this less technically. The most likely situation seems to be that, first, network members have significant resistance to forming attachments, and vary significantly in that resistance; second, they care about matching their attachments to the return attachments from their partners; and, third, they care about forming stronger attachments with their preferred partners. Network members, however, are likely to perceive all potential partners as roughly equal in quality, probably with some disagreement on who is higher, or else they vary considerably in their perceptions of quality. To generalize still further, this suggest a situation of considerable individuality: little social influence, little and unimportant agreement on quality, and important variation in how much individuals value having attachments.

It is interesting to consider why hypotheses 1, 3, and 4 are supported least for the hospital departments, in particular the hospital care unit. This conjuncture of failures only obtains in a small portion of the parameter space, specifically, where reciprocity (s) and attribute homophily (h) are both 0. We note that the care unit was a nursing department, in which many employees worked different shifts and worked different shifts different days (van de Bunt, 1999). The staff may have been relatively homogeneous, which may have made attribute homophily less important for attachments. Work in different shifts may have led direct reciprocity to be even less important than usual. Unfortunately, we do not have the data to test the inferred parameter values.

5. Discussion and conclusion

In this paper, we have presented a network model of continuous attachment formation in networks, used that model to generate propositions concerning inequalities in network structures, and tested those predictions on network data from five organizational settings involving four kinds of attachments. Our network model is based on an actor model that posits that in establishing their attachments network members may be concerned with their partners' quality, the extent to which their partners reciprocate their attachments (reciprocity), others' attachments to their partners (social influence), and the extent to which their partners are like them (attribute homophily). It assumes, in addition, that network members also have a resistance to attachments in addition to their interests in attachments. The network model also supposes there may be variation among network members: they may vary in how important attachment resistance and social influence are to them, in their quality, and in their perceptions of others' quality. It also allows for different distributions of actor quality and for transitivity in social influence, such that people are more influenced by those with whom they have stronger attachments. A computer algorithm finds the single, robust Nash equilibrium, the attachment levels in the network such that no single member can improve his welfare by unilaterally changing any attachment.

We systematically explored the parameter space and examined the patterns in the equilibrium attachments. From this we determined that allowing transitivity in social influence, permitting variation in the weight of social influence, and using a truncated normal distribution rather than a uniform distribution of network members' quality made no qualitative difference to the outcomes. Consequently, we dropped these elements of the model.

From the equilibrium outcomes, we inferred six propositions that concerned inequalities in attachments of individuals, within dyads, within triads, and for the network as a whole. We tested the propositions on our empirical networks and found support for them. Specifically, we found that if ego is high than alter in total attachment received, then ego's attachment to alter will tend to be less than alter's to ego (proposition 2), but ego's average attachment to third parties will tend to be greater than alter's (proposition 3). The average level of bilateral asymmetry in attachments tends to be low (proposition 5). In-degree centralization, a measure of inequality in attachments received, tends to be less that out-degree centralization, a measure of inequality in attachments made (proposition 6).

More generally, we found support for the network model, including the actor model, in that the empirical data for the most part conformed to the six propositions. These empirical results held not only across all five organizational settings, but also across the four kinds of attachment: trust, communication, cooperation, and advice received. This supports our assumption, following Gould (2002), that a single network model may suffice for a variety of kinds of attachment.

A generative model such as ours, if supported, has several uses. We note and then discuss three. First, it can be used to infer characteristics of particular empirical cases, based on the pattern of parameters in the model that duplicates empirical measures. Second, more generally, it can be used to predict variation in outcomes, based on measured variation in conditions that correspond to parameters. Third, the model can be used to generate new propositions by adding new features to the model and generalizing outcomes that result.

Inference of case characteristics can be used either to test the network model or simply descriptively. We can test the network model by treating the deduced set of parameter values as a hypothesis to be tested on further data from the empirical case in question. This is likely to require data concerning the relative importance of different elements of the actor model, which probably differ by the kind of attachment. For example, reciprocity may be more important for certain kinds of attachments, and social influence more important for others. If we are fairly confident in the network model, we could use a deduced set of parameter values for an empirical case as descriptive characteristics of that case. An example of this use is our earlier discussion of why support for the propositions was weakest in the hospital departments, in which we hypothesized that attribute homophily and reciprocity were unimportant determinants of attachments in the hospital care department.

A second use of the network model also would be for the deduction of parameter values, but it would generate hypotheses in the opposite direction. We would gather data for different kinds of networks in different settings, use those conditions to infer likely values of parameters, and then treat predicted model outcomes for those parameters as hypotheses to be tested. This also would require some understanding of the relationship between network conditions and parameter values, that is, weights of components in the actor model.

A third use of the network model is to add a new element to the model, and employ the expanded model to generate additional propositions and hypotheses to be tested. For example, certain network member characteristics, age or gender, for example, may be associated with the weights of some parameters in the actor model, such the importance of attachment resistance or the importance of social influence. We could add such associations to the model and see whether it predicts effects such as an association between the characteristics and place in the inequality structure, or a tendency for network members more similar on the characteristics to have stronger mutual attachments.

In conclusion, our approach to the investigation of inequalities in network structures expands our understanding of this phenomenon. What we contribute is a theoretically based and mathematically sound generative model that predicts important inequality-related features of networks at several levels, that does so generally for networks that meet the scope conditions, and that does not require knowledge of specific actor attributes such as quality. We have shown the model's potential for prediction, explanation, and informed description of features of empirical networks.

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