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CHAPTER FIFTEEN

A Multilevel p_2 Model with Covariates for the Analysis of Binary Bully–Victim Network Data in Multiple Classrooms

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INTRODUCTION

Many studies have been aimed at defining the exact nature of bullying, identifying bullies and their victims in school classes, investigating the personal and developmental characteristics of bullies and victims, and evaluating intervention programs to prevent bullying (see, e.g. Espelage & Swearer, 2003). Children have different roles in bullying (Schwartz, 2000), and some pairs of children lead to more bullying than others (Coie et al., 1999). Relatively little is known about the dyadic properties of bullies and victims (Rodkin & Berger, in press). Recently, a dual perspective theory of bullying was proposed, focusing on the dyadic nature of the bully-victim relationship (R. Veenstra et al., 2007).

This theory is tested on pre-adolescent data from TRAILS (Tracking Adolescents' Individual Lives Survey). TRAILS is designed to chart and explain the development of mental health and social development from preadolescence into adulthood (De Winter et al., 2005; Oldehinkel, Hartman, De Winter, Veenstra, & Ormel, 2004). Students were asked to report about several of their ties with classmates. This round robin design yields in principle two observations for each

relationship between two children A and B, one from the perspective of child A (the nominator or 'sender'), reporting whether or not s/he bullies child B (the target or 'receiver'), and vice versa. These two reports may not always coincide and are less likely to be in agreement for a bullying tie than for a friendship tie. The set of dyadic data collected in a closed group forms a social network. Many methods and models have been proposed for social network analysis (see Wasserman & Faust, 1994). For a review on the intricacies of dyadic designs and dyadic data analysis, see Kenny, Kashy, and Cook (2006).

We use a multilevel p_2 model (Zijlstra, Van Duijn, & Snijders, 2006) to analyze bully network data from 54 classes collected in the TRAILS study. This model takes into account the dependent nature of the data and employs the characteristics of sender and receiver individually and as a dyad. Moreover, class characteristics can be used to explain differences per classroom; for instance, between prevalence rates of bullying in school classes. We follow the dual perspective theory as laid out by Veenstra et al. (2007) but slightly modify the covariates used in the analysis. In the next section we start with the definition and interpretation of the simple p_2 model, followed by the multilevel p_2 model, and its relation to other models for social network data. In Section 3, we present the data and theory to be tested. After a section introducing the interpretation of p_2 model results, we present the results obtained for the dual perspective theory. The final section summarizes and discusses the findings.

The (Multilevel) p₂ Model

The p_2 model (Lazega & Van Duijn, 1997, Van Duijn, Snijders, & Zijlstra, 2004) analyzes a single binary network Y of size n, modeling the probability of the four possible dyadic tie outcomes (Y_{ij}, Y_{ji}), where Y_{ij} denotes the presence (1) or absence (0) of a tie (i.e. nomination) between sender (nominator) i and receiver (target) j for all n actors in the network. In the p_2 model, Y represents the bully network in a classroom of n students (i.e. the actors in the network). Although the p_2 model is aimed at the analysis of complete networks where all actors report their ties with all others in the network, the model does not require all observations of the network to be present, and thus only analyzes the available dyads.

The p_2 model builds on the p_1 model (Holland & Leinhardt, 1981), which characterizes the dyadic outcome probability by four important parameters: μ (density), ρ (reciprocity), α (sender) and β (receiver). In the p_1 model, given in equation (1), the probability that child i bullies child j is determined by i's sender parameter α_i, j's receiver parameter β_j, overall density μ, and if j also bullies i, overall reciprocity ρ. Likewise, the probability that child j bullies child

i is determined by j's sender parameter, i's receiver parameter, overall density μ, and if i also bullies j, overall reciprocity ρ.

$$P(Y_{ij} = y_1, Y_{ji} = y_2) = \tag{1}$$

$$\frac{\exp\{y_1(\mu + \alpha_i + \beta_j) + y_2(\mu + \alpha_j + \beta_i) + y_1 y_2 \rho\}}{1 + \exp\{\mu + \alpha_i + \beta_j\} + \exp\{\mu + \alpha_j + \beta_i\} + \exp\{2\mu + \alpha_i + \alpha_j + \beta_i + \beta_j + \rho\}}$$

where $y_1, y_2 \in \{0, 1\}, i \neq j$.

The interesting part of the p_1 formula is the numerator. The denominator is needed to ensure that the four outcome probabilities sum to 1. It is informative because it contains the four possible numerator outcomes. Ignoring the first term in the denominator (i.e. the 1), the second term in the denominator contains the parameters in the numerator involved in modeling the asymmetric (1,0) dyadic outcome. This unreciprocated tie from i to j where child i bullies child j, but j does not bully i, corresponds to the values $y_1 = 1$ and $y_2 = 0$. The probability of this tie depends on i's sender parameter α_i, j's receiver parameter β_j, and the network density μ. The third term in the denominator, obtained for the opposite asymmetric (0,1) dyad, includes μ, α_j, and β_i, and therefore depends on j as a sender and i as a receiver. For the case where both ties are reported, (1,1), corresponding to the fourth term in the denominator, the reciprocity parameter ρ is present in addition to the parameters involved in the single asymmetric dyadic outcomes. Thus, the density parameter is included twice (2μ). In friendship networks the concept of reciprocity reflects the increased probability of reciprocal ties. In bullying networks, we expect an absence of reciprocity or possibly even a reversed reciprocity. Finally, the first term in the denominator (1) corresponds to the (0,0) outcome, the null tie, which serves as a reference category, against which the probabilities of the other outcomes are compared. The dyadic outcome probabilities according to the p_1 model are summarized in Table 15.1.

Through the use of the exponential function in numerator and denominator, the p_1 model is reminiscent of a logistic regression model, and the interpretation of the parameters are indeed similar. For instance, the higher the sender and receiver parameters, the higher the probability of the presence of a (sent and/or received) tie. Holland and Leinhardt (1981) define log-odds ratios for the interpretation of the density and reciprocity parameters. The density parameter, μ, represents the log-odds of a tie (i.e., an asymmetric dyad (1,0) or (0,1) vs. (0,0), the reference outcome). This value is equal to the log ratio of the off-diagonal

Table 15.1
Dyadic Outcome Probabilities in the p_1 Model

Dyad (y_{ij},y_{ji})		Observed tie from j to i, y_{ji}	
		0	**1**
Observed tie from i to j, y_{ij}	**0**	1	$\exp(\mu+\alpha_j+\beta_i)$
	1	$\exp(\mu+\alpha_i+\beta_j)$	$\exp(\mu+\alpha_i+\beta_j)\exp(\mu+\alpha_j+\beta_i)\exp(\rho)$

Note. All entries are to be divided by the sum of the four elements in the table; $(1 + \exp(\mu + \alpha_i + \beta_j) + \exp(\mu + \alpha_j + \beta_i) + \exp(2\mu + \alpha_i + \alpha_j + \beta_i + \beta_j + \rho)$

elements and the top-left element in Table 15.1. The reciprocity parameter, ρ, represents the log-odds of a symmetric dyad $(0,0)$ and $(1,1)$, vs. an asymmetric dyad, $(1,0)$ and $(0,1)$, which is equal to the log ratio of the diagonal elements and the off-diagonal elements in Table 15.1. If there is no reciprocity (ρ equal to zero), the two ties (from i to j and from j to i) are independent in the p_1 model and become a simple product of two logistic regression models. When applied to bullying networks, the log-odds of an asymmetric tie from child i to child j vs. the reverse asymmetric tie is an informative measure which we will call the asymmetric log-odds. This log ratio of the bottom left element and the top right element in Table 15.1 is equal to $(\alpha_i+\beta_j)$-$(\alpha_j+\beta_i)$, the difference in sender parameters minus the difference in receiver parameters.

The p_1 model can be regarded as a (saturated) loglinear model for a cross-table representing the n x n adjacency matrix Y (Fienberg & Wasserman, 1981). For identification purposes, the p_1 model needs a restriction on the sender and receiver parameters, for instance $\Sigma_i \alpha_i = \Sigma_i \beta_i = 0$. When further restrictions are put on the sender and receiver parameters (e.g., distinguishing several categories of senders and receivers), the loglinear model can be viewed as a multinomial logistic regression model for four possible outcomes (see also Agresti, 2002). Another way to classify the p_1 model is as a fixed effects version of the Social Relations Model (SRM; Kenny & La Voie, 1984) for binary network data (Kenny et al., 2006).

The desire to use additional (covariate) information about the actors in a network (e.g., the child's sex) and the undesirable statistical properties of a saturated model (see Van Duijn et al., 2004) led to the development of the p_2 model. In this model individual sender and receiver parameters are replaced by regression equations:

$$\begin{aligned} \alpha_i &= X_{1i}\gamma_1 + A_i, \\ \beta_i &= X_{2i}\gamma_2 + B_i. \end{aligned} \tag{2}$$

Actor covariates, X_1 and X_2, can thus be used to predict individual sender and receiver effects. Either categorical or continuous covariates can be used, which may or may not be the same for explaining sender and receiver parameters. The residuals in the regression equations represent unexplained differences among the actors in the network. These residuals can also be viewed as latent variables or random effects and interpreted as individual tendencies to send or receive ties. A bivariate normal distribution is assumed for the pairs of random sender and receiver effects (A_i, B_i), with zero mean and common covariance matrix with three distinct elements: sender variance, receiver variance, and sender-receiver-covariance. The bivariate distribution reflects the association between the tendencies to bully and to being bullied. The sender and receiver variances are fairly interpretable. For instance, if the sender variance is larger than the receiver variance, then the variation between actors as senders is much larger (which makes the tendency to send ties less predictable) than the variation between actors as receivers.

Where the p_1 model has $2n$ parameters (an individual sender and receiver effect for each actor in the network), the p_2 model has fewer parameters in addition to the density and reciprocity parameter, equal to the number of regression parameters $\dim(\gamma_1)+\dim(\gamma_2)$ plus the three elements of the covariance matrix of the random sender and receiver effects. Due to this reduction in the number of parameters to be estimated, dyad specific density and reciprocity parameters are regressed on dyadic covariates Z_1 and Z_2 in the p_2 model:

$$
\begin{aligned}
\mu_{ij} &= \mu + Z_{1ij}\delta_1, \\
\rho_{ij} &= \rho + Z_{2ij}\delta_2.
\end{aligned}
\tag{3}
$$

Parameters μ and ρ now denote the mean density and reciprocity when the values of the dyadic covariates are zero. The dyadic covariates can be derived from actor specific covariates, indicating, for example, whether both children have the same sex. In that case the covariate matrix Z is symmetric. Z may also be asymmetric, such as when the difference between actor covariates are used or when a network of different tie relations (e.g., friendship) is used to predict the density. Due to the inherent symmetry of reciprocity and its definition as an interaction effect in the p_2 model, the choice of Z_2 is restricted to the subset of symmetric dyadic covariates used as Z_1. In the present application, we will not model reciprocity with covariates.

The multilevel p_2 model (Zijlstra, Van Duijn, & Snijders, 2006) is a straightforward extension of the p_2 model for the analysis of multiple networks. It assumes the same p_2 model for a sample of networks of possibly different size. Similar to sample size considerations in normal multilevel analysis, a reasonable number of classes, say at least 15 but preferably more, is required to apply the multilevel p_2 model meaningfully.

Similar to the simplest normal multilevel model with two levels, the intercept varies over the higher level. In the multilevel p_2 model the mean density parameter μ is normally distributed over the networks (expressed by the random effect M_k) and possibly related to network characteristics (through Z_{1ijk} with constant values for each dyad in network k):

$$\mu_{ijk} = \mu + M_k + Z_{1ijk}(\delta_1 + D_{1k}). \tag{4}$$

Note that in (4) the effect of dyad characteristics is also allowed to vary, by including random effects D_{1k}, comparable to the random slope model. The estimated mean μ can be viewed as the grand mean density (over all networks) when all covariate values equal zero. The estimated variance of the mean density indicates how much the mean density varies over networks. More generally, the multilevel p_2 model may include random effects for μ, ρ, γ_1, γ_2, δ_1, and δ_2. We will only apply a random effect for μ. The p_2 model and the multilevel p_2 model can be viewed as the binary counterparts of the SRM expressed as random effects (i.e.; as a multilevel model; Snijders & Kenny, 1999).

The parameters of the p_2 model and the multilevel p_2 model are estimated using Markov Chain Monte Carlo (MCMC) algorithms implemented in specialized software for social network analysis (StOCNET; Boer et al., 2006). In addition to the MCMC specifications (we used the standard option of 4,000 burn-in iterations and a sample size of 8,000), the estimation time depends on the number of networks to be analyzed and their size. On the present data set (54 networks, with a median of 15 students in roughly 7,500 dyads) it took approximately 12 hours on a standard pc to run an analysis. The estimation results in estimated posterior means and posterior standard deviations of the p_2 model parameters. The p-values of covariate effect parameters are derived from approximate t-ratios. Bayesian credibility intervals are available as approximate confidence intervals. The software is freely available for download at http://stat.gamma.rug.nl/stocnet, with manuals for both StOCNET and for the p_2 model. Details of the estimation procedures can be found in Zijlstra, Van Duijn, Snijders (in press) for the p_2 model, and in Zijlstra et al. (2006) for the multilevel p_2 model.

Bullying Networks - Theory and Data

The bullying network data were collected as part of TRAILS, a large longitudinal study among (pre)adolescents in the Netherlands. Due to the set-up of the study, in each participating classroom some but not all of the students provided information on several aspects of their dyadic relations with classmates. Thus, a not necessarily complete subnetwork of each classroom with varying

size is available. Due to incomplete data the sample size varies from 7 to 33 students per class.

The dyadic nature of the bullying network data was theoretically investigated in Veenstra et al. (2007). They used a dual goal-framing approach to highlight the opposite perspectives of bullies and victims and the inherent asymmetric or hierarchical nature of bullying. For self-proclaimed bullying it was hypothesized that bullies are likely to be dominantly aggressive boys whose victims are vulnerable, rejected, and not aggressive. Additional hypotheses were formulated about the positive effect of disliking on bullying. The influence of same-sex or mixed-sex bullying was not quite clear a priori. Boy-boy bullying ties might be more likely, because boys are more aggressive and because they may achieve more prestige by bullying boys instead of girls. On the other hand, girls, being less aggressive and more vulnerable may be more likely to be victimized. Before we set up the p_2 model to investigate (several) operationalizations of the expectations based on the dual perspective theory, the available network, dyadic and individual data are described in more detail.

Individual (Actor) Covariates

Teachers were asked to rate each pupil on aggressiveness and vulnerability, using an adapted version of the Revised Class Play instrument (Masten, Morison, & Pellegrini, 1985, see also R. Veenstra et al., 2007). We constructed 6 ordinal categories of approximately equal size from the skewed distributions (within and over all classes) to enhance comparability between classrooms and prevent too much influence of the relatively few aggressive pupils. In addition, sex was used as an actor covariate.

Network Variables and Dyadic Covariates

The dependent variable Y is the network on self-reported bullying based on the question "Who do you bully?". The network with dislike relations ("Who do you not like at all?") is used as a dyadic covariate. The asymmetric dyadic variable rejection R was derived for each dyad from the dislike relation D by computing the percentage of classmates other than the nominator who dislike the target: $R_{ij} = \Sigma_{k \neq i} D_{kj}/(n-1)$. Thus this variable separates the effects of child i's dislike D_{ij} from dislike by fellow classmates.

From child's sex, two symmetric dyadic covariates were constructed, the first to indicate mixed-sex (boy-girl and girl-boy) dyads and the second to indicate only boy-boy dyads (girl-girl dyads are the reference group), thereby distinguishing the two kinds of same-sex dyads. The first type of dyadic covariate is a standard dyadic similarity variable, generated in the p_2 model software by

obtaining the absolute difference of the binary sex indicator variable and takes on the value 1 for mixed-sex dyads and 0 for same-sex dyads. Thus a negative parameter estimate indicates a preference for same-sex bully ties. The second dyadic variable expresses the difference between boy-boy dyads and girl-girl dyads, the latter being the reference dyad if the mixed-sex dyadic variable is included.

From the individual aggression scores of nominator and target, a dyadic variable is obtained by taking the difference score, also a standard option in the p_2 model software. The derived dyadic variable is always asymmetric: $Z_{ij} = X_i - X_j$, and thus $Z_{ji} = -Z_{ij}$. A positive parameter estimate indicates a dominance effect where larger (i-j) differences increase the probability of i being the bully and j the victim in the (i,j) dyad. In this case, the (1,0) dyad is more likely than the (0,1) outcome.

ILLUSTRATION

We start with three simple models presented in Table 15.2 to illustrate the interpretation of p_2 model results. The first model is without covariates (an 'empty' or 'basic' model), and the other two models investigate the effect of sex. For the 'basic' p_2 model parameters, no significance level is indicated. The density parameter estimate is highly negative, indicating that the probability of a bully tie is much smaller than 0.50, making the null (0,0) dyadic outcome most likely. This is a phenomenon often observed in the analysis of social network data, even when the outcome variable is a positive relationship. Especially in larger networks, it is quite improbable that an actor would choose half of the other actors in the network. Thus, density is related to network size, which is also indicated by the rather large class density variance. As expected for bullying, the estimated reciprocity parameter is low. This low estimate facilitates the interpretation of the other covariate parameters because both ties in the dyad are approximately independent from a statistical point of view. The large sender variance indicates that the tendency to report bullying is highly variable over children.

The expected probabilities of the four dyadic outcomes can be derived from the model parameter estimates using (1). They are equal to 99.5% for the null dyad, and slightly lower than 0.25% for the two asymmetric dyads. The probability of a mutual dyad is negligible (approximately 0.25*0.25=0.0625%). Note that in calculating these expected probabilities the sender and receiver variance values are disregarded. Thus, the probabilities concern dyads consisting of the 'average' nominator and target (i.e., with zero sender and receiver random effects) and for the 'average' network (with zero density random effect). Just for illustration, we can also compute the expected probabilities for child A with

Table 15.2

Parameter Estimates of Multilevel p_2 Models Investigating the Effect of Sex on Bullying in 54 Classes

Effect	Model 0: No Effects		Model 1A: Individual and Dyadic Effects		Model 1B: + Class Effects	
	Posterior mean (Posterior *S.D.*)		Posterior mean (Posterior *S.D.*)		Posterior mean (Posterior *S.D.*)	
Density	-6.02	(0.20)	-7.00	(0.32)	-5.12	(0.55)
Reciprocity	0.55	(0.37)	0.58	(0.31)	0.57	(0.34)
Sender covariates						
Being a boy			0.93	(0.20) *	0.91	(0.28) *
Receiver covariates						
Dyadic covariates						
Mixed-sex			0.98	(0.20) *	1.02	(0.19) *
Boy-boy			0.75	(0.29) *	0.81	(0.30) *
Class covariates						
Percentage boys					-4.10	(1.49) *
Random effects						
Class density variance	1.41	(0.52)	1.58	(0.62)	1.79	(0.49)
Sender variance	5.51	(0.65)	5.07	(0.66)	5.24	(0.70)
Receiver variance	1.68	(0.25)	1.55	(0.23)	1.56	(0.25)
Sender-receiver covariance	0.47	(0.34)	0.29	(0.27)	0.29	(0.29)
Number of dyads	7668		7668		7668	

Note. * $p < 0.05$.

a large sender random effect (equal to 3) and an average receiver effect, and child B with an average sender effect and a small negative receiver effect (equal to -0.5) in a network with an above-average density (random effect equal to 1). This parameter configuration changes the estimated probabilities to 92% for a null dyad, 0.085% for the mutual dyad, 7.4% for the asymmetric tie from child A to child B, and 0.61% for the reverse asymmetric tie. Approximately, the odds of an asymmetric tie (compared to 'average' children) from child A to child B have increased with a factor 12 (this can also be determined without computing the probabilities first, by taking the exponent of 2.5, i.e., the sum of sender and receiver effects). The odds of the reverse tie have not changed. The asymmetric log-odds of A bullying B instead of B bullying A is also equal to 2.5 (the sum of A's sender and B's receiver effect minus the sum of A's receiver and B's sender effect, where the latter two terms are equal to zero, hence the correspondence with the odds of an asymmetric tie).

The second model (model 1A in Table 15.2) has a sender effect for boys and the mixed-sex and boy-boy dyadic covariates defined previously. Note that we choose three parameters to contrast the four possible dyadic sex combinations with the girl-girl dyad as the reference group. Significant covariate parameter estimates, based on a t-ratio larger than 2, are indicated with an asterisk. Slight changes in the basic p_2 model parameters with respect to Model 0 are observed. The sender effect of being a boy is positive and significant, in line with the expectation that boys report more bullying ties than girls. The two dyadic sex covariates are also positive and significant, indicating that both mixed-sex and boy-boy ties are more likely than girl-girl bully ties. Disregarding the most common null dyad, the asymmetric boy-girl bully tie is slightly more likely than the asymmetric boy-boy bully tie, yet only low probabilities of 0.6% vs. 0.5%, for 'average' children in an 'average' network. The numerators are essential for the computations and comparisons of these probabilities. The numerator of the (1,0) boy-boy dyad is equal to the exponent of the sum of the density effect (-7.00), the sender effect of being a boy (0.93) and the boy-boy dyadic effect (0.75), exp(-5.32)=0.0049. In the numerator of the (1,0) boy-girl dyad, the dyadic boy-boy effect is replaced by the mixed-sex effect of 0.98, and therefore slightly larger: exp(-5.09)=0.0062. The estimated asymmetric girl-boy and girl-girl bully tie probabilities are 0.24% and 0.1%, respectively. The null dyad probabilities for all sex-combinations are larger than 99% and the mutual dyad probabilities smaller than 0.01%. The asymmetric log-odds ratio is equal to the sender effect of being a boy: 0.93. This (possibly too) simple model would support the hypothesis that boys bully more than girls, and also that boys bully each other more than girls bully each other. However, no support is found for the hypothesis that boys bully boys more than they bully girls. Note that all

probabilities of dyads with at least one bully tie reported are extremely low, making it difficult to predict the outcome on the basis of sex alone.

Mainly for illustrative purposes, a class covariate related to sex (i.e., percentage of boys) is added in model 1B. It assesses whether the varying tendency to report bullying can be explained by the sex composition of the classes. The percentage of boys in each class is found to have a negative effect (see Table 15.2), implying that a higher percentage of boys in a class leads to a lower propensity to bully. The variability of the density parameter is not reduced but increased, suggesting that in spite of the relationship between sex composition and general tendency to report bullying in a classroom, many deviations are found in the sampled classes. The posterior standard deviation of the density parameter is increased compared to model 1B. This type of result likely indicates that the parameter estimates are not stable yet due to non-convergence of the MCMC estimation procedure. In such situations, a possible solution is to run the model again with more iterations (more details can be found in the p_2 manual). Therefore, the interpretation of this model is explicitly preliminary.

RESULTS

The hypotheses generated by the dual perspective theory are tested step-wise, in three models, presented in Table 15.3. In Model 2 we investigate the effects of aggressiveness and vulnerability, without sex effects (as they may be related to these individual characteristics). On the basis of the theory, we expect a sender effect of aggressiveness and a receiver effect of vulnerability. We first investigated in some preliminary model specifications (not reported) the linearity of the 6 categories of aggressiveness and vulnerability which turned out to be tenable. Although the expected non-aggressiveness of the target could be defined as a receiver effect, we chose a model with a dyadic difference effect of nominator and target aggressiveness. As expected, the sender effect of aggressiveness and the receiver effect of vulnerability are positive. The effect of the difference in aggressiveness is negative, which can be interpreted as reducing the effect of aggressiveness for highly aggressive nominators vs. less aggressive targets. This effect is illustrated in the asymmetric log-odds where the effect of the dyadic difference in aggressiveness counts twice. Because the sender effect of aggression (0.76) is more than twice as large as the minus dyadic difference effect of aggressiveness (-0.21), the log-odds are equal to 0.34 times the difference in aggressiveness minus 0.37 times the difference in vulnerability. The largest asymmetric log odds, 35, is obtained for maximally aggressive and minimally vulnerable bullies and minimally aggressive, but maximally vulnerable targets. This dyadic combination does not lead to an extremely large probability of an asymmetric bully tie: 1.4%. Due to the strong sender effect of aggressiveness,

Table 15.3

Parameter Estimates of Several Multilevel p_2 Models Testing the Dual Perspective Theory of Bullying in 54 Classes

Effect	Model 2: Aggression and Vulnerability Effects		Model 3: + Rejection		Model 4: + Sex + Dislike	
	Posterior mean (Posterior *S.D.*)		Posterior mean (Posterior *S.D.*)		Posterior mean (Posterior *S.D.*)	
Density	-9.96	(0.53)	-9.37	(0.44)	-10.3	(0.50)
Reciprocity	0.46	(0.31)	0.56	(0.31)	0.53	(0.32)
Sender covariates						
Being a boy					1.07	(0.26) *
Aggressiveness	0.76	(0.09) *	0.68	(0.09) *	0.62	(0.08) *
Receiver covariates						
Vulnerability	0.37	(0.06) *	0.23	(0.06) *	0.18	(0.06) *
Dyadic covariates						
Mixed-sex					0.48	(0.20) *
Boy-boy					-0.05	(0.29)
Diff. aggressiveness	-0.21	(0.05) *	-0.12	(0.05) *	-0.08	(0.05)
Rejection			4.06	(0.58) *	4.00	(0.68) *
Dislike					1.98	(0.16) *
Random effects						
Class density variance	1.55	(0.55)	1.19	(0.46)	1.28	(0.49)
Sender variance	4.75	(0.66)	4.48	(0.59)	4.69	(0.68)
Receiver variance	1.32	(0.22)	0.76	(0.19)	0.89	(0.20)
Sender-receiver covariance	0.23	(0.26)	0.10	(0.24)	0.12	(0.27)
Number of dyads	6841		6838		6838	

Note. * $p < 0.05$.

the maximum probability of an asymmetric bully tie is from a nominator with maximum score (6) on aggression and minimum score (1) on vulnerability to a target with the maximum score on vulnerability and aggression: 4.0%. This probability is still as large as 3.8% for nominators who are maximally vulnerable themselves. On the opposite of the scale for minimally aggressive and vulnerable nominators and targets, the probability of a mutual or asymmetric bully tie is negligible (less than 0.1%). Although the probabilities are still rather small, they are much larger than the predicted probabilities from model 1A. We can interpret this difference as the increased predictive power of model 2.

Thus, Model 2, although incomplete, supplies support for the dual perspective theory in the sense that bullies are likely to be aggressive boys and their victims vulnerable. The hypothesis about bullies seeking less aggressive victims, is not quite supported in the sense that the model predicts bullying between aggressive children as much (or even a little more) than between aggressive and less aggressive children. On the other hand, given a dyad with one more aggressive and one less aggressive child, it is more likely to observe that the aggressive child bullies the less aggressive child than the opposite.

In Model 3, reported in Table 15.3, we add the effect of rejection of the target. It is positive, supporting the hypothesis that the more children are rejected by others, the more they are prone to being bullied. All other effects are slightly smaller, but still significant. Calculating the probabilities of a bullying tie between a maximally aggressive and vulnerable nominator and a target where neither are rejected gives only a 2.0% probability of an asymmetric bullying tie, most likely because the value of 0 for rejection is unrealistic. It is now somewhat more difficult to find meaningful combinations of characteristics for nominators and targets. The negative effects of the difference in aggression level and of the level of vulnerability and the positive effect of percentage of rejection offset each other. Aggression, vulnerability, and rejection are all moderately positively correlated with each other (see R. Veenstra et al., 2007). If we choose child A with aggression level 6, vulnerability level 2, and rejection level 0.1 (i.e. disliked by 10% of the class mates) and child B with aggression level 6, vulnerability level 3, and rejection level 0.3, the probability that child A reports a bully tie with child B is 3.0%, that child B reports a bully tie with child A is 1.0%, and that they both report to bully each other is 0.05%.

In the final model (Model 4 in Table 15.3), sex effects are added according to Model 1A as well as the effect of dyadic dislike. Not surprisingly, dislike turns out to be an important dyadic covariate The strength of the effect is comparable to a rejection value of 0.50, but it is clearly separate, given that the estimate for the effect of rejection was unchanged. A comparison of Models 1A and 3 indicates subtle changes in the parameter estimates, where especially the reduced effects of the mixed-sex and boy-boy dyads are of interest. These

changes can be interpreted as a form of (partial) mediation. Recomputing the probabilities for the dyad with the same configuration as in Model 3, rather large differences are found between the different sex combinations. Focusing on the two asymmetric bully ties $(1,0)$ and $(0,1)$ - 3% and 1% in Model 2 - and assuming that the children do not like each other, we find the probabilities to be equal to 12.7% and 4.8% for boy-boy dyads, 20.1% and 2.6% for boy-girl dyads, 7.6% and 8.2% for girl-boy dyads, and 5.2% and 2.0% for girl-girl dyads. These probabilities are all rather high, and we might question whether the configuration of a girl with aggression level 6, and vulnerability level 1 or 3 is realistic. Therefore, interpretation of p_2 model results requires a lot of careful assumptions and considerations. In summarizing and further interpreting the results in the next section we avoid the complex language involved in discussing odds and probabilities, and highlight the support found for the dual perspective theory in Model 4.

SUMMARY AND DISCUSSION

In this chapter the multilevel p_2 model was used to test several hypotheses coming from the dual perspective theory of bullying put forward by Veenstra et al. (2007) on self-proclaimed bullying data in 54 Dutch school classes. The multilevel p_2 model is a good choice for the analysis of this large amount of dyadic ties in multiple social networks because it distinguishes the roles of sender and receiver in a dyadic link, and their individual characteristics, while at the same time taking into account the dependence between the two ties within the dyad and between the ties to and from the same actors. The multilevel p_2 model also incorporates dyadic and network information and, just like any regression model, gives estimates of the effects of several covariates simultaneously. Complete network data is not required for a p_2 model analysis, but the usual concerns about missing data do apply (see, e.g., Little & Rubin, 2002).

Testing the significance of effects is straightforward, whereas interpretation of p_2 model results is more difficult, because they have to be translated into four dyadic outcome probabilities. This translation process is similar to, but a bit more complex than in logistic regression, due to the interdependence of the dyadic outcomes and sensitivity to the dyadic covariate configuration. The random effects of senders and receivers (and potentially more parameters in the multilevel p_2 model) adds another level of complexity. This complexity can be overcome, as long as one keeps in mind that the results are about 'average' children (in an 'average' classroom).

We found that bullies have an advantage over the children they victimize by being more dominantly aggressive than their victims. These results are consistent with earlier results at the individual (see also Vaillancourt, Hymel, &

McDougall, 2003) and at the dyadic level (Dodge, Price, Coie, & Christopoulos, 1990). As we expected, bullies pick on targets that they do not like and who are vulnerable (i.e. fearful/isolated) and rejected by others. This last point is also consistent with earlier findings at the individual level (Boivin, Hymel, & Bukowski, 1995; Hodges, Boivin, Vitaro, & Bukowski, 1999; Hodges & Perry, 1999). We also found that boys tend to bully more than girls. Moreover, controlling for aggression, vulnerability, rejection, and dislike, a bully ties is most likely from a boy to a girl. The analyses with the p_2 model also revealed that the dual perspective theory explained some of the highly variable tendency to report bullying as well as part of the much lower target variance.

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