

*Decomposition Methods in Demography*

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In memory of Anton Kuijsten.

ISBN 90 5170 727 4  
NUR 740

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Tel.: + 31 20 625 54 29, Fax: + 31 20 620 33 95 E-mail: [info@rozenbergps.com](mailto:info@rozenbergps.com)

Rijksuniversiteit Groningen

Decomposition Methods in Demography

Proefschrift

ter verkrijging van het doctoraat in de  
Ruimtelijke Wetenschappen  
aan de Rijksuniversiteit Groningen  
op gezag van de  
Rector Magnificus, dr. F. Zwarts,  
in het openbaar te verdedigen op  
maandag 30 juni 2003  
om 16:00 uur

door

Vladimir Canudas Romo

geboren op 22 november 1972  
te Parijs, Frankrijk

Promotores: Prof. dr. J.W. Vaupel  
Prof. dr. ir. F.J. Willekens

Beoordelingscommissie: Prof. dr G. Caselli  
Prof. dr G. Feichtinger  
Prof. dr. J. Oosterhaven

# CONTENTS

**List of Tables**

**List of Figures**

**Preface**

<b>I</b>	<b>Introduction</b>	<b>1</b>
<b>1</b>	<b>Demography</b>	<b>3</b>
1.1	Demography . . . . .	3
1.2	Decomposition Techniques . . . . .	4
1.3	Research Question . . . . .	5
1.4	Organization of the Book . . . . .	5
<b>2</b>	<b>Notation</b>	<b>7</b>
2.1	Introduction . . . . .	7
2.2	Demographic Averages . . . . .	7
2.3	Demographic Measures: Ratios, Proportions, Rates, and Probabilities . . . . .	8
2.4	Operators . . . . .	10
2.5	Data and Applications . . . . .	12
2.5.1	Data Sources . . . . .	12
2.5.2	Tables of Applications . . . . .	13

<b>II</b>	<b>Decomposition Methods</b>	<b>15</b>
<b>3</b>	<b>Standardization and Decomposition Techniques</b>	<b>17</b>
3.1	Introduction . . . . .	17
3.2	Methods of Standardization . . . . .	17
3.3	First Decomposition Method . . . . .	18
3.4	Further Decomposition Research . . . . .	20
3.4.1	Cho and Retherford's Decomposition . . . . .	20
3.4.2	Kim and Strobino's Decomposition . . . . .	23
3.4.3	Das Gupta's Decomposition . . . . .	25
3.4.4	Structural Decomposition Analysis . . . . .	26
3.5	Conclusion . . . . .	29
<b>4</b>	<b>Applications of Decomposition Methods</b>	<b>31</b>
4.1	Introduction . . . . .	31
4.2	Decomposition of Mortality Measures . . . . .	32
4.2.1	Decomposing Life Expectancy . . . . .	32
4.2.2	Conclusion . . . . .	36
4.3	Decomposition of Fertility Measures . . . . .	36
4.3.1	Decomposing the Total Fertility Rate . . . . .	39
4.3.2	Conclusion . . . . .	42
4.4	Decomposition of Growth Measures . . . . .	43
4.4.1	Decomposing the Crude Growth Rate . . . . .	46
4.4.2	Conclusion . . . . .	48
<b>5</b>	<b>Alternative Decomposition Methods</b>	<b>51</b>
5.1	Introduction . . . . .	51
5.2	Regression Decomposition . . . . .	52
5.3	The Purging Method . . . . .	57
5.4	The Delta Method . . . . .	60
5.5	Conclusion . . . . .	62
<b>III</b>	<b>Decomposing Demographic Change Into Direct Versus Compositional Components</b>	<b>65</b>
<b>6</b>	<b>Decomposing Demographic Averages</b>	<b>67</b>
6.1	Introduction . . . . .	67
6.2	Derivatives of Averages . . . . .	68
6.3	Averages over Subpopulations . . . . .	70
6.4	Derivatives of Demographic Differences and Additions . . . . .	72
6.5	Relative Derivatives of Averages . . . . .	74
6.6	Relative Derivative of Products and Ratios . . . . .	75
6.7	Decomposition of the Change in Life Expectancy . . . . .	78
6.7.1	The Derivative of Life Expectancy . . . . .	78

6.7.2	The Difference in Male and Female Life Expectancy . . . . .	81
6.8	Conclusion . . . . .	82
<b>7</b>	<b>Age, Categorical and Cause of Death Decomposition</b>	<b>85</b>
7.1	Introduction . . . . .	85
7.2	Age Decomposition . . . . .	86
7.3	Single Age Decomposition . . . . .	89
7.4	Categorical Decomposition . . . . .	92
7.5	Cause of Death Decomposition . . . . .	94
7.6	Conclusion . . . . .	98
<b>8</b>	<b>Multidimensional Decompositions</b>	<b>101</b>
8.1	Introduction . . . . .	101
8.2	Averages of Averages . . . . .	102
8.2.1	Average Age of the Population . . . . .	102
8.2.2	Averages of Averages, Generalization . . . . .	103
8.3	Simplifying a Complex Average . . . . .	105
8.3.1	Crude Death Rate of a Group of Countries . . . . .	105
8.3.2	Simplifying a Complex Average, Generalization . . . . .	107
8.4	Decomposing the Compositional Component . . . . .	108
8.5	Separating the Weighting Functions . . . . .	109
8.5.1	Two Compositional Factors . . . . .	109
8.5.2	Decomposing the Average Age at Death . . . . .	110
8.5.3	Three Compositional Factors or More . . . . .	112
8.5.4	Purging the Weighting Functions . . . . .	114
8.6	Categorical Decomposition in a Two-Way Table . . . . .	115
8.7	Conclusion . . . . .	117
<b>9</b>	<b>Estimation of the Decomposition Formula</b>	<b>119</b>
9.1	Introduction . . . . .	119
9.2	Vector Formulation of Direct vs. Compositional Decomposition . . . . .	120
9.2.1	Matrix Notation in Demography . . . . .	120
9.2.2	The Direct vs. Compositional Decomposition . . . . .	120
9.2.3	Matrices of Change . . . . .	123
9.3	Estimation of Derivatives in Demography . . . . .	124
9.3.1	Changes over Time . . . . .	124
9.3.2	Mortality Measures . . . . .	126
9.3.3	Fertility Measures . . . . .	128
9.3.4	Growth Measures . . . . .	128
9.4	Conclusion . . . . .	130

<b>IV</b>	<b>Evaluation of the Decomposition Methods</b>	<b>131</b>
<b>10</b>	<b>Decomposition Techniques, Revisited</b>	<b>133</b>
10.1	Introduction . . . . .	133
10.2	Kitagawa's Decomposition Method, Revisited . . . . .	133
10.3	Further Decomposition Research, Revisited . . . . .	135
10.4	Applications of Decomposition Methods, Revisited . . . . .	137
10.4.1	Mortality Measures, Revisited . . . . .	137
10.4.2	Fertility Measures, Revisited . . . . .	139
10.4.3	Growth Measures, Revisited . . . . .	140
10.5	Alternative Decomposition Methods, Revisited . . . . .	140
10.6	Conclusions . . . . .	142
<b>11</b>	<b>Conclusions</b>	<b>143</b>
11.1	Introduction . . . . .	143
11.2	Properties of Decomposition Methods . . . . .	144
11.3	Concluding Remarks . . . . .	146
	<b>References</b>	<b>147</b>
	<b>Index</b>	<b>157</b>
	<b>Summary in Dutch</b>	<b>159</b>



# LIST OF TABLES

2.1	Example of how the results are presented in the tables. . . . .	14
3.1	Crude death rate, $d(t)$ , per thousand, and Kitagawa's decomposition of the annual change over time in 1965-1975, 1975-1985 and 1985-1995 for Mexico. .	19
3.2	Crude death rate, $\bar{d}_E(t)$ , per thousand, and Cho and Retherford's decomposition of the annual change over time in 1960-1970, 1975-1985 and 1992-1996. The crude death rate is the average of crude death rates of selected European countries.	22
3.3	Crude death rate, $\bar{d}_E(t)$ , per thousand, and Kim and Strobino's decomposition of the annual change over time in 1960-1970, 1975-1985 and 1992-1996. The crude death rate is the average of crude death rates of selected European countries. .	25
3.4	Crude death rate, $\bar{d}_E(t)$ , per thousand, and Das Gupta's decomposition of the annual change over time in 1960-1970, 1975-1985 and 1992-1996. The crude death rate is the average of crude death rates of selected European countries. .	27
3.5	Crude death rate, $\bar{d}_E(t)$ , per thousand, and Oosterhaven and Van der Linden's decomposition of the annual change over time in 1960-1970, 1975-1985 and 1992-1996. The crude death rate is the average of crude death rates of selected European countries. . . . .	28
4.1	Life expectancy at birth, $e^o(0, t)$ , and Arriaga's decomposition of the annual change over time from 1900 to 1905, from 1950 to 1955 and from 1995 to 2000, in Sweden. . . . .	34
4.2	Age and cause of death decomposition for the annual change over time in life expectancy, in percentages, for Japan for the period 1980-1990, following Pollard's decomposition methods. . . . .	35
4.3	Crude birth rate of married women and Zeng's decomposition of the annual change over time for Denmark, the Netherlands and Sweden from 1992 to 1997.	37
4.4	Crude birth rate of unmarried women and Zeng's decomposition of the annual change over time for Denmark, the Netherlands and Sweden from 1992 to 1997.	38

4.5	Total fertility rate as a product of five factors, $TFR(t)$ , and Bongaarts and Potter's decomposition of the annual relative change over the period 1975-1993/94 for Bangladesh. . . . .	40
4.6	Total fertility rate as a product of five factors, $TFR(t)$ , and Gertler and Molyneaux's decomposition of the relative change over time in the period 1975-1993/94 for Bangladesh. . . . .	42
4.7	The population growth rate, $r(t)$ , rates in percentages, and the balancing equation in 1985-1995 for France, Japan and the USA. . . . .	43
4.8	Age-specific growth rates, $r(a, t)$ , rates in percentages, and decompositions as suggested in the systems of Preston and Coale, and Arthur and Vaupel for France at ages 20, 50 and 80 in 1990. . . . .	45
4.9	Population growth rate of the world, $r(t)$ , and Keyfitz's estimation of the annual change, around January 1, 1979 and around January 1, 1982. . . . .	46
4.10	The population growth rate, $r(t)$ , rates in percentages, and Vaupel and Canudas Romo's decomposition of the growth rates in the period 1985-1995 for France, Japan and the USA. . . . .	47
4.11	The population growth rate, $r(t)$ , rates in percentages, and Horiuchi's decomposition of the growth rates in the period 1985-1990 for France, Japan and the USA. . . . .	49
5.1	Number of children regressed by age at first child, education achievement, sex and the number of unions in life, for Mexicans from the center, north and south of the country. . . . .	54
5.2	Regression decomposition of the differential in achieved number of children between Mexicans from the center and those from the north and south. Number of children is regressed on educational achievement, number of unions in life, age at first child and sex. . . . .	55
5.3	Observed Mexican migration by education achievement and region of origin. . . . .	59
5.4	Mexican migration adjusted by using Clogg's method for purging the confounding influence of the interaction between region of origin and educational achievement. . . . .	59
5.5	Gompertz trajectories calculated for males and females in Hungary and Japan, for 1989 and 1999. The force of mortality at age 75 is calculated together with the Delta decomposition of the change in the period. . . . .	62
6.1	Crude death rate, $d(t)$ , per thousand, and decomposition of the annual change over time from 1985 to 1995 for Canada, Mexico and United States. . . . .	70
6.2	Life expectancy at birth, $\bar{e}_o(t)$ , and decomposition of the annual change over time in 1960-1970, 1975-1985 and 1992-1996. Life expectancy is calculated as the average over selected European countries. . . . .	71
6.3	Difference between the male and female crude death rates, $\Delta d(t)$ , per thousand, and decomposition of the annual change over time between 1980-1985, 1985-1990 and 1990-1995 for Mexico. . . . .	73
6.4	Crude death rate, $d(t)$ , per thousand, and decomposition of the annual relative change over time in 1965-1975, 1975-1985 and 1985-1995 for Mexico. . . . .	75

6.5	Crude birth rate, $CBR(t)$ , in percentage, and decomposition of the annual relative change over time, in percentage, from 1992 to 1997, for Denmark, the Netherlands and Sweden. . . . .	77
6.6	Ratio of total fertility rates for married over unmarried women and the decomposition of the annual relative change over time, in percentage, for Denmark, the Netherlands and Sweden from 1992 to 1997. . . . .	78
6.7	Life expectancy at birth, $e^o(0, t)$ , and the decomposition of the annual change from 1990 to 1999 for Japan, Sweden and the United States. . . . .	80
7.1	Age decomposition of the annual change over time of the crude death rate, $d(t)$ , per thousand, in 1965-1975, 1975-1985 and 1985-1995 for Mexico. . . . .	87
7.2	Population growth rate of the world, $\bar{r}(t)$ , and decomposition of the annual change over time around January 1, 1979 and January 1, 1982. . . . .	94
7.3	Regional decomposition of the annual change over time in the world's population growth rate around 1979, per 10,000. . . . .	95
7.4	Causes of death distribution for Japan in 1980 and 1990. . . . .	97
7.5	Cause of death decomposition for the annual change over time in life expectancy, for Japan from 1980 to 1990. . . . .	98
8.1	Average age of United States population, $\tilde{a}(t)$ , calculated as the average over age and race, and decomposition of the annual change over time from 1990 to 2000. . . . .	103
8.2	Crude death rate, $\bar{d}_E(t)$ , per thousand, and decomposition of the annual change over time in 1960-1970, 1975-1985 and 1992-1996 for selected European countries. . . . .	107
8.3	Crude death rate, $\bar{d}_E(t)$ , per thousand, and decomposition of the annual change over time in 1960-1970, 1975-1985 and 1992-1996 for selected European countries. Decomposing by direct change and two compositional components: age and country. . . . .	110
8.4	Average age at death, $\bar{a}_D(t)$ , and decomposition of the annual change over time for Japan from 1980 to 1990. Decomposing by distribution of deaths over age and by cause of death. . . . .	111
8.5	Crude death rate, $\bar{d}_E(t)$ , per thousand, and decomposition of the annual change over time in 1960-1970, 1975-1985 and 1992-1996 for selected European countries. Decomposing by direct change and three compositional components: age, sex and country. . . . .	113
8.6	General fertility rate, $\bar{b}_E(t)$ , per hundredth, and decomposition of the annual change over time in 1992-1997 for selected European countries. Decomposing by direct change and three compositional components: age, marital status and country. . . . .	114
9.1	Crude death rate, $d(t)$ , per thousand, and decomposition of the annual change over time from 1985 to 1995 for the United States. The values in the columns are derived from the assumption that the age-specific death rates change exponentially and linearly over time. . . . .	127

10.1	Crude death rate, $d(t)$ , per thousand, and the annual change over time in 1985-1995 for Mexico. The change is decomposed by the direct vs. compositional decomposition and the method suggested by Kitagawa. . . . .	134
10.2	Crude death rate, $\bar{d}_E(t)$ , per thousand, and decomposition of the annual change over time in 1992-1996 for selected European countries. The decompositions are direct vs. compositional decomposition and the methods suggested by Cho and Retherford, Kim and Strobino, Das Gupta and Oosterhaven and Van der Linden. . . . .	135
10.3	Life expectancy at birth, $e^o(0, t)$ , and decomposition suggested by Vaupel and Canudas and Arriaga for the annual change over time from 1995 to 2000 for Sweden. . . . .	138
10.4	Crude birth rate, $CBR(t)$ , in percentage, for the total population and by marital status (married, unmarried and total). The decompositions are direct vs. compositional decomposition for the relative annual change over time in $CBR$ and the method suggested by Zeng for the difference over time, in percentage, from 1992 to 1997, for the Netherlands. . . . .	139
10.5	Population growth rate of the world, $\bar{r}(t)$ , and direct vs. compositional decomposition and the method suggested by Keyfitz for the annual change over time around January 1, 1982. . . . .	141

# LIST OF FIGURES

5.1	Causal diagram for a linear recursive model. . . . .	57
6.1	Five-year moving average of the improvement in mortality and the remaining life expectancy at ages 2 to 100 for Japan, Sweden and the United States in 1995. . . . .	81
6.2	Life expectancy for Swedish males and females from 1895 to 1995, and the difference in life expectancies. . . . .	82
6.3	Decomposition of the annual change in the Swedish male and female life expectancy difference from 1895 to 1995. . . . .	83
7.1	Age decomposition of the annual change over time in the Mexican crude death rate from 1985 to 1995. . . . .	88
7.2	The direct effect and the compositional effect, in percentages, of the annual change over time in the Mexican male and female crude death rates from 1985 to 1995. . . . .	89
7.3	The age-specific death rates and the Mexican crude death rate for males and females in 1990. . . . .	90
7.4	The Mexican male and female age-specific growth rates together with the level-2 effect of the annual change over time in the crude death rate. . . . .	91
7.5	Cause of death decomposition of the annual change over time in life expectancy, from 1980 to 1990 for Japan. . . . .	99
8.1	Lexis surface of an age and country decomposition of the direct component of the annual change in the crude death rate of selected European countries from 1992 to 1996. . . . .	116
8.2	Lexis surface of an age and country decomposition of the compositional component of the annual change in the crude death rate of selected European countries from 1992 to 1996. . . . .	117

9.1	Lexis surface of change in the death rates by age groups and selected European countries from 1992 to 1996. . . . .	123
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## Preface

The adventure of decomposition methods is the most challenging responsibility that I have encountered so far. I accepted this formidable task because, I believe, it is the key idea of all the methods founded in the core of demography. Decomposition theory is based on the simple principle of separating demographic measures into components that contribute to an understanding of the phenomena under study. Most of the decompositions presented here refer to changes over time in demographic variables. For this reason, this research contributes to the understanding of population dynamics. The principle of decomposition methods is not only applicable to demography, however. All sciences, in one or other way, use decomposition as a research tool. As such, other social and natural sciences could benefit from the results presented in this book.

My background in actuarial science and population studies provided me with the skills to learn, develop and enjoy the decompositions presented here. The formulas are simple but they explain the fundamental relations existing among demographic variables. An understanding of differential equations and algebra and an interest in population phenomena are useful in understanding the power of decomposition.

I had special interest in looking at the developments of different decomposition methods and finding out which questions these methods could help answer. From reviewing previous methods, insight was gained into the desired properties of decomposition methods and progress in decomposition theory was made.

Stimulated by the early work of James Vaupel on decomposition methods, I began studying the research carried out in this field. As I became familiar with the state of the art in decomposition analysis, I was inspired with many ideas for future developments of Vaupel's formulas. Advice from Frans Willekens in other areas of demography helped me further develop my work. I am grateful to both James Vaupel and Frans Willekens for their roles as "Doktorvaters". Their guidance and friendship was very valuable during the process of this Ph.D. I am particularly grateful to James Vaupel because his dedication and enthusiasm for formal demography were very inspiring for me. I particularly enjoyed those demographic lessons on the ferry from Rostock to Denmark. I am particularly grateful to Frans Willekens because his suggestions and encouragement allowed me to expand my research to a broader area of study. I was delighted when the meetings were held at his house and included the wonderful hospitality of Maria Willekens.

This Ph.D. was carried out as a collaborative work between two institutions. I came to the Max Planck Institute for Demographic Research in Rostock, Germany, as the first Sergio Camposortega Cruz Ph.D. Fellow in 1999. As a member of this institution, I had the opportunity to meet many well-established demographers, as well as the promising future generation of demographers. Exchanges with all these people who are dedicated to demography were very stimulating for my work. My appreciation of support goes to Gunnar, Sara, Jan, Vladka, Roland, Michaela, Dirk, Lena, Annette B., Rasmus, Iliana, Hans Peter, Jonathan, Ines, Konstantin, Edlira, Nika, Alexia, Heiner, Gunde, Rainer, Ulrike, Patricio, Elizabetha, Francesco, Nadege, Jutta, and many others.

During my time in Rostock, I also had the chance to participate in courses of the International Max Planck Research School for Demography, first as a student and later as a lecturer. Concerning these courses, I would like to thank the students for their patience and for their

feed-back. The courses of this School provided me with the opportunity to visit my second host institution, the University of Groningen. During my Ph.D. studies I had several extended visits to the Population Research Centre (PRC) in Groningen, which also provided me with a scholarship for my final year. I enjoyed the stimulating international environment of the PRC. My gratitude for support goes to Stiny, Karen, Mamun, Moury, Sarbani, Hideko, José, Maaïke, Luisa, Ajay, Zula, Elda, Marloes, Andrea, Susan, Vanessa, Rob, Miriam, Bettina, Thomas, Inge, among many others. Mrs. Marijke Steenstra, my Dutch host, also deserves special thanks for letting me feel at home.

Both institutions were very welcoming and I enjoyed the homely environment, especially among friendly colleagues who contributed with their energetic spirit to create perfect places for developing new ideas. I have reached this far, in academic terms and in distance, thanks to the education received in my former universities in México the UNAM and FLACSO. Thank you to all of you, dear Demographers. Finally, I wish to thank the Consejo Nacional de Ciencia y Tecnología of México for supporting me with a complementary scholarship.

I am grateful to Annette Erlangsen, Jenae Tharaldson, Gina Rozario and Sara Grainger for their English editing of early versions of the book and to Rozenberg Publishers for publishing the book. My appreciation also goes to the master painter Daniel Ponce Montuy for allowing me to use his art on the cover of the book.

Before closing this preface I would like to acknowledge all those who were at all times in my thoughts and helped me through the difficult times when days were too short and work too intense. All my admiration goes to my friends, some far some near - I keep on learning from your delight for life. All my love goes to my family who has taught me how to keep smiling in hard times. In those hard cold days, when even the demographic spirit was giving up, a warm hug from a loved one reminded me my compromise with life, Thanks Annettita.