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SL$(2,\mathbb{R})$-invariant IIB brane actions

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Abstract: We give a universal SL$(2,\mathbb{R})$-invariant expression for all IIB p-brane actions with $p=-1,1,3,5,7,9$. The Wess-Zumino terms in the brane actions are determined by requiring (i) target space gauge invariance and (ii) the presence of a single Born-Infeld vector. We find that for $p=7$ ($p=9$) brane actions with these properties only exist for orbits that contain the standard D7-brane (D9-brane). We comment about the actions for the other orbits.

Keywords: Supersymmetric Effective Theories, D-branes, p-branes.

Contents

1. Introduction

It is well-known that the duality group of the classical IIB string theory is $\mathrm{SL}(2,\mathbb{R})$ and that this group of duality transformations gets broken to $\mathrm{SL}(2,\mathbb{Z})$ at the quantum level. At the level of the low energy limit of IIB string theory the $\mathrm{SL}(2,\mathbb{R})$ symmetry manifests itself as a (non-linear) symmetry that acts on the fields of the IIB supergravity multiplet [1−3]. In particular, the two scalars (the dilaton and the axion) parametrize the coset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{SO}(2) \equiv \mathrm{SU}(1,1)/\,\mathrm{U}(1)$.

When the Dp-branes of IIB supergravity were discovered [4] a somewhat unsatisfactory situation arose: the formulations of the worldvolume actions for the Dp-branes broke the $\mathrm{SL}(2,\mathbb{R})$ symmetry of the theory. This applies for instance to the actions of [5−7]. For special cases there have been attempts to rectify this situation. For instance, an $\mathrm{SL}(2,\mathbb{R})$-invariant formulation of (p,q)-strings [8] has been given [9] even including kappa-symmetry [10]. This formulation made use of the fact that in two spacetime dimensions the Born-Infeld vector is equivalent to an integration constant describing the tension of a string. Similarly, the case of 3-branes has been discussed [16]. In this case one makes use of the fact that in 4 spacetime dimensions the electric-magnetic dual of a Born-Infeld vector is again a vector. Such special properties do not occur for the branes with $p > 3$ and indeed constructing an $\mathrm{SL}(2,\mathbb{R})$-invariant formulation of 5-branes turns out to be problematic [17].

In this paper we will fill this gap and provide an $\mathrm{SL}(2,\mathbb{R})$-invariant expression for all the branes of IIB string theory. In doing this we make crucial use of the fact that only recently the supersymmetry and gauge transformations for all p-form fields compatible with the IIB algebra have been derived [18].[1] These fields are a doublet of 2-forms, a singlet 4-form, a doublet of 6-forms, a triplet of 8-forms, a quadruplet of 10-forms and a doublet of 10-forms:

$$A^{\alpha}_{(2)}\,, \quad A_{(4)}\,, \quad A^{\alpha}_{(6)}\,, \quad A^{\alpha\beta}_{(8)}\,, \quad A^{\alpha\beta\gamma}_{(10)}\,, \quad A^{\alpha}_{(10)}\,. \tag{1.1}$$

Here we have used the $\mathrm{SU}(1,1)$ notation with $\alpha = 1,2$.[2] In [19] the gauge transformations and supersymmetries of these p-form fields were given in a manifestly $\mathrm{SU}(1,1)$-invariant form.

These results opened up the possibility of formulating, for all p, p-brane actions in an $\mathrm{SL}(2,\mathbb{R})$-invariant way. A first step in this direction was taken in [20], where all possible branes for IIB were classified and their tensions determined in an $\mathrm{SL}(2,\mathbb{R})$-covariant way. In particular, it was found that the D7-brane and D9-brane belong to *nonlinear* doublets of $\mathrm{SL}(2,\mathbb{R})$. This is unlike the (p,q)-strings that form a linear doublet [8].

In this paper we continue the construction of the brane actions by including the Born-Infeld worldvolume vector. This vector is part of a doublet of vectors:

$$V^{\alpha}_{(1)}\,, \tag{1.2}$$

where the existence of the two different worldvolume vectors corresponds to the fact that either an F-string or a D-string (or, more generally, a (p,q)-string) can end on the brane. The challenge is now to construct a WZ term that at the same time involves a single worldvolume vector and p-form fields that are in non-trivial representations of $\mathrm{SU}(1,1)$. In particular, at first sight the triplet of 8-forms and the quadruplet of 10-forms suggest that we introduce corresponding charges that transform as a triplet $q_{\alpha\beta}$ and quadruplet $q_{\alpha\beta\gamma}$ of $\mathrm{SU}(1,1)$, respectively. Assuming that the worldvolume vector that occurs on the brane is given by the combination $q_\alpha V^{\alpha}_{(1)}$ for certain constants q_α we will show in this paper that, given certain requirements, a Wess-Zumino (WZ) term can only be constructed for the restricted charges given by

$$q_{\alpha\beta} = q_\alpha q_\beta\,, \qquad\qquad q_{\alpha\beta\gamma} = q_\alpha q_\beta q_\gamma\,. \tag{1.3}$$

These charges include those of the standard D7-brane and D9-brane. Note that, in the case of the D7-brane, the above restriction is equivalent to the condition that

$$\det\, q_{\alpha\beta} = 0\,. \tag{1.4}$$

For charges that belong to the other conjugacy classes of $\mathrm{SL}(2,\mathbb{R})$, with $\det\,(q_{\alpha\beta}) \neq 0$, it is not possible to construct a brane action of the required form.

[1] The same has been done for the IIA case [19].

[2] We use both (complex) $\mathrm{SU}(1,1)$ and (real) $\mathrm{SL}(2,\mathbb{R})$ notation. The connection between the two is explained in appendix A which also contains our further conventions.

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In this paper we will derive a universal formula for the WZ term valid for all branes. Furthermore, given the above restrictions on the charges we will show that the brane tension that occurs in the kinetic terms of all brane actions can also be given by an elegant universal formula. In this way we obtain a unified expression of all p-brane actions. These brane actions are $\mathrm{SU}(1,1)$-invariant provided we also rotate the constants q_α at the same time we rotate the worldvolume and target space fields. In other words, we propose an $\mathrm{SU}(1,1)$-covariant family of actions.

This paper is organized as follows. In section 2 we derive the general $\mathrm{SU}(1,1)$-invariant expression for all IIB brane actions. We will do this first for the Wess-Zumino terms and, next, for the kinetic terms. Special cases will be discussed in section 3 where we will compare with other results in the literature. We give our conclusions in section 4. In appendix A we give our conventions and in appendix B we list the gauge transformations and invariant field strengths of the different IIB p-form gauge potentials.

2. $\mathrm{SL}(2,\mathbb{R})$-invariant IIB brane actions

The standard branes of IIB string theory, that is a doublet of strings, a singlet of three-branes, a doublet of five-branes, a nonlinear doublet of 7-branes and a non-linear doublet of 9-branes [20], all carry a world-volume vector-field. The need for this can easily be seen by counting (bosonic and fermionic) worldvolume degrees of freedom and requiring supersymmetry. The fact that only one vector field is involved is related to the fact that only one type of string can end on these branes. Since this string belongs to a doublet of strings it is natural to introduce an $\mathrm{SU}(1,1)$ doublet of worldvolume vectors $V^\alpha_{(1)}$, see eq. (1.2), and next require that only a particular combination occurs on the brane. Motivated by the case of D-branes we define a gauge-invariant field strength $\mathcal{F}^\alpha_{(2)}$ as follows:[3]

$$\mathcal{F}^\alpha_{(2)} = F^\alpha_{(2)} + A^\alpha_{(2)}\,, \qquad\qquad F^\alpha_{(2)} = 2\partial V^\alpha_{(1)}\,, \tag{2.1}$$

where $A^\alpha_{(2)}$ denotes the pull-back of the target space 2-form field. Whenever that does not cause confusion we will use the same symbol for the target space fields and their pull-backs. In particular, we do not indicate the worldvolume scalars that are involved in the pull-backs. The worldvolume curvature (2.1) is invariant under the gauge transformations

$$\delta_g V^\alpha_{(1)} = \partial\Sigma^\alpha - \Lambda^\alpha_{(1)}\,, \qquad\qquad \delta_g A^\alpha_{(2)} = 2\partial\Lambda^\alpha_{(1)}\,, \tag{2.2}$$

where Σ^α is the worldvolume gauge parameter and $\Lambda^\alpha_{(1)}$ is the (pull-back of the) gauge parameter of the target space two form $A^\alpha_{(2)}$.

To characterize which type of string ends on the brane we require that only the combination $q_\alpha V^\alpha_{(1)}$ occurs on the brane. In the following two subsections we will derive expressions for the WZ terms and for the kinetic terms.

[3] In this paper we use the notation of [19], denoting n-forms $F_{\mu_1\ldots\mu_n}$ by $F_{(n)}$. Antisymmetrization (with weight one) of the indices is always understood.

2.1 Wess-Zumino terms

The WZ-term of the brane actions are determined by writing down the most general Ansatz for a WZ-term and then demanding[4]

1. Target space gauge invariance
2. A single worldvolume vector field ($p \neq 1$)

The second requirement is needed to have worldvolume supersymmetry. As we already discussed the requirement of a single worldvolume vector is satisfied by requiring that only the combination $q_\alpha V^\alpha_{(1)}$ occurs on the brane. By convention we assume that the case of D-branes, i.e., an F-string ending on the brane, is covered by taking $q_{\alpha'} = (0, -1)$ where we work in the $\mathrm{SL}(2,\mathbb{R})$-basis,[5] see appendix A. All the other cases, i.e., a general (p,q)-string ending on the brane, are then covered by an $\mathrm{SL}(2,\mathbb{R})$ transformation of the D-brane case and are obtained by taking a general q-vector.

Our aim is to find a unified and $\mathrm{SU}(1,1)$-invariant WZ term for all IIB p-branes. In order to do this it is useful to recall the universal formula for WZ terms in the case of the (non $\mathrm{SU}(1,1)$-invariant) D-branes:

$$\mathcal{L}_{\mathrm{WZ}}(\mathrm{D}p\text{-brane}) = C\, e^{\mathcal{F}_{(2)}}\,, \tag{2.3}$$

where C is defined as the formal sum[6]

$$C = \sum_n C_{(n)} = C_{(0)} + C_{(2)} + C_{(4)} + C_{(6)} + C_{(8)} + C_{(10)}\,, \tag{2.4}$$

and $C_{(n)}$ are the usual RR n-forms. It is understood here that after expanding the exponential in eq. (2.3) in each term that particular $C_{(n)}$ is chosen such that the product of forms adds up to a $(p+1)$-form. Using this notation one can check that the WZ term is invariant under the gauge transformations

$$\delta C = d\lambda + F_{(3)}\,\lambda\,, \tag{2.5}$$

where $F_{(3)}$ is the curl of the NS-NS 2-form field B and λ is the formal sum

$$\lambda = \sum_n \lambda_{(n)} = \lambda_{(1)} + \lambda_{(3)} + \lambda_{(5)} + \lambda_{(7)} + \lambda_{(9)}\,. \tag{2.6}$$

The $\lambda_{(n)}$ are the different RR gauge parameters. To prove that the WZ term is gauge-invariant requires a one line calculation where one uses that $d\mathcal{F}_{(2)} = F_{(3)}$. For this to work it is important that the gauge transformation of C is of the above form, i.e., it

[4]Note that the case $p = 1$ is special since in 2 spacetime dimensions a vector does not carry any worldvolume degree of freedom. Instead, it can be integrated out to yield an integration constant. Indeed, the action of [9], for instance, contains *two* worldvolume vectors. The $p = 1$ case will be treated separately, see below.

[5]To distinguish we use $\alpha = 1,2$ in the $\mathrm{SU}(1,1)$ basis and $\alpha' = 1,2$ in the $\mathrm{SL}(2,\mathbb{R})$ basis.

[6]Note that this sum also contains a term involving $C_{(0)} \equiv \ell$ which is not required by gauge invariance.

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contains the usual $d\Lambda$ term and terms that are all proportional to $F_{(3)}$. This is not the case for the SU$(1,1)$-covariant gauge potentials we have introduced in [18]. Their gauge transformations are listed in appendix B and it can be seen that they contain $d\Lambda$ terms, terms proportional to $F^\alpha_{(3)}$ but also additional terms proportional to $\Lambda^\alpha_{(1)}$.

Inspired by the case of D-branes we make the following field redefinitions to remove the extra terms from the gauge transformations:

$$
\begin{aligned}
\mathcal{C}^\alpha_{(2)} &= A^\alpha_{(2)}\,, &(2.7)\\
\mathcal{C}_{(4)} &= A_{(4)} - \tfrac{3}{8}\tilde{q}_\alpha q_\beta A^\alpha_{(2)} A^\beta_{(2)}\,, &(2.8)\\
\mathcal{C}^\alpha_{(6)} &= A^\alpha_{(6)} + 20 A_{(4)} A^\alpha_{(2)} - \tfrac{15}{2} q_\beta \tilde{q}_\gamma A^\alpha_{(2)} A^\beta_{(2)} A^\gamma_{(2)}\,, &(2.9)\\
\mathcal{C}^{\alpha\beta}_{(8)} &= A^{\alpha\beta}_{(8)} + \tfrac{7}{4} A^{(\alpha}_{(6)} A^{\beta)}_{(2)} + 35 A_{(4)} A^\alpha_{(2)} A^\beta_{(2)} - \tfrac{105}{8} q_\gamma \tilde{q}_\delta A^\alpha_{(2)} A^\beta_{(2)} A^\gamma_{(2)} A^\delta_{(2)}\,, &(2.10)\\
\mathcal{C}^{\alpha\beta\gamma}_{(10)} &= A^{\alpha\beta\gamma}_{(10)} - 3 A^{(\alpha\beta}_{(8)} A^{\gamma)}_{(2)} - \tfrac{21}{4} A^{(\alpha}_{(6)} A^\beta_{(2)} A^{\gamma)}_{(2)} - 105 A_{(4)} A^\alpha_{(2)} A^\beta_{(2)} A^\gamma_{(2)} \\
&\quad + \tfrac{315}{8} q_\delta \tilde{q}_\epsilon A^\alpha_{(2)} A^\beta_{(2)} A^\gamma_{(2)} A^\delta_{(2)} A^\epsilon_{(2)}\,, &(2.11)
\end{aligned}
$$

where $\tilde{q}_\alpha$ is another doublet that satisfies

$$\tilde{q}_{[\alpha} q_{\beta]} = \tfrac{i}{2}\epsilon_{\alpha\beta}\,. \tag{2.12}$$

We choose a basis such that for the case of D-branes we have $\tilde{q}_{\alpha'} = (1,0)$ and $q_{\alpha'} = (0,-1)$. Note that in this case $\tilde{q}$ is the S-dual of q, see appendix A.

We have not included the doublet of 10-forms since they do not seem to fit in this family of potentials and require a separate discussion, see below. After these redefinitions we end up with the desired form of the gauge transformations:

$$\delta_g \mathcal{C} = d\Lambda + F_{(3)}\ \Lambda\,, \tag{2.13}$$

where $F_{(3)} = 3\partial\mathcal{C}_{(2)}$ and $\mathcal{C}, \Lambda$ are defined by the formal sums[7]

$$
\begin{aligned}
\mathcal{C} &= \sum_{n,\alpha} \mathcal{C}^{(\alpha)}_{(n)} = \mathcal{C}_{(0)} + \mathcal{C}^\alpha_{(2)} + \mathcal{C}_{(4)} + \mathcal{C}^\alpha_{(6)} + \mathcal{C}^{\alpha\beta}_{(8)} + \mathcal{C}^{\alpha\beta\gamma}_{(10)}\,, &(2.14)\\
\Lambda &= \sum_{n,\alpha} \Lambda^{(\alpha)}_{(n)} = \Lambda^\alpha_{(1)} + \Lambda_{(3)} + \Lambda^\alpha_{(5)} + \Lambda^{\alpha\beta}_{(7)} + \Lambda^{\alpha\beta\gamma}_{(9)}\,. &(2.15)
\end{aligned}
$$

Eq. (2.13) applies to all the forms of rank higher than 4, while for $\mathcal{C}_{(4)}$ it has to be replaced by

$$\delta\mathcal{C}_{(4)} = 4\partial\Lambda_{(3)} + \tfrac{1}{2} q_\alpha F^\alpha_{(3)} \tilde{q}_\beta \Lambda^\beta_{(1)} \quad . \tag{2.16}$$

The notation in (2.13) indicates that all SU$(1,1)$ indices in the second Λ term are symmetrised with the α index of F, and all the terms have the same rank and the same number of SU$(1,1)$ indices. Special cases are worked out in detail in section 3.

We find that in the new basis the WZ term can be cast into the following universal form

$$\mathcal{L}_{\mathrm{WZ}}(p\text{-brane}) = q \cdot \mathcal{C}\, e^{q\mathcal{F}_{(2)}}\,, \tag{2.17}$$

[7] Note that in the expression for $\mathcal{C}$ the first term in the sum, which will be discussed at the end of this subsection, is not required by gauge-invariance of the WZ term.

where $q\mathcal{F}_{(2)}$ stands for $q_\alpha \mathcal{F}^\alpha_{(2)}$ and $q \cdot \mathcal{C}$ denotes contraction of all SU(1,1) indices of the $\mathcal{C}_{(n)}$ with as many q's as are required, except for $\mathcal{C}_{(2)}$ that must be contracted with $\tilde{q}$. Therefore, all terms in the WZ term (2.17) are either independent of $\tilde{q}$ or at most linear in $\tilde{q}$ where $\tilde{q}$ always occurs in the combination $\tilde{q}_\alpha \mathcal{C}^\alpha_{(2)}$. Note that the formula (2.17) implies, as anticipated in the introduction, that in the leading term of the WZ terms the restrictions on the charges given in (1.3) hold. We will show in subsection (3.4) that without these restrictions on the charges it is not possible to construct a WZ term that satisfies the criteria given in (2.1).

We close this subsection with two comments. First, we have not yet specified the first term $\mathcal{C}_{(0)}$ in the formal sum $\mathcal{C}$. This term leads to an expression in the WZ term that is gauge-invariant by itself. For Dp-branes with p odd this expression is given by $\ell\, \mathcal{F}^{(p+1)/2}_{(2)}$. To reproduce this expression we must take

$$\mathcal{C}_{(0)} = -\frac{q_\alpha \tilde{q}_\beta \mathcal{M}^{\alpha\beta}}{q_\gamma q_\delta \mathcal{M}^{\gamma\delta}}\,, \tag{2.18}$$

where the matrix $\mathcal{M}$ is given, in the SL$(2,\mathbb{R})$ basis, in (A.9). Secondly, the doublet of 10-forms $A^\alpha_{(10)}$ is not included by the universal formula (2.17). The reason is that the construction of a WZ term for these 10-forms does not require the introduction of a worldvolume vector field. This is due to the fact that their gauge transformations only contain the leading term $\delta A^\alpha_{(10)} = d\Lambda^\alpha_{(10)}$ and therefore a WZ term of the form

$$q_\alpha A^\alpha_{(10)} \tag{2.19}$$

is already gauge-invariant by itself. However, without a worldvolume vector it is not clear how to obtain an equal number of bosonic and fermionic worldvolume degrees of freedom and establish worldvolume supersymmetry. We will not consider this case further in this paper.

2.2 Kinetic terms

We next discuss the construction of the kinetic terms. It is convenient to work in Einstein frame, since the metric g_E is SU(1,1)-invariant. The action for a p-brane in Einstein frame can be written as

$$\mathcal{L}_{\text{kinetic}}(\,p\text{-brane}) = \tau_{p,E}\,\sqrt{\det\left(g_E + s\,q\mathcal{F}\right)}\,, \tag{2.20}$$

where $\tau_{p,E}$ is the brane tension for the p-brane in Einstein frame and s is a scalar function of the scalar fields. Using the expressions for the tensions, already obtained in [20] from supersymmetry, together with our universal formula for the WZ term (2.17) we find the following general formula for the p-brane tensions:[8]

$$\tau_{p,E} = \left(qq\mathcal{M}\right)^{\frac{p-3}{4}} \qquad p \neq 1\,, \tag{2.21}$$

$$\tau_{1,E} = \left(\tilde{q}\tilde{q}\mathcal{M}\right)^{\frac{1}{2}} \qquad p = 1\,, \tag{2.22}$$

[8] For the $p = 1$ case, see subsection (3.2).

where $qq\mathcal{M}$ stands for $q_\alpha q_\beta \mathcal{M}^{\alpha\beta}$.

Further, we find we can write the kinetic terms as

$$\mathcal{L}_{\text{kinetic}}(p\text{-brane}) = \tau_{p,E}\sqrt{\det\left(g_E + \frac{q\mathcal{F}}{(qq\mathcal{M})^{1/2}}\right)}\,. \tag{2.23}$$

Summarizing, we find that the SU(1,1)-invariant Lagrangian for general IIB p-branes is given by ($p \neq 1$)

$$\boxed{\mathcal{L}(p\text{-brane}) \;=\; \tau_{p,E}\sqrt{\det\left(g_E + \frac{q\mathcal{F}}{(qq\mathcal{M})^{1/2}}\right)} \;+\; q\cdot\mathcal{C}\,e^{q\mathcal{F}_{(2)}}}\,, \tag{2.24}$$

with the tension given by eq. (2.21), the scalar matrix $\mathcal{M}$ given by (A.9) and the formal sum $\mathcal{C}$ defined in (2.14). The case $p = 1$ is special in the sense that in that case the worldvolume vectors can be integrated away, see subsection (3.2).

Note that all factors of $q_\alpha, \tilde{q}_\alpha$ in (2.24) are such that, if we assign a dimension $\Delta = 1, \Delta = -1$ to q_α and $\tilde{q}_\alpha$, respectively, all terms in a given p-brane action have dimension $\Delta = \frac{1}{2}(p-3)$.

3. Special cases

In this section we give explicit details for special cases and compare with the literature.

3.1 (-1)-branes

This case corresponds to the orbit of D-instantons and is special in the sense that we now work with Euclidean IIB supergravity. The SU(1,1)-invariant instanton action is given by

$$\mathcal{L}_{(\text{-1})\text{-brane}} = (qq\mathcal{M})^{-1} \;-\; \frac{q\tilde{q}\mathcal{M}}{qq\mathcal{M}}\,. \tag{3.1}$$

For $q_{\alpha'} = (0,-1)$ and $\tilde{q}_{\alpha'} = (1,0)$ we recover the standard D-instanton Lagrangian $\mathcal{L}_{\text{ D-instanton}} \sim e^{-\phi} + \ell$.

3.2 1-branes

The brane action for strings is [8]

$$\mathcal{L}_{\text{1-brane}} = (\tilde{q}\tilde{q}\mathcal{M})^{1/2}\sqrt{\det g_E} \;+\; \tilde{q}_\alpha \mathcal{C}^\alpha_{(2)}\,. \tag{3.2}$$

We use here a form of the Lagrangian where there are no Born-Infeld vectors. Unlike all other branes, we use $\tilde{q}_\alpha$ instead of q_α for the leading term in the WZ terms. This fits with the fact that the dimension of the Lagrangian for strings is given by $\Delta = -1$. The constants q_α are absent. Note that the construction of a $p = 1$ gauge-invariant WZ term does not require the introduction of a worldvolume vector, unlike the $p > 1$ branes. The $p = 1$ Lagrangian is equivalent to the one given in [9] if we identify $\tilde{q}_{\alpha'} = (p, q)$ with the two integration constants that follow from integrating out the two worldvolume vectors that occur in the formulation of [9].

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3.3 3-branes

The brane action for the $p = 3$ case is given by

$$\mathcal{L}_{\text{3-brane}} = \sqrt{\det\left(g_E + \frac{q\mathcal{F}}{(qq\mathcal{M})^{1/2}}\right)} \;+\; \mathcal{C}_{(4)} + \tfrac{3}{4}\tilde{q}_\alpha q_\beta \mathcal{C}^\alpha_{(2)}\mathcal{F}^\beta_{(2)} + \mathcal{C}_{(0)}(q\mathcal{F}_{(2)})^2\,, \tag{3.3}$$

with $\mathcal{C}_{(0)}$ defined in (2.18). This is precisely the action that one obtains by dimensional reduction of the PST action [11, 12] for a self-dual tensor in six dimensions [13 − 15]. It is interesting to apply an electric-magnetic duality transformation to the worldvolume vector $q_\alpha V^\alpha_{(1)}$ and to compare with [16]. We find that after an electric-magnetic duality transformation we end up with the same action but with the electric potential $q_\alpha V^\alpha_{(1)}$ replaced by a magnetic one, say $M_{(1)}$, and with everywhere else q_α replaced by $\tilde{q}_\alpha$. On the other hand, in our basis (A.7) the effect of an S-duality transformation is to replace q_α by $\tilde{q}_\alpha$ everywhere, including the term $q_\alpha V^\alpha_{(1)}$. Identifying

$$M_{(1)} = \tilde{q}_\alpha V^\alpha_{(1)} \tag{3.4}$$

we see that the two operations coincide, i.e., an S-duality acts on the worldvolume vector like an electric-magnetic (Hodge) duality transformation. This agrees with [16].

3.4 5-branes

The action for 5-branes is given by

$$\begin{aligned}\mathcal{L}_{\text{5-brane}} = {} & (qq\mathcal{M})^{1/2}\sqrt{\det\left(g_E + \frac{q\mathcal{F}}{(qq\mathcal{M})^{1/2}}\right)} \\ & + q_\alpha\left(\mathcal{C}^\alpha_{(6)} - 60\mathcal{C}_{(4)}\mathcal{F}^\alpha_{(2)} - \tfrac{45}{2}\tilde{q}_\beta q_\gamma \mathcal{C}^\beta_{(2)}\mathcal{F}^\gamma_{(2)}\mathcal{F}^\alpha_{(2)}\right) + \mathcal{C}_{(0)}(q\mathcal{F}_{(2)})^3\,.\end{aligned} \tag{3.5}$$

Gauge invariance of the WZ term implies that the 6-form transforms as

$$\delta_g \mathcal{C}^\alpha_{(6)} = 6\partial\Lambda^\alpha_{(5)} - 80 F^\alpha_{(3)}\Lambda_{(3)} \quad , \tag{3.6}$$

which is indeed like in eq. (2.13).

A different attempt to construct an SU(1, 1)-invariant 5-brane action was undertaken in [17]. Although the formula of [17] is not complete, it would be interesting to see whether there is any relation between the result of [17] and (3.5).

3.5 7-branes

The case of 7-branes is more subtle due to two reasons. First of all there are different conjugacy classes of 7-branes solutions which are distinguished by the value of $\det(q_{\alpha\beta})$. Secondly, to define 7-branes globally, one needs to consider other 7-branes at different positions in space.[9] The $\det(q_{\alpha\beta}) = 0$ conjugacy class contains the D7-brane. Ignoring

[9] For a careful discussion of the global properties and the role of the three different conjugacy classes, see [21].

global properties, i.e., restricting to the dynamics of small fluctuations, this class has the following brane action:

$$\begin{aligned}\mathcal{L}_{7\text{-brane}} &= qq\mathcal{M}\sqrt{\det\left(g_E + \frac{q\mathcal{F}}{(qq\mathcal{M})^{1/2}}\right)} \\ &+ q_\alpha q_\beta\left[\mathcal{C}^{\alpha\beta}_{(8)} - 7\mathcal{C}^{\alpha}_{(6)}\mathcal{F}^{\beta}_{(2)} + 210\mathcal{C}_{(4)}\mathcal{F}^{\alpha}_{(2)}\mathcal{F}^{\beta}_{(2)} + \tfrac{105}{2}\tilde{q}_\gamma q_\delta \mathcal{C}^{\gamma}_{(2)}\mathcal{F}^{\delta}_{(2)}\mathcal{F}^{\alpha}_{(2)}\mathcal{F}^{\beta}_{(2)}\right] \\ &+ \mathcal{C}_{(0)}(q\mathcal{F}_{(2)})^4\,.\end{aligned} \tag{3.7}$$

The gauge transformation of the 8-form is

$$\delta\mathcal{C}^{\alpha\beta}_{(8)} = 8\partial\Lambda^{\alpha\beta}_{(7)} - 14F^{(\alpha}_{(3)}\Lambda^{\beta)}_{(5)} \quad , \tag{3.8}$$

which again is of the form (2.13).

The above WZ-term has a 7-brane "charge" matrix $q_{\alpha\beta} = q_\alpha q_\beta$. The determinant of this matrix is, by construction, zero (the matrix has two linearly dependent columns). It is natural to ask whether brane actions for the other conjugacy classes, i.e., with $\det(q_{\alpha\beta}) \neq 0$ can also be constructed. Assuming such a charge matrix we write down the first few terms for the most general ansatz for a WZ-term:

$$WZ_{(8)} = q_{\alpha\beta}\left[A^{\alpha\beta}_{(8)} + a_1 A^{\alpha}_{(6)}A^{\beta}_{(2)} + a_2 A^{\alpha}_{(6)}F^{\beta}_{(2)} + \ldots\right] \tag{3.9}$$

where a_1, a_2 are to be determined. Demanding that there be only one vector field on the brane requires $a_2 = 0$. This can be seen by assuming $a_2 \neq 0$. Then the second column of the matrix $q_{\alpha\beta}$ must be zero, because otherwise we would introduce two gauge fields in the a_2-term. But the second column being trivial implies $\det(q_{\alpha\beta}) = 0$, in contradiction to our assumption, and so we must have $a_2 = 0$. We now take a look at the gauge transformation of (3.9). The terms of the type $\partial A^{\alpha}_{(6)}\Lambda^{\beta}_{(1)}$ and of the type $\partial A^{\alpha}_{(2)}\Lambda^{\beta}_{(5)}$, both of which are produced by the $A^{\alpha\beta}_{(8)}$ and the $A^{\alpha}_{(6)}A^{\beta}_{(2)}$-terms in our ansatz (remember that we already have eliminated the last term in (3.9), which would also have been a source of such terms), cannot be canceled at the same time for any choice of a_1. This shows that it is not possible to construct a brane action containing a single Born-Infeld vector, for any 7-brane with $\det(q_{\alpha\beta}) \neq 0$.

Of course, the above analysis does not exclude non-standard brane actions. For instance, recalling that the monodromy of a $\det(q_{\alpha\beta}) > 0$ brane can be obtained as the product of monodromies corresponding to two $\det(q_{\alpha\beta}) = 0$ branes one could view a $\det(q_{\alpha\beta}) > 0$ brane as a bound state of two $\det(q_{\alpha\beta}) = 0$ branes. This suggests that we might consider a (non-Abelian) brane action containing two vector fields.[10]

3.6 9-branes

Finally, we consider the case of 9-branes. The 9-branes related to the D9-brane (the

[10] We thank Jelle Hartong for a discussion of this possibility.

nonlinear doublet of 9-branes) have the following brane action

$$\begin{aligned}\mathcal{L}_{\text{9-brane}} = &(qq\mathcal{M})^{3/2}\sqrt{\det\left(g_E + \frac{q\mathcal{F}}{(qq\mathcal{M})^{1/2}}\right)}\\ &+ q_\alpha q_\beta q_\gamma\Big[\mathcal{C}^{\alpha\beta\gamma}_{(10)} + 15\mathcal{C}^{\alpha\beta}_{(8)}\mathcal{F}^{\gamma}_{(2)} - \tfrac{105}{2}\mathcal{C}^{\alpha}_{(6)}\mathcal{F}^{\beta}_{(2)}\mathcal{F}^{\gamma}_{(2)}\\ &+ 1050\mathcal{C}_{(4)}\mathcal{F}^{\alpha}_{(2)}\mathcal{F}^{\beta}_{(2)}\mathcal{F}^{\gamma}_{(2)} + \tfrac{1575}{8}\tilde{q}_\delta q_\epsilon \mathcal{C}^{\delta}_{(2)}\mathcal{F}^{\epsilon}_{(2)}\mathcal{F}^{\alpha}_{(2)}\mathcal{F}^{\beta}_{(2)}\mathcal{F}^{\gamma}_{(2)}\Big]\\ &+ \mathcal{C}_{(0)}(q\mathcal{F}_{(2)})^5\,. \end{aligned} \tag{3.10}$$

The WZ term is gauge invariant provided that

$$\delta\mathcal{C}^{\alpha\beta\gamma}_{(10)} = 10\partial\Lambda^{\alpha\beta\gamma}_{(9)} + 40F^{(\alpha}_{(3)}\Lambda^{\beta\gamma)}_{(7)} \quad , \tag{3.11}$$

which is of the form (2.13). Note that, unlike the case of 7-branes, the other conjugacy classes, not containing the D9-brane, are not supersymmetric [20].

4. Conclusion

In this paper we have presented an elegant $\mathrm{SU}(1,1)$-invariant expression for all p-brane actions of the IIB theory, see eq. (2.24). We only considered the bosonic terms in the action. It is natural to also consider the fermionic terms and require kappa-symmetry. This requires a $\mathrm{SU}(1,1)$-covariant superspace formulation of IIB supergravity.

Concerning the 7-branes, it would be interesting to perform a zero mode analysis on the 7-brane solutions for all conjugacy classes and from that point understand why only the 7-brane solution belonging to the $\det(q_{\alpha\beta}) = 0$ conjugacy class has a single vector field zero mode. Furthermore, one could then determine what the zero modes are, if any, for the other conjugacy classes.

As far as the 9-branes are concerned, it remains unclear what the interpretation is of the doublet of 10-form potentials. We have seen that a gauge-invariant WZ term does not contain a worldvolume vector field. One would therefore expect that also the kinetic term does not contain such a vector field. Nevertheless, if kappa-symmetry is going to work we expect to have 8 fermionic worldvolume degrees of freedom like all the other branes and they need to be matched by 8 bosonic degrees of freedom. In ten dimensions such bosonic degrees of freedom can only be described by a vector belonging to a vector multiplet.

Finally, to construct our central formula (2.24) it was crucial to perform the field redefinitions (2.7). These field redefinitions involve the vectors q_α and $\tilde{q}_\alpha$. It would be interesting to obtain a better understanding of these redefinitions and of the role of the $\mathrm{SU}(1,1)$-covariant $\mathcal{C}$-potentials in IIB string theory.

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A. Conventions

We raise and lower SU$(1,1)$ indices with the two-dimensional Levi-Civita tensor ϵ:

$$q^\alpha = \epsilon^{\alpha\beta} q_\beta \,, \qquad\qquad q_\beta = q^\alpha \epsilon_{\alpha\beta} \,. \tag{A.1}$$

A SU$(1,1)$-doublet q_α satisfies the following reality condition:

$$(q_1)^\star = q_2 \,. \tag{A.2}$$

Instead of using the SU$(1,1)$-basis, with complex components q_α, it is sometimes convenient to use the SL$(2,\mathbb{R})$-notation with real components $q_{\alpha'}$. The two bases are related via the following transformation:

$$q_{1'} = \tfrac{1}{\sqrt{2}}(q_1 + q_2) \,, \qquad\qquad q_{2'} = \tfrac{i}{\sqrt{2}}(q_1 - q_2) \,. \tag{A.3}$$

With these conventions we have that $\epsilon^{\alpha\beta} q_\alpha r_\beta = \epsilon^{\alpha'\beta'} q_{\alpha'} r_{\beta'}$ with $\epsilon^{12} = 1$ and $\epsilon^{1'2'} = i$. Note that under S-duality we have

$$q_1 \xrightarrow{S} -iq_1 \,, \qquad\qquad q_{1'} \xrightarrow{S} -q_{2'} \,, \tag{A.4}$$

$$q_2 \xrightarrow{S} +iq_2 \,, \qquad\qquad q_{2'} \xrightarrow{S} +q_{1'} \,. \tag{A.5}$$

In the text we have also defined a doublet $\tilde{q}_\alpha$ that satisfies the relation

$$\tilde{q}_{[\alpha} q_{\beta]} = \tfrac{i}{2} \epsilon_{\alpha\beta} \,. \tag{A.6}$$

We use an SL$(2,\mathbb{R})$-basis where the case of D-branes is recovered by making the choices:

$$\tilde{q}_{\alpha'} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \,, \qquad\qquad q_{\alpha'} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \,. \tag{A.7}$$

Then we have under S-duality

$$q_{\alpha'} \xrightarrow{S} \tilde{q}_{\alpha'} \,. \tag{A.8}$$

In our basis the 2×2 scalar matrix $\mathcal{M}$ is given by

$$\mathcal{M}^{\alpha'\beta'} = e^\phi \begin{pmatrix} \ell^2 + e^{-2\phi} & \ell \\ \ell & 1 \end{pmatrix} \,. \tag{A.9}$$

For the convenience of the reader we give the value of some general SU$(1,1)$-invariant expressions for the choices (A.7):

$$\tilde{q}_\alpha A^\alpha_{(2)} \to C_{(2)} \,, \qquad\qquad q_\alpha A^\alpha_{(2)} \to B_{(2)} \,, \tag{A.10}$$

$$\tilde{q}_\alpha A^\alpha_{(n)} \to B_{(n)} \,, \qquad\qquad q_\alpha A^\alpha_{(n)} \to C_{(n)} \,, \qquad n \neq 2 \,. \tag{A.11}$$

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B. The p-form gauge fields of IIB supergravity

For convenience, we provide the gauge transformations and field strengths for all IIB p-form gauge fields as they were determined in [18].

The p-form gauge fields of IIB supergravity are a singlet 4-form, a doublet of 2-forms, 6-forms and 10-forms, a triplet of 8-forms and a quadruplet of 10-forms. The gauge transformations of these gauge fields are:

$$\begin{aligned}
\delta A^{\alpha}_{\mu_1\mu_2} &= 2\partial_{[\mu_1}\Lambda^{\alpha}_{\mu_2]} \quad , && \text{(B.1)}\\
\delta A_{\mu_1\ldots\mu_4} &= 4\partial_{[\mu_1}\Lambda_{\mu_2\mu_3\mu_4]} - \tfrac{i}{4}\epsilon_{\gamma\delta}\Lambda^{\gamma}_{[\mu_1}F^{\delta}_{\mu_2\mu_3\mu_4]} \quad , && \text{(B.2)}\\
\delta A^{\alpha}_{\mu_1\ldots\mu_6} &= 6\partial_{[\mu_1}\Lambda^{\alpha}_{\mu_2\ldots\mu_6]} - 8\Lambda^{\alpha}_{[\mu_1}F_{\mu_2\ldots\mu_6]} - \tfrac{160}{3}F^{\alpha}_{[\mu_1\mu_2\mu_3}\Lambda_{\mu_4\mu_5\mu_6]} \quad , && \text{(B.3)}\\
\delta A^{\alpha\beta}_{\mu_1\ldots\mu_8} &= 8\partial_{[\mu_1}\Lambda^{\alpha\beta}_{\mu_2\ldots\mu_8]} + \tfrac{1}{2}F^{(\alpha}_{[\mu_1\ldots\mu_7}\Lambda^{\beta)}_{\mu_8]} - \tfrac{21}{2}F^{(\alpha}_{[\mu_1\mu_2\mu_3}\Lambda^{\beta)}_{\mu_4\ldots\mu_8]} \quad , && \text{(B.4)}\\
\delta A^{\alpha}_{\mu_1\ldots\mu_{10}} &= 10\partial_{[\mu_1}\Lambda^{\alpha}_{\mu_2\ldots\mu_{10}]} \quad , && \text{(B.5)}\\
\delta A^{\alpha\beta\gamma}_{\mu_1\ldots\mu_{10}} &= 10\partial_{[\mu_1}\Lambda^{\alpha\beta\gamma}_{\mu_2\ldots\mu_{10}]} - \tfrac{2}{3}F^{(\alpha\beta}_{[\mu_1\ldots\mu_9}\Lambda^{\gamma)}_{\mu_{10}]} + 32F^{(\alpha}_{[\mu_1\mu_2\mu_3}\Lambda^{\beta\gamma)}_{\mu_4\ldots\mu_{10}]} \quad . && \text{(B.6)}
\end{aligned}$$

The expressions for the corresponding field strengths are given by:

$$\begin{aligned}
F^{\alpha}_{\mu_1\mu_2\mu_3} &= 3\partial_{[\mu_1}A^{\alpha}_{\mu_2\mu_3]} \quad , && \text{(B.7)}\\
F_{\mu_1\ldots\mu_5} &= 5\partial_{[\mu_1}A_{\mu_2\ldots\mu_5]} + \tfrac{5i}{8}\epsilon_{\alpha\beta}A^{\alpha}_{[\mu_1\mu_2}F^{\beta}_{\mu_3\mu_4\mu_5]} \quad , && \text{(B.8)}\\
F^{\alpha}_{\mu_1\ldots\mu_7} &= 7\partial_{[\mu_1}A^{\alpha}_{\mu_2\ldots\mu_7]} + 28A^{\alpha}_{[\mu_1\mu_2}F_{\mu_3\ldots\mu_7]} - \tfrac{280}{3}F^{\alpha}_{[\mu_1\mu_2\mu_3}A_{\mu_4\ldots\mu_7]} \quad , && \text{(B.9)}\\
F^{\alpha\beta}_{\mu_1\ldots\mu_9} &= 9\partial_{[\mu_1}A^{\alpha\beta}_{\mu_2\ldots\mu_9]} + \tfrac{9}{4}F^{(\alpha}_{[\mu_1\ldots\mu_7}A^{\beta)}_{\mu_8\mu_9]} - \tfrac{63}{4}F^{(\alpha}_{[\mu_1\mu_2\mu_3}A^{\beta)}_{\mu_4\ldots\mu_9]} \quad , && \text{(B.10)}\\
F^{\alpha}_{\mu_1\ldots\mu_{11}} &= 11\partial_{[\mu_1}A^{\alpha}_{\mu_2\ldots\mu_{11}]} = 0 \quad , && \text{(B.11)}\\
F^{\alpha\beta\gamma}_{\mu_1\ldots\mu_{11}} &= 11\Big(\partial_{[\mu_1}A^{\alpha\beta\gamma}_{\mu_2\ldots\mu_{11}]} - \tfrac{1}{3}F^{(\alpha\beta}_{[\mu_1\ldots\mu_9}A^{\gamma)}_{\mu_{10}\mu_{11}]} + 4F^{(\alpha}_{[\mu_1\mu_2\mu_3}A^{\beta\gamma)}_{\mu_4\ldots\mu_{11}]}\Big) = 0 \quad . && \text{(B.12)}
\end{aligned}$$

Note that all curvature terms that occur at the right-hand-side of the above equations (both the gauge transformations and the expressions for the curvatures) are related to the doublet of 2-form gauge fields, i.e. they are proportional to Λ^{α}_{μ}, $A^{\alpha}_{\mu\nu}$ or $F^{\alpha}_{\mu\nu\rho}$.

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