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Electromagnetic off-shell effects in proton-proton bremsstrahlung

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Abstract

We study the influence of the off-shell structure of the nucleon electromagnetic vertex on proton-proton bremsstrahlung observables. Realistic choices for the off-shell behavior are found to have considerable influences on observables such as cross sections and analyzing powers. The rescattering contribution diminishes the effects of off-shell modifications in negative-energy states. © 1998 Elsevier Science B.V.

1. Introduction

The on-shell nucleon-nucleon T-matrix can be determined rather accurately in elastic scattering experiments, but in calculations of nuclear properties a knowledge of the off-shell interaction is required. One of the simplest processes that can be used to extract from experiment information on these off-shell effects is proton-proton bremsstrahlung (ppγ). The influence of off-shell effects in the interaction on ppγ observables is intimately related to off-shell effects in the electromagnetic nucleon vertex as investigated in this work. These off-shell effects enter in addition to corrections due to meson-exchange currents, isobar components and relativity which have been considered elsewhere [1–3]. Since off-shell effects cannot be measured directly, their interpretation is not unambiguous. In the concluding remarks this point is addressed.

The problem of off-shell form factors in the γ*NN* vertex has been faced in different ways. In one approach [4,11–13,5] one uses dispersion integrals to relate the imaginary part of the vertex, arising from inelastic processes, to the real part which now obtains an explicit dependence on the invariant mass of the nucleon. An alternative approach [6–9] is to dress the vertex explicitly with one-loop corrections. Also from chiral-perturbation theory estimates can be obtained on the dependence of the off-shell form factor near the on-shell point [10]. The different approaches give considerable differences in their predictions. For this reason we propose a parametrization of the off-shell form factor based on a low-energy behavior.

We investigate effects of off-shell form factors on observables in proton-proton bremsstrahlung within the fully relativistic framework developed in Ref. [2] using the nucleon-nucleon T-matrix of Fleischer and

Tjon [14]. The effect of the form factor on observables is discussed in terms of the coefficients of our parametrization and it is found to be large for realistic parameters.

2. The electromagnetic off-shell form factors

The most general structure of the nucleon-nucleon electromagnetic vertex for the real photon reads [4] [1]:

$$
\Gamma_\mu(W,W') = \sum_{rs=+,-} \Lambda^r(W) \left(e\gamma_\mu F_1^{rs}(W,W') \right.
$$
$$
+ i\kappa \frac{\sigma_{\mu\nu} q^\nu}{2m} F_2^{rs}(W,W')
$$
$$
\left. + q^\mu F_3^{rs}(W,W') \right) \Lambda^s(W'), \quad (1)
$$

or equivalently

$$
\Gamma_\mu(W,W') = \sum_{kl=0,1} \not{p}^k \left(e\gamma_\mu A_1^{kl}(W^2,W'^2) \right.
$$
$$
+ i\kappa \frac{\sigma_{\mu\nu} q^\nu}{2m} A_2^{kl}(W^2,W'^2)
$$
$$
\left. + q^\mu A_3^{kl}(W^2,W'^2) \right) \not{p}'^l, \quad (2)
$$

where $e(m)$ is the proton charge (mass) and κ is the anomalous magnetic moment. In the formulas above the initial (final) proton has momentum $p'(p), q = p' - p$ is photon momentum, $W = \sqrt{p^2}, W' = \sqrt{p'^2}$. The operators $\Lambda^\pm(W) = (\pm\not{p} + W)/(2W)$ project on positive- and negative-energy states of the off-shell proton with invariant mass W. The functions $F_i^{rs}(W,W')$ or $A_i^{kl}(W^2,W'^2)$ are the electromagnetic off-shell form factors of the nucleon. In the present discussion we present the results for the general fully off-shell form factors since these are required in the calculation of the rescattering diagrams.

Time reversal requires symmetries among the form factors, $F_{1,2}^{rs}(W,W') = F_{1,2}^{sr}(W',W), F_3^{rs}(W,W') = -F_3^{sr}(W',W)$. From the equivalence of Eq. (1) and Eq. (7) one deduces immediately that

$$
F_i^{rs} = A_i^{00} + rW A_i^{10} + s W' A_i^{01} + rs WW' A_i^{11},
$$

with $\quad r,s = +,-,$ \hfill (3)

where A_i^{kl} are functions of W^2 and $(W')^2$. F_i^{rs} thus obey the following symmetry relation

$$
F_i^{rs}(W,W') = F_i^{++}(rW,sW'). \quad (4)
$$

Therefore, it suffices to know F_i^{++} only.

For calculations of physical processes with a real photon, $F_3^{rs}(W,W') = 0$, due to transversality of the photon. In addition, for the half-off-shell vertex ($W = m$ or $W' = m$), $F_1^{rs}(W,W') = 1$, as a consequence of the Ward-Takahashi identity [13]. We will adopt this value as an approximation for $F_1^{rs}(W,W')$ also for the full-off-shell vertex. Therefore, from now on we will focus on the magnetic form factors $F_2^{rs}(W,W')$ and call them $F^{rs}(W,W')$ for brevity.

In our bremsstrahlung calculations we are interested in photon energies up to 150 MeV. Although this region extends beyond the applicability of low-energy theorems, one may still use a Taylor series expansion to determine the most important aspects of off-shell form factors. From a Taylor expansion of the bremsstrahlung matrix element around the on-shell point ($W = W' = m$) one finds that the leading order contributions are determined by two parameters: the slope

$$
k = \frac{\partial F^{++}(W,W')}{\partial(W/m)}\Bigg|_{W=W'=m}
$$
$$
= \frac{\partial F^{++}(W,W')}{\partial(W'/m)}\Bigg|_{W=W'=m}, \quad (5)
$$

and the value of $F^{+-}(W,W')$ and $F^{-+}(W,W')$ at the on-shell point,

$$
\kappa^- = F^{+-}(m,m) = F^{-+}(m,m). \quad (6)
$$

Henceforth, we will use the values for k and κ^- to compare the different models for off-shell form factors. In all our calculations the experimental value for the anomalous magnetic moment of the proton will be used, $\kappa = 1.79$. We will also impose that Eq. (1) reproduces the standard electromagnetic vertex with all the particles on-shell and thus $F^{++}(m,m) = 1$.

One approach to calculate off-shell form factors is through the application of dispersion relations [11–13]. The real part of a form factor is related to the

---

[1] We follow the notations of [15] for the γ-matrices and $\sigma_{\mu\nu}$

integral of the imaginary part which in turn is related to a total cross section by unitarity. This should give a model independent estimate of the form factor since the total cross section can in principle be measured. However, for the evaluation of the integral the cross section at very high energies is required, which is unknown. Related to this, the convergence of the dispersion integral is unclear and several subtractions may be necessary. These unknowns are reflected in a rather large range of values deduced for k and κ. Threshold dominance assumption was adopted in Refs. [11,13] to compute imaginary parts of the form factors. It led to the values $\kappa^- = 24$ and $k = 2.2$ in [11]. In Ref. [13] instead $\kappa^- = 14$ was obtained. The value of k is not given in this reference, but seems considerably larger than the value quoted in [11]. If more subtractions are needed to make the dispersion integrals convergent, these values are not even predicted anymore [12].

A self-consistent model for the form factors near the on-shell point can be obtained from next-to-leading-order Chiral Perturbation Theory. The isovector part of k was calculated [10] to be equal to 2, with the isoscalar part being negligible in the chiral limit. This approach does not predict κ^-.

Near the on-shell point one expects that off-shell corrections to the vertex are calculable in a one-loop model. Naus and Koch did calculations with a dominant contribution from the one-pion loop [6], where the pseudoscalar pion-nucleon coupling was used. Values $k = -0.38$ and $\kappa^- = 6.88$ are inferred from this model. Another qualitative model was developed in Ref. [9]. There, the loop comprises nucleon and a "dressing" scalar meson of mass 0.8 GeV. This model predicts values $k = -0.24$ and $\kappa^- = 1.53$. A more realistic approach including the vector-meson dominance model was presented by Tiemeijer and Tjon [7]. It leads to $k = -0.23$ for the pseudoscalar coupling and $k = -0.32$ for the pseudovector one. (The value of κ^- is not presented in Ref. [7].) In an analogous framework [8] also the Δ-isobar degree of freedom was included in the one-pion loop. This model gives rise to values of approximately zero for both k and κ^-.

The main conclusion we draw from this short overview of the results on the form factors is that a consensus is lacking. For this reason we have adopted an ad hoc parametrization of the form factors, based

on the low energy expansion, with k and κ^- as parameters. As was shown above, it is enough to parametrize the form factor $F^{++}(W,W')$ only,

$$F^{++}(W,W')$$
$$= 1 + \left(\frac{W - m}{m} \frac{a_1 W + a_2 W'}{m} \right.$$
$$\left. + \frac{W' - m}{m} \frac{a_1 W' + a_2 W}{m} \right)$$
$$\times \exp\left(- \frac{\left(|W|^2 - m^2\right)^2 + \left(|W'|^2 - m^2\right)^2}{2 \, d m^4} \right).$$
$$(7)$$

The constants a_1 and a_2 are easily expressed through the parameters k and κ^- discussed above

$$k = a_1 + a_2, \quad \kappa^- = 1 + 2(a_1 - a_2).$$

The exponentially decreasing factor was included to ensure that the form factors do not grow like W^2 at infinity. At the value of the constant d used in the calculations, $d = 0.5$, this exponent is of a minor importance for the observables (see discussion of the results below).

Since the values of k and κ^- differ considerably for the various approaches to the form factors, we performed calculations for the following two different choices of the parameters (note that $k = 0$, $\kappa^- = 1$ and $1/d = 0$ corresponds to ignoring all off-shell dependence);

F^- $k = 0$ $\kappa^- = 0$. For this choice of parameters the role of the negative energy states is emphasized since near the on-shell point F^{++} is not affected but F^{+-} and F^{-+} are. This parametrization will be referred to as "F^-" and is close to that following from the one-loop model of Ref. [8].

∂F^+ $k = 2$ $\kappa^- = 1$. For this case the anomalous magnetic moment for negative- and positive-energy states is the same, but with a linear dependence on the invariant mass near the on-shell point. This parametrization is thus referred to as "∂F^+". The value $k = 2$ is taken from chiral perturbation theory [10]. Since this approach does not predict κ^-, we put $\kappa^- = 1 = F^{++}(m,m)$ in order to reduce the number of independent parameters and to make it complementary to the previous choice.

3. Results and discussion

In the calculations we used the approach developed in Ref. [2], based on a fully relativistic t-matrix of Ref. [14] in which a complete account is given of relativistic effects. The rescattering contribution was included in the calculations, which was shown to be crucial for a correct treatment of negative-energy states. Corrections due to meson-exchange currents and the isobar degree of freedom were not included. These are not important for investigating the influence of possible off-shell effects, but will be for a definitive comparison with the data.

Fig. 1 shows that the off-shell effects change the cross section up to $\approx$ 15% for the ∂F^+-case and $\approx$ 30% for the F^--case. The effect is even larger for

the analyzing power at a wide range of photon angles. Corrections are seen of the order of $\approx$ 30% for the ∂F^+- and $\approx$ 25% for the F^--calculations. We checked the effect of the parameter d as well. Increasing d has negligible effects on the observables: less than 2% for d varying from 0.5 to 1000. The latter value corresponds to the practical absence of the exponent in the parametrization of the form factors.

To investigate whether the off-shell effects found are due to the one-boson-exchange kernel or they originate from the full complexity of the t-matrix, we calculated the cross section with and without off-shell corrections in Born approximation using a one-boson-exchange interaction including π-, ρ-, σ-, ω-, η-, and δ-mesons, referred to as the OBE-model.
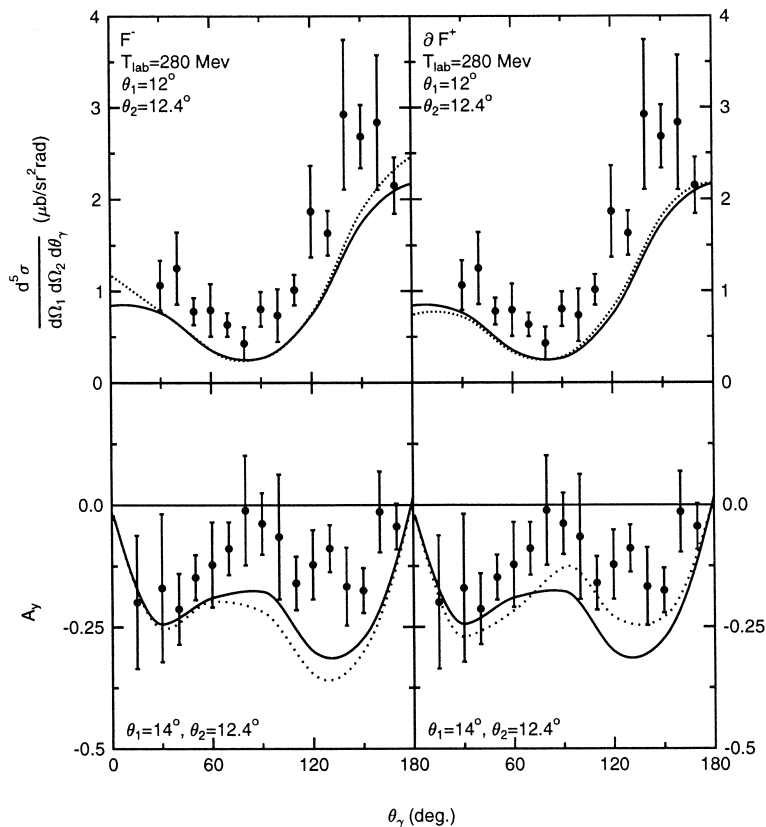

Fig. 1. Influence of the off-shell nucleon electromagnetic form factor on the observables as function of the photon angle. T_{lab} is the incoming proton energy in the laboratory frame, θ_1 and θ_2 are the outgoing proton angles. The two upper panels show cross sections; the two lower ones show the analyzing powers. The solid (dotted) lines correspond to the calculations without (with) including off-shell effects. The two left panels correspond to the F^--case of parametrization; the two right ones to the ∂F^+-case. The data are taken from the TRIUMF experiment [16].

The coupling constants were taken the same as in the kernel of the t-matrix. The result for the cross section is shown in Fig. 2 (the analyzing power is zero in the OBE model). It is seen that the effects are of the same order of magnitude as in the full calculations shown in Fig. 1. In detail there are however large differences, while in the full calculations the effects of κ^- are large this is not the case in the OBE-model. It should be noted that in the OBE-model rescattering diagrams should not be included to satisfy current conservation.

To obtain a better understanding of these off-shell effects we have investigated them in a low-energy approach. The low-energy theorem for the bremsstrahlung processes [17] states that in the expansion of the cross section in the small energy q of the radiated photon,

$$\sigma = \frac{\sigma_0}{q} + \sigma_1 + q\sigma_2 + q^2\sigma_3 + \cdots , \qquad (8)$$

the terms σ_0 and σ_1 are uniquely determined by on-shell quantities such as the nucleon-nucleon phase shifts. It means that any off-shell effects may contribute only to the coefficients σ_2, σ_3, and terms of higher order in q. Since the derivation of the low-energy theorem relies primarily on current conservation it applies to both the full and the schematic OBE-model calculations. Within the one-boson-exchange model, we investigated analytically the q-dependence of the difference $\Delta\sigma$ between cross sections

with and without the off-shell corrections. In a low energy expansion of $\Delta\sigma$, we were retaining only leading orders in q and p_{ext}/m, p_{ext} being the general notation for the initial and final proton three momenta. The leading off-shell contribution was found to be proportional to q for both the ∂F^+ and F^- cases, i.e. $\Delta\sigma = q\sigma_2 + q^2\sigma_3 + \ldots$, in accordance with the low-energy theorem. Let $\Delta\sigma(meson)$ denote a part of $\Delta\sigma$ that comes from the exchange of a particular *meson*. We found that for every meson, $\Delta\sigma(meson)$ is proportional to q with coefficients strongly dependent on the structure of the meson-nucleon coupling. More specifically, for the ∂F^+-case of parametrization, the ratio $\Delta\sigma(\rho)/\Delta\sigma(\omega) \approx \Delta\sigma(\rho)/\Delta\sigma(\pi) \approx 2$. For the F^--case, the ω-meson dominates, with $\Delta\sigma(\omega)/\Delta\sigma(\rho) \approx \Delta\sigma(\omega)/\Delta\sigma(\pi) \geq 10$, depending on photon angle. At the same time $\Delta\sigma(\omega)$ is of the same order of magnitude for both the ∂F^+- and the F^--cases. Since the contributions of the different mesons add up constructively for the ∂F^+-case, after summing up all the mesons, $\Delta\sigma$ in the ∂F^+-case is much larger than $\Delta\sigma$ in the F^--case as also seen in Fig. 2.

The large differences in the effects of the various mesons-exchange contributions can be traced back to the structure of the nucleon vertex. For example, both the ω and the ρ are vector particles but the ω couples to the nucleon via a rather pure vector-coupling (γ_μ) while for the ρ-meson there is a strong tensor component ($\sigma_{\mu\nu}q^\nu$). It can be shown analyti-
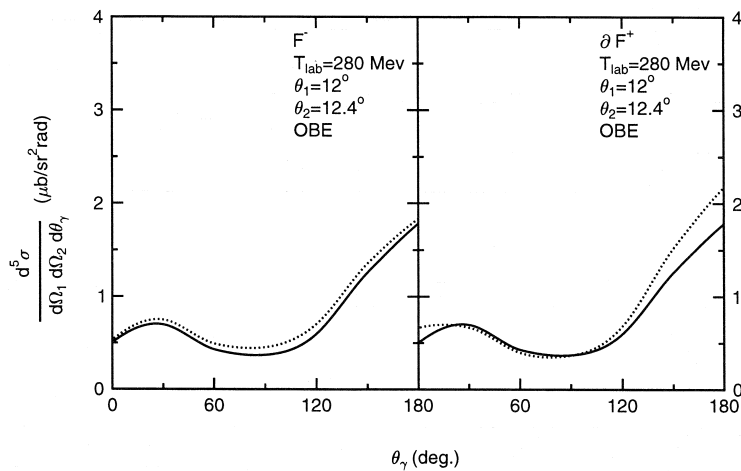

Fig. 2. The same as the upper panels of Fig. 1, but calculated in Born approximation using a one-boson-exchange interaction.

cally that for a pure vector coupling the two choices for the off-shell form factor considered have identical effects on the cross section. Adding a tensor component breaks this equivalence of the ∂F^+- and F^--parametrizations already for the vector mesons.

For a proper description of the off-shell effects in the electromagnetic vertex it is necessary to include the rescattering contributions. This is demonstrated in Fig. 3, where the dependence is shown of $\Delta\sigma$ on the photon energy for the calculations based on the full t - matrix. These calculations were done at $T_{lab} = 280$ MeV and photon angle $\theta_\gamma = 5°$. The photon energy was varied by changing the (equal) angles of the outgoing protons from 33° to 43°. Comparing the full calculations for the F^- and ∂F^+ cases, we see that at low photon energies the off-shell effect coming form the negative-energy part of the vertex is much smaller than that from the positive-energy part, which is consistent with our findings in the one-boson-exchange model and appears to be due to the smallness of the ω-exchange contribution for the full result. Even though in the present calculations rescattering gives a similar suppression of the effect in negative-energy states as that of Ref. [2], the situation is qualitatively different here. In Ref. [2] the effect of negative-energy states in the convection current was studied, where current conservation was shown to put strong constraints on the leading order effect. In our case the magnetic part of the vertex is addressed, where arguments based on current conservation do not apply. In spite of this, still the rescattering contribution is responsible for a strong cancellation of the effect of negative-energy states. At the same time, the contributions to $\Delta\sigma$ from negative- and positive-energies are comparable with each other when the photon energy becomes larger than 100 MeV/c, which corresponds to the kinematics in Figs. 1 and 2. It also follows from Fig. 3 that no contribution of the off-shellness shows up either in the constant σ_1 or in the singular term σ_0/q in Eq. (8), which implies that the calculations fulfill the low-energy theorem.

We also did similar calculations for $T_{lab} = 500$ MeV (not shown here). We found that the off-shell effects on the observables are larger, but only by at most a factor 1.5, compared to the calculations at $T_{lab} = 280$ MeV.
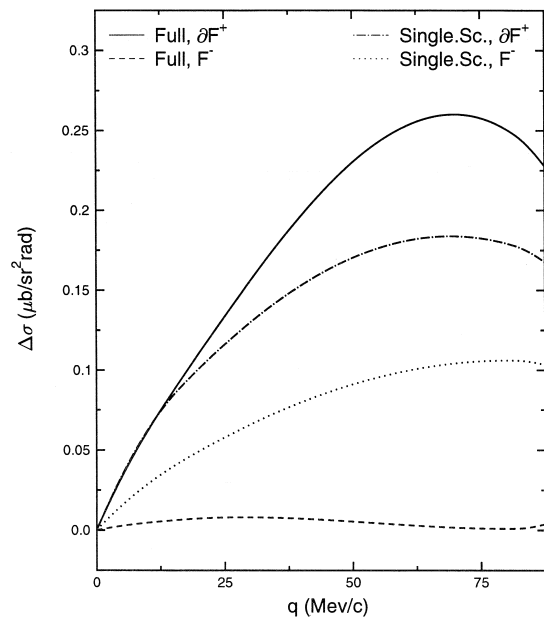
In the present investigation we focussed attention

Fig. 3. The dependence on the photon energy of $\Delta\sigma$, the difference between cross sections with and without the off-shell corrections to the γNN-vertex. All plots are obtained in the calculations with the full t-matrix. The dash-dotted (dotted) line corresponds to the calculation where only the single scattering diagrams were taken into account, for the case of the ∂F^+ (F^-)-parametrization. The solid (dashed) line is the calculation including the rescattering contribution, for the ∂F^+ (F^-) case.

to the off-shell structure in the nucleon-photon vertex. The off-shell structure of a vertex can however not be linked directly to observables. As was shown in [5,18] it is possible, by field transformations, to transform off-shell effects in a three-point vertex into higher-point vertices (contact terms). For the present case this implies that in observables one is sensitive to a product of the off-shell structure of the T-matrix and that of the photon vertex. One could obtain the same predictions with some sort of (new) meson-photon-N-N four-point vertex. In this paper we have shown that observables in proton-proton bremsstrahlung are sensitive to the off-shell structure, an effect which has not been considered before.

In conclusion, we investigated the effects of the off-shell electromagnetic structure of the nucleon on observables in proton-proton bremsstrahlung. We used a parametrization of the nucleon electromagnetic off-shell form factors based on its general properties and with values consistent with model

calculations. We found a sizable influence of these off-shell effects on the observables, in particular on the analyzing power.

Acknowledgements

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