Effect of the unpolarized spin state in spin-correlation measurement of two protons produced in the 12C(d,2He) reaction

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Effect of the unpolarized spin state in spin-correlation measurement of two protons produced in the ^{12}C(d,^{2}He)reaction

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Abstract

In this note we discuss the effect of the unpolarized state in the spin-correlation measurement of the $^1S_0$ two-proton state produced in ^{12}C(d,^{2}He) reaction at the KVI, Groningen. We show that in the presence of the unpolarized state the *maximal* violation of the CHSH-Bell inequality is lower than the classical limit if the purity of the state is less than $\sim$ 70%. In particular, for the KVI experiment the violation of the CHSH-Bell inequality should be corrected by a factor $\sim$ 10% from the pure $^1S_0$ state.

1 Introduction

In an experiment performed at the Kernfysisch Versneller Instituut (KVI), Groningen [5] with the goal to test Bell inequality violation in Nuclear Physics (perhaps to be applied in quantum information physics), the experimental group, by bombarding a ^{12}C target with 170 MeV d, was able to generate a singlet-spin, two-proton state coupled to unpolarized state with $\sim 10\%$ contribution. In this paper we will analyze the experimental results of this experiment and we will show that the effect of the unpolarized state is important and could not be neglected.

2 CHSH inequalities and entanglement in a mixed ensemble

Bell-type inequalities relating averages of four random dichotomic variables a, a' and b, b' representing measurements of spin in directions $\hat{a}$, $\hat{a'}$ and $\hat{b}$, $\hat{b}'$. The Clauser, Horne, Shimony and Holth (CHSH) [3] form of Bell-type inequalities for spin 1/2 case could be written in this form

$$|E(\phi_1, \phi_1^{'}, \phi_2, \phi_2^{'})| = |E(\phi_1, \phi_2) + E(\phi_1, \phi_2^{'}) + E(\phi_1^{'}, \phi_2) - E(\phi_1^{'}, \phi_2^{'})| \leq 2, \quad (1)$$

where ϕ_i is the analyzer angular setting for the i^{th} particles ($i = 1,\ 2$) and $E(\phi_i, \phi_j)$ is the correlation function defined as

$$E(\phi_i, \phi_j) = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{total}}\,. \quad (2)$$

In quantum-theory language the CHSH operator corresponding to the CHSH inequality is represented by an operator

$$\mathcal{B} = \hat{a} \cdot \sigma \otimes (\hat{b} + \hat{b}')\cdot\sigma + \hat{a'}\cdot\sigma \otimes (\hat{b} - \hat{b}')\cdot\sigma\,, \quad (3)$$

acting in Hilbert space $\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}$ in $2 \otimes 2$ dimension. The correlation function is given by the mean value of the operator $\hat{a}\sigma \otimes \hat{b}\sigma$. For a pure state this correlation function could be easily computed, *e.g.* for singlet state we have

$$E(\phi_i, \phi_j) = -cos(\phi_i - \phi_j)\,. \quad (4)$$

For mixed state, however, the mean value should be averaged over the ensemble and therefore the CHSH inequality not a sufficient condition to test the presence of entanglement[7]. Different measures of the entanglement have been proposed in the literature for mixed state[1], e.g. entanglement of formation, distillation, relative entropy of entanglement, negativity, etc... Here we will use the entanglement of formation as our measure of the entanglement.

[1]Any measurement of the entanglement should not increase by local operation (*e.g.* unitary transformation) and classical communication (*e.g.* phone calls.), known as LOCC

In a mixed ensemble any bipartite quantum state ρ_{AB} can be written as:

$$\rho_{AB} = \frac{1}{4}\left(I \otimes I + \mathbf{A} \cdot \sigma \otimes I + I \otimes \mathbf{P} \cdot \sigma + \sum_{i,j=1}^{3} D_{ij} \sigma_i \otimes \sigma_j \right) . \tag{5}$$

σ_i are the pauli matrices, I is the identity operator, $\mathbf{A}$ and $\mathbf{P}$ are vectors in $\mathcal{R}^3$. The D_{ij} form a 3×3 matrix called the correlation matrix D. In this representation of the density matrix the mean value of the CHSH-Bell operator is given by [4]

$$\langle \mathcal{B} \rangle = \hat{a} \cdot \left[D(\hat{b} + \hat{b}') \right] + \hat{a}' \cdot \left[D(\hat{b} - \hat{b}') \right] . \tag{6}$$

Using the representation of the density matrix given in Eq. (5), we characterize any bipartite quantum state ρ_{AB} by

- The entanglement measured by the "tangle", τ, of the entanglement of formation [6] and defined by

$$\tau = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\} , \tag{7}$$

  where the λ's are the square roots of the eigenvalues, in decreasing order, of the matrix, $\rho_{AB}(\sigma_y \otimes \sigma_y \rho^{\star}_{AB} \sigma_y \otimes \sigma_y)$ and $\rho^{\star}_{AB}$ is the complex conjugation of ρ_{AB} in the computational basis $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$.

- The maximum amount of the CHSH-Bell violation of the state ρ_{AB} [4]

$$\langle \mathcal{B} \rangle^{max} = 2\sqrt{M(\rho_{AB})} . \tag{8}$$

  $M(\rho_{AB})$ is the sum of the two larger eigenvalues of $DD^{\dagger}$ [2].

- The purity of the state that measures how far the state is from pure state

$$S_L = \mathrm{Tr}(\rho^2_{AB}) . \tag{9}$$

3 Analysis of the experimental data of the KVI experiment

The spin state of the two protons produced in the ^{12}C(d,^{2}He) reaction at $E_d =$ 170 MeV at KVI [5] is a singlet state mixed with the unpolarized (random contamination) state with γ ($0 \leq \gamma \leq 1$) controlling the degree of mixing (the details of the experimental setup and analysis method of the (d,^{2}He) reaction

[2] In this case the directions $\hat{b}$ and $\hat{b}'$ of the analyser setting are equal to $\cos(\theta)\hat{c}_{\max} \pm \sin(\theta)\hat{c}'_{\max}$ and the direction $\hat{a}$, $\hat{a}'$ are equal to $\frac{D\hat{c}_{\max}}{||D\hat{c}_{\max}||}$, $\frac{D\hat{c}'_{\max}}{||D\hat{c}'_{\max}||}$, respectively. $\hat{c}_{\max}$ and $\hat{c}'_{\max}$ are two unit (not unique) and mutually orthogonal vectors in $\mathcal{R}^3$ that maximize the function $||D\hat{c}||^2 + ||D\hat{c}'||^2$ (see Ref. [4] for more detail).

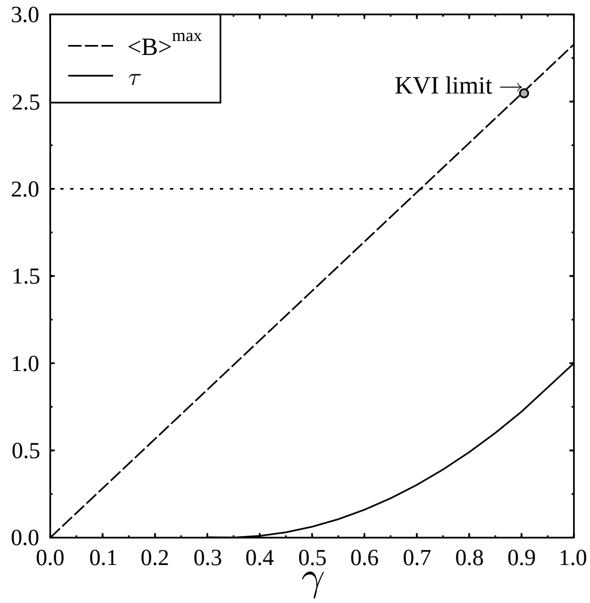

Figure 1: Plot of $\langle\mathcal{B}\rangle^{\max}_{Werner}$ (dashed line) and τ (solid line) versus the purity γ. The dotted line is the Bell limit, the circle is the KVI limit for $\gamma \sim 0.9$.

were described in detail in Ref. [5]). Given all that, we can write the density matrix of such state as

$$\rho_W = (1-\gamma)\frac{I}{4} + \gamma|\Psi^-\rangle\langle\Psi^-| \tag{10}$$

which interpolates between the unpolarized state $I/4$ and singlet state $|\Psi^-\rangle = (|+-\rangle - |-+\rangle)/\sqrt(2)$. This class of states is called Werner states [7]. The purity of Werner states is a monotonic function of γ. Thus, in this paper we use γ as our measure of purity. Also, for Werner state it is easy to prove using the condition noted above that

$$\langle\mathcal{B}\rangle^{max}_{Werner} = \gamma\langle\mathcal{B}\rangle^{max}_{pure}\,. \tag{11}$$

Note that, a violation of the modified Bell-inequality does not exclude an explanation with a hidden variable theory. In Fig. 1 we plot $\langle\mathcal{B}\rangle^{max}_{Werner}$ and the tangle τ versus the purity γ. As we can see in this figure the Werner state does not violate the Bell inequality if its purity γ is less than $1/\sqrt{2} \sim 70\%$. However, the entanglement is still non-zero in the Werner state until $\gamma > 1/3 \sim 33\%$. Therefore, some quantum correlation cannot be seen only

Table 1: Experimental data and quantum theory predictions for a pure singlet states (case 1) and mixed Werner states (case 2) for several violating cases of the CHSH-Bell inequality according to the definition given in Eq. 1.

CHSH-Bell Inequality	QM case 1	QM case 2	Exp. Data
$E(50°, 0°, 25°, 75°)$	2.46	2.21	0.67± 2.30
$E(60°, 0°, 30°, 90°)$	2.60	2.34	1.21± 2.42
$E(70°, 0°, 35°, 105°)$	2.72	2.45	1.54± 2.76
$E(80°, 0°, 40°, 120°)$	2.80	2.52	2.11± 2.86
$E(90°, 0°, 45°, 135°)$	2.83	2.55	2.23± 2.48
$E(100°, 0°, 50°, 150°)$	2.79	2.51	2.39± 2.87
$E(110°, 0°, 55°, 165°)$	2.69	2.34	2.58± 2.91
$E(120°, 0°, 60°, 180°)$	2.50	2.25	2.75± 2.95
χ^2	1.26	0.85	

by measuring the violation of the Bell-type inequality because some of them (Werner states) are entangled but still do not violate Bell inequality. Note that there is a possible experimental measurement of the entanglement based on the entanglement witness [8] that we think to implement in the future experiment.

In Tab. 1 we compare the quantum theory predictions assuming a pure singlet state (case 1) and mixed Werner states (case 2) for the spin of the two detected protons for several violating cases of the CHSH-Bell inequality. The value of $\chi^2 = \sum_i (\frac{R^i_{th} - R^i_{exp}}{\Delta R^i_{exp}})^2$ is given in the bottom of the table for both cases. We have found that $\chi^2_{Werner} < \chi^2_{Singlet}$ as expected. However, we cannot judge this result as evidence of the mixing of the singlet with the unpolarized state because the experimental data suffer from large errors.

4 Conclusion

In this paper we have discussed the effect of the unpolarized state in the spin correlations measurement of the $^1S_0$ two proton state produced in ^{12}C(d,^{2}He) reaction at KVI. We have shown that even a small coupling (less than 10%) of the pure singlet state with the unpolarized state changes dramatically the Bell-violation value. After introducing the contribution of the unpolarized state we have found a better χ^2. The experimental results are suffering from a large statistical error and therefore not conclusive for testing Bell's inequality, but with a modified experimental setup, measurements with significantly improved precision will become feasible.

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