A valve stiction tolerant formulation of MPC for industrial processes

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Abstract: This paper presents three different formulations of MPC to face static friction in control valves for industrial processes. A pure linear formulation, a stiction embedding structure, and a stiction inversion controller are designed. The controllers are derived for SISO systems with linear process dynamics, where valve stiction is the only nonlinearity present in the control loop. A novel smoothed stiction model is introduced to improve and fasten the dynamic optimization module of stiction embedding MPC. A stiction compensation method is revised and used as a warm-start to build a suitable trajectory for the predictive controller. The different MPC formulations are tested and compared on some simulation examples.

Keywords: Process control; model predictive control; control valve; stiction; stiction compensation

1. INTRODUCTION

Control valves are the most commonly used actuators in the process industries. Unfortunately, in many cases valves not only contain static nonlinearity (e.g. saturation), but also dynamic nonlinearity including backlash, friction, and hysteresis. Deadband due to backlash and mostly static friction (stiction) is a root source of the valve problems. As a consequence, these malfunctions would produce a sustained oscillation in the process variables, decrease the life of control valves, and generally, lead to inferior quality end products by causing reduced profitability (Jelali and Huang, 2010). Therefore, it seems that the potential benefit of advanced control algorithms, as model predictive control (MPC), could be reduced because of poor valves, if their faults and malfunctions are not expressly considered.

MPC has in fact been also used as a compensation strategy for several types of valve malfunctions. The first MPC-based formulation was developed by Zabiri and Samyudia (2006), by using a mixed-integer quadratic programming (MIQP) on constraints of the input. An inverse backlash model and valve saturation are incorporated in the controller to overcome the deadband associated with backlash. Later, this structure is applied to a system with stiction in Zabiri and Samyudia (2009). In Rodríguez and Heath (2012) a formulation which reduces the bounds on optimization variables computed by the MPC, by trying to delete different types of valve nonlinearity, and by reducing the problem to a purely linear structure has been proposed. Recently, Durand and Christofides (2016) have presented an economic MPC structure which includes a detailed physical stiction model, constraints on the magnitude and rate of change of the input, and is combined with a slave controller of PI-type that regulates the valve output to its MPC set-point.

When stiction is present, the valve is not successful in following the input signals imposed by the controller. Consequently, a limit cycle is typically generated around the steady-state operating points. As suggested by previous works, one way of reducing stiction effects is to explicitly take this malfunction into account in MPC design so that an improved performance could be obtained. As many other fault tolerant approaches, an estimate of stiction amount is needed, and the sticky valve must be identified within the closed loop, especially when the system is multidimensional. For this purpose, well-established techniques of stiction detection and quantification could be used and adapted as necessary (Jelali and Huang, 2010).

This paper is focused on designing an MPC formulation that considers valve stiction explicitly, in order to compensate for its undesired effects on control systems. The controller will be derived for single-input single-output (SISO) systems with linear process dynamics, as the nonlinearity comes only from the valve. In order to improve the numerical optimization performance, a suitable smoothing of the discontinuous valve stiction model and an appropriate input sequence, derived from a stiction compensation method and used as warm-start for MPC, will be necessary. This novel methodology will be compared against standard and advanced MPC formulations using as test bench several simplified numerical examples.

The remainder of the paper is organized as follows. Different MPC approaches and valve stiction models are presented in Section 2. The proposed stiction-tolerant MPC formulation is detailed in Section 3. Some simulation examples as basis of comparison are then presented in Section 4. Finally, conclusions are drawn in Section 5.

2. PROBLEM DEFINITION

The whole plant is formed by the control valve followed by the process dynamics as depicted in Figure 1. In detail, χ is the process input, that is, the valve output; y is the process output; u is the MPC output, while w and v are two sequences of white Gaussian noise. For the sake of simplicity, the case of SISO system is studied: a nonlinearity for the valve followed by a linear dynamics for the process, thus forming a Hammerstein structure for the whole plant. Extensions to MIMO systems and nonlinear processes will be investigated in future research.

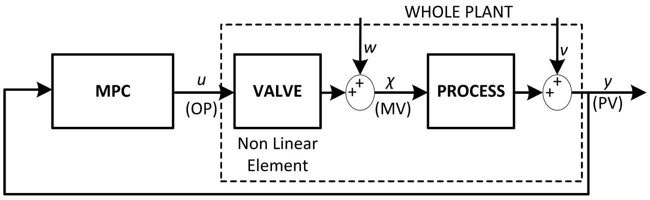

Fig. 1. The closed-loop system with the (sticky) control valve followed by the process.

Valve dynamics is described by a data-driven stiction model, while the linear process dynamics is expressed by a state-space model. The whole plant dynamics in standard state-space formulation can be written as:

$$\begin{aligned} z_{k+1} &= f(z_k, u_k) + w_k \\ y_k &= h(z_k) + v_k \end{aligned} \quad (1)$$

The valve output χ represents the first component of the state vector of whole plant $z_k = [\chi_{k-1}, \xi_k]^T$, so that:

$$z_{k+1} = \begin{bmatrix} \chi_k \\ \xi_{k+1} \end{bmatrix} = \begin{bmatrix} \varphi(\chi_{k-1}, u_k) \\ \mathbf{A}\xi_k + \mathbf{B}\varphi(\chi_{k-1}, u_k) + w_k \end{bmatrix} \quad (2)$$
$$y_k = \mathbf{C}\xi_k + v_k$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times m}$, $\mathbf{C} \in \mathbb{R}^{p\times n}$, n is the process model dimension, and $m = p = 1$, respectively. Note that the first component of state equation is given by the stiction nonlinearity, expressed by the discontinuous function $\varphi(\cdot)$: $\mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$, later discussed.

2.1 Possible MPC approaches

Three different approaches of MPC are presented and compared in this work. The first formulation is a stiction unaware controller, with a pure linear MPC formulation since it completely disregards the valve dynamics and uses only a linear process model for the whole plant (see Figure 2). Secondly, a stiction embedding MPC is considered, as shown in Figure 3. This controller is aware of the stiction presence, as it employs an extended model – both of valve and process dynamics – thus forming a nonlinear formulation (NMPC). Finally, the third approach is also aware of stiction, but it has an explicit model for the inverse dynamics of stiction ($\tilde{\varphi}^{-1}$), where $\tilde{u}$ is the MPC output, subject to optimization, which forms input to stiction inverse model, and $u = \tilde{\varphi}^{-1}(\tilde{u})$ is the output of the whole controller. Note that, in the case of perfect stiction inversion, one should get $\varphi(\tilde{\varphi}^{-1}(\tilde{u})) = \tilde{u}$, and then $\tilde{u} \equiv \chi$. This type of formulation, introduced by Rodríguez and Heath (2012), has the advantage of not only considering expressly stiction dynamics, but also reducing the controller to a linear structure, which is simply based on the process model, as in Figure 4.

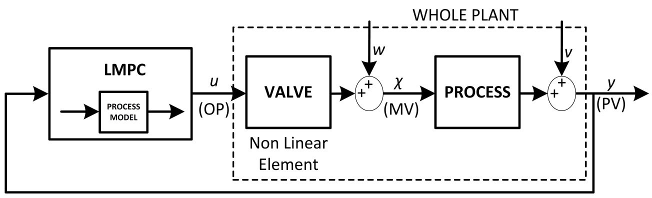

Fig. 2. Closed-loop system with stiction unaware MPC.

2.2 Valve stiction modeling

Stiction in pneumatic sliding stem control valves can be described both by detailed physical models and by empirical

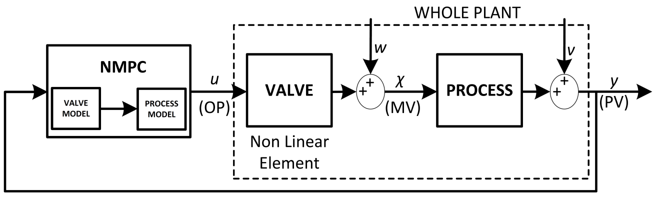

Fig. 3. Closed-loop system with stiction embedding MPC.

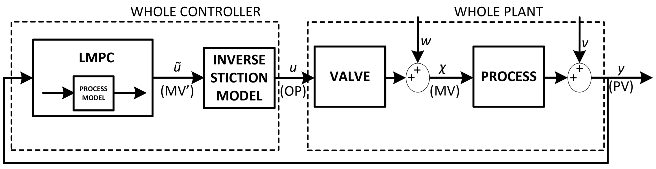

Fig. 4. Closed-loop system with stiction inversion MPC.

(data-driven) models. If fast response from the valve is assumed, the transient response can be ignored and a static – but with memory – nonlinear function can be used to approximate the valve's dynamic response, that is, only the stationary-state values of stem position are considered. Therefore, the standard empirical (He et al., 2007) or the semi-physical model (He and Wang, 2010) by He and coworkers are suitable to reproduce the valve response generated by physical stiction models without involving computationally intensive numerical integration.

In this paper, we choose to use the He's standard model (He et al., 2007), thus including stiction in every valve move. The sticky valve has a nonlinear dynamics $\chi_k = \varphi(\chi_{k-1}, u_k)$ expressed by the following two relations:

$$\chi_k = \begin{cases} \chi_{k-1} + [e_k - \mathrm{sign}(e_k) f_D] & \text{if} \quad |e_k| > f_S \\ \chi_{k-1} & \text{if} \quad |e_k| \le f_S \end{cases} \quad (3)$$

where f_S and f_D are static and dynamic friction parameters, respectively, and $e_k = u_k - \chi_{k-1}$. Note that e_k is a sort of valve position error, and $f_S \ge f_D$ by definition. By substituting e_k and then by separating the nonlinear *sign* function, three different input-output relations are possible:

$$\chi_k = \begin{cases} u_k - f_D & \text{if} \quad |u_k - \chi_{k-1}| > f_S, \quad u_k - \chi_{k-1} > 0 \\ u_k + f_D & \text{if} \quad |u_k - \chi_{k-1}| > f_S, \quad u_k - \chi_{k-1} < 0 \\ \chi_{k-1} & \text{if} \quad |u_k - \chi_{k-1}| \le f_S \end{cases} \quad (4)$$

Then, by solving the first two inequalities, one gets:

$$\chi_k = \begin{cases} u_k - f_D & \text{if} \quad u_k - \chi_{k-1} > f_S \\ u_k + f_D & \text{if} \quad u_k - \chi_{k-1} < -f_S \\ \chi_{k-1} & \text{if} \quad |u_k - \chi_{k-1}| \le f_S \end{cases} \quad (5)$$

Therefore, the stiction nonlinearity $\varphi(\cdot)$ is formed by a set of three, relatively simple, linear and parallel relations, thus constituting a sort of switching "multi-mode" model to be integrated along with the dynamics of the process, to form a *discontinuous* model. Note that the proposed methodology and formulations of MPC are valid also for other types of stiction models.

3. MPC DESIGN

In this section the considered formulations of MPC are detailed, by introducing an empirical stiction inverse model, a novel smoothed stiction model, some specific choices for modules and tuning parameters, and a suitable warm-start based on a stiction compensation method.

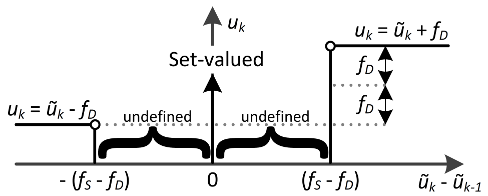

Fig. 5. Inverse function $\tilde{\varphi}^{-1}$ for He's stiction model.

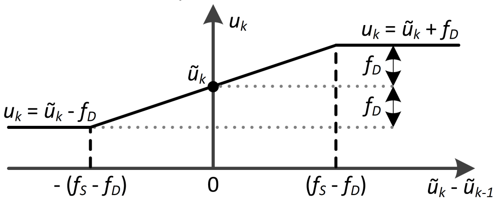

Fig. 6. Approximated inverse stiction function $\hat{\varphi}^{-1}$.

3.1 Stiction inverse model

The stiction inverse MPC formulation presented in Figure 4 requires to invert the stiction nonlinearity to obtain the control sequence, that is, $u = \tilde{\varphi}^{-1}(\tilde{u})$. Starting from He's model in (5), by assuming $\tilde{u} = \chi$ and knowing at each sampling $\tilde{u}_k$ and $\tilde{u}_{k-1}$, which compose inputs to stiction inverse model, one can write:

- if $\tilde{u}_k \neq \tilde{u}_{k-1}$ then:
  - $u_k = \tilde{u}_k + f_D$ if and only if $u_k - \tilde{u}_{k-1} > f_S$;
  - $u_k = \tilde{u}_k - f_D$ if and only if $u_k - \tilde{u}_{k-1} < -f_S$;
- if $\tilde{u}_k = \tilde{u}_{k-1}$ then $u_k \in [\tilde{u}_{k-1} - f_S, \tilde{u}_{k-1} + f_S]$

Then, by substituting u_k for the two inequalities, one gets:

$$u_k \begin{cases} = \tilde{u}_k + f_D & \text{if} \quad \tilde{u}_k - \tilde{u}_{k-1} > f_S - f_D \\ = \tilde{u}_k - f_D & \text{if} \quad \tilde{u}_k - \tilde{u}_{k-1} < f_D - f_S \\ \in [\tilde{u}_{k-1} - f_S, \tilde{u}_{k-1} + f_S] & \text{if} \quad \tilde{u}_k - \tilde{u}_{k-1} = 0 \\ \text{is undefined} & \text{otherwise} \end{cases} \tag{6}$$

where $f_S - f_D \geq 0$ and $f_D - f_S \leq 0$. Figure 5 shows a schematic representation of the function $\tilde{\varphi}^{-1}$. This stiction inverse model has an incomplete domain in $\mathbb{R}$, admits unique values for $\tilde{u}_k - \tilde{u}_{k-1} > f_S - f_D$ and for $\tilde{u}_k - \tilde{u}_{k-1} < f_D - f_S$, is multivalued for $\tilde{u}_k - \tilde{u}_{k-1} = 0$, while otherwise is not defined. Note that, by implementing this exact model of stiction inverse in the MPC formulation, one should theoretically impose the following non connected domain $\tilde{\mathbb{U}}$ for values of $\tilde{u}_k$:

$$\tilde{\mathbb{U}} = \{\tilde{u}_k : \;\; \tilde{u}_k > \tilde{u}_{k-1} + (f_S - f_D) \;\cup \\ \tilde{u}_k < \tilde{u}_{k-1} - (f_S - f_D) \;\cup\; \tilde{u}_k = \tilde{u}_{k-1}\} \tag{7}$$

which is not actually implementable, since it implies a non connected set of constraints on $\tilde{u}_k$, apart from the special case of pure dead-band, that is, $f_S = f_D$.

Therefore, an approximated inverse model ($\hat{\varphi}^{-1} \approx \tilde{\varphi}^{-1}$) is needed to get an implementable MPC. A possible simple solution is to turn the model into a continuous function with linear junctions, as the following:

$$u_k = \begin{cases} \tilde{u}_k + f_D & \text{if} \quad \tilde{u}_k - \tilde{u}_{k-1} > f_S - f_D \\ \tilde{u}_k - f_D & \text{if} \quad \tilde{u}_k - \tilde{u}_{k-1} < -(f_S - f_D) \\ \tilde{u}_k + \dfrac{f_D}{f_S - f_D}(\tilde{u}_k - \tilde{u}_{k-1}) & \text{if} \quad |\tilde{u}_k - \tilde{u}_{k-1}| \leq f_S - f_D \end{cases} \tag{8}$$

Figure 6 shows a schematic representation of this approximated stiction inverse. Note that for $f_S = f_D$ the third condition, that is, when $\tilde{u}_k - \tilde{u}_{k-1} = 0$, has to be reduced to $u_k = \tilde{u}_k$.

Extensive simulations have verified that approximated model (8) equals the exact one (6), that is, $\hat{\varphi}^{-1} \equiv \tilde{\varphi}^{-1}$, only when the difference $\tilde{u}_k - \tilde{u}_{k-1}$ is always within the domain of the exact

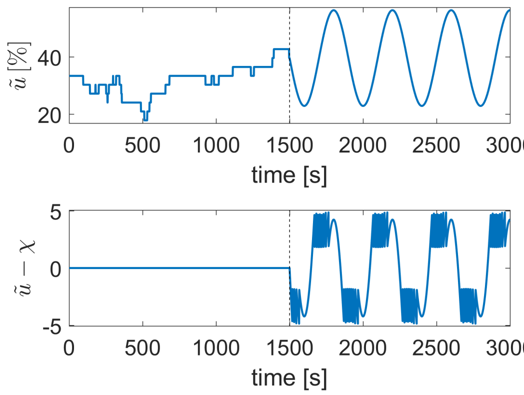

Fig. 7. Test signal: behavior of stiction inverse ($f_S = 5$, $f_D = 2$).

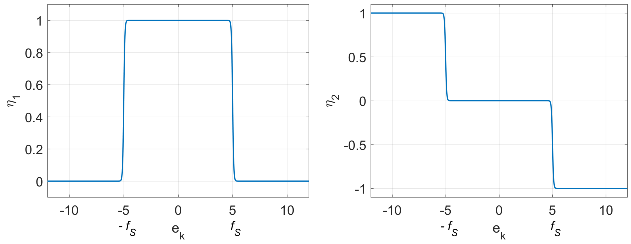

Fig. 8. Hyperbolic functions for the smoothed stiction model with $f_S = 5$ and $\tau = 10$.

inverse, and then one gets $\tilde{u} \equiv \chi$. Otherwise, this MPC formulation has a structural mismatch and its performance tends to degrade. Figure 7 shows the output $\tilde{u}_k$ of an ideal MPC. This signal respects constraints in (7) until 1500 s and thus allows a perfect stiction inversion; then, once the signal assumes the shape of a sine curve, the stiction inversion becomes incomplete and the process input differs from the MPC output.

3.2 A smoothed stiction model

It is worth noting that (5) are stiff equations, thus the optimization problem of MPC might be a difficult task, due to the presence of *if-else* statements which imply two hard discontinuities in the input-output relation of the valve. A different stiction model is thus used in the optimizer of the dynamic module of NMPC in order to get a smoother problem. Equations (5) are expressly approximated by using a single *smoothing* function:

$$\chi_k = \eta_1(e_k)\chi_{k-1} + (1 - \eta_1(e_k))u_k + \eta_2(e_k)f_D \tag{9}$$

where $\eta_1(e_k)$ and $\eta_2(e_k)$ are the sum of two hyperbolic functions, defined as below:

$$\begin{aligned} \eta_1(e_k) &= \frac{1}{2}\tanh(\tau(e_k + f_S)) + \frac{1}{2}\tanh(\tau(-e_k + f_S)) \\ \eta_2(e_k) &= \frac{1}{2}\tanh(-\tau(e_k + f_S)) + \frac{1}{2}\tanh(\tau(-e_k + f_S)) \end{aligned} \tag{10}$$

where τ is a smoothing parameter, such that the higher is its value, the larger is the sharpness of the functions (see Figure 8). Extensive simulations have verified that for $\tau \geq 10^4$ the valve signature given by the proposed smoothed model (9) reproduces exactly the results of He's model.

3.3 Other features of compared formulations of MPC

In this section the main features common to all formulations of MPC presented in Section 2.1 are detailed. The canonical offset-free MPC is used for all three formulation (Pannocchia et al., 2015). A linear disturbance model is used, so that the whole plant model becomes:

$$\begin{aligned}\hat{z}_{k+1|k} &= f(\hat{z}_k, u_k) + B_d\,\hat{d}_{k|k}\\ \hat{d}_{k+1|k} &= \hat{d}_{k|k}\\ \hat{y}_k &= h(\hat{z}_k) + C_d\,\hat{d}_{k|k}\end{aligned} \tag{11}$$

where B_d is the state disturbance matrix ($\in \mathbb{R}^{n\times n_d}$), and C_d is the output disturbance matrix ($\in \mathbb{R}^{p\times n_d}$), where $n_d = p = 1$. The three modules (*Estimator*, *Steady-State Optimizer*, *Dynamic Optimizer*) implemented in the proposed MPC formulations are briefly described below. Note that all modules are executed at the same frequency as it typically happens in the process industry (Qin and Badgwell, 2003).

State estimation. The state estimator receives current output measurement (y_k), and updates state ($\hat{z}_{k|k-1}$) and disturbance ($\hat{d}_{k|k-1}$) predictions. The *Luenberger observer* is used, so that prediction update is made by:

$$\begin{bmatrix}\hat{z}_{k|k}\\ \hat{d}_{k|k}\end{bmatrix} = \begin{bmatrix}\hat{z}_{k|k-1}\\ \hat{d}_{k|k-1}\end{bmatrix} + K\,e_k \tag{12}$$

where $e_k = y_k - \hat{y}_k$ is the *prediction error*, and $K = [K_z^T, K_d^T]^T$ is the observer matrix $\in \mathbb{R}^{(n+n_d)\times p}$.

Steady-state optimization. The steady-state optimizer computes the state (z_{ss}), input (u_{ss}), and output (y_{ss}) targets to match the desired external set-points (u_{sp}, y_{sp}) while respecting the imposed constraints. The optimization problem is as follows:

$$[z_{ss}, u_{ss}, y_{ss}] = \arg\min_{u,y}\ \ell_{ss}(u,y) \tag{13a}$$

subject to

$$\begin{aligned} z_{\min} &\le z \le z_{\max}\\ y_{\min} &\le y \le y_{\max}\\ u_{\min} &\le u \le u_{\max}\\ c_{ss}(z,u,y) &= 0\end{aligned} \tag{13b}$$

The objective function is quadratic:

$$\ell_{ss}(u,y) = (y-y_{sp})^T Q_{ss}(y-y_{sp}) + (u-u_{sp})^T R_{ss}(u-u_{sp}) \tag{14}$$

where Q_{ss} is the output penalty matrix ($\in \mathbb{R}^{p\times p}$), and R_{ss} is the control penalty matrix ($\in \mathbb{R}^{m\times m}$). The considered constraints are:

- *Bounds*: on state, input, and output vectors;
- *Equilibrium point* $c_{ss}(z,u,y)$: on the state map $\Rightarrow$ $z_{ss} - f(z_{ss}, u_{ss}) - B_d\hat{d}_{k|k} = 0$, and on the output map $\Rightarrow$ $y_{ss} - h(z_{ss}) - C_d\hat{d}_{k|k} = 0$.

Dynamic optimization. The dynamic optimizer finds optimal trajectory ($\mathbf{z}$,$\mathbf{u}$) from current state and input to targets and computes $u_k = \mathbf{u}_k(0)$. The problem is formulated as follows:

$$[\mathbf{z}_k, \mathbf{u}_k] = \arg\min_{\mathbf{z},\mathbf{u}}\ \ell_{dyn}(\mathbf{z},\mathbf{u}) = \sum_{i=0}^{N-1} \ell(z_i, u_i) + V_f(z_N) \tag{15a}$$

subject to

$$\begin{aligned} z_{\min} &\le z_i \le z_{\max}\\ y_{\min} &\le y_i \le y_{\max}\\ u_{\min} &\le u_i \le u_{\max}\\ \Delta u_{\min} &\le \Delta u \le \Delta u_{\max}\\ c_{eq}(z_i,u_i) &= 0\end{aligned} \tag{15b}$$

where N is the prediction horizon length, and $V_f(z_N) = (z_N - z_{ss})^T Q_N(z_N - z_{ss})$ is the terminal weight. Also this objective function is quadratic:

$$\ell(z_i,u_i) = (z_i - z_{ss})^T Q(z_i - z_{ss}) + \Delta u_i^T S \Delta u_i \tag{16}$$

where $\Delta u_i = u_i - u_{i-1}$ is the input rate of change, Q is the state penalty matrix ($\in \mathbb{R}^{n\times n}$), S is the control difference penalty matrix ($\in \mathbb{R}^{m\times m}$). The considered constraints are:

- *Bounds*: on the state, input, input rate of change, and on output;
- *Dynamic map* $c_{eq}(z,u,y)$: on the state map $\Rightarrow$ $z_{i+1} - f(z_i,u_i) - B_d\hat{d}_{k|k} = 0$, and on the output map $\Rightarrow$ $y_i - h(z_i) - C_d\hat{d}_{k|k} = 0$.

Controller tuning. Some details about tuning parameters are given. The state penalty matrix is chosen as follows:

$$Q = \frac{\alpha}{\max c_{ij}{}^2}\mathbf{C}^T\mathbf{C} + I \tag{17}$$

where $\max c_{ij}$ is the maximum element of matrix $\mathbf{C}$, α is the actual tuning parameter. The steady-state matrices are chosen as $Q_{ss} = 1$ and $R_{ss} = 0$, while the ratio Q/S is around $1/10$, with $\alpha = 1$ and $Q_N = 10^3$. The standard output disturbance model and the standard observer are here adopted: $B_d = 0$, $C_d = I$, and $K_z = 0$, $K_d = I$.

3.4 2-move compensation: a warm-start for NMPC

In order to get good tracking performance and move variables to their targets by avoiding oscillations induced by valve stiction, a suitable *warm-start* should be given to the dynamic optimizer of MPC. This first-guess trajectory is inspired by a novel 2-move stiction compensation method.

Introduced by Srinivasan and Rengaswamy (2008), the "two-move compensator" ought to remove oscillations on control variable, and keep the valve output at its steady-state value, by performing at least two moves in opposite directions. The proposed sequence of valve input signal is as follows:

$$\begin{aligned} u_k &= \begin{cases} u_{k-1} + a f_S & \text{if} \quad u_{k-1} \ge \chi_{ss}\\ u_{k-1} - a f_S & \text{if} \quad u_{k-1} < \chi_{ss}\end{cases}\\ u_{k+1} &= \begin{cases} \chi_{ss} - f_D & \text{if} \quad u_{k-1} \ge \chi_{ss}\\ \chi_{ss} + f_D & \text{if} \quad u_{k-1} < \chi_{ss}\end{cases}\\ u_{k+j} &= u_{k+1} (= u_{ss}) \qquad \text{if} \quad j > 1\end{aligned} \tag{18}$$

The *first* input u_k (for $j = 0$) moves the valve stem away from its stuck position, if $a > 2$. Note that, according to (5), at maximum $|u_{k-1} - \chi_{k-1}| = f_S$, therefore, if $a > 2$, one gets $|u_k - \chi_{k-1}| > f_S$ and can move the valve: $\chi_k \ne \chi_{k-1}$ (see Figure 9). Then, the second signal u_{k+1} (for $j = 1$) brings the stem position to its steady-state value (χ_{ss}) in order to eliminate error on control variable. After this second movement ($j > 1$), the stem cannot move from steady-state position since the input signal is kept constant.

It is worth reminding that the first version of two-move stiction compensation presents several drawbacks, which heavily hinder its on-line implementation (Bacci di Capaci et al., 2016). Among others issues, the steady-state value of valve position (χ_{ss}) is assumed to be known, while this variable is not usually measurable in process plants.

In the proposed formulation (1), the valve output represents the first component of the state vector of whole plant model. Therefore, at each sampling time, the steady-state optimization module of NMPC can compute a suitable steady-state target (χ_{ss}) also for the valve output:

$$\begin{aligned}\xi_{ss} &= \mathbf{A}\xi_{ss} + \mathbf{B}\varphi(\chi_{ss}, u_{ss})\\ y_{ss} &= \mathbf{C}\xi_{ss} = y_{sp}\end{aligned} \tag{19}$$

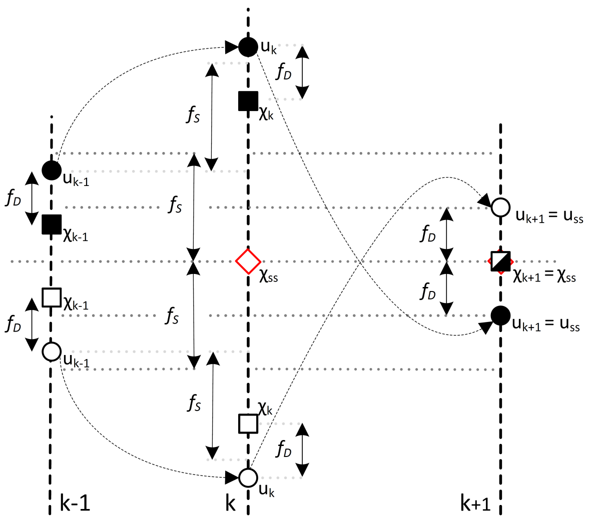

Fig. 9. Sequence of two moves for stiction compensation.

Therefore, the proposed compensation method (18) represents a valid *warm-start* for the NMPC, which improves significantly performance of dynamic optimization module:

$$\begin{aligned} u^0_{1:N} &= [u^0_1, u^0_2, u^0_3, \ldots, u^0_{N-1}, u^0_N] \\ &= [u_{-1} \pm a \cdot f_S, \chi_{ss} \mp f_D, \chi_{ss} \mp f_D, \ldots, \chi_{ss} \mp f_D, \chi_{ss} \mp f_D] \end{aligned} \tag{20}$$

Note that (18) represents just a particular input sequence. A general formulation of *warm-start* can be obtained by imposing the following dynamic optimization problem:

$$\min_{\hat{\chi}_k, u_k, u_{k+1}} \quad (\hat{\chi}_{k-1} - \hat{\chi}_k)^2 + (\hat{\chi}_k - \hat{\chi}_{ss})^2 \tag{21}$$

subject to

$$\hat{\chi}_k = \varphi(\hat{\chi}_{k-1}, u_k) \tag{22}$$

$$\hat{\chi}_{ss} = \varphi(\hat{\chi}_k, u_{k+1})$$

which computes two moves (u_k, u_{k+1}) by optimizing on $\hat{\chi}_k$, and by assuming $\hat{\chi}_{k+1} = \hat{\chi}_{ss}$.

4. SIMULATION ANALYSIS

The objective of this section is to investigate and compare the performance of the three considered formulations of MPC. A third order transfer function for the process model is used:

$$P(s) = \frac{1}{(10s+1)(5s+1)(s+1)}$$

which corresponds to the following state-space model in discrete time domain with sampling period $T_s = 1$:

$$A = \begin{bmatrix} 2.0914 & -0.6874 & 0.2725 \\ 2.0000 & 0 & 0 \\ 0 & 0.5000 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0625 \\ 0 \\ 0 \end{bmatrix},$$

$$C = [0.0391 \;\; 0.0575 \;\; 0.0204]$$

The three formulations of MPC are compared under equivalent conditions in terms of tuning parameters, state observer, disturbance model as discussed in Section 3.3. The only differences lay in the *smoothed* stiction model of (9) and in the stiction compensation sequence of (20), which are respectively used within dynamic optimization module of the NMPC formulation as valve model and as warm-start. In the sequel, stiction unaware MPC and stiction inversion MPC are labeled as LMPC-0 and LMPC-1, respectively. Simulations are performed on a code (Vaccari and Pannocchia, 2016), written in Python 2.7 with the use of symbolic framework offered by CasADi 3.1. Both optimization modules of MPC implement `IPOPT`, the standard in the class of nonlinear programming solvers.

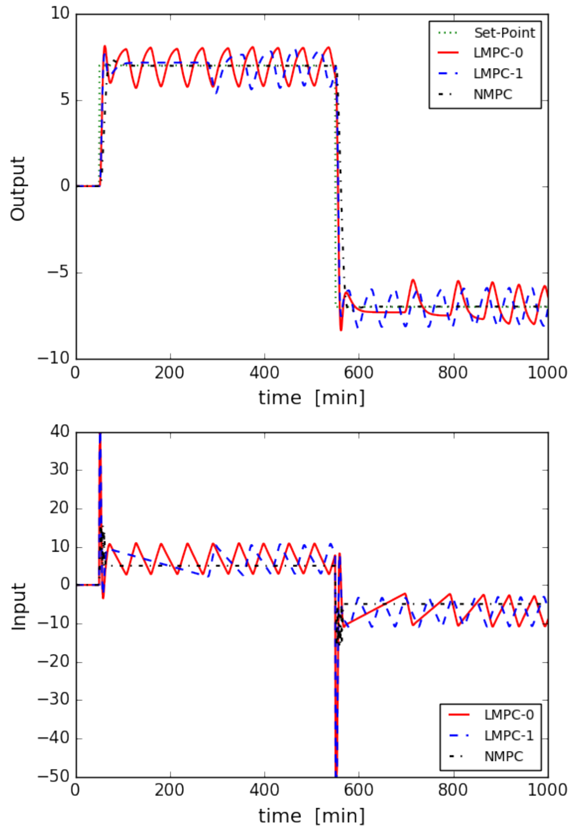

Fig. 10. Output and input response for different MPC formulations (with $N = 50$).

4.1 *Nominal comparison*

In this first simulation, nominal performance is evaluated, since no error in process and valve dynamics is present, and no noise is added. Stiction is described by He's model with $f_S = 5$ and $f_D = 2$. Output and input responses for the same set-point trend – a series of two step changes – are shown in Figure 10. It can be observed that stiction embedding formulation (NMPC) is the only controller which guarantees very good tracking performance and also an effective stiction compensation. On the opposite, the other two formulations show lower performance and do not remove oscillations induced by stiction. Note that not even stiction inversion MPC, despite being aware of the valve malfunction, can yield good control, since the conditions on input sequence, shown in (7), are not verified.

4.2 *Effect of noise*

The noise effect is here presented by considering all the same parameters used in previous analysis. Ten simulations in response to a single step change are performed with different magnitude of the output white noise covariance matrix R_{wn}, where $v = R_{wn}^{1/2} v_{rnd}$, and v_{rnd} is a random sequence with uniform distribution over $[0, 1)$. The performance is evaluated by using the following closed-loop objective function:

$$J_{CL} = \sum_k \frac{(y_k - y_{sp})^2}{\max c_{ij}{}^2} + S\Delta u_k{}^2$$

Table 1 summarizes the complete results. It can be observed that for NMPC rather constant values of J_{CL} are obtained until $R_{wn} = 10^{-2}$, that is, an acceptable tracking performance and a good stiction compensation is still possible for significant levels

Table 1. Effect of noise for three MPC formulations. Values of the objective function J_{CL} $[\times 10^3]$.

Noise Level (R_{wn})	0	10^{-10}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}
LMPC-0	69.6	72.0	70.8	67.6	69.9	71.1	70.6	48.8	41.8	154.6
LMPC-1	45.9	42.8	40.8	12.2	41.6	64.6	59.2	44.1	19.6	140.3
NMPC	0.055	0.055	0.056	0.063	0.041	0.034	0.418	2.20	16.8	143.0

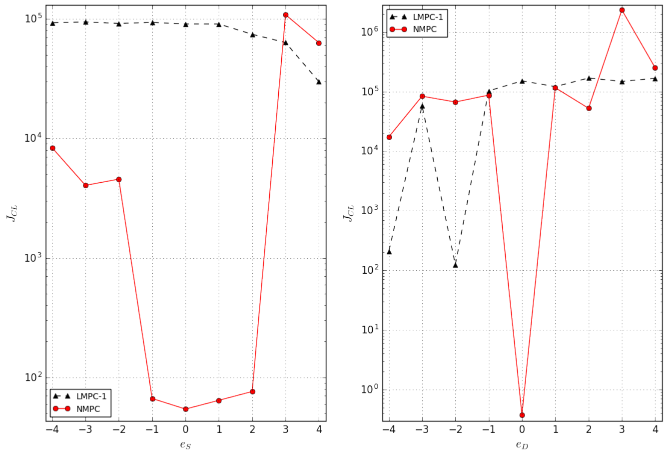

Fig. 11. Effect of mismatch on stiction parameters: left) f_S, right) f_D.

of noise. The other two formulations produce larger values of J_{CL} and a lower robustness to noise.

4.3 *Effect of mismatch on stiction parameters*

Finally, the effect of the wrong values of stiction parameters ($\hat{f}_S$, $\hat{f}_D$) in the valve model of two stiction aware MPC formulations is studied. Mismatched values on static and dynamic friction are considered separately. In the first case, actual values are $f_S = 6$, $f_D = 2$ and $\hat{f}_S$ is varied; in the second case, process values are $f_S = 8$, $f_D = 4$ and $\hat{f}_D$ is changed. Figure 11 summarizes the whole results, by showing values of J_{CL} with respect to single errors: $e_S = f_S - \hat{f}_S$ and $e_D = f_D - \hat{f}_D$. For NMPC, as awaited, minimum values of the objective function are obtained for null errors; acceptable performance are also possible when the estimated parameter of static friction ($\hat{f}_S$) is bigger than the actual value, that is, when $e_S < 0$. On the opposite, performance can significantly degrade when stiction parameters are underrated. Therefore, a robust stiction embedding MPC can be designed conservatively by considering a large amount of stiction in the plant model. Stiction inversion MPC shows overall a lower robustness to errors on stiction parameters apart from the case of large underestimation and for the specific case of overestimation into deadband, that is, $\hat{f}_d = \hat{f}_S = f_S(= 8)$, for which a fair stiction inversion is possible.

5. CONCLUSIONS

This paper has presented three different formulations of MPC to face static friction in control valves for industrial processes. A pure linear formulation, a stiction embedding structure, and a stiction inversion controller are designed. It has been observed that stiction embedding MPC is the only formulation which guarantees very good tracking performance and also stiction compensation. A robust behavior is verified also in the presence of significant amount of white noise on the output, and even for conservative errors in the nonlinear part of plant model, that is, mismatches on valve dynamics parameters. Anyway, the better performance of stiction embedding MPC is possible at the expense of using a nonlinear formulation which solves a more complex and heavy optimization problem. On the opposite, the other two formulations show globally lower performance and do not remove oscillations induced by valve stiction. Note that stiction inversion MPC, despite being aware of the valve fault, cannot generally yield a good control, since conditions of discontinuity on input sequence are hardly verified when this controller is implemented in closed-loop.

REFERENCES

Bacci di Capaci, R., Scali, C., and Huang, B. (2016). A revised technique of stiction compensation for control valves. In *Proceedings of the 11th IFAC DYCOPS*, 1038–1043. Trondheim, Norway, 6–8 June.

Durand, H. and Christofides, P. (2016). Actuator stiction compensation via model predictive control for nonlinear processes. *AIChE Journal*, 62, 2004–2023.

He, Q.P. and Wang, J. (2010). Valve stiction modeling: First-principles vs. data-drive approaches. In *Proceedings of the 7th American Control Conference*, 3777–3782. Baltimore, MD, USA, 30 June–2 July.

He, Q.P., Wang, J., Pottmann, M., and Qin, S.J. (2007). A curve fitting method for detecting valve stiction in oscillating control loops. *Ind. Eng. Chem. Res.*, 46, 4549–4560.

Jelali, M. and Huang, B. (2010). *Detection and Diagnosis of Stiction in Control Loops: State of the Art and Advanced Methods*. Springer-Verlag, London.

Pannocchia, G., Gabiccini, M., and Artoni, A. (2015). Offset-free MPC explained: novelties, subtleties, and applications. In *Proceedings of 5th IFAC NMPC Conference*, 342–351. Seville, Spain, 17–20 September.

Qin, S.J. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Contr. Eng. Pract.*, 11, 733–764.

Rodríguez, M. and Heath, W. (2012). MPC for plants subject to saturation and deadzone, backlash or stiction. In *Proceedings of the 4th IFAC NMPC Conference*, 418–423. Noordwijkerhout, The Netherlands.

Srinivasan, R. and Rengaswamy, R. (2008). Approaches for efficient stiction compensation in process control valves. *Comput. Chem. Eng.*, 32, 218–229.

Vaccari, M. and Pannocchia, G. (2016). A multipurpose, easy-to-use Model Predictive Control design and simulation code. In *Proceedings of the 4th EUCCO*, http://easychair.org/smart–program/EUCCO2016/. Leuven, Belgium, 12–14 September.

Zabiri, H. and Samyudia, Y. (2006). A hybrid formulation and design of model predictive control for systems under actuator saturation and backlash. *J. Proc. Contr.*, 16, 693–709.

Zabiri, H. and Samyudia, Y. (2009). MIQP-based MPC in the presence of control valve stiction. *Chemical Product and Process Modeling*, 4, 85–97.