An Analysis of Mathematical Tasks Used at Second-Level in Ireland

Brendan O'Sullivan B.A (Hons), M.A., M.Ed., H.Dip.Ed.

Thesis submitted to Dublin City University for the degree of Doctor of Philosophy

School of Mathematical Sciences, Dublin City University

September 2017

Research Supervisors

Dr. Sinéad Breen
Dr. Ann O'Shea (Maynooth University)

Declaration

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work, that I have exercised reasonable care to ensure that the work is original, and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed:_______________________

ID No:_______________________

Date: _______________________

Acknowledgements

To begin, I would like to thank my supervisors, Sinéad Breen and Ann O'Shea, for their constant encouragement and support. Over the six years of this journey, they always showed patience and understanding for a part-time student while challenging me to produce the best research possible. I was extremely lucky to work with such a formidable team!

Special thanks to Siún Nic Mhuirí who offered thoughtful insights in relation to my work. I would also like to thank Elizabeth Oldham, always offering good advice when I needed a wise ear. In addition to Elizabeth, I must express gratitude to Ailish Hannigan, Catherine Kiely, Laura O'Dwyer and Maurice OReilly; each of whom gave time and assistance despite busy schedules.

Majella Dempsey is also deserving of mention for facilitating the workshop in Maynooth, along with the many Pre-Service Teachers that participated so willingly. The assistance of the researchers who took part in the Novelty framework is also appreciated. I must also mention the students that I have been privileged to teach over the years and I am very thankful for how the experience has moulded me as a teacher.

All my friends deserve thanks for their encouraging words; I would like to acknowledge those that gave me old mathematics textbooks at the beginning of the analysis. Such generosity saved me precious time and ended a frustrating search. Many thanks to Elisabet Mellroth for providing me with material from Sweden when I visited her family. I would also like to dedicate this work to the memory of Maureen Clarke. Her contribution to mathematics education in Cork should never be forgotten.

Finally, and most importantly, thank you to both of my parents: Dan and Marian who have been with me every step of the way and always showed confidence in my ability to finish this work.

Contents

List of Abbreviations

HL = Higher Level

OL = Ordinary Level

LL = Lower Level

HLD = Higher Level Demand

LLD = Lower Level Demand

LCD = Level of Cognitive Demand

PM = Project Maths

CR = Creative Reasoning

IR = Imitative Reasoning

PMO = Project Maths syllabus problem-solving Objectives

DM = Doing Mathematics

HP = Procedures with connections to meaning

LP = Procedures without connections to meaning

LM = Memorization

Table of Figures

Table of Tables

Abstract

An Analysis of Mathematical Tasks at Second-Level in Ireland

by Brendan O'Sullivan

This thesis is concerned with the analysis of mathematical textbook tasks at second-level in Ireland, in the context of the introduction of the revised curriculum entitled 'Project Maths'. This work aims to gain greater insight into the mathematical tasks that students and teachers encounter on a daily basis, to attain some understanding of the teaching and learning taking place in Irish classrooms. A total of 7635 tasks on the topics of Pattern, Sequences and Series and Differential Calculus contained in three textbook series for senior cycle, in editions available before and those available after the curriculum change were analysed. The analysis was informed by the use of five frameworks: an amended version of the Project Maths problem-solving objectives, a novelty framework designed during the course of this research, Levels of Cognitive Demand (Smith and Stein, 1998), Lithner's reasoning framework (Lithner, 2008) and Usiskin's model of mathematical understanding (Usiskin, 2012). My findings suggest that the post-Project Maths textbook tasks offer greater opportunities in all five areas when compared to those in the older textbooks, but that there is still scope for further development. Based on my analysis, it would appear that all three textbook series have neglected important objectives of the Project Maths curriculum such as justifying conclusions and communicating mathematically. Furthermore, the findings indicate that there is a need for more attention to be paid to fostering novelty in textbook tasks, increasing the level of cognitive demand, and more opportunities should be provided for creative reasoning. Greater effort should also be made to diversify the dimensions of student understanding offered by tasks. Following this analysis, some sample tasks were designed, paying particular attention to the areas my findings indicate are in need of improvement. This was achieved by building on existing textbook tasks as well as creating completely new ones.

Chapter 1 Introduction

My research is concerned with the analysis of mathematical tasks at second level in Ireland, during a period of significant reform of post-primary mathematics. The importance of the textbook, as a source of tasks, in Irish mathematics classrooms has been noted; very often it is the only resource which students have access to during the lesson aside from the teacher, while most of the tasks assigned for classwork and homework come directly from the textbook (Project Maths, 2017). The aim of this research is to gain greater insight into the nature of tasks that students and teachers work with in Irish classrooms. With this goal in mind, my work has centred on analysing tasks contained in three textbook series (at Higher and Ordinary Level) currently used in Ireland (O'Sullivan, Breen and O'Shea 2013, O'Sullivan 2014).

In Ireland, secondary education is divided into two parts – a junior cycle and a senior cycle. The former spans three years, building on the education received at primary level while preparing students for the sitting of the Junior Certificate examination. The senior cycle involves two years of study and leads to the Leaving Certificate examination; students typically study seven subjects and entrance to higher education is decided using the results of this set of examinations. At junior cycle, subjects are normally studied at either Ordinary or Higher Level, although three subjects, Irish, English and mathematics, can also be studied at Foundation Level (DES/NCCA, 2000). At senior cycle the subjects of Irish and mathematics can also be pursued at Foundation Level. The Higher course is aimed at the more able students with a special interest in the subject, the Ordinary course provides knowledge and techniques that will be necessary for the study of scientific, economic, business and technical subjects at third level, while the Foundation course provides students with the mathematical tools needed in their daily life and work (DES/NCCA, 2000). In the junior cycle, the study of mathematics is mandatory and thus there is a high participation rate.. While mathematics is not compulsory in the senior cycle, the number sitting for examination in the subject remains high. All of the courses are assessed by means of a terminal examination. In 2017, 93% of students sitting the Leaving Certificate took the subject of mathematics for examination. The majority of these candidates, 59%, sat the ordinary level, 30% sat the higher level papers while 12% took the foundation level (SEC, 2017).

1.1 Research Rationale

Prior to the introduction of the curricular reform, the seemingly low problem-solving ability of Irish school children was noted as being a major area of concern in a number of publications (OECD 2007, 2010; Forfás 2009; EGFSN 2008, Shiel, Surgenor, Close and Millar 2006). It was also noted that Ireland had 'comparatively few very high achievers' (NCCA 2005, pg. 188). A growing perception existed that students were being prepared to pass examinations in mathematics but were not given an understanding of the subject or of how to apply its concepts (Elwood and Carlisle, 2003). Lyons, Lynch, Close, Boland and Sheerin (2003) observed an over-riding preoccupation with examinations in second-level mathematics classrooms and identified the teaching of mathematics in Ireland as encouraging procedural learning, rather than promoting the ability to problem solve. Leaving Certificate examiners commented that there was a 'noticeable decline in the capacity of candidates to engage with problems that were not of a routine and well-rehearsed type' (SEC, 2005, p. 72). Similarly examiners at Junior Certificate called for a change of approach in the hopes of 'developing a deeper understanding' (SEC, 2006, p. 53). Concerns about the lack of application of mathematics in real-world contexts were also expressed across the educational sphere (NCCA 2006).

Against this background, the curricular revision of second-level Mathematics in Ireland, an initiative known as 'Project Maths', aims to place a greater emphasis on student understanding of mathematical concepts, together with increased use of contexts and applications that should enable students to relate mathematics to everyday experience. It also hopes to create a classroom environment where a greater amount of the teaching and learning taking place is done through problem-solving (Project Maths, 2009). Given that Project Maths was designed to help address the issues described in this section, an analysis of mathematical tasks in textbooks designed for the new syllabus will give some insight as to whether such concerns are indeed being addressed.

1.2 Textbook Analysis as a Research Method

Robitaille and Travers (1992) have made the point that a dependence upon textbooks is 'perhaps more characteristic of the teaching of mathematics than of any other subject' (p. 706). Fan and Kaeley (2000) argued that teachers using different types of textbooks made use of different styles of teaching strategies. Fan (2011) defined mathematics textbook research 'as disciplined inquiry into issues about mathematics textbook and the relationships between mathematics textbooks and other factors in mathematics education' (p.9). Having conducted a survey of mathematics textbook literature published over the previous six decades, Fan, Zhu and Miao (2013) concluded that the most popular area of research is to be found on textbook analysis and comparison. Textbook analysis, as described by Fan et al. (2013), is a broad term including 'analysis of a single textbook or a series of textbooks, which often focuses on how a topic or topics are treated or how a particular idea or aspect of interest is reflected in textbooks' (p.636). Fan et al. (2013) point out that textbook analysis can also involve the analysis of different series of textbooks, often with a focus on identifying their similarities and differences.

Fan (2013) conceptualises textbooks as a variable, specifically an intermediate variable in the context of education, such a variable 'causes variation in the dependent variable and is itself caused to vary by the independent variable' (p. 771). An intermediate variable can cause variation in dependent variables such as student learning or the promotion of problem-solving. The textbook, as an intermediate variable, is also caused to vary by independent variables such as the integration of information technology or the composition of textbook writing teams. Put simply, the independent variables in this case are the factors affecting the development of textbooks. Overall, the textbook is affected by independent variables but can also influence dependent variables of its own. Using this conceptual framework, textbook research can be viewed as consisting of three broad areas as shown in Figure 1.1 below: 1) textbooks as the subject of research (textbooks as an intermediate variable); 2) how different factors affect the development or production of textbooks (textbooks as a dependent variable) and 3) how other factors are affected by the textbook (textbooks as an independent variable).

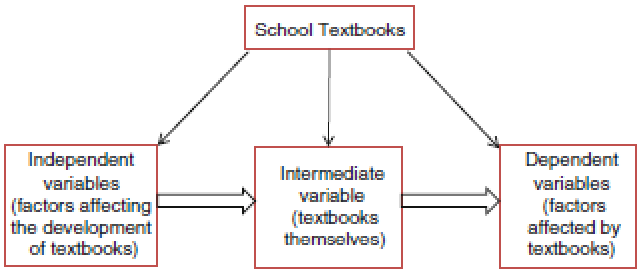


Figure 1.1: Textbooks viewed as an intermediate variable in the context of education (Fan, 2013, p. 771)

As will be seen in chapter 3 on Theoretical Frameworks, another theoretical framework known as the instructional triangle (Cohen, Raudenbush and Ball, 2003) looks at the relationship between teachers, students and the knowledge or specifically mathematical content that makes up instruction. The textbook as a source of tasks concerned with mathematical content is again an intermediate variable within this framework. Different textbooks offer different material for mathematical instruction and assignment by teachers and these in turn can affect the opportunities for learning offered to students. The instructional triangle and its extension will be discussed in greater detail in chapter 3, however it complements Fan's conceptualisation of textbooks and underlines their importance as a focus of research.

Fan (2011, p.8) points out that a large majority of the literature in relation to mathematics textbooks is not based on empirical methods or experimental methods. Most research publications are based on document analysis, which Fan (2013) describes as 'relatively well-established and accepted as a research method' (p.773). Fan (2013) also points out that this practice of document analysis and coding has a long tradition, citing Pingel commenting on general textbook research that 'during the early years of textbook studies, textbooks were often regarded as quasi-independent entities. The examination focused almost exclusively on the content and the written text' (Pingel, 2010, p.42) in Fan (2013, p. 774). The research presented in this study views the textbook, containing mathematical tasks, as an intermediate variable and presents an analysis and comparison of two topics in three textbook series that were in use both before and after the reform of the curriculum. It does not make use of empirical methods such as classroom observations, interviews or field surveys as it does not attempt to draw conclusions on how textbooks are being used in Irish post-primary classrooms. Instead it analyses the nature of the mathematical tasks that are contained in popular Irish mathematics textbooks.

1.3 Recent Textbook Analysis in Ireland

The following is a short overview of the recent textbook analysis that has been conducted in Ireland. More details will be given in chapter 2, the Literature Review. Over 75 per cent of Irish second-level teachers use a textbook on a daily basis (O'Keeffe, 2011, p.262). There is very little published work in relation to the classification of textbook tasks in Ireland before the implementation of Project Maths. Moreover, much of the other work involving textbook analysis that has been undertaken since Project Maths has also had a different focus looking at different aspects of the textbooks available. O'Keeffe and O'Donoghue (2009) analysed four series of textbooks used in Irish junior cycle classrooms using Rivers' framework which focuses on motivational factors, comprehension cues, technical aids and philosophical position.

A later study - O'Keeffe and O'Donoghue (2011) analysed a number of textbooks in the area of fractions. This focused on content, structure, expectation and language. The findings of this study were then used to create a 'model chapter' for teaching the topic of fraction addition. O'Keeffe and O'Donoghue (2012) conducted a larger study of the textbooks published in response to Project Maths that were available at the time (ten in all), consisting of textbooks used at both junior and senior cycle. The TIMSS mathematics curriculum framework, was used and adapted for this analysis. This conceptual framework looks at the textbook as a whole, under three broad headings: intended curriculum, implemented curriculum and attained curriculum. The conclusion of the analysis was that the textbooks display a genuine attempt to match Project Maths expectations but no one textbook meets all of these.

Davis (2013) examined the prevalence of reasoning-and-proving in the topic of complex numbers in six Irish textbooks and one teaching and learning plan produced for teachers during the introduction of Project Maths. The results from Davis' study suggest that there is a 'misalignment between the six textbook units and the Leaving Certificate syllabus' (Davis, 2013, p. 54). Given the amount of change that accompanied Project Maths, I identified the scope for further research in the area of Irish mathematical textbooks. It would be expected that the tasks contained in the new texts produced to meet the needs of the new curriculum would be more conducive to intrinsic learning and problem-solving than the publications that they were replacing. It would also be necessary to identify any deficiencies if present. I also set out to design a small number of tasks in order to address any shortcomings identified. The research contained in this thesis goes some way towards adding insight into this area.

1.4 Outline of the Research Undertaken

This study is concerned with analysing mathematical textbook tasks from two topics studied at Higher and Ordinary Level at senior cycle in use from the pre- and post-Project Maths eras. To complete this analysis effectively and to gather as much useful information as possible, it was necessary to consider how the analysis would be conducted and what immediate use would be made of the analysis findings.

Having considered these issues, the following emerged as the primary stages of the study:

- To review the literature in relation to the background issues that prompted the introduction of the new Project Maths curriculum and what concerns it aimed to address. Another activity undertaken was to review the literature on the classification of tasks in textbooks and how the findings of such studies prove to be useful, also giving consideration in such literature to the role of the textbook in classrooms internationally and in Ireland.

- To identify suitable frameworks that could be used for the analysis of textbook tasks, adapting existing frameworks where necessary to make them fit for the purpose of the analysis. Emerging from work completed on two existing frameworks; the Levels of Cognitive Demand and Lithner's reasoning framework, it was decided to create a new framework to identify 'novel' tasks and to enable differentiation between the degrees of novelty based on students' prior experience and exposure to exemplary material in a textbook chapter.

- To gather quantitative data on tasks in the topics of Pattern, Sequences and Series, and Differential Calculus, by conducting analyses using five frameworks, on textbooks available both before and after the introduction of the Project Maths curriculum.

- To design some sample tasks, informed by the textbook analysis, to address any deficiencies identified by the five frameworks in order to better realise the goals of the Project Maths curriculum. This was informed by looking at material giving guidelines on the design of mathematical tasks and the benefits available to teachers when engaged in designing their own tasks.

1.5 Research Questions

In the past, mathematics textbooks in Ireland have been accused of being too procedural and dependent on the use of algorithms when students are solving tasks (NCCA, 2012b, p. 18). This analysis aims to give some indication as to whether this situation persists and whether the Project Maths problem-solving syllabus objectives are being realised. Four main research questions have emerged with the first (what is the nature of the mathematical tasks that are being used in Irish classrooms?) consisting of five parts:

RQ 1 (a) Are these tasks fulfilling the expectations of Project Maths in terms of its problem-solving objectives?

RQ 1 (b) What degree of novelty is incorporated in these tasks?

RQ 1 (c) What is the level of cognitive demand of the tasks analysed on the topics contained in the textbook chapters?

RQ 1(d) What kind of reasoning do students need to use when completing these tasks?

RQ 1 (e) What kind of understanding (using Usiskin's dimensions) is being promoted in these textbook tasks?

Another research question (RQ 2) that is investigated relates to the textbook series and whether differences exist between them.

RQ 2 Is there a difference between textbook series?

Similarly, given that tasks from textbooks in use from both before and after the introduction of the Project Maths curriculum were analysed, a third question (RQ 3) relates to whether there are changes between these sets of textbook series.

RQ 3 Is there a difference between the pre-Project Maths and post-Project Maths textbook series?

Finally my last research question (RQ 4) considers whether textbook tasks can be created in order to better address the intentions of the Project Maths curriculum than those tasks currently available.

RQ 4 Can textbook tasks be designed to better meet the goals of Project Maths?

1.6 Overview of the Thesis

Subsequent to the introduction to the research outlined in this chapter, the work undertaken is discussed in greater detail over the next seven chapters.

Chapter 2 provides some background information about mathematics education in Ireland including the recent revision of the second-level syllabus and the implementation of curriculum reforms. The chapter then focuses on the importance of the classification of tasks in textbooks, which is central to this thesis. Consideration is given to the role of the textbook in classrooms internationally and in Ireland. Specific textbook studies are examined, discussing how they were conducted as well as the findings for each study. Literature on task design is also considered and how it can prove to be beneficial to teacher development.

Chapter 3 outlines the theoretical frameworks that are utilised in this study. Herbst and Chazan's (2012) extension of the instructional triangle (Cohen et al., 2003) is discussed in relation to the importance of analysing mathematical tasks. Mathematical content, in the form of tasks, is an important aspect of the study of the teaching and learning. The type of tasks that are offered to students by teachers allows us to gain some insight into the opportunities available for the learning of mathematics. The Levels of Cognitive Demand (as described in Smith and Stein, 1998) and mathematical reasoning (Lithner, 2008) frameworks are outlined. The Project Maths syllabus problem-solving objectives are described to give some insight into the goals that the syllabus aspires to. An explanation is provided of how these objectives are adapted for use as a framework for the analysis of tasks in this study. Usiskin's (2012) multidimensional model of mathematical understanding is described. After examining the criteria for task analysis using each of these four frameworks, the kind of data that each framework offers to researchers is outlined. A framework to classify the 'novelty' of tasks has been designed as part of this thesis and is presented in this chapter; this framework can assist with distinguishing between certain categories in the Project Maths objectives, Levels of Cognitive Demand and reasoning frameworks.

Chapter 4 describes the methodology used for the classification of tasks using the frameworks described in chapter 3. Once the methodology employed in the textbook tasks analysis is explained, examples of classifications using the above frameworks are provided. This chapter also describes a workshop used to gain external validation for the Novelty framework outlined in chapter 3.

Chapter 5 provides the results of the classification of 7635 textbook tasks on two topics from a number of textbooks in use before and after the introduction of the Project Maths curriculum. The textbooks are from three textbook series, namely Active Maths, Text and Tests, and Concise Maths. Tasks are analysed from the areas of Pattern, Sequences and Series, and Differential Calculus using the methodology described in chapter 4.

Chapter 6 gives examples of the types of tasks that I have designed, informed by the results of the classification of textbook tasks in chapter 5. The tasks designed are accompanied by a classification of these tasks using four of the frameworks described in chapter 3; the Levels of Cognitive Demand framework, Lithner's reasoning framework, Usiskin's mathematical understanding framework and the Project Maths problem-solving objectives framework. A workshop at which the tasks designed for this study were independently evaluated with respect to the Levels of Cognitive Demand framework and the Project Maths problem-solving Objectives framework is also described. Amendments to the workshop tasks are then provided arising from how the workshop participants classified the tasks. Classifications for these amended tasks are also given.

Chapter 7 discusses the findings from the analysis and design of tasks in order to answer the research questions posed in section 1.5. The limitations of the research described in this thesis are examined at the end of the chapter.

Chapter 8 concludes the thesis by making a number of recommendations arising from the research completed. Directions for future research are also outlined in this section.

Chapter 2 Literature Review

This chapter will give some background information about mathematics education in Ireland including the recent revision of the second-level syllabus and the implementation of curriculum reforms. The importance attached to the classification of tasks in textbooks is then examined. Consideration is given to the role of the textbook in classrooms internationally and in Ireland. Specific textbook studies are outlined with attention given to how they were conducted and what each one found. Task design is also considered and how it can be beneficial to teacher development.

2.1 Curriculum Change

Project Maths is an initiative, led by the National Council for Curriculum and Assessment (NCCA), to bring about positive change in the teaching and learning of mathematics at second-level in Ireland. Its inception came about as a result of a number of concerns raised about the state of mathematics education in the Irish system. One major worry for policy-makers was the level of performance of students in an international context. PISA 2003 rated Ireland 20^{th} out of 40 for mathematical literacy (Cosgrove, Shiel, Sofroniou, Zastrutzki and Shortt, 2005), and it brought attention to the relatively low performance of higher achieving students in mathematics in Ireland. Cosgrove et al. (2005, p. 223) pointed out that there was a weak match between PISA mathematics and the existing syllabus in Ireland and suggested that any future review of mathematics education should consider whether important mathematical content was absent from the syllabus. There was a growing perception that students were being prepared to pass examinations in mathematics but were not being given an understanding of the subject or of how to apply its concepts (Lyons et al., 2003, p. 6). In 2005, a discussion paper outlining some of the pressing issues was published by the NCCA (NCCA, 2005). The concerns raised included an emphasis on procedural skills rather than understanding, poor application of mathematics to real-world contexts, low uptake of Higher Level mathematics, low grades achieved at Ordinary Level, gender differences in uptake and achievement, and difficulties experienced by some students in third-level courses. A subsequent questionnaire sought the views of students, teachers, principals, parents, lecturers and employers. Following this consultation, worries were expressed by many of these stakeholders about the lack of application of mathematics in real-world contexts (NCCA, 2006). Teaching and learning practices were cited in the consultation report as having the greatest influence on students' understanding of mathematics. While the report acknowledged the need for change in the syllabus and the assessment of same to take into account the need for more use of contexts and more applications, there was broad agreement that a deeper understanding of mathematical concepts would be achieved if more time was given to the teaching and learning of the 'why', 'what if' and 'how' of mathematics. Over 90% of those consulted felt that an improved textbook could play a strong role in improving students' performance on mathematics examinations.

Following this review, the NCCA began the process of revising the curriculum and the new curriculum that emerged was named Project Maths. The new curriculum emphasised the development of student problem-solving skills (NCCA, 2012a, p. 1). One of the key aims of Project Maths is to encourage students to think about their strategies, to explore possible approaches and evaluate these, and so build up a body of knowledge and skills that they can apply in both familiar and unfamiliar situations. The new syllabuses were introduced on a gradual basis. There are five strands in total: 1) Statistics and Probability, 2) Geometry and Trigonometry, 3) Number, 4) Algebra and 5) Functions. Strands 1 and 2 were introduced in 24 Pilot Schools in 2008 and in all other schools in 2010. Strands 3 and 4 followed in 2009 for the pilot schools and in 2011 for the remaining cohort, while the final strand was introduced in 2010 and 2012 respectively.

Change was not limited to syllabus content; Project Maths also advocated different learning and teaching practices. A series of workshops, to which all mathematics teachers in the pilot schools were invited, looked at specific topics from the different syllabus strands to demonstrate a more investigative approach to teaching and learning. Looking at the experiences of teachers in the pilot schools, it has been noted that teachers have observed a change in their teaching practices, with a move away from didactic approaches to more active methodologies (NCCA, 2012a, p. 20). As the Project Maths curriculum becomes more embedded in Irish classrooms, it has been necessary for some teachers to adopt different methodologies in order to attain the learning outcomes outlined by the syllabus (Department of Education and Skills, 2010). These include the use of ICT to enable greater exploration of mathematical ideas within the classroom. Teachers have also been encouraged to engage in class discussion through the brainstorming of mathematical problems and to consider alternative solutions to similar questions. Similar to the pilot schools, an extensive programme of in-service workshops encouraging different approaches to teaching and learning has been made available to mathematics teachers.

A number of reports have been published in relation to Project Maths and its impact. The NCCA (2012b) in its response to the debate on Project Maths notes that textbooks have traditionally supported practising routine questions with solutions based on illustrative examples. The report calls for more emphasis to be given to students engaging in problem-solving approaches and justifying or explaining their solutions (NCCA, 2012b, p. 18). It does not make explicit reference to how textbooks can be used to support this; however it concludes that teachers in the initial schools reported that their classroom practice now relies less heavily on the textbook as the sole teaching resource. The implication is that teachers should use the textbook less in the Project Maths classroom.

The NCCA also commissioned two reports (interim and final) (Jeffes, Jones, Cunningham, Dawson, Cooper, Straw, Sturman and O'Kane, 2012; Jeffes, Jones, Wilson, Lamont, Straw, Wheater and Dawson, 2013) exploring the impact of Project Maths on student learning and achievement in the initial pilot schools (called phase one in this report) that introduced the syllabus in 2008 and the remaining schools in the country (called non-phase one schools) that introduced it in 2010. As the revised mathematics syllabuses were introduced incrementally in schools, a cohort of phase one Junior Certificate students in 2010 had studied Strands 1-4 of the revised mathematics syllabus, while the same cohort of students in non-phase one schools had followed the previous mathematics syllabus introduced in 2000. The latter group was therefore included as a comparison group for this research. Similarly, Leaving Certificate students in phase one schools had studied all five strands of the revised mathematics syllabus, while students in non-phase one schools had followed revised syllabuses for Strands 1 and 2 only. Research was carried out into students' performance on different mathematical topics using assessment booklets, and attitude surveys were conducted to gain insight into how students felt about mathematics.

Key findings of the first (interim) report (Jeffes et al., 2012) show that students following the revised syllabus were performing well in many aspects of the Project Maths syllabus, and there were many parallels between students' achievement and their attitudes. It was suggested that phase one students were more reflective about their experiences of learning mathematics, and were now more able to identify their own areas of strengths and weaknesses. The research presented in the interim report did not identify any real differences between the skills of students following the Project Maths syllabus and those following the previous syllabus, but students following the Project Maths syllabuses reported that they were regularly engaging with a broader range of teaching techniques and experiences than their counterparts following the older curriculum. Such activities included: the application of mathematics to real-life situations; making connections and links between mathematics topics; using mathematical language and verbal reasoning to convey ideas; and planning and conducting investigations. The interim report (Jeffes et al., 2012, p. 132) found that both Junior Certificate and Leaving Certificate students following the new curriculum were performing particularly well in relation to Strand 1 (Statistics and Probability), for example, which was accompanied by a high degree of confidence reported in relation to this strand. Students who had followed the revised syllabus appeared to find Strand 4 (Algebra) more difficult and this was identified as an area in which students lack confidence.

The final report (Jeffes et al., 2013, p. 5) found that schools having greater experience of teaching the revised syllabuses did not appear to be associated with any improvement in students' achievement and confidence. Achievement was highest in Strand 1 (Statistics and Probability) and lowest in Strand 5 (Functions), confidence was highest in both of these strands and lowest in Strand 3 (Number) and Strand 4 (Algebra) for all schools (both phase one and non-phase one). It was observed that confidence in mathematics did not always correspond to achievement. Although students who participated in the research towards the end of the academic year performed better than those tested at the start of the school year, higher levels of confidence were not associated with students who had almost completed their Leaving Certificate studies.

One of the main findings of the second (final) report (Jeffes et al., 2013, p. 3) is that while students are frequently undertaking activities like applying mathematics to real-life situations and making connections between topics as described in the interim report, more traditional approaches like using textbooks and copying from the whiteboard continue to be widespread. The report notes that while some processes of the revised mathematics syllabuses are visible in the work reviewed, there does not appear to have been a substantial change in what teachers are asking students to do, and it recorded few differences between the phase one and non-phase one students. It is suggested that teachers were, at that time, emphasising the content of the revised syllabuses rather than the processes promoted within it. The research suggests that students are building up mastery of mathematical procedures and, to a slightly lesser extent, are problem-solving and making mathematical representations. However, there is very little evidence in the work reviewed that students are engaging in reasoning and proof, communicating mathematically, or making connections between mathematics topics. It would appear that the students' reports on their experience do not necessarily match what was observed in the students' actual work. The report suggests that students need to be regularly given high quality tasks that require them to engage with the processes promoted by Project Maths, including: problem-solving; drawing out connections between mathematics topics; communicating more effectively in written form; and justifying and providing evidence for their answers.

Research on Project Maths has not been confined to the experience of students; the opinions of teachers have also been sought. Questionnaires were given to mathematics teachers and mathematics department co-ordinators in Ireland as part of PISA 2012 (Cosgrove, Perkins, Shiel, Fish and McGuinness, 2012). This survey aimed to obtain empirical and qualitative information on the views of a nationally representative sample of teachers on the implementation of Project Maths, and to compare this information across teachers in pilot schools and non-pilot schools. Close to half of the teachers

(47.5%) indicated that they did not know if Project Maths was having a positive impact at junior cycle. One interpretation of this is that it is still too early in the implementation of the syllabus for teachers to have formed an opinion. Overall teachers at this level indicated a high level of confidence in teaching aspects of the Project Maths syllabus. However they also identified a number of challenges: time pressures (time to become familiar with coursework, to prepare classes, for group work and investigations); the staggered implementation of Project Maths; and the literacy demands of the new courses. Teachers expressed the opinion that there had been positive changes in a number of aspects in students' learning, in particular, their understanding of key concepts in probability and statistics and, in geometry and trigonometry. It was also reported that teachers felt that students have improved in their ability to solve real-life problems and their ability to work collaboratively with one another.

In 2013, the NCCA made contact with the two post primary teacher unions (the Association of Secondary Teachers, Ireland and Teachers Union of Ireland), the Irish Mathematics Teachers' Association (IMTA) and the Project Maths Development Team (PMDT) with a view to establishing a group to engage in discussions around the new syllabus and how teachers could be supported in its implementation. To this end, a Maths in Practice group was convened, comprising nominees from these organisations as well as representation from the Department of Education and Science and the State Examinations Commission. A series of meetings were held between September 2013 and January 2014, culminating in the publication of a report (NCCA, 2014). This document contains a number of recommendations, in particular that the Leaving Certificate syllabus should be reviewed following its complete implementation and the final syllabus revisions that were introduced. It was noted that teachers were still in need of support with the development and use of tasks that involve working with connections between topics. (NCCA, 2014) It also called for the exploration of design-based research, whereby teachers would design and enact interventions in the classroom, as a focus of continuing professional development for mathematics teachers in Ireland. It suggested that such research would not only be beneficial to individual teachers but would also contribute to improving mathematics pedagogy in a wider sense. The NCCA promised that recommendations of the report would also be taken into account when considering the new mathematics curriculum specification for junior cycle, which is due to be published in 2017 and introduced to schools in 2018.

The IMTA produced a policy document in 2013 that outlined problems experienced by teachers implementing the revised syllabuses and suggested possible solutions. Under the heading of resources, the IMTA (2013, p. 7) reported that teachers are still quite dependent on textbooks, and that these are used as a means to interpret the syllabi. A key problem identified with the textbooks was that the examples often provided proved to be poor or there was an insufficient quantity provided. The report also suggested that the textbook tasks had poor gradation, with the level of challenge progressing from easy to very demanding while neglecting moderately difficult tasks. It also called for more guidance from the NCCA to be given to publishers and textbook authors when preparing the textbooks.

A Chief Examiner's Report (SEC, 2016) in Leaving Certificate mathematics was published in 2016, the first of its kind published after the introduction of the Project Maths syllabus. It reviewed candidates' performance in the 2015 examinations and set itself the goal of identifying strengths and challenges in order to provide guidance for teachers and students in the future (SEC, 2016). It found that at Higher Level, the numbers achieving a grade C or below has shown a marked increase when compared with examinations before the introduction of the Project Maths curriculum and for Ordinary Level, the proportion of students securing a grade B or above has declined. Those that might have studied Ordinary Level in the old syllabus are now staying at Higher Level instead (SEC, 2016). It also noted that the syllabus expectations are more ambitious than previously and are not always easy to achieve; the authors commented that there has been a deliberate attempt to emphasise higher order thinking skills but acknowledged that this presents difficulties for both students and teachers alike (SEC, 2016, p. 8). The report recommended that students should become more familiar with describing, explaining, justifying and providing examples. It noted that these skills assist with improving understanding (SEC, 2016, p. 9). It also cautioned that teachers need to provide students with the opportunity to develop strategies for working with unfamiliar problems (SEC, 2016, p. 30). Teachers were also reminded to encourage students to practise solving problems involving real-life applications of mathematics. As part of this process, students should be asked to model these situations by constructing algebraic expressions or equations and/or representing them differently by drawing diagrams (SEC, 2016, p. 30).

Lubienski (2011, p. 46) noted that the absence of a textbook specifically designed for the new syllabus was problematic at the introduction of the new syllabus as it was very time consuming for teachers to plan lessons. Also students were having difficulty keeping the amount of paperwork organised due to the teachers' dependency on the use of photocopied material to substitute for the lack of a textbook. The publishers of mathematics textbooks have gradually produced new texts in response to the changed needs of the classroom. However, it has also been reported that teachers are not fully satisfied with the textbooks that have been produced, 45.3% of teachers surveyed in a study on schools that participated in the Programme for International Student Assessment (PISA) 2012 felt that the textbooks produced did not support the approach required by the curriculum appropriately (Cosgrove et al., 2012, p. 49). In addition, it was found that 31.6% of teachers in this cohort feel that the content and range of textbooks available are a major challenge. The teachers complained that there was no textbook available for students studying at Foundation Level and that the content did not satisfactorily address what was needed for students following the Junior Certificate Higher Level course. Cosgrove et al. (2012) also state that some teachers in their study felt that the style of the material covered by the new textbooks differed to what was covered during the professional development workshops for Project Maths. But, despite the changes, value was still being placed on the textbook by students with teachers in the pilot schools feeling pressurised to use the textbooks in the classroom (NCCA, 2012a, p. 8).

The Educational Research Centre (ERC) produced a report in 2016, intended for teachers of mathematics in post-primary schools, focussing on the PISA 2012 outcomes for students in Ireland and their implications for teaching and learning (Perkins and Shiel, 2016). In the outcomes from PISA 2012, students in Ireland were found to have performed significantly better than their OECD peers, although Ireland's average performance has not changed considerably since 2003. Instead, there has been a decrease in the OECD average between 2003 and 2012, leaving Ireland ranked 13th of the 34 OECD countries that participated. It must be noted that many of the Irish students participating in the PISA 2012 assessment had not yet encountered the Project Maths curriculum due to its phased implementation. Aside from the initial schools, those sitting the Junior Certificate mathematics examination in 2012 had not been exposed to Project Maths material. The report noted that there are several themes that have emerged from the PISA 2012 mathematics findings that are particularly important for improving performance for students in Ireland: 1) the relatively poor performance of girls, 2) high levels of mathematics anxiety among students, 3) addressing the needs of lower performing students, 4) the relative underperformance of higher-achieving students, 5) opportunity to learn mathematics and 6) the use of Information and Communication Technologies (ICTs) in the teaching and learning of mathematics.

A number of recommendations and practical suggestions are contained in the report. These include encouraging higher achieving girls, to engage with more demanding and complex tasks and to explore problem-solving in novel ways (Perkins and Shiel, 2016, p. 4). To support students, it is suggested that teachers could raise 'students' cognitive engagement in mathematics by 1) allowing students to decide on their own procedures when solving problems, 2) assigning problems that can be solved in different ways, 3) presenting problems in different contexts, 4) giving problems with no immediate solution, and 5) asking students to explain how they solved a problem' (Perkins and Shiel, 2016, p. 8). For lower-achieving students, it is recommended that more opportunities would be presented to solve more complex problems so that they can develop flexibility in applying what they have learned. Strategies suggested to assist higher-achieving students include providing opportunities to engage with problems in novel contexts and to explore different solutions to problems, including through the use of technology. This would ensure that, where students are assigned procedural mathematical tasks, they fully understand the underlying concepts. Students are also encouraged to reflect on problems, asking them to explain their answers, and supporting them in learning from their mistakes (Perkins and Shiel, 2016, p. 12).

The OECD produced a broader report to offer advice to teachers in relation to the findings from PISA 2012 and strategies for teaching and learning. It found that Ireland ranked 2nd out of 65 participating countries in relation to the reliance on memorisation strategies for learning mathematics, this assessment was based on students' self-reports. The use of memorisation is cautioned against as students dependent on committing material to memory are less likely to solve more challenging problems (OECD, 2016). In relation to the most difficult question on the 2012 PISA examination, students who were identified as being the most reliant on memorisation when studying mathematics were found to be four times less likely to solve this problem correctly than the cohort of students who reported using memorisation the least (OECD, 2016, p. 38). The report recommends that teachers should encourage students to think more deeply about what has been learned and encourage the establishment of connections with real-world problems (OECD, 2016, p. 38).

2.2 Role of the Textbook

Textbooks are widely accepted as a commonly used resource in mathematical classrooms. According to Jones, Fujita, Clarke and Lu (2008, p. 142) over 60 per cent of teachers, on average, internationally, identify the use of a textbook as the primary basis of their lessons-with a further 30 per cent or more using a textbook as a supplementary resource. Robitaille and Travers (1992, p. 706) suggest that the dependency on textbooks is more characteristic of mathematics teaching than any other subject. Historically, there has not been a lot of research carried out on the nature of post primary mathematics textbooks in Ireland (Conway and Sloane, 2005). However, there is some evidence that textbooks play an important role in Irish classrooms. Even in early-childhood mathematics classrooms, it was found that teachers use the textbook for guidance and giving structure to the programme of work (Dunphy, 2009, p. 118). Harbison (2009, p. 131) points out that textbooks have a role in proposing potential pathways for navigating through the strands of the primary Mathematics curriculum); and O'Keeffe and O'Donoghue (2009) found that over 75 per cent of Irish second level mathematics teachers use a textbook on a daily basis. It has also been reported that a lot of the time in the classroom appears to be related to the use of a textbook and very often it is the only resource which students have access to during the lesson aside from the teacher, with most of the problems assigned for classwork and homework coming from the textbook (Project Maths, 2017).

However, Moffet (2009, p. 265) acknowledges that different textbooks lend themselves to varying types of instruction and these ultimately result in differing learning outcomes. The latter may explain why the use of textbooks can prove to be problematic. Fan and Kaeley (2000) conducted a study investigating the influence of textbooks on teaching strategies. Their findings show that textbooks can impact not only on the content of teachers' lessons but also how teachers actually teach. They concluded that it would be difficult to reform teachers' teaching methods without corresponding reform of the textbooks being used due to the important role that textbooks play in affecting teaching strategies. Indeed, Valverde, Bianchi, Wolfe, Schmidt and Houang (2002, p. 2) have suggested that textbooks act as mediators between the teacher in the classroom and those who design curriculum policy. Howson (1995) suggests that textbooks are closer to the reality of the classroom than a national curriculum. As the textbook remains as a popular resource with teachers in Irish classrooms, it is important to see what kind of teaching and learning is promoted through the use of particular textbooks. As well as revising the curriculum, the Project Maths Development Team (Project Maths, 2017) has cautioned teachers in their choice of textbook: the PMDT points out that there is no single textbook which can suit the learning needs of all students and it has advised schools, when choosing a textbook, to take into accounts the abilities, needs and interests of their students, as well as the quality of the book. It had been noted, before the implementation of Project Maths, that mathematics textbooks in the Irish system promoted retention and practice as opposed to active learning (O'Keeffe and O'Donoghue, 2009, p.290).

Similarly classroom inspections in Ireland, before the introduction of Project Maths, have shown that teaching was highly dependent on the class textbook which had a tendency to reinforce this drill and practice style (NCCA, 2006). Ireland was not unique in this, and similar inspections in Swedish lower secondary schools found that the teaching of mathematics relied on the use of textbooks more than was the case for any other subject (Johansson, 2006). Johansson (2006) also reported that, in her study of Swedish classrooms, students were working individually with textbook tasks for more than half of each lesson observed, and the students' homework was also assigned from the textbook. Swedish teachers in the study were also found to take examples and definitions directly from the textbook. Viholainen, Partanen, Piiroinen, Asikainen and Hirvonen (2015) indicated that textbooks are used by teachers as mediators of the curriculum in upper secondary mathematics in the Finnish classrooms that they observed. The teachers involved in their study tended to use the textbook as the main source in planning their teaching. Teachers reported strong confidence in the pedagogical solutions offered by the textbooks, particularly the selection of content, the order of topics and exercises. Students viewed the textbook primarily as a source of exercises. (Viholainen et al., 2015, p. 174). Lepik (2015), when surveying Estonian teachers, found that textbooks exerted a strong influence on the content that is taught and learned. However, the teachers claimed that their teaching approaches were not driven by the instructional approaches of the textbook; rather they reported using the textbook only as a source of exercises within the classroom.

2.3 Textbook Studies in Ireland

Up until recently there has not been a lot of research conducted on mathematics textbooks in Ireland, especially at post-primary level. This situation has now changed with the introduction of Project Maths. This section will further consider the work that has been carried out on textbook analysis involving Irish mathematics textbooks at both primary and post-primary level.

Dunphy (2009) looked at how successful it was to teach mathematics for understanding to lower primary school students in Ireland by relying on textbooks as the main method of pedagogy. Her evidence of the use of mathematics textbooks was obtained from questionnaires about practice, completed by 48 classroom teachers across four class levels in primary school. The research showed that textbooks were used by these teachers in the lower primary years as one of the main resources in teaching the curriculum. Teachers in her study perceived textbooks as a means of consolidating and extending learning. They also used them to structure the programme of learning and as an aide to classroom management. Dunphy advises that future textbooks should have more meaningful tasks; this would help the motivation of students as they would view the tasks as relevant and worthwhile.

Studying textbooks used in different countries can reveal similarities and differences in the teaching and learning of mathematics. Such analyses can reveal differences in the performance expected of students in different countries, the extent to which a selected textbook series from a country prioritizes conceptual understanding or procedural fluency, and how the treatment of the mathematical content differs among certain countries. Charalambous, Delaney, Hsu and Mesa, (2010) report on a comparison of the treatment of addition and subtraction of fractions in primary mathematics textbooks used in Cyprus, Ireland, and Taiwan. Their research looks at what similarities and differences can be seen in the presentation of addition and subtraction of fractions in primary mathematics textbooks in these three countries. It also examines what expectations of student performance on these topics are embedded in the primary textbook tasks on this topic. When considering what was required of students, they focussed on the potential cognitive demands of textbook tasks and the type of response asked for using the Levels of Cognitive Demand framework. Their textbook analysis drew heavily on the work of the QUASAR (Quantitative Understanding: Amplifying Student Attainment and Reasoning) project team as outlined in Smith and Stein (1998). Each of the tasks was classified into one of the four levels of cognitive demand (LCD): lower level demands of memorization and procedures without connections to meaning, and higher level demands of procedures with connections to meaning and doing mathematics. This framework will be examined in more detail later in chapter 3.

In their analysis, they looked at one textbook from Cyprus, two textbooks from Ireland and two textbooks from Taiwan. It was found that tasks in both Irish textbooks appeared to be organized on a continuum, from tasks with lower demands at the start of a section to tasks with higher demands at the end, whereas in the Cypriot and Taiwanese textbooks tasks of lower and higher demands were interspersed throughout the relevant sections. They found that more than 85% of tasks in the Cypriot and the Irish textbooks represented low cognitive demand (procedures without connections to meaning), while they classified 71% of the tasks in the first Taiwanese textbook and 81%

in the second Taiwanese textbook as having high cognitive demands (procedures with connections to meaning or doing mathematics).

Only one textbook out of the five analysed, a Taiwanese textbook, had tasks that required students to explain their solutions. Tasks in the Irish and Cypriot textbooks required students to simply supply the answer. Both Taiwanese textbooks placed a higher expectation on students than the other textbooks, specifically requesting the writing of a mathematical sentence as part of the solution to a problem. The authors point out that writing such sentences or providing explanations is important for helping students clarify their thinking and solution processes.

O'Keeffe and O'Donoghue (2009) looked at the effect of the textbook content structure on student comprehension and motivation. Eight Irish textbooks, two each from four different textbook series, used at junior cycle were looked at in total for the study. In order to analyse the effectiveness of these textbooks for pupil motivation and comprehension, the Rivers Matrix was applied. This framework looks at a number of factors: motivational factors - historical notes, scientist and mathematician biographies, career information, applications and photographs; comprehension cues - colour and graphics; technical aids - inclusion of material related to calculators and computers; philosophical position - emphasis and predominant philosophy. The TIMSS framework outlined earlier in section 1.3 was also used as a framework for the analysis of these textbooks.

The presence of historical notes, biographies, career information and photographs was found to be almost non-existent across most of the textbooks. The percentage of tasks which were viewed as problems, where a problem was defined as a situation where a direct route to a desired goal is blocked (O'Keeffe and O'Donoghue, 2009, p. 285) was particularly low in each case. The use of attractive colours, which is important for student comprehension, was found to be limited throughout all the textbooks. Concern was expressed about the low number of graphics employed to assist with the visualisation of real-life problems and the lack of real-life graphics throughout the textbooks. The use of technical aids was not evident in any of the textbook series analysed. It was noted that all the textbooks highlighted retention and practice, with little focus on active learning. It is pointed out that none of the textbooks fostered an environment of understanding. All the textbooks were found to have an emphasis on proficiency and logic. This means that the books focussed on students being able to complete a number of questions. Thus understanding was not given the same importance as procedure and method in each of these textbooks.

According to this study's findings, the most popular textbook series being used by teachers at the time for teaching and planning was not the most beneficial available to aid learning. The authors advocated a need for standards and/or a checklist for textbooks in order to enhance learning at junior cycle. At the time of this study it was noted that the failure of textbooks to include motivational material and comprehension cues highlighted a need for improved textbooks. Further research of this kind was called for in the hopes of developing standards in mathematics textbook design at junior cycle which would lead to better teaching and learning resources, and would contribute to an increase in the uptake of Higher Level Mathematics at junior cycle.

In 2011, O'Keeffe and O'Donoghue (2012) conducted a larger study of the textbooks published in response to Project Maths that were available at the time (ten in all). The TIMSS mathematics curriculum framework, was used and adapted for this analysis. The Third International Mathematical and Science Study (TIMSS) was a cross-national survey of student achievement in mathematics and science in which forty-five countries took part. Valverde et al. (2002) reported on the TIMSS study which analysed six hundred and thirty science and mathematics textbooks. It remains one of the only studies to examine the textbook as a whole, even considering the number of pages in each textbook. The conceptual framework for TIMSS focussed on curriculum as a broad explanatory factor underlying student achievement (Beaton, Martin and Mullis, 1997), considering three parts in its model, namely intended curriculum, implemented curriculum and attained curriculum. For TIMSS the textbook was viewed as the potential implemented curriculum.

The study found that all textbooks analysed fell short of the standard needed to support the Project Maths (intended) curriculum effectively, as outlined in the Project Maths Syllabus documents for junior cycle (including the Common Introductory Course (CIC)), and senior cycle. However, the study acknowledged that some of the new textbooks are better aligned to Project Maths expectations than others. The individual profiles of textbooks as developed in this study demonstrate different strengths and weaknesses. Topic omissions were observed even though the Project Maths syllabus treats all topics as compulsory. One example given was 'Exponents, Roots and Radicals' which was excluded from one textbook's treatment of strand 2. Project Maths expectations in relation to teaching for understanding, problem-solving, using real life applications, and integration of ICT were found to be addressed to varying degrees within the textbooks. Some textbooks showed greater consideration for teaching for understanding in terms of narration and related narration; while no one textbook series was found to address problem-solving satisfactorily. The conclusion of the analysis was that the textbooks display a genuine attempt to match Project Maths expectations but no one textbook meets all the Project Maths expectations.

It is clear that the most significant finding of the report is the mismatch between the textbook expectations and the Project Maths expectations. The report recommends that an exemplar textbook series for Project Maths should be produced by a specially selected writing team appointed and funded by the Department of Education and Science. All commercially produced textbooks for Project Maths should then be reviewed against this exemplar textbook series. Such a review procedure would lead to a list of mathematics textbooks for Project Maths approved by the DoES.

Davis (2013) examined the prevalence of reasoning-and-proving in the topic of complex numbers in six Irish textbooks and one teaching and learning plan produced for teachers during the introduction of Project Maths. His study uses a framework consisting of five main components: namely pattern identification, conjecture development, argument construction, technological tools, and reasoning-and-proving objects. Only 1.4% of tasks in Ordinary Level textbooks and 1.3% of tasks in Higher Level textbooks involved pattern identification or conjecture development. There were no opportunities to test conjectures, construct counterexamples or develop proof subcomponents in any of the materials examined. The results from Davis' study suggest that the six textbook units do not align with the Project Maths syllabus (Davis, 2013, p. 54). The Project Maths Leaving Certificate syllabus requires students to explore patterns and formulate conjectures, explain findings, and justify conclusions. However, it would appear that the textbooks are not meeting this aspect of the Project Maths syllabus and it will fall to the teacher to supplement this shortfall with suitable activities.

2.4 Importance of Tasks and a Wise Choice of Tasks

This section outlines the importance of tasks and how classifying tasks can highlight their role in the teaching and learning of mathematics. There are many different definitions of problems and exercises. For this study, a task is considered to be an activity where a student interacts with a mathematical topic by attempting to solve a textbook exercise or problem either as homework or within the classroom. An exercise is treated as something that can be solved using a familiar method and a problem involves the finding of a solution where the solution method is not immediately obvious. Mason and Johnston-Wilder (2006, p. 4) define a task to be what learners are asked to do in the mathematics classroom. Mathematics classroom instruction is generally organised around and delivered through students' activities on mathematical tasks (Doyle, 1988, p. 168). In all of the seven countries that participated in the TIMSS 1999 Video Study, eighth-grade mathematics was most commonly taught by spending at least 80% of lesson time in mathematics classrooms working on mathematical tasks (Hiebert, Gallimore, Garnier, Givvin, Hollingsworth, Jacobs, Chui, Wearne, Smith, Kersting, Manaster, Tseng, Etterbeek, Manaster, Gonzales and Stigler, 2003, p. 42). Doyle (1983, p. 161) makes the case that what students learn is largely defined by the tasks they are given. He believes that tasks specify ways of processing information and influence learners by directing their attention to particular aspects of content. Sullivan, Clarke and Clarke (2012, p. 14) give great importance to mathematical tasks by saying that mathematical thinking stems from students' engagement with problem like tasks as opposed to following a series of detailed instructions from the teacher.

Henningsen and Stein (1997, p. 525) point out that tasks by their nature have the potential to influence and structure the way students think and this can either limit or broaden their views on the subject matter being studied. Shimizu, Kaur, Huang and Clarke (2010, p. 1) describe the role of mathematical tasks as being an important aspect of study when attempting to understand teaching and learning for those researching classroom practices. Marx and Walsh (1988, p. 208) also affirm the advantages of analysing tasks due to their link between teaching and learning. Teachers choose the tasks that they will use in the classroom with specific learning objectives in mind. Through the use of different pedagogical methods the same task can be used to achieve different learning outcomes (Marx and Walsh, 1988, p. 211). It is not possible to determine exactly how a teacher will use a task from its structure alone. However Marx and Walsh believe that it is important that tasks should be of a sufficiently high quality so as to allow teachers to achieve their desired learning objectives when the tasks are given to students. If the tasks are too narrow in their scope, then teachers, seeking to enrich their students' mathematical experience, will have to augment them or look elsewhere to meet the needs of their classroom (Marx and Walsh, 1988, p. 211).

There is evidence to suggest that tasks convey information to students on the nature of the subject of mathematics. Examples and selected tasks influence how students ultimately perceive and use mathematics (Anthony and Walshaw, 2009, p. 155). Through engagement with tasks, students not only gain mathematical competence but also develop ideas about the nature of mathematics. By working with tasks, students get an opportunity to develop their own mathematical interests (Hodge, Zhao, Visnovska and Cobb, 2007). Henningsen and Stein (1997) argue that effective teachers carefully choose tasks so that all students are given assistance in making progress with their cumulative understanding in a mathematical topic and are also encouraged to engage in high-level mathematical thinking.

Several frameworks have been suggested to examine tasks in relation to their use in the classroom as well as the implementation of the curriculum. Cohen et al. (2003) developed a framework which places teachers, students and content as three elements situated at the vertices of a triangle where each element interacts with the others and the environments (including parents, textbook publishers, examination bodies and state agencies) in which they are immersed. Herbst (2008, p. 125) views the tasks in the mathematics classroom as common ground that links each of the vertices of the instructional triangle. In terms of content, the task is a representation of mathematical ideas. For the students, the task is an opportunity to learn more or to think differently. The teacher manages the task as a representation of the content to be learned and the teacher is also responsible for using the task as an opportunity for students to study and learn mathematics. Each element in the instructional triangle is important as is the relationship and interplay between them; the task is at the centre of the activity that permeates these relationships and is worthy of study when considering the quality of instruction.

Pepin, Xu, Trouche and Wang (2016) examined the resource systems, including tasks, of three Chinese mathematics 'expert' teachers. This was completed using the Documentational Approach to Didactics, which is a framework that acknowledges the important and central role of resources for the work of teachers. Teachers' resources are defined as the curriculum/textbook and personal resources that teachers make use of as part of their teaching routine, as well as the material used when preparing for teaching in the classroom. Pepin et al. (2016, p. 15) describe the influence of resources on teachers' instructional practice and identity as crucial. They argue that viewing resources as a lens enables the researcher to examine mathematics teaching expertise and to identify different aspects of such expertise by examining the link between teaching and the materials that teachers interact with on a daily basis (Pepin et al., 2016, p. 16).

Jones and Pepin (2016, p. 106) point out that teachers' selection of tasks are very often linked to their perceptions of teaching and learning. They believe that tasks are often interpreted as influencing or even determining the kind of opportunities afforded to students for learning. Haggarty and Pepin (2002) examined mathematics textbooks in England, France and Germany and observed how a small sample of teachers from each country used the textbooks to teach the concept of 'angle'. They also interviewed the teachers in relation to their use of tasks. It was found that the students were offered very different learning opportunities which were influenced by both the teacher and textbooks and the culture in which they were situated. They suggest that not only tasks but also their use shaped what students could learn and how they could access and use mathematical content.

Remillard (1996, 1999) identified three arenas of curriculum development activity that teachers engaged in through a study of two fourth-grade teachers' use of a textbook published after curriculum reform. The term 'curriculum development' originated from Ben-Peretz's (1990) description of the role of the teacher in interpreting and adapting resources for use in the classroom. Remillard (1999, p. 328) refers to the design, construction and mapping arenas. The design arena involves selecting and designing mathematical tasks, the construction arena involves enacting these tasks in the classroom and responding to students' interactions with such tasks. In the study, it was found that no matter how teachers selected tasks from the textbook, teachers would make improvised adaptations to tasks in response to students' interactions with the tasks. This is described as adapting and adjusting tasks, in an unrehearsed manner, so as to facilitate students' work with these tasks (Remillard, 1999, p. 328). The final arena, mapping, involves making decisions and/or choices that ultimately determine the content of the mathematics curriculum for the academic year.

Remillard (2005) offers a framework for characterising and studying teachers' interactions with curriculum materials, specifically resources and guides. Gehrke, Knapp and Sirotnik (1992) is used by Remillard (2005) to distinguish between the 'formal curriculum', 'intended curriculum' and 'enacted curriculum'. The formal curriculum referring to the goals and activities espoused by national policies, school policies or formulated in textbooks. The intended curriculum makes reference to teachers' aims while the enacted curriculum describes what actually takes place in the classroom. The framework of Gehrke et. al (1992) uses terminology differently from that found in many other publications, notably ones for cross-national studies such as TIMSS. For TIMSS, the 'intended curriculum' is that prescribed by the state, while that taking place at classroom level is called the 'implemented curriculum'. The U.S.A. does not have a national intended curriculum and this had led to different usages of these terms when considering the American context. Remillard (2005, p. 238) provides a new framework

that adds another dimension when considering the implementation of the curriculum referred to as the 'planned curriculum'. In effect it suggests that the curriculum enacted is the result of the interactions between the teacher and the curriculum. It is different from the intended curriculum as it goes beyond just the aims for instruction held by the teacher. The framework acknowledges that the planned curriculum can shape what the teacher seeks and draws on when interacting with resources. The enacted curriculum then results from how these plans manifest in certain contexts with students Using this framework, Remillard (2005, p. 239) cautions that the process involving the use of a mathematics curriculum guide is both dynamic and complex, affected by teachers' beliefs, knowledge and dispositions. She also points out that the decision to adopt a single curriculum does not necessarily result in uniform mathematics instruction.
Tasks that require different cognitive processes are likely to induce different kinds of learning. Ainley, Pratt and Hansen (2006) suggest that by providing tasks and learning experiences that allow students to think, in an original fashion, about mathematical concepts and relationships, teachers help learners to develop efficient ways of learning about mathematics. Watson and De Geest (2005) caution that tasks should involve more than practicing taught algorithms. Effective tasks should provide opportunities for students to question their reasoning activity and to struggle with mathematical ideas. Posing tasks with a high level of mathematical challenge enables students to employ an increasingly sophisticated range of mathematical thinking. Jonsson, Kulaksiz and Lithner (2016) point out that practising tasks requiring creative reasoning is superior to practising those necessitating algorithmic reasoning in terms of later memory retrieval.

Results from Stein and Lane (1996) suggest the importance of incorporating high-level, complex tasks if teachers wish to promote students' capacity to think, reason and problem solve. Their research found that the greatest improvement on performance assessments were recorded when students had been exposed to instructional tasks that involved students 'doing mathematics' or using 'procedures with connections to meaning' as opposed to more traditional tasks. In particular, students' performance gains were greater when tasks were set up to encourage the use of explanations, multiple solution strategies and different types of representation.

Hiebert and Wearne (1993) analysed teaching approaches and tasks in six classrooms. Four of the teachers used a conventional approach dependent mainly on the solving of routine exercises and rarely employing problem situations; two teachers used fewer tasks but explored them at a greater depth allowing for more exploration and discussion. It was found that students in the classrooms using the alternative approach fared better than those taught in the conventional classrooms when given mathematical assessments. They concluded that tasks designed to encourage higher-order thinking are more likely to foster such thinking rather than tasks that promote the practice of skills.

Xin (2007) conducted a study to examine the learning opportunities for problem-solving provided in 1 U.S. and 1 Chinese mathematics textbook series. It found that there was an unbalanced word problem distribution in the U.S. textbook in comparison to the Chinese one and this resulted in the American students being able to solve certain word problems more easily than the Chinese due to the textbook coverage. Guven, Aydin-Guc and Ozmen (2016) conducted a study to examine the relationship between the tasks that teachers preferred to use in mathematics lessons and student achievement when solving different types of tasks. Nine mathematics teachers were interviewed, and corresponding tasks were prepared and administered to 225 eighth-grade students. It was found that teachers preferred tasks that matched the goals of the curriculum and were routine in nature. Students were found to be more successful with solving tasks that involve missing data in contrast to tasks with irrelevant data, tasks that are visual and do not require the use of different strategies. They were found to be less successful with tasks that require the use of different strategies. It was found that the type of tasks at which students were successful and which teachers preferred were related. From these studies, it would appear that tasks can impact on teaching and affect the learning opportunities offered to students.

2.5 International Studies on Textbook Tasks

As seen in section 2.2, it has been reported that textbook tasks can influence teachers' pedagogy and can either encourage or discourage the use of different teaching strategies. A number of studies analysing mathematics textbooks in different ways have taken place internationally. For instance, as will be seen in this section, attention has been given to the cognitive skills that textbook tasks require. Similarly, work has been completed on the type of reasoning that is necessary for solving textbook tasks. Researchers have investigated the relationship between textbooks and the curriculum, investigating if the textbook actually reflects the intended curriculum. Work has also been completed on the mathematical topics covered in textbooks and how well tasks promote its learning. This section will look at some key international textbook studies in each of these areas.

Jones and Tarr (2007) looked at the cognitive level of tasks in a number of American middle grade textbooks. They looked at twelve textbook series which were published over a fifty year period. Their analysis made use of the LCD framework mentioned earlier. The findings of Jones and Tarr suggested the majority of tasks over the fifty year span required a low level of cognitive demand, in particular involving procedures without connections to meaning. Just one textbook series proved to be an exception where the majority of its tasks required a high level of cognitive demand. They found that there were a greater number of tasks, but not necessarily a greater percentage of tasks, that required higher levels of cognitive demand in the more recently published textbooks. The analysis revealed a greater percentage of tasks requiring a high level of cognitive demand in the 1970s than those published between 1994 and 2004 (Jones and Tarr, 2007, p. 18).

Bayazit (2013) analysed three Turkish elementary school textbooks in relation to the quality of tasks promoting students' proportional reasoning. Some of the tasks were classroom activities while others were intended as homework. The analysis was carried out at both a macro and micro level and included examining the level of cognitive demand of the tasks. Like Jones and Tarr (2007) above, the analysis was based on the level of cognitive demand that textbooks tasks present using the four categories of memorization, procedures without connections to meaning, procedures with connections to meaning and doing mathematics. It was found that the majority of tasks (75%) had a high level of cognitive demand (LCD) and most of the tasks were presented in multiple representations and made use of non-mathematical contexts.

Kim (2014) conducted a study in order to examine how Korean post primary textbooks support students' mathematical thinking and learning. For the analysis, the topics of functions and geometry were selected from 5 textbooks aimed at grades 7-9 and the entire contents from 2 textbooks used for grade 10. Kim's study used the LCD framework suggested by Smith and Stein (1998). The findings suggested that 94% of the tasks analysed in the selected textbooks were at a low level of cognitive demand, with a limited number of tasks requiring 'procedures with connections to meaning' (5%) and very few tasks involving 'doing mathematics' (1%). Kim (2014, p. 285) concluded that students are not given many opportunities to think non-algorithmically and are not offered sufficiently many tasks in order for them to develop an understanding of mathematical processes and relationships.

Yang and Lin (2014) examined the topic of functions in mathematics textbooks used in Finland, Singapore and Taiwan for grades 7-9. The most popular textbook series from each country was selected. Using the LCD framework, it was found that the Taiwanese series had the greatest number of HLD tasks (63.9%), the Singaporean series had less (53.7%) while the Finnish series had the least (37.3%). The main difference being that a lot of Finnish textbook tasks were placed in the 'memorization' category (14.1%) compared to the Singaporean series (0%) and Taiwanese series (1.9%). Yang and Lin (2014, p. 509) suggest that the Finnish textbooks could increase the complexity of their tasks.

Son and Hu (2016) completed a similar analysis investigating the treatment of the concept of function in selected post-primary school textbooks from the U.S. and China. For the study, 1 U.S. reform curriculum textbook, 2 U.S. traditional curriculum textbooks and 1 Chinese reform curriculum textbook were used. They found that the U.S. curricula examined as part of their study introduce the concept of function one year earlier than the Chinese curriculum. The U.S. textbooks also provide far more problems for students to work on than is the case with the Chinese textbooks. In contrast, the Chinese curriculum emphasises developing both procedures and concepts and it also includes more problems that necessitate visual representations, explanations and problem-solving. The Chinese textbooks contain several worked out examples that encourage students to design multiple methods of solution. The U.S. reform curriculum textbook had the highest percentage of problems set in an illustrative context with 93.2% while the 7th grade U.S. traditional curriculum textbook had the highest proportion of problems set in a pure mathematics context with 88.2%. The Chinese textbook was more balanced with 51.5% of problems set in a pure mathematics context and the remainder in a real-life context. Son and Hu, (2016, p. 18) suggest that the students using the U.S. reform curriculum textbook are given more opportunities to connect function concepts to real-life than those using the other textbooks. In relation to cognitive expectation, it was found that the two U.S. traditional curriculum textbooks (7th grade: 10%, 8th grade: 21.9%) and the Chinese textbook (36.3%) had a smaller proportion of tasks requiring problem-solving or mathematical reasoning compared to the reformed curriculum textbook (51%). Another finding of this study was that students using the U.S. curriculum reform textbook would be offered better learning opportunities than those relying on traditional texts.

Bergqvist, Lithner and Sumpter (2008) looked at tasks intended for upper secondary school students in Sweden, similar to the ones encountered in textbooks. The students' reasoning while solving tasks was analysed, with a focus on the use of different strategy choices and how they were implemented. The reasoning framework developed by Lithner, will be outlined in more detail in chapter 3. Fifteen students were videotaped solving tasks, showing their written work while thinking aloud. The results from Bergqvist et al.'s study indicate that mathematically well-founded considerations were rare. The dominating reasoning types observed were algorithmic reasoning (where students tried to remember a suitable algorithm) and guided reasoning (where progress was possible only when essentially all important strategy choices were made by the interviewer).

Sidenvall, Lithner and Jader (2015) completed an analysis of students' textbook task-solving in Swedish upper secondary school, focussing on the types of mathematical reasoning required and the rate of correct task solutions. The data was gathered by studying video-recordings, transcripts and students' notebooks taken from normal classwork. As part of the study, 15 students broken into 7 groups drawn from four different classes attempted 86 textbook tasks. A typical lesson commenced with a teacher's presentation followed by student work with textbook tasks. Students were free to work on their own or collaboratively. Sidenvall et al. (2015, p. 533) warn that rote learning is a cause for concern because it relates to the tendency for students to use imitative strategies, which are mathematical superficial, instead of creating their own solution methods through reasoning. They found that rote learning and superficial reasoning was a common feature, and 80% of all attempted tasks were correctly solved using imitative reasoning strategies. In the few cases where mathematically founded reasoning was used, all tasks were correctly solved. One feature of the study suggests that student collaboration and dialogue does not necessarily lead to mathematically founded reasoning or deeper learning. Students were found to copy solutions from each other without receiving or seeking mathematical justification. They concluded that collaboration of such a nature could actually be a disadvantage to learning. It was also evident that the worked examples and theory sections from the textbook were not used as an aid by the students when solving tasks.

Boesen, Lithner and Palm (2010) compared tasks in a Swedish upper secondary school national test to those that students would have encountered previously when using textbooks. Their results show that when confronted with tasks similar to those in the textbooks, they mostly used imitative reasoning by trying to recall facts or algorithms. When using this imitative reasoning, they were successful in completing the tasks. However, using this type of reasoning did not require the consideration of intrinsic mathematical properties. When solving test tasks that were wholly different to those contained in the textbooks, successful solutions were based on the use of creative reasoning and they tended to be connected to the relevant mathematical foundations.

Jader, Lithner and Sidenvall (2015), in an unpublished manuscript, completed an analysis of school mathematics textbooks from 12 countries (including the Active Maths textbook series from Ireland), looking at the reasoning that selected tasks matching the descriptions "equations and formulas" and "perimeter, area and volume" from algebra and geometry topics required. This study considered whether solutions could be modelled on worked examples provided in the textbooks or if it was necessary to construct a method of solution. The results show that an average of 79 % of the tasks examined required imitative reasoning. It was found that the percentage of tasks requiring creative reasoning was higher among geometry tasks than algebra tasks in all textbooks. It was also discovered that 13 % of the tasks could be solved by mimicking solution methods provided but required some minor modification, and the remaining 9 % of the tasks require that the main part of the solution is created without the guidance of a modelled solution. In the case of Ireland, 86% of tasks were found to require imitative reasoning, 10% followed a modelled solution with some alteration and 4% necessitated creative reasoning.

The 1999 TIMSS Video study (Givvin, Hiebert, Jacobs, Hollingsworth and Gallimore, 2005) analysed teaching patterns in seven different countries, including Australia. Mathematics lessons were examined from several viewpoints, including the types of problems that students solved. It was found that Australia had the second highest proportion of real-life contexts in the lessons observed but that three quarters of the tasks were repetitions of preceding similar problems in the lesson. Australian lessons also had the highest proportion of problems with low procedural complexity and had virtually no use of proof or reasoning. Since then researchers have been keen to see if changes implemented in Australia have had the desired effect on pedagogy and textbook content.

Vincent and Stacey (2008) examined whether tasks presented in textbooks in 2006 were still broadly aligned with the results of the 1999 study for Australia. They looked at three topics (addition and subtraction of fractions, solving linear equations and plane geometry concerning triangles and quadrilaterals) in nine eight grade textbooks in Australia.
Each of the textbook tasks was classified according to five of the TIMSS Video Study criteria: procedural complexity, type of solving processes, degree of repetition, proportion of application tasks and the proportion of tasks requiring deductive reasoning. Procedural complexity was classified as either low, medium or high according to the number of decisions required by the student when solving the task. The type of solving processes used in problems involved categories such as using procedures, stating

concepts or making connections. A task was classified as repetition if it was the same or mostly the same as a previous problem in the lesson. Vincent and Stacey (2008) classified ‘exercises’ as practising procedures on a set of similar tasks while ‘application’ involved students applying procedures learned in one context to solve problems involving a different context. The majority of tasks in all textbooks analysed were of low procedural complexity. For fractions, the percentage of tasks of low procedural complexity ranged from 56% to 83%, for equations there was a similar range from 58% to 83% for equations. It was much higher for geometry, with the proportion of tasks of low procedural complexity ranging from 73% to 96%. They found that there was a broad similarity between the textbook tasks analysed and the lessons presented in the TIMSS Video Study. However, their findings point towards there being too much emphasis on repetitive tasks of low procedural complexity and they call for students to be exposed to the full range of task types in textbooks.

Nie, Freedman, Hwang, Wang, Moyer and Cai (2013) studied teachers’ intentions for and reflections on their use of Standards based Connected Mathematics Programme (CMP) textbooks and traditional textbooks to guide instruction at 6^{th} grade in 14 U.S. middle schools. They report that even when teachers make serious attempts to teach in ways aligned with the Standards based curriculum, teachers often keep many practices inconsistent with reform due to the influence of tradition. For the analysis, they focussed on learning goals, instructional tasks, teachers’ anticipation of students’ difficulties and perceptions of students’ achievement in relation to learning goals. Their study found that the CMP textbooks provided more high cognitive level tasks to implement than the traditional textbooks. Having observed 305 CMP lessons using a corresponding textbook and 274 traditional lessons with a non-CMP textbook, it was found that many more of the former lessons were executed with a high level of cognitive demand, meaning that it incorporated at least one instructional task classified as procedures connected to meanings or doing mathematics. By comparing the teachers’ intended learning goals to the implemented learning goals as perceived by observers, they found that the percentage of standards based lessons that were actually implemented while maintaining a high level cognitive demand for students was lower than originally planned. Nie et al. (2013, p. 707) suggest that while the textbook emphasis may shape instructional goals, this influence is affected by the realities of implementation. They also found that the CMP teachers were more likely to follow the guidance of their textbooks when selecting instructional tasks for a lesson than their non-CMP contemporaries.

Herbel-Eisenmann (2007) analysed a U.S. middle school 6th grade mathematics textbook to see if it achieved the ideological goal of the intended curriculum to move the authority away from the teacher and the textbook and to promote student reasoning and justification. It was found that there was a mismatch between the textbook and this goal. Thompson and Senk (2014) explored the treatment of U.S. high school geometry in 12 teachers' classrooms using the same textbook. Their analysis suggests that factors, other than the textbook, such as the planned curriculum of the teacher, account for differences in the enacted curriculum in terms of the material taught or skipped, the nature in which the topic is taught, the tasks that are assigned to students for homework, and the use of instructional technology applied in the classroom. Tran (2016) examined the alignment of three selected U.S. high school textbooks series with the Common Core State Standards for Mathematics (CCSSM) regarding the treatment of the topic of statistical association. The textbook content was compared with the CCSSM learning expectations (LEs). All 22 CCSSM LEs were covered by two of the three series. However 4 CCSSM LEs were addressed with only 1 task, while 6 CCSSM LEs were addressed by 2 tasks in a particular textbook series. Seeley (2003) is cited as warning that publishers, in order to maximize sales and profits, fill textbooks with content to meet as many state requirements as possible with little consideration as to how topics are treated.

Stylianides (2009) examined a popular American textbook series used in middle grades (6 to 8) in terms of opportunities offered to students to engage in reasoning-and-proving in the topic areas of algebra, geometry and number theory. He found that more than half of the tasks analysed gave no opportunity for students to engage in reasoning-and-proving. For the tasks that were classified as requiring reasoning-and-proving, the number that were designed to engage students in empirical arguments or conjectures was quite low, while there was a high proportion of such tasks encouraging students to use rationales (that is, use an argument which does not make explicit reference to key accepted truths that it uses).

ZDM, the International Journal on Mathematics Education (Fan, Jones, Wang and Xu, 2013), produced a special edition on the theme of textbook research in mathematics education. Fan, Zhu and Miao (2013, p. 633), in the edition's lead article, reviewed relevant research pertaining to mathematics textbooks. They conducted a survey of the literature published over the last six decades and classified the focus of this literature using four different categories: 1) role of textbooks in teaching and learning, 2) textbook analysis and comparison, 3) textbook use and 4) other areas. The survey found that the majority (63%) of empirical studies on mathematics textbooks focussed on textbook analysis and comparison. It highlights that there has been an imbalance in different areas in relation to the development of research on mathematics textbooks. They call for the existence of textbooks to be viewed from a broader perspective instead of being treated as an isolated identity, for more research about the relationship of the textbook and students' learning outcomes to be completed, and for more work to be completed on issues related to the development and production of textbooks. Researchers are also urged to use more advanced methodology in textbook research and to complete more work in the area of electronic textbooks.

2.6 Task Design

This section will look at some key texts on the design of tasks. Then in the next section, more recent developments in relation to task design having a role in teacher development will be discussed. Jones and Pepin (2016, p. 107) define task design as referring 'to mathematical tasks (including tasks in the form of digital resources and tools) that are developed and designed in, or for, mathematics teaching, or in, or for, mathematics teacher education. Hence, task design could include designing tasks for teaching specific mathematical topics to specific learners, designing tasks for textbooks (including digital platforms and e-books), designing learning sequences, and designing tasks for the professional learning of mathematics teachers.'

In recent times, there has been a repeated call for more work to be completed in relation to task design. Brown (2009, p. 23) views the idea of teacher as designer as useful because it brings attention to the constructive interaction between agent (teachers) and tools (curriculum materials) that ensues during the process of instruction. Schoenfeld (2009) noted that there was a need for increased communication between educational designers and researchers. De Araujo and Singletary (2011, p. 1207) made the observation that there was a lack of teachers' perspectives on tasks. Similarly, Geiger, Goos, Dole, Forgasz and Bennison (2014, p. 240) note that there is a potential for improving teaching and learning practices through partnerships between teachers and researchers. This can occur where principles of task design are explored, refined and documented. Also this process should be accompanied by an examination of how to effectively integrate tasks with pedagogical approaches. Watson and Ohtani (2015, p. 11) comment that although task design and teaching are often viewed as separate activities conducted by separate groups, the communities involved in task design are diverse in composition and naturally overlap.

There is substantial research providing lists of design principles for mathematical tasks (e.g. Ahmed (1987), Hamilton, Lesh, Lester and Brilleslyper (2008) and Foster (2015)). Foster (2015), as an example of such lists, advises that tasks should be 1) enticing, 2) accessible yet challenging and 3) naturally extendable. Swan (2008) working with teachers developed five task types that encourage concept formation. It should be noted that the tasks to which he refers are intended as classroom activities rather than for completion as homework. He envisions students working collaboratively to solve tasks and verbalise their thinking. The five task types are classifying mathematical objects, interpreting multiple representations, evaluating mathematical statements, creating problems and analysing reasoning and solutions. The mathematical objects referred to can range from shapes to quadratic equations. By classifying mathematical objects, Swan suggests that students learn to distinguish and identify the properties of these objects. Swan asserts that these types of tasks assist students to develop mathematical language and retain definitions. He suggests that by encountering different representations of the same mathematical idea, students can draw links between representations and develop new mental images for concepts. When evaluating mathematical statements, students are encouraged to develop mathematical arguments, justify their viewpoint and devise relevant examples and counterexamples to defend their reasoning. In his view, requiring students to create problems reveals the processes behind mathematics rather than just focussing on solving exercises. Swan believes that the analysis of reasoning and solutions encourages students to recognise that there are several ways to approach and solve a mathematical problem. Swan (2005, p. 21) also calls for students to be asked to convince, explain and prove when engaged in mathematical activity.

In their book, Mason and Johnston-Wilder (2004, p. 109) provide a list of words they believe denote processes and actions that mathematicians employ when they are working with mathematical problems: "exemplifying, specializing, completing, deleting, correcting, comparing, sorting, organizing, changing, varying, reversing, altering, generalizing, conjecturing, explaining, justifying, verifying, convincing, refuting". It is suggested that the use of these words would enable students to gain a richer experience of the aspects of mathematical thinking. For designing tasks, language is important and it is important to encourage students to verbalise their mathematical thinking using terms like 'justify', 'explain' or 'verify'. Furthermore, Mason and Johnston-Wilder (2004, p. 6) recommend the use of a 'mixed economy' of tasks as no single strategy or task type has proved to be universally successful in developing mathematical thinking.

Swan and Burkhardt (2012) have suggested principles for mathematical task design suitable for use as assessment. They suggest that tasks should reflect the curriculum in a balanced way, meaning that students would ideally be given the opportunity to encounter all types of performance that the curriculum goals espouse. It is also recommended that tasks have 'face validity' and are fit for purpose; students should view them as problems which are worthy of solving due to being interesting or having a potential use. They also believe that tasks should be accessible yet challenging, thus enabling students of all different abilities to be able to demonstrate what they can do. Their principles call for reasoning to be rewarded rather than results, in other words that students should be encouraged to engage in a process of reasoning when considering how to solve a task. Designers are also advised to use authentic contexts, encouraging students to make connections within mathematics and other subjects. Interacting with such contexts ensures that students would better understand life and the world around them as a result. They also believe that students should be encouraged to select and choose their own methods with tasks that provide opportunities for making decisions. Finally they recommend that tasks should be transparent in their demands, so that students are clear as to what kind of response is expected and valued.

Watson and Thompson (2015) focus on design issues related to written tasks, typically found in textbooks. Three interrelated aspects are considered: 1) nature and structure of tasks, 2) pedagogic purpose of their design and 3) intended mathematical activity as embedded in tasks. In textbooks, the topic sequence is suggested by the authors and consideration must be given to necessary prerequisites and how tasks might build on each other. Watson and Thompson (2015, p. 150) point out that text-based tasks rarely support learners to develop the skill of self-checking through the use of mathematics to verify solutions. Very often the textbook authors retain the mathematical authority by providing an answer book. Watson and Thompson suggest that by creating tasks that encourage learners to self-check or incorporate multiple approaches, students are facilitated to gain some mathematical authority. They also recommend that tasks should lend themselves to conjecture and exploration so that the student can establish relationships and connections and engage in the dynamic world of mathematics. Watson and Thompson believe that pedagogic purpose can influence how a designer creates a sequence of tasks. However, the designer must concentrate on introducing learners to new ideas and be aware of how tasks can convey a view of what is valued in mathematics. In relation to intended mathematical activity, Watson and Thompson (2015, p. 170) call on designers to create tasks which allow for the key idea to be varied and learners are able to see this along with the effects of such variation in subsequent activity. They also observe that tasks can introduce teachers and learners to new ways of engaging with mathematics even if such opportunities are not availed of.

Sullivan, Clarke and Clarke (2013, p. 135) found that a teacher's choice and use of tasks are very important for effective mathematics teaching, and a task's characteristics can influence its potential for learning. They identified a number of key characteristics of tasks that should be incorporated in the design of tasks. In their view, students should be engaged in fostering meaning-making and establishing connections to other aspects of mathematics, when solving tasks. They recommend that tasks are challenging for most of the class, with the pathway to the solution not being clear to the students. Tasks should require students to think, make decisions and communicate. Additionally the authors recommend that contexts or situations should be relevant to the students so that the tasks are seen as potentially useful or connected to their lives. If a task is appropriately designed it will promote students' conceptual understanding of mathematics while catching and maintaining their interest and it will optimize their learning (Chapman, 2013 cited in Jonsson, Norqvist, Liljekvist and Lithner 2014).

2.7 Teacher Professional Development in the Creation and Use of Tasks

Task design has traditionally been viewed as curriculum developers creating mathematical tasks that ultimately get implemented by teachers as part of their classroom instruction (Jones and Pepin, 2016, p. 108). Pepin, Gueudet and Trouche (2013, p. 934) suggest that there may be several processes at work when teachers interact with curriculum resources such as tasks, these can be summarised as adoption, genesis and transformation. They state that the potential integration/inclusion into a teacher's 'normal' practice is an essential condition for the adoption of a textbook or piece of software. This view of an individual teacher's autonomy is shared by Ball and Cohen (1996, p. 6), they outline that teachers' understanding of material, their personal beliefs as to what is important, and their thoughts about students and the role of the teacher all influence and shape their practice. Silver, Ghousseini, Charamlambous and Mills (2009) suggest that a given text resource portrays mathematics in a particular way, which in turn can be rejected by a teacher if this does not correspond to his/her own views. However, Remillard, Herbel-Eisenmann and Lloyd (2011) believe that features of the curriculum resources can contribute to teacher development, and this in turn can bring about an evolution in the teacher's beliefs. Transformation refers to how tasks can promote teacher learning. Davis and Krajcik (2005, p. 5) suggest that tasks have the potential to promote a teacher's pedagogical design capacity, or his ability to use personal or curricular resources in order to adapt the curriculum so as to achieve productive instructional results.

Mathematical tasks have also been used as a focus of teacher professional development. Sullivan and Mousley (2001) have made the case that there should be more focus on supporting teachers in gaining an understanding of the complexity surrounding decision-making about classroom tasks. Clarke, Grevholm and Millman (2009) took up the cause in relation to primary mathematics teacher education, while Zaslavsky and Sullivan (2011) have taken a similar approach with post-primary teacher education. Arbaugh and Brown (2005) believed that classifying exercises in terms of their cognitive demand would enable teachers to examine their own practice. It was found that teachers amended their classifications of tasks over time to better reflect levels of cognitive demand. Stein, Smith, Henningsen and Silver (2009) prepared a case book for use in professional development that included a number of exemplary tasks, criteria for analyzing task properties, and several protocols to facilitate the discussion of tasks among teachers. Further work completed using such an approach (Boston and Smith 2009; Boston 2013) has identified changes in teachers' choice of task after completing professional development in relation to classifying tasks in terms of their level of cognitive demand. It also found that some teachers sustained such choice after a period of time had elapsed (Boston and Smith 2011).

Swan (2007) designed professional development which put emphasis on a number of task types that allowed participating post-primary mathematics teachers to examine their beliefs in relation to teaching. In several cases, it facilitated a transition towards a more student-centred approach to teaching. It also encouraged some teachers to establish more connections between mathematical concepts when teaching the subject. These studies demonstrate the transformative nature of tasks when used in a professional development situation.

Askew and Canty (2013) examined how primary school teachers, in collaboration with a task designer, worked with, and developed, classroom tasks as starting points for promoting students' reasoning in mathematics. They concluded that the introduction of a framework for working on, and with these tasks, and the treatment of this work as a joint venture, promoted teachers' professional practice and also fostered greater collegiality and stronger ties amongst the staff within a school. Knott, Olson, Adams and

Ely (2013) examined the characteristics of task design to which teachers needed to give attention when adjusting or modifying their existing lessons in order to engage students in learning new forms of generalisation and justification. It was found that once teachers were supported appropriately, in the context of professional development, the teachers could then independently create rich tasks which facilitated students gaining confidence in the areas of justifying and generalising. Even and Olsher (2014) studied teachers as participants in textbook task development and they found that teachers had divergent views on the preparation of a textbook. Most teachers that they worked with would happily take on the role of writing a textbook, mainly informed by the knowledge that they had built up from their own teaching experience. In contrast, they found that some of the teachers were cautious and uncomfortable with making changes to a textbook written by expert curriculum developers due to deference to the expertise of the textbook authors. Coles and Brown (2016), working in a teacher–researcher partnership, recommended particular design principles based around 'the making of distinctions' accompanied by an explicit language of mathematical thinking in task design. Their aim was to bridge the persistent gap that exists between teachers' intentions and students' activity. Johnson, Severance, Penuel and Leary (2016, p. 169) pointed out that when teachers engage in task analysis, it becomes apparent that designers and participating teachers have different goals in terms of professional development. Through working together on mathematical tasks, teachers and researchers participated as stakeholders in a co-design process. They noted that tensions were evident due to a lack of design consensus and such tensions affected participation and learning. In particular, Johnson et al. (2016, p. 182) highlight that the reality of a teacher's instructional situation can make the implementation of tasks difficult. In order to understand tasks in the institutional context, an approach that goes beyond simple delivery of professional development from researchers to teachers is necessary However, they viewed these tensions as healthy and recommended that the successful confrontation of design tensions as part of a collaborative design process could yield new task adaptation and implementation practices that could prove to be sustainable across an educational system.

Chapter 3 Theoretical Frameworks

3.1 Introduction

This chapter will outline the theoretical frameworks that are employed in this study. I will begin by establishing the importance of mathematical tasks within instruction by looking at Herbst and Chazan's (2012) extension of the instructional triangle (Cohen et al., 2003), based on the concepts of didactical contract and milieu (Brousseau, 1997) which are used to describe the relationship between teachers, students and knowledge at stake through instruction. It is this theoretical framework that motivates my overall study of mathematical tasks at second level in Ireland. Five frameworks used in this PhD study's analysis of tasks and their sub classifications are also outlined in this chapter. The Levels of Cognitive Demand (as described in Smith and Stein, 1998) and mathematical reasoning (Lithner, 2008) frameworks are described first. The Project Maths syllabus problem-solving objectives are then described to give some insight into the goals of the syllabus, followed by an outline of how these objectives were adapted for use as a framework for the analysis of tasks. An account of Usiskin's (2012) multidimensional model of mathematical understanding is also provided. After examining the criteria for each of these four frameworks, the kind of insight that each framework offers to the researcher is discussed. Consideration is given to the similarities and differences between these frameworks. The need for the 'novelty' of tasks to be given special consideration becomes apparent and thus a framework to classify the novelty of tasks has been designed.

3.2 The Instructional Triangle

Herbst and Chazan (2012, p.601) argue that by looking at the nature of instruction, it is possible to justify actions that are taken in mathematics teaching. Cohen et al. (2003, p. 122) describe instruction as the 'interactions among teachers and students around content in environments'. This definition puts the focus, when examining instruction, on how resources are used rather than identifying whether resources are present or not. Predating this definition, according to Herbst and Chazan (2012, p.601), instruction was traditionally conceptualised as the set of resources that could be found in the environment where learning was intended to take place, such resources could include features like student characteristics, teacher quality or materials used in the classroom.

The presence or lack of these resources could then be used to account for any perceived differences in learning. Accompanying Cohen et al.'s definition of instruction is a diagram, shown below in Figure 3.1, that has become popularly known as the 'instructional triangle'.

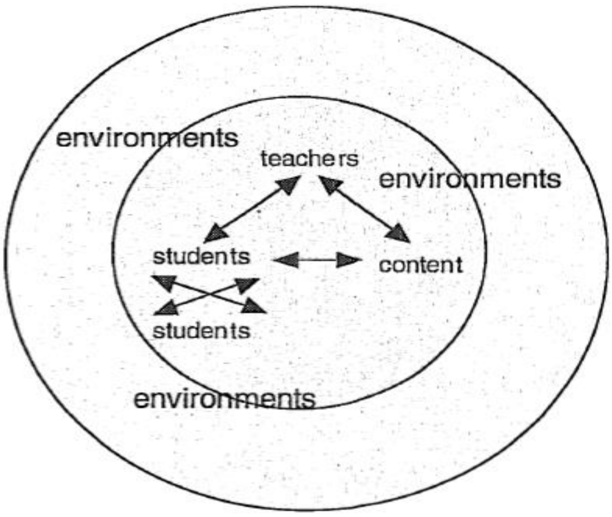


Figure 3.1 Instructional triangle from Cohen et al. (2003, p. 124)

As can be seen from the diagram, teachers, students and mathematical content are situated at the three vertices of this instructional triangle. However, they are also influenced by and interacting with the environments in which they are immersed.

The instructional triangle can be situated within a distinct tradition of mathematics education. Herbst and Chazan (2012, p. 602) describe how 'Didactique des Mathematiques' is a French theory of mathematical education which is used to refer to the study of the dissemination of mathematical knowledge, with a particular emphasis on the study of teaching. The term also encompasses the study of the transformations produced on mathematical knowledge by those learning it in an institutional setting. Brousseau (1997) put forward a theory of didactical situations, which studies the complexity inherent in any situation involving the interaction of teacher, student and content. This is in effect a three way schema. Brousseau's theory aims to single out relationships that emerge in the interaction between learners, mathematical knowledge to be learned and the milieu. Sriraman and English (2010, p. 22) describe these milieu as typically including 'other learners, the concepts (previously) learned by students as well as prior conceptual machinery present in the student's repertoire and available for use'. Brousseau (1999), as cited in Sriraman and English (2010, p. 23), described his theory as a 'means not only of understanding what teachers and students are doing, but also of producing problems or exercises adapted to knowledge and to students, and finally a means of communication between researchers and with teachers.'

Tasks have a special role within this theoretical model. The didactical contract, as described by Brosseau (1997, p.32) and cited in Herbst and Chazan (2012, p. 602), views the teacher as creating 'sufficient conditions for the appropriation of knowledge' and this appropriation must be recognised when it occurs. The student is meant to satisfy these conditions and learn the intended material. Successful instruction is thought to occur when all aspects of the contract work together. For Herbst and Chazan (2012), tasks are viewed as the means by which student interact and work with mathematical content. They define a mathematical task 'as the engagement of students in actions with and against a milieu' (2012, p. 607). The milieu in this conceptualization includes the goal that students are working toward and the resources (including tasks) with which students are operating. They add that as students work on mathematical tasks, the role of the teacher includes supporting that work by 'ensuring that the milieu functions as expected' (Herbst and Chazan, 2012, p. 607). Teachers' actions in this regard can be judged according to the norms of the task. They provide the example of a calculator to illustrate this point. If a task requires the students to use a calculator then it may be necessary for the teacher to ensure that the calculator has batteries and the keypad is functional. In this sense, the teacher performs actions to support the function of the milieu. Such actions can then be justified or critiqued based on the norms of a task. Herbst and Chazan (2012) modify the instructional triangle to demonstrate how a mathematical task and its norms may justify a teacher's action.

The instructional triangle shows the importance attached to tasks in the form of mathematical work that must be completed by students. Herbst and Chazan further developed the concept when considering the exchange between students' mathematical work and the knowledge at stake and how this exchange is managed. They suggest that teachers must not only sustain students' work but also design or choose the mathematical task that students will work on. This also includes observing the work, gathering evidence that students have studied and ultimately learned a specific item of knowledge. The norms of an instructional situation serve to justify a teacher's preference for one action over another. An example of such norms would be when a teacher encourages a student's efforts by inserting an extra step into the solution method when solving an equation, yet cautions against oversimplification. Figure 3.2 represents this kind of intervention by introducing the norms of an instructional situation into the existing instructional triangle model.

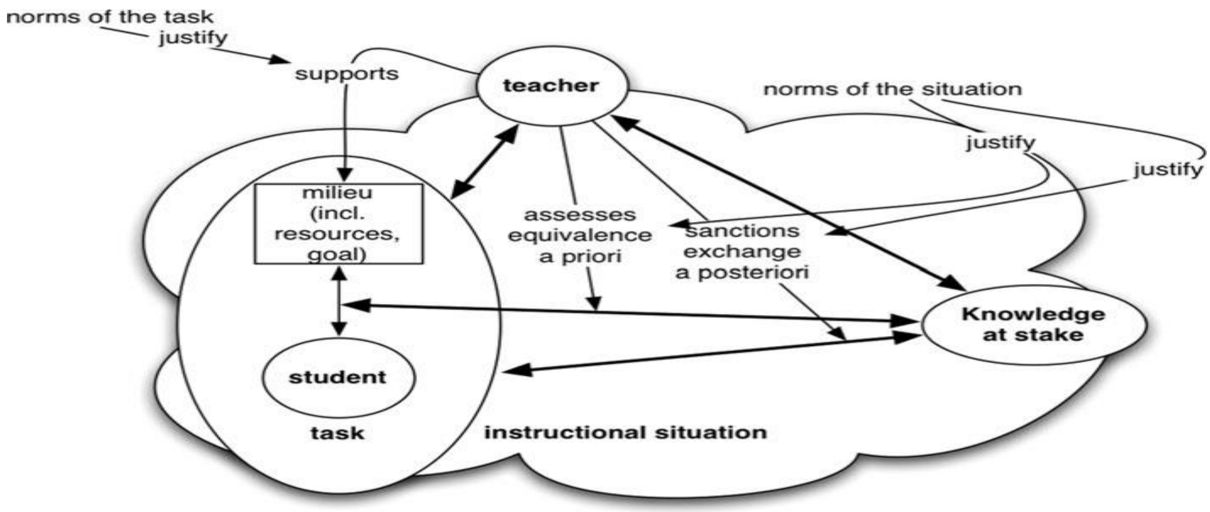


Figure 3.2 Elaboration of the instructional triangle (Herbz and Chazan, 2012, pg. 609)

This theoretical framework brings particular attention to new instructional situations¸ that are developed through teacher and students' negotiation of how to deal with the breaches that a novel task creates in an existing instructional situation. One of the main recommendations of Herbst and Chazan's elaboration of the instructional triangle is that attempts to improve instruction could be operationalised in terms of engaging students in novel tasks. For them, a novel task is a means for encountering new ideas and operating with them within the milieu. They suggest that novel tasks would both build 'on existing instructional situations and breaching with some of their norms' (p. 611). The novelty of tasks is an important consideration in this study and a framework I developed for the identification of novelty will be described later in this chapter.

Ellis, Hanson, Nunez and Rasmussen (2015), influenced by the adapted instructional triangle of Herbst and Chazan, studied universities with calculus programmes perceived to be successful and considered the role of homework systems within these programmes. They compared these programmes with other universities outside of their study in an effort to identify differences in their homework systems. They situated the system of giving and completing homework within the broader context of the instructional environment. This enabled them to consider how relationships between students, the calculus course instructor, and the mathematical content interact with the homework system, and how all of these interactions are governed by the didactical contract between the instructor and students.

Their findings indicated that homework and group projects were assigned more frequently in the universities that they selected compared to those that were not chosen for the study. Also their study suggested that online homework systems were availed of more commonly in the selected institutions, such homework was graded more frequently and returned with feedback from instructors thus creating more interaction between teacher and students and allowing for instruction outside the classroom, and the content assigned tended to focus on material that involved more novel, cognitively demanding tasks. Ellis et al. (2015, pg. 285) suggested that allowing students to attempt homework tasks multiple times, helped students to gain confidence as it demonstrated that they could persist and eventually find the correct solution. Students were able to experience heightened success through the completion of homework, something that might not have been possible within the classroom due to time constraints. Instructors also valued homework because it not only gave further opportunities for engagement with mathematical concepts and applications, but it also allowed students opportunities to explain their thinking when solving tasks. Ellis et al. point out that at second-level in the United States, the majority of the interactions between student, teacher, and content occur within the classroom. However they suggest that at university, many of these interactions take place outside of the lecture hall. 'Thus, the homework system plays a heightened role in undergraduate mathematics because it acts as the milieu for these interactions to occur both inside and outside the classroom' (Ellis et al., 2015, p.286).

In the context of my PhD study, it is assumed that Irish second level teachers generally guide students through the development of knowledge with tasks mainly completed in class but some are also assigned for homework. Given the findings of Ellis et al. (2015) above, tasks are worthy of analysis because they are an important part of the milieu of mathematics instruction that takes place both inside and outside the classroom. However unlike Ellis et al., the analysis here will focus on the nature of mathematical tasks as opposed to how they are used or assigned by teachers.

Taking the instructional triangle adapted by Herbst and Chazan (2012) as a conceptual framework, this study is concerned with the importance of mathematical tasks and their role in supporting the construction of knowledge. At this level, tasks are used by teachers to provide students with opportunities to develop understanding and/or skills and the responsibility for the construction of knowledge mainly rests with the teacher. The relationships that exist between the tasks, the teacher and the knowledge at stake all function within the larger environment of the school. Teachers use the tasks to plan class activity and set homework. It is the teacher who decides what aspects of content to emphasise through the tasks assigned and what content will be covered. Studying the nature of mathematical tasks readily available at second level in Ireland is important because it gives some insight into how students are expected to construct their knowledge and the interactions that might take place between the teacher, the student and the content.

3.3 Description of Frameworks

3.3.1 Levels of Cognitive Demand

It has been argued that it is important to examine the cognitive demand required by tasks because of their influence on student learning as it determines ‘how they come to think about, develop, use and make sense of mathematics’ (Stein, Grover and Henningsen, 1996, p. 459). The framework used by Stein et al. describes four levels of cognitive demand for tasks: lower level demands of ‘memorization’ and ‘procedures without connection to meaning’, and higher level demands of ‘procedures with connection to meaning’ and ‘doing mathematics’. A description of each level of the framework has been taken from Smith and Stein (1998) and is outlined in Figures 3.3 and 3.4. These descriptions were carefully adhered to when looking at the tasks selected from each textbook.

3.3.1.1 Lower-level demands

Tasks of lower-level cognitive demand can fall into one of two categories; those of 'memorization' and 'procedures without connection to meaning'. Tasks involving 'memorization' require the reproduction of previously learned facts or definitions or committing rules and formulae to memory. They cannot be solved using procedures either because a procedure does not exist or because the timeframe is too short in order to employ one. Such tasks involve the exact reproduction of previously seen material and there is often a clear unambiguous direct statement as to what should be reproduced. A distinctive feature of this category is that there is no connection to the concepts or meaning that underlies the facts, rules or definitions reproduced in response to that task.

Levels of Demands

Lower-level demands (memorization)

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connections to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.

Lower-level demands (procedures without connections to meaning)

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlies the procedure being used.
- Are focused on producing correct answers instead of developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.

Figure 3.3 Lower-level cognitive demand (Smith and Stein, 1998, p.348)

Higher-level demands (Procedures with connections to meaning)

- Focus students' attention on the use of procedures for the purposes of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and develop understanding.

Higher-level demands (Doing Mathematics)

- Require complex and nonalgorithmic thinking – a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instruction, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experience and make appropriate use of them in working through the task.
- Require students to analyse the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Figure 3.4 Higher-level cognitive demand (Smith and Stein, 1998, p.348)

The second category 'procedures without connection to meaning' may request the use of a particular procedure or such use is evident from prior experience or the phrasing of the task. There is limited cognitive demand required and little ambiguity surrounds what the task entails and how it should be completed. There is no connection to the mathematical concepts involved and the meaning underlying a procedure is not drawn out. Such procedural tasks are more focused on producing correct answers than on developing mathematical understanding. Explanations, if asked for, focus only on the procedure being used and not the thinking behind it.

3.3.1.2 Higher-level demands

Higher-level demands are also divided into two different areas; 'procedures with connections to meaning' and 'doing mathematics'. Again the focus is on the level of cognitive engagement that the task demands. Such tasks cannot be completed without a degree of cognitive effort, and the underlying conceptual ideas must be engaged with. A task described by the framework as involving procedures with connections to meaning tends to focus the users' attention on using procedures in such a way as to develop deeper understanding of mathematical concepts. Multiple representations could be required, including the use of visual diagrams, manipulatives, symbols and situations involving the solving of problems. The development of meaning is or may be made possible by making connections between several representations of the same concept.

'Doing mathematics' is the most demanding of the levels identified in this framework. A task requires the doing of mathematics when complex thinking is required. A predictable well-rehearsed approach is not suggested by the task; rather the exploration and understanding of mathematical concepts are encouraged. The solution process can be unpredictable and very often requires further analysis of the task and its constraints. Such constraints could limit the possible approaches to finding a solution.

3.3.2 Creative and Imitative Mathematical Reasoning

Lithner (2008) characterises key aspects of reasoning. His work provides a dichotomy between what is termed as imitative reasoning and creative reasoning as outlined in Figure 3.5. Creative reasoning must fulfil some of the following requirements – namely novelty, flexibility, plausibility and being fixed in appropriate mathematical foundations. Imitative reasoning instead relies on using well-rehearsed methods or the memorisation of procedures. This type of reasoning is further broken down into three subcategories: familiar memorized or algorithmic reasoning, delimiting algorithmic reasoning, and guided algorithmic reasoning.

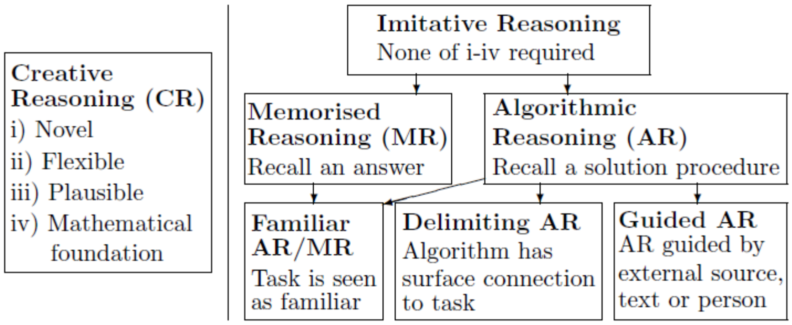


Figure 3.5 An overview of reasoning types (Lithner, 2006, p.5)

3.3.2.1 Creative Reasoning

For a task to be deemed to require creative reasoning for its successful completion, it should display at least one of these four criteria: novel, flexible, plausible and mathematical foundation. It can be seen that creative reasoning shares certain similarities with the higher-level demands of the LCD framework. In Lithner's framework, when a task is novel, it requires complex thinking in order to come up with an approach or suitable method to find a solution. A flexible approach is required when unexpected constraints are encountered or different approaches to its solution must be incorporated. Reasoning is plausible when it is supported by arguments in favour of a particular solution strategy choice and the conclusions drawn from the solution of a task are based on mathematical foundation. The term 'mathematical foundation' refers to a connection being made to the concepts or ideas underlying a task.

3.3.2.2 Imitative Reasoning

If a task does not require any of the four criteria for creative reasoning, then it is said to necessitate the use of imitative reasoning. The framework provides further classifications for imitative reasoning. The first of these categories is named memorised reasoning, where the task requires the recollection of an answer. The second category is termed algorithmic reasoning and these two categories have a common subcategory known as familiar memorised reasoning/familiar algorithmic reasoning. When the task is seen as familiar, there is little thought put into how to attempt to find the solution, a well-rehearsed procedure or algorithm is evident for familiar algorithmic reasoning while a common answer or fact that has been committed to memory is given for familiar memorised reasoning. Algorithmic reasoning has two further subcategories, namely delimiting algorithmic reasoning and guided algorithmic reasoning. Delimiting algorithmic reasoning is used when an algorithm has a surface connection to the task and is used once the required link is established. Guided algorithmic reasoning is used if the task has an approach guided by a particular source. For example there is a hint provided on how to approach the task or the first line of the solution has been provided. Thus, this framework allows tasks to be categorised in terms of the type of reasoning that is required when seeking a solution.

3.3.3 Project Maths Syllabus Objectives

The National Council for Curriculum and Assessment has set out seven overarching objectives for students of Leaving Certificate mathematics (NCCA, 2012, p. 6). Figure 3.6 lists these and it identifies what learners are meant to attain having studied the syllabus.

Project Maths Syllabus Objectives

O. 1 The ability to recall relevant mathematical facts.

O.2 Instrumental understanding ("knowing how") and necessary psychomotor skills (skills of physical coordination).

O.3 Relational understanding ("knowing why").

O.4 The ability to apply their mathematical knowledge and skills to solve problems in familiar and unfamiliar contexts.

O.5 Analytical and creative powers in mathematics.

O.6 An appreciation of mathematics and its uses.

O.7 A positive disposition towards mathematics.

Figure 3.6 List of Project Maths learning objectives in the Leaving Certificate syllabus.

Given the aspirational nature of these learning objectives, it is difficult to quantify a response to these objectives or use them as a framework. However, the development of synthesis and problem-solving skills is outlined in every content strand of the syllabus through the list shown in Figure 3.7 below (NCCA, 2012, p. 20). This lends itself more easily to the classification of mathematical tasks.

1) Explore patterns and formulate conjectures.
2) Explain findings.
3) Justify conclusions.
4) Communicate mathematics verbally and in written form.
5) Apply their knowledge and skills to solve problems in familiar and unfamiliar contexts.
6) Analyse information presented verbally and translate it into mathematical form.
7) Devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Figure 3.7 List of synthesis and problem-solving objectives in the Leaving Certificate syllabus.

Information presented by textbook tasks is generally encountered using the written form and so the sixth objective shown in Figure 3.7 is modified by changing "verbally" to 'in written form' before using it for classification. To classify the tasks as precisely as possible, it is also necessary to break some of the other objectives down further, especially those which cover multiple situations. In particular with reference to contexts, it is more useful to make the division between familiar and unfamiliar as it allows one to see if students are working with familiar or unfamiliar material and/or settings rather than just identifying that the task is set in a context.

One of the goals of the revised curriculum was to introduce more unfamiliar contexts into the teaching and learning of mathematics. An important separation also has to be made between the very different actions of devise, select and use. Again, this would give more insight into the composition of a set of tasks rather than just working with one general heading. An amended set of objectives is outlined in Figure 3.8.

Using the amended list allows one to classify what kind of learning opportunities are offered to the student when completing tasks. It is possible for a task to meet more than one of these objectives and completing analysis using the amended list gives a clear picture of whether tasks are meeting the Project Maths curricular goals in relation to synthesis and problem-solving.

<table>
<tr><th>Original List</th><th>Amended List</th></tr>
<tr><td>1) Explore patterns and formulate conjectures.</td><td>1) Explore patterns and formulate conjectures.</td></tr>
<tr><td>2) Explain findings.</td><td>2) Explain findings.</td></tr>
<tr><td>3) Justify conclusions</td><td>3) Justify conclusions.</td></tr>
<tr><td>4) Communicate mathematics verbally and in written form.</td><td>4) Communicate mathematics in written form.</td></tr>
<tr><td>5) Apply their knowledge and skills to solve problems in familiar and unfamiliar contexts.</td><td>5) Apply knowledge and skills to solve problems in familiar contexts.
6) Apply knowledge and skills to solve problems in unfamiliar contexts.</td></tr>
<tr><td>6) Analyse information presented verbally and translate it into mathematical form.</td><td>7) Analyse information and translate it into mathematical form.</td></tr>
<tr><td rowspan="3">7) Devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.</td><td>8) Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.</td></tr>
<tr><td>9) Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions</td></tr>
<tr><td>10) Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions</td></tr>
</table>

Figure 3.8: Original/Amended list of Project Maths problem-solving syllabus Objectives

3.3.4 Multidimensional Model of Mathematical Understanding

Usiskin (2012) deals with the understanding of a concept in mathematics from the standpoint of the learner. Five dimensions of this understanding are outlined in his framework: the Skill-Algorithm dimension, the Property-Proof dimension, the Use-Application (modelling) dimension, the Representation-Metaphor dimension, and the

History-Culture dimension. These dimensions are not presented in a hierarchy and one aspect is not meant to precede another.

The first dimension Skill-Algorithm looks at the algorithms that are required in learning a concept and the choice of a particular algorithm because it is more efficient than other algorithms known. This dimension acknowledges that there is more to procedural understanding than just applying an algorithm. Usiskin argues that 'the understanding of procedures is not so lower-level at all' (Usiskin, 2012, p.5). While skill and procedural understanding are often thought of as lower order forms of thinking, this framework recognises the deeper level of understanding required to choose a particular algorithm when many are known, due to it being more efficient than others. Property-Proof understanding identifies the mathematical properties that underlie a concept. This aspect of learning looks beyond arbitrary rules and considers the mathematical theory behind them. The Use-Application understanding focuses on how a concept can be used in some way or its applications. The Representation-Metaphor dimension encourages the representation of a concept in some way. Finally History-Culture understanding concerns itself with the how and why of the development of a mathematical concept over time. The History-Culture dimension is an aspect that is identified as necessary for the 'real true' understanding. The premise being that those who study the history of mathematics or cross-cultural mathematics obtain an understanding of mathematical concepts that is different from the other dimensions.

3.4 Existing Frameworks:

It is useful to consider each individual framework and the kind of information that it provides to the researcher. A better understanding of the information gathered through the frameworks gives an insight into the results that are produced. It should be noted that Usiskin's model and the Project Maths problem-solving objectives were not originally intended to be used as a classification scheme but were adapted for the purposes of this PhD study. In contrast, the LCD and reasoning frameworks are established frameworks which have been used elsewhere. As outlined in chapter 2, the LCD framework has been used to analyse textbooks in the US (Jones and Tarr, 2007), Turkey (Bayazit, 2013) and Korea (Kim, 2014). Lithner's reasoning framework has been confined mainly to Sweden, where Bergvist et al. (2008), Boesen et al. (2010) and Sidenvall et al. (2015) have made use of it to analyse upper secondary school tasks or their equivalent. Although Jader et al. (2015) conducted an analysis of tasks taken from textbooks in 12 different countries. While there are many similarities between the frameworks, they often approach ideas in different ways depending on the particular focus or interest of the specific framework. The benefit of using several frameworks is that the researcher gets a much richer account of the demands set and opportunities provided by a particular task.

3.4.1 Level of Cognitive Demand

The LCD framework has two advantages in terms of the information that it provides. It shows the level of cognitive demand that an exercise has, and it encourages the researcher to determine why an exercise is lower or higher level specifically. For example a student could be using procedures in any exercise but the framework allows you to specify whether the procedure requires the student to actively engage in mathematics or if the procedure employed is simply manipulating something to produce an answer without a true meaning for the student.

The aspects of tasks that justify their categorisation into lower or higher level are also identified. For example, in order to settle on a classification it is necessary to determine whether an explanation of thought processes in finding a solution is required, if multiple representations are employed or whether a task can be completed easily without great effort or not. However, the LCD framework does not give enough insight into the experience of the user when solving a task. It makes reference to previously learned facts and previous instruction or experience but is not explicit on how this should be determined. It focuses on the structure of the task and what it demands of the user, and thus allows the classification of tasks on their own merits. If a task is looked at in isolation, it can appear to be very demanding but if a user has completed several such tasks previously, then the true level of cognitive demand must be diminished to some degree and this is not taken into account in the framework. A measure of familiarity or novelty is needed so as to determine the previous experience that the user brings to a task.

3.4.2 Imitative/Creative Reasoning

While Lithner's reasoning framework considers the type of reasoning that a task promotes, it does not take into account the experience that is brought to the task. While it speaks about a task being novel, it does not provide much detail in terms of how a task is categorised as such. Similar to the LCD framework, the use of the reasoning framework in classifying textbook tasks does not address the impact that other material such as examples or previous exercises in a textbook can have on the classification in terms of the user's experience and how these may influence the reasoning applied to a task. Again, a measure of novelty or familiarity is needed to ensure that the solver's prior experience is taken into account.

3.4.3 Multidimensional Model of Mathematical Understanding

Applying Usiskin's framework considers the understanding of the learner as opposed to simply the structure of the task. In practice it is more informative to use this framework in terms of all tasks on a topic rather than a specific task. The framework can be very narrow when looked at in terms of one task while more of an overview is possible when it is used in connection with a whole set of tasks. As each of the dimensions looks at a different aspect of understanding, it is important to look at an entire set of tasks as a single one does not give an accurate account of the kind of understanding that is being created. The first two dimensions of the framework match well with categories of the LCD and IR/CR frameworks. The Skill-Algorithm dimension ties in with the lower-level demand of 'procedures without connections to meaning' and is broadly in line with imitative reasoning. The Property-Proof dimension is most closely aligned to the higher level demands of the LCD framework and brings the student to the mathematical properties that underpin a task. It is useful to consider the different ways that a question can be approached using procedures and algorithms and whether the user actually requires or develops an understanding of the mathematics that underpins a topic.

Usiskin's multi-dimensional framework goes beyond the how and why of understanding. It is not limited to a consideration of instrumental and relational understanding. A number of other dimensions are introduced to achieve this. The Use-Application dimension involves identifying the use of concepts or the understanding that accompanies knowing when to use or apply a particular concept. It is clear within the framework that applications involve a different kind of thinking, not necessarily a higher order one. The use of such modelling is not something that is explicitly identified in the LCD or reasoning frameworks. The LCD framework makes reference to problem situations but it is not clear if this refers to applications or real-world contexts.

The other two dimensions, Representation-Metaphor and History-Culture, look at the different ways that students gain understanding. Using this framework it is necessary to consider whether the student makes a connection between existing representations and using a different kind of representation or metaphor such as a graph or diagram and whether he/she is given historical background or cultural reference for the material covered. Usiskin's framework puts the learner's experience specifically into the spotlight and also looks at more aspects than simply the divide of higher versus lower cognitive demand.

The History-Culture dimension is not identified in any way in either the LCD or creative reasoning frameworks. However, it could be seen to contribute to one of the Project Maths overarching objectives of the development of 'a positive disposition towards mathematics' (see Figure 3.6).

3.4.4 Project Maths Problem-Solving Syllabus Objectives

The learning objectives of the Project Maths problem-solving syllabus allow the researcher to see which aspects of the syllabus goals are being addressed by tasks. It would be reasonable to expect that all problem-solving objectives would be attended to adequately by a textbook. It would be useful to have a method to distinguish between familiar and unfamiliar contexts. For the problem-solving syllabus objectives, this would allow easier classification between the objectives of 'Apply knowledge and skills to solve problems in familiar contexts' and 'Apply knowledge and skills to solve problems in unfamiliar contexts'. Similarly when distinguishing between the three objectives use, select or devise 'appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions', it would be useful to have some measure of familiarity. 'Devise' would be associated with novel scenarios while 'select' and 'use' would be used with more familiar tasks.

3.5 Task and Solver

The majority of the frameworks described are very much concerned with the structure of tasks; it would be useful to give more consideration to the experience of the user when classifying tasks. Berry, Johnson, Maull and Monaghan (1999, p. 109) encounter an interesting issue when attempting to characterize routine questions. In their words, they see the implication that routineness is 'located in a question rather than being a psychological construct of the relation between an individual, or group and a question' as problematic and conclude that the psychological relation is likely to be a socio-psychological one. In other words what is routine for one individual is not necessarily so for another. Selden, Mason and Selden (1989, p. 45) go some way towards acknowledging this aspect of human experience by identifying two components in a problem namely task and solver. A problem is defined by Selden et al. as a non-routine or novel task if completing it requires *finding* a method of solution and carrying out such a method. Selden et al. (1989, p.45) describe a solver as 'usually a person, but possibly a group of persons or a machine'. Selden, Selden and Mason (1994, p.67) take the view that while some problem-solving studies do not explicitly mention the solver, it is essential to consider the solver and the skills and information that is brought to a task when examining a mathematical task. They point out that 'tasks cannot be classified as problems independent of knowledge of the solver's background' (p. 67).

They also make a distinction between problems and exercises, stating that exercises are typically found at the end of sections in textbooks and claim that traditional calculus courses contain few cognitively nontrivial problems. The implication is that exercises test routine skills while problems examine the ability to cope with non-routine material. The term 'cognitively nontrivial' is given to tasks where the solver does not begin knowing a method of solution. Selden et al. (1989, p. 46) note that the solving of such cognitively nontrivial tasks or problems, as opposed to exercises, necessitate the use of skills and they identify that there is a tendency when such tasks are presented in textbooks for the problems to be divided into smaller parts whenever possible. These smaller parts are accompanied by the use of algorithms, sample solutions and examples wherever possible. The use of these devices results in problems, that were intended to be novel and challenging, becoming routine. Selden et al. describe solving a problem as finding a method of solution, possibly an algorithm, and carrying it out. Most importantly, Selden et al. (1989, p.45) highlight the fact that the solver 'comes equipped with information and skills, perhaps misconceptions, for attempting the task.' The result being that novel problems cannot be solved twice by the same person without a loss of novelty, as the solver would possess a method for solution the second time.

Berry et al.'s (1999) and Selden et al.'s (1989) work suggests that a solver's previous experience is worthy of attention. It is clear that this is something that should be considered when attempting to classify tasks using various frameworks. The Levels of Cognitive Demand framework acknowledges the notion of the solver's familiarity with a task by making reference to 'prior experience' but does not provide explicit criteria for judging how such experience can be gauged. Similarly, Lithner's characterisation of imitative and creative reasoning refers to the presence of novelty and tasks being seen as familiar by the solver. However an explicit measure of familiarity is not provided. When looking at the Project Maths Objectives, it is useful to have an insight into the novelty of a task when determining whether it is a problem or not. The objective 'apply knowledge and skills to solve problems presented in familiar/unfamiliar contexts' has been divided into two objectives as to whether the contexts involved are familiar or unfamiliar. A measure of novelty would facilitate making such a distinction as to whether contexts are considered familiar or unfamiliar. At the start of this chapter, I identified one of the main recommendations of Herbst and Chazan's (2012) elaboration of the instructional triangle is that improvement in instruction could be brought about by engaging students in novel tasks. Given the importance attached to the solver's previous experience by Selden et al. and the use of novel tasks by Herbst and Chazan, it appears that there is a need for greater consideration of novelty and its measurement in terms of what experience the solver brings to a task.

3.6 Creation of Novelty Framework

The motivation to design a framework which allows for the novelty in a task to be measured, based on the experience built up by the solver, has been described above. I searched for a framework to measure novelty that could be used to classify textbook tasks but could find no evidence of one in existence at that time. Prior to this, I constructed a simpler index for familiarity of questions on Mathematics state examinations, using the number of times a topic had occurred in preceding years (O'Sullivan, Breen and O'Shea, 2012).This experience was then built upon to create a novelty framework. In doing so, it was necessary to consider a number of elements before finalising the criteria for classifying tasks.

To start a pilot set of tasks from each of the textbooks was considered. For each task in this set, the features of the task were identified clearly. These included whether the response to a task involved using a procedure or formula, proving a mathematical statement, constructing or interpreting a diagram, using a definition, investigating mathematical properties and so on. Having considered these features of the tasks and examined the skills necessary for solution, the experience gained by the solver from expository material, the preceding examples, and previous exercises within the same textbook chapter was considered. Definitions, explanation of key words provided, exemplars demonstrating key concepts and illustration of methods of solution for problems were all examined. Previous chapters were not inspected as, anecdotally, it is not usual in Irish schools for the teacher to follow the order of the textbook in a linear fashion and thus it is not possible for us to say which chapters a student would already have encountered. Also for practical reasons, a limitation had to be placed on the amount of material that was to be examined. In all three textbook series studied here, the topic of Pattern, Sequences and Series is covered in a single chapter while Differential Calculus was spread over several chapters. Of course, students could have other experiences but the taxonomy presented here is confined to the material contained in one textbook chapter only. My two supervisors and I classified the pilot tasks independently. Then the three sets of individual classifications were compared and elements influencing the choice of classifications were discussed. This analysis was carried out repeatedly using other sample sets of tasks on various topics, and in different chapters of each textbook in order to consider all scenarios that could be encountered when classifying tasks for novelty. The criteria for classification were revised as necessary to account for all new aspects of novelty observed. In this way, the three classifications of 'novel', 'somewhat novel' and 'not novel' as presented in Figure 3.9 were eventually agreed upon. It was decided that individual questions which were presented as multiple parts of a textbook task would be considered as stand-alone tasks. Only preceding tasks and exemplary material are to be considered when looking at the impact of earlier material on tasks that make up multiple parts of a single question.

3.7 Description of Framework for Measuring Novelty

For this framework, skills are taken to refer to the methods and techniques used in the solutions to tasks.

Novel (i) Skills involved in finding the solution are not familiar from preceding exercises or from any previous point in the chapter being analysed. (ii) The mathematical concept involved is not familiar from previous exercises or examples. (iii) Significant adaption of the method outlined in examples and exercises must be made in order to get the required solution.
Somewhat Novel (i) The presentation of the task makes the question appear unfamiliar. However its solution requires the use of familiar skills. (ii) The context (perhaps the use of an unfamiliar real-world situation) makes the task appear unfamiliar but familiar skills are used in its solution. (iii) A new feature or aspect of a concept is encountered but the solution to the task only involves the use of familiar skills. (iv) A minor adaption of the method outlined in the examples has to be made in order to get the required solution. The skills required are familiar but the use or application of such skill is slightly modified.
Not Novel (i) The presentation, context and concepts of the task are familiar. (ii) The solution to the exercise or problem has been modelled in preceding exercises or has been encountered earlier in the same chapter. (iii) The skills required are very familiar to the user and the method of solution is clear due to the similarity between the exercise and preceding examples and exercises.

Figure 3.9: Framework for classification of novelty in tasks encountered in textbooks.

Please note it is not necessary for all characteristics in the description of the categories to apply in order for a task to be classified under a particular label. However as many characteristics as possible should be identified before settling on a particular classification and there should be sufficient evidence to justify the placement of the task in one category rather than another.

The following is an outline of each of the three terms used in the novelty framework and described briefly in Figure 3.9. It gives a description of how the researcher can differentiate between the different categories.

Novel

A task is said to be 'novel' when, to find its solution, the solver requires skills or mathematical concepts that have not been covered in preceding tasks or at any previous point in the chapter being analysed. There cannot be any substantial similarity between the given task and the previous examples or tasks in this case.

A 'novel' task should be original and essentially demand a new form of thinking from the solver that has not been encountered before within that chapter. This includes a situation where a task requires a different method of solution to the one the solver is familiar with from the contents of the chapter, or when the solver has to make a significant change or alteration to the method that is familiar in order to find the required solution.

Once a 'novel' question has been encountered, it diminishes the novelty of any similar question that follows it. While a task may be 'novel' when first encountered, any similar task that follows it or which requires a similar approach can no longer be classified as 'novel'.

Somewhat Novel

If there is only a superficial difference between preceding examples and the task in hand, then it is labelled as 'somewhat novel'. For instance, the presentation of the question may be different to preceding examples but when the solver goes about solving the task, it is apparent that the skills required are quite familiar from the preceding exposition, examples or earlier tasks in the chapter. Such differences can occur when a task is presented in a different context, such as a real-world scenario, which can serve to render the task unfamiliar to the solver initially. There can be some difficulty for the solver when first encountering the task but this is diminished once the familiar material is identified. Such variation necessitates the creation of an intermediate category to acknowledge that a task can have unfamiliar aspects but relies on skills that are actually quite familiar at that point in the student's experience.

Not Novel

An exercise or task is 'not novel' if its solution has been dealt with in preceding exercises or other examples in previous parts of the chapter. The task could be a direct repetition of material covered in examples or very similar to it. If the material is not covered in the examples immediately preceding a set of exercises, but yet the solver has experience of it from an earlier part of the chapter, then the task would be classified as 'not novel'.

By using these criteria, it is suggested that it is possible for the researcher to get a clearer picture of what a solver has been exposed to in terms of the textbook examples and previous tasks. This gives an insight into the degree of novelty present when completing tasks. Such data complements the information provided by the other frameworks as it allows one to look not only at a task but also at the background against which it is set. This framework allows one to consider the previous experiences of the solver within a mathematics textbook chapter and how this impacts upon the solver when solving a task. A workshop was conducted for the validation of the novelty framework described here, it will be reported on in chapter 4.

Chapter 4 Methodology

4.1 Introduction

This chapter will outline the methodology used for the classification of tasks using the frameworks described in chapter 3. It will also describe a workshop that was used to gain external validation for the Novelty framework. I will define the term 'task' as used in this study, explain the choice of mathematical topics and textbook series, discuss the coding and the efforts made to ensure consistency and reliability of results. Once the methodology employed in the task analysis has been explained, examples of classifications using the five frameworks will be provided.

4.2 Definition of a Task

For this work, a task is considered to be an activity where a student interacts with a mathematical topic by attempting to solve a question either as homework or within the classroom. This is in keeping with Mason and Johnston-Wilder's definition (2006, p.4). In this analysis of tasks, particular attention will be paid to the structure and presentation of tasks and the skills that students utilise in their interaction with concepts and topics when attempting to derive a solution. This builds up a picture of the kind of learning that is promoted by these tasks. Polya (1957, p. 171) and Henningsen and Stein (1997, p.525) both acknowledge that students' sense of mathematics develops from their experiences with the tasks with which they are asked to engage. For the purposes of this study, tasks are not seen as not being confined to the classroom but are also interacted with when students complete homework.

4.3 Choice of Topics

This study looks at the topics of Pattern, Sequences and Series, and Differential Calculus: these two topics were chosen because they are present on both Higher and Ordinary Level Leaving Certificate Mathematics syllabuses. Also these topics were both present on the old syllabus and this allows for further comparison between the Pre- and Post-Project Maths eras. As they are present at both levels, this gives several options for comparison. It is possible to compare the treatment of the topics in different ways: across syllabus levels within a textbook series or between different series. The tasks from this topic are analysed from three textbooks series available on the Irish market: 'New Concise Maths', 'Text and Tests' and 'Active Maths'. These three textbook series were selected because they were the first to be published in response to the new curriculum, while they have also traditionally been the most popular in Irish classrooms. Publishers declined to release any data in relation to textbook choice and/or usage as it was judged to be sensitive in a competitive market. Two of these textbooks series have the same author(s) as before the introduction of Project Maths and for the third textbook series the pre-Project Maths author was retained as an advisor. This allows for comparison between the older textbooks and the textbooks introduced in response to Project Maths. One other textbook entitled 'Numbers' was available but it appears that very few schools opted to use it, judging from a small sample of school book lists available online, and it is only available for Ordinary level which would limit the options for comparison. Also it was new to the market which would not allow for comparison with an older edition. Since the choice of textbooks was made, two textbook series called 'Effective Maths' and 'Power of Maths' have been published. The 'Effective Maths' series is only available at Higher Level with no Ordinary Level edition. The 'Power of Maths' series is a recent entry to the market and does offer Higher and Ordinary Level editions. However the three textbook series analysed in this study have become established popular choices in Irish classrooms. Although each series of textbooks has been introduced at both junior and senior cycle, it was decided to focus on the senior cycle material in particular because of the high-stakes (Leaving Certificate) examination that accompanies it.

4.4 Selecting Tasks

From the six Pre-Project Maths and six Post-Project Maths textbooks, each chapter relating to Patterns, Sequences and Series and Differential Calculus was analysed. A total of 7635 tasks (3584 Pre-Project Maths and 4051 Post-Project Maths) were classified from the chapters chosen. Tasks from revision sections at the end of chapters were excluded as anecdotal evidence suggests that teachers tend to focus on the material within a chapter and use these sections for revising after returning to a topic after some time. Once the revision sections were excluded, the tasks from all other exercise sections were analysed. These questions would normally be assigned as classwork or given to students for homework (Hourigan and O'Donoghue, 2007, p. 471), thus these tasks provide an insight into the teaching and learning taking place in Irish classrooms. It was necessary to discard some tasks when ambiguity was encountered, for example where a misprint made it difficult to interpret what a task required. A task was also omitted from the analysis if the wording led to the possibility of the task being interpreted in more than one way and such readings led to different solutions. In all, 16 tasks were excluded from classification due to these kinds of ambiguity. Questions sometimes consisted of several parts and it was necessary to break these up and treat them as multiple tasks. Several checks were made over time to ensure that what was treated as a task in one textbook was consistent with the other five textbooks regardless of how exercises were structured or presented.

4.5 Coding

Each of the 7635 tasks were classified using each of the five frameworks outlined earlier. Each individual task was examined separately. This involved carefully considering the features (symbols, wording, context etc.) of a task relevant to a particular framework and determining the likely solution methods that students would employ. As each framework focuses on different aspects of a task, it was necessary to conduct each classification separately to ensure that the tasks were interpreted without any confusion. The criteria for each framework were applied separately to each task. For the Novelty framework, each task was placed in one of the three categories: 'novel', 'somewhat novel' or 'not novel' using the criteria described in Figure 3.9 from section 3.7; these classifications were made only after taking into account the preceding examples and exercises that the student would have encountered in the chapter before attempting the task. When classifying tasks using the LCD framework, decisions were made using the list of criteria from figures 3.3 and 3.4 outlined in section 3.3.1, dividing tasks into four categories HP (*procedures with connections to meaning*) or DM (*doing mathematics*) for Higher Level Demand (HLD) and LP (*procedures without connections to meaning*) or LM (*memorization*) for Lower Level Demand (LLD). With the mathematical reasoning framework, tasks were labelled as CR (Creative Reasoning) or IR (Imitative Reasoning). These classifications were determined using the characteristics given in Figure 3.5 and which were expanded on in section 3.3.2. The tasks were examined very carefully as to which Project Maths objectives they addressed using Figure 3.8 from section 3.3.3, as it was important to take into account that more than one objective could be in evidence. Similar care was taken with the dimensions of mathematical understanding framework, taking note of the experience that the student was gaining from completing the task. For this latter framework, it was also important to observe that a task could be placed in more than one category as described in section 3.3.4. Each task was given a unique code and all classifications and reasons for such classifications were entered into an SPSS file. The data generated from these classifications were analysed using SPSS.

The process of coding began during the first year of this study, with the topic of Pattern, Sequences and Series in the post-Project Maths textbooks using the LCD and reasoning frameworks. Chapters were broken into manageable sections and the classifications were reviewed and revised over this time. It took some time to calibrate and establish a consistent satisfactory understanding of the coding system with these two frameworks. The key was not to classify too many tasks in a short space of time so that discussion on the classifications could be facilitated and also ensuring that all relevant aspects of a task were noted. The experience of working with these two frameworks assisted with the development of the Novelty framework. Usiskin's multidimensional model and the Project Maths problem-solving Objectives frameworks were also introduced during the second year. By the end of the third year, 1838 post-Project Maths tasks from Pattern, Sequences and Series taken from the three textbooks series at both Higher and Ordinary Level had been coded using the five frameworks. During the fourth year, the 1568 pre-Project Maths tasks on Pattern, Sequences and Series were classified. The topic of Differential Calculus was then examined towards the end of the fourth year in both the pre- and post-Project Maths textbooks. This accounted for 2016 and 2217 tasks respectively, each analysed using the five frameworks. At the end of the fifth year, 7635 tasks had been analysed.

4.6 Inter-rater Reliability

After I had coded the textbook tasks, at least one of my two supervisors also looked at each task separately and we compared our classifications after each framework analysis was complete. We then discussed any of the classifications that we had differences on and gave our perspective on why we analysed them as we did, coming to agreement on how the coding should be applied. Having clarified and resolved our coding, we made any necessary revisions and reviewed the existing classifications of previous tasks in light of these revisions, in order to ensure consistency throughout the analysis. This led to a final set of classifications. This form of internal refinement and quality control is common practice for this kind of work (Alafaleq and Fan 2014, Edwards 2011).

4.7 Examples of Classification of Tasks using Frameworks

The following provides examples of how the tasks were classified using the five different frameworks described earlier. Key information justifying why each classification was chosen is provided after listing the decision that was reached for each framework. The exemplar material that was provided in the textbooks before each task is listed, as this can be informative as to the kind of procedures and algorithms that the student has been exposed to. Moreover, it is necessary to be aware of this material when considering how the degree of novelty is determined. The formulae available in the mathematical tables were not considered relevant as any formula encountered by the students was treated as familiar if it was provided in the exemplar material. I am presenting examples of tasks from both topic areas. The material for Pattern, Sequences and Series is taken from Text and Tests and the Differential Calculus examples are taken from Concise Maths.

4.7.1 Pattern, Sequences and Series

Before classifying any of the tasks in a textbook, it is necessary to consider all of the previous exercises completed and any expository material that is available. For each of the sample tasks that follow, the preceding exemplar material is provided.

Example 1 Find the sum of the series 4 + 11 + 18 + 25 + ... + 144

Examining the series, we find: a = 4

d = 11 − 4 = 7

To find n the number of terms up to the term 144, we use

$$T_n = a + (n-1)d$$

$$144 = 4 + (n-1)7 = 4 + 7n - 7$$

$$144 = 7n - 3$$

$$7n = 147$$

$n = \left(\dfrac{147}{7}\right) = 21$ ... i.e. there are 21 terms in this sequence

$$\therefore S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{21}{2}[2(4) + (21-1)7]$$

$$S_n = 1554$$

Figure 4.1: Exemplar material 1 from Text and Tests 6 (Higher Level) pg. 145

Example 2

To celebrate the birth of his niece, an uncle offers to open a savings account with a deposit of €50. He also offers to every year add €10 more than he did the previous year until his niece is 21 years of age.

(i) Find an expression for S_n, the sum of money on deposit after n years.

(ii) Find S_{21}, the total saved after 21 years.

a=€50 $\qquad S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

d=€10 $\qquad S_n = \dfrac{n}{2}[2(50) + (n-1)10]$

$$= \frac{n}{2}[100 + 10n - 10]$$

$$\Rightarrow S_n = \frac{n}{2}[10n + 90]$$

$$S_{21} = \frac{21}{2}[10(21) + 90] = €3150$$

Figure 4.2: Exemplar material 2 from Text and Tests 6 (Higher Level) pg. 145

4.7.1.1 Classification Example 1:

Find S_n and S_{20} of the following arithmetic sequence: $1+5+9+13+...$

Figure 4.3: Text and Tests 6 Exercise 4.3 Q 1 pg. 149

Project Maths Objectives:

There is one of the Project Maths learning objectives in evidence here.

- *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*: the student can use the relevant formula in order to get the required solution.

Novelty:

- 'Not Novel': This task is 'not novel' as it is very similar to example 2 in figure 4.2 in terms of finding the nth sum and example 1 in figure 4.1 for evaluating the sum. These examples outline the algorithm for finding the sum of terms in an arithmetic series. This question could be answered using the same algorithm.

Levels of Cognitive Demand:

- Lower level demand: ('Procedures without connections to meaning') This could involve the use of an algorithm. The student might substitute in the values in order to get the required solution.

Imitative v Creative Reasoning:

- Imitative Reasoning: The student has an algorithm that could be used to find the required solution.

Dimensions of Mathematical Understanding:

This exercise makes use of one of Usiskin's dimensions, which is Skill-Algorithm.

- Skill-Algorithm: The user could replace the terms in the formula with the values observed for a, d and n. To get the second part of the answer, the value of n could also be substituted.

Example 3

Given $S_n = n^2 - 4n$, find an expression for T_n and hence determine if the sequence is arithmetic.

$$S_n = n^2 - 4n$$

$$S_{n-1} = (n-1)^2 - 4(n-1) \quad \ldots \text{ replace } n \text{ with } n-1$$
$$= n^2 - 2n + 1 - 4n + 4$$
$$\Rightarrow \quad S_{n-1} = n^2 - 6n + 5$$

$$T_n = S_n - S_{n-1} = n^2 - 4n - (n^2 - 6n + 5)$$
$$= n^2 - 4n - n^2 + 6n - 5.$$
$$T_n = 2n - 5$$

If a sequence is arithmetic, $T_n - T_{n-1}$ must be a constant.

$$\Rightarrow \quad T_n - T_{n-1} = 2n - 5 - [2(n-1) - 5]$$
$$= 2n - 5 - (2n - 7)$$
$$= 2n - 5 - 2n + 7$$
$$= 2 \text{ , i.e. a constant.}$$

Therefore the sequence is arithmetic.

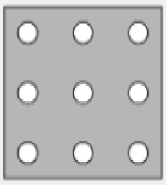
Figure 4.4: Exemplar material 3 from Text and Tests 6 (Higher Level) pg. 146

Example 4

A lighting company is making a sequence of light panels with the number of bulbs per panel in arithmetic sequence.

For the first 10 panels, 165 bulbs were used.

If the third panel is as shown in the diagram, find a, the first term of the sequence, and d, the common difference.



3rd panel (9 bulbs)

Hence draw a diagram of the first four panels.

$T_n = a + (n-1)d$ Also, $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$T_3 = a + (3-1)d$

$T_3 = a + 2d = 9$ ….. we know that $S_n = 165$ when $n = 10$

$\Rightarrow S_{10} = \frac{10}{2}\{2a + (10-1)d\}$

$S_{10} = 10a + 45d = 165.$

$$\begin{aligned} a + 2d &= 9 \\ \Rightarrow \quad 10a + 20d &= 90 \\ \text{and} \quad 10a + 45d &= 165 \\ -25d &= -75 \quad \text{... subtracting} \\ \Rightarrow \quad d &= \left(\frac{-75}{-25}\right) = 3 \end{aligned}$$

If $d = 3$, then $a + 2(3) = 9$

$\Rightarrow a = 3$

The sequence of bulbs in the panels is $3, 6, 9, 12$.

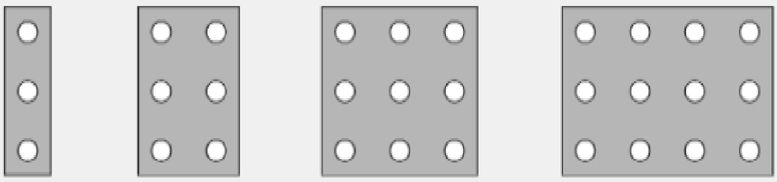


Figure 4.5: Exemplar material 4 from Text and Tests 6 (Higher Level) pg. 146-147

Note that the exemplar material (examples 1 and 2) shown in figures 4.1 and 4.2 occurred before the following exercise also.

4.7.1.2 Classification Example 2:

Show that $S_n = \dfrac{n(a+l)}{2}$ is the sum to n terms of an arithmetic sequence where l is the last term.

Figure 4.6: Text and Tests 6 (Higher Level) Exercise 4.3 Question 15 pg. 150

Project Maths Objectives:

There are two Project Maths learning objectives relevant here.

- *Apply knowledge and skills to solve problems in unfamiliar contexts*: The mathematical context might not be familiar to the student and further investigation might be necessary to determine how to solve it.

- *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*: The student might find it necessary to think carefully and devise a suitable approach to the question.

Novelty:

- 'Novel'. This exercise does not draw on anything specific from the examples or the preceding exercises. It may be unfamiliar to the user and require a high degree of investigation and experimentation before solving the task in the manner required.

Levels of Cognitive Demand:

- Higher level demand ('doing mathematics'): Complex thought may be required on the part of the user in terms of how to approach the exercise and find the necessary expression.

Creative vs. Imitative Reasoning:

- Creative Reasoning: The user might look at the terms of a series in terms of a and d. The link between l and a+ (n-1) d could be established by the student after investigation. The task is novel as the user does not have a clear way into the question. It is flexible as the user chooses how to approach the question. There is also a mathematical foundation present as the user might explore the properties of a series and how the sum of a series is found.

Dimensions of Mathematical Understanding:

Two of Usiskin's dimensions are relevant here, namely Skill-Algorithm and Property-Proof.

- Skill-Algorithm: the user might write out each term of the arithmetic series in terms of a and d.
- Property-Proof: The user could look at the arithmetic series in terms of a and d and include the nth terms. It may be necessary for the user to think about the last term and how it would be expressed in terms of a, d and n.

Example 5

(i) Use the sigma notation ($\sum$) to represent $2 + 6 + 10 + 14 + \ldots\ldots$ for 45 terms.

(ii) For what value of n is $\sum_{r=1}^{n} 3r - 5 = 90$?

(i) $2 + 6 + 10 + \ldots\ldots \quad \left.\begin{aligned} a &= 2 \\ d &= 4 \end{aligned}\right\} \quad \begin{aligned} T_n &= a + (n-1)d \\ &= 2 + (n-1)4 \end{aligned}$

$$T_n = 4n - 2$$

$$\Rightarrow \quad T_r = 4r - 2.$$

$$\therefore \quad 2 + 6 + 10 + \ldots\ldots \text{ for 45 terms} = \sum_{r=1}^{r=45} (4r - 2)$$

(ii) $\sum_{r=1}^{n} (3r - 5) = [3(1) - 5] + [3(2) - 5] + [3(3) - 5)] + \ldots\ldots 3n - 5$

$$= \quad -2 \quad + \quad 1 \quad + \quad 4 \quad + \ldots\ldots 3n - 5.$$

$$\Rightarrow \left.\begin{aligned} a &= -2 \\ d &= 1 - (-2) = 3 \\ S_n &= 90 \end{aligned}\right\} \qquad \begin{aligned} S_n &= \tfrac{n}{2}\{2a + (n-1)d\} \\ 90 &= \tfrac{n}{2}\{2(-2) + (n-1)(3)\} \\ 180 &= n(-4 + 3n - 3) \\ 180 &= n(3n - 7) \end{aligned}$$

$$\Rightarrow \quad 3n^2 - 7n - 180 = 0$$

$$(3n + 20)(n - 9) = 0$$

$$\Rightarrow \quad n - 9 = 0 \quad \text{or} \quad 3n + 20 = 0$$

$$\therefore \quad n = 9 \quad \text{or} \quad n = \frac{-20}{3}$$

$$\therefore \quad n = 9 \quad \text{since} \quad n \in N$$

Figure 4.7: Exemplar material 5 from Text and Tests 6 (Higher Level) pg. 148

4.7.1.3 Classification Example 3

Write the following sequence in sigma notation 4+8+12+16+…+124

Figure 4.8: Text and Tests 6 (Higher Level) Exercise 4.3 Question 7 pg. 149

Project Maths Objectives:

- *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*: A model is used to rewrite the series in sigma notation

  Novelty:
- 'Not Novel': This task is 'not novel' as it is very similar to example 5 in figure 4.7. The example outlines an algorithm for using sigma notation to represent a sequence. This question could be answered by using the same algorithm.

Levels of Cognitive Demand:

- Lower level demand ('Procedures without connections to meaning'): Student could use an algorithm to write the sequence in the required format.

Creative vs. Imitative Reasoning:

- Imitative Reasoning: The user can use an algorithm to write the sequence as requested.

Dimensions of Mathematical Understanding:

One of Usiskin's dimensions is relevant here, namely Skill-Algorithm.

- Skill-Algorithm: As above, the user can use an algorithm demonstrated in the text to write the sequence as requested.

__4.7.1.4 Classification Example 4__

In an Art class, a student experiments with a design for a dreamcatcher using rings and threads. The first three designs are shown below.
He wishes to continue his pattern of designs. How many rings will he need for

(i) design 10 (ii) design 20?

How many rings in total will he need to make all 20 designs?

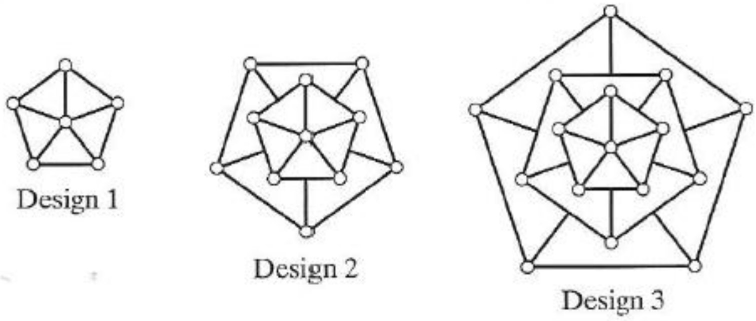


Figure 4.9: Text and Tests 6 (Higher Level) Exercise 4.3 Question 10 pg. 149

This is the classification for part (i) How many rings will he need for design 10?

Project Maths Objectives:

There are three Project Maths learning objectives relevant here.

- *Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*: The student might find it necessary to choose between different available models. The user could use a formula or continue to draw designs to find the required number of rings.

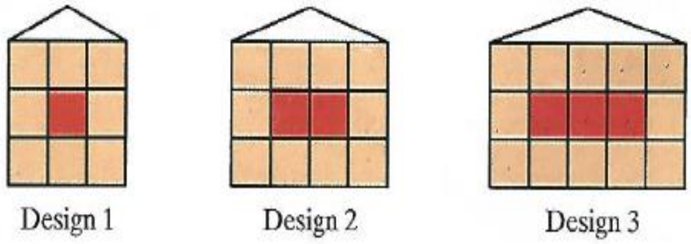
- *Analyse information and translate it into mathematical form:* The information given in the question is read and it is interpreted mathematically so that the required solution can be found.

- *Explore patterns and formulate conjectures:* The student could explore the given designs and identify the pattern in order to determine the number of rings in later designs.

Novelty:

- 'Not Novel'. The student is reliant on familiar skills established from solving tasks involving patterns in the preceding exercise set 4.2. At face value, the context could be viewed as unfamiliar but given the previous exposure to this type of task, the novelty is diminished. An example of this kind of similar task, which would have been solved by the student previously, is given below in figure 4.10.

Niamh made wall hangings using the following designs:



(i) How many red and orange tiles will she need for design 8?

Figure 4.10: Text and Tests 6 (Higher Level) Exercise 4.2 Question 5 pg 142-143

Levels of Cognitive Demand:

- Lower level demand ('procedures without connections to meaning'): The student could complete this by counting the number of rings and then using an algorithm to find the value of the tenth term without considering the underlying mathematical foundations.

Creative vs. Imitative Reasoning:

- Imitative Reasoning: The user might complete this without giving due consideration to the mathematics involved by imitating the previous exercises that were solved in the tasks for exercise set 4.2, here the student identifies the numerical value of the first three terms and then calculates the value of the tenth term using a formula.

Dimensions of Mathematical Understanding:

Three of Usiskin's dimensions are relevant here, namely Skill-Algorithm, Use-Application and Representation-Metaphor.

- Skill-Algorithm: the student might use the formula for an arithmetic sequence to find the rings needed for the tenth design.
- Use-Application: The dreamcatcher and its design are set in a real life situation.
- Representation-Metaphor: The question is presented using pictures of designs 1-3 rather than a numerical sequence. The student might draw more of the designs to explore the pattern and determine how many rings are needed for the tenth design.

4.7.2 Differential Calculus

Example

If $y = \frac{2x}{1-x}$, show that $\frac{dy}{dx} > 0$ for all $x \neq 1$.

Solution:

$$y = \frac{2x}{1-x}$$

$$\frac{dy}{dx} = \frac{(1-x)(2)-(2x)(-1)}{(1-x)^2} \quad \text{(quotient rule)}$$

$$= \frac{2-2x+2x}{(1-x)^2}$$

$$= \frac{2}{(1-x)^2}$$

Since $2 > 0$ and $(1-x)^2 > 0$ for all $x \neq 1$, it follows that $\frac{2}{(1-x)^2} > 0$ for all $x \neq 1$.

$\therefore \frac{dy}{dx} > 0$ for all $x \neq 1$.

Note: $(\text{any real number})^2$ will always be a positive number unless the number is zero. $\therefore (1-x)^2 > 0$ for all $x \neq 1$.

Figure 4.11: Exemplar material 6 from New Concise Project Maths 4 (Higher Level) pg. 416

4.7.2.1 Classification Example 5

Let $f(x) = x^3 + 4x + 2$ Explain why this function is bijective.

Figure 4.12: New Concise Project Maths 4 (Higher Level) Exercise 15.2 Question 2 (ii) pg. 416

Project Maths Objectives:

- *Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*: The student has several models to choose from when establishing that the function is bijective. The property could be established by examining the derivative or drawing a graph of the function.
- *Explain findings*: An explanation is requested
- *Apply knowledge and skills to solve problems in familiar contexts:* It is not immediately obvious as to how to determine that the function is bijective. However the student has built up knowledge of derivatives and how to use them to establish that functions are increasing so this can be applied to explain why f(x) is bijective.

Novelty:

- 'Novel': The task has not been modelled in the chapter.

Levels of Cognitive Demand:

- Higher level demand ('Procedures with connections to meaning'): Student could decide to use a procedure like finding the derivative to show that the function is always increasing or the 'Horizontal Line Test' to show that it is both injective and surjective.

Creative vs. Imitative Reasoning:

- Creative Reasoning: The student may have to draw on creative reasoning to establish the properties of the function. The student may find the derivative of the function and establish that it is always increasing. This will then have to be linked to the criteria for a bijective function.

Dimensions of Mathematical Understanding:

Two of Usiskin's dimensions are relevant here, namely Skill-Algorithm and Property-Proof.

- Skill-Algorithm: The student might possibly find the derivative and establish that the function is always increasing using the skills developed earlier in the chapter.
- Property-proof: The student will have to establish some property to show it is bijective, this could be done without drawing the graph of the function

4.7.2.2 Classification Example 6

An artificial ski slope is described by the function $h = 165 - 120s + 60s^2 - 10s^3$ where s is the horizontal distance and h is the height of the slope. Show that the ski slope never rises.

Figure 4.13: New Concise Project Maths 4 (Higher Level) Exercise 15.2 Question 8 pg. 417

Project Maths Objectives:

- *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*: A model could be used to find the required derivative and obtain the required form.
- *Apply knowledge and skills to solve problems in unfamiliar contexts*: The student has experience working with this kind of function but has not encountered this kind of real life context before.

Novelty:

- 'Somewhat Novel': The skills required are similar to a previous question in this exercise set whereby the student was asked to show that $\frac{dy}{dx} \leq 0$ for all $x \in R$ where y = $10 - 3x + 3x^2 - x^3$.

  The unfamiliar presentation comes from the task being set in a real life context.

Levels of Cognitive Demand:

- Higher level demand ('Procedures with connections to meaning'): Student can use an algorithm to find the required derivative and get the negative square form but must link it to the function's decreasing. By establishing that the function is always decreasing, they are interacting with the mathematical foundations.

Creative vs. Imitative Reasoning:

- Creative Reasoning: An algorithm can be used to differentiate and factorise the expression, however to use this is not immediately obvious as the context requires creative thought. The student has not encountered the idea of 'never rises' previously and must consider how to establish this mathematically. Creative reasoning is required to resolve the requirements of the task and then answer it.

Dimensions of Mathematical Understanding:

Three of Usiskin's dimensions are evident here.

- Skill-Algorithm: the student can use a model to find the derivative and to prove that the function is always decreasing.
- Property-Proof: Proving that the ski slope never rises. (Function always decreasing).
- Use-Application: The ski slope is a real life situation.

4.7.2.3 Classification Example 7

Let $f(x) = x - \sin x$. Show that $f'(x) > 0$ for $0 < x < \frac{\pi}{2}$.

Figure 4.14: New Concise Project Maths 4 (Higher Level) Exercise 15.2 Question 7 pg. 417

Project Maths Objectives:

- *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*: A model can be used to get the required derivative. Students then need to consider when a particular inequality is satisfied.

- *Apply knowledge and skills to solve problems in familiar contexts*: The student knows how to differentiate the function. However, it is necessary to link knowledge of derivative to that of the behaviour of the trigonometric function cos (x) in order to determine whether the inequality holds or not for the given domain.
- Novelty:

  'Somewhat Novel': Differentiating and solving is familiar but deciding where the inequality holds is a new aspect of the solution method.

Levels of Cognitive Demand:

- Higher level demand ('Procedures with connections to meaning'): Student would need to connect their knowledge of derivatives to their knowledge of the behaviour of certain functions. Although it is possible to find the derivative by rule, it is still necessary to consider the behaviour of the cosine function.

Creative vs. Imitative Reasoning:

- Creative Reasoning: Creative reasoning could be needed to check the extent of the solution set of the inequality; this could require creative reasoning especially when no prior experience of this type of analysis has been established.

Dimensions of Mathematical Understanding:

Two of Usiskin's dimensions is relevant here, namely Skill-Algorithm and Property-Proof.

- Skill-Algorithm: the student uses a model to find the derivative.
- Property-Proof: the student proves that the derivative is increasing within a certain domain.

In this section, I have outlined the methodology used in the analysis of tasks from the textbook series, and given some examples of the type of analysis carried out. In the next chapter, I will present the results of this analysis.

4.8 Validation of the Novelty Framework

This section outlines a workshop that was conducted to validate the Novelty framework. The framework was designed to assist with addressing the research question: what degree of novelty is incorporated into the textbook tasks? The validation workshop was held to determine whether the Novelty framework was clearly described and could be used effectively and reliably. In the workshop, five researchers with an interest in mathematics education independently classified tasks drawn from two textbook chapters used in the main study. They were then invited to complete a questionnaire in relation to their experience of using the framework. Participants were asked if the framework was easy to use and they were invited to describe any difficulties encountered in its use. The questionnaire also asked if they agreed with the descriptions of 'novel', 'somewhat novel' and 'not novel' used in the framework. This was followed by a group discussion where participants expanded on their views and gave a more detailed account of using the Novelty framework.

4.8.1 Format of the Workshop

4.8.2 Codebook

Codes are defined as ''tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study'' (Miles and Huberman, 1994, pg. 56). Prasad (2008, pg. 187) informs us that 'coding the unit of analysis into a content category is called coding. Individuals who do coding are called coders.' A codebook is a set of codes, definitions, and examples used as a guide to help analyse data (DeCuir-Gunby, Marshall and McCulloch, 2011, pg. 138). Lin and Jeng (2015, pg. 91) emphasise that the researcher conducting a coding session with a group 'will have to create a procedure instruction and guide book for the participating coders'. A codebook provides not only a guide for coding responses but it also serves as documentation for the layout and code definitions of a data file (Lavrakas, 1988, p.100). DeCuir-Gunby, Marshall and McCulloch (2011, pg. 138) advise that the 'more specificity in a codebook, the easier it is for coders to distinguish between codes and to determine examples from nonexamples of individual codes'. They also recommend that the more detailed the codebook, the more consistency there will be among coders when using it to code data.

The codebook that I created described the three different levels of novelty in my framework and provided a guide as to how to distinguish between each category (see Appendix A). Definitions of the terms to be used during the coding were provided. The units of data collection were identified as a textbook exercise or a smaller part of a multi-part exercise. The codebook also outlined how each task or unit of data was identified. It was important to have two identifiers for the tasks involved. One was a chronological list of the tasks to be used within the workshop so that it would be easy to locate a particular task. The original exercise numbers were also retained, to easily compare the classifications of the workshop participants with earlier classifications by the author. The codebook also contained expository material and worked examples on one topic from one of the textbooks. I decided to use the topic of Pattern, Sequences and Series in the codebook because it was more compact than Differentiation in terms of expository material and thus deemed more suitable for a workshop in which time was limited. Five sample tasks were included, accompanied by worked solutions and suggested classifications. A sample coding form was also included which demonstrated how the classification for each sample task could be recorded.

4.8.3 The Coders

In order to identify a suitable number of independent coders, I reviewed relevant literature in relation to coding. Alafaleq and Fan (2014) investigated how the national middle school mathematics textbooks in Saudi Arabia represent problem-solving heuristics and they established a framework for coding heuristics into different categories relating to how students would be encouraged to solve the textbook problems. The reliability of their coding was checked with a single external coder who also coded all the problems. The coding result by the independent coder was compared with that obtained by the researchers. Inter-rater agreement was measured using the Intra-class Correlation Coefficient (ICC), a statistic of 0.98 was achieved on heuristics and 0.97 was recorded on heuristics existence in each book, indicating a high agreement in coding. Similarly, Mailizar and Fan (2014) also made use of one external coder when examining how mathematics textbooks in Indonesia reflect authentic learning. They designed a framework that classified tasks into categories such as authentic, semi-authentic, real-authentic or non-authentic. Again the coder was invited to code all the problems, in an effort to establish the reliability of the original coding. Like the previous study, the ICC was used to get some indication of the inter-rater agreement. For authentic tasks, an ICC of 0.789 was recorded, an ICC of 0.771 was found for semi-authentic tasks while real-authentic tasks had an ICC of 0.702. As the ICC between the researchers and the coder, for each type of task, was higher than 0.7, this suggested that the results were reasonably reliable. Neither of these two studies, unlike the Novelty framework workshop, made use of a questionnaire or discussion, instead focusing solely on the reliability of the original coding.

Edwards (2011) in his unpublished PhD thesis examined how successful students are at accurately generating examples of sequences satisfying certain combinations of properties, while also investigating the qualitative variation in students' experiences of sequence generation. To address these research questions, he designed a coding scheme. Two of his colleagues were asked to independently code interview data in order to validate his coding scheme. As part of his validation exercise, the two mathematics education researchers were presented with new data to code within the framework, and were then asked to reflect on their chosen classifications and the validity of the framework. Both researchers provided feedback on the structure and content of the dimensions presented in the scheme and expressed opinions as to how appropriate the coding was to the data and whether it succeeded in providing insight into the data. Unlike the two previous studies, he did not calculate a reliability coefficient.

Seven researchers with a background in mathematics education were invited to participate in a Novelty framework validation workshop. This was to ensure as many different perspectives as possible were incorporated. Five were able to attend on the selected day, each of whom had some level of exposure to post-primary mathematics education in Ireland. Four of the participants had taught mathematics at second level in Ireland and would have some familiarity with mathematics textbooks at second level. None of the coders had previous experience of working with the Novelty framework. Each coder was sent a copy of the codebook in advance of the workshop. The coders will be known individually hereafter as Coder 1, Coder 2 … Coder 5 when discussing findings from the workshop.

4.8.4 Outline of the Workshop

The face-to-face workshop lasted for three hours, with all participants present at the same time. It was broken into five stages - namely, a presentation on the Novelty framework criteria, working with practice tasks, the coding of tasks using the Novelty framework, the completion of a questionnaire and a discussion on participants' experience of using the framework. There was a short break between stages. The presentation lasted forty five minutes, twenty minutes was spent working with practice tasks while ninety minutes was given to the remaining stages. The coding of tasks and completion of the questionnaire was completed within seventy minutes while twenty minutes was given to the discussion.

The presentation on the Novelty framework provided the opportunity for coder training, whereby each participant in the workshop was informed of the framework's coding scheme and how it should be applied. Working with practice tasks was also considered a critical stage in the workshop as it acted as a pilot test. This provided an important opportunity to ensure consistency before coders were asked to apply the framework to the full data. The result of pilot testing helps to identify inconsistencies between coders or to discover other inadequacies in terms of category construction or its description (Prasad, 2008). Triangulation lends credibility to the findings by incorporating multiple sources of data, methods, investigators, or theories (Erlandson, Harris, Skipper and Allen, 1993). Aside from the coding of tasks by the five participants, the questionnaire and group discussion also provided valuable insight into the description of the Novelty framework and how easy it was to apply.

4.8.5 Presentation on the Novelty Framework Criteria

The workshop began with a presentation where I explained the criteria necessary for classification using the Novelty framework. I outlined a number of issues that coders should be conscious of when classifying tasks. In particular I emphasised that the term 'skills', in the framework, is taken to refer to the methods and techniques used in the solutions to tasks. I made it clear that it was not necessary for all characteristics in the description of a category to apply when deciding on a classification. Each coder was encouraged to establish sufficient evidence so as to distinguish between the different categories, by finding as many relevant characteristics as possible before settling on a particular classification. Attention was also brought to the fact that when classifying with the framework, commonly used skills like calculating slope or finding area with a formula are taken to be familiar from junior cycle.

The five sample tasks were then presented to participants in order to demonstrate how the author had applied the Novelty framework to classify these tasks. A method of solution appropriate for sample task 1 (see appendix A.2) had been modelled in a previous example in the expository material and so the skills were very familiar, thus the suggested coding was 'not novel'. Sample task 2 was very similar to the previous task and the presentation, context and concepts of the task were familiar, which would suggest that it should be coded as 'not novel'. Sample task 3 was used to show how the presentation of a task could provide an unfamiliar appearance yet the solution was reliant on familiar skills once a slight modification was made to the solution method already demonstrated in the text. According to the framework, this would place the task in the 'somewhat novel' category. A new aspect of the concept (the presence of powers) was present in sample task 4. As the solution required the use of familiar skills, this task was classified as 'somewhat novel'. The final example, sample task 5, required the use of unknown values and simultaneous equations which necessitated a significant adaption of the method given in the examples. Using the framework, I recommended that the task be classified as 'novel'. The coders were invited to ask questions at this point in order to clarify any confusion or discuss any of the five suggested classifications in more detail.

4.8.6 Working with Practice Tasks

The coders were then given the opportunity to code four practice tasks and record their classifications on a coding form. The expository material was taken from the Ordinary Level textbook in the Concise Maths series. These four tasks were chosen to be representative of what the coders would encounter in the actual coding exercise in the next phase of the workshop. I had previously coded each of these four tasks in the main study. The first was intended to be regarded as not novel, with the second being interpreted as somewhat novel. The third task in this set was an important inclusion because, had it appeared before the second task, it would have been expected to be labelled as 'somewhat novel' but given its similarity to the second task and the experience garnered in completing that task, the novelty would be diminished. The coders were expected to recognise this and opt for a lower level of novelty. It was important to bring coders' attention to this explicitly because it demonstrates the impact of the experience of completing previous tasks and how this is accounted for in the framework. Finally the last task provided was one which had been classified by the author as 'novel'.

After the coders had completed their analysis of the practice tasks, the group shared their classifications and discussed what features of the skills, presentation and concept influenced their coding. This was in effect our pilot coding, in the sense that the coders were allowed to interact and share their experiences and highlight any ambiguities in the descriptions of the framework categories. It also allowed the opportunity to emphasise that the three categories were intended to be mutually exclusive and coders should strive to distinguish between neighbouring categories such as 'novel' and 'somewhat novel' or 'somewhat novel' and 'not novel' as much as possible.

4.8.7 Choice of Tasks

To choose the final set of tasks which would be used for the validation of the framework, I conducted an overview of the novelty of the two topics over the three textbook series. It was important to choose tasks that would be readily accessible to new coders and allow them to identify skills quickly without feeling overburdened or encountering confusion when identifying skills in the worked examples.

When selecting the tasks, I observed that an early section on Differential Calculus of the Higher Level Text and Tests textbook contained a representative mixture of the three novelty categories, according to my own results, for coding. To balance this I chose the Ordinary Level Concise Maths textbook on the topic of Patterns, Sequences and Series. This featured tasks that were predominantly 'not novel' but also contained tasks that I had previously classified as 'somewhat novel'. Having chosen expository material and worked examples, I had to decide on the number of tasks that the coders would be asked to work on. If too many were chosen, it would risk coder fatigue and if an insufficient number was selected it would be unrepresentative. I opted for 30 tasks. I made the decision to abridge the section from Text and Tests by removing three tasks. This gave a total of fifteen tasks on differential calculus. I limited the tasks from Concise Maths to fifteen on patterns, sequences and series. The final subsample contained tasks whose classifications were expected to span the three categories in the Novelty framework; from the coding of these thirty tasks with my supervisors originally, we had agreed that 20 were Not Novel, 8 were Somewhat Novel and 2 were Novel. This suggested that the coders in the workshop would have the opportunity to classify tasks from each of the three categories of novelty as well as classifying tasks from both Higher and Ordinary Level. The 30 tasks that were coded on the day of the workshop, along with the relevant expository material can be found in Appendix A.

4.9 Coding Results

Table 1 indicates the classification that each coder gave to the specified task.

1= Novel, 2 = Somewhat Novel, 3 = Not Novel, NC = Not Classified

Task	Original Coding by author and supervisors	Coder 1	Coder 2	Coder 3	Coder 4	Coder 5	Levels of Agreement amongst coders
1	3	3	3	2	3	3	4 out of 5
2	2	2	2	2	2	2	5 out of 5
3	2	2	2	2	2	2	5 out of 5
4	3	3	3	3	3	3	5 out of 5
5	2	2	2	2	2	2	5 out of 5
6	3	3	3	3	3	3	5 out of 5
7	3	3	2	2	3	3	3 out of 5
8	3	3	3	3	2	3	4 out of 5
9	3	2	3	3	3	3	4 out of 5
10	3	3	3	3	3	3	5 out of 5
11	3	2	2	3	2	3	3 out of 5
12	3	3	3	3	3	3	5 out of 5
13	2	2	2	3	2	2	4 out of 5
14	3	3	3	3	3	3	5 out of 5
15	3	3	3	3	3	2	4 out of 5
16	2	3	3	2	2	3	3 out of 5
17	3	3	3	3	3	3	5 out of 5
18	3	2	2	3	3	2	3 out of 5
19	3	2	3	3	3	3	4 out of 5
20	3	3	3	2	3	3	4 out of 5
21	3	3	3	2	3	3	4 out of 5
22	2	2	2	2	3	3	3 out of 5
23	1	1	1	2	1	1	4 out of 5
24	2	3	3	2	1	2	Split
25	3	3	2	NC	2	2	3 out of 5
26	2	2	2	2	3	2	4 out of 5
27	3	3	3	3	3	3	5 out of 5
28	3	3	2	3	2	3	3 out of 5
29	3	3	3	3	2	3	4 out of 5
30	1	1	1	2	1	1	4 out of 5

Table 1: Classifications of coders from Novelty framework validation workshop

Coder	Novel	Somewhat Novel	Not Novel
Author and Supervisors	2 Tasks (6.66%)	8 Tasks (26.66%)	20 Tasks (66.66%)
Coder 1	2 Tasks (6.66%)	10 Tasks (33.33%)	18 Tasks (60%)
Coder 2	2 Tasks (6.66%)	11 Tasks (36.66%)	17 Tasks (56.66%)
Coder 3	0 Tasks (0%)	13 Tasks (43.33%)	16 Tasks (53.33%)
Coder 4	3 Tasks (10%)	8 Tasks (26.66%)	19 Tasks (63.33%)
Coder 5	2 Tasks (6.66%)	9 Tasks (30%)	19 Tasks (63.33%)

Table 2: percentage of the 30 tasks in each of the three categories for each coder

The coders were unanimous on 10 tasks out of the total 30, 8 of these were taken from the Ordinary Level textbook on Pattern, Sequences and Series while 2 belonged to the differential calculus section taken from the Higher Level textbook. Of the remaining tasks, 4 coders out of the 5 agreed on 12 of the tasks with 3 agreeing on 7 tasks. This meant that a majority of coders classified 29 tasks in the same way. Task 24 was unusual in that it was the only one that was classified with all three of the novelty categories and it was the only task not to achieve a majority consensus in its classification. Coder 3 did not classify any tasks as 'novel', this resulted in the 'novel' category not receiving a unanimous classification for any task unlike the other two categories. The most common disagreement in the coding was between labelling tasks as 'somewhat novel' or 'not novel'. Coder 5 deviated from the majority the least with a differing code just twice, coder 1 was in the minority four times and Coder 2 had a differing classification to the majority three times. In contrast Coder 4 was in the minority ten times while coder 3 disagreed with the majority's classification nine times.

When comparing the coders' classifications with my original classification completed in conjunction with my two supervisors, 10 tasks had absolute agreement between the coders and ourselves. It was found that 4 coders agreed with our classification on another 12 tasks. For 3 tasks, 3 coders agreed with our original analysis. This meant that a majority of the coders agreed with our original classification on 25 of the 30 tasks. There was much less agreement on tasks 11, 16, 18, 24 and 25. Each of these will be discussed individually.

We originally classified task 11 (see exercise 11, section 9.1, appendix A.5) as 'not novel', only two coders agreed with this and the majority coded it as 'somewhat novel'. In our classification, we felt that the concept, skills and method of solution had been modelled in the worked example provided. The coders appeared to feel that the requirement for subtraction involved the introduction of a new concept.

With task 16 (see exercise 1(i), section 2.1, appendix A.5) only 2 coders agreed with our classification that it was 'somewhat novel', with the majority of 3 coders regarding it as 'not novel'. When originally classifying this task, we felt that the presentation of the task made the task appear unfamiliar yet its solution was dependent on the use of familiar skills. A new feature or aspect of a concept is encountered in the sense that the student has to find the points on the curves given which is not modelled in the examples, but the solution to the task then only involves the use of familiar skills. A minor adaptation of the method outlined in the examples has to be made in order to get the required solution. The skills required are familiar but the use or application of such skills is slightly modified. It is likely that the majority of coders felt that it was closely aligned to the worked examples and the skills were familiar.

The majority of coders also disagreed with our classification of task 18 (see exercise 1 (iii), section 2.1, appendix A.5) as 'not novel', 3 coders classified it instead as 'somewhat novel'. In the original analysis, we felt that the experience garnered from the two previous tasks (16, 17) (see exercises 1 (i) and 1 (ii), section 2.1, appendix A.5) meant that the presentation, context and concepts of the task were now quite familiar. Its solution had been modelled and the skills required were also familiar. It is likely that the coders focused on the negative rate of change involved, interpreting it as the introduction of a new aspect of the concept here.

In the original analysis, task 24 (see exercise 5 (ii), section 2.1, appendix A.5) was labelled as 'somewhat novel'. 2 coders felt that it was 'not novel' and 1 felt that it was 'novel', showing a lot of disagreement. When we classified the task originally, we felt that the skills required were familiar from preceding exercises but the requirement to give an estimate of the average rate of growth involved the introduction of a new aspect of the concept, which had not been modelled previously. It is likely that the 2 coders labelling it as 'not novel' did not view the requirement for an estimate in the same way, while the coder labelling it as 'novel' did not consider the skills involved in finding the estimate as familiar.

Task 25 (see exercise 6 (i), appendix A.5) which was based on material familiar from Junior Cycle brought the greatest disagreement with our classification. Only 1 coder agreed with our label of 'not novel' due to the skills and solution being very familiar from Junior Cycle. 3 coders felt that it should be classified as 'somewhat novel' most likely due to the introduction of a formula not modelled in the examples and 1 did not classify it on the grounds that the skills were not familiar from the chapter.

4.10 Inter-rater Reliability

Generalisability, in the sense that results can be replicated and reproduced by others, is important when seeking to establish that the ratings found are not the idiosyncratic results of any one individual's subjective judgement (Mosmery and Barzegar, 2015, Tinsley and Weiss, 1975). To establish generality, it is necessary to have knowledge of both inter-rater agreement and inter-rater reliability (Tinsley and Weiss, 1975). Inter-rater agreement represents the extent to which the different coders tend to make the exact same judgements about the subject being rated. Inter-rater reliability relates to the extent of variability and error inherent in a measurement (Gisev, Bell and Chen, 2013, Tinsley and Weiss, 2000).

Recall that the data from the validity workshop is ordinal because there is a natural order to the terms 'novel', 'somewhat novel' and 'not novel', and that five coders participated in the validation exercise. The choice of inter-rater agreement and inter-rater reliability indices to be used here was influenced by the nature of the data and the number of coders involved. For instance, some indices like Cohen's Kappa are only suitable for exactly 2 raters, while the Intraclass Correlation Coefficient (ICC) requires interval or continuous data and is not suitable for categorical data or where a rating has been omitted. Cohen's Kappa is suitable only for the analysis of nominal ratings. With nominal ratings, raters classify subjects into response categories that have no order structure. That is, two consecutive nominal categories are considered as being as different as the first and last categories. If categories can be ordered or ranked, then the Kappa coefficient could dramatically understate the extent of agreement among raters as it treats any disagreement as total disagreement. Because it does not account for partial agreement, Kappa as proposed by Cohen (1960) is inefficient for analysing ordinal ratings. Cohen (1968) proposed the weighted version of Kappa to fix this problem but this only applies to the case of two raters and not multiple ones. Fleiss (1971) and Janson and Olsson (2001) extended Cohen's Kappa so that it could be used with multiple raters, but this is not suitable for ordinal data. Hence, I decided to use average pairwise percent agreement to examine inter-rater agreement and Krippendorff's alpha to examine inter-rater reliability. Hripcsak and Heitjan (p.108, 2002) recommend that pairwise agreement should be reported as descriptive agreement measures but not as a formal reliability measure. Hayes and Krippendorff (p. 82, 2007) outline several advantages in using Krippendorff's alpha: it can be used for any number of coders; it can be used for different kinds of variables including nominal and ordinal ones; it can be used for large or small sample sizes (with no minimum), with the advantage that it can also be used for incomplete or missing data.

Average pairwise percent agreement is easily calculated and gives some indication as to how well the coders agreed on the data coded. To calculate pairwise agreement, the agreement between a pair of coders is calculated. When working with multiple coders, it is necessary to find the average pairwise agreement among all possible coder pairs. For example, Coder 1 and 2 agreed on 25 cases out of 30 giving 83.33%. Table 3 summarises the percent agreement for each pair.

Pair	Percent %
Coders Original and 1	80
Coders Original and 2	76.667
Coders Original and 3	73.333
Coders Original and 4	73.333
Coders Original and 5	83.333
Coders 1 and 2	83.333
Coders 1 and 3	53.333
Coders 1 and 4	63.333
Coders 1 and 5	76.667
Coders 2 and 3	60
Coders 2 and 4	73.333
Coders 2 and 5	80
Coders 3 and 4	50
Coders 3 and 5	60
Coders 4 and 5	70

Table 3: Average Pairwise Percent Agreement

To calculate the overall average pairwise agreement, each of the above values was totalled and divided by the number of pairs: $\frac{1056}{15} = 0.70444 \text{ or } 70.444\%$. Neuendorf (p.145, 2002) indicates that such a statistic would suggest that there was some agreement amongst the coders but this calculation would not be sufficient to draw conclusions on its own. We can see from the table that the greatest agreement between the original coding and the external coders was with coder 5 while the least was with coders 3 and 4.

Calculating percent agreement has some drawbacks. For example, there is no comparative reference point to indicate whether the rate of agreement is higher or lower than chance. As percent agreement is an average, it can neglect to highlight significant disagreement such as that seen in task 24. Krippendorff (2004, p.426) warns that it is possible to 'hide unreliable categories behind reliable ones' due to the lack of attention given to significant disagreement. Lombard, Snyder-Duch and Bracken (2002 p.601) also recommend that researchers do not recommend solely on percent agreement due to this weakness.

Krippendorff's statistic measures both observed and expected disagreement, unlike Cohen's Kappa which only looks at expected agreement. The basic definition of alpha is given by $\alpha = 1 - \frac{D_o}{D_e}$, where D_o is the observed disagreement among values assigned to the units of analysis and D_e is the disagreement that would be expected when the coding of units is attributable to chance rather than to the properties of these units (Krippendorff, 2011, p.1). The values range between α = 0 which would indicate no agreement and α =1 indicating perfect agreement. Krippendorff (2011, p.1) explains that when 'observers agree perfectly, observed disagreement D_o = 0 and α =1, which indicates perfect reliability. When observers agree as if chance had produced the results, $D_o = D_e$ and α = 0, which indicates the absence of reliability.

Gwet (2011) developed a piece of software known as AgreeStat 2011, which allows for the calculation of inter-rater reliability coefficients including Krippendorff's alpha using Excel. Using this program, the result for Krippendorff's alpha for the tasks analysed during the workshop was given as 0.45, which would suggest that there was a moderate level of reliability between the coders. This value of 0.45 is unweighted and does not take into account the nature of the data, which is ordinal. Krippendorff (2011) gives an account of a weighting scheme which takes into account the metric difference for ordinal data as opposed to other types of data. Antoine, Villaneau and Lefeuvre (2014, p. 8) point out the advantages of using such weighting as it allows a more accurate reliability calculation without being affected by the influence 'of both the number of categories and the number of coders'. Using Gwet's program to calculate Krippendorff's alpha with ordinal weightings, a value of 0.56 was obtained still indicating moderate reliability.

Landis and Koch (1977) provide guidelines for interpreting such coefficients with values from 0.0 to 0.2 indicating slight agreement, 0.21 to 0.40 indicating fair agreement, 0.41 to 0.60 indicating moderate agreement, 0.61 to 0.80 indicating substantial agreement, and 0.81 to 1.0 indicating almost perfect agreement. The use of these values is debated however, and Krippendorff (1980) provides a more conservative interpretation suggesting that conclusions should be discounted for variables with inter-rater values less than 0.67, conclusions cautiously be made for values between 0.67 and 0.80, and definite conclusions be made for values above 0.80. 'In practice, however, coefficients below Krippendorff's conservative values are often retained in research studies, and Krippendorff offers these cutoffs based on his own work in content analysis while recognizing that acceptable inter-rater reliability estimates will vary depending on the study methods and the research question' (Hallgren, 2012). The literature makes use of both Krippendorff's and Landis and Koch's interpretation when looking at Krippendorff's alpha. Arstein and Poesio (2008) complain that the lack of consensus on how to interpret the values of agreement and reliability coefficients is a serious problem with current practice in reliability testing and they doubt that a single cutoff point is appropriate for all purposes.

To explain the relatively low Krippendorff's alpha in relation to the higher pairwise percentage agreement, it is necessary to consider prevalence. Hripcsak and Heitjan (2002) caution that an 'unbalanced' sample can have an adverse effect on reliability coefficient like kappa and Krippendorff's alpha. As explained previously, the coders were given 30 tasks with 2 intended as 'novel', 8 as 'somewhat novel' and 20 as 'not novel'. This particular selection for the coding sample was made to mirror the larger study of the textbook analysis that had been completed using the Novelty framework. It would not have been as representative if an equal 10 tasks from each category had been selected. The coders were given sections from actual textbooks being used in Irish classrooms and 10 'novel' tasks were not encountered in close proximity. This resulted in a low prevalence of 'novel' tasks and a high prevalence of 'not novel' tasks. Byrt, Bishop and Carlin (1993), along with Cicchetti and Feinstein (1990) acknowledge that a balanced sample is not always possible, which is the case with my sample used for the validation exercise. Viera and Garrett (2005) and Gwet (2008) point out that agreement measures can be affected by the prevalence of the finding under consideration. Very low values of alpha may not necessarily reflect low rates of overall agreement. As can be seen in table 1, only coder 3 failed to class any task as ‘novel’ and 3 coders were in complete agreement with the classification originally given by myself and my two supervisors which would point to high agreement as supported by the pairwise percentage calculation. It would appear that two of the coders encountered some difficulty in relation to distinguishing between the categories of ‘somewhat novel’ and ‘not novel’.

However I believe that there is a high level of agreement evident despite the statistic obtained by Krippendorf’s alpha, which is sensitive to an unbalanced sample and the high prevalence of certain categories (Feng, 2012, Byrt et al., 1993). Cichetti and Feinstein (1990), in addition to Feinstein and Cichetti (1990), discuss having a high level of agreement but an apparently lower level of reliability as a paradox. Gwet (2008) has developed a different coefficient known as AC_1 intended to be a ‘paradox-robust alternative’ (Gwet, 2011, p. 9), by using new variance estimators that, according to Gwet, give a more realistic measure for agreement by chance and thus solve the issue of the apparent paradox. This coefficient has been further refined to take into account various weightings, including those needed for ordinal data. This is known as AC_2 and it has the added advantage in that it can also handle missing ratings. Using the AgreeStat program, I found the unweighted AC_1 = 0.61 and the weighted AC_2 = 0.8 which would indicate substantial agreement. It should be noted that the weighted ordinal percent agreement was found to be 0.9 using this program. The differences in the classifications assigned by the five coders will be discussed using their responses to the questionnaire and the discussion that ensued after the classification was complete.

4.11 Questionnaire

The questionnaire consisted of three open ended questions and was designed to gain insight into the opinions and experiences of the workshop participants in terms of using the Novelty framework. The three questions were as follows:

- Did you find the Novelty framework easy to use? Please outline any difficulties you encountered.

- Do you agree with the descriptions (novel, somewhat novel, not novel) used in this framework?
- Any other comments?

The feedback in relation to the Novelty framework was very positive. All five respondents indicated that the framework was easy to use, noting that the framework was clearly laid out and that it was easy to identify the features within each category. Given that it was their first time using the framework, several reported that it was easy to follow and offered both useful and interesting insights. However, there was some difficulty reported in aspects of its application. Coder 3 found it challenging to distinguish between context and presentation when using the framework. Coders 1 and 4 felt that it was difficult to distinguish between 'not novel' and 'somewhat novel' when there is just a slight difference in the presentation of a task. They also reported difficulty at times with identifying a new feature or aspect of a mathematical concept when considering the 'somewhat novel' category. Coder 2 suggested that an extra criterion for the 'novel' category could be added for the case when the task requires a connection between different mathematical topics or strands of the syllabus. Coder 5 commented on the use of the word 'method', that its meaning or intention could be made more explicit: 'I usually think of method as meaning a procedure of some sort to be implemented, rather than (for example) an idea or an observation or a decision.'

All five respondents agreed with the three descriptions of 'novel', 'somewhat novel' and 'not novel' as presented. They did not suggest that the Novelty framework should be seriously modified, which suggests that the participants were able to follow the descriptions offered and use the criteria as intended.

Coder 1 felt that the framework would be useful for Pre-Service Teachers (PSTs) who often believe that 'textbooks are infallible'. Coder 2 also felt that the framework had a lot to offer not only PSTs but also practicing teachers, textbook authors and educational researchers. Coder 3 felt that the framework offered a great opportunity to 'sit and analyse the construction of tasks in textbooks and to critique the choice and order of tasks.' Coders 4 and 5 felt that the framework was interesting and could see potential for its extension to other work. Coder 5 was particularly interested in identifying a relationship between 'novel' tasks and difficulty.

4.12 Group Discussion

The discussion was intended to allow for the expansion of ideas raised in the questionnaire and to allow for comments that might not have been expressed in the questionnaire. At the beginning of the discussion, the group looked at the number of categories in the Novelty framework and felt that three was sufficient. Those who spoke on this felt that extending the number of categories to four would be excessive and would bring added difficulty in applying it, and that the existing three allowed for clearer distinctions between the categories.

Coder 1 discussed a potential difficulty in trying to abandon personal experience when using the framework and getting into the mind-set of the student adequately: 'I was kind of thinking – Oh to me they look all the same but maybe to a fifteen or sixteen year old they mightn't you know. I might say it's not novel but to them it might be. I found that difficult.' Coder 5 agreed that it was important to put oneself into the perspective of the student rather than the experienced mathematician. The group concluded that the post primary teacher's perspective can be very valuable when considering, for example, the impact of the appearance of negative numbers rather than positive numbers in an exercise. This is because researchers at third level could be far removed from the students' experience. It could be difficult for someone outside the second level classroom to judge how a student would react to such material.

The merits of the framework were also noted by the group. They felt that aside from textbook authors and educational researchers, it could be beneficial to teachers and teacher educators in particular. Coder 1 felt that it would be useful for PSTs to assist with the selection of tasks for the classroom and to also consider their potential strengths or weaknesses. Coder 3 added that it could be used as an empowering tool for teachers when looking at tasks.

4.13 Conclusion

From the Novelty framework validation workshop, it appears that the participants felt that the framework is clearly described and the results show that it can be used reasonably effectively and reliably. It must be noted that this workshop was held after the classifications (which will be presented in the next chapter) had been completed and so the recommendations made by participants have not yet been taken into account. However, the experience has given some indications for modification in the future. Although a value of 0.8 for Gwet's AC_2 indicates substantial agreement, consideration must still be given to possible improvements that might be made to the framework to attain greater reliability in coding.

From the feedback provided, I would implement the following changes to the Novelty framework. I would add an additional criterion to the 'novel' category and provide an extra example in the 'somewhat novel' classification. A more explicit exploration of earlier syllabus coverage would also be beneficial, especially for coders not familiar with the content area being analysed with the framework.
It would be useful to add an extra criterion for when a task requires a connection between different mathematical topics or strands of the syllabus in the 'novel' classification. For example: 'The user must independently make a connection between different syllabus strands or different mathematical concepts or topics'. This is something that the student would not have encountered within the current chapter and it would benefit the Novelty framework to anticipate such a scenario.

Distinguishing between the categories of 'somewhat novel' and 'not novel' is a key concern. The difference between these two classifications can be subtle and it is important that coders are able to identify features and apply the relevant classifications consistently. It appears that the participants were content with the criteria but needed further examples of how it is applied. Some coders were uncertain as to how to deal with 'presentation' and 'context' consistently. Currently an example for 'context' has been provided in the criteria with 'perhaps the use of an unfamiliar real-world situation'. It would be beneficial to provide a similar example for 'presentation' like 'different phrasing being used from what was encountered before so that it is not immediately obvious that the question involves the same mathematical solution method as previous questions'. This would assist coders with coming to terms with the 'somewhat novel' category and how to distinguish it from 'not novel'.

In future, when illustrating how I intend the framework is to be applied, it would be important not only to show how examples and preceding exercises offer modelled solutions for exercises but also to emphasise how a new aspect of a concept can be introduced and how a subtle adaption of the solution method can come about. It would be helpful to highlight when a task is essentially familiar but a new aspect of the concept has been introduced. Building this into the framework description would assist with identifying tasks that should be classified as 'somewhat novel'.

I think it would be beneficial to encourage future researchers using the framework to record their understanding of terminology when working with practice tasks so that the definition of 'skills' as provided in the criteria is not forgotten when coding and allow initial users to have greater certainty when applying the framework's criteria. It would also be important to remind this cohort to become familiar with material that students would encounter on a regular basis in their earlier mathematical experiences so as to inform their coding. This could be done encouraging researchers to obtain a list of relevant syllabus coverage from preceding years rather than assuming that users will be familiar with this kind of material. Steps such as these should help ensure greater reliability in coding in future. It must also be noted that there may be a flaw in the way Krippendorff's alpha works with data that is not evenly distributed over categories. I think it would be useful for future researchers to ensure that the sample of tasks given to coders is structured so that there are roughly equal numbers of tasks in each equal of the three categories of 'novel', 'somewhat novel' and 'not novel' so that the effects of prevalence are not felt.

Chapter 5 Results

5.1 Introduction

For this thesis a total of 7635 tasks on two topics from a number of textbooks were analysed. The textbooks were from three textbook series Active Maths, Text and Tests, and Concise Maths. Tasks were analysed from the areas of Pattern, Sequences and Series, and Differential Calculus. At HL, there are more tasks in the area of Differential Calculus than Pattern, Sequences and Series. The reverse is true at OL, with the exception of the Text and Tests Pre-PM textbook, where a greater number of tasks are assigned in the Pattern, Sequences and Series topic. The number of tasks generally increased between the Pre-PM and Post-PM eras with the exception of the Text and Tests HL textbook recording a decrease in the amount of tasks for both topics and the Concise Maths textbook experiencing a slight decrease in the number of Differential Calculus tasks at OL.

	Active Maths		Text and Tests		Concise Maths	
	Higher	Ordinary	Higher	Ordinary	Higher	Ordinary
Pattern, Sequences and Series	316	317	199	175	298	263
Differential Calculus	536	219	445	211	387	218

Table 4: Pre-PM Number of tasks per topic, level and textbook series

	Active Maths		Text and Tests		Concise Maths	
	Higher	Ordinary	Higher	Ordinary	Higher	Ordinary
Pattern, Sequences and Series	325	351	194	236	360	368
Differential Calculus	566	296	437	235	471	212

Table 5: Post-PM Number of tasks per topic, level and textbook series

In this chapter, the results for each framework will now be presented for each of the textbook series, both before and after the introduction of the Project Maths syllabus.

Each of the frameworks used in this thesis has a specific purpose when it comes to analysing tasks. Recall that the LCD framework gives insight into how much cognitive engagement a particular task demands. It allows for the classification of the cognitive load that is required for the successful completion of a task. The reasoning framework facilitates an examination of the thought employed by students in relation to the arguments used in the solving of tasks, characterizing such reasoning as either imitative or creative reasoning. The Novelty framework looks at the notion of novelty and measures it in terms of the experience the solver brings to a task. The use of the PM syllabus problem-solving objectives when classifying tasks sheds light on what qualities the textbooks are trying to develop in relation to the goals of the new syllabus. Finally Usiskin's model looks at the type of understanding that a task promotes in terms of the learning of a mathematical concept.

5.2 Level of Cognitive Demand

In this section, tasks on Differential Calculus and Patterns, Sequences and Series from each of the textbook series (Active Maths, Text and Tests, and Concise Maths) are classified using the Level of Cognitive Demand framework. It should be noted that the Active Maths textbook series was known as Discovering Maths before the new syllabus was introduced. The other two series retained their original names, to avoid confusion the Discovering Maths series will be referred to under the Active Maths headings.

5.2.1 Active Maths

5.2.1.1 Higher Level

A total of 852 tasks were analysed from the pre-Project Maths textbook Discovering Maths 4 (Higher Level). The analysis found that 69 (8.1%) were HLD tasks and the remaining 783 (91.9%) were LLD tasks. For the post-PM textbook Active Maths 4 Book 1 (Higher Level), a total of 891 tasks were examined. The analysis categorised these tasks into 197 (22.1%) HLD tasks and 694 (77.9%) LLD tasks. Table 6 shows the results of the classification.

LCD Classification Active Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	***Pre-PM Total***	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	***Post-PM Total***
HL Demand	*35 (11.1%)*	*34 (6.3%)*	***69 (8.1%)***	*97 (29.8%)*	*100 (17.7%)*	***197 (22.1%)***
HL Subcategory: Doing Mathematics	4 (1.3%)	2 (0.4%)	***6 (0.7%)***	3 (0.9%)	21 (3.7%)	***24 (2.7%)***
HL Subcategory: Procedures with connections to meaning	31 (9.8%)	32 (5.9%)	***63 (7.4%)***	94 (28.9%)	79 (14%)	***173 (19.4%)***
Lower Level (LL) Demand	*281 (88.9%)*	*502 (93.7%)*	***783 (91.9%)***	*228 (70.2%)*	*466 (82.3%)*	***694 (77.9%)***
LL Subcategory: Procedures without connections to meaning	281 (88.9%)	502 (93.7%)	***783 (91.9%)***	226 (69.5%)	466 (82.3%)	***692 (77.7%)***
LL Subcategory: Memorization	0 (0%)	0(0%)	***0 (0%)***	2(0.7%)	0 (0%)	***2 (0.2%)***
Total	**316**	**536**	**852**	**325**	**566**	**891**

Table 6: Active Maths Textbook Series (Higher Level) Levels of Cognitive Demand

A number of chi-square tests of independence were completed with $\alpha = 0.05$ in order to examine relationships between variables. For instance in the Active Maths HL textbook series, there is no significant difference between the old and new textbooks in relation to the proportions of tasks on each topic, the proportion of tasks is independent of topic $X^2(1, N = 1743) = 0.07,\ p = 0.79$. Results of all chi-square tests of independence are shown in tables 50-61 in Appendix B. Only those for which results were of particular interest will be commented on throughout this chapter.

As the proportions of tasks on the two topics were not significantly different in the pre- and post-PM textbooks, I combined the tasks from the two topics and tested to see if there was a difference in the proportion of HLD tasks in the old and new textbooks. The post-PM textbook has a higher incidence of HLD tasks than the pre-PM textbook and this difference has been found to be statistically significant. (see table 50, appendix B).

5.2.1.2 Ordinary Level

For the pre-PM textbook Discovering Maths 3 (Ordinary Level), 536 tasks were analysed. Of these tasks, 23 (4.3%) were categorised as HLD and 513 (95.7%) were labelled as LLD. For the Post-PM OL textbook Active Maths 3 Book 1, 109 tasks (16.8%) were found to be HLD with 538 (83.2%) labelled as LLD.

LCD Classification Active Maths Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	***Pre-PM Total***	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	***Post-PM Total***
HLD	6 (1.9%)	17 (7.8%)	***23 (4.3%)***	62 (17.7%)	47 (15.9%)	***109 (16.8%)***
HL Subcategory: Doing Mathematics	0 (0%)	0 (0%)	***0 (0%)***	0 (0%)	7 (2.4%)	***7 (1.1%)***
HL Subcategory: Procedures with connections to meaning	6 (1.9%)	17 (7.8%)	***23 (4.3%)***	62 (17.7%)	40 (13.5%)	***102 (15.7%)***
LLD	311 (98.1%)	202 (92.2%)	***513 (95.7%)***	289 (82.3%)	249 (84.1%)	***538 (83.2%)***
LL Subcategory: Procedures without connections to meaning	311 (98.1%)	202 (92.2%)	***513 (95.7%)***	289 (82.3%)	249 (84.1%)	***538 (83.2%)***
LL Subcategory: Memorization	0 (0%)	0 (0%)	***0 (0%)***	0 (0%)	0 (0%)	***0 (0%)***
Total	**317**	**219**	**536**	**351**	**296**	**647**

Table 7: Active Maths Textbook Series (Ordinary Level) Levels of Cognitive Demand

Similar to the Higher Level series, the proportion of HLD tasks is greater in the post-Project Maths textbook.

5.2.1.3 Comparison of Higher Level and Ordinary Level

Figure 5.1 below compares the proportion of tasks at each LCD across syllabus levels for both pre and post-PM textbooks. There is a statistically significant higher proportion of HLD tasks in the HL textbook than in the OL textbook for the post-PM Active Maths textbook series (see table 50, appendix B). Similar results were found for the pre-PM Active Maths textbook.

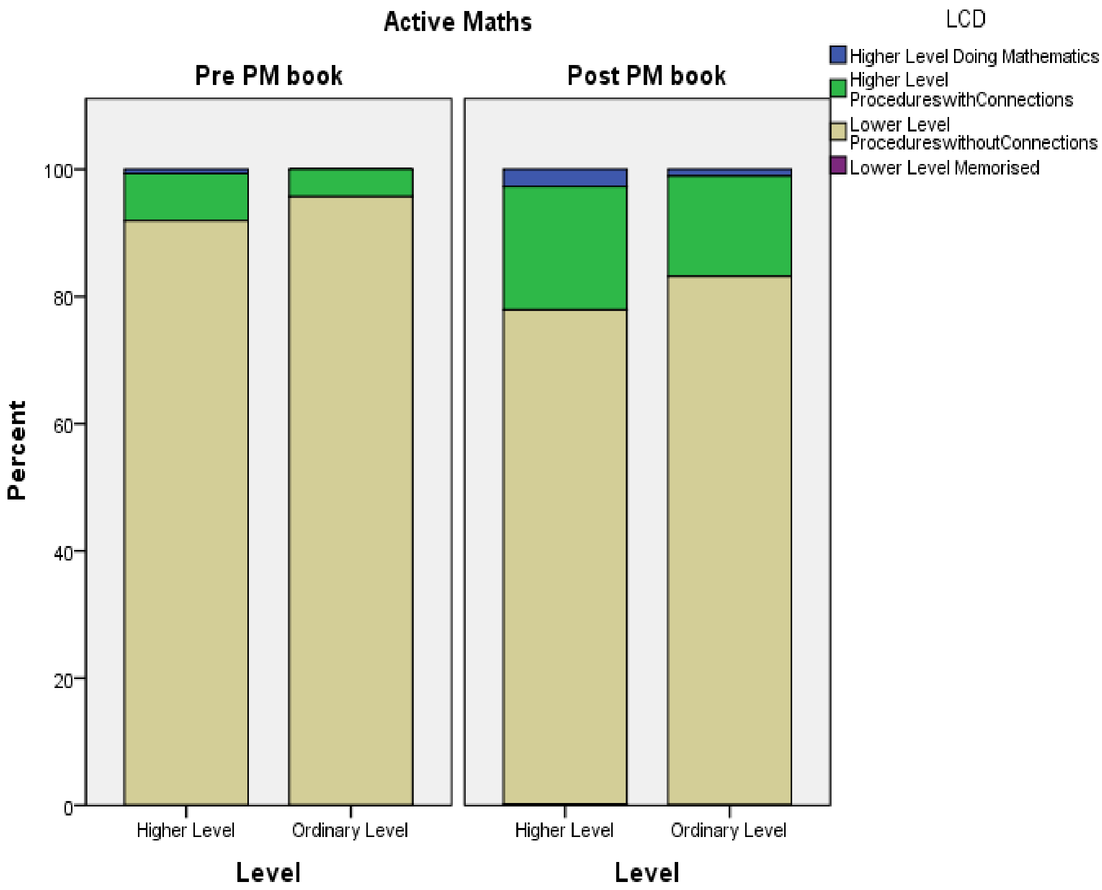


Figure 5.1: Comparison of LCD category proportions in HL and OL Active Maths textbooks for both pre and post-PM eras

5.2.1.4 Comparison of two topics

There is a statistically significant higher proportion of HLD tasks in the topic of Pattern, Sequences and Series than the topic of Differential Calculus in the HL post-PM textbooks.

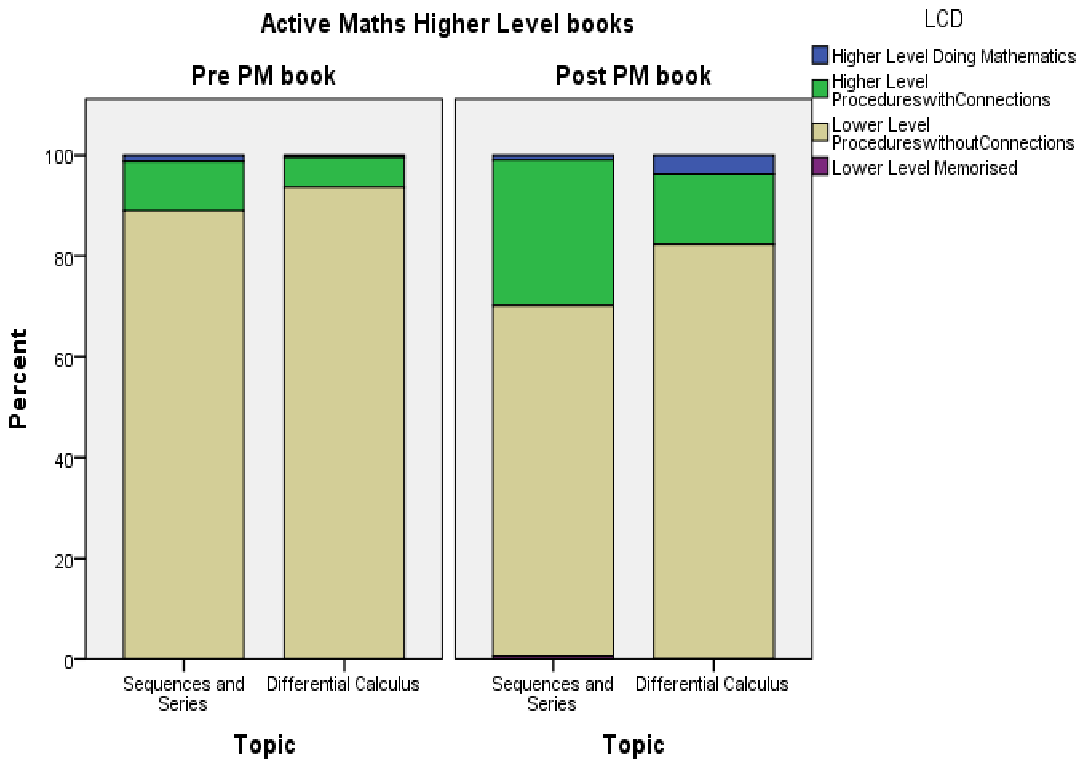


Figure 5.2: Comparison of LCD category proportions per topic in HL Active Maths textbooks for both pre and post-PM eras

Similar results were found in the pre-PM HL textbook. However the higher proportion of HLD tasks in Pattern, Sequences and Series is not statistically significant in the OL post-PM textbooks. For the OL pre-PM textbook there was actually a statistically significant higher proportion of HLD tasks on the topic of differential calculus (see table 50, appendix B for details). Figure 5.2 illustrates the proportions of tasks on each topic in the HL books at the various levels defined in the LCD framework.

5.2.2 Text and Tests

5.2.2.1 Higher Level

For the pre-PM era, the topic of Pattern, Sequences and Series was contained in Text and Tests 5 (HL). Differential Calculus was covered in Text and Tests 4 (HL). A total of 199 tasks were classified in Pattern, Sequences and Series with 445 tasks analysed in the topic Differential Calculus. Of these 27 (4.2%) were classified as 'procedures with connections to meaning' and the remaining 617 (95.8%) were placed in the category of 'procedures without connections to meaning'. I found tasks in all four LCD categories in the post-PM textbooks. A total of 194 tasks on Pattern, Sequences and Series were classified from Text and Tests 6 (HL). In Text and Tests 7 (HL), 437 tasks were analysed on the topic of Differential Calculus. Of these 22 (3.5%) were classified as 'doing mathematics', 125 (19.8%) were found to require 'procedures with connections to meaning', 482 (76.4%) tasks were classified as 'procedures without connections to meaning' and the remaining 2 (0.3%) necessitated the use of 'memorization'.

The two topics were combined and tested to see if there was a difference in the proportion of HLD tasks in the old and new textbooks. As can be seen in table 51, appendix B the post-PM textbook has a statistically significant higher incidence of HLD tasks than the pre-PM textbook.

LCD Classification Text and Tests Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	***Pre-PM Total***	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	***Post-PM Total***
HLD	14 (7%)	13 (2.9%)	***27 (4.2%)***	75 (38.7%)	72 (16.5%)	**147 (23.3%)**
HL Subcategory: Doing Mathematics	0 (0%)	0 (0%)	***0 (0%)***	19 (9.8%)	3 (0.7%)	**22 (3.5%)**
HL Subcategory: Procedures with connections to meaning	14 (7%)	13 (2.9%)	***27 (4.2%)***	56 (28.9%)	69 (15.8%)	**125 (19.8%)**
LLD	185 (93%)	432 (97.1%)	***617 (95.8%)***	119 (61.3%)	365 (83.5%)	**484 (76.7%)**
LL Subcategory: Procedures without connections to meaning	185 (93%)	432 (97.1%)	***617 (95.8%)***	119 (61.3%)	363 (83.1%)	**482 (76.4%)**
LL Subcategory: Memorization	0 (0%)	0 (0%)	***0 (0%)***	0 (0%)	2 (0.4%)	**2 (0.3%)**
Total	**199**	**445**	**644**	**194**	**437**	**631**

Table 8: Text and Tests Textbook Series (HL) Levels of Cognitive Demand

5.2.2.2 Ordinary Level

In the pre-Project Maths textbook, Text and Tests 3 (Ordinary Level), 386 tasks were analysed. Of these tasks, 8 (2.1%) of the tasks were categorised as HLD and 378 (97.9%) of the tasks were labelled as LLD. None of the tasks were categorised as 'doing mathematics'. The dominant sub-category applied was 'procedures without connections to meaning' with 378 tasks and just 8 tasks involved the use of 'procedures with connections to meaning'. After the introduction of PM, in the OL textbook Text and Tests 3, it was found that 37 tasks (7.8%) were categorised as HLD with 'procedures with connections to meaning' and 434 (92.2%) were classified as LLD with 'procedures without connections to meaning'.

The proportion of tasks on each topic in the Text and Tests series has not changed significantly over time. For the Text and Tests OL textbooks, similar to the HL series, the post-PM textbook has a statistically significant higher proportion of HLD tasks than the pre-PM textbook (table 51, appendix B).

LCD Classification Text and Tests Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	***Pre-PM Total***	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	***Post-PM Total***
HLD	8 (4.6%)	0 (0%)	***8 (2.1%)***	13 (5.5%)	0 (0%)	***37 (7.8%)***
HL Subcategory: Doing Mathematics	0 (0%)	0 (0%)	***0 (0%)***	0 (0%)	0 (0%)	***0 (0%)***
HL Subcategory: Procedures with connections to meaning	8 (4.6%)	0 (0%)	***8 (2.1%)***	13 (5.5%)	24 (10.2%)	***37 (7.8%)***
LLD	167 (95.4%)	211 (100%)	***378 (97.9%)***	223 (94.5%)	211 (89.8%)	***434 (92.2%)***
LL Subcategory: Procedures without connections to meaning	167 (95.4%)	211 (100%)	***378 (97.9%)***	223 (94.5%)	211 (89.8%)	***434 (92.2%)***
LL Subcategory: Memorization	0 (0%)	0 (0%)	***0 (0%)***	0 (0%)	0 (0%)	***0 (0%)***
Total	**175**	**211**	**386**	**236**	**235**	**471**

Table 9: Text and Tests Textbook Series (OL) Levels of Cognitive Demand

5.2.2.3 Comparison of Higher and Ordinary Level

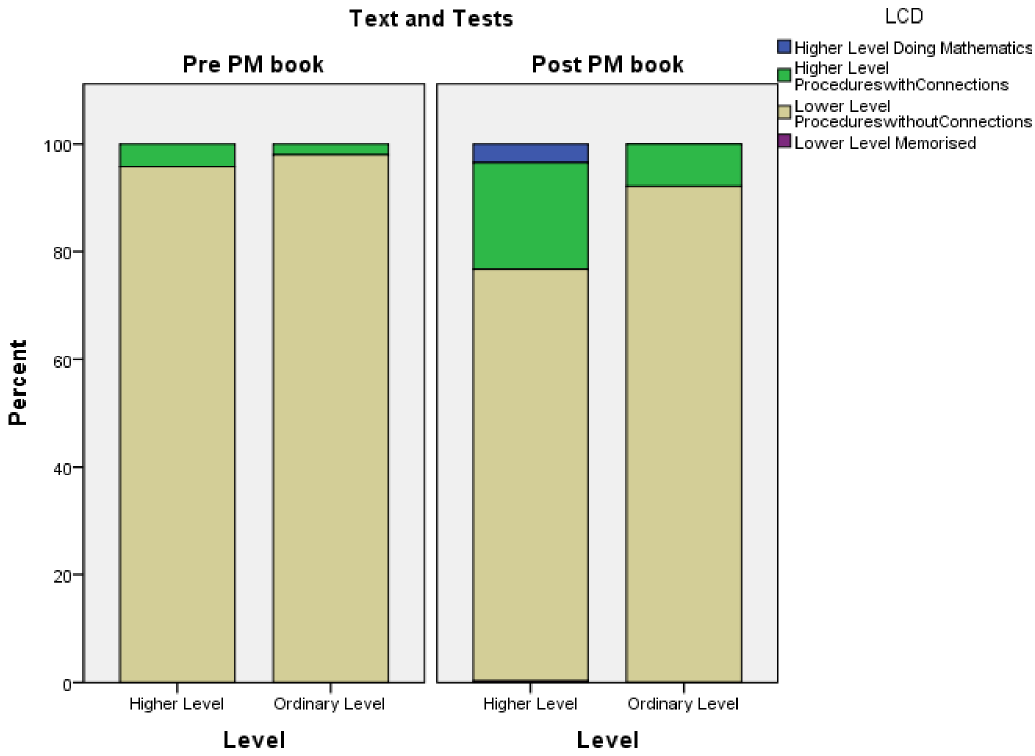


Figure 5.3: Comparison of LCD category proportions for Higher and Ordinary Level Text and Tests textbooks for both pre and post-PM eras

I found a statistically significant higher proportion of HLD tasks in the HL textbook than in the OL textbook for the post-PM Text and Tests textbook series. The pre-PM Text and Tests textbook yielded different results; no significant difference was apparent in the proportion of HLD tasks between the HL and OL of the pre-Project Maths textbooks (for details see table 51, appendix B), in both cases the percentage of HLD tasks was very low. Figure 5.3 compares the pre and post-PM textbooks in relation to the different categories of the LCD framework broken down by syllabus level.

5.2.2.4 Comparison of two topics

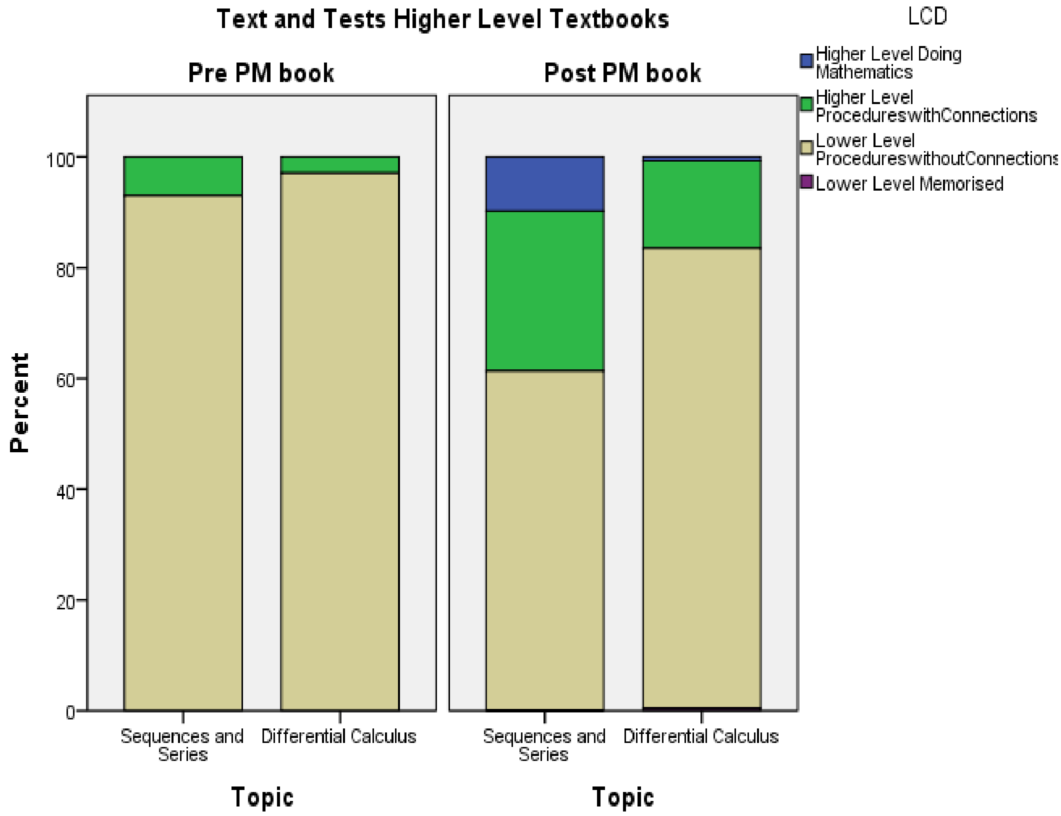


Figure 5.4 Comparison of LCD category proportions per topic in the HL Text and Tests textbooks for both pre and post-PM eras

There is a statistically significant higher proportion of HLD tasks in the topic of Pattern, Sequences and Series than the topic of Differential Calculus in the HL post-PM textbook. Similarly, there is a statistically significant higher proportion of HLD tasks in the topic of Pattern, Sequences and Series than the topic of Differential Calculus in the HL pre-Project textbook. In contrast to the post-PM textbooks, there is a statistically significant higher proportion of HLD tasks in the topic of Pattern, Sequences and Series than the topic of Differential Calculus in the OL pre-PM textbook (see table 51, appendix B). Figure 5.4 displays data for the HL textbooks.

5.2.3 Concise Maths

5.2.3.1 Higher Level

A total number of 685 tasks were examined from the Concise Maths series in use before the syllabus change. Out of these tasks, 1 (0.1%) was categorised as 'doing mathematics'. The remaining 24 (3.5%) HLD were classified as 'procedures with connections to meaning' and all of the 660 (96.4%) LLD tasks were placed in the category of 'procedures without connections to meaning'. A total of 831 tasks were classified from the post-Project Maths textbooks. The 'doing mathematics' classification had the lowest incidence with 3 (0.4%) tasks, the remaining 109 (13.1%) HLD tasks were classified as 'procedures with connections to meaning'. The majority of the tasks were LLD and 719 (86.5%) tasks were classified as 'procedures without connections to meaning'.

LCD Classification Concise Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	***Pre-PM Total***	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	***Post-PM Total***
HLD	17 (5.7%)	8 (2.1%)	***25 (3.6%)***	37 (10.3%)	75 (15.9%)	***112 (13.5%)***
HL Subcategory: Doing Mathematics	1 (0.3%)	0 (0%)	***1 (0.1%)***	0(0%)	3 (0.6%)	***3 (0.4%)***
HL Subcategory: Procedures with connections to meaning	16 (5.4%)	8 (2.1%)	***24 (3.5%)***	37 (10.3%)	72 (15.3%)	***109 (13.1%)***
LLD	281 (94.3%)	379 (97.9%)	***660 (96.4%)***	323 (89.7%)	396 (84.1%)	***719 (86.5%)***
LL Subcategory: Procedures without connections to meaning	281 (94.3%)	379 (97.9%)	***660 (96.4%)***	323 (89.7%)	396 (84.1%)	***719 (86.5%)***
LL Subcategory: Memorization	0(0%)	0(0%)	***0 (0%)***	0 (0%)	0 (0%)	***0 (0%)***
Total	**298**	**387**	**685**	**360**	**471**	**831**

Table 10: Concise Maths Textbook Series (HL) LCD

The post-PM textbook has a statistically significant higher incidence of HLD tasks than the pre-PM textbook (see table 52, appendix B).

5.2.3.2 Ordinary Level

From the pre-PM textbook, 481 tasks were analysed. Of these, 22 (4.6%) were classified as HLD and the remaining 459 (95.4%) were categorised as LLD. For the post-Project Maths textbook, a greater number of tasks were analysed with 68 (11.7%) labelled as HLD and 512 (88.3%) were found to be LLD.

LCD Classification Concise Maths Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	***Pre-PM Total***	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	***Post-PM Total***
HLD	15 (5.7%)	7 (3.2%)	***22 (4.6%)***	22 (6%)	46 (21.7%)	***68 (11.7%)***
HL Subcategory: Doing Mathematics	0 (0%)	1 (0.4%)	***1 (0.2%)***	0 (0%)	2 (0.9%)	***2(0.3%)***
HL Subcategory: Procedures with connections to meaning	15 (5.7%)	6 (2.8%)	***21 (4.4%)***	22 (6%)	44 (20.8%)	***66 (11.4%)***
LLD	248 (94.3%)	211 (96.8%)	***459 (95.4%)***	346 (94%)	166 (78.3%)	***512 (88.3%)***
LL Subcategory: Procedures without connections to meaning	248 (94.3%)	211 (96.8%)	***459 (95.4%)***	346 (94%)	165 (77.8%)	***511 (88.1%)***
LL Subcategory: Memorization	0 (0%)	0 (0%)	***0 (0%)***	0 (0%)	1 (0.5%)	***1 (0.2%)***
Total	**263**	**218**	**481**	**368**	**212**	**580**

Table 11: Concise Maths Textbook Series (OL) LCD

In the Concise Maths textbook OL series, the proportions of tasks per topic is not independent of the textbook era. Also, the post-PM textbook has a statistically higher incidence of HLD tasks than the pre-PM textbook (see table 52, appendix B). The same is true at HL.

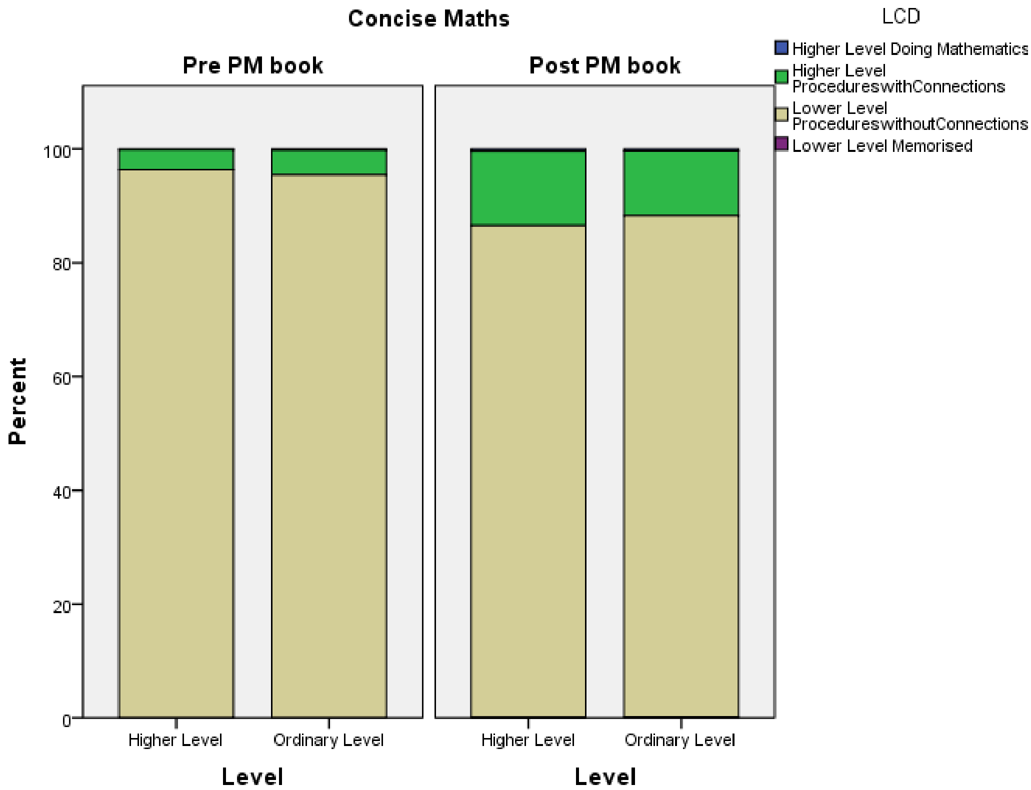


Figure 5.5: Comparison of LCD category proportions for HL and OL Concise Maths textbooks for both pre and post-PM eras

5.2.3.3 Comparison of Higher and Ordinary Level

There is no statistically significant difference between the proportion of HLD tasks in the HL and OL textbooks for the post-PM Concise Maths textbook series (see table 52, appendix B) or for the pre-PM series. Figure 5.5 shows that 3.6% of the HL tasks and 4.6% of the OL tasks were classified as HLD for the pre-PM textbook. This is in contrast to 13.5% of HL tasks and 11.7% of OL tasks for the post-PM textbook.

5.2.3.4 Comparison of two topics

A statistically significant higher proportion of HLD tasks in the topic of Differential Calculus than the topic of Pattern, Sequences and Series was found in the HL post-PM textbook. Similar results were found for the OL post-PM textbooks. In contrast, a statistically significant higher proportion of HLD tasks in the topic of Pattern, Sequences and Series than the topic of Differential Calculus was found in the HL pre-PM textbook. However the higher proportion of HLD tasks in Pattern, Sequences and Series is not statistically significant in the OL post-PM textbooks (see table 52, appendix B). Figure 5.6 compares the pre and post-PM textbooks at HL.

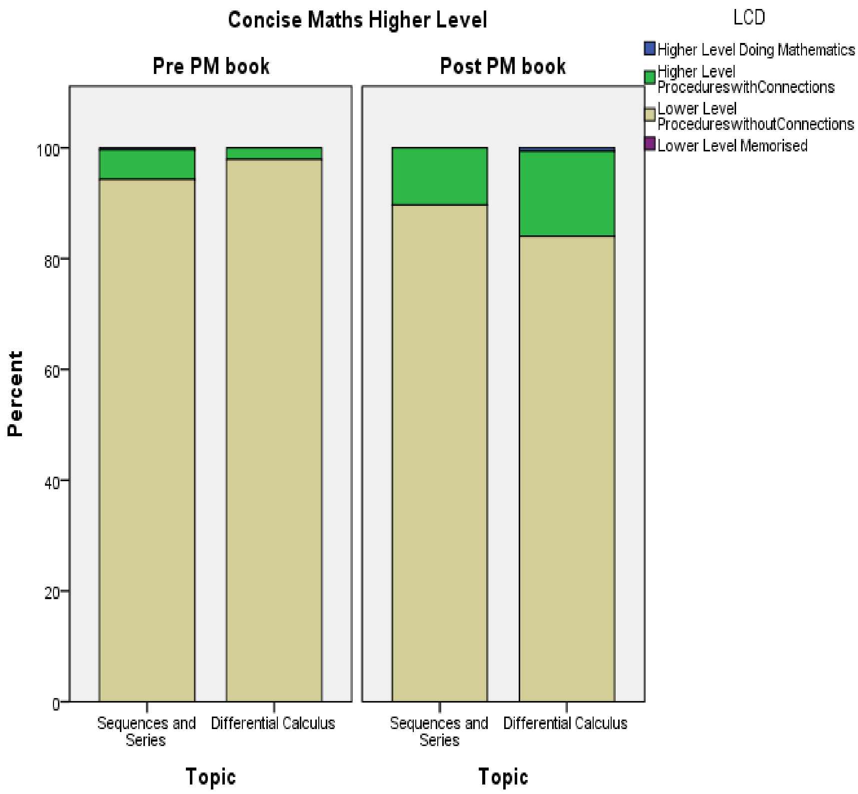


Figure 5.6: Comparison of LCD category proportions per topic in the Concise Maths HL textbooks for both pre and post-PM eras

5.2.4 Comparing the Textbook Series

For all three series, more tasks were labelled as HLD in the post-PM textbooks than the pre-PM textbooks for both syllabus levels. While there has been an increase in the cognitive demand of the tasks offered by post-PM textbooks compared to their pre-PM counterparts, particularly in the number of tasks available that involve the use of 'procedures with connections to meaning', the results presented here also suggest that teachers will have to look elsewhere or develop their own tasks in order to expose students to experiences which would correspond to the classification of 'doing mathematics'.

For all three series, a higher proportion of tasks were labelled as HLD in the HL textbooks than in the OL textbooks for both textbook eras. However the majority of tasks in all three post-PM textbooks series at both HL and OL were still classified as LLD as shown in figure 5.7, with almost all LLD tasks being in the category ‘procedures without connections to meaning’. Five out of six of these textbooks had no memorisation tasks. For the HL books, a statistically significant relationship was found between the proportion of HLD tasks and the post-PM textbook series to which the book belonged. A similar result was found for the OL textbooks (see table 53, appendix B). Here, the Active Maths series has the greatest proportion of HLD tasks with 16.8%, the Concise Maths series has 11.7% while the Text and Tests series has just 7.8% at this level. There are very few tasks classified as ‘doing mathematics’, the greatest proportion was found in Text and Tests HL with 3.5% and the lowest in Text and Tests OL with 0%.

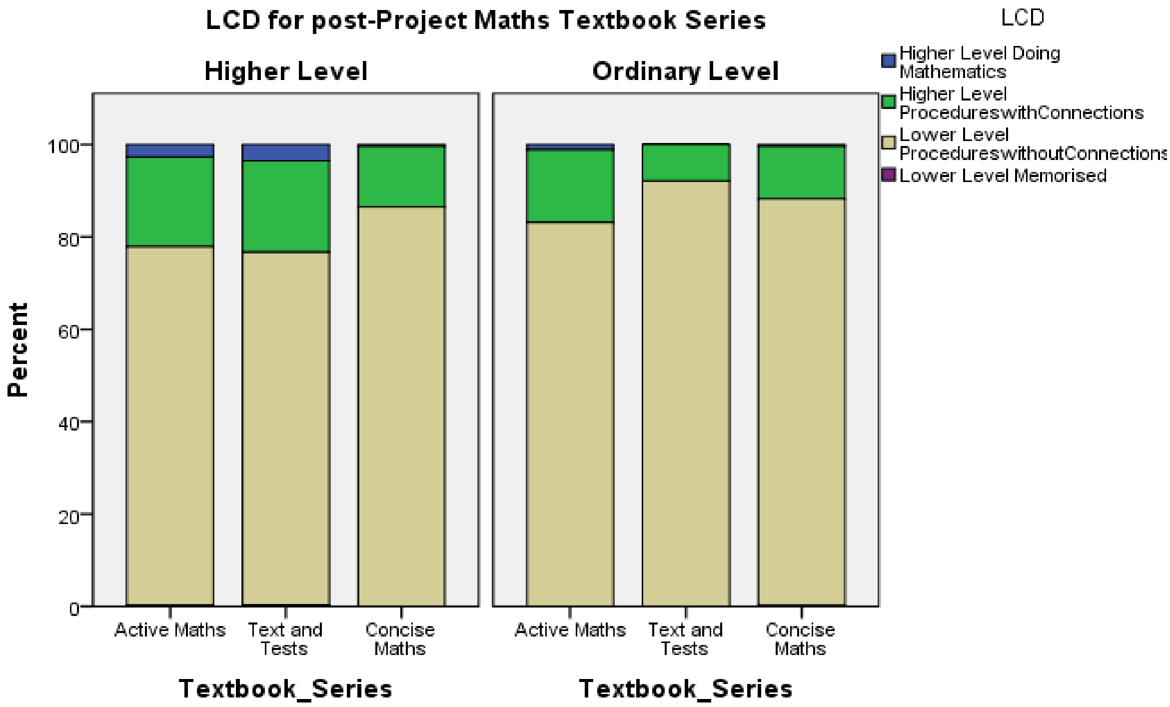


Figure 5.7: Comparison of LCD category proportions per textbook series for the post-PM era

For the HL pre-PM series, Active Maths had the highest proportion of HLD tasks with 8.1%. In the OL pre-PM textbooks, the Concise Maths series had the greatest proportion of HLD tasks with 4.6%. In the case of the pre-PM HL textbooks, the relationship between the textbook series and the proportion of HLD tasks was statistically significant. However, for the OL textbooks the connection between the textbook series and proportion of HLD tasks was not found to be statistically significant (see table 53, appendix B).

5.3 Creative/Imitative Reasoning

5.3.1 Active Maths

5.3.1.1 Higher Level

In Discovering Maths 4 (HL) a total of 852 tasks were analysed. A total of 56 (6.6%) tasks were classified as CR and 794 (93.4%) as IR. For the textbook in use after the introduction of Project Maths, 891 tasks were examined with 135 (15.2%) classified as CR and 756 (84.8%) as IR (table 12).

Reasoning Classification Active Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Creative Reasoning (CR)	22 (7%)	34 (6.3%)	**56 (6.6%)**	52 (16%)	83 (14.7%)	**135 (15.2%)**
Imitative Reasoning (IR)	294 (93%)	502 (93.7%)	**796 (93.4%)**	273 (84%)	483 (85.3%)	**756 (84.8%)**
Total	**316**	**536**	**852**	**325**	**566**	**891**

Table 12: Active Maths Textbook Series (HL) Types of Reasoning

In the Active Maths HL textbook series, (as shown in table 54, appendix B), there is a statistically significant higher incidence of CR tasks in the post-Project Maths textbook when compared to the pre-PM textbook.

5.3.1.2 Ordinary Level

For the textbooks in use before the introduction of PM, a total of 536 tasks were analysed. From these a total of 23 (4.3%) were categorised as CR with 513 (95.7%) as IR. For the Ordinary Level edition published after the introduction of Project Maths, a total of 647 tasks were analysed. From this total, 76 (11.7%) tasks were found to involve CR and 571 (88.3%) tasks were labelled as IR (table 13).

Reasoning Classification Active Maths Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
CR	6 (1.9%)	17 (7.8%)	**23 (4.3%)**	45 (12.8%)	31 (10.5%)	**76 (11.7%)**
IR	311 (98.1%)	202 (92.2%)	**513 (95.7%)**	306 (87.2%)	265 (89.5%)	**571 (88.3%)**
Total	**317**	**219**	**536**	**351**	**296**	**647**

Table 13: Active Maths Textbook Series (OL) Types of Reasoning

For the Active Maths OL textbook series, similar to the HL series, the post-PM textbook has a statistically significant higher incidence of CR tasks than the pre-PM textbook (see table 54, appendix B).

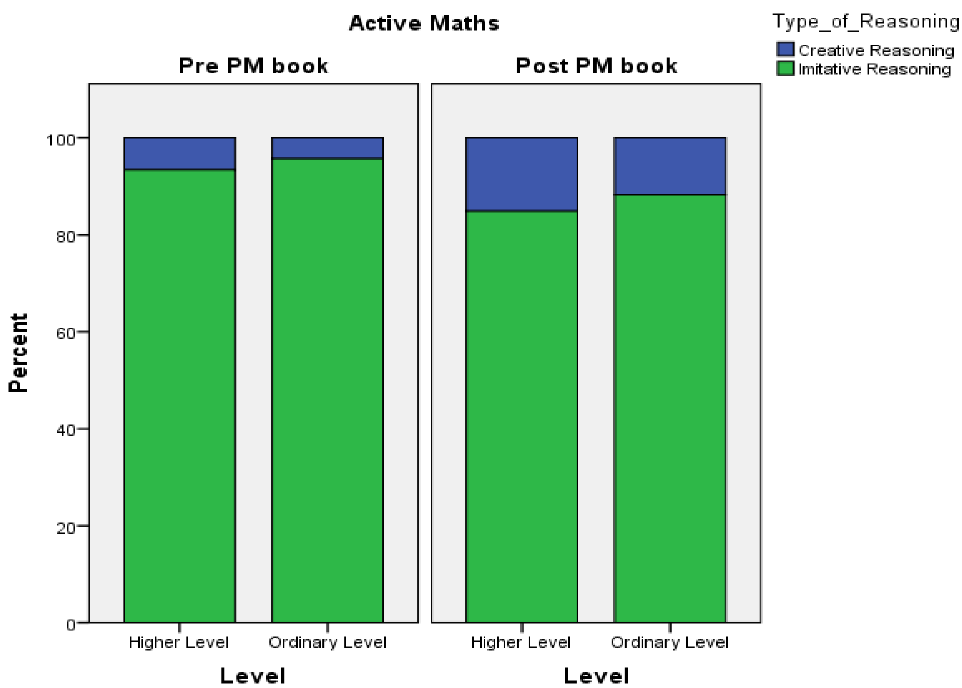


Figure 5.8: Comparison of reasoning proportions for HL and OL Active Maths textbooks for both pre and post-PM eras

5.3.1.3 Comparison of Higher Level and Ordinary Level

There is no significant difference in the proportion of CR tasks between the HL and OL textbooks for the post-PM Active Maths textbook series. Similar results were found for the pre-PM Active Maths textbook, (as shown in table 54, appendix B). Figure 5.8 shows the proportion of tasks for both levels in the pre and post-PM textbooks.

5.3.1.4 Comparison of two topics

The OL textbook had a statistically significant higher proportion of CR tasks in the topic of differential calculus (table 54, appendix B). Figure 5.9 shows the proportion of CR and IR tasks for the Active Maths OL textbooks broken down by topic.

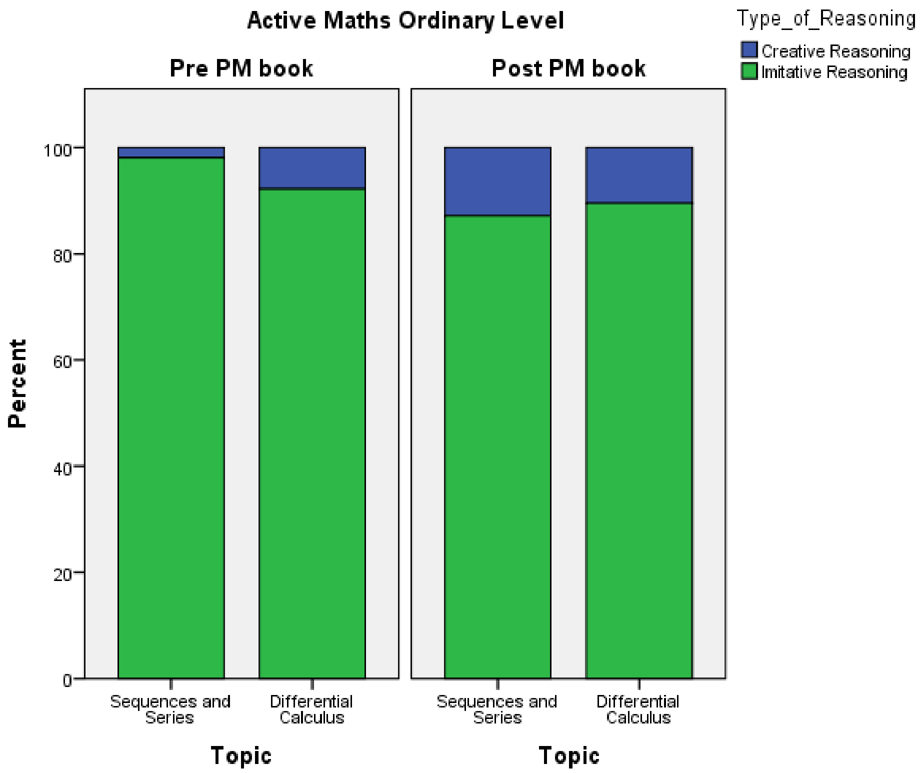


Figure 5.9: Comparison of reasoning per topic for OL Active Maths textbooks for both pre and post-PM eras

5.3.2 Text and Tests

5.3.2.1 Higher Level

For the HL textbook, in the pre-PM era, 644 tasks were analysed in total. These tasks were classified as 28 CR (4.3%) and 616 IR (95.7%). In the chapters for the post-PM era, a total of 631 tasks were examined, the classifications were 65 CR (10.3%) and 566 IR (89.7%) (table 14).

Reasoning Classification Text and Tests Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
CR	15 (7.5%)	13 (2.9%)	**28 (4.3%)**	42 (21.6%)	23 (5.3%)	**65 (10.3%)**
IR	184 (92.5%)	432 (97.1%)	**616 (95.7%)**	152 (78.4%)	414 (94.7%)	**566 (89.7%)**
Total	**199**	**445**	**644**	**194**	**437**	**631**

Table 14: Text and Tests Textbook Series (HL) Types of Reasoning

The post-PM textbook has a statistically significant higher incidence of CR tasks than the pre-PM textbook (table 55, appendix B).

5.3.2.2 Ordinary Level

In relation to the OL series, when material from the post-PM period was examined, 471 tasks were subdivided into 23 CR (4.9%) and 448 (95.1%) IR. The pre-PM textbooks had 386 tasks analysed, just 8 (2.1%) were classified as CR and 378 (97.9%) as IR (table 15).

Reasoning Classification Text and Tests Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
CR	8 (4.6%)	0 (0%)	**8 (2.1%)**	7 (3%)	16 (6.8%)	**23 (4.9%)**
IR	167 (95.4%)	211 (100%)	**378 (97.9%)**	229 (97%)	219 (93.2%)	**448 (95.1%)**
Total	**175**	**211**	**386**	**236**	**235**	**471**

Table 15: Text and Tests Textbook Series (OL) Types of Reasoning

In the Text and Tests OL textbook series, the post-PM textbook has a higher incidence of CR tasks than the pre-PM textbook and this was statistically significant (see table 55, appendix B).

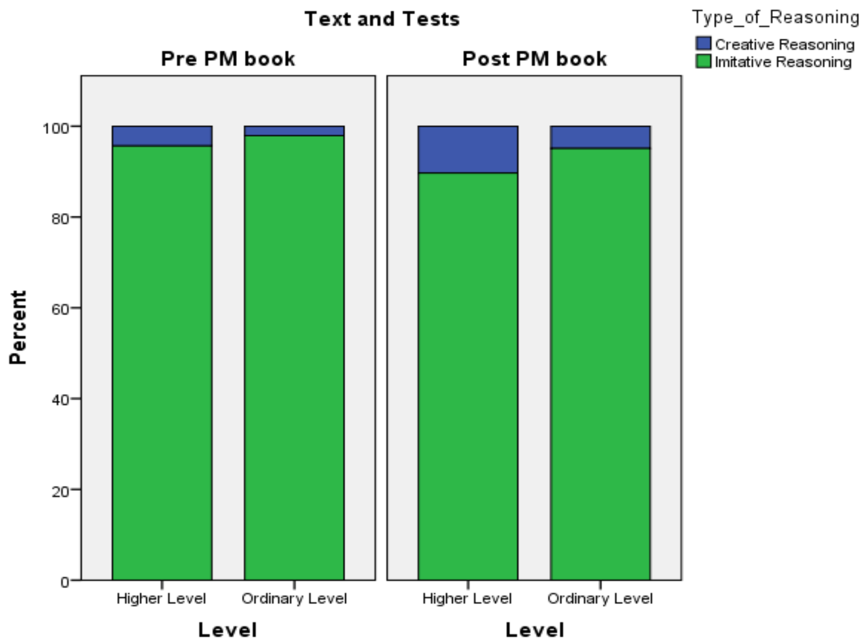


Figure 5.10: Comparison of reasoning proportions for HL and OL Text and Tests textbooks for both pre and post-PM eras

5.3.2.3 Comparison of Higher Level and Ordinary Level

There is a statistically significant higher proportion of CR tasks in the HL textbook than in the OL textbook for the post-PM Text and Tests textbook series (table 55, appendix B). Figure 5.10 shows the proportion of CR and IR tasks for each level and era in the Text and Tests textbook series.

5.3.2.4 Comparison of two topics

There is a statistically significant higher proportion of CR tasks in the topic of Pattern, Sequences and Series than the topic of differential calculus in the HL post-Project Maths textbooks. Similar results were found in the pre-PM textbooks in both the HL and OL texts (see table 55, appendix B). Figure 5.11 outlines the proportions of CR and IR for each topic in the HL Text and Tests textbook series for both the pre and post-PM eras.

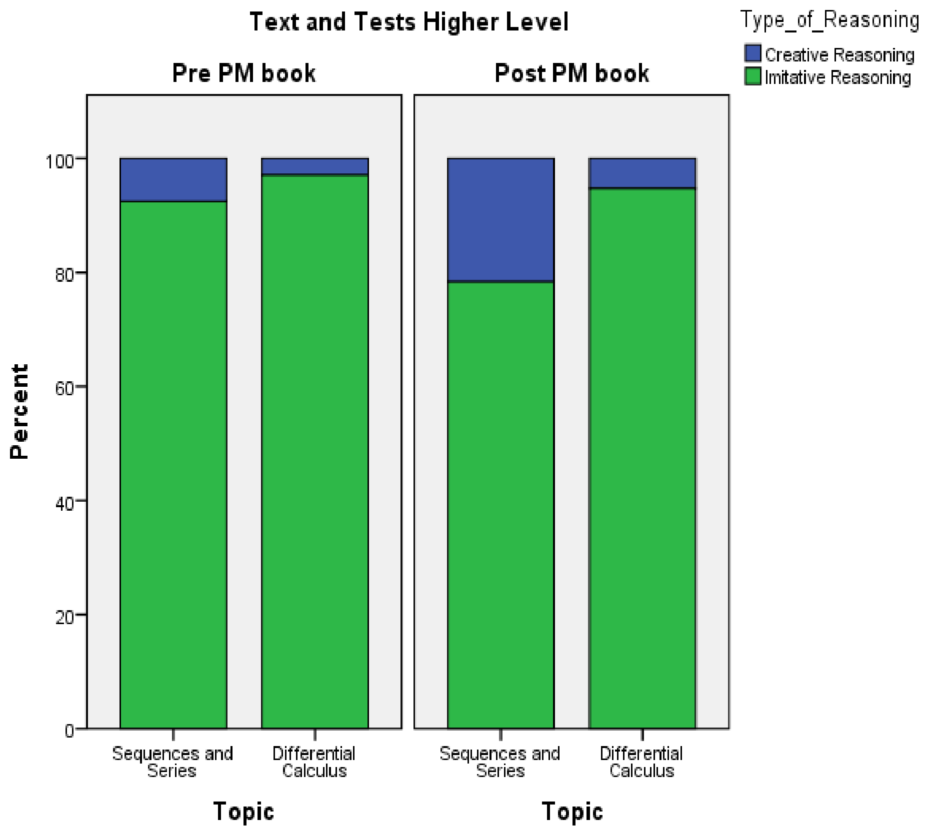


Figure 5.11: Comparison of reasoning proportions per topic for HL Text and Tests textbooks for both pre and post-PM eras

5.3.3 Concise Maths

5.3.3.1 Higher Level

For the HL (Pre-PM) textbook series, 685 tasks were divided into 25 (3.6%) CR and 660 (96.4%) IR. As for the post-PM textbook series, the total number of tasks analysed came to 831, which were further classified as 84 (10.1%) CR and 747 (89.9%) IR (table 16).

Reasoning Classification Concise Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
CR	17 (5.7%)	8 (2.1%)	**25 (3.6%)**	24 (6.7%)	60 (12.7%)	**84 (10.1%)**
IR	281 (94.3%)	379 (97.9%)	**660 (96.4%)**	336 (93.3%)	411 (87.3%)	**747 (89.9%)**
Total	**298**	**387**	**685**	**360**	**471**	**831**

Table 16: Concise Maths Textbook Series (HL) Types of Reasoning

The post-PM textbook has a higher incidence of CR tasks than the pre-PM textbook (table 56, appendix B).

5.3.3.2 Ordinary Level

When considering the pre-PM era, 481 tasks from OL textbook chapters were subdivided into 22 CR (4.6%) and 459 IR (95.4%). In the post-PM edition, a total of 580 tasks, were subdivided into 62 (10.7%) CR and 518 (89.3%) IR (table 17).

Reasoning Classification Concise Maths Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
CR	15 (5.7%)	7 (3.2%)	**22 (4.6%)**	22 (6%)	40 (18.9%)	**62 (10.7%)**
IR	248 (94.3%)	211 (96.8%)	**459 (95.4%)**	346 (94%)	172 (81.1%)	**518 (89.3%)**
Total	**263**	**218**	**481**	**368**	**212**	**580**

Table 17: Concise Maths Textbook Series (OL) Types of Reasoning

In the Concise Maths OL textbook series, the post-PM textbook has a statistically significant higher incident of CR tasks than the pre-PM textbook (see table 56, appendix B).

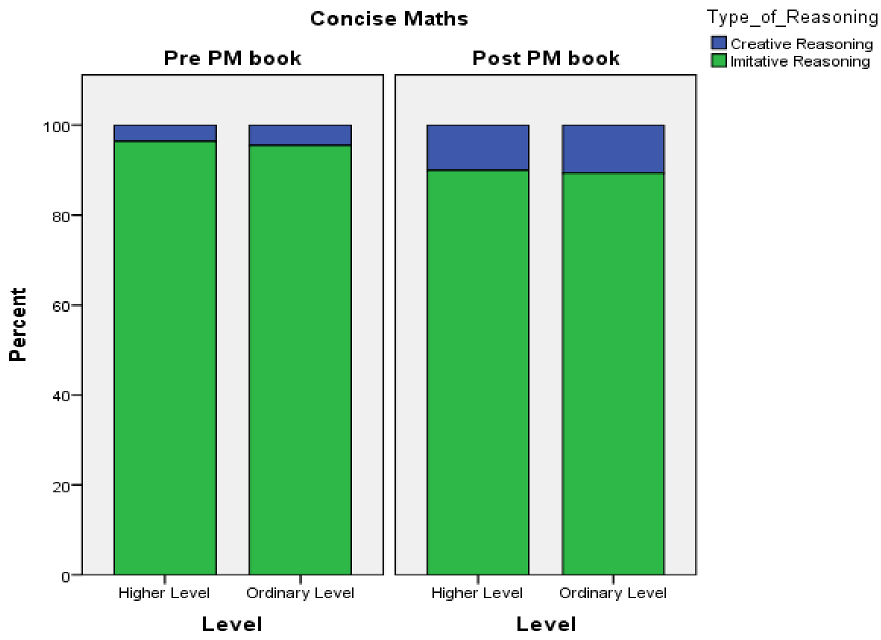


Figure 5.12: Comparison of reasoning proportions for HL and OL Concise Maths textbooks for both pre and post-PM eras

5.3.3.3 Comparison of Higher Level and Ordinary Level

There is no statistically significant difference in the proportion of CR tasks between the Higher Level and Ordinary Level textbooks for the post-Project Maths Concise Maths textbook series. Similar results were found for the pre-Project Maths Concise Maths textbook, (table 56, appendix B). Figure 5.12 shows the proportion of creative and imitative reasoning for the pre and post Project Maths Concise Maths textbooks at each level.

5.3.3.4 Comparison of two topics

There is a statistically significant higher proportion of CR tasks in the topic of Differential Calculus than the topic of Pattern, Sequences and Series in the HL post-PM textbook. A similar result was found for the OL post-PM textbook. In relation to the pre-PM HL textbooks a statistically higher proportion of CR tasks in the topic of Pattern, Sequences and Series than the topic of Differential Calculus was found (see table 56, appendix B for details). Figure 5.13 shows the proportion of different reasoning types for each topic in both eras of the Concise Maths HL textbook series.

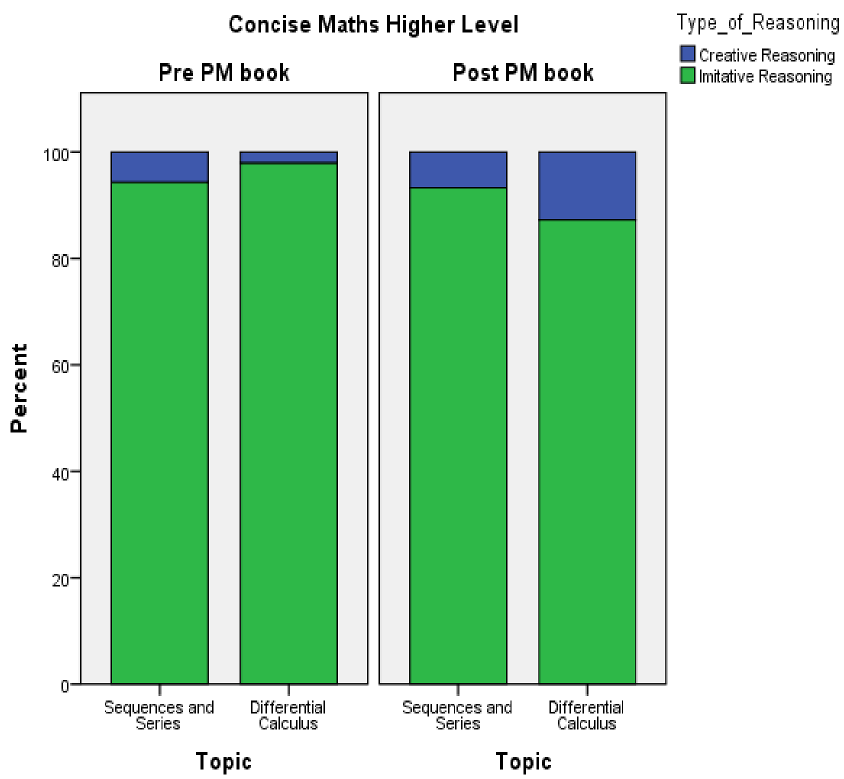


Figure 5.13: Comparison of reasoning proportions per topic for Higher Level Concise Maths textbooks for both pre and post-Project Maths eras

5.3.4 Comparing the Textbook Series

For the HL post-Project Maths textbooks, as shown in figure 5.14, the Active Maths textbook series had the highest proportion of CR. Concise Maths had the least and this difference was found to be statistically significant (table 57, appendix B).

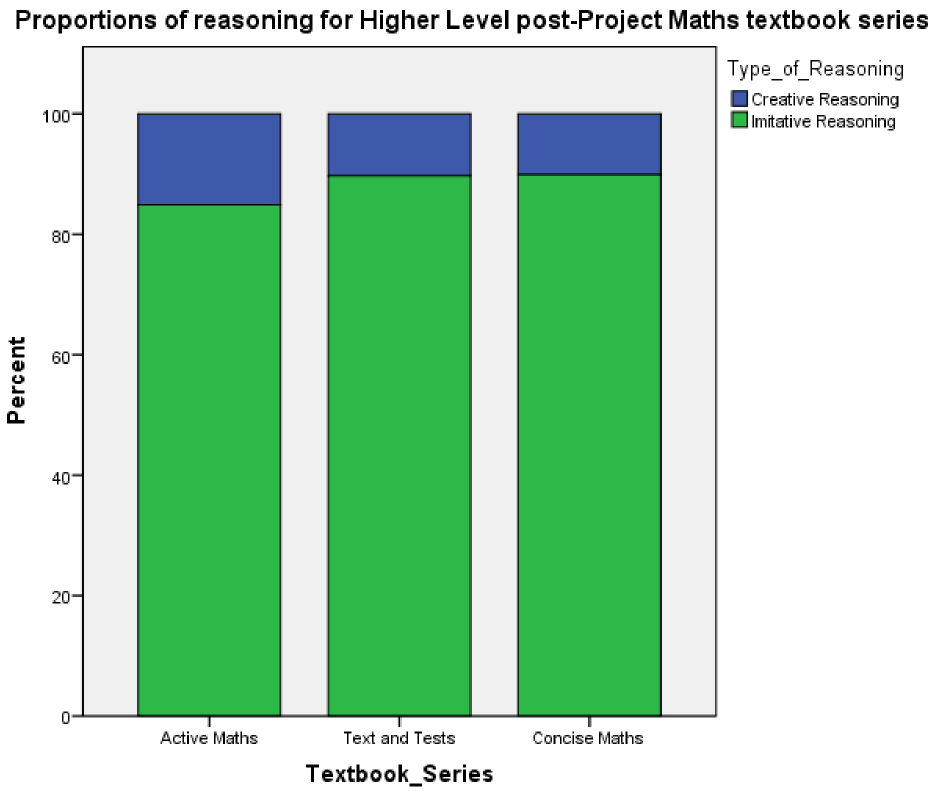


Figure 5.14: Comparison of reasoning proportions for HL post-PM textbook series

A similar result, (as shown in table 57, appendix B) and figure 5.15, was found for the OL textbooks.

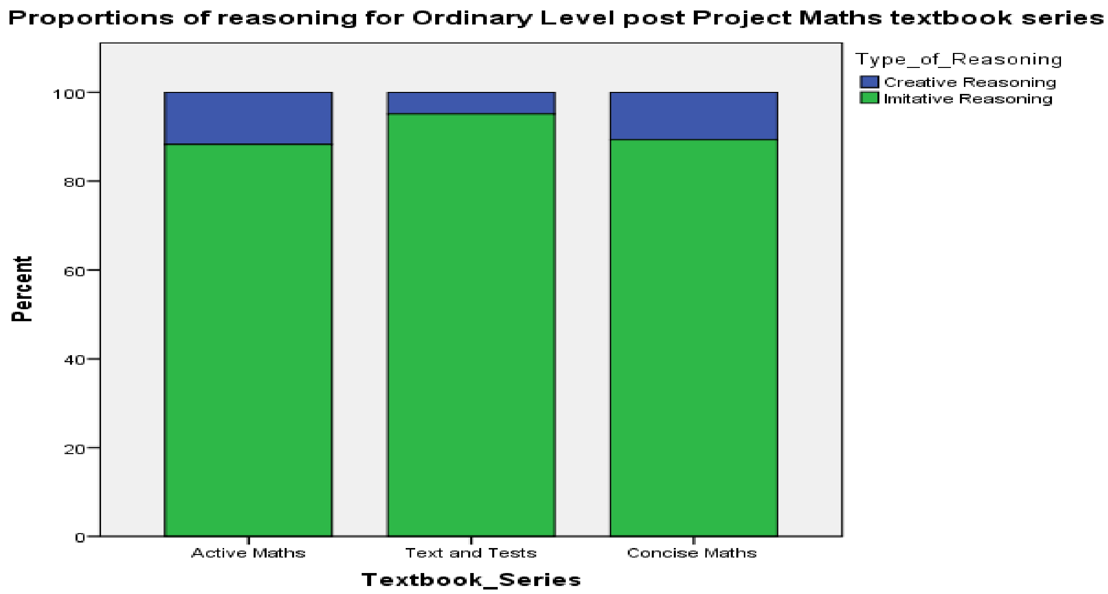


Figure 5.15: Comparison of reasoning proportions for OL post-PM textbook series

For the pre-PM HL textbooks, the greatest proportion of CR tasks was found in the Active Maths series, the lowest incidence of CR tasks was found in the Concise Maths series - again this difference across textbooks was found to be statistically significant (table 57, appendix B).

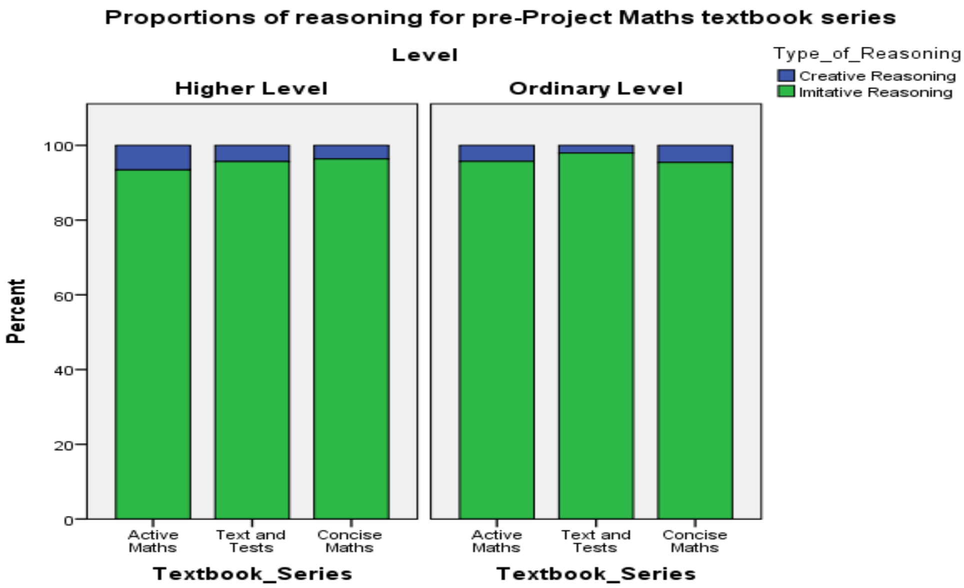


Figure 5.16: Comparison of reasoning proportions for pre-PM textbook series

The greatest proportion of CR tasks in the OL pre-PM textbook series was found in Concise Maths with 4.6%. The proportion of tasks requiring CR has increased since the introduction of the PM syllabus, rising from an average over the three textbook series of 4.5% to 11%.

5.4 Novelty

5.4.1 Active Maths

5.4.1.1 Higher Level

The Active Maths series which was in use for HL before the introduction of PM had a large number of tasks classified as 'not novel' with 77.5%. The other two categories of 'somewhat novel' and 'novel' had an incidence of 19% and 3.5% respectively. The new textbooks recorded a marked increase in the proportion of 'novel' tasks at 8.8% (table 18): the relationship between the 'novelty' of tasks and the textbook era is statistically significant (table 58, appendix B).

Novelty Classification Active Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Novel	9 (2.8%)	21 (3.9%)	**30 (3.5%)**	23 (7.1%)	55 (9.7%)	**78 (8.8%)**
Somewhat Novel	59 (18.7%)	103 (19.2%)	**162 (19%)**	73 (22.5%)	97 (17.1%)	**170 (19.1%)**
Not Novel	248 (78.5%)	412 (76.9%)	**660 (77.5%)**	229 (70.4%)	414 (73.2%)	**643 (72.1%)**
Total	**316**	**536**	**852**	**325**	**566**	**891**

Table 18: Active Maths Textbook Series (HL) Types of Novelty

5.4.1.2 Ordinary Level

For OL, a similar situation to the HL was observed (table 19). The proportions for 'not novel', 'somewhat novel' and 'novel' were 88.4%, 10.1% and 1.5% respectively. For the textbooks introduced to meet the needs of the new syllabus, the incidence of 'novel' tasks increased to 4% and the 'somewhat novel' tasks increased to 17.2%, this was accompanied by a fall in the 'not novel' tasks to 78.8%.

Novelty Classification Active Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Novel	1 (0.3%)	7 (3.2%)	**8 (1.5%)**	14 (4.0%)	12 (4.1%)	**26 (4.0%)**
Somewhat Novel	29 (9.2%)	25 (11.4%)	**54 (10.1%)**	41 (11.7%)	70 (23.6%)	**111 (17.2%)**
Not Novel	287 (90.5%)	187 (85.4%)	**474 (88.4%)**	296 (84.3%)	214 (72.3%)	**510 (78.8%)**
Total	**317**	**219**	**536**	**351**	**296**	**647**

Table 19: Active Maths Textbook Series (OL) Types of Novelty

For the Active Maths OL textbook series (as seen in table 58, appendix B), the textbook series has a statistically significant relationship between the 'novelty' of tasks and the era of the textbook.

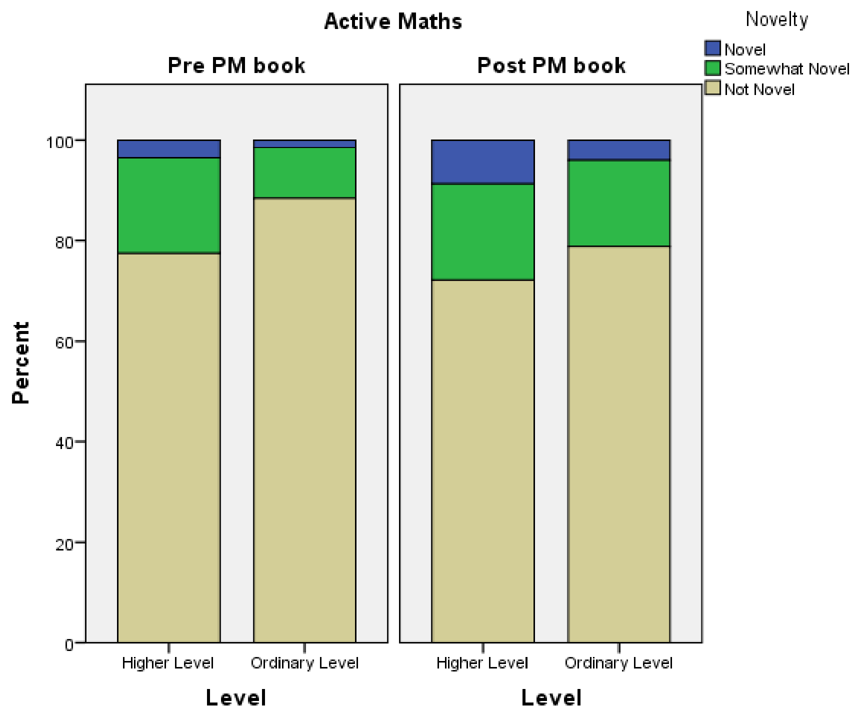


Figure 5.17: Comparison of novelty proportions in HL and OL Active Maths textbooks per PM era

5.4.1.3 Comparison of Higher Level and Ordinary Level

The proportion of tasks across the 'novelty' categories in the post-PM Active Maths textbooks is not independent of the textbook level. Similar results were found for the pre-PM Active Maths textbooks, (see table 58, appendix B). Figure 5.17 shows a comparison of the novelty proportions at HL and OL per PM era.

5.4.1.4 Comparison of two topics

For the OL post-PM textbooks, it was found that the proportion of tasks at different levels of 'novelty' was not independent of the topic with fewer differential calculus tasks classified as 'not novel'. The result for the OL pre-PM textbook was similar (see table 58, appendix B). Figure 5.18 gives a comparison of the proportion of tasks at different levels of 'novelty' in the two topics in the Active Maths OL textbook series.

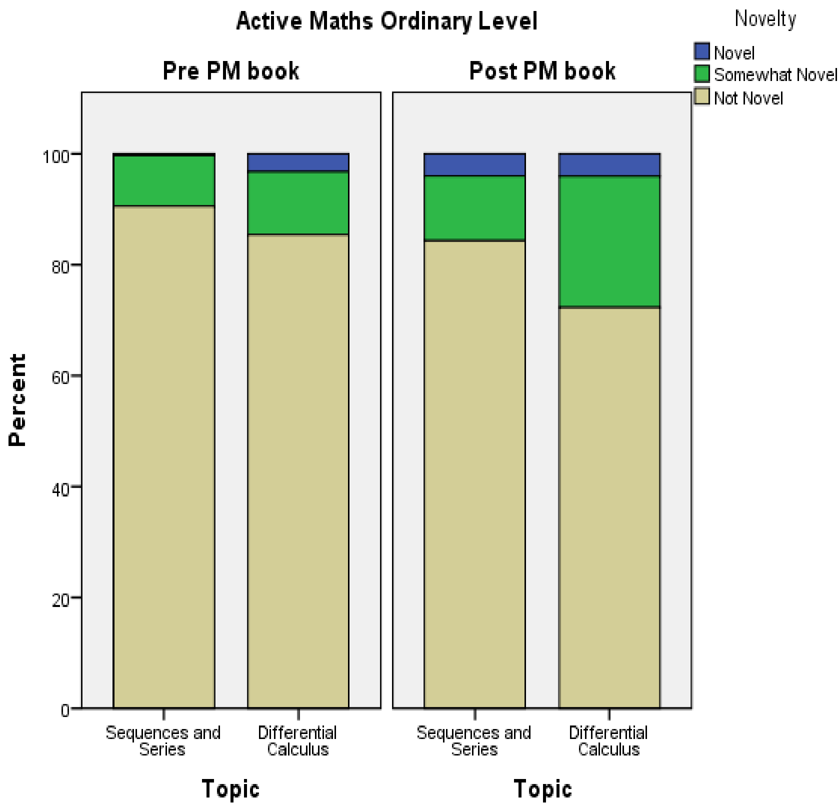


Figure 5.18: Comparison of proportion of novelty categories per topic in the OL Active Maths pre- and post-PM textbooks

5.4.2 Text and Tests

5.4.2.1 Higher Level

For the HL Text and Tests series, there was a large increase in the proportion of 'somewhat novel' tasks, rising from 17.5% in the Pre-PM series to 28.5% in the textbooks in use after the implementation of the new syllabus. This corresponded to a similar decrease in the 'not novel' classification, falling from 80.9% to 67.7%, while the proportion of 'novel' tasks did not change significantly.

In the Text and Tests HL textbook series, there is a statistically significant relationship between the novelty of tasks and the textbook era (table 59, appendix B). This is a similar result to the Active Maths series.

Novelty Classification Text and Tests Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Novel	12 (6.0%)	11 (2.5%)	**23 (3.6%)**	8 (4.1%)	16 (3.7%)	**24 (3.8%)**
Somewhat Novel	39 (19.6%)	74 (16.6%)	**113 (17.5%)**	54 (27.8%)	126 (28.8%)	**180 (28.5%)**
Not Novel	148 (74.4%)	360 (80.9%)	**508 (78.9%)**	132 (68.1%)	295 (67.5%)	**427 (67.7%)**
Total	**199**	**445**	**644**	**194**	**437**	**631**

Table 20: Text and Tests Textbook Series (HL) Types of Novelty

5.4.2.2 Ordinary Level

For the OL series, the difference between the two editions was much more modest. The 'novel' category increased from 0.3% to 3.2% and the 'somewhat novel' classification increased from 13% to 13.6%. There was a corresponding small decrease in the 'not novel' category, falling from 86.7% to 83.2%. However, the relationship between 'novelty' and the textbook era was still statistically significant for these OL textbooks (table 59, appendix B).

Novelty Classification Text and Tests Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	Pre-PM Total	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	Post-PM Total
Novel	1 (0.6%)	0 (0%)	**1 (0.3%)**	1 (0.4%)	14 (6.0%)	**15 (3.2%)**
Somewhat Novel	35 (20%)	15 (7.1%)	**50 (13.0%)**	25 (10.6%)	39 (16.6%)	**64 (13.6%)**
Not Novel	139 (79.4%)	196 (92.9%)	**335 (86.7%)**	210 (89%)	182 (77.4%)	**392 (83.2%)**
Total	**175**	**211**	**386**	**236**	**235**	**471**

Table 21: Text and Tests Textbook Series (OL) Types of Novelty

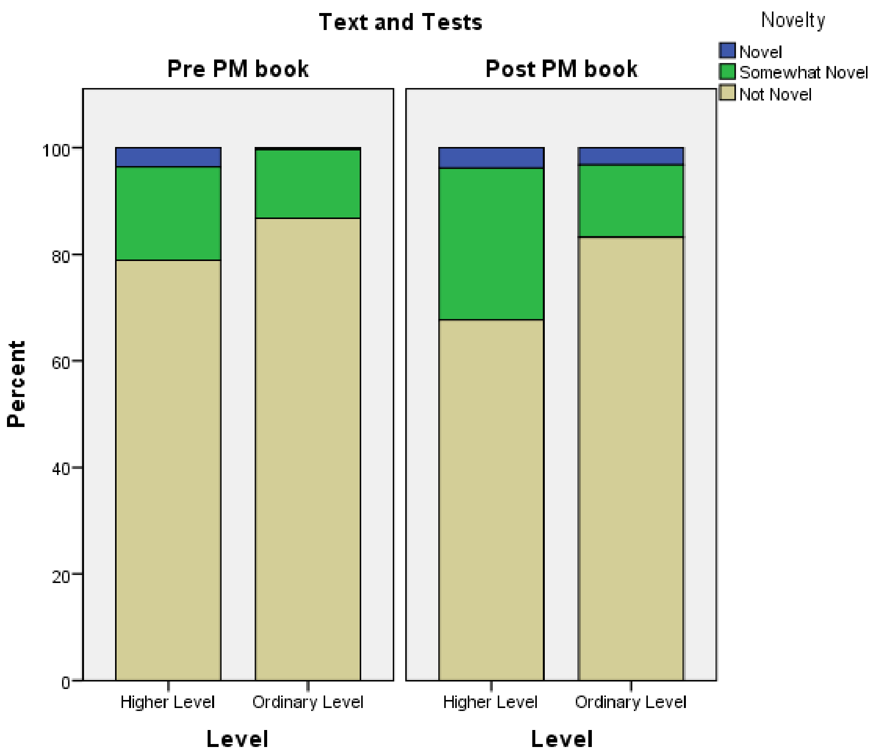


Figure 5.19: Comparison of the proportions of novelty in HL and OL Text and Tests textbooks per PM era

5.4.2.3 Comparison of Higher Level and Ordinary Level

The distribution of tasks across the 'novelty' categories is not independent of the textbook level for the Text and Tests post-PM textbook series. Similar results were found for the pre-PM Text and Tests textbooks, (see table 59, appendix B). Figure 5.19 gives a comparison of the proportions of 'novelty' in the Higher and Ordinary Level textbooks per PM era.

5.4.2.4 Comparison of two topics

For the OL post-PM textbooks, it was found that the proportion of tasks at different levels of 'novelty' was not independent of the topic with fewer differential calculus tasks classified as 'not novel'. In the case of the pre-PM textbooks: for the HL textbook, the different levels of 'novelty' were found not to be independent of the topic with more tasks from Pattern, Sequences and Series classified as 'novel' and 'somewhat novel', a similar statistically significant result was found for the OL pre-Project Maths textbook, (see table 59, appendix B for details). Figure 5.20 below compares the proportions of 'novelty' in the Text and Tests Higher Level textbooks for both the pre and post-PM periods.

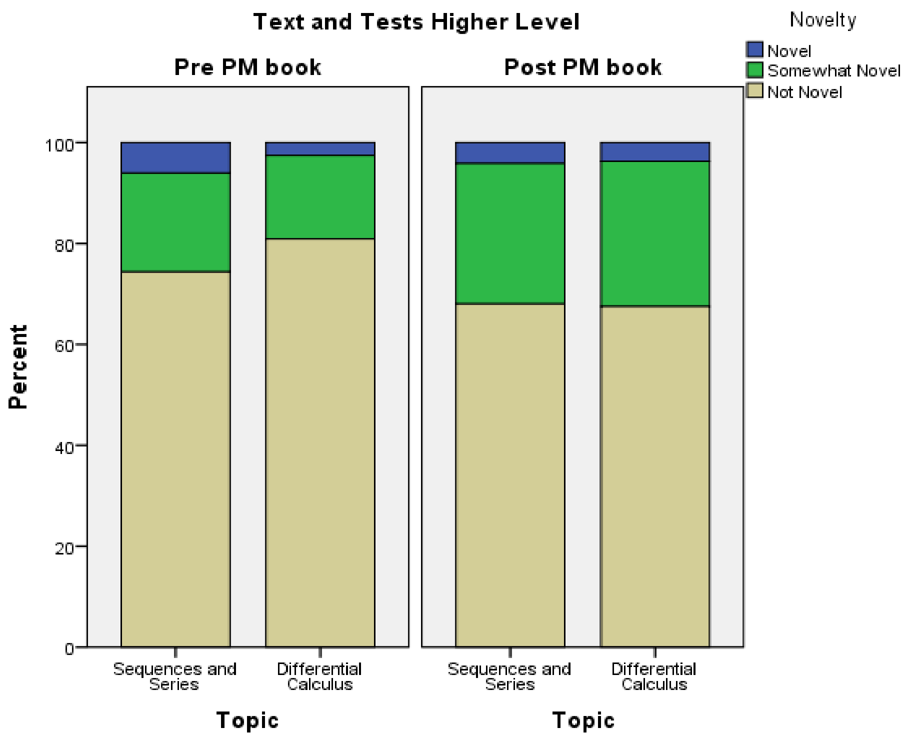


Figure 5.20: Comparison of novelty proportions per topic in the Text and Tests Higher Level textbooks per Project Maths era

5.4.3 Concise Maths

5.4.3.1 Higher Level

For the HL textbook series, the 'somewhat novel' category showed the greatest difference between the tasks used before (17.4%) and after (22.7%) the syllabus was introduced. As with the other series, there was a small increase in the 'novel' tasks and a small decrease in the 'not novel' classifications.

Novelty Classification Concise Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Novel	9 (3%)	8 (2.1%)	**17 (2.5%)**	16 (4.4%)	31 (6.6%)	**47 (5.7%)**
Somewhat Novel	61 (20.5%)	58 (15.0%)	**119 (17.4%)**	79 (22.0%)	110 (23.3%)	**189 (22.7%)**
Not Novel	228 (76.5%)	321 (82.9%)	**549 (80.1%)**	265 (73.6%)	330 (70.1%)	**595 (71.6%)**
Total	**298**	**387**	**685**	**360**	**471**	**831**

Table 22: Concise Maths Textbook Series (HL) Types of Novelty

In the Concise Maths Higher Level textbook series, there is a statistically significant relationship between the 'novelty' of tasks and textbook era (table 60, appendix B).

5.4.3.2 Ordinary Level

For the OL textbook series, the increase in classifications for the 'novel' category was slightly greater than the HL textbook going from 2.9% to 6.7%. The 'somewhat novel' category also increased going from 14.8% to 19.3%. The proportion of 'not novel' tasks fell from 82.3% to 74%.

Novelty Classification Concise Maths Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Novel	7 (2.7%)	7 (3.2%)	**14 (2.9%)**	19 (5.2%)	20 (9.4%)	**39 (6.7%)**
Somewhat Novel	34 (12.9%)	37 (17.0%)	**71 (14.8%)**	63 (17.1%)	49 (23.1%)	**112 (19.3%)**
Not Novel	222 (84.4%)	174 (79.8%)	**396 (82.3%)**	286 (77.7%)	143 (67.5%)	**429 (74%)**
Total	**263**	**218**	**481**	**368**	**212**	**580**

Table 23: Concise Maths Textbook Series (OL) Types of Novelty

For the Concise Maths OL textbook series, there is a statistically significant relationship between the 'novelty' of tasks and the textbook era (table 60, appendix B).

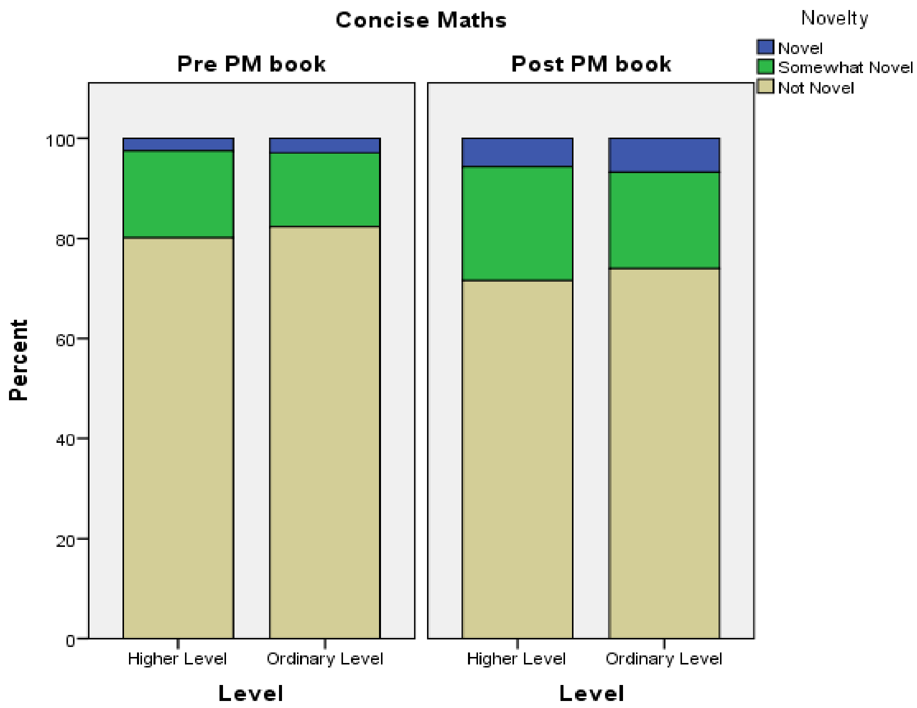


Figure 5.21: Comparison of the proportions of novelty in HL and OL Concise Maths textbooks per PM era

5.4.3.3 Comparison of Higher Level and Ordinary Level

The proportion of tasks in each of the 'novelty' categories is independent of the textbook level for the post-PM Concise Maths textbook series. Similar results were found for the pre-PM Concise Maths textbook (table 60, appendix B). Figure 5.21 in the previous section shows proportions of 'novelty' in both HL and OL Concise Maths textbooks for both the pre and post-PM eras.

5.4.3.4 Comparison of two topics

For the OL post-PM textbooks, it was found that the proportion of tasks at different levels of 'novelty' was not independent of the topic with fewer differential calculus tasks classified as 'not novel' (see table 60, appendix B). Figure 5.22 gives a comparison of the proportions of 'novelty' for the Concise Maths textbook series in each PM era.

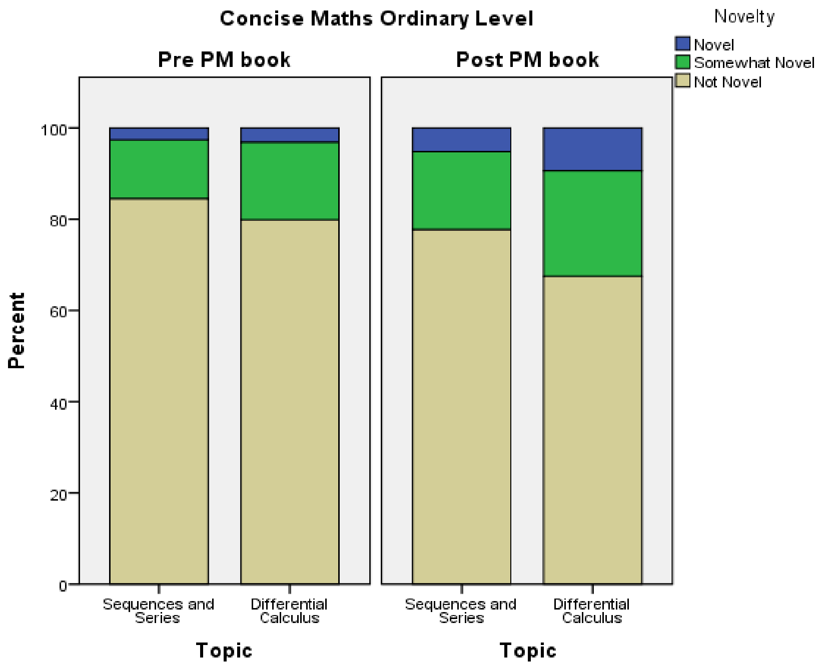


Figure 5.22: Comparison of the proportion of novelty per topic in the OL Concise Maths textbook series

5.4.4 Comparing Textbook Series

For the HL textbooks, the highest proportion of 'novel' tasks was in Active Maths with 8.8%. Text and Tests had the greatest proportion of 'somewhat novel' tasks at 28.5%. Active Maths also had the highest incidence of 'not novel' tasks with 72.1%, slightly more than Concise Maths with 71.6%. For the corresponding textbooks in use during the period prior to the introduction of PM, Text and Tests had the highest proportion of ‘novel’ tasks with 3.6%. The Active Maths series had the highest proportion of ‘somewhat novel’ tasks at this time with 19.2%. The Concise Maths series had the greatest percentage of tasks labelled ‘not novel’ with 80.1%.

Every textbook analysed showed an increase in the ‘novel’ and ‘somewhat novel’ categories between the pre-PM textbooks and those introduced for the PM syllabus. This corresponded to a decrease in the ‘not novel’ classification. The greatest increase for ‘novel’ tasks at Higher Level was recorded in Active Maths which rose from 3.5% to 8.8%, while Text and Tests had the largest difference in both the ‘somewhat novel’ and ‘not novel’ categories. Rising from 10.1% to 17.2% for ‘somewhat novel’ and falling from 78.9% to 67.7% in the classification of ‘not novel’. For the HL books, a statistically significant relationship was found between the proportion of tasks in each of the three ‘novelty’ categories and the three post-PM textbook series. A similar result was found for the OL textbooks (see table 61, appendix B).

For OL, the Concise Maths textbook had the highest share of ‘novel’ tasks with 6.7% and the greatest quantity placed in the ‘somewhat novel’ category with 19.3%. The Text and Tests textbook had the most tasks classified as ‘not novel’ with 83.2%. As with the HL textbook series, each of the ‘novel’ and ‘somewhat novel’ classifications increased in the PM textbooks when compared to those used previous to the introduction of the syllabus. Correspondingly the number of tasks assigned to the ‘not novel’ category fell. The Concise Maths textbook showed the greatest improvement in the ‘novel’ classification, rising from 2.9% to 6.7%. The Active Maths series showed the biggest increase in the ‘somewhat novel’ category going from 10.1% to 17.2%. It also had the largest reduction in the ‘not novel’ classification, falling from 85.4% to 78.8%. The post-PM textbook series were found to have 229 tasks (5.7%) in the ‘novel’ category, 826 (20.4%) in the ‘somewhat novel’ category and 2996 tasks (74%) in the ‘not novel’ category. It would appear that there was some increase in the ‘novel’ and ‘somewhat novel’ categories, accompanied by a decline in the ‘not novel’ classification.

For the pre-PM textbook series, 93 tasks (2.6%) were found to be ‘novel’, 569 tasks (15.9%) were classified as ‘somewhat novel’ and the remaining 2922 tasks (81.5%) were categorised as ‘not novel’. For the HL books, no relationship was found between the ‘novelty’ of tasks and the three pre-PM textbook series. A statistically significant relationship was found between the proportion of tasks placed in each of the three ‘novelty’ classifications and the pre-PM OL textbook series, (table 61, appendix B).

5.5 Project Maths Syllabus Problem-solving Objectives

It should be noted that since the PMO categories are not mutually exclusive and thus tasks could be classified in more than one category the percentages in tables in this section will not total to 100%.

5.5.1 Active Maths

For the pre-PM Active Maths textbooks, none of the tasks addressed the objectives *Explain findings* or *Communicate mathematics in written form*. The most common objective was *Use mathematical models, formulae or techniques to process information and to draw relevant conclusions* with 93.5% of tasks classified in this category. The other objectives had a much lower incidence with *Explore patterns and formulate conjectures* at 8.3% and *Apply knowledge and skills to solve problems in familiar contexts* at 6.8%.

For the post-PM textbook series, all objectives were represented in the set of tasks classified. There was an increase in the incidence of all of the objectives except for two; *Explore patterns and formulate conjectures* fell to 7.4% and the *Use of mathematical models* decreased to 84.7%. The objective *Apply knowledge and skills to solve problems in unfamiliar contexts* experienced the greatest increase going from 0.8% to 8.1%. The objective *Communicate mathematics in written form* was still quite low at 3.1%, similarly *Explain findings* was required by just 3.9% of the tasks. Aside from the *Use of mathematical models to draw relevant conclusions*, none of the Project Maths problem-solving objectives that were identified exceeded 10%. (See table 24).

For convenience, each of the objectives will be assigned a number for ease of representation in the tables as follows:

- Objective 1 = Explore patterns and formulate conjectures
- Objective 2 = Explain findings
- Objective 3 = Justify conclusions
- Objective 4 = Communicate mathematics in written form
- Objective 5 = Apply knowledge and skills to solve problems in familiar contexts,
- Objective 6 = Apply knowledge and skills to solve problems in unfamiliar contexts,
- Objective 7 = Analyse information and translate it into mathematical form,
- Objective 8 = Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions,
- Objective 9 = Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions
- Objective 10 = Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

To distinguish between objectives 2 and 3 for classification: explain findings was chosen if a task asked students to interpret their solutions, while tasks that required students to provide some proof or evidence were classified in the category *justify conclusions*. For objectives 5 and 6, a 'problem' is viewed as something that contains an element of non-routine material and requires some engagement on the part of the student in order to solve it. In relation to the solution method, it involves the use of skills in a new or at least different way.

Active Maths (HL) Problem-Solving Objectives	Pattern, sequences and series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, sequences and series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Objective 1	30 (9.5%)	41 (7.6%)	**71 (8.3%)**	57 (17.5%)	9 (1.6%)	**66 (7.4%)**
Objective 2	0 (0%)	0 (0%)	**0 (0%)**	9 (2.8%)	26 (4.6%)	**35 (3.9%)**
Objective 3	1 (0.3%)	5 (0.9%)	**6 (0.7%)**	5 (1.5%)	18 (3.2%)	**23 (2.6%)**
Objective 4	0 (0%)	0 (0%)	**0 (0%)**	7 (2.2%)	21 (3.7%)	**28 (3.1%)**
Objective 5	29 (9.2%)	29 (5.4%)	**58 (6.8%)**	21 (6.5%)	44 (7.8%)	**65 (7.3%)**
Objective 6	7 (2.2%)	0 (0%)	**7 (0.8%)**	37 (11.4%)	35 (6.2%)	**72 (8.1%)**
Objective 7	5 (1.6%)	0 (0%)	**5 (0.6%)**	16 (4.9%)	14 (2.5%)	**30 (3.4%)**
Objective 8	21 (6.6%)	30 (5.6%)	**51 (6.0%)**	25 (7.7%)	39 (6.9%)	**64 (7.2%)**
Objective 9	0 (0%)	4 (0.7%)	**4 (0.5%)**	12 (3.7%)	34 (6.0%)	**46 (5.2%)**
Objective 10	295 (93.4%)	502 (93.7%)	**797 (93.5%)**	282 (86.8%)	473 (83.6%)	**755 (84.7%)**

Table 24: Classification of PMO for HL Active Maths series

In the pre-PM OL textbook (see table 25) only 5 objectives were addressed by the set of tasks analysed. In particular, there was no evidence of the following: *Explain findings, Justify conclusion, Communicate mathematics in written form, Apply knowledge and skills to solve problems in unfamiliar contexts, Analyse information and translate it into mathematical form*. *Explore patterns and formulate conjectures* was the most common objective in evidence at 17.2%. This can be seen in figure 5.23. Note that in this figure and in the later figures in this section, Objective 10 *Use appropriate mathematical* *models, formulae or techniques to process information and to draw relevant conclusions* has been omitted. This is to ensure that the scale allows the other nine objectives to be compared adequately.

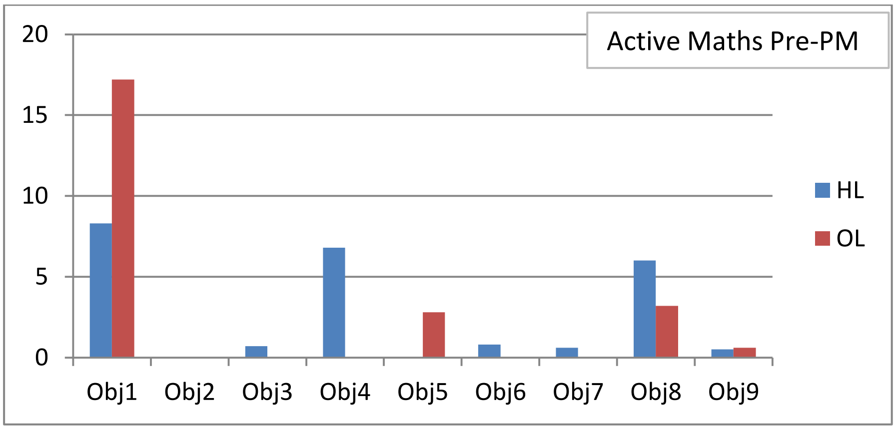


Figure 5.23: Percentage of PMO in the HL and OL pre-PM Active Maths textbook series

For the post-PM OL textbook series all 10 objectives were observed and the incidence of each of these increased except for the following three objectives: *Explore patterns and formulate conjectures* and the two objectives *Devise/Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*. The most common objective was the *Use of appropriate mathematical models* at 94% and the objective that was encountered the least was *Select appropriate mathematical models* with 1.2%.

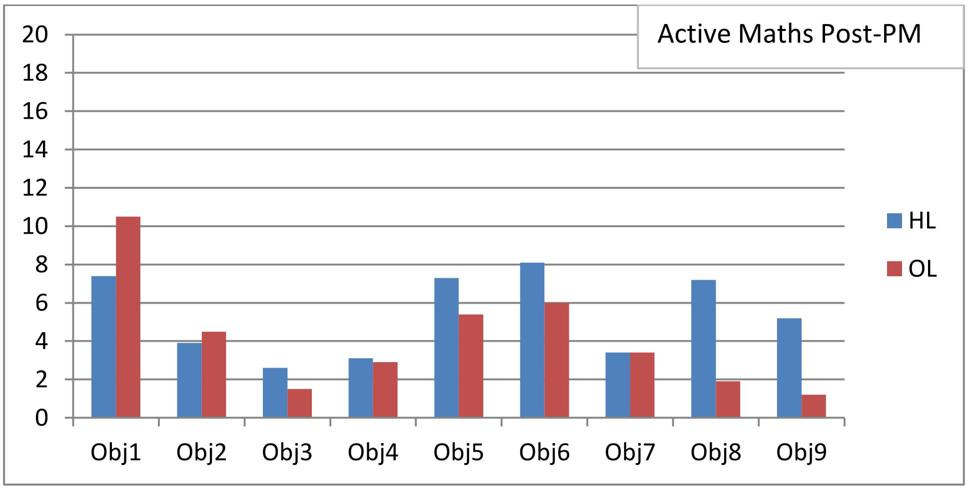


Figure 5.24: Percentage of PMO in the HL and OL post-PM Active Maths textbook series

Several objectives had a low incidence (less than 5%), these were: *Explain findings, Justify conclusions, Communicate mathematics in written form, Analyse information and translate it into mathematical form, Devise and Select mathematical models, formulae or techniques to process information and to draw relevant conclusions.*

Active Maths (OL) Problem-Solving Objectives	Pattern,(Pre PM)	Differential Calculus (Pre PM)	**Pre PM Total**	Pattern, (Post PM)	Differential Calculus (Post PM)	**Post PM Total**
Objective 1	92 (29.0%)	0 (0%)	**92 (17.2%)**	64 (18.2%)	4 (1.4%)	**68 (10.5%)**
Objective 2	0 (0%)	0 (0%)	**0 (0%)**	14 (4.0%)	15 (5.1%)	**29 (4.5%)**
Objective 3	0 (0%)	0 (0%)	**0 (0%)**	1 (0.3%)	9 (3.0%)	**10 (1.5%)**
Objective 4	0 (0%)	0 (0%)	**0 (0%)**	8 (2.3%)	11 (3.7%)	**19 (2.9%)**
Objective 5	6 (1.9%)	9 (4.1%)	**15 (2.8%)**	5 (1.4%)	30 (10.1%)	**35 (5.4%)**
Objective 6	0 (0%)	0 (0%)	**0 (0%)**	25 (7.1%)	14 (4.7%)	**39 (6.0%)**
Objective 7	0 (0%)	0 (0%)	**0 (0%)**	9 (2.6%)	13 (4.4%)	**22 (3.4%)**
Objective 8	6 (1.9%)	11 (5.0%)	**17 (3.2%)**	8 (2.3%)	4 (1.4%)	**12 (1.9%)**
Objective 9	0 (0%)	3 (1.4%)	**3 (0.6%)**	4 (1.1%)	4 (1.4%)	**8 (1.2%)**
Objective 10	311 (98.1%)	205 (93.6%)	**516 (96.3%)**	332 (94.6%)	276 (93.2%)	**608 (94.0%)**

Table 25: Classification of PMO for OL Active Maths series

5.5.1.1 Comparison of era

For Active Maths HL, an increase in the incidence of all objectives was recorded between the pre- and post-PM eras save for *Explore patterns and formulate conjectures* (which decreased from 8.3% to 7.4%) and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions (*which decreased from 93.5% to 84.7%). Figure 5.25 on the next page compares the topic and PM era for the Active Maths HL textbook.

At OL, an increase was noted in the classification of all objectives with the exception of three: there was a sizeable decrease in *Explore patterns and formulate conjectures* (17.2% to 10.5%) and very small decreases in the other two objectives *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* and *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.* It would appear that the new Active Maths textbooks have moved towards embracing the problem-solving objectives but a decline has been noted in *Explore patterns and formulate conjectures* at both levels. The decline at Ordinary Level in *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* is of note, suggesting that more tasks meeting this objective will be required. The decline in *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* could be explained by the increased diversity of the objectives present in the post-PM era.

5.5.1.2 Comparison of topic

At HL, in the post-PM era, the incidence of five objectives *Explain findings, Justify conclusions, Communicate mathematics in written form, Apply knowledge and skills to solve problems in familiar contexts* and *Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* was higher in the topic of Differential Calculus than in Pattern, Sequences and Series. In the OL textbook of the same era, the same five objectives with the addition of the objective *Analyse information and translate it into mathematical form* was found to be more common in the topic of Differential Calculus. Figure 5.26 below compares the topic and PM era for the Active Maths OL textbooks.

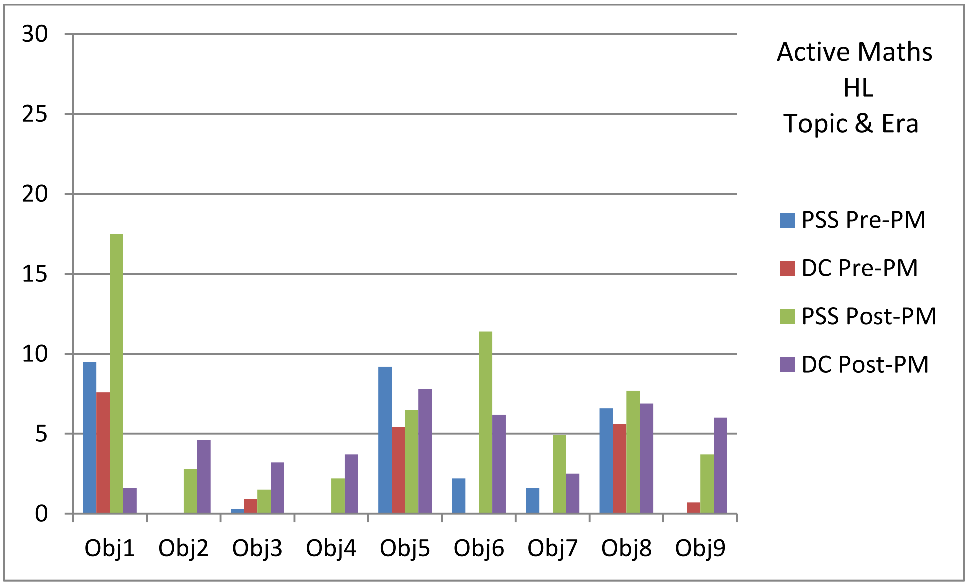


Figure 5.25: Active Maths (HL) comparison of topic and PM era

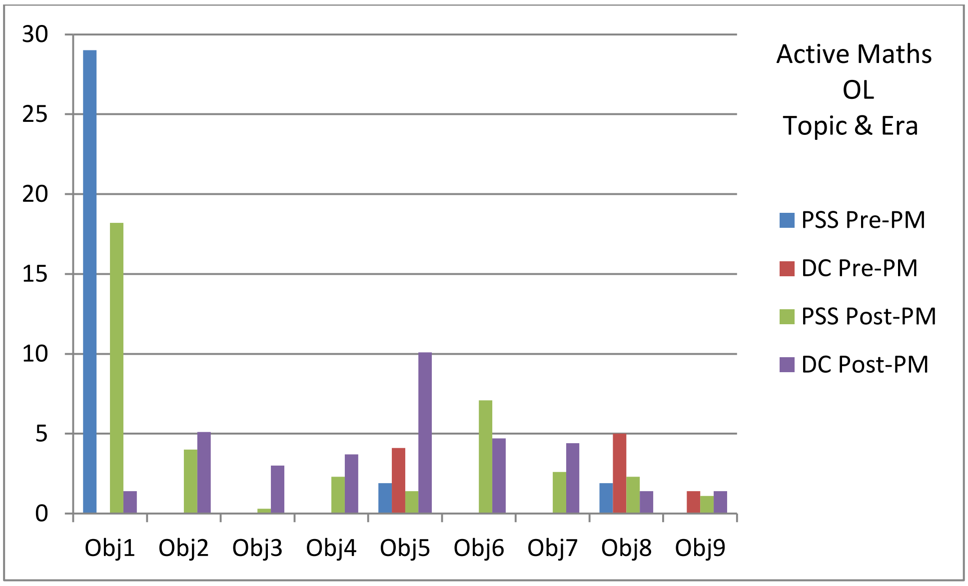


Figure 5.26: Active Maths (OL) comparison of topic and PM era

At HL, the greatest difference between topics was found in the objective *Explore patterns and formulate conjectures* with 17.5% in Pattern, Sequences and Series and 1.6% in Differential Calculus. Similarly at OL the greatest difference was also to be found in *Explore patterns and formulate conjectures* with 18.2% in Pattern, Sequences and Series and 1.4% in Differential Calculus. Given the subject matter of Pattern, Sequences and Series, it is to be expected that there would be a greater incidence of this objective. In contrast, the objective *Apply knowledge and skills to solve problems in familiar contexts* was much more common at OL in the topic of Differential Calculus (10.1%) when compared to Pattern, Sequences and Series (1.4%).

5.5.1.3 Higher Level vs Ordinary Level

All the objectives in the post-PM textbook era were more common at HL than OL with the exception of three; *Explore patterns and formulate conjectures* (10.5% compared to 7.4%), *Explain findings* (4.5% compared to 3.9%) and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* (94% compared to 84.7%). It is surprising that the OL textbook offers more opportunities for the exploration of patterns, the formulation of conjectures and the explanation of findings when compared to the HL textbook. Again the difference in the objective *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* could be accounted for by the increased diversity of the other problem-solving objectives in the HL textbook.

5.5.2 Text and Tests

For the Text and Tests HL textbooks (see table 26), all 10 objectives were found to be present in the material examined from the post-PM era. The objective involving the use of mathematical models was the most commonly encountered at 89.2%. *Explore patterns and formulate conjectures* was recorded at 13% and the proportion of tasks involving the objective *Apply knowledge and skills to solve problems in familiar contexts* was 7.6%. Three of the objectives had incidences of less than 1% in the tasks examined. Within the tasks analysed, 0.8% required the explanation of findings, 0.3% needed the justification of conclusions while 0.5% involved the communication of mathematics in written form. The incidence of the objective *Devise mathematical models* was 3.5% while that of *Select mathematical models* was 4.8%.

Text and Tests (HL) Problem-Solving Objectives	Pattern, (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Objective 1	5 (2.5%)	0 (0%)	**5 (0.8%)**	56 (28.9%)	26 (5.9%)	**82 (13.0%)**
Objective 2	0 (0%)	1 (0.2%)	**1 (0.2%)**	3 (1.5%)	2 (0.5%)	**5 (0.8%)**
Objective 3	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	2 (0.5%)	**2 (0.3%)**
Objective 4	0 (0%)	0 (0%)	**0 (0%)**	1 (0.5%)	2 (0.5%)	**3 (0.5%)**
Objective 5	15 (7.5%)	11 (2.5%)	**26 (4.0%)**	28 (14.4%)	20 (4.6%)	**48 (7.6%)**
Objective 6	0 (0%)	0 (0%)	**0 (0%)**	6 (3.1%)	14 (3.2%)	**20 (3.2%)**
Objective 7	0 (0%)	4 (0.9%)	**4 (0.6%)**	10 (5.2%)	4 (0.9%)	**14 (2.2%)**
Objective 8	15 (7.5%)	11 (2.5%)	**26 (4.0%)**	21 (10.8%)	1 (0.2%)	**22 (3.5%)**
Objective 9	1 (0.5%)	3 (0.7%)	**4 (0.6%)**	14 (7.2%)	16 (3.7%)	**30 (4.8%)**
Objective 10	183 (92.0%)	431 (96.9%)	**614 (95.3%)**	148 (76.3%)	415 (95.0%)	**563 (89.2%)**

Table 26: Classification of PMO for HL Text and Tests

For the post-PM OL Text and Tests textbook (see table 27), all 10 objectives were found in the tasks analysed. As with the Higher Level textbooks the *Use mathematical models* objective was the most common at 90.2%, while *Justify Conclusions* was the least at 0.2%. However, *Explain findings* had a higher incidence at 6.4%.

Text and Tests (OL) Problem-Solving Objectives	Pattern, (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Objective 1	18 (10.3%)	0 (0%)	**18 (4.7%)**	64 (27.1%)	16 (6.8%)	**80 (17.0%)**
Objective 2	1 (0.6%)	0 (0%)	**1 (0.3%)**	20 (8.5%)	10 (4.3%)	**30 (6.4%)**
Objective 3	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	1 (0.4%)	**1 (0.2%)**
Objective 4	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	12 (5.1%)	**12 (2.5%)**
Objective 5	1 (0.6%)	0 (0%)	**1 (0.3%)**	5 (2.1%)	14 (6.0%)	**19 (4.0%)**
Objective 6	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	3 (1.3%)	**3 (0.6%)**
Objective 7	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	4 (1.7%)	**4 (0.8%)**
Objective 8	8 (4.6%)	0 (0%)	**8 (2.1%)**	3 (1.3%)	2 (0.9%)	**5 (1.1%)**
Objective 9	0 (0%)	0 (0%)	**0 (0%)**	1 (0.4%)	17 (7.2%)	**18 (3.8%)**
Objective 10	167 (95.4%)	211 (100%)	**378 (97.9%)**	223 (94.5%)	202 (86.0%)	**425 (90.2%)**

Table 27: Classification of PMO for OL Text and Tests

5.5.2.1 Comparison of era

For Text and Tests HL, an increase in the incidence of all objectives was recorded between the pre- and post-Project Maths eras save for *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions (*which decreased from 4% to 3.5%) and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions (*which decreased from 95.3% to 89.2%).

At OL, a similar situation was noted with a decrease in the incidence of *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* from 2.1% to 1.1%. The lower incidence of this objective in both the HL and OL Text and Tests textbooks is of concern as it might offer students less opportunities to experiment and create their own solution methods.

5.5.2.2 Comparison of topic

At HL, in the post-PM era, the incidence of six objectives *Explore patterns and formulate conjectures*, *Explain findings, Apply knowledge and skills to solve problems in familiar contexts*, *Analyse information and translate it into mathematical form, Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* and *Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* was more common in the topic of Pattern, Sequences and Series than Differential Calculus. Figure 5.27 below compares the topic and PM era for the Text and Tests Higher Level textbook.

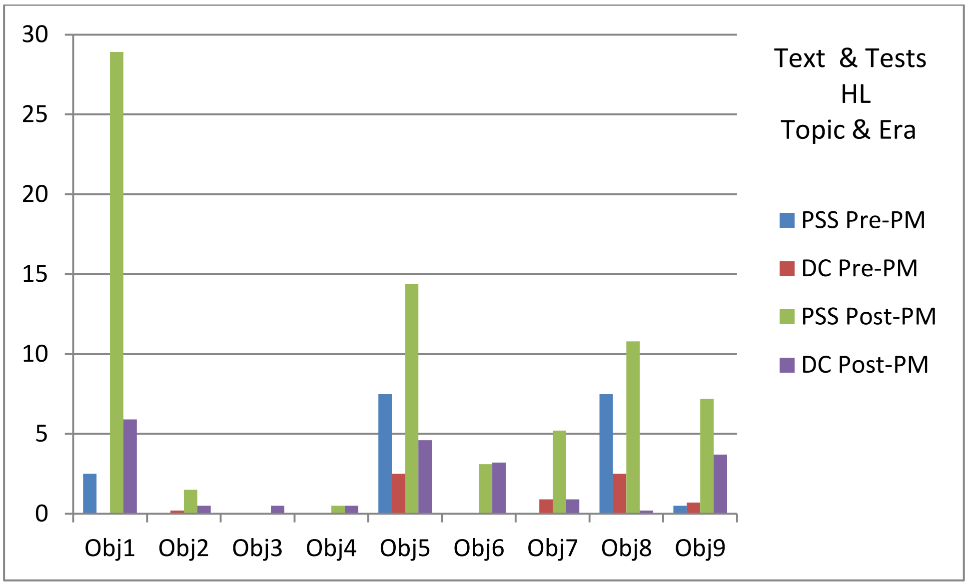


Figure 5.27: Text and Tests (HL) comparison of topic and PM era

In the post-PM OL textbook, four of these objectives (*Explore patterns and formulate conjectures, Explain findings, Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* and *Use appropriate* *mathematical models, formulae or techniques to process information and to draw relevant conclusions)* were also found to be more common in the topic of Pattern, Sequences and Series. Figure 5.28 below compares the topic and PM era for the Active Maths OL textbooks.

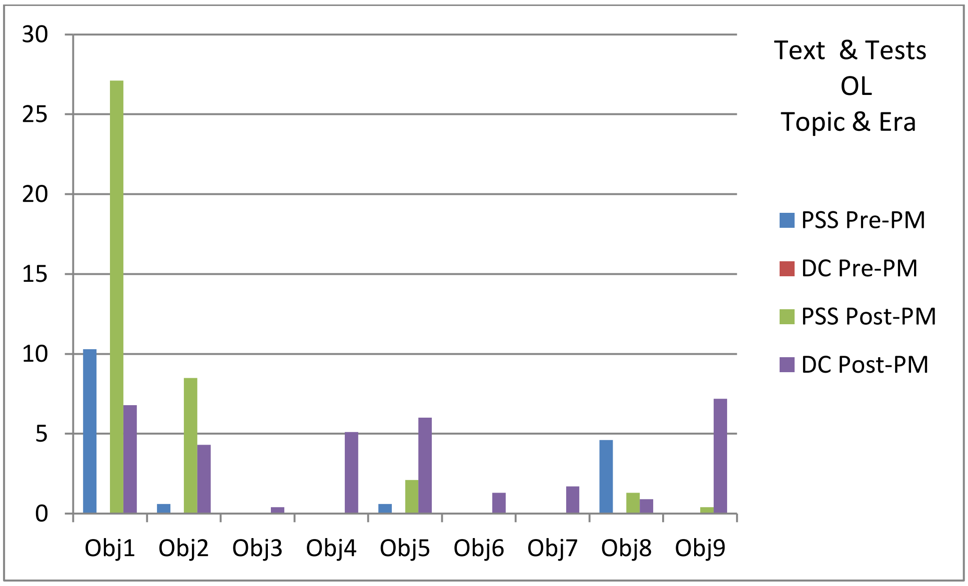


Figure 5.28: Text and Tests (OL) comparison of topic and PM era

At both HL and OL, the greatest difference between topics was found in the objective *Explore patterns and formulate conjectures* (Higher Level: 28.9% in Pattern, Sequences and Series, 5.9% in Differential Calculus; Ordinary Level: 27.1% in Pattern, Sequences and Series, 6.8% in Differential Calculus). This may be due to the material contained in the topic.

5.5.2.3 Higher Level vs Ordinary Level

Five of the objectives in the post-PM textbook era were more common at HL than OL; *Justify conclusions* (0.3% compared to 0.2%), *Apply knowledge and skills to solve problems in unfamiliar contexts* (3.2% compared to 0.6%), *Analyse information and translate it into mathematical form* (2.2% compared to 0.8%) Devise *appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* (3.5% compared to 1.1%) and *Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* (4.8% compared to 3.8%). It is surprising that the OL textbook appears to better embrace the problem-solving objectives of *Explore patterns and formulate conjectures,* *Explain findings, Communicate mathematics in written form, Apply knowledge and skills to solve problems in familiar contexts* and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.*

5.5.3 Concise Maths

All 10 objectives were present in the material analysed from the Concise Maths Higher Level textbooks in use after the introduction of PM. The objective with the highest incidence was *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* at 92.8%. The next most common was *Apply knowledge and skills to solve problems in familiar contexts* with 6.7%. *Communicate mathematics in written form* was the least common with 0.7%.

The material analysed from the OL textbooks, in use after the syllabus change, had all 10 objectives present (see table 29). *Use appropriate mathematical models* was the most common at 86.6% and the next most common was *Explore patterns and formulate conjectures* at 14%. The objective *Communicate mathematics in written form* was the least common with 0.3%.

Concise Maths (HL) Problem-Solving Objectives	Pattern (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Objective 1	0 (0%)	0 (0%)	**0 (0%)**	7 (1.9%)	9 (1.9%)	**16 (1.9%)**
Objective 2	2 (0.7%)	2 (0.5%)	**4 (0.6%)**	0 (0%)	11 (2.3%)	**11 (1.3%)**
Objective 3	1 (0.3%)	0 (0%)	**1 (0.1%)**	2 (0.6%)	6 (1.3%)	**8 (1.0%)**
Objective 4	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	6 (1.3%)	**6 (0.7%)**
Objective 5	15 (5.0%)	6 (1.6%)	**21 (3.1%)**	21 (5.8%)	35 (7.4%)	**56 (6.7%)**
Objective 6	2 (0.7%)	2 (0.5%)	**4 (0.6%)**	14 (3.9%)	23 (4.9%)	**37 (4.5%)**
Objective 7	2 (0.7%)	0 (0%)	**2 (0.3%)**	13 (3.6%)	8 (1.7%)	**21 (2.5%)**
Objective 8	17 (5.7%)	8 (2.1%)	**25 (3.6%)**	5 (1.4%)	4 (0.8%)	**9 (1.1%)**
Objective 9	0 (0%)	4 (1.0%)	**4 (0.6%)**	2 (0.6%)	33 (7.0%)	**35 (4.2%)**
Objective 10	281 (94.3%)	375 (96.9%)	**656 (95.8%)**	344 (95.6%)	427 (90.7%)	**771 (92.8%)**

Table 28: Classification of PMO for HL Concise Maths

Concise Maths (OL) Problem-solving Objectives	Pattern,(Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Objective 1	15 (5.7%)	1 (0.5%)	**16 (3.3%)**	71 (19.3%)	10 (4.7%)	**81 (14.0%)**
Objective 2	0 (0%)	0 (0%)	**0 (0%)**	8 (2.2%)	6 (2.8%)	**14 (2.4%)**
Objective 3	0 (0%)	0 (0%)	**0 (0%)**	10 (2.7%)	3 (1.4%)	**13 (2.2%)**
Objective 4	0 (0%)	0 (0%)	**0 (0%)**	1 (0.3%)	1 (0.5%)	**2 (0.3%)**
Objective 5	15 (5.7%)	6 (2.8%)	**21 (4.4%)**	16 (4.3%)	29 (13.7%)	**45 (7.8%)**
Objective 6	0 (0%)	1 (0.5%)	**1 (0.2%)**	22 (6.0%)	11 (5.2%)	**33 (5.7%)**
Objective 7	1 (0.4%)	0 (0%)	**1 (0.2%)**	16 (4.3%)	8 (1.7%)	**24 (4.1%)**
Objective 8	15 (5.7%)	7 (3.2%)	**22 (4.6%)**	12 (3.3%)	3 (1.4%)	**15 (2.6%)**
Objective 9	0 (0%)	0 (0%)	**0 (0%)**	6 (1.6%)	25 (11.8%)	**31 (5.3%)**
Objective 10	248 (94.3%)	211 (96.8%)	**459 (95.4%)**	324 (88%)	178 (84.0%)	**502 (86.6%)**

Table 29: Classification of PMO for OL Concise Maths

5.5.3.1 Comparison of era

In the Concise Maths HL textbooks, all objectives were found to have increased between the two eras with the exception of *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions (*which decreased from 3.6% to 1.1%) and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions (*which decreased from 95.8% to 92.8%).

At OL, an increase in all objectives was noted except for the two objectives *Devise* and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* (decrease from 4.6% to 2.6% and 95.4% to 86.6% respectively). The lower incidence of the *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* objective in both the HL and OL Concise Maths textbooks is of concern as it might make students more dependent on the selection and use of solution methods rather than experimenting to construct their own.

5.5.3.2 Comparison of topic

At HL, in the post-PM era, the incidence of three objectives *Analyse information and translate it into mathematical form*, *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* were more common in the topic of Pattern, Sequences and Series than Differential Calculus. Figure 5.29 below compares the topic and PM era for the Text and Tests HL textbook.

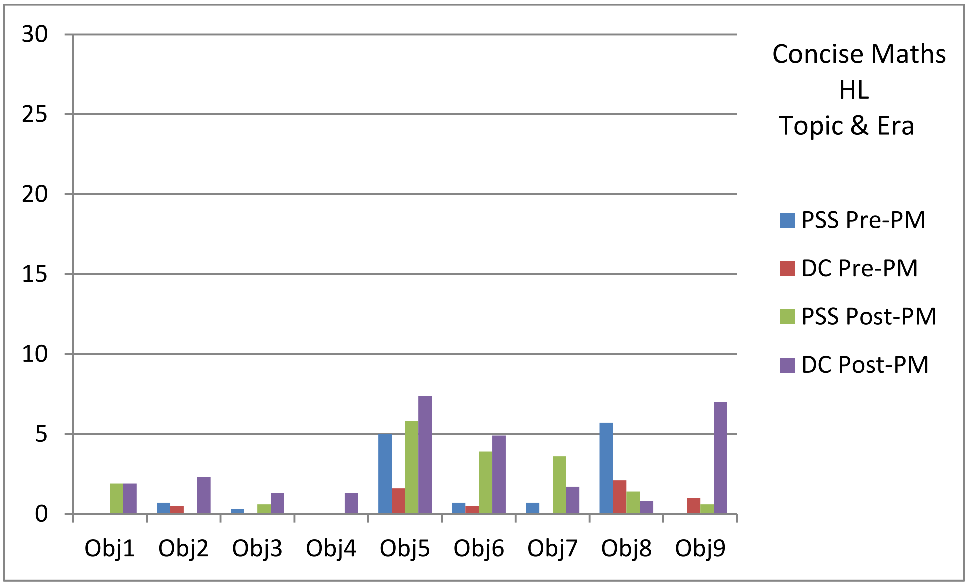


Figure 5.29: Concise Maths (HL) comparison of topic and PM era

In the post-PM OL textbook; six objectives *Explore patterns and formulate conjectures, Justify conclusions, Apply knowledge and skills to solve problems in unfamiliar contexts, Analyse information and translate it into mathematical form, Devise appropriate* *mathematical models, formulae or techniques to process information and to draw relevant conclusions* and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* were found to be more common in the topic of Pattern, Sequences and Series. Figure 5.30 below compares the topic and PM era for the Active Maths OL textbooks.

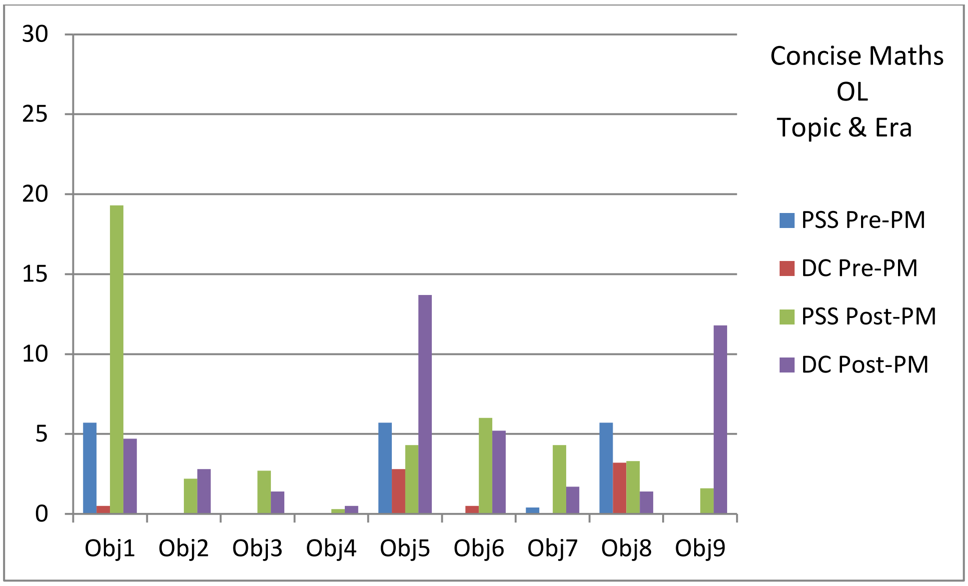


Figure 5.30: Concise Maths (OL) comparison of topic and PM era

At HL, the greatest difference between topics was found in the objective *Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* with 0.6% in Pattern, Sequences and Series and 7% in Differential Calculus. At OL the greatest difference was found in the objective *Explore patterns and formulate conjectures* with 19.3% in Pattern, Sequences and Series and 4.7% in Differential Calculus. As observed with the other textbook series, this may be due to the material contained in the topic.

5.5.3.3 Higher Level vs Ordinary Level

Just two of the objectives in the post-PM textbook era were more common at HL than OL; *Communicate mathematics in written form* (0.7% compared to 0.3%) and *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* (92.8% compared to 86.5%). Overall, it would appear that the OL textbook appears to better embrace the problem-solving objectives than the HL textbook.

5.5.4 Comparison of Textbook Series

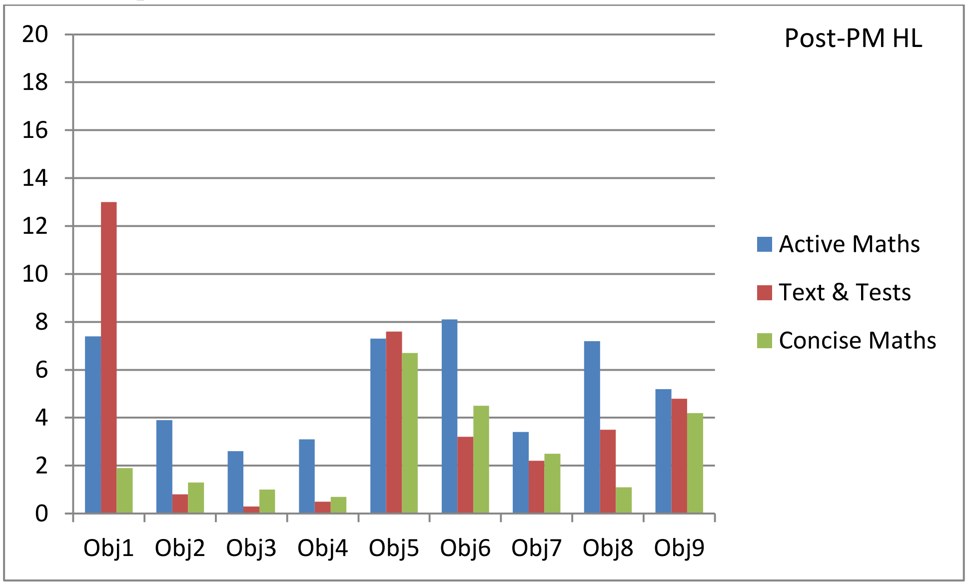


Figure 5.31: Comparison of PMO (percentages) post-PM HL textbook series

Figure 5.31 above gives a comparison of the PMO for the post-PM HL textbook series. The HL Active Maths textbook had the greatest incidence of 7 of the objectives, Text and Tests HL had 2 and Concise Maths Higher Level had the highest percentage of one objective *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant* conclusion which is not displayed in the figure. This would suggest that out of the three textbook series, Active Maths is the most rounded in terms of problem-solving objectives at Higher Level.

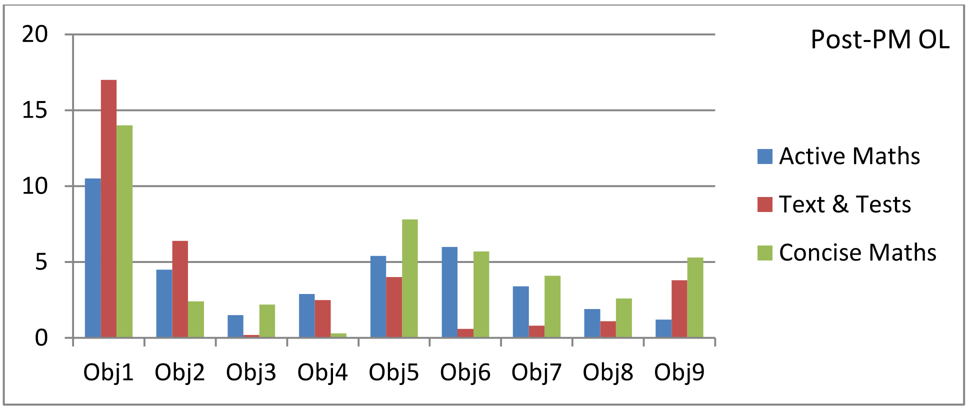


Figure 5.32: Comparison of post-PM PMO (percentages) OL textbook series

Figure 5.32 gives a comparison of the PMO for the post-PM OL textbook series.

It would appear that Concise Maths OL has the greatest blend of problem-solving objectives.

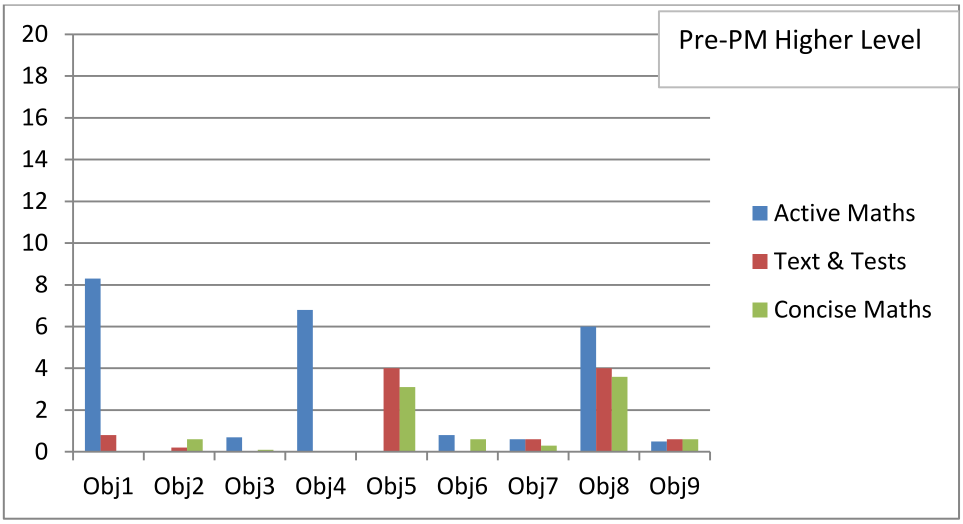


Figure 5.33: Comparison of pre-PM PMO (percentages) HL textbook series

Figure 5.33 gives a comparison of the PMO for the pre-PM HL textbook series. In the pre-PM HL textbook series, Active Maths had the greatest percentages in five objectives while Concise Maths had the greatest in two. The classification *Communicate mathematics in written form* was not recorded in any of the series. In this era, the Active Maths textbook was the most rounded in terms of problem-solving objectives.

Figure 5.34 gives a comparison of the PMO for the pre-PM OL textbook series. The objectives *Justify conclusions* and *Communicate mathematics in written form* were not recorded in any of the OL textbooks. Concise Maths OL had the greatest percentages in four objectives, Active Maths OL and Text and Tests OL had the greatest in two objectives each. It would suggest that the Concise Maths OL textbook was the most rounded in terms of PMO at this time.

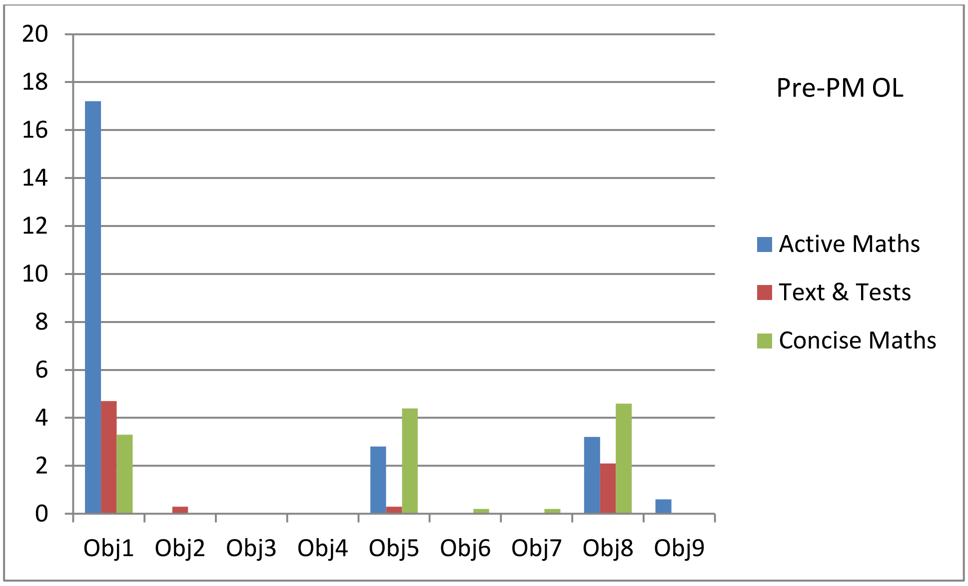


Figure 5.34: Comparison of pre-PM PMO (percentages) OL textbook series

For the three textbook series, all of the material analysed experienced an increase in the percentage of tasks for seven of the problem-solving objectives between the pre- and post-PM eras. In contrast a percentage decrease was found in three objectives. The objective *Explore patterns and formulate conjectures* was found to have decreased for both the HL and OL textbooks in the Active Maths series, while increasing in the Text and Tests and Concise Maths series. With the exception of the Active Maths HL textbook, a decrease in the objective *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* was found. Finally a decrease in the objective *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* was noted in all textbooks. These results would suggest that the textbook series have all improved in terms of embracing the problem-solving objectives. The decrease in *Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* suggests that the objectives have become more diverse and a greater proportion of tasks require students to select appropriate solution methods rather than just using given techniques. However, it is a cause for concern that *Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* has not been given more attention. It is likely that those using the material from the Active Maths OL textbook or any of the textbooks from the Text and Tests and Concise Maths series will need to develop the ability to create their own solution methods and techniques in some other way.

5.6 Usiskin's Multidimensional Model of Mathematical Understanding

I did not find any tasks in any of the textbook series, either pre-PM or post-PM editions, which addressed the History-Culture dimension. For completion, the dimension has been included in all of the tables that follow even though its incidence is 0%. As was the case with the PMO framework, it is possible for tasks to be classified in more than one category here and so the percentages in the tables that follow do not add to 100%.

5.6.1 Active Maths

For the HL Pre-PM textbooks, the proportion of tasks classified in the Skill-Algorithm dimension was quite high (98%) while the proportions of tasks in the Property-Proof (8.2%), Use-Application (9%) and Representation-Metaphor (7.4%) dimensions were much lower. After the introduction of the new syllabus, the proportion of tasks in the Skill-Algorithm dimension fell slightly to 95%. The greatest increase was seen in the Representation-Metaphor dimension which rose to 21%. The Property-Proof and Use-Application dimensions recorded a more modest increase by rising to 13.1% and 14.9% respectively (see table 30). For these two dimensions, the result is surprising: given the focus of PM, it might be expected that a greater increase should be noted.

Usiskin Multidimensional model Classification Active Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Skill-Algorithm	302 (95.6%)	533 (99.4%)	**835 (98.0%)**	316 (97.2%)	530 (93.6%)	**846 (95.0%)**
Property-Proof	38 (12%)	32 (6%)	**70 (8.2%)**	75 (23.1%)	42 (7.4%)	**117 (13.1%)**
Use-Application	9 (2.8%)	68 (12.7%)	**77 (9.0%)**	31 (9.5%)	102 (18%)	**133 (14.9%)**
Representation-Metaphor	0 (0%)	63 (11.8%)	**63 (7.4%)**	55 (16.9%)	132 (23.3%)	**187 (21%)**
History-Culture	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	0 (0%)	**0 (0%)**
Total Tasks	**316**	**536**	**852**	**325**	**566**	**891**

Table 30: Classification of Usiskin's Multidimensional Model for HL Active Maths

The situation was very similar with the OL textbooks. Again the Skill-Algorithm dimension fell slightly in the post-PM editions. The biggest increase was recorded with the Representation-Metaphor dimension rising to 27.8% from 1.3%, while more modest increases were noted in the Property-Proof and Use-Application dimensions (see table 31). It would appear that the Skill-Algorithm is still the dominant dimension in the post-PM textbooks while a strong increase is evident in the Representation-Metaphor dimension.

Usiskin Multidimensional model Classification Active Maths Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Skill-Algorithm	317 (100%)	216 (98.6%)	**533 (99.4%)**	348 (99.1%)	282 (95.3%)	**630 (97.4%)**
Property-Proof	20 (6.3%)	9 (4.1%)	**29 (5.4%)**	49 (14%)	18 (6.1%)	**67 (10.4%)**
Use-Application	0 (0%)	47 (21.5%)	**47 (8.8%)**	21 (6%)	82 (27.7%)	**103 (15.9%)**
Representation-Metaphor	0 (0%)	7 (3.2%)	**7 (1.3%)**	95 (27.1%)	85 (28.7%)	**180 (27.8%)**
History-Culture	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	0 (0%)	**0 (0%)**
Total Tasks	**317**	**219**	**536**	**351**	**296**	**647**

Table 31: Classification of Usiskin's Multidimensional Model for OL Active Maths

5.6.1.1 Higher Level vs Ordinary Level

The HL post-PM textbook had a greater percentage in just one dimension Property-Proof. The OL post-PM textbook had a greater proportion of tasks categorised in the Skill-Algorithm, Use-Application and Representation-Metaphor dimensions.

5.6.1.2 Comparison of topic

For the pre-PM Active Maths HL textbook series, only the Property-Proof dimension had a higher incidence in the topic of Pattern, Sequences and Series when compared to Differential Calculus. In the post-PM textbook, the Skill-Algorithm and Property Proof dimensions had a greater percentage in the Pattern, Sequences and Series topic than Differential Calculus. All the dimensions in each topic recorded an increase in percentages with the exception of Skill-Algorithm which decreased slightly in Differential Calculus. Figure 5.35 compares the classification of Usiskin's Multidimensional Model for

HL Active Maths for topic and textbook era. It should be noted that the Skill-Algorithm dimension has been omitted to preserve the scale of the figure.

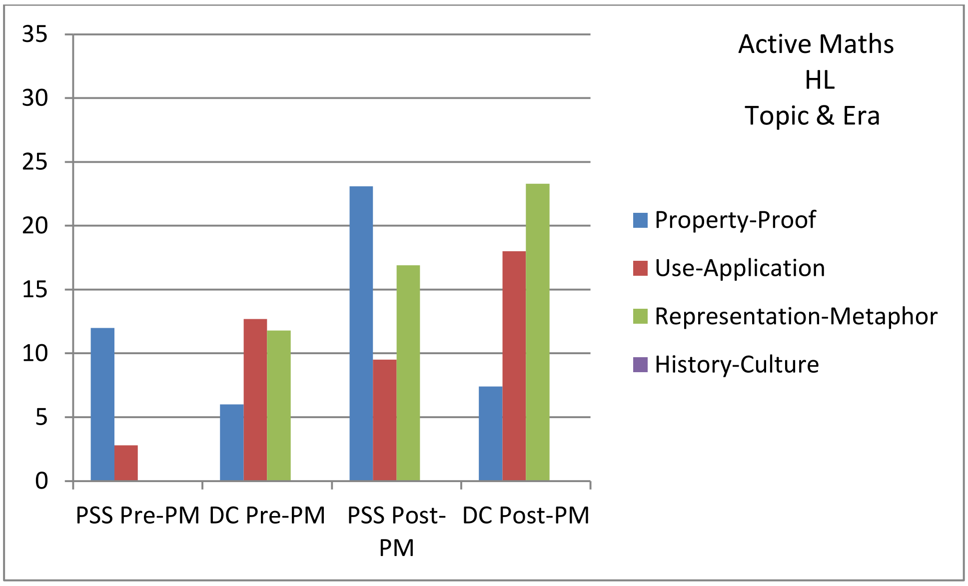


Figure 5.35: Usiskin's Multidimensional Model for HL Active Maths comparing topic and textbook era.

For the OL Active Maths textbook series, a similar situation was noted. The dimensions of Use-Application and Representation-Metaphor were more common in the topic of Differential Calculus for both pre and post-PM textbook series. The Property-Proof dimension was more frequently encountered in the pattern, sequence and series tasks than the Differential Calculus topic for both eras.

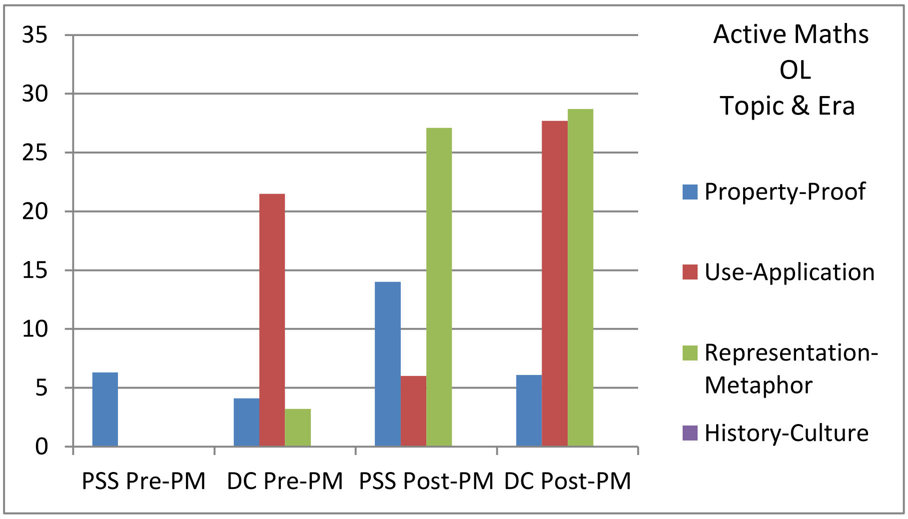


Figure 5.36: Usiskin's Multidimensional Model for OL Active Maths comparing topic and textbook era.

5.6.2 Text and Tests

For the post-PM HL Text and Tests textbooks the most common dimension encountered was Skill-Algorithm with 95.9% (see table 32). The Representation-Metaphor dimension is much less common at 12.8% while the Use-Application and Property-Proof dimensions are similar with 10.5% and 8.7% respectively.

Usiskin Multidimensional model Classification Text and Tests Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Skill-Algorithm	192 (96.5%)	445 (100%)	**637 (98.9%)**	173 (89.2%)	432 (98.9%)	**605 (95.9%)**
Property-Proof	16 (8%)	6 (1.3%)	**22 (3.4%)**	44 (22.7%)	11 (2.5%)	**55 (8.7%)**
Use-Application	0 (0%)	0 (0%)	**0 (0%)**	24 (12.4%)	42 (9.6%)	**66 (10.5%)**
Representation-Metaphor	0 (0%)	26 (5.8%)	**26 (4.0%)**	26 (13.4%)	55 (12.6%)	**81 (12.8%)**
History-Culture	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	0 (0%)	**0 (0%)**
Total Tasks	**199**	**445**	**644**	**194**	**437**	**631**

Table 32: Classification of Usiskin's Multidimensional Model for HL Text and Tests

At OL, the post-PM textbook had the presence of the Skill-Algorithm dimension in almost every task with 97% (see table 33). The Use-Application dimension was found in 19.3% of the analysed tasks. The presence of the Representation-Metaphor and Property-Proof dimensions were lower at 17.8% and 7.4% respectively.

Usiskin Multidimensional model Classification Text and Tests Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Skill-Algorithm	175 (100%)	211 (100%)	**386 (100%)**	236 (100%)	221 (94.0%)	**457 (97.0%)**
Property-Proof	11 (6.3%)	0 (0%)	**11 (2.8%)**	19 (8.1%)	16 (6.8%)	**35 (7.4%)**
Use-Application	0 (0%)	40 (19.0%)	**40 (10.4%)**	13 (5.5%)	78 (33.2%)	**91 (19.3%)**
Representation-Metaphor	0 (0%)	1 (0.5%)	**1 (0.3%)**	51 (21.6%)	33 (14%)	**84 (17.8%)**
History-Culture	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	0 (0%)	**0 (0%)**
Total Tasks	**175**	**211**	**386**	**236**	**235**	**471**

Table 33: Classification of Usiskin's Multidimensional Model for OL Text and Tests

5.6.2.1 Higher vs Ordinary

The percentages for the Skill-Algorithm, Use-Application and Representation-Metaphor dimensions were greater in the OL post-PM Text and Tests textbook than the HL textbooks of the same era. The Property-Proof dimension percentage was greater in the HL Text and Tests textbooks.

5.6.2.2 Comparison of topic

For the Text and Tests pre-PM HL textbook series, the proportion of tasks classified in the Skill-Algorithm and Representation-Metaphor dimensions were more common in the topic of Differential Calculus, while the percentage for the Property-Proof dimension was greater in the topic of Pattern, Sequences and Series. In the post-PM era, the Property-Proof, Use-Application and Representation-Metaphor dimensions had greater percentages in the topic of Pattern, Sequences and Series. The Skill-Algorithm dimension was greater in Differential Calculus. Figure 5.37 compares the topic and era for the Multidimensional Model for the HL Text and Tests textbook series.

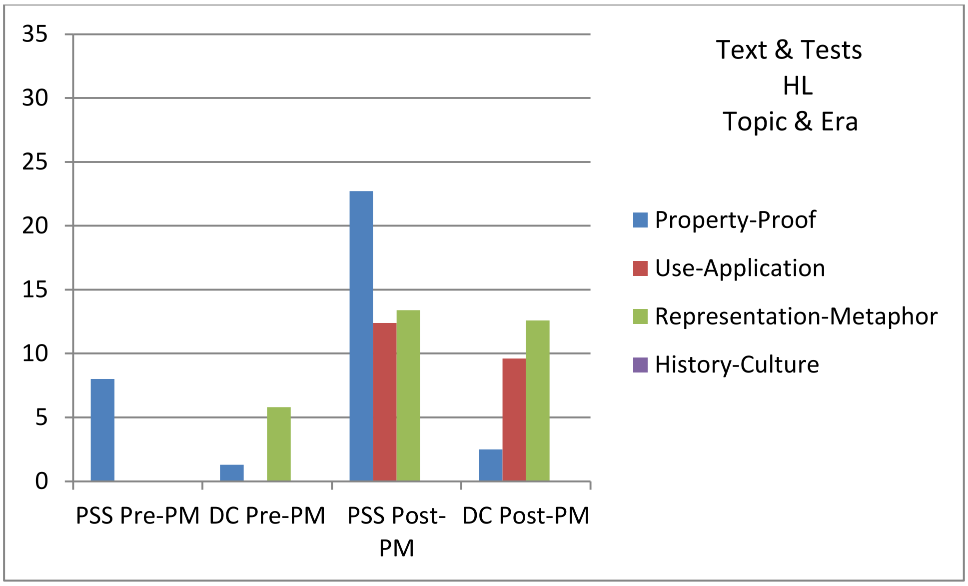


Figure 5.37: Usiskin's Multidimensional Model for HL Active Maths comparing topic and textbook era.

The dimensions Use-Application and Representation-Metaphor dimensions had a greater incidence in the topic of Differential Calculus for the pre-PM Text and Tests OL textbook. The Property Proof dimension was the only dimension which had a higher percentage in the topic of Pattern, Sequences and Series. A much higher incidence of Use-Application in Differential Calculus (33.2%) was found in the post-PM Text and Tests OL textbook when compared to that of Pattern, Sequences and Series (5.5%). However for the other three dimensions, a higher percentage was recorded in the topic of Pattern, Sequences and Series. Figure 5.38 compares the topic and era for the Multidimensional Model for the OL Text and Tests textbook series.

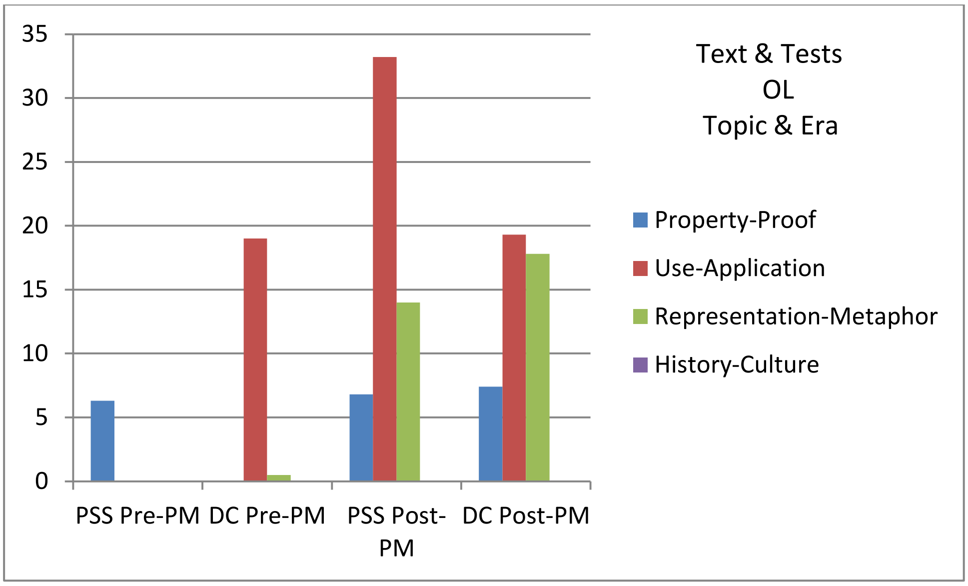


Figure 5.38: Usiskin's Multidimensional Model for HL Active Maths comparing topic and textbook era.

5.6.3 Concise Maths

Just as with the other two HL post-PM textbook series, the proportion of Skill-Algorithm tasks was quite high for Concise Maths with 98.9% (see table 34). The Use-Application dimension was much lower with 16.4%. The two remaining dimensions were much lower with Property-Proof at 7% and Representation-Metaphor at 6.4%.

Usiskin Multidimensional model Classification Concise Maths Textbook Series (HL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Skill-Algorithm	297 (99.7%)	387 (100%)	**684 (99.9%)**	358 (99.4%)	464 (98.5%)	**822 (98.9%)**
Property-Proof	12 (4%)	0 (0%)	**12 (1.8%)**	26 (7.2%)	32 (6.8%)	**58 (7.0%)**
Use-Application	2 (0.7%)	29 (7.5%)	**31 (4.5%)**	18 (5%)	118 (25.1%)	**136 (16.4%)**
Representation-Metaphor	1 (0.3%)	7 (1.8%)	**8 (1.2%)**	7 (1.9%)	46 (9.8%)	**53 (6.4%)**
History-Culture	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	0 (0%)	**0 (0%)**
Total Tasks	**298**	**387**	**685**	**360**	**471**	**831**

Table 34: Classification of Usiskin's Multidimensional Model for HL Concise Maths

Similar to the pre-PM OL Text and Tests textbooks, the corresponding Concise Maths textbooks were all found to involve the Skill-Algorithm dimension. The remaining dimensions were distributed as followed: Property-Proof with 7.7%, Use-Application with 8.9% and Representation-Metaphor at 4.2%. As with the other post-PM textbook series, the proportion of Skill-Algorithm fell and the other three dimensions rose. The Skill-Algorithm dimension decreased to 94.7%. A slight increase in the Property-Proof dimension was recorded, rising from 7.7% to 8.6%. The other two dimensions showed much larger increases: Use-Application rose to 21.7% from 8.9% and Representation-Metaphor climbed to 20.9% from 4.2% (see table 35).

Usiskin Multidimensional model Classification Concise Maths Textbook Series (OL)	Pattern, Sequences and Series (Pre-PM)	Differential Calculus (Pre-PM)	**Pre-PM Total**	Pattern, Sequences and Series (Post-PM)	Differential Calculus (Post-PM)	**Post-PM Total**
Skill-Algorithm	263 (100%)	218 (100%)	**481 (100%)**	355 (96.5%)	194 (91.5%)	**549 (94.7%)**
Property-Proof	33 (12.5%)	4 (1.8%)	**37 (7.7%)**	22 (6%)	28 (13.2%)	**50 (8.6%)**
Use-Application	0 (0%)	43 (19.7%)	**43 (8.9%)**	69 (18.8%)	57 (26.9%)	**126 (21.7%)**
Representation-Metaphor	0 (0%)	2 (0.9%)	**2 (4.2%)**	85 (23.1%)	36 (17%)	**121 (20.9%)**
History-Culture	0 (0%)	0 (0%)	**0 (0%)**	0 (0%)	0 (0%)	**0 (0%)**
Total Tasks	**263**	**218**	**481**	**368**	**212**	**580**

Table 35: Classification of Usiskin's Multidimensional Model for OL Concise Maths

5.6.3.1 Higher Level vs Ordinary Level

For the post-PM HL Concise Maths textbooks, only the Skill-Algorithm dimension had the greatest incidence when compared to the OL textbooks. The OL textbooks appear to offer a greater range of learning dimensions than the HL ones.

5.6.3.2 Comparison of topic

In the HL Pre-PM Concise Maths textbook series, the Property-Proof dimension was the only one with a greater percentage in the topic of Pattern, Sequences and Series. The dimension of Use-Application was much more evident in the topic of Differential Calculus (25.1%) when compared to that of Pattern, Sequences and Series (5%) for the post-PM Concise Maths Higher Level textbook series. Similarly the Representation-Metaphor dimension had a greater percentage in Differential Calculus for the same era.

Figure 5.39 compares the topic and era for the Multidimensional Model for the HL Concise Maths textbook series.

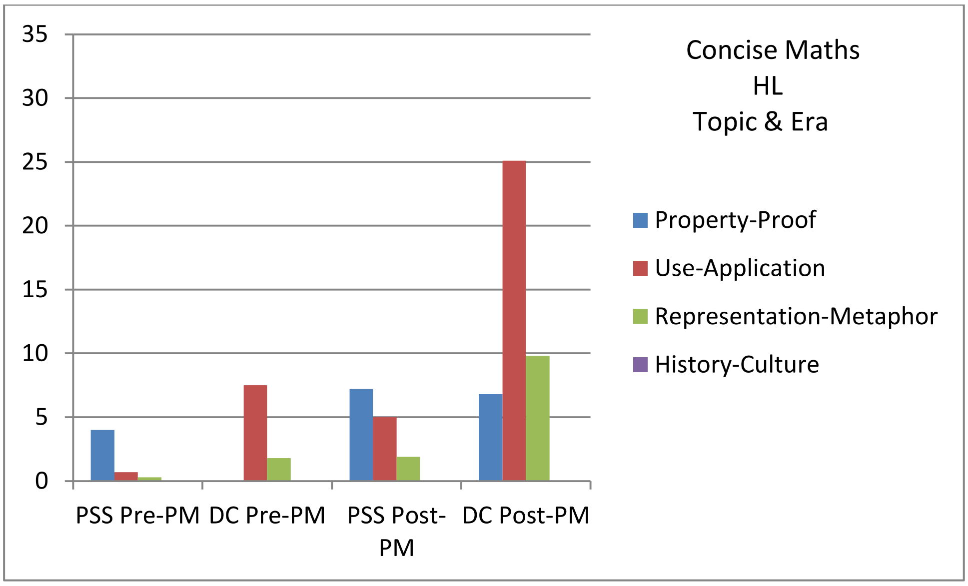


Figure 5.39: Usiskin's Multidimensional Model for HL Concise Maths comparing topic and textbook era.

A similar situation exists for the OL Concise Maths post-PM textbook series. The percentage of tasks classified in the Use-Application dimension was greater in the Differential Calculus topic (26.9%) when compared to the Pattern, Sequences and Series topic (18.8%). The Property-Proof dimension was also more evident in Differential Calculus. In the pre-PM Concise Maths textbook series, the percentage of Use-Application dimension was relatively high at 19.7% and was much greater than the corresponding proportion of tasks on the topic of Pattern, Sequences and Series at 0%. Figure 5.40 compares the topic and era for the Multidimensional Model for the OL Concise Maths textbook series.

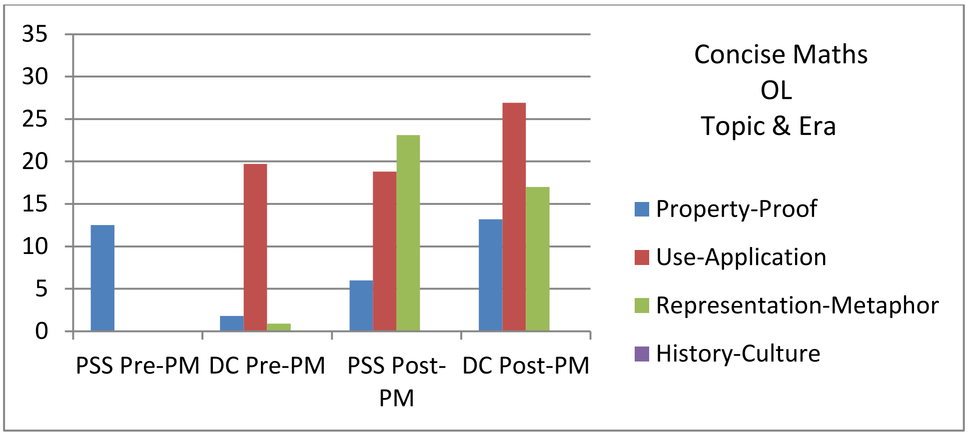


Figure 5.40 Usiskin's Multidimensional Model for OL Concise Maths comparing topic and textbook era.

5.6.4 Comparison of Textbook Series

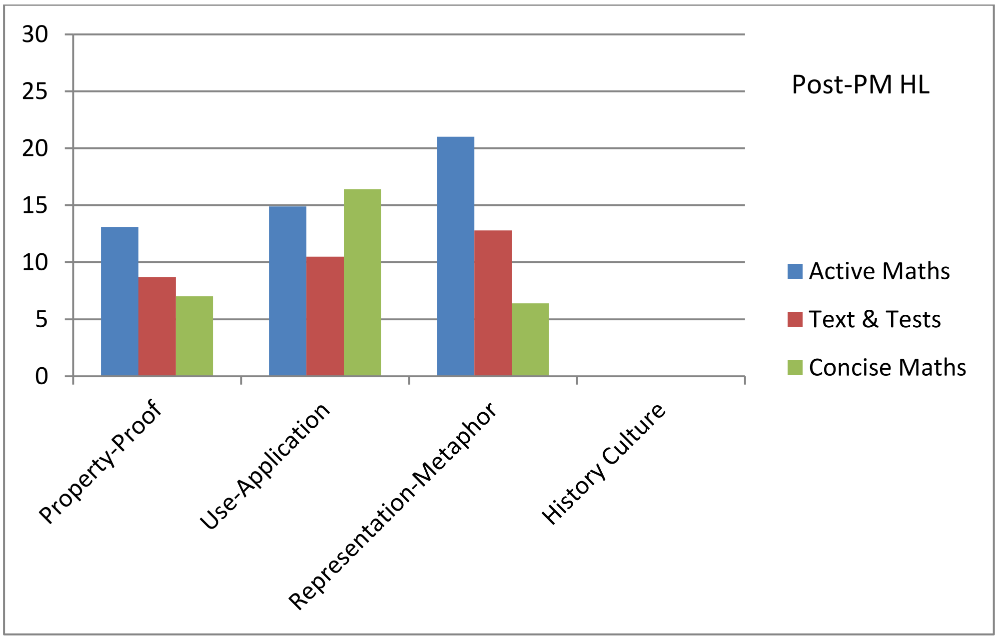


Figure 5.41: Comparison of Usiskin's dimensions (percentages) in post-PM HL textbook series

Figure 5.41 compares the incidence of Usiskin's dimensions in the three HL textbook series for the post-PM era. The Skill-Algorithm dimension had the greatest incidence in Concise Maths, the Property-Proof dimension was encountered most often in Active Maths, the greatest proportion of tasks involving the Use-Application dimension was found in Concise Maths.

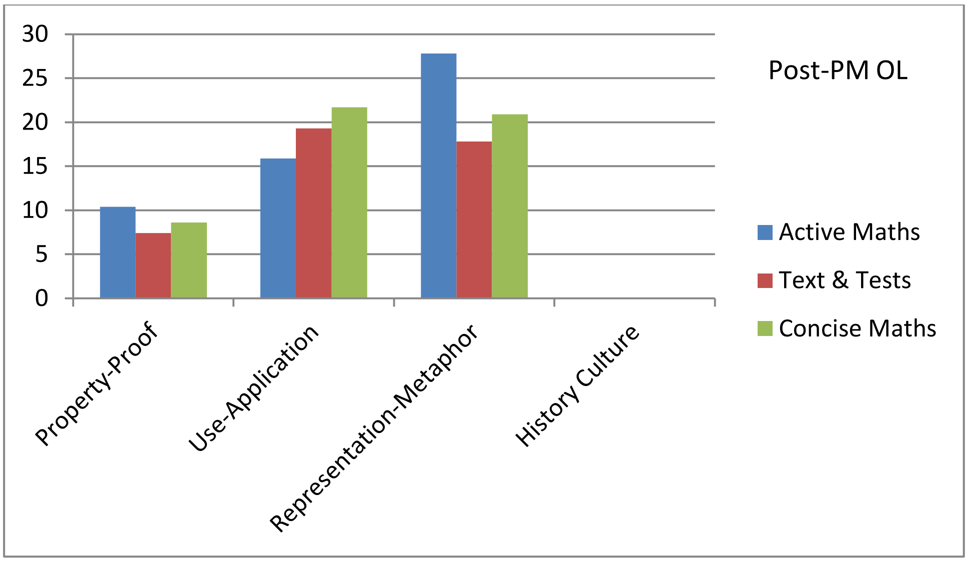


Figure 5.42: Comparison of Usiskin's dimensions (percentages) in post-PM OL textbooks

Figure 5.42 gives a comparison of Usiskin's dimensions in the post-PM OL textbooks. Active Maths had the greatest proportion of the Skill-Algorithm, Property-Proof and Representation-Metaphor dimensions. The Concise Maths series had the greatest percentage of tasks classified in the Use-Application dimension.

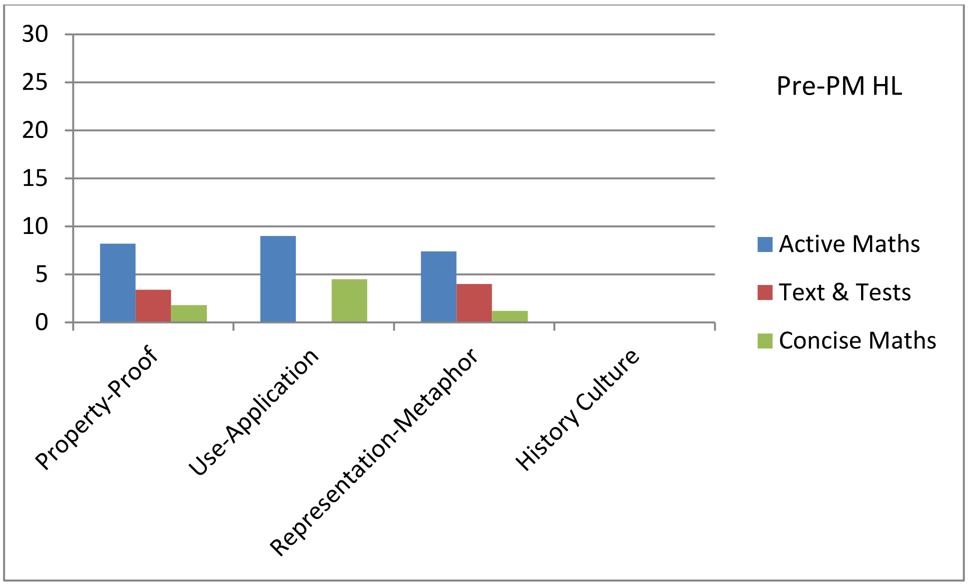


Figure 5.43: Comparison of Usiskin's dimensions (percentages) for pre-PM HL textbook series

Figure 5.43 above gives a comparison of Usiskin's dimensions for the pre-PM HL textbook series. In the pre-PM HL textbook series, Active Maths had the greatest incidence of the Property-Proof, Use-Application and Representation-Metaphor dimensions, while Concise Maths had the greatest incidence of the Skill-Algorithm dimension. Figure 5.44 gives a comparison of Usiskin's dimensions for the pre-PM OL textbook series. The greatest percentage of tasks classified as Property-Proof and Representation-Metaphor were to be found in the Concise Maths textbook series. The highest proportion of the Skill-Algorithm dimension was to be found in the Text and Tests pre-PM OL textbook series.

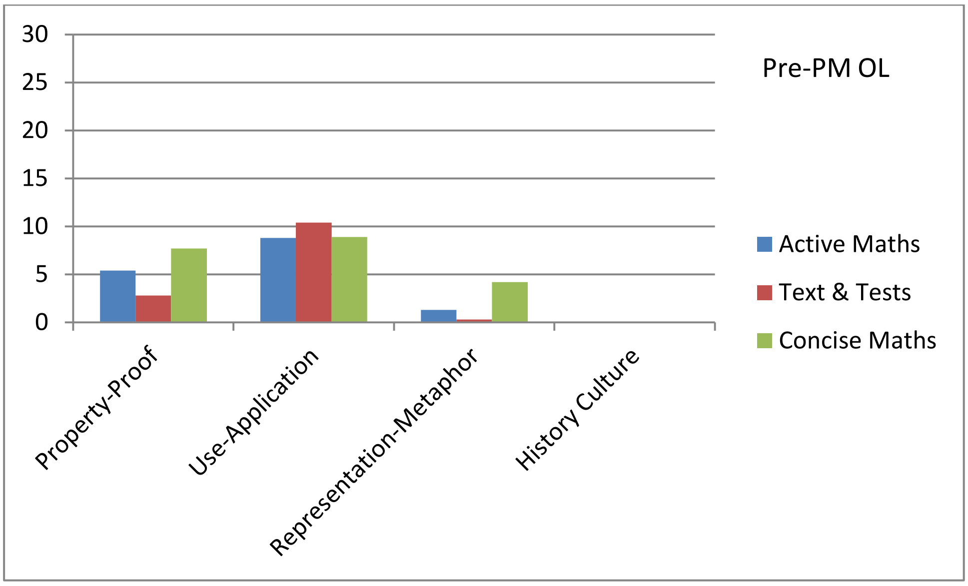


Figure 5.44: Comparison of Usiskin's dimensions (percentages) for pre-PM OL textbook series

Each of the post-PM textbook series, regardless of level, recorded a decrease in the proportion of the Skill-Algorithm dimension when the tasks from the post-Project Maths series were compared to those from the pre- Project Maths series. None of the tasks in any of the textbook series examined were found to incorporate the History-Culture dimension.

In this chapter, I have given the results of my analysis of textbook tasks using the five frameworks outlined in Chapter 3. In chapter 7, I will discuss these results.

Chapter 6 Task Design

6.1 Introduction

In this chapter, I will provide examples of the types of tasks that I have designed in response to my findings on current textbook tasks, accompanied by a classification of these tasks using the Levels of Cognitive Demand framework, Lithner's reasoning framework, Usiskin's mathematical understanding framework and the Project Maths problem-solving Objectives framework. I will describe a workshop at which a selection of the tasks designed for this project was independently evaluated with respect to the Levels of Cognitive Demand framework and the Project Maths problem-solving Objectives framework. Preparations for the workshop and how the workshop was conducted are described. The results of the workshop participants' task classifications are presented and analysed. Amendments to the workshop tasks are also outlined in light of feedback from the participants along with rationales for the changes. Classifications for these amended tasks are also provided.

When contemplating the design of tasks, I considered the results of the classification of textbook tasks. As can be seen in the previous chapter, my analysis suggests that there is a need for greater opportunities to engage in creative reasoning, to *do mathematics* and work with procedures connected to underlying mathematical meaning. Students need more opportunities to work with proofs and consider the history and/or culture of mathematics. My classification of tasks in textbooks currently available showed that several Project Maths objectives are neglected in the textbook tasks and more tasks need to encourage students to justify their conclusions, explain their findings and communicate mathematically. Similarly tasks need to engage students with analysing information in written form and to ask them to translate this into mathematical form. It is also important that students are required to select and devise mathematical models rather than just using very familiar models.

6.2 Designed Tasks

Modifications of existing Textbook Tasks

I began by augmenting existing textbook tasks in order to address neglected areas. Before re-designing the tasks being attended to, I identified the areas that I wanted to target in each such as increasing the level of cognitive demand, incorporating creative reasoning, involving the Property-Proof or History-Culture dimensions, or placing a greater emphasis on explaining findings and justifying conclusions. Often, in order to scaffold tasks or to introduce the student to a concept, it was necessary to begin with more straightforward procedural parts before increasing the complexity of the later parts. The design effort was an iterative process where I repeatedly considered whether the new tasks actually achieved what was intended and whether further alterations would lead to greater opportunities to address the deficiencies that were identified in the textbook tasks.

The example that follows (to which I added four newly designed sub-tasks) is drawn from the topic of Pattern, Sequences and Series. The original exercise is given and a description of how it has been adapted is provided. My classifications for the designed tasks are provided in a table directly after the given task.

Example 1 (Workshop Task 2) Question 10 Text and Tests 6 Exercise 4.1 pg. 138

The first terms of the Fibonacci sequence are given below.

0, 1, 1, 2, 3, 5, 8, 13, 21, … .

(i) *Describe in words how the sequence is formed.*

Hence write out the next four terms in the sequence

I extended this task by adding on four new parts (ii)-(v).

(ii) *Choose any four consecutive Fibonacci numbers. Add the first and last terms from your selection, then divide by 2. Repeat the process again with four other consecutive Fibonacci numbers, and then another four.*

(iii) *What do you notice in part (ii)?*

(iv) *Can you justify your observation mathematically?*

(v) In nature, the head of certain flowers and vegetables can also involve Fibonacci numbers, this involves counting the number of seed heads in the plant that form a spiralling pattern. Using the internet, find the name of some such flowers/vegetables and complete the table below.

Flower/Vegetable Name			
Number of seed heads in spiral			

Task Label	LCD	Reasoning	Usiskin's Model	PMO
1 (ii)	Procedures without	Imitative	Skill-	Use Model

	connections to meaning	Reasoning	Algorithm	
1 (iii)	Procedures with connections to meaning	Creative Reasoning	Property-Proof	Communicate Mathematics Explore Patterns
1 (iv)	Doing Mathematics	Creative Reasoning	Property-Proof	Justify Conclusions Apply Knowledge Devise Model
1 (v)	No classification	No classification	History-Culture	No classification

Table 36: Classifications of example 1

6.2.1 Original Designed Tasks

I designed eight original tasks, four from the topic of Pattern, Sequences and Series and four from Differential Calculus. My aim was to provide greater opportunities for students to engage in tasks requiring the categories 'doing mathematics' and 'procedures with connections to meaning' (LCD framework). I also wanted to address less frequently observed categories of Usiskin's framework such as Property-Proof and History-Culture, as well as those categories of the PMO framework that appeared to be neglected such as *Explain findings*, *Justify conclusions* and *Communicate mathematics in written form*. It should be noted that it would be very difficult for one task to meet all such criteria, so very often a certain category was targeted when designing a particular task. To augment the task in section 6.2 and also to design new tasks, I made use of Swan and Burkhardt's (2012) principles as described in section 2.6, giving particular attention to making the tasks of interest to students. This feature of task design has also been recommended by Chapman (2013). For my tasks I encouraged students to use the internet, observe patterns in nature and write a line of poetry. These features not only incorporate authentic contexts but are intended to show students how the mathematics involved can be of use in the world around them and the tasks are linked to other subjects that might be of interest. Another concern was to take into account the different abilities of students that would be attempting the tasks and to ensure that the material in the tasks would be accessible. I was influenced by Watson and Thompson's (2015) advice here to structure the tasks 'so that the desired key idea is varied and learners can see this and the effects of such variation' (pg. 170). I varied the activity across the set of tasks so that it did not become repetitive yet it was not overly difficult while encouraging the use and application of mathematical thought. This often required careful scaffolding so that students could be guided towards the appropriate investigation of concepts, for example beginning a set of tasks with imitative reasoning before introducing the requirement for creative reasoning. Mason and Johnston-Wilder (2004, p. 6) recommend to use a 'mixed economy' of tasks in order to realise as many goals as possible. I found this to be true; for example, when designing tasks to address the lack of the History-Culture dimension, often it was not possible to meet the deficiencies from the other frameworks, thus it was necessary to include other material in preceding parts of the task to promote creative reasoning and provide opportunities to meet a greater variety of Project Maths problem-solving objectives as can be seen in example 1. However, as can be observed in example 2, it was not always possible to achieve this and targeting the History-Culture dimension here did not allow for an extensive inclusion of other framework categories.

Example 2

Line 1
Line 2 Now
Line 3 Here,
Line 4 Always,
Line 5 Powerful,
Line 6 Growing gracefully:
Line 7 Maths nurturing the poetry

(i) *Count the number of syllables in each line of the above poem, write them as a sequence.*

(ii) *What type of sequence is this?*

(iii) *Create a suitable line for the poem so as to continue the sequence.*

Task Label	**LCD**	**Reasoning**	**Usiskin's Model**	**PMO**

2 (i)	Procedures without connections to meaning	Imitative Reasoning	Skill-Algorithm	Use Model
2(ii)	Memorization	Imitative Reasoning	History-Culture	Explore Patterns
2 (iii)	Procedures without connections to meaning	Imitative Reasoning	Skill-Algorithm History-Culture	Use Model

Table 37: Classifications of example 2

Example 3 (Workshop task 1)

A display of cans on a supermarket shelf consists of 15 cans on the bottom, 13 cans in the next row, and so on in an arithmetic sequence until the top row has 9 cans.

(i) *Can you suggest two other arithmetic sequences for arranging the cans on the shelf?*

(ii) *Explain how you found the sequence*

Task Label	**LCD**	**Reasoning**	**Usiskin's Model**	**PMO**
3 (i)	Doing Mathematics	Creative Reasoning	Property-Proof Use-Application	Apply knowledge Analyse Information Explore patterns Devise Model
3 (ii)	Doing Mathematics	Creative Reasoning	Property-Proof	Explain Findings Communicate Mathematics

Table 38: Classifications of example 3

Example 4

The first and fourth terms of an arithmetic series are a and b respectively. The sum of the first n terms of this series is denoted by S_n*. Find* S_6 *in terms of a and b.*

Task Label	Level of Cognitive Demand	Reasoning	Usiskin's Model	PMO
4	Procedures with connections to meaning	Creative Reasoning	Skill-Algorithm	Devise Model

Table 39: Classifications of example 4

<u>Example 5 (Workshop task 3)</u>

A ball is thrown upwards from ground level and rises to a height of 15 metres. Once it reaches this height, it falls and strikes the ground and bounces to 60% of its previous height. It repeats the process, each time bouncing to 60% of the previous height.

(i) *Find the total distance travelled by the ball by the time it bounces for the fifth time. Give your answer to two decimal places.*

(ii) *Rewrite the question above so that the ball's height is INCREASING each time instead of decreasing. What would be the total height after five bounces in this situation? (Give your answer to two decimal places)*

(iii) *If the ball from part (i) was caught on the third bounce, how high would the ball have to rise to on the first bounce in order for the total distance travelled to be 98 metres after three bounces?*

Task	LCD	Reasoning	Usiskin's	PMO

Label			Model	
5 (i)	Procedures with connections to meaning	Imitative Reasoning	Skill-Algorithm Use-Application	Apply Knowledge Analyse Information Select Model
5 (ii)	Procedures with connections to meaning	Imitative Reasoning	Skill-Algorithm Use-Application	Apply Knowledge Analyse Information Select Model
5 (iii)	Procedures with connections to meaning	Creative Reasoning	Skill-Algorithm Use-Application	Apply Knowledge Analyse Information Select Model

Table 40: Classifications of example 5

Example 6

Let f be the function f(x) = $x^2 - 2x - 3$ *and g be the function g(x) = 2x-2*

(i) *Using the same axes and scales, draw the graph of f and the graph of g, for* $-4 \le x \le 5$.

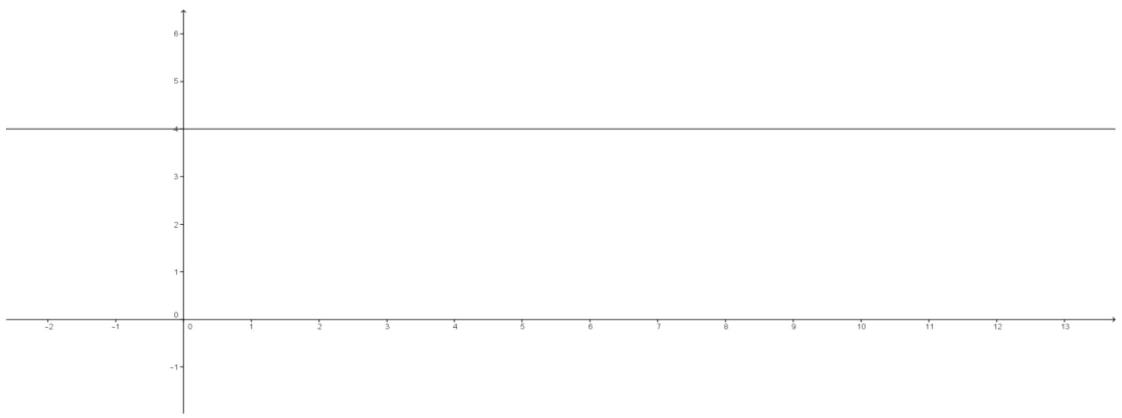
(ii) *You now have a graph of a function and its derivative. Identify which is the function and which is the derivative, giving at least two reasons to support your choice.*

(iii) *Describe the behaviour of the curve y=f(x) when:*
g > 0
g < 0
g = 0

Task Label	LCD	Reasoning	Usiskin's Model	PMO
6 (i)	Procedures without connections to meaning	Imitative Reasoning	Representation-Metaphor Skill-Algorithm	Use Model
6 (ii)	Procedures with connections to meaning	Creative Reasoning	Representation-Metaphor Property-Proof	Communicate Mathematics Justify Conclusions
6 (iii)	Procedures with connections to meaning	Creative Reasoning	Representation-Metaphor	Devise Model Explore Patterns

Table 41: Classifications of example 6

<u>Example 7 (Workshop task 4)</u>



(i) *You are given the graph of f ″(x) above, use your knowledge of derivatives to draw three possible graphs of f′(x).*

(ii) *Use your knowledge of derivatives to draw three possible graphs of f(x).*

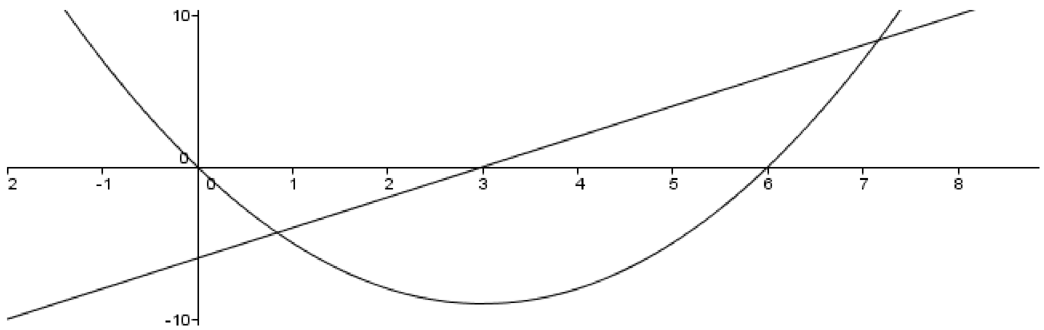
(iii) *Justify why you have drawn the graphs in this way.*

Task Label	LCD	Reasoning	Usiskin's Model	PMO
7 (i)	Doing Mathematics	Creative Reasoning	Representation-Metaphor	Explore Patterns Apply Knowledge Devise Model
7 (ii)	Doing Mathematics	Creative Reasoning	Representation-Metaphor	Explore Patterns Apply Knowledge Devise Model
7 (iii)	Doing Mathematics	Creative Reasoning	Property-Proof	Justify Conclusions Communicate Mathematics

Table 42: Classifications of example 7

Example 8 (Workshop task 5)

The function $f(x)$ is defined for all $x \in R$.
Graphs of $f'(x)$ [the curve] and $f''(x)$ [the line] are shown.



(i) *Using the diagram above, find the stationary points of f (x)*

(ii) *Identify them as maximum or minimum points.*

(iii) *Justify your answer.*

Task Label	LCD	Reasoning	Usiskin's Model	PMO
8 (i)	Doing Mathematics [Students could find the x co-ordinates of the stationary points from the graph of the derivative and where it cuts the x axis. Students most likely do not have an algorithm or procedure for this]	Creative Reasoning	Representation-Metaphor	Apply Knowledge Devise Model
8 (ii)	Doing Mathematics [Students could investigate the sign of the second derivative here by looking at the behaviour of its graph.]	Creative Reasoning	Property-Proof Representation-Metaphor	Apply Knowledge Devise Model
8 (iii)	Doing Mathematics [Students must provide a sufficient justification for what they have found, based on their discoveries with the graphs, in the previous two parts.]	Creative Reasoning	Property-Proof	Justify Conclusions

Table 43: Classifications of example 8

Example 9 (Workshop task 6)

(i) *Show that* $f(x) = x + \frac{1}{x}$ *does not have any points of inflection.*

(ii) *What can be altered in the given function in order to ensure that the function has a point or points of inflection?*

(iii) *Give an example of another function which does not have any points of inflection.*

(iv) *Examine the following statement: 'If* $(x, f(x))$ *is a point of inflection, then* $f''(x) = 0$ *or* $f''(x)$ *does not exist.'*

Write down the converse of this statement. Is it true? Justify your conclusion using a relevant example showing that points of inflection either exist or do not exist.

Task Label	LCD	Reasoning	Usiskin's Model	PMO
9 (i)	Procedures without connections to meaning	Imitative Reasoning	Skill-Algorithm	Use Model
9 (ii)	Doing Mathematics	Creative Reasoning	Property-Proof	Apply Knowledge
9 (iii)	Doing Mathematics	Creative Reasoning	Property-Proof	Apply Knowledge
9 (iv)	Doing Mathematics	Creative Reasoning	Property-Proof	Communicate Mathematics Justify Conclusions

Table 44: Classifications of example 9

6.3 The Tasks Workshop

It was decided to hold a workshop with Pre-Service Teachers (PSTs) in order to allow newly designed tasks to be evaluated using the Levels of Cognitive Demand framework and the Project Maths Objectives framework. This would allow me to gather independent evidence as to whether the tasks achieved my aims or not. I chose these two frameworks as I felt that they would be the most beneficial for future teachers' to develop a familiarity with.

Participation in the workshop was voluntary and it was made clear from the outset that the surveys and classifications completed during the course of the workshop would be kept anonymous. The group that agreed to participate was drawn from second and third year PSTs at Maynooth University enrolled either on the BSc in Mathematics Education, or on the BSc in Science Education who have chosen Mathematics as one of their subjects. In terms of teaching experience, the second year students had observed teachers in the classroom and the third year students had one year's experience which consisted of teaching one day per week. The PSTs were invited to attend the workshop and were sent a copy of a plain language statement, which described the purpose of the workshop and my project, in advance.

A number of decisions had to be made in advance of the tasks workshop in relation to which of the designed tasks would be chosen for classification. The tasks that were designed addressed several concerns such as giving greater attention to the Property-Proof and History-Culture dimensions of Usiskin's framework. It is difficult for one task to meet several criteria so priority was given to selecting tasks which addressed the Project Maths problem-solving Objectives most infrequently observed in current textbooks and which exhibited a high level of cognitive demand. There was also a limit on the time available for the workshop so the tasks could not be overly long. It was decided to use tasks that would be suitable not only for completion in classrooms but also for assignment as homework to students and could be classified using the two frameworks within the time available. The parts that were to be classified were marked with the term 'classify'. The PSTs participating in the workshop were presented with six tasks, consisting of sixteen parts in total, to classify. These can be found in appendix C, where the tasks are labelled from W1 (i) to W6 (iv).

6.4 Preparation for the Workshop

Several documents had to be prepared for the workshop. A codebook was created to contain all the essential information required by the participants during the coding process (see appendix C). The first section provided a list of criteria for classification with the Levels of Cognitive Demand framework. This was followed by six sample tasks. Each of these sample tasks was accompanied by an outline of the likely method of solution, some commentary discussing how the task might be classified and the actual classification assigned to it by me. Each sample task was selected from the textbook series that were analysed earlier in this study. In the next section of the codebook, the same six sample tasks were then analysed, in a similar manner, using the Project Maths Objectives framework. The final part of the codebook was created to assist coders when attempting to classify the practice tasks and the tasks designed for the workshop. This section gave an account of the material that students would have been likely to have been exposed to in school before attempting such tasks. It was important to provide the coders with this information as it would impact on their decisions in relation to the possible use of procedures in the Levels of Cognitive Demand framework. In relation to the Project Maths Objectives framework, it is necessary to be aware of what kind of mathematical models students have been exposed to when considering whether students are using, selecting or devising a model when solving a task. The codebook was the main point of reference for the coders when classifying both the practice and workshop tasks.

Two coding forms were created so that participants could record their classifications (see appendix C). This was to not only ensure consistency when collecting the data but to also assist the coders with choosing the classification in an efficient manner. The coding forms allowed the coders to see clearly the different options for the two frameworks and to tick which classifications they felt were applicable to the task under consideration. The coding forms also made it easier for data analysis as each coder recorded their classification in a similar manner.

A questionnaire consisting of two parts was also prepared in order to elicit the workshop participants' views in relation to the two frameworks and the tasks that were designed for the workshop. The framework consisted of ten questions; four pertaining to the frameworks and six examining the pre-service teachers' experience with the designed tasks. The first question sought to determine whether participants felt that the LCD framework was useful to teachers in their work. They were then invited to outline any difficulties that they might have experienced when using the LCD framework. For the third question, the PSTs were asked if they felt that it was important to classify tasks using the Project Maths Objectives framework. Again the participants were invited to outline any difficulties that they might have encountered when using the PMO framework. The second half of the questionnaire focused on the designed tasks that the PSTs classified. The first three questions in this section looked for information on the tasks and the PSTs' experience of classifying these tasks using the two frameworks. Specifically the participants were asked if they felt that the tasks were clearly described and if they found it easy to identify the LCD and PMO of the designed tasks. Finally the PSTs were asked if they felt that the tasks were suitable for the use in the classroom and if they would be suitable for assignment as homework. The final question in the questionnaire invited the participants to add any further comments that they might have on the tasks.

6.5 The Workshop

The workshop was divided into two parts. The first session lasted for an hour and it was attended by 19 PSTs. A presentation was given providing an introduction to the project and it also gave an outline of the two frameworks to be used. The Levels of Cognitive Demand framework was explained with examples of classification chosen from the topic of pattern, sequences and series. To ensure consistency in classifying tasks, when considering the Project Maths Objectives framework, care was taken to differentiate between 'explain findings' and 'justify conclusions': the participants were informed that the 'justify conclusions' objective is similar to providing a proof, while the 'explain findings' objective would be used where those solving a task were asked to interpret their solution. Attention was also given to the 'apply their knowledge and skills to solve problems in familiar and unfamiliar contexts' objective as different interpretations are possible for the terms 'problem' and 'contexts'. For classification in the workshop, problems were to be treated as something containing an element of non-routine material with a requirement for some engagement on the part of the student in order to solve it and the context could be mathematical rather than a real-life scenario or application. The participants were also advised of the need to make a clear distinction between 'Use', 'Select' and 'Devise'. It was advised that the objective 'Use' should be interpreted as involving the implementation of a well-known procedure or algorithm. 'Select' was described as making a choice from an existing repertoire of models and techniques known by the student solving the task, in order to determine how best to approach the exercise. Finally, it was recommended that the objective 'Devise' should be applied if the workshop participants felt that a student solving a task had to experiment in terms of finding a method of solution and effectively created their own model when answering the exercise. For the remaining part of the session, candidates were given the opportunity to practice classifying textbook tasks on pattern, sequences and series using the two frameworks and the group discussed how the classifications were made.

The second session lasted for ninety minutes and it took place on the same day as the first session. There was a break of one hour between the two sessions as the PSTs had to attend a lecture in their timetable. For the second session, 18 participants from the original 19 were present and they were again given a brief outline of this PhD project and its study of textbook tasks. The PSTs were reminded of key aspects of the two frameworks and how classifications should be applied. Examples of classifications were provided, this time taken from the topic of differential calculus. Participants were given the opportunity to classify practice textbook tasks from the topic of differential calculus using the two frameworks. The PSTs were then given time to discuss the classifications as a group before being given the first half of the questionnaire relating to the frameworks. Once the questionnaire was completed, participants were given a short break. After the break, candidates were given some of the tasks I designed as part of this project (those labelled Workshop Tasks 1-6 in Appendix C) and asked to classify them. As each participant finished classifying the tasks, the last part of the questionnaire was administered. An opportunity was given at the end for a discussion in relation to the two frameworks and the designed tasks.

6.6 Results

Individual results for each coder using the LCD framework can be found in appendix C.

Classification	Frequency	Percent
Higher Level Doing Mathematics	74	25.7
Higher Level Procedures with Connections	157	54.5
Lower Level Procedures without Connections	49	17.0
Lower Level Memorised	8	2.8

Table 45: Classifications for the LCD framework from the Tasks Workshop

Using the Levels of Cognitive Demand framework, the majority of the 18 participants agreed that 14 of 16 of the designed tasks were of high cognitive demand. However, the majority only agreed with the intended category classification 4 times. It was intended that 12 of the tasks would involve *doing mathematics* while 4 would involve the use *of procedures with connection to meaning.* This would have resulted in 75% of the tasks being classified as *doing mathematics* and 25% as *procedures with connection to meaning*. As can be seen in table 45, 25.7% of all tasks classified by all participants were classified as *doing mathematics*, 54.5% as *procedures with connection to meaning*, 17.8% were classified as *procedures without connection to meaning* and 2.8% as *memorisation*.The LCD of tasks W2 (iii) and W2 (iv) were classified as intended with the majority classifying W2 (iii) *as procedures with connection to meaning* and W2 (iv) as *doing mathematics*. The majority also classified W3 (i) and W6 (iv) with the classification intended. Five participants classified W3 (ii) as *doing mathematics* rather than the expected *procedures with connection to meaning*. It is possible that these coders did not consider there to be any procedure that could be used in its solution. The workshop participants consistently classified W5 (i), W5 (ii) (and to a lesser extent W6 (iii)) as having a low level cognitive demand. At second-level, I expected that finding the stationary points of a function and identifying them as maximum and minimum points from just the graph would be quite demanding for students. The workshop participants disagreed with this and mainly labelled the finding of the stationary points as using a *procedure without connection to meaning*.

Label	Original Classification	Workshop Classification: Doing Mathematics	Workshop Classification: Procedures with connection to meaning	Workshop Classification: Procedures without connection to meaning	Workshop Classification: Memorisation	Majority agreed with original classification	Majority agreed with Higher Level
W1 (i)	DM	4 (22.2%)	13 (72.2%)	1 (5.6%)	0	No	Yes
W1 (ii)	DM	3 (16.7%)	15 (83.3%)	0	0	No	Yes
W2 (iii)	HP	2 (11.1%)	13 (72.2%)	3 (16.7%)	0	Yes	Yes
W2 (iv)	DM	11 (61.1%)	7 (38.8%)	0	0	Yes	Yes
W3 (i)	HP	2 (11.1%)	9 (50%)	7 (38.9%)	0	Yes	Yes
W3 (ii)	HP	5 (27.8%)	8 (44%)	5 (27.8%)	0	No	Yes
W3 (iii)	HP	15 (83.3%)	3 (16.7%)	0	0	No	Yes
W4 (i)	DM	2 (11.1%)	14 (77.8%)	2 (11.1%)	0	No	Yes
W4 (ii)	DM	3 (16.7%)	12 (66.7%)	3 (16.7%)	0	No	Yes
W4 (iii)	DM	3 (16.7%)	14 (77.8%)	1 (5.6%)	0	No	Yes
W5 (i)	DM	1 (5.6%)	5 (27.8%)	12 (66.7%)	0	No	No
W5 (ii)	DM	1 (5.6%)	4 (22.2%)	10 (55.6%)	3 (16.7%)	No	No
W5 (iii)	DM	5 (27.8%)	12 (66.7%)	0	1 (5.6%)	No	Yes
W6 (ii)	DM	3 (16.7%)	12 (66.7%)	3 (16.7%)	0	No	Yes
W6 (iii)	DM	3 (16.7%)	10 (55.6%)	4 (22.2%)	1 (5.6%)	No	Yes
W6 (iv)	DM	11 (61.1%)	7 (38.8%)	0	0	Yes	Yes

Table 46: Analysis of workshop task classifications for the LCD framework

Identifying the maximum and minimum points was largely classified as procedure without connection to meaning with some regarding it as memorisation. Despite there not being any procedure identified in the codebook for this, the majority of participants felt that such a task would be routine or memorised material for students at this level. With W6 (iii), several participants felt that students would have memorised an example of a function which does not have any points of inflection. This was not something that was outlined in the textbooks or regarded as normal practice in the codebook but participants may have felt that a teacher would ask students to commit this kind of material to memory.

It was hoped to provide a greater variety of Project Maths Objectives within the designed tasks than was available in the textbook exercises. Looking at the objectives individually, the majority of workshop participants agreed that *explore patterns and formulate conjectures* was present in tasks W1 (i) and W2 (iii) but did not feel the same about W4 (i) and W4 (ii). Participants may have felt that it was not necessary to explore patterns in order to complete this task. Interestingly, the majority of coders felt that *explore patterns and formulate conjectures* was also present in W1 (ii), W2 (iv), W3 (i), W3 (ii), W3 (iii) and W4 (iii) despite this not being planned for when designing the tasks. One task (W1 (ii)) aimed to address the objective of *explain findings* and all but one of the students agreed that this had been achieved. The tasks W2 (iv), W4 (iii), W5 (iii) and W6 (iv) were designed to include the objective *justify conclusions*, and the majority of coders felt that these tasks addressed it successfully.

The inclusion of the objective *communicate mathematics* was less successful and yielded more mixed results, 50% felt that it was present in W1 (ii), 44.4% thought it was in W2 (iii), 55.6% felt it was in W4 (iii) and 33.3% used the classification for W6 (iv). It may have been that the PSTs thought that asking students to give the converse of a statement does not involve the *communicating mathematics* objective.

Project Maths Objective	W1 (i)	W1 (ii)	W2 (iii)	W2 (iv)
Explore Patterns	18 100% (Intended)	9 50%	16 88.9% (Intended)	9 50%
Explain Findings	-	17 94.4% (Intended)	11 61.1%	6 33.3%
Justify Conclusions	-	5 27.8%	4 22.2%	14 77.8% (Intended)
Communicate Mathematics	3 16.7%	9 50% (Intended)	8 44.4% (Intended)	6 33.3%
Apply Knowledge	12 66.7% (Intended)	3 16.7%	5 27.8%	6 33.3% (Intended)
Analyse Information	15 83.3% (Intended)	3 16.7%	5 27.8%	8 44.4%
Devise Model	5 27.8% (Intended)	-	-	10 55.6% (Intended)
Select Model	5 27.8%	-	-	2 11.1%

Table 47: Analysis of workshop task classifications for the PMO framework W1 (i) – W2 (iv)

The objective *apply knowledge and skills to solve problems in familiar/unfamiliar contexts* was judged to be included successfully in all intended tasks except for W2 (iv) where students were asked to justify their observations mathematically. It is possible that the PSTs might not have regarded this as a problem; instead viewing it as a yes/no question. The tasks designed to meet the *objective analyse information presented verbally and translate it in mathematical form* were classified by most coders as hoped with 83.3% for W1 (i), 94.4% for W3 (i), 88.9% for W3 (ii) and 88.9% for W3 (iii) also.

Project Maths Objective	W3 (i)	W3 (ii)	W3 (iii)	W4 (i)	W4 (ii)	W4 (iii)
Explore Patterns	12 66.7%	10 55.6%	11 61.1%	2 11.1% (Intended)	2 11.1% (Intended)	11 61.1%
Explain Findings	-	1 5.6%	2 11.1%	-	-	-
Justify Conclusions	-	-	-	-	-	16 88.9% (Intended)
Communicate Mathematics	1 5.6%	6 33.3%	-	-	-	10 55.6% (Intended)
Apply Knowledge	12 66.7% (Intended)	12 66.7% (Intended)	18 100% (Intended)	15 83.3% (Intended)	15 83.3% (Intended)	5 27.8%
Analyse Information	17 94.4% (Intended)	16 88.9% (Intended)	16 88.9% (Intended)	5 27.8%	4 22.2%	2 11.1%
Devise Model	4 22.2%	7 38.9%	13 72.2%	1 5.6% (Intended)	2 11.1%% (Intended)	1 5.6%
Select Model	7 38.9% (Intended)	4 22.2% (Intended)	4 22.2% (Intended)	7 38.9%	7 38.9%	3 16.7%

Table 48: Analysis of workshop task classifications for the PMO framework W3 (i) – W4 (iii)

Project Maths Objective	W5 (i)	W5 (ii)	W5 (iii)	W6 (ii)	W6 (iii)	W6 (iv)
Explore Patterns	2 11.1%	1 5.6%	-	3 16.7%	2 11.1%	3 16.7%
Explain Findings	-	1 5.6%	7 38.9%	2 11.1%	2 11.1%	5 27.8%
Justify Conclusions	-	2 11.1%	14 77.8% (Intended)	3 16.7%	2 11.1%	13 72.2% (Intended)
Communicate Mathematics	-	1 5.6%	5 27.8%	5 27.8%	1 5.6%	6 33.3% (Intended)
Apply Knowledge	10 55.6% (Intended)	11 61.1% (Intended)	6 33.3%	14 77.8% (Intended)	14 77.8% (Intended)	16 88.9%
Analyse Information	5 27.8%	3 16.7%	1 5.6%	4 22.2%	4 22.2%	8 44.4%
Devise Model	1 5.6% (Intended)	1 5.6% (Intended)	3 16.7%	4 22.2%	5 27.8%	10 55.6%
Select Model	7 38.9%	3 16.7%	4 22.2%	4 22.2%	1 5.6%	1 5.6%

Table 49: Analysis of workshop task classifications for the PMO framework W5 (i) – W6 (iv)

There was less consistency with the three objectives *devise, select and use: appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.* To apply these classifications correctly, it is necessary to know what methods or techniques the students are familiar with and the classification crucially depends on when in the teaching sequence the tasks are used. Tasks W1 (i), W2 (iv), W4 (i), W4 (ii), W5 (i) and W5 (ii) were designed with the objective *devise model* in mind. In the classification, 55.6% felt that W2 (iv) achieved this goal. W1 (i) received the intended classification 27.8% of the time, W4 (i) 5.6%, W4 (ii) 11.1%, W5 (i) and W5 (ii) 5.6% each. As discussed earlier, the participants had different solution methods in mind for tasks W4 (i), W4 (ii), W5 (i) and W5 (ii) than what was intended when designing the tasks. *Select model* was identified more frequently than devise but again the workshop participants were slow to classify tasks using this objective. Tasks W3 (i), W3 (ii) and W3 (iii) were intended to address this objective with W3 (i) gaining 38.9%, W3 (ii) getting 22.2% and W3 (iii) receiving 22.2%. Overall, it would appear that the participants considered that the tasks were successfully designed in order to provide a greater variety of Project Maths objectives. However, in any future design process I would strive to include more tasks relating to the three objectives of *communicate mathematics, devise model and select model.*

In terms of levels of cognitive demand, it would appear that the tasks succeeded in providing more opportunities for higher-level engagement but students viewed the tasks as being more procedural than what was intended. Similarly a wider variety of Project Maths Objectives were addressed than was the case, relatively speaking, for the existing textbook tasks but some objectives would still require greater representation.

6.7 Questionnaire

All 18 coders felt that the Levels of Cognitive Demand framework would be useful to teachers in their work. Several reported that it would help avoid teaching in such a way that would prioritise memorising formulae and practicing procedures. Coder 1 stated that ‘it could prevent teaching such a way that students do not think about solving questions and just memorise formulae and methods.’ Many also felt that it would help to determine how suitable an exercise would be for a group of students and how tasks could challenge the more able students and expand their mathematical knowledge. Coder 3 said that ‘it allows them to see how challenging the questions are to students. It will allow them to see what questions will push the Higher Levelled students and what questions will most likely trouble the weaker student.’ Two felt that it would help structure the class for a mixed ability group by ensuring that there was an appropriate mix of tasks that would enable differentiated learning to take place. Coder 14 felt that teachers ‘can ensure that they are providing their class with all types of questions under this framework. This would be particularly important in mixed ability classes.’

Overall, none of the participants felt that the Level of Cognitive Demand framework was difficult to use. However, some qualified this by saying that it was difficult at times to distinguish between the classifications for higher level cognitive demand. Coder 8 explained that it was ‘just between distinguishing the higher order demands as they appear quite similar’. Several added that they found the framework straightforward once they were certain of what kind of material the students had covered before completing the tasks. Coder 14 stated that ‘it is essential to know what knowledge the students already have’.

All the coders felt that it was important to classify tasks using the Project Maths syllabus framework. The main reason being offered was that it ensured that teachers were covering the prescribed material properly and it would help teachers to avoid serious omissions in relation to students’ problem-solving skills. Coder 4 felt that it was important ‘so that students will cover all types of questions in the PMO. If tasks were not classified perhaps some categories may not be covered.’ Another reason offered was that it enabled students to receive a more rounded mathematical education by exposure to several different types of questions and this would help them to be better prepared for working with mathematics later in life. According to coder 3, ‘It allows the teacher to implement the Project Maths syllabus to the best level it can be at. It also allows for a wider scope of questions/activities to be implemented.’

Four expressed the opinion that the Project Maths syllabus framework was more difficult to apply than the Level of Cognitive Demands framework in general. This was because there were so many different objectives and it involved making several choices in comparison to just one with the LCD framework. Two felt that this framework required more time and this would make it more arduous. Coder 9 pointed out that ‘due to the extended choice this framework would be more time consuming.’ One coder reported that more teaching experience would be required in order to apply the framework effectively. Two participants stated that they had difficulty distinguishing between the devise, select and use objectives. Coder 7 felt that ‘sometimes it was difficult to decide between devise, select and use.’ This is something that was borne out in the task classifications.

In relation to the designed tasks¸ all 18 coders indicated that the tasks were clearly described. Several added that they were clear and concise with the layout of the tasks being well presented. Two students felt that identifying the level of cognitive demand required some thought in order to go back to the frame of mind of a Leaving Certificate student. Coder 5 pointed out that ‘it’s hard to remember that something we think might be easier, they might find hard.’ Six participants, four that expressed concerns about the framework initially and an additional two after the process of classification was complete; felt that identifying the Project Maths Objectives of the designed tasks was more difficult than determining the Levels of Cognitive Demand. They reported that it was more challenging to use for classification due to the number of objectives and it took more thought in order to determine which applied and which did not.

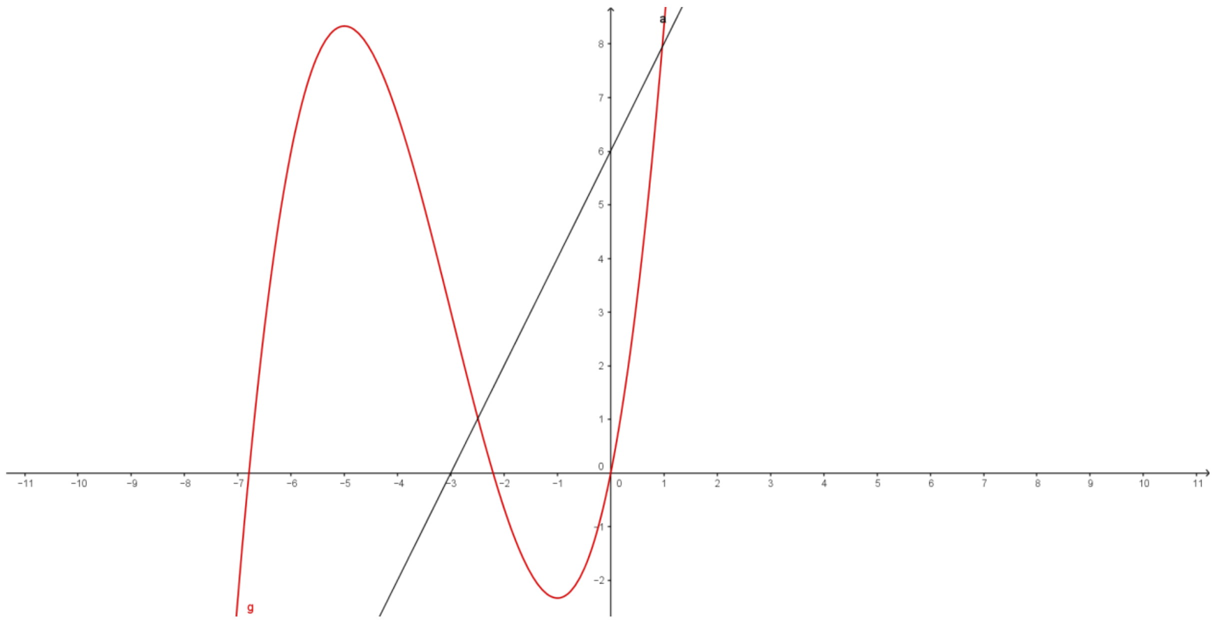
All respondents felt that the tasks were suitable for use in the classroom, particularly at the end of a topic or later on when revising material. Three participants felt that the designed tasks would be very beneficial as part of a group problem-solving session. Twelve pre-service teachers felt that the tasks would be suitable for homework, especially as a challenge. Coder 3 stated that ‘it will allow students extra time to sit down and work through the questions and would be a good challenge to them.’ Six coders felt that the tasks were suitable for use in the classroom but not for homework as the tasks were quite difficult and could be disheartening for weaker students if they were attempting them on their own. They felt that such students would benefit from assistance from a teacher in order to make further progress with the tasks. Coder 19 expressed the view that ‘a teacher’s help would encourage students to go a step beyond their prior knowledge with the tasks.’

6.8 Conclusion

As was seen earlier, the majority of PSTs only agreed with the intended category classification for the LCD framework 4 times. The results for the tasks classified using the PMO framework were more consistent in comparison but issues were encountered particularly in relation to the three categories: *Devise, Select* or *Use mathematical models, formulae or techniques to process information and to draw relevant conclusions*. It should be acknowledged that the workshop involved a lot of technical detailed work over a short space of time for the PSTs. This might have resulted in it being difficult to apply the classifications as intended. It would have been beneficial if they had more time to work on examples and were able to avail of more opportunities to discuss their classifications. Another issue is that the PSTs had to work with two different frameworks at the same time, whereas when my supervisors and I were coding, we usually only worked with one. Also the PMO framework has 10 categories and this can be very demanding to apply with limited experience. The classification process for both frameworks involves knowing what material students have seen before. When working with tasks from a textbook, this is a straightforward process but the PSTs were given the workshop tasks in isolation with only a list of topics that the intended students may have seen previously. It is likely that these factors may have had some impact on the results that were found.

Looking at the 16 tasks that were classified I felt that W5 (i), W5 (ii) and W6 (iii) would benefit from some modification.

In relation to W5, this time I would not provide a graph of the derivative but instead provide the second derivative in an effort to avoid any familiar procedure being applied. I would also make the graph more difficult to interpret so that the student would have to make a greater effort in order to determine the derivative. I would also avoid asking for a determination of the stationary points using the wording 'the maximum and minimum points' as this appears to have become procedural for students.



Modified Task 5

The function $f(x)$ is defined for all $x \in R$.

Graphs of $f(x)$ [the cubic] and $f''(x)$ [the line] are shown.

The line intercepts the y-axis at the point (0,6) and also contains the point (1,8)

M5 Find an expression for f'(x). Determine the stationary points of f(x).

Expected Classification for M5:

Procedures with connections to meaning.

Apply knowledge to solve problems presented in familiar/unfamiliar contexts.

Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Modified Task 6

M6 (i) Show that $f(x) = x^4$ does not have any points of inflection.

M6 (ii) *State and draw a sketch of another function which does not have any points of inflection.*

M6 (iii) (a) *Next draw a graph of a function with:*

One point of inflection

Two points of inflection.

M6 (iii) (b) *Describe what a point of inflection is using your graphs.*

The original function in W6 (i) had no points of inflection because it was not defined at 0. This modified task M 6 (i) would be more beneficial to the student in terms of preparing them for the examination of graphs that follows in the subsequent tasks. For M6 (ii) if the students have memorised a function which does not have any points of inflection, they will still have to do some mathematics in order to draw a graph of the function. Similarly they will have to engage with the mathematics in order to produce graphs with one point of inflection or two points of inflection.

Expected Classification for M6 (ii)

Determining the sketch of the graphs even if the functions have been memorised would still involve the use of procedures with connections to meaning.

Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions. People can go about sketching graphs in different ways. Some might determine the roots first and draw from there. Others might sub in values to the shape of the graph.

Expected Classification for M6 (iii) (a)

Determining the sketch of the graphs even if the functions have been memorised would still involve the use of procedures with connections to meaning.

Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

M6 (iii) (b)

Doing Mathematics

Apply knowledge to solve problems in familiar/unfamiliar context.

Communicate mathematics in written form.

6.9 Summary

Following efforts to design tasks that would demand more of students cognitively, offer more opportunities for creative reasoning, incorporate neglected dimensions of mathematical understanding and provide more diversity in terms of the Project Maths problem-solving objectives addressed, I have presented 9 examples consisting of 30 tasks. One of these examples was an extension of an existing textbook task, ameliorating the deficiencies identified using the four frameworks. A workshop was prepared to gain independent verification from PSTs as to whether 16 of these tasks did indeed provide greater opportunities for higher level cognitive engagement and meet an increased number of Project Maths problem-solving objectives. The results from the workshop indicate that the tasks were successful overall in requiring a high level of cognitive demand, although the PSTs felt that the tasks were more procedural than what was intended. Their classifications affirmed that a greater number of Project Maths Objectives were addressed proportionally in the tasks that I had designed than the existing textbook tasks that were analysed. I also outlined amendments to the workshop tasks in light of how the tasks were classified in the workshop, in order to make the tasks as effective as possible.

Chapter 7 Discussion

This dissertation examines the nature of mathematical tasks from textbooks that are being used in Irish second-level classrooms. To facilitate the analysis, four frameworks were used while a fifth was designed to measure the phenomenon of novelty. Due to the established role of the textbook as a source of tasks (Project Maths, 2017, O'Keeffe and O'Donoghue, 2009, Jones, Fujita, Clarke and Lu**,** 2008, p. 142), the three most popular textbook series were classified using the frameworks. The results from this classification informed the design of tasks that might be used in the classroom or assigned as homework.

It will be recalled from chapter 3 that Cohen et al.'s Instructional Triangle provides the overarching conceptual framework in which this study of mathematical tasks is situated Teachers, students and mathematical content are positioned at the three vertices of this instructional triangle. Herbst and Chazan (2012) have extended this conceptualization and point to tasks as a means through which students interact with the mathematical content They further describe the teacher's role as including supporting students' work with tasks (Herbst and Chazan, 2012). My findings offer insights into how teachers could provide such support – for example, by choosing textbook tasks carefully, searching for tasks from other sources to supplement these when necessary and possibly augmenting these tasks and/or designing new tasks to achieve the goals of the Project Maths classroom. When discussing the results in the following sections, I am concentrating on the post-Project Maths textbook series as these give some indication of the tasks that are available for use in post-primary classrooms currently.

7.1 Research Questions

7.1.1 RQ 1 (a) Are These Tasks Fulfilling the Expectations of Project Maths in Terms of its Problem-Solving Objectives?

The most common Project Maths problem-solving objective encountered in all three textbook series at both levels involves the *use of appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions*. For Higher Level, its greatest frequency is in the Concise Maths series and at Ordinary Level, it is greatest in the Active Maths series. As described in chapter 3, I divided the Project

Maths Objective *devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions* into three separate parts. This was necessary to identify the difference between using and devising techniques and provide greater detail for the analysis. As seen earlier, the use and selection of models, formulae or techniques is aligned to algorithmic reasoning while devising is a key feature of creative reasoning (Lithner, 2008, p.266).

The *objective explore patterns and formulate conjectures* is also relatively common across the three series and it is likely that this is influenced by the particular topic of Pattern, Sequences and Series. It is to be expected given the subject matter, that this objective should have a strong presence within the chapters analysed. The objectives of *justify conclusions* and *communicate mathematics in written form*, in particular, are rarely addressed. It would appear that the textbooks have yet to fully embrace the goals of Project Maths and there is still a tendency for exercises to emphasise the practice of skills and algorithms rather than asking students to devise new techniques or apply their knowledge in unfamiliar contexts. This supports Davis' (2013) view that there is a misalignment between these textbooks and the Project Maths syllabus. However, my analysis looks at all of the Project Maths Objectives while Davis' study focused exclusively on the area of reasoning-and-proving. In Davis' study, he found that there was little opportunity for students to engage in reasoning-and-proving and very little evidence of tasks requiring the explanation of findings or the justification of conclusions. This shows the difficulty that Irish teachers have in finding a textbook to effectively support their teaching of the revised curriculum.

Cosgrove et al. (2012) found that 31.6% of teachers found the availability of teaching materials (like textbooks) to support the Project Maths curriculum to be a major challenge, while 45.8% of teachers described it as a challenge (p.54). O'Keeffe and O'Donoghue (2011) found that all the textbooks included in their study fell short of the standard needed to support Project Maths effectively, even though some are better aligned to the syllabus expectations than others. Their work makes use of a modified version of the TIMSS mathematics curriculum framework known as the TIMSS+ instrument, which is applied to a number of mathematics textbooks or series of textbooks by dividing textbooks into chunks of curricula or strands identified as units called specially constructed curricula (SCC). Their report presents data from 10 textbooks and 6 SCC. My analysis focusses on two topics (Pattern, Sequences and Series and Differential Calculus) at senior cycle but it analyses the chapters relating to those topics in six textbooks using five different frameworks. The findings of my analysis would support the views expressed by O'Keeffe and O'Donoghue (2011).

There has been some attempt to meet the Project Maths objectives in the textbooks considered here but my results indicate that more attention needs to be given to the design of tasks that meet the goals of Project Maths. If not, the more traditional mathematics classroom where the emphasis is on the development of procedural skills rather than applying mathematics in real-life contexts or considering properties of mathematical concepts and how they interconnect might persist if the teacher does not supplement these tasks (NCCA, 2006, p.7). The Chief Examiner's Report cautioned that teachers need to provide students with the opportunity to 'solve unfamiliar problems and to develop strategies to deal with questions for which a productive approach is not immediately apparent' (SEC, 2016, p.30). This will be a challenge for teachers to incorporate sufficient tasks to achieve this in their classrooms. It must be recalled that 'an important feature of Project Maths is the reduction in emphasis on practising routine or procedural questions and solutions based on illustrative examples, with more emphasis being given to students engaging in problem-solving approaches and justifying or explaining their solutions' (NCCA, 2012b, p.18). The Project Maths curriculum designers want students to be encouraged to think about their strategies, to explore possible approaches and evaluate these. This would allow them to build up a body of knowledge and appropriate skills that they can apply in both familiar and unfamiliar situations. However my findings suggest that the current textbooks being used in Irish classrooms do not facilitate this because the tasks do not incorporate sufficient 'novel' tasks which in turn lead to a reliance on the use of algorithms and procedures that are not linked to the mathematical meaning. It will fall to the teacher to find or create suitable tasks outside of the textbook series in order to make sure that the Project Maths curriculum is implemented fully.

On textbooks and curricular goals more generally Houang and Schmidt (2008) have cautioned that textbooks can have varying interpretations of curricular intentions. These findings are similar to Herbel-Eisenmann (2007) who analysed a mathematics textbook to see if it achieved the ideological goal of the enacted curriculum to move the authority away from the teacher and the textbook and to promote student reasoning and justification. It was found in their study that the textbook fell short of this goal. Also Tran (2016) examined the alignment of a U.S. high school textbook with the Common Core curriculum and found that there were limitations in terms of reaching the learning outcomes expected.

Fan and Kaeley (2000) have shown that textbooks can impact on how teachers teach. They hold the view that it would be challenging to bring about change in teachers' teaching methods without corresponding reform of the textbooks being used in the classroom. This is due to the important role that textbooks play in affecting teaching strategies. It would appear that it will be difficult for the Project Maths curriculum to be realised effectively until further development takes place in these textbooks in light of the deficiencies highlighted by the research presented in this thesis.

7.1.2 RQ 1 (b) What Degree of Novelty is Incorporated in the Tasks?

It appears that all the textbook series have a tendency to reinforce material presented in the exemplary sections: there does not appear to be a great degree of novelty across the exercises that were analysed. Of all the textbook tasks analysed, 5.7% were classified as 'novel', 20.4% as 'somewhat novel' and 74% as 'not novel'. The comparison of Higher and Ordinary level textbooks suggests that there is a greater proportion of tasks in the Higher Level textbooks classified as 'somewhat novel' when compared to Ordinary Level. It also appears that fewer tasks at Higher Level are classified as 'not novel' when compared to the Ordinary Level textbooks. At Higher Level, the Active Maths series has the greatest proportion of both 'novel' and 'not novel' exercises, the Text and Tests series has the highest overall number of 'somewhat novel' tasks. At Ordinary Level, Concise Maths has the greatest proportion of tasks in both the 'novel' and 'somewhat novel' categories, while Active Maths has the most 'not novel' at this level. Overall these findings for novelty would indicate that students are not getting sufficient exposure to non-routine situations when applying their skills. O’Keeffe and O’Donoghue (2012), in their review of Project Maths textbooks, classified 71.4% of textbook tasks analysed at senior cycle as routine and 28.6% as non-routine. It would appear that the results for the ‘not novel’ category are similar to those for the routine classification. These results appear to support the earlier view outlined in O’Keeffe and O’Donoghue (2009) that mathematics textbooks in the Irish system promote “retention and practice” (p. 290). If teachers are to remain dependent on such textbooks as a source of classroom tasks then it is likely that the much criticized ‘drill and practice’ style of teaching will continue to be a feature of Irish mathematics classrooms. It is important to avoid the solution process becoming too routine and the over use of familiar tasks deprives students of opportunities to explore concepts or develop the skills necessary for solving non-routine problems. Lithner (p. 273, 2008) reminds us that while it is possible to get far with these algorithmic reasoning strategies, it leads to a search for algorithms becoming ‘mathematics instead of being a part of it’. The ERC in its report on the results of PISA 2012 calls on teachers to incorporate more novel tasks for higher-achieving students to solve, in particular to provide students with ‘opportunities to engage with problems in novel contexts and to explore different solutions to problems (Perkins and Shiel, 2016, p. 12).

It would not be possible or indeed desirable for all tasks to be novel as students would not get the opportunity to develop necessary procedural skills. In 2001, an expert U.S. committee reviewed and synthesised relevant research on mathematics education from pre-kindergarten through grade 8 and published a report named ‘Adding It Up’ on how teaching, curricula, and teacher education should change to improve mathematics learning during these critical years. Kilpatrick, Swafford and Findell (2001) in this report view pitting skill against understanding as a ‘false dichotomy’ and instead see the two as interwoven and state that procedural fluency is necessary for ‘learning many mathematical concepts with understanding and using procedures can help to strengthen and develop that understanding’ (p. 112). However a greater degree of novelty in the textbooks would be beneficial for promoting greater conceptual understanding.

7.1.3 RQ 1(c): What is the Level of Cognitive Demand of the Tasks Analysed on the Topics Contained in the Textbook Chapters?

In the textbook chapters analysed, a much greater proportion of exercises were classified as having a Lower Level of cognitive demand than having a Higher Level of cognitive demand, with most tasks involving the use of procedures without connection to meaning. The Concise Maths series has the greatest proportion of such exercises at Higher Level, this is mirrored by the Text and Tests series at Ordinary Level. There is a greater share of Higher Level demand exercises to be found in the Higher Level textbooks when compared with the Ordinary Level textbooks within each respective textbook series. Of the Higher Level demand exercises, 'procedures with connection to meaning' were far more common than the category of 'doing mathematics'. When comparing pre-Project Maths textbook series with post-Project Maths textbook series, a significant increase in the proportion of Higher Level demand tasks can be observed between eras. The NCCA (2005) raised concerns about an emphasis on developing procedural skills in Irish mathematics classrooms and that this was reinforced by the teaching taking place. It would appear that these textbooks will perpetuate the issue to some degree, should teachers remain dependent on the textbook as the primary source of tasks. More recently, when commenting on the implications of the PISA 2016 findings for teaching and learning in Ireland, Perkins and Shiel (2016) recommended that teachers could raise students' cognitive engagement in mathematics by allowing students to decide on their own procedures when solving problems. Moreover they suggest that when teachers assign procedural tasks to students, it should be ensured that students understand the underlying concepts. The textbook tasks analysed in the study reported here would not offer a wide selection for teachers to choose from as 83.3% of tasks were classified as requiring a low cognitive demand.

The situation is not confined to second level; Charalambous et al. (2010) undertook a study of the topic of addition of fractions in primary textbooks and found that the majority of tasks (85%) in two Irish primary school mathematics textbooks required a low level of cognitive demand. The predominance of textbook tasks requiring a low level of cognitive demand is similar to the findings of Jones and Tarr (2007) for American Middle school textbooks (83% LLD for the most recent popular textbook series analysed), Kim (2014) for Korean post primary textbooks (94% LLD) and unlike Bayazit (2013) (25% LLD) for Turkish elementary school textbooks. My results are similar to that of Jones and Tarr (2007) and Charalambous et al. (2010) but the Irish textbooks on average have a higher proportion of tasks requiring a high level of cognitive demand than Kim (2014). Bayazit (2013) has a substantially higher proportion of HLD tasks than all other textbooks analysed using the LCD framework contained in this study.

The results of my research in relation to the levels of cognitive demand required for task completion are disappointing because they suggest that students are not being sufficiently challenged when completing textbook tasks. It could be argued that teachers may use these tasks as a starting point and adjust them in such a way that they are more demanding cognitively. However, it has been found that when teachers are implementing tasks in the classroom, they are more likely to maintain or reduce the cognitive demand of a task rather than increase it (Stein, Smith, Henningsen, & Silver, 2009; Charalambous et al., 2010). Similarly Nie et al. (2013) caution that lessons, that were intended to have a high level cognitive demand, can often be implemented with a much lower level than originally planned. Their study also points out that teachers are more likely to follow the textbook when implementing a reformed curriculum than colleagues following an older more traditional one. This would suggest that Irish teachers coming to terms with the Project Maths curriculum may be quite dependent on textbooks and thus would likely struggle to find tasks for students with a high level of cognitive demand.

7.1.4 RQ 1(d) What Kind of Reasoning do Students Need to Use when Completing These Tasks?

Another question addressed by the analysis refers to the type of reasoning that students need to use when completing tasks. The vast majority of the exercises classified in terms of mathematical reasoning were found to require the use of imitative and algorithmic reasoning. Out of the three textbook series at Higher Level, Concise Maths has the greatest proportion of such tasks and the Active Maths series has the greatest percentage of tasks requiring creative reasoning. At Ordinary level, the Active Maths series again offers the greatest opportunity to engage in tasks requiring creative reasoning while the Text and Tests series has the least. The ratio of creative reasoning to imitative reasoning tasks differs between the Higher and Ordinary Level textbooks with a greater occurrence of opportunities for creative reasoning in the Higher Level textbooks.

These findings, showing the high number of tasks classified as requiring algorithmic reasoning, would suggest that students are not getting sufficient exposure to tasks that would help them attain the skills necessary for solving non-routine problems. Such an emphasis on algorithmic reasoning is also unlikely to encourage the development of conceptual understanding. Boesen, Lithner and Palm (2010, p. 89) remarked, when looking at assessment tasks that the dependency they observed on imitative reasoning 'seems insufficient for enhancing the learning of more mathematically founded reasoning , for problem-solving and for the attainment of conceptual understanding'. Moreover, as students move through the material on the syllabus, the number of algorithms required for solving tasks grows unwieldy and might prove unmanageable. This view is shared by Bergqvist et al. (2008) who point out that as a student moves through 'secondary and perhaps into tertiary mathematics, the mere number of algorithms may at some stage be insurmountable' (p. 11). The OECD has called on teachers to avoid this situation and recommended that they should encourage their students to 'think more deeply about what they have learned and make connections with real-world problems' (OECD, 2016, p. 38).

The results of my analysis indicate that 89% of the tasks from the three Irish mathematics textbook series could be solved using imitative reasoning, this suggests that it is important to design tasks that would promote creative reasoning so as to move beyond surface considerations and get a better grasp of the concepts involved. Jader et al. (2015), using the Lithner framework, analysed post-primary mathematics textbooks from 12 different countries including Ireland. Their results show that an average of 79 % of tasks analysed in total required imitative reasoning. The Irish tasks came from two Ordinary Level Active Maths textbooks, and were on the topics of geometry and algebra; Jader et al. (2015) found that just 4% of these tasks necessitated creative reasoning. Of the tasks taken from Irish textbooks in their study which were classified as necessitating imitative reasoning, 10% were found to require memorised reasoning and 86%

algorithmic reasoning. The results of my analysis for the same textbook series on the topics of Differential Calculus and Pattern, Sequences and Series had 11.7% of tasks classified as requiring creative reasoning, along with 88.3% categorised as algorithmic reasoning, and no tasks were found to require memorised reasoning. Although the proportion of CR tasks is still relatively low, these topics appear to offer more opportunities for engagement with creative reasoning than those studied by Jader et al. (2015).

My results are similar to Lithner's (2004) findings on undergraduate exercises (10% CR) although lower than Bergqvist et al.'s (2008) findings on upper secondary school tasks (30% CR), which show that mathematically well-founded reasoning is not common. Sidenvall et al. (2015), echoing the results of Jader et al. (2015), in their analysis of students' textbook task-solving in Swedish upper secondary school, found that 80% of all attempted tasks could be correctly solved using imitative reasoning strategies. The dominant reasoning type in the Swedish textbook tasks that they analysed is algorithmic reasoning, where students need to remember a suitable algorithm. The results of my classification using the reasoning framework are in agreement with those discussed previously in section 2.5 in relation to Lithner's framework.

7.1.5 RQ 1 (e) What Kind of Understanding (Using Usiskin's Dimensions) is Being Promoted in These Textbook Tasks?

Usiskin's model of mathematical understanding framework describes the kind of understanding that students encounter when learning mathematics. As a framework for this analysis, it is used to consider the understanding that is encountered when completing textbook tasks. The skill-algorithm dimension dominates the tasks which were analysed for all of the textbooks. I found that the proof-property dimension was the least common in tasks from all Ordinary Level textbooks and two of the Higher Level textbooks. Slightly more importance appears to have been placed on proof in the Higher Level textbooks than in the Ordinary Level textbooks for the Active Maths and Text and Tests textbook series although the proportion of the tasks in the proof-property dimension was still quite low overall at both levels. This is similar to the results of Stylianides' (2009) study using American textbooks and Davis' (2013) findings in the Irish context. None of the textbooks analysed had any exercises corresponding to the history-culture dimension, which is similar to the findings of O'Keeffe and O'Donoghue's (2009) study of Irish junior cycle textbooks which was carried out before the introduction of Project Maths. Ju, Moon and Song (2016) argue for the importance of including the history of mathematics in textbooks and lessons. They point out that exposing students to the history of mathematics, emphasising its role in the development of civilisation, both humanises and demystifies the discipline. This may serve to motivate students' learning and improve their attitudes toward the subject. Jankvist (2009) suggests that teachers use history as a means to establish the connection between the cultural life of a certain period of time and mathematics, the benefits of this would be to show to students the rich dynamic culture that accompanies mathematics. Given the results of the analysis reported in this thesis, it would appear that the textbooks have neglected this aspect of tasks and teachers, interested in promoting this type of understanding, will have to look elsewhere to expose students to this aspect of mathematics. The representation-metaphor category is much more frequent at Ordinary Level when compared to Higher Level. It would also appear that the textbooks do not give enough attention to real life applications (through the use-application dimension), preferring the practising of skills and the use of algorithms.

The predominance of the skill-algorithm dimension indicates that there is a need to include tasks which involve other kinds of understanding. As mentioned previously, reasoning and proving appears to be neglected in the current textbooks and requires more attention if students are to establish proper mathematical foundations. The Chief Examiner's Report recommends that students should become more familiar with describing, explaining, justifying and providing examples. It noted that 'these are skills that are worth practising, as they will improve understanding' (SEC, 2016, p. 30).Kilpatrick et al. (2001) view reasoning and proving as an important aspect of adaptive reasoning, the capacity to think logically about the relationship between concepts and situations. They call for students to be able to 'justify and explain their ideas in order to make their reasoning clear, hone their reasoning skills and improve their conceptual understanding' (p. 129). Swan (2005, p. 21) also calls for students to be asked to 'explain, convince and prove' when engaged in mathematical activity, while Mason and Johnston-Wilder (2006, p. 76) point out that it is important to establish the status of assertions in mathematics as to whether they are 'always true, sometimes true, never true or are they still just conjectures?’. The low proportion of the representation-metaphor and use-application dimensions indicates that students are not currently encouraged to consider mathematical concepts in different ways and establish relationships between these concepts. The Chief Examiner’s Report encourages teachers to give students opportunities to practise solving problems involving real-life applications of mathematics. As part of this process, students should be asked ‘to construct algebraic expressions or equations to model these situations, and/or to draw diagrams to represent them’ (SEC. 2016, p.30). In a broader context, the OECD’s report on PISA 2012, recommends that teachers should encourage students to ‘think more deeply about what they have learned and make connections with real-world problems’ (OECD, 2016, p. 38). Furthermore, real life applications are an important consideration within the new curriculum (NCCA, 2012a, p.6) and more of such tasks need to be included, especially given the concerns that were raised in the consultation document, that predated the introduction of the Project Maths curriculum, published by the NCCA (2005) that there was ‘a poor application of mathematics in real-world contexts’ in post-primary mathematics education (p.7). A similar call is made in O’Keeffe and O’Donoghue (2011) when they identify real life applications as one of the areas in need of development so as to meet the expectations of the Project Maths syllabus. The final report on the impact of Project Maths on student learning in the initial pilot schools found that students ‘enjoy applying mathematics to real-life contexts and find this beneficial for their learning’ (Jeffes et al., 2013, p.25). O’Keeffe and O’Donoghue (2009) found that none of the textbooks in their study of Junior Cycle textbooks fostered an environment of understanding, emphasising instead mathematical skills proficiency. It would appear that this situation persists in the senior cycle textbooks that were analysed as part of this study. Overall it is necessary to move away from tasks reliant on algorithms and familiar procedures and promote tasks that give a more rounded understanding.

7.1.6 RQ 2 Is There a Difference Between Textbook Series?

The Active Maths series appears to have the greatest proportion of creative reasoning tasks at both Higher and Ordinary Level. This series also has the greatest proportion of tasks at higher levels of cognitive demand at Ordinary Level. At Higher Level, Active Maths has the greatest percentage of tasks classified as ‘novel’ and ‘not novel’, while Text and Tests has the highest number of tasks classified as ‘somewhat novel’. All three series have a high incidence of the skill-algorithm dimension at both Higher and Ordinary level in Usiskin’s dimensions of mathematical understanding. Of the remaining dimensions, the Active Maths series has the greatest incidence of the property-proof and representation-metaphor dimensions at Higher and Ordinary level. The Concise Maths series has the greatest amount of tasks corresponding to the use-application category at both Higher and Ordinary level. The Active Maths series at Higher Level appears to have the closest alignment to the Project Maths problem-solving curriculum, having the greatest proportion of tasks for 8 of the 10 objectives. At Ordinary Level, the picture is much more mixed with Concise Maths having the greatest proportion for 5 Project Maths problem-solving objectives.

Based on these results, the Active Maths series appears to be the better choice of textbook for Higher Level in terms of the opportunities it offers for creative reasoning and developing different dimensions of mathematical understanding, and meeting the expectations of the Project Maths problem-solving syllabus. At Ordinary Level, the Active Maths series also has more opportunities for creative reasoning and HLD tasks. It also gives more exposure to the mathematical understanding dimensions with the exception of use-application, but it does not align with the Project Maths problem-solving curriculum as well as the Concise Maths textbook. In comparison the Concise Maths series ranks lowest for level of cognitive demand and creative reasoning in its textbook tasks at Higher Level, while the same was found for the Text and Tests textbook series at Ordinary Level. The Ordinary Level Text and Tests textbook has a greater proportion of exercises classified as ‘not novel’, accompanied by the lowest incidence of the proof-property and representation-metaphor dimensions. It appears to be least aligned to the Project Maths problem-solving curriculum as judged by having the greatest incidence in just two objectives. Based on these findings, use of the Higher Level Concise Maths textbook and the Ordinary Level Text and Tests textbook would present the greatest challenges for supporting teaching, promoting learning and reaching the problem-solving goals. The other textbooks offer tasks that are more helpful but there is room for improvement in all areas. This supports the view of O'Keeffe and O'Donoghue (2011) that some of the textbooks are 'better aligned to Project Maths expectations than others' (p.21). They also point out that all the textbooks fall short of the standard required to support the expectations of the Project Maths syllabus.

The website belonging to the Project Maths Development Team cautions that 'there is no single textbook which can suit the learning needs of all students' (Project Maths, 2017). One of the challenges for teachers, in using textbooks currently, is to determine the extent to which tasks need to be supplemented by additional material. The Maths in Practice report noted that 'teachers still need support in developing and using appropriate tasks that exploit the connections between topics rather than planning for individual lessons that focus on isolated areas of mathematics in a linear fashion' (NCCA, 2014, p.8).

7.1.7 RQ3 Is There a Difference between the Pre-Project Maths and Post-Project Maths Textbook Series?

My analysis looked at textbook tasks in use both before and after the introduction of the Project Maths syllabus. For each of the frameworks, there was an improvement noted in the post-Project Maths textbooks. The proportion of tasks requiring creative reasoning and a high level of cognitive demand has increased since the introduction of the Project Maths syllabus. However, the number of tasks in the textbooks dependent on the use of familiar algorithms is still quite high. There was some increase in the proportion of tasks classified as 'novel' and 'somewhat novel', accompanied by a decline in the 'not novel' classification for the post-Project Maths textbooks. As might be expected a greater incidence of tasks addressing the Project Maths Syllabus problem-solving objectives was recorded in the post-Project Maths textbook with the exception of the two objectives involving using and devising 'appropriate mathematical models formulae or techniques to process information and to draw relevant conclusions', however the presence of tasks more closely aligned to the Project Maths Syllabus problem-solving objectives still remains low overall.

With Usiskin's multidimensional model, an increase was recorded in the three dimensions of Representation-Metaphor, Property-Proof and Use-Application yet a much greater incidence would be desirable. Despite modest improvements, it would appear that students would benefit from greater exposure to more varied tasks. The Chief Examiner's Report has recommended that students should become more familiar with the processes of description, explanation, justification and the provision of examples. It noted that 'these are skills that are worth practising, as they will improve understanding' (SEC, 2016, p. 30). The NCCA in its report responding to the debate on the Project Maths curriculum and its introduction called for more emphasis to be given to students engaging in problem-solving approaches and justifying or explaining their solutions (NCCA, 2012b, p. 18). Similarly the final report on the impact of Project Maths in pilot schools (Jeffes et al., 2013) observed that students are building up expertise with the use of procedures. The report also noted that students are problem-solving and making mathematical representations but to a lesser extent than the use of procedures. An absence of engagement with reasoning and proof, communicating mathematically, or making connections between mathematics topics was also observed. It would appear that the textbooks do not support this goal adequately and teachers will need to augment existing tasks to achieve it. The Chief Examiner's Report has acknowledged that the syllabus expectations are more ambitious than previously and that they are not necessarily easy to achieve; 'there has been a deliberate attempt to increase the emphasis on higher-order thinking skills. These are skills that students find difficult to master and teachers may find difficult to instil' (SEC, 2016, p. 9).

7.1.8 RQ4 Can Textbook Tasks be Designed to Better Meet the Goals of Project Maths?

In chapter 6, I reported on the design of a sample of 17 textbook tasks that could address the deficiencies identified by the results of my analysis. Of these, 3 were existing textbook tasks on the topic of Pattern, Sequences and Series that were augmented to increase the level of cognitive demand, to require more creative reasoning when solving the tasks and to also give more varied exposure to the different learning dimensions and Project Maths problem-solving objectives. The other 14 were original tasks that I designed with 6 on the topic of Pattern, Sequences and Series and the remaining 8 on Differential Calculus.

When the designed tasks were evaluated in the workshop, the participants agreed that the tasks presented were closely aligned to the Project Maths problem-solving objectives as intended and on the whole, looking at their classifications, the tasks did succeed in providing greater cognitive demand for those solving them. Given the results of my textbook analysis, teachers may have to look elsewhere when considering resources for the classroom. One option may be to augment existing textbook tasks or design new ones as I have done in chapter 6. This is something that would be encouraged by both Remillard, Herbel-Eisenmann and Lloyd (2011) and Davis and Krajcik (2005) as something that would be beneficial to teachers and their development. Knott, Olson, Adams and Ely (2013) have also demonstrated that once teachers are supported appropriately in the context of professional development, teachers can successfully create their own rich tasks for use in the classroom. This could in turn encourage teachers to reflect on their teaching practice and bring about changes to their methodologies in the classroom such as establishing more connections between mathematical concepts as experienced by Swan (2007). I have demonstrated that tasks can be both augmented and new ones designed to meet the needs of the reformed curriculum classroom, informed by the findings of my analysis. This is something that could be facilitated as part of the Maths in Practice group's (NCCA, 2014) recommendation that opportunities for exploration of design-based research as part of professional development for mathematics teachers should be explored.

From this experience, I learned that no single task can meet every criterion that is aspired to. For example, while it is possible to create a task that has a high level of cognitive demand and requires creative reasoning in the required solution method it is not possible to incorporate every learning dimension or problem-solving objective in one task. Thus it is important to take heed of Mason and Johnston-Wilder's (2004, p. 6) recommendation to use a 'mixed economy' of tasks in order to realise as many goals as possible.

Given the findings by Davis (2013) in relation to reasoning and proving, as well as my findings from the multidimensional learning framework and the Project Maths problem - solving objectives framework, I gave attention to designing tasks that required students to explore patterns, form conjectures, explain their observations and attempt to justify them mathematically. A lot of these were to mirror the actions of a mathematician, as described by Mason and Johnston-Wilder (2004). I also made use of Swan and Burkhardt's (2012) principles, especially to make the tasks of interest to students. This feature of task design was also recommended by Chapman (2013). I linked the tasks to using the internet, observations in nature, researching history and writing poetry in order to incorporate authentic contexts and encourage students to make them feel that they were useful and linked to other subjects that might be of interest to them. It was also important to try and vary the activity in the tasks while encouraging as much mathematical thought as possible. This often required careful scaffolding so that students could be guided towards the appropriate investigation of concepts. I was influenced by Watson and Thompson's advice here to structure the tasks 'so that the desired key idea is varied and learners can see this and the effects of such variation' (pg. 170) . Another concern was to take into account the different abilities of students that would be attempting the tasks and to ensure that all students could engage with the material presented.

7.2 Limitations

7.2.1 Textbook Task Analysis

Remillard (1999) places great importance on the textbook when teachers are deciding how to teach the curriculum. 'Textbooks offer a curriculum map that organizes mathematical topics into sections, each including specific concepts or skills. Teachers map the curriculum when they decide how or whether to use these structures' (Remillard, 1999, p. 334). However we are also reminded that no two classrooms are alike; Remillard (2005, p. 239) cautions that the 'process of using a mathematics curriculum guide is complex and dynamic and is mediated by teachers' knowledge, beliefs, and dispositions suggests that the decision to adopt a single curriculum in a school or district will not alone result in uniform mathematics instruction'.

Thompson and Senk (2014) explored the treatment of high school geometry in 12 teachers' classrooms using the same textbook. Their analysis suggests that there are several different factors that can impact on the material that is actually enacted in the classroom. Teachers make decisions on what material is covered or skipped based on personal circumstances. It is important to acknowledge that tasks available in a textbook are just one aspect of what is taking place in terms of teaching and learning; teachers can adapt and use them in different ways.

Hence, one limitation of the research reported here is that it cannot definitively comment on what is taking place in Irish mathematics classrooms. My research does not claim nor imply that textbooks are the only source of tasks that teachers use in the classroom, however such tasks do give some insight into what is available to teachers and what is commonly used in the Irish classroom. The Project Maths Development Team reminds us that most of the time in classrooms tends to be related to textbooks in some way. 'Most problems for student's classwork and homework are taken from them' (Project Maths, 2017). Even without observation of classrooms, my research complements the analysis by Jeffes et al. (p.5, 2013) which states that students have 'a good mastery of mathematical procedures and, to a slightly lesser extent, problem-solving and making mathematical representations'. The textbook tasks analysed certainly support the development of proficiency with mathematical procedures but do not offer the same opportunity for supporting problem-solving or using different representations. Students interviewed for the report state they are getting exposure in the classroom to making connections between mathematics topics, as well as applying mathematics to real-life situations, something which is not discernible from an analysis of textbook tasks. This demonstrates a limitation of my study in that it is not possible to get a full picture of classroom activity based solely on the analysis of textbooks. Students may be given different tasks by the teacher or exposed to different activities not covered by the textbooks.

7.2.2 Time Constraints

Two workshops were conducted as part of this thesis. It is possible that more exposure, for the participants, to the frameworks involved would have proved advantageous. The availability of time had an impact on the conduct of the workshops. For the validation of the Novelty framework, (see chapter 4 for a full description) researchers with an interest in post-primary mathematics education were assembled and such a cohort is only available at certain times of the year due to their busy schedules. In the case of the tasks workshop, pre-service teachers volunteered time in the middle of their timetable. For both groups, this meant that there was a limited opportunity to introduce them to the frameworks involved and provide adequate training for conducting the classifications. It is likely that both groups would have benefitted from additional time and greater experience in using the frameworks but this was not possible given the workplace pressures involved.

7.2.3 Choice of Topics

The topics of pattern, sequences and series and differential calculus were selected for study. Every task from both the pre-Project Maths and the post-Project Maths eras for these three textbook series at both Higher and Ordinary Level were analysed. It could be argued that alternative topics may give different results, however these are the topics chosen are taught at both levels and involve the use of a number of different skills by the student. The Project Maths curriculum was introduced incrementally through three phases with pattern, sequences and series introduced in the first phase and differential calculus introduced in the last phase. The choice of these two topics is representative of the material that students and teachers encounter in the Irish mathematics classroom. It should also be noted that my results are similar to the findings of other studies yet different to others. My results are similar to Charalambous et al. (2010) and Jones and Tarr (2007) in relation to the level of cognitive demand. My results are different to Jader et al. (2015) in relation to the kind of reasoning that students employ when solving textbook tasks, whereby I found a greater requirement for creative reasoning and no tasks necessitating memorised reasoning. Also, my results from the novelty framework on my two chosen topics are similar to O'Keeffe and O'Donoghue's results on routine/non-routine problems and my results support their contention that the textbook tasks fall short of the standard required to realise the expectations of the Project Maths syllabus. Finally, Davis (2013) looked at the topic of complex numbers and found that there is a misalignment between the textbook tasks in relation to the opportunities afforded for reasoning-and-proving and the Project Maths syllabus, something that my textbook analysis also asserts.

Chapter 8 Recommendations

It must be acknowledged from the outset that the task analysis is unable to take into account how a teacher or student might use the textbook tasks. The recommendations that follow are being made on the assumption that the tasks will be used at 'face value', in the sense that they are assigned and completed as given in the original textbook.

8.1 Providing More Balance in Sets of Tasks

A key concern arising from my analysis is the lack of balance in the textbook tasks analysed. Swan and Burkhardt (2012) have suggested principles for task design suitable for use as assessment. These principles could also be used when designing tasks suitable for use in the classroom and/or for homework. They suggest that a balanced series of tasks should meet all the different goals and objectives that the curriculum aspires to. This is something that the textbook tasks do not currently achieve in relation to the Project Maths curriculum according to my analysis. It is also recommended that tasks should be viewed by students as something interesting or having a potential use outside of the classroom. The lack of Usiskin's History-Culture dimension and the relatively low incidence of the Use-Application dimension in my analysis would suggest that the Irish textbook tasks do not currently achieve this. Swan and Burkhardt also believe that tasks should be accessible to every student yet simultaneously challenging, thus enabling students of all different abilities to be able to demonstrate some level of understanding in relation to the task. Their principles call for reasoning to be rewarded rather than results, meaning that students should be encouraged to engage in a process of reasoning and understanding rather than just advance towards a final result. Designers are also advised to use authentic contexts as well as mathematical contexts, encouraging students to make connections within mathematics and other subjects. In the analysis reported here, the dominance of tasks requiring the use of procedures without connections to meaning and the dependence on algorithms would suggest that the sets of tasks in each textbook require more balance. Similarly, the results from the analysis using the Project Maths framework and Usiskin's multidimensional model point towards the need for the employment of more authentic contexts in order for students to appreciate the mathematics that can be found in other subjects and the surrounding world.

Watson and Thompson (2015) also recommend the inclusion of more authentic contexts for the study of mathematics. In their chapter on design issues related to text-based tasks, they discuss mathematical authority and empowering students by allowing them to self-check their work through the processes of making observations, justifying conjectures or verifying solutions. The results of my analysis for the Project Maths problem-solving framework, in conjunction with the LCD and Lithner's reasoning frameworks suggest that the textbook tasks do not offer students enough opportunities to engage in such activity. In particular, the tasks analysed appear to be deficient of processes involving the explanation of findings, the justification of conclusions and the formulation of conjectures.

Based on the findings of my analysis, the tasks available in the textbooks appear to be unbalanced as explained on the previous page and the tasks would benefit students more by giving a broader, more engaging and dynamic experience of mathematics as a discipline. I would recommend that tasks be designed that incorporate the history of mathematics as well as other subject areas that would be of interest to students. Real life applications should relate to the world of students as much as possible in an effort to be as authentic as possible to their experience. It is also important that all students, despite individual ability, are able to engage with the tasks presented in the textbooks. To achieve this, it will be necessary to have a good mixture of tasks, in order to appeal to the different interests and abilities that may be present in classrooms and facilitate differentiated learning.

8.2 Making Greater Use of Unfamiliar Contexts and Novel Tasks

Given my findings using the 'novelty' framework and the Project Maths problem-solving objectives framework, more attention needs to be paid to solving problems in unfamiliar contexts and to avoid the overuse of repetitive well-rehearsed task types. My results show that the majority of tasks are 'not novel' and I observed a greater proportion of tasks involving the 'application of knowledge and skills to solve problems' in familiar contexts rather than unfamiliar ones. A task can be initially complex and require a process of creative reasoning when considering its solution. However if several similar such tasks are encountered, then the novelty is diminished and the reasoning employed moves from being creative towards imitative. If a context is overused, then it is no longer unfamiliar. I would recommend that teachers currently using the textbooks analysed in this study, or considering their use, to supplement tasks with a greater number of unfamiliar contexts in order to achieve the desired variety. Furthermore, it would be beneficial to students if teachers ensured that a greater proportion of both 'novel' and 'somewhat novel' tasks were included in classwork and homework.

8.3 Giving More Attention to Reasoning-And-Proving

The area of reasoning-and-proving is worthy of attention, particularly considering the results of the analysis presented here using the LCD framework, Lithner's reasoning framework, the Project Maths objectives and Usiskin's model of mathematical understanding. Reasoning-and-proving can be found in the category of 'doing mathematics' in the LCD framework, in creative reasoning for Lithner's reasoning framework, in the property-proof dimension for Usiskin's model and in the explain findings and justify conclusions for the Project Maths problem-solving objectives. The incidence of such tasks in the analysis was low. Lithner (2008, p.273) points out that 'conceptual understanding, creative reasoning, and insights into the central roles of mathematics in our society are not enhanced by rote learning'. An overreliance on imitative reasoning leads to students becoming dependent on algorithms and memorisation and this leads to a consequent inability to solve unfamiliar problems or to transfer mathematical knowledge to other areas competently and appropriately. Without being encouraged to use creative reasoning more regularly, for example by being required to find different methods of solution on their own, students might build up the incorrect impression that there is only one possible method of solution when approaching mathematical tasks. This could deprive them of the opportunity to become more flexible in their approach to solving tasks.

Another result of neglecting creative reasoning opportunities in favour of imitative reasoning is that students will not gain an appreciation of mathematics and the potential for its application in different areas. I found that there was a low incidence of tasks classified in the property-proof dimension of Usiskin's framework and in general, very few of the tasks classified required the explanation of findings or the justification of conclusions. These results are also supported by both Jader et al.'s (2015) and Davis' (2013) analyses. It appears that more tasks are required in order to encourage students to engage in creative reasoning, explain findings, justify conclusions and communicate their mathematical thoughts. Jeffes et al. (2013) also highlight this when they call for tasks that involve students in 'communicating more effectively in written form; and justifying and providing evidence for their answers'. In fact, they called for high quality tasks 'to engage with the processes promoted by the revised syllabuses, including: problem-solving; drawing out connections between mathematics topics' (p.32). Teachers will have to take care in the classroom, if not doing so already, to encourage students to verify their solutions, solve tasks using several methods and to explain their mathematical thinking when completing tasks.

8.4 Increasing the Level of Cognitive Demand in Tasks

Based on my findings, I believe that there is a need for the inclusion of more cognitively demanding tasks in textbooks in order to stimulate students' thinking. My findings suggest that there is a dependency on procedures in the current textbooks, which provide very little connection to the underlying concepts. This gives cause for concern. Given the emphasis placed on conceptual understanding in the revised curriculum, it is important that more tasks which establish a connection to the meaning behind procedures are provided in textbooks and employed by teachers in the classroom. Similarly, if students are to be able to problem solve effectively then they need more exposure to tasks which correspond to the 'doing mathematics' category. This category is not dependent on procedures and encourages complex mathematical thought, including reasoning activities such as forming and verifying conjectures, searching for patterns and investigating the parameters of problems.

Henningsen and Stein (1997, p. 525) have cautioned that 'the nature of tasks can potentially influence and structure the way students think and can serve to limit or broaden their views on the subject matter with which they are engaged'. If a student works predominantly with tasks that have a low level of cognitive demand then several issues arise. Students without exposure to tasks with a high level of cognitive demand risk being unable to transfer their mathematical skills to other subject areas and being unable to problem solve. Also, it is likely that they will suffer with confidence about the subject and only be comfortable with familiar material. Any learning that takes place is likely to be reliant on memorising operations, that will ultimately be forgotten over time and students will gain little understanding of the underlying meanings.

Jeffes et al. (2013, p. 32) suggest that 'students need to be regularly given high quality tasks that require them to engage with the processes promoted by the revised syllabuses'. The results of my analysis suggest that the textbooks examined do not currently promote enough complex and non-algorithmic thinking. This in turn makes students dependent on a predictable solution process and they are unable to discover relationships between concepts. Teachers in Irish classrooms should be made aware of such pitfalls when choosing a textbook to support teaching and learning. A higher proportion of textbook tasks with a higher level of cognitive demand would encourage students to explore the nature of mathematical concepts and provide opportunities to become more confident at analysing problems, identifying potential constraints and discovering different approaches to their solution. If teachers do not expose students to tasks requiring more complex thought then difficulties may persist with problem-solving and students will be deprived of opportunities to broaden their understanding of mathematics. It would be beneficial to have textbooks with a greater number of tasks that require a higher level of cognitive engagement. I would recommend that tasks such as those that I designed in chapter 6 could be included in textbooks or classroom activities as a means to address this deficit. Very often, existing textbook tasks can be extended or adjusted in order to raise the level of cognitive demand required. If teachers could augment existing tasks to increase the level of cognitive demand then they would be less dependent on using tasks directly from the textbook. Augmenting tasks could involve requiring students to examine patterns, formulate conjectures, expound on their observations and justify such observations mathematically.

8.5 Availing of Greater Expertise When Designing Textbook Tasks

From my experience, I would recommend that publishers and authors should give more attention to the design of textbook tasks in Ireland. This echoes Lubienski's (2011, p. 47) concern as to 'how a teacher can focus on both teaching full time and authoring a textbook, particularly one that needs to be completed within a year'. From my experience with textbook analysis and the designing of tasks, it is clear that the process of creating a textbook would benefit from the input of researchers in mathematics education. Lubienski (2011, p. 47) suggests that 'both mathematics education scholars and mathematicians have a greater role to play in at least reviewing books, if not actually co-authoring them'. Not only would this build in an element of quality assurance, but it would also assist with the creation of tasks that encourage students to think mathematically. I would also recommend that there should be some input from the primary sector. Greater contributions from primary and tertiary practitioners could ensure that the textbook tasks build on the experience of students garnered before entering post-primary school and assist with greater preparation for the demands of third level. Geiger et al. (2014) note that partnerships between teachers and researchers hold potential for improving teaching and learning practices in mathematics, especially through the design of tasks for use in the classroom. I believe that this could also apply to designing tasks for inclusion in mathematics textbooks.

One of the main recommendations in the report by O'Keeffe and O'Donoghue (p.21, 2012) is that an exemplar textbook series for Project Maths be produced by 'a specially selected and constituted writing team' appointed and funded by the Department of Education and Skills. They suggest that this exemplar textbook series would then be used as a model for all commercially produced textbooks and any new textbook should be reviewed by the DoES against this series leading to an approved list of mathematics textbooks for the Project Maths classroom. Lubienski has noted that the issue of textbooks in Ireland appears to be politically sensitive. She adds that Project Maths leaders appear to be unwilling to comment on the suitability of textbooks and appear to be 'circumventing textbooks as opposed to leveraging them' (Lubienski, p.45). I believe that the current situation of practicing teachers writing textbooks could be extended or supported by the involvement of people with additional expertise. Valverde et al. (p, 2, 2002) have suggested that textbooks act as 'mediators between the intentions of the designers of curriculum policy and the teachers that provide instruction in classrooms'. Houang and Schmidt (p. 3, 2008) point out that in many countries students may not be aware of the existence of curriculum guides and teachers may not make regular use of them. Yet, evidence from TIMSS suggests that textbooks were present in almost every classroom in the countries that participated and were regularly used in instruction. As a result, while textbooks were often not officially national in character, they were a common element in most classrooms. This shows the importance of the textbook in terms of implementing a curriculum. Thus, I think that more engagement with textbook publishers and authors is required in light of the issues that have been highlighted in my analysis and elsewhere.

The creation of an expert team to write a textbook series could certainly further the aims of implementation of the Project Maths curriculum and support teachers in their efforts to change their practices. It must be acknowledged that the textbooks analysed here were produced to meet an extremely tight deadline while the new curriculum was being introduced on a phased basis. Now that Project Maths has had an opportunity to become more embedded, it is possible to consider how best textbook tasks should be produced in the future.

8.6 Establishing Professional Development to Support Teachers in Analysing/Creating Tasks

Swan (2007) has shown that a programme of professional development employing carefully designed tasks and supported by relevant guidance could enable teachers to re-examine their beliefs in relation to the teaching and learning of mathematics. Over time a general shift by participating teachers away from transmission practices and towards a more student centred model was reported in his study. This was not due to the tasks alone but the fact that the tasks used allowed for such things as sustained collaborative work or the use of more challenging examples. Teachers, in this study, also importantly moved away from emphasising the completion of tasks and instead focused on 'comprehending' tasks by appreciating what opportunities such tasks offered to students and how they could be beneficial for their learning and building an appreciation of mathematics (Swan, 2007). Arbaugh and Brown (2005) believed that classifying exercises in terms of their cognitive demand would enable teachers to examine their own practice. Stein et al. (2009) prepared a case book for use in professional development that included a number of exemplary tasks, criteria for analyzing task properties, and several protocols to facilitate the discussion of tasks among teachers. Further work completed using such an approach (Boston and Smith 2009; Boston 2013) has identified changes in teachers' choice of task for use in the classroom after completing professional development in relation to classifying tasks in terms of their level of cognitive demand. It also found that some teachers maintained this change after a period of time had elapsed (Boston and Smith 2011).

Given my experience with the workshop to evaluate the designed textbook tasks, I believe that a programme of professional development whereby teachers were made familiar with the Levels of Cognitive Demand framework would be useful in Ireland. Teachers could be exposed to tasks with varying levels of cognitive demand. The participants at the workshop, detailed in chapter 6, described the potential benefits of such an exercise in terms of reflection and planning. However, such a programme could possibly empower teachers to adjust existing tasks or even design tasks in order to offer students more challenging opportunities. This would have to be supported by exposure to the principles and frameworks of task design, so that appropriate tasks were constructed. Over time, this could be extended to the other frameworks used in this analysis. Such professional development, if successful, could be used to build up a bank of tasks that could be shared with teachers across the country.

The report of the 'Maths in Practice' group (NCCA, 2014) recommended the exploration of opportunities for design-based research for Irish mathematics teachers. They suggested a model of professional development where teachers would design and conduct interventions in the classroom. If teachers were provided with such opportunities for professional development in the future, I would recommend the design of tasks as described in chapter 6 as a possibility that could prove beneficial to teachers.

8.7 Advice for Individual Teachers

Given the results of the textbook analysis reported here, consideration should be given to what an individual teacher might do in order to support the teaching and learning taking place in his/her classroom. It would appear that no one textbook studied here meets all the needs of the students and I am mindful of the fact that to augment or design tasks can take a substantial amount of time. The Project Maths Development Team has made resources available on its website but these are limited and might not always address the topic area that the teacher is about to teach. Similarly looking for resources online might not always yield fruitful results.

My analysis of the three textbook series has found that there are deficiencies in each individual textbook. I would recommend that teachers build up their own bank of tasks by choosing the best tasks from a variety of textbooks. In this way, it will be possible to give the student an array of good quality tasks quickly. This is important to retain the engagement of students who wish to be challenged or may wish to work ahead independently. Given that the supply of textbook tasks in any single textbook necessitating novel creative reasoning and/or requiring a high level of cognitive demand can be exhausted quickly, it would be wise to combine such tasks from as many textbooks as possible to ensure rich learning opportunities for students. Attention should also be given to ensuring that a mixture of tasks is assigned to students so that several learning dimensions are encountered and the desired problem-solving objectives are met in the course of solving tasks. It is important that teachers are conscious that while attaining mastery of procedural skills is beneficial, it must also be accompanied by tasks that require creative mathematical thinking that encourage students to solve problems in unfamiliar contexts.

8.8 Advice for Researchers

Five frameworks were used in this thesis. The Levels of Cognitive Demand framework has been readily used internationally as part of textbook content analysis. Lithner's reasoning framework is less common outside of Sweden but has also been used for textbook analysis. The other three frameworks are unique to my research currently. Usiskin's multidimensional model of mathematical understanding was not intended for analysing textbook tasks but has provided valuable data. Similarly, the Project Maths problem-solving objectives framework was not intended for such analysis as it was adapted from Irish syllabus documents. Finally, the 'novelty' framework was created given my experiences with the LCD and Lithner's reasoning frameworks and a desire to measure students' familiarity with tasks. Having gathered considerable experience working with frameworks, I would recommend that researchers engaged in the classification of tasks build in opportunities for reflection and certainly ensure that they work with others before finalising classifications. It is important that researchers realise that they carry 'baggage' in the sense that they hold views which might influence their decisions on classifications. It is important that they reflect deeply on their decisions and when deciding on classifications, they do so consistently and apply the criteria as accurately as possible. As a failsafe, I would recommend having at least one other researcher classify a sample of the tasks independently so that you can compare classifications and discuss decisions in order to reach consensus on how the framework is used.

Each of the frameworks gives a different insight into the textbook tasks that were analysed. Of the five, I believe that the Level of Cognitive Demand framework is the easiest to use for someone wishing to begin textbook task analysis. Unlike some of the other frameworks, the categories are mutually exclusive and this would lend itself to being more straightforward in terms of application. Gaining an understanding of the categories of 'doing mathematics', 'procedures with connection to meaning', 'procedures without connection to meaning' and 'memorization' should not be overly challenging for someone embarking on this type of research while building up expertise in distinguishing between the categories would be established over time.

8.9 Future Research

The analysis of tasks looked at the three textbook series that were widely used in Irish post-primary schools at Higher and Ordinary Level. Another series named Effective Maths was published in 2014 but it is only available at Higher Level; the series was in use during the pre-Project Maths era under the name Maths. In 2016, a further series entitled Power of Maths was launched at Higher and Ordinary Level. This series was not in use before the introduction of the Project Maths syllabus. It would be interesting to analyse tasks from these two textbook series using the five frameworks to examine whether any difference exists between the current analysis and these newer series.

Teachers often use past examination questions as a source of tasks. It would be worthwhile to use the frameworks to establish whether the findings for the assessment tasks are similar to the textbook tasks. Given the nature of the examination tasks, it would be necessary for some adaption to be completed in order to use the frameworks effectively. For instance, the Novelty framework would not have exemplary material or modelled solutions but instead would look at the amount of repetition of previous material. The degree of 'novelty' would be determined by the occurrence of assessment tasks and how similar they are to the preceding years. Decisions would also have to be made as to what constitutes familiar and unfamiliar contexts when considering the Project Maths problem-solving objectives. As with the textbooks, the examination papers in use during the pre-Project Maths years could be compared to the post-Project Maths era.

8.10 Other Work

The use of the five frameworks enables the gathering of very specific details as to the nature of mathematical tasks and the understanding that students gain from their completion. Teachers could benefit from exposure to these frameworks as a means to reflect upon their practice in the classroom. I would prepare workshops that would introduce them to the classification of tasks using a particular framework and then follow up by outlining the principles of task design so that they would be empowered to design tasks for use in the classroom. By making teachers aware of perceived areas of deficiency and familiarising them with methods to address such shortcomings, teachers would be able to produce high quality tasks without being dependent on others. I would also research the effects of these interventions and what kind of impact that they had on the practice of teachers, as well as examining the nature of the tasks that they would produce.

In the area of task design, I would like to undertake to extend the work completed on the sample tasks into the creation of a model textbook chapter. This would allow the perceived deficiencies identified by the task analysis presented here to be addressed directly. My aim would be to develop tasks which move away from the current focus on procedural, well-practised exercises and instead provide more opportunities for creative reasoning and engagement with unfamiliar problems. Such a chapter could be used as an exemplar for textbook publishers when considering what kind of tasks would be beneficial in future publications.

Bibliography

Ahmed, A. (1987). *Better mathematics: A curriculum development study*. London: HMSO.

Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal*, *32*(1), 23-38.

Alafaleq, M., & Fan, L. (2014). Problem solving heuristics in middle school mathematics textbooks in Saudi Arabia. In Jones, K., Bokhove, C., Howson, G., & Fan, L (Eds). *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014)*. (pp. 121-126): University of Southampton.

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the West. *Journal of Mathematics Education*, *2*(2), 147-164.

Antoine, J. Y., Villaneau, J., & Lefeuvre, A. (2014). Weighted Krippendorff's alpha is a more reliable metrics for multi-coders ordinal annotations: experimental studies on emotion, opinion and coreference annotation. In *The 14th Conference of the European Chapter of the Association for Computational Linguistics (EACL 2014).*

Arbaugh, F., & Brown, C. A. (2005). Analyzing mathematical tasks: a catalyst for change? *Journal of Mathematics Teacher Education*, *8*(6), 499-536.

Artstein, R., & Poesio, M. (2008). Inter-coder agreement for computational linguistics. *Computational Linguistics*, *34*(4), 555-596.

Askew, M., & Canty, L. (2013). Teachers and researchers collaborating to develop teaching through problem solving in primary mathematics. In C. Margolinas (Ed.), *Task design in mathematics education: Proceedings of ICMI Study 22*, (pp. 531–540): Oxford.

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is: Or might be: The role of curriculum materials in teacher learning and instructional reform? *Educational researcher*, *25*(9), 6-14.

Bayazit, I. (2013). Quality of the Tasks in the New Turkish Elementary Mathematics Textbooks: the Case of Proportional Reasoning. *International Journal of Science and Mathematics Education*, 11 (3), 651-682.

Beaton, A. E., Martin, M. O., Mullis, I. V. (1997). Providing data for educational policy in an international context: The third international mathematics and science study (TIMSS). *European Journal of Psychological Assessment*, 13(1), 49- 58.

Ben-Peretz, M. (1990). *The teacher-curriculum encounter: freeing teachers from the tyranny of texts*. Albany: State University of New York Press.

Bergqvist, T., Lithner, J., & Sumpter, L. (2008). Upper secondary students' task reasoning. *International journal of mathematical education in science and technology*, *39*(1), 1-12.

Berry, J., Johnson, P., Maull, W. & Monaghan J. (1999). *Routine questions and examination performance*. In *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*, ed. O Zaslavsky, 23(2), 105-112. Israel Institute of Technology

Boesen, J., Lithner, J., & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational studies in mathematics*, *75*(1), 89-105.

Boston, M. D. (2013). Connecting changes in secondary mathematics teachers' knowledge to their experiences in a professional development workshop. *Journal of Mathematics Teacher Education*, *16*(1), 7-31.

Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education*, 119-156.

Boston, M. D., & Smith, M. S. (2011). A 'task-centric approach'to professional development: Enhancing and sustaining mathematics teachers' ability to implement cognitively challenging mathematical tasks. *ZDM*, *43*(6-7), 965-977.

Brousseau, G. (2006). *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970–1990* (Vol. 19). Springer Science & Business Media.

Brown, M. W. (2009). The teacher-tool relationship. *Mathematics teachers at work: Connecting curriculum materials and classroom instruction*, 17-36.

Byrt, T., Bishop, J., & Carlin, J. B. (1993). Bias, prevalence and kappa. *Journal of clinical epidemiology*, *46*(5), 423-429.

Carley-Baxter, L.R. (2008). Codebook. In Lavrakas, P.J. (Ed.), *Encyclopedia of Survey Research Methods* . Thousand Oaks, CA: Sage Publications, Inc.

Chapman, O. (2013). Investigating teachers' knowledge for teaching mathematics. *Journal of Mathematics Teacher Education*, *16*(4), 237-243.

Charalambous, C. Y., Delaney, S., Hsu, H. Y., & Mesa, V. (2010). *A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. Mathematical Thinking and Learning,* 12(2), 117-151.

Cicchetti, D. V., & Feinstein, A. R. (1990). High agreement but low kappa: II. Resolving the paradoxes. *Journal of clinical epidemiology*, *43*(6), 551-558.

Clarke, B., Grevholm, B., & Millman, R. (2009). *Tasks in primary mathematics teacher education*. New York: Springer.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement*, 20, 37–46.

Cohen, J. (1968). Weighted kappa: Nominal scale agreement with provision for scaled disagreement of partial credit. *Psychological Bulletin*, 70, 213–220.

Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational evaluation and policy analysis*, *25*(2), 119-142.

Coles, A., & Brown, L. (2016). Task design for ways of working: making distinctions in teaching and learning mathematics. *Journal of Mathematics Teacher Education*, *19*(2-3), 149-168.

Conway, P. & Sloane, F. (2005). I*nternational Trends in Post Primary Maths Education: Perspectives on Learning*, Teaching and Assessment, NCCA, Dublin.

Cooke, P., Morris, O.D. & O'Regan, F. (2012). *Text and Tests 6,* The Celtic Press, Dublin.

Cosgrove, J., Shiel, G., Sofroniou, N., Zastrutzki, S. & Shortt, F. (2005). *Education for Life: The achievements of 15-year-olds in Ireland in the Second Cycle of PISA*, Dublin: Educational Research Centre.

Cosgrove, J., Perkins, R., Shiel, G., Fish, R., & McGuinness, L. (2012). *Teaching and learning in Project Maths: Insights from teachers who participated in PISA 2012*, Dublin: Educational Research Centre.

Davis, E. A., & Krajcik, J. S. (2005). Designing educative curriculum materials to promote teacher learning. *Educational researcher*, *34*(3), 3-14.

Davis, J. D. (2013). Examining reasoning-and-proving in the treatment of complex numbers in Irish secondary Mathematics textbooks. *Bulletin of the Irish Mathematical Society*, 71, 41-57.

De Araujo, Z., & Singletary, L. M. (2011). Secondary mathematics teachers' conceptions of worthwhile tasks. In *Proceedings of the 33rd annual meeting of the North American* *chapter of the international group for the psychology of mathematics education*, (pp. 1207-1215).

DeCuir-Gunby, J. T., Marshall, P. L., & McCulloch, A. W. (2011). Developing and using a codebook for the analysis of interview data: An example from a professional development research project. *Field methods*, *23*(2), 136-155.

Department of Education and Science and National Council for Curriculum and Assessment. 2000. Mathematics Syllabus: Higher, Ordinary and Foundation Level. Dublin: Stationery Office.

Department of Education and Skills (2010). *Report of the Project Maths Implementation Support Group*. Dublin: Department of Education and Skills.

Doyle, W. (1983). Academic work. *Review of educational research*, *53*(2), 159-199.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist*, *23*(2), 167-180.

Dunphy, E. (2009). Do mathematics textbooks or workbooks enhance the teaching of mathematics in early childhood? : Views of teachers of four and-five-year old children in primary schools in Ireland. In D. Corcoran, T. Dooley, S. Close, R. Ward (Eds.), *Proceedings of Third National Conference on Research in Mathematics Education*, 114-122, Dublin: St. Patrick’s College.

Edwards, A. (2011) *Using example generation to explore undergraduates’ conceptions of real sequences: a phenomenographic study*. Ph.D. Thesis.

EGFSN (2008). *Statement on Raising National Mathematical Achievement*
Retrieved May 19, 2017, from
http://www.skillsireland.ie/media/egfsn081215_raising_mathematical_achievement.pdf

Ellis, J., Hanson, K., Nuñez, G., & Rasmussen, C. (2015). Beyond plug and chug: An analysis of calculus I homework. *International Journal of Research in Undergraduate Mathematics Education*, *1*(2), 268-287.

Elwood, J. & Carlisle, K. (2003). *Examining gender: Gender and achievement in the Junior and Leaving Certificate Examinations 2000/2001*. Dublin: National Council for Curriculum and Assessment.

Erlandson, D. A., Harris, E. L., Skipper, B. L., & Allen, S. D. (1993). Doing naturalistic inquiry: a guide to methods. Newbury Park, CA: Sage Publications, Inc.

Even, R., & Olsher, S. (2014). Teachers as participants in textbook development: the integrated mathematics wiki-book project. In *Mathematics curriculum in school education* (pp. 333-350). Springer Netherlands.

Fan, L. (2011). *Textbook research as scientific research: Towards a common ground for research on mathematics textbooks.* Paper presented at the 2011 International Conference on School Mathematics Textbooks, Shanghai.

Fan, L. (2013). Textbook research as scientific research: towards a common ground on issues and methods of research on mathematics textbooks. *ZDM*, *45*(5), 765-777.

Fan, L., Jones, K., Wang, J., Xu, B. (Eds.)(2013). Textbook research in mathematics education [special issue] *ZDM*, *45*(5), 633-777.

Fan, L., & Kaeley, G. S. (2000). The influence of textbooks on teaching strategies: An empirical study. *Mid-Western Educational Researcher,* 13(4), 2-9.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *International Journal of Science and Mathematics Education*, *45*(5), 633-646.

Feinstein, A. R., & Cicchetti, D. V. (1990). High agreement but low kappa: I. The problems of two paradoxes. *Journal of clinical epidemiology*, *43*(6), 543-549.

Feng, G. C. (2013). Factors affecting intercoder reliability: A Monte Carlo experiment. *Quality & Quantity*, 1-24.

Fleiss, J. L. (1971). Measuring nominal scale agreement among many raters. *Psychological Bulletin*, 76, 378–382.

Forfás (2009). *Expert Report on Future Skills Needs Statement of Activity 2008* Retrieved May 19, 2017, from http://www.skillsireland.ie/media/egfsn090616_statement_on_activity.pdf

Foster, C. (2015). The Convergent–Divergent Model: an opportunity for teacher–learner development through principled task design. *Educational Designer*, *2*(8), 1-25.

Gehrke, N. J., Knapp, M. S., & Sirotnik, K. A. (1992). In search of the school curriculum. *Review of research in education*, *18*, 51-110.

Geiger, V., Goos, M., Dole, S., Forgasz, H., & Bennison, A. (2014). Devising principles of design for numeracy tasks. In *Curriculum in focus: Research-guided practice: Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 239-246).

Gisev, N., Bell, J. S., & Chen, T. F. (2013). Interrater agreement and interrater reliability: key concepts, approaches, and applications. *Research in Social and Administrative Pharmacy*, *9*(3), 330-338.

Givvin, K. B., Hiebert, J., Jacobs, J. K., Hollingsworth, H., & Gallimore, R. (2005). Are there national patterns of teaching? Evidence from the TIMSS 1999 video study. *Comparative Education Review*, *49*(3), 311-343.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, *71*(3), 199-218.

Gueudet, G., Pepin, B., & Trouche, L. (2012). *From text to 'lived resources': curriculum material and mathematics teacher development*. New York: Springer.

Guven, B., Aydin-Guc, F., & Ozmen, Z. M. (2016). Problem types used in math lessons: the relationship between student achievement and teacher preferences. *International Journal of Mathematical Education in Science and Technology*, 1-14.

Gwet, K. L. (2008). Computing inter-rater reliability and its variance in the presence of high agreement. *British Journal of Mathematical and Statistical Psychology*, *61*(1), 29-48.

Gwet, K. L. (2011). On the Krippendorff's Alpha Coefficient Retrived on May 16, 2017 from http://www.agreestat.com/research_papers/onkrippendorffalpha_old.pdf

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: Who gets an opportunity to learn what?. *British Educational Research Journal*, *28*(4), 567-590.

Hallgren, K. A. (2012). Computing inter-rater reliability for observational data: an overview and tutorial. *Tutorials in quantitative methods for psychology*,*8*(1), 23-34.

Hamilton, E., Lesh, R., Lester, F. R. A. N. K., & Brilleslyper, M. (2008). Model-eliciting activities (MEAs) as a bridge between engineering education research and mathematics education research. *Advances in Engineering Education*, *1*(2), 1-25.

Harbison L. (2009). The use of Mathematics Textbooks to promote understanding in the lower primary years. In D. Corcoran, T. Dooley, S. Close, R. Ward (Eds.), *Proceedings of Third National Conference on Research in Mathematics Education*, 123-132: St. Patrick's College, Dublin.

Hayes, A. F., & Krippendorff, K. (2007). Answering the call for a standard reliability measure for coding data. *Communication methods and measures*, 1(1), 77-89.

Henningsen M. and Stein M. (1997). Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning, *Journal for Research in Mathematics Education*, 28 (5), 524-549.

Herbel-Eisenmann, B. A. (2007). From intended curriculum to written curriculum: Examining the" voice" of a mathematics textbook. *Journal for Research in Mathematics Education*, 344-369.

Herbst, P. (2008). The teacher and the task. In *Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 125-131).

Herbst, P., & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *ZDM*, *44*(5), 601-612.

Hiebert, J., Gallimore, R., Garnier, H., Bogard Givvin, K., Hollingsworth, H., Jacobs, J., Chui, A.M.Y., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C., Gonzales, P. & Stigler, J (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study*, Washington, DC: National Centre for Education Statistics.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American educational research journal*, *30*(2), 393-425.

Hodge, L. L., Visnovska, J., Zhao, Q., & Cobb, P. (2007). What does it mean for an instructional task to be effective? In *Proceedings of the 30th annual meeting of the MERGA* (Vol. 1, pp. 392-401).

Houang, R. T. and Schmidt, W. H. (2008). *TIMSS international curriculum analysis and measuring educational opportunities*. Paper presented at the 3rd IEA International Research Conference. Retrieved February 8th, 2014 from http://www.iea.nl/fileadmin/user_upload/IRC/IRC_2008/Papers/IRC2008_Houang_Schmidt.pdf

Hourigan, M., & O'Donoghue, J. (2007). Mathematical under-preparedness: the influence of the pre-tertiary mathematics experience on students' ability to make a successful transition to tertiary level mathematics courses in Ireland. *International Journal of Mathematical Education in Science and Technology*, *38*(4), 461-476.

Howson, G. (1995). *Mathematics Textbooks: A Comprehensive Study of Grade 8 Texts* Vancouver: Pacific Educational Press.

Hripcsak, G., & Heitjan, D. F. (2002). Measuring agreement in medical informatics reliability studies. *Journal of biomedical informatics*, *35*(2), 99-110.

Humphrey, G. (2003). *New concise Project Maths 3 for leaving cert ordinary level* Dublin: Gill and Macmillan Press.

Humphrey, G. (2003). *New concise Project Maths 4 for leaving cert higher level* part one Dublin: Gill and Macmillan Press.

Humphrey, G. (2004). *New concise Project Maths 4 for leaving cert higher level part two* Dublin: Gill and Macmillan Press.

Humphrey, G., Guildea, B., Reeves, G. & Boylan, L. (2012). *New concise Project Maths 4 for leaving cert higher level* Dublin: Gill and Macmillan Press.

Humphrey, G., Guildea, B., Reeves, G. & Boylan, L. (2012). *New concise Project Maths 5 for leaving cert higher level* Dublin: Gill and Macmillan Press.

Huntley, M. A., & Terrell, M. S. (2014). One-step and multi-step linear equations: a content analysis of five textbook series. *ZDM*, *46*(5), 751-766.

Irish Mathematics Teachers' Association (IMTA) (2013). *Project Maths and the Irish Maths Teachers Association* Retrieved 19th May 2017 from http://www.imta.ie/wp-content/uploads/2016/02/PM-IMTA-doc.pdf

Jäder, J., Lithner, J., & Sidenvall, J. (2015). *A cross-national textbook analysis with a focus on mathematical reasoning–The opportunities to learn.* Unpublished manuscript.

Jankvist, U. T. (2009). A categorization of the "whys" and "hows" of using history in mathematics education. *Educational Studies in Mathematics*, *71*(3), 235-261.

Janson, H., & Olsson, U. (2001). A measure of agreement for interval or nominal multivariate observations. *Educational and Psychological Measurement*, *61*(2), 277-289.

Jeffes, J., Jones, E., Cunningham, R., Dawson, A., Cooper, L., Straw, S., Sturman, L. & O'Kane, M. (2012). *Research into the impact of Project Maths on student achievement, learning and motivation: first interim report.* Slough: National Foundation for Education Research.

Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheater, R. & Dawson, A. (2013). *Research into the impact of Project Maths on student achievement, learning and motivation: final report.* Slough: National Foundation for Educational Research.

Johansson, M. (2006). *Textbooks as instruments. Three teachers' way to organize their mathematics lessons.* Nordic Studies in Mathematics Education, 11(3), 5–30.

Johnson, R., Severance, S., Penuel, W. R., & Leary, H. (2016). Teachers, tasks, and tensions: lessons from a research–practice partnership. *Journal of Mathematics Teacher Education*, *19*(2-3), 169-185.

Jones, D. L., & Tarr, J. E. (2007). *An examination of the levels of cognitive demand required by probability tasks in middle grades mathematics textbooks*. Statistics Education Research Journal, *6*(2), 4-27.

Jones K., Fujita T., Clarke N. & Lu Y. (2008). *BSRLM Geometry working group: proof and proving in current classroom materials.* In Joubert, M. (Ed.) Proceedings of the British Society for Research into Learning Mathematics 28(3), 142-146.

Jones, K., & Fujita, T. (2013). Interpretations of National Curricula: the case of geometry in textbooks from England and Japan. *ZDM*, *45*(5), 671-683.

Jones, K., & Pepin, B. (2016). Research on mathematics teachers as partners in task design. *Journal of Mathematics Teacher Education*, *19*(2-3), 105-121.

Jonsson, B., Kulaksiz, Y. C., & Lithner, J. (2016). Creative and algorithmic mathematical reasoning: effects of transfer-appropriate processing and effortful struggle. *International Journal of Mathematical Education in Science and Technology*, 1-20.

Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. *The Journal of Mathematical Behavior*, *36*, 20-32.

Ju, M. K., Moon, J. E., & Song, R. J. (2016). History of Mathematics in Korean Mathematics Textbooks: Implication for Using Ethnomathematics in Culturally Diverse School. *International Journal of Science and Mathematics Education*, *14*(7), 1321-1338.

Kapur, M. (2008). Productive failure. *Cognition and instruction*, *26*(3), 379-424.

Kapur, M. (2011). A further study of productive failure in mathematical problem solving: Unpacking the design components. *Instructional Science*, *39*(4), 561-579.

Keating, M., Mulvany, D. & O'Loughlin, J. (2012). Active Maths 3: Book 1: Leaving certificate ordinary level, Dublin: Folens.

Keating, M., Mulvany, D. & O'Loughlin, J. (2012). Active Maths 4: Book 1: Leaving certificate higher level, Dublin: Folens.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

Kim, G. (2014). Textbook analysis: examining how Korean secondary mathematics textbooks support students' mathematical thinking. In Jones, K., Bokhove, C., Howson, G., & Fan, L (Eds). *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014)*. (pp. 283-286): University of Southampton.

Knott, L., Olson, J., Adams, A., & Ely, R. (2013). Task design: Supporting teachers to independently create rich tasks. *Task design in mathematics education*, 599-608.

Krippendorff, K. (1980). Content Analysis: An Introduction to Its Methodology. Newbury Park, CA: Sage.

Krippendorff, K. (2004). Reliability in content analysis. *Human Communication Research*, *30*(3), 411-433.

Krippendorff, K. (2011). Computing Krippendorff's Alpha-Reliability. Retrieved on May 16, 2017 from http://repository.upenn.edu/asc_papers/43

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics*, 159-174.

Lavrakas, P. J. (2008). *Encyclopedia of Survey Research Methods: AM* (Vol. 1). Sage.

Lepik, M. (2015). Analyzing the use of textbook in mathematics education: the case of Estonia. *Acta Paedagogica Vilnensia*, *35*(35), 90-102.

Lin, C. S., & Jeng, W. (2015). Using Content Analysis in LIS Research: Experiences with Coding Schemes Construction and Reliability Measures. *Qualitative & Quantitative Methods in Libraries*, 4, 87-95.

Lithner, J. (2004). Mathematical reasoning in calculus textbook exercises. *The Journal of Mathematical Behavior*, *23*(4), 405-427.

Lithner, J. (2008). A research framework for creative and imitative reasoning, *Educational Studies in Mathematics,* 67(3), 255-276.

Lombard, M., Snyder-Duch, J., & Bracken, C. C. (2002). Content analysis in mass communication: Assessment and reporting of intercoder reliability. *Human communication research*, *28*(4), 587-604.

Lubienski, S. (2011). Mathematics Education and Reform in Ireland: An Outsider's Analysis of Project Maths*, Bulletin of the Irish Mathematical Society*, 67, 27-55.

Lyons, M., Lynch, K., Close, S., Boland, S. & Sheerin E. (2003). *Inside Classrooms: a study of teaching and learning in Irish second-level schools,* Dublin: IPA.

Mailizar, M., & Fan, L. (2014). Assessing a new Indonesian secondary mathematics textbook: how does it promote authentic learning? In Jones, K., Bokhove, C., Howson, G., & Fan, L (Eds). *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014)*. (pp. 319-326): University of Southampton.

Mason, J., & Johnston-Wilder, S. (2006). *Designing and using mathematical tasks*. St. Albans: Tarquin Publications.

Marx, R. W., & Walsh, J. (1988). Learning from academic tasks. *The Elementary School Journal*, 207-219.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Beverly Hills: Sage Publications.

Moffett, P. (2009). Reform via textbooks: Lessons from a cross-national collaboration. In D. Corcoran, T. Dooley, S. Close, R. Ward (Eds.), *Proceedings of Third National Conference on Research in Mathematics Education*, 262-272, Dublin: St. Patrick's College.

Morris, O.D. (2003). *Text and Tests 4*, Dublin: The Celtic Press.

Morris, O.D. (2004). *Text and Tests 3*, Dublin: The Celtic Press.

Morris, O.D. (2004). *Text and Tests 5*, Dublin: The Celtic Press.

Morris, O.D., & Cooke, P. (2013). *Text and Tests 7*, Dublin: The Celtic Press.

Morris, O.D., Cooke, P. & Behan, P. (2012). *Text and Tests 3*, Dublin: The Celtic Press.

Mosmery, P., & Barzegar, R. (2015). The effects of using peer, self and teacher-assessment on Iranian EFL learners' writing ability at three levels of task complexity. *International Journal of Research Studies in Language Learning*, *4*(4), 15-27.

Murphy, O. (2001). *Discovering Maths 3*, Dublin: Folens Publishers.

Murphy, O. (2002). *Discovering Maths 4*, Dublin: Folens Publishers.

The National Council for Curriculum and Assessment (NCCA) (2005). *Review of Mathematics in Post-Primary Education a discussion paper* Retrieved October 16, 2011, from http://www.ncca.ie/uploadedfiles/MathsReview/MathsDiscusPaperEng.pdf

NCCA (2006). *Review of Mathematics in Post-Primary Education, Report on the Consultation*

Retrieved October 16, 2012, from http://www.ncca.ie/uploadedfiles/mathsreview/Maths_Consult_Report.pdf

NCCA (2012). *Leaving Certificate Mathematics Syllabus* Retrieved October 16, 2012 from http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Syllabuses_and_Assessment/Leaving_Cert_Maths_syllabus_for_examination_in_2013.pdf

NCCA (2012a). *Project Maths: Reviewing the project in the initial group of 24 schools – report on school visits* Retrieved November 18, 2012 from http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Resources/Experiences_of_teachers.pdf

NCCA (2012b). *Project Maths Responding to current debate* Retrieved May 19, 2017 from http://www.ncca.ie/en/Publications/Other_Publications/Project-Maths-Responding-to-Current-Debate.pdf

NCCA (2014). *Maths in Practice. Report and Recommendations* Retrieved June 21, 2016 from http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Information/Reports/Maths-in-Practice-Report-.pdf

Neuendorf, K. A. (2002). The content analysis guidebook. Thousand Oaks, CA: Sage.

Nie, B., Freedman, T., Hwang, S., Wang, N., Moyer, J. C., & Cai, J. (2013). An investigation of teachers' intentions and reflections about using Standards-based and traditional textbooks in the classroom. *ZDM*, *45*(5), 699-711.

OECD PISA (2007). *PISA 2006: Science Competencies for Tomorrows World Executive Summary* Retrieved October 16, 2012, from http://www.oecd.org/pisa/pisaproducts/39725224.pdf

OECD PISA (2010). *PISA 2009: What Students Know and Can Do Executive Summary* Retrieved May 19, 2017, from http://www.oecd.org/pisa/pisaproducts/46619703.pdf

OECD (2016). *Ten Questions for Mathematics Teachers ... and how PISA can help answer them,* PISA, OECD Publishing, Paris.

O'Keeffe, L., (2011). *An Investigation into the Nature of Mathematics Textbooks at Junior Cycle and their Role in Mathematics Education*, Unpublished Dissertation, University of Limerick.

O'Keeffe, L., & O'Donoghue, J. (2009). Assessing the effect of mathematics textbook content structure on student comprehension and motivation. In D. Corcoran, T. Dooley, S. Close, R. Ward (Eds.), *Proceedings of Third National Conference on Research in Mathematics Education*, 284-295, Dublin: St. Patrick's College.

O'Keefe, L., & O'Donoghue, J. (2011). The use of evidence based research on mathematics textbooks to increase student conceptual understanding. *International Journal for Cross-Disciplinary Subjects in Education*, *2*(1), 304-311.

O'Keeffe, L., & O'Donoghue, J. (2012). *A Review of School Textbooks for Project Maths*. National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL), Limerick.

O'Sullivan, B., Breen, S., & O'Shea, A. (2012). A consideration of familiarity in Irish mathematics examinations. In Smith, C. (Ed.) *Proceedings of the British Society for Research into Learning Mathematics* 32(2): 70-75.

O'Sullivan, B., Breen, S., & O'Shea, A. (2013). We never did this: A framework for measuring novelty of tasks in Mathematics textbooks In T. Dooley, S. Nic Mhuirí, M. O'Reilly, R. Ward (Eds.), *Proceedings of Fifth National Conference on Research in Mathematics Education*, 284-295, Dublin: St. Patrick's College.

O'Sullivan, B. (2014). Change comes slowly: Using textbook tasks to measure curriculum implementation in Ireland. In Jones, K., Bokhove, C., Howson, G., & Fan, L (Eds). *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014)*. (pp 371-376): University of Southampton.

Pepin, B., Gueudet, G., & Trouche, L. (2013). Re-sourcing teachers' work and interactions: a collective perspective on resources, their use and transformation. *ZDM*, *45*(7), 929-943.

Pepin, B., Xu, B., Trouche, L., & Wang, C. (2016). Developing a deeper understanding of mathematics teaching expertise: an examination of three Chinese mathematics teachers' resource systems as windows into their work and expertise. *Educational Studies in Mathematics*, 1-18.

Perkins, R., Moran, G., Cosgrove, J., Shiel, G. (2009). *PISA 2009: The performance and progress of 15-year-olds in Ireland*. Dublin: Education Research Centre.

Perkins, R., & Shiel, G. (2016). Implications for the teaching and learning of mathematics in Ireland. Retrieved May 19, 2017 from http://www.erc.ie/wp/wp-content/uploads/2016/10/PISA-in-ClassroomsWeb.pdf

Pingel, F. (2010). *UNESCO guidebook on textbook research and textbook revision* (2nd revised and updated edition). Retrieved May 19, 2017 from http://unesdoc.unesco.org/images/0011/001171/117188E.pdf

Polya, G. (1957). *How to Solve it: A New Aspect of Mathematical Method. 2d Ed*. Doubleday.

Prasad, B. D. (2008). Content analysis. *A method in Social Science Research. Teoksessa: Lal Das, DK and Bhaskaran, V (toim.).(2008): Research methods for Social Work. New Delhi: Rawat*, 173-193.

Project Maths (2009). Retrieved April 9, 2011 from http://www.projectmaths.ie/for-teachers/junior-certificate/problem-solving-at-junior-cycle/

Project Maths (2017). Retrieved May 16, 2017, from http://www.projectmaths.ie/for-teachers/explanation-of-resources/textbooks/

Project Maths Implementation Support Committee (2010). *Report of the Project Maths Support Group*. Retrieved June 21, 2016 from https://www.education.ie/en/Publications/Policy-Reports/Report-of-the-Project-Maths-Implementation-Group.pdf

Rabardel, Pierre. "Les hommes et les technologies; approche cognitive des instruments contemporains." (1995): 239. Retrieved June 21, 2016 from https://hal.archives-ouvertes.fr/file/index/docid/1017462/filename/Hommes_et_technologie_Rabardel1995.pdf

Remillard, J. T. (1996). *Changing texts, teachers, and teaching: The role of curriculum materials in mathematics education reform*. Ph.D. Thesis.

Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry*, *29*(3), 315-342.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, *75*(2), 211-246.

Remillard, J. T., Herbel-Eisenmann, B. A., & Lloyd, G. M. (Eds.). (2011).*Mathematics teachers at work: Connecting curriculum materials and classroom instruction*. London: Routledge.

Rezat, S., & Sträßer, R. (2012). From the didactical triangle to the socio-didactical tetrahedron: artifacts as fundamental constituents of the didactical situation. *ZDM*, *44*(5), 641-651.

Robitaille, D.F., & Travers, K. J. (1992). International studies of achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 687-709). New York: Macmillan.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). Many visions, many aims (Vol. 1). Dordrecht: Kluwer.

Schoenfeld, A. H. (2009). Working with schools: The story of a mathematics education collaboration. *American Mathematical Monthly*, *116*(3), 197-217.

Seeley, C. L. (2003). Mathematics textbook adoption in the United States. In G. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (pp. 957–988). Reston, VA: National Council of Teachers of Mathematics.

Selden, J., Mason A., and Selden A. (1989). Can average calculus students solve nonroutine problems? *Journal of Mathematical Behaviour,* 8 (1), 45-50.

Selden, J., Selden, A., & Mason, A. (1994*). Even good calculus students can't solve nonroutine problems. MAA Notes*, 19-28.

Shiel, G., Surgenor, P., Close, S., & Millar, D. (2006). The 2004 National Assessment of Mathematics Achievement. Dublin: Educational Research Centre.

Shimizu, Y., Kaur, B., Huang, R., & Clarke, D. (Eds.). (2010). *Mathematical tasks in classrooms around the world*. Rotterdam: Sense Publishers.

Sidenvall, J., Lithner, J., & Jäder, J. (2015). Students' reasoning in mathematics textbook task-solving. *International journal of mathematical education in science and technology*, *46*(4), 533-552.

Silver, E. A., Ghousseini, H., Charalambous, C. Y., & Mills, V. (2009). Exploring the curriculum implementation plateau: An instructional perspective. *Mathematics teachers at work: Connecting curriculum materials and classroom instruction*, 245-265.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(4): 268-275.

Son, J. W., & Hu, Q. (2016). The initial treatment of the concept of function in the selected secondary school mathematics textbooks in the US and China. *International Journal of Mathematical Education in Science and Technology*, *47*(4), 505-530.

Sriraman, B., & English, L. (2010). Surveying theories and philosophies of mathematics education. In *Theories of mathematics education* (pp. 7-32). Berlin Heidelberg: Springer.

Stacey, K. & Vincent, J. (2009). Modes of reasoning in explanations in Australian eight-grade mathematics textbooks. *Educational Studies in Mathematics*, 72 (3), 271-288.

State Examinations Commission (2005). *Leaving Certificate Examination 2005 Mathematics Chief Examiner's Report for Higher, Ordinary and Foundation Levels*. Retrieved May 19, 2017, from http://www.examinations.ie/archive/examiners_reports/cer_2005/LCMathematics.pdf

State Examinations Commission (2006) *Junior Certificate Examination 2006 Mathematics Chief Examiner's Report for Higher, Ordinary and Foundation levels*. Retrieved May 19, 2017, from http://www.examinations.ie/archive/examiners_reports/cer_2006/JC_Maths_2006.pdf

State Examinations Commission (2016*). Leaving Certificate Examination 2015 Mathematics. Chief Examiner's Report* Retrieved June 21, 2016 from https://www.examinations.ie/misc-doc/EN-EN-53913274.pdf

State Examinations Commission (2017). *Percentage breakdown of candidates by grade awarded in each subject* Retrieved August 26, 2017 from https://www.examinations.ie/misc-doc/BI-ST-30831573.pdf

Stein, M. K., Grover, M. & Henningsen, M. (1996). *Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms*. American Educational Research Journal, 33(2), 489-509.

Stein, M. K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, *2*(1), 50-80.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction*. New York: Teachers College Press.

Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical Thinking and Learning*, 11(4), 258-288.

Sullivan, P., Clarke, D., & Clarke, B. (2012). *Teaching with tasks for effective mathematics learning* (Vol. 9). Berlin: Springer Science & Business Media.

Sullivan, P., & Mousley, J. (2001). Thinking teaching: Seeing mathematics teachers as active decision makers. In *Making sense of mathematics teacher education* (pp. 147-163). Springer Netherlands.

Swan, M. (2005). Standards Unit-Improving learning in mathematics: challenges and strategies. *London: Department for Education and Skills Standards Unit*.

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education*, *10*(4-6): 217-237.

Swan, M. (2008). Designing a multiple representation learning experience in secondary algebra. *Educational Designer*, *1*(1), 1-17.

Swan, M., & Burkhardt, H. (2012). Designing assessment of performance in mathematics. *Educational Designer*, *2*(5).

Thompson, D. R., & Senk, S. L. (2014). The same geometry textbook does not mean the same classroom enactment. *ZDM*, *46*(5), 781-795.

Tinsley, H. E., & Weiss, D. J. (1975). Interrater reliability and agreement of subjective judgments. *Journal of Counseling Psychology*, *22*(4), 358-376.

Tinsley, H. E., & Weiss, D. J. (2000). Interrater reliability and agreement. In H. E. A. Tinsley & S. D. Brown (Eds.), Handbook of applied multivariate statistics and mathematical modelling. San Diego, CA: Academic Press, 95-124.

Törnroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation*, *31*(4), 315-327.

Tran, D. (2016). Statistical Association: Alignment of Current US High School Textbooks with the Common Core State Standards for Mathematics. *School Science and Mathematics*, *116*(5), 286-296.

Usiskin, Z. (2012). *What does it mean to understand some mathematics?* Retrieved October 16 2012 from http://www.icme12.org/data/ICME%2012%20Usiskin%20Regular%20lecture%20print%20final.doc

Valverde, G. A,. Bianchi, L. J., Wolfe, R. G., Schmidt, W. H. & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the transition of policy into practice through the world of textbooks.* Dordrecht, Netherlands: Kluwer.

Viera, A. J., & Garrett, J. M. (2005). Understanding interobserver agreement: the kappa statistic. *Fam Med*, *37*(5), 360-363.

Viholainen, A., Partanen, M., Piiroinen, J., Asikainen, M. & Hirvonen, P. (2015). The role of textbooks in Finnish upper secondary school mathematics: theory, examples and exercises. *Nordic Studies in Mathematics Education*, *20*(3-4), 157-178.

Vincent, J., & Stacey, K. (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS video study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal*, *20*(1): 82-107.

Watson, A., & De Geest, E. (2005). Principled teaching for deep progress: Improving mathematical learning beyond methods and materials. *Educational Studies in Mathematics*, *58*(2), 209-234.

Watson, A., & Ohtani, M. (2015). Themes and Issues in Mathematics Education Concerning Task Design: Editorial Introduction. In *Task Design In Mathematics Education* (pp. 3-15). Springer International Publishing.

Watson, A., & Thompson, D. R. (2015). Design Issues Related to Text-Based Tasks. In *Task Design In Mathematics Education* (pp. 143-190). Springer International Publishing.

Xin, Y. P. (2007). Word problem solving tasks in textbooks and their relation to student performance. *The Journal of Educational Research*, *100*(6), 347-360.

Yang, D. C., & Lin, Y. C. (2014). A comparison of functions in middle school textbooks among Finland, Singapore and Taiwan. In Jones, K., Bokhove, C., Howson, G., & Fan, L (Eds). *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014)*. (pp. 505 – 510): University of Southampton.

Zaslavsky, O., & Sullivan, P. (Eds.). (2011). *Constructing knowledge for teaching secondary mathematics: Tasks to enhance prospective and practicing teacher learning* (Vol. 6). Springer Science & Business Media.

Appendices

Appendix A Documents for Novelty Workshop

A.1 Codebook – Information for Textbook Tasks Analysis in Relation to Novelty

The table given below describes three levels of Novelty: Novel, Somewhat Novel and Not Novel. In order to classify textbook tasks as Novel, Somewhat Novel or Not Novel, expository material and examples shown prior to a task within a single chapter are examined and the key skills required for deriving a solution to each task are identified. Skills are taken to refer to the methods and techniques used in the solutions to tasks. Please note that it is not necessary for all characteristics in the description of the categories of novelty shown in the table to apply in order for a task to be classified in that category. However there should be sufficient evidence in order to distinguish between the different categories and as many characteristics as possible should be identified before settling on a particular classification. Common skills like, for example, factorising or calculating slope are taken to be familiar from junior cycle.

Novel	(iv)	Skills involved in finding the solution are not familiar from preceding exercises or from any previous point in the chapter being analysed.
	(v)	The mathematical concept involved is not familiar from previous exercises or examples.
	(vi)	Significant adaption of the method outlined in examples and exercises must be made in order to get the required solution.
Somewhat Novel	(v)	The presentation of the task makes the question appear unfamiliar. However its solution requires the use of familiar skills.
	(vi)	The context (perhaps the use of an unfamiliar real-world situation) makes the task appear unfamiliar but familiar skills are used in its solution.
	(vii)	A new feature or aspect of a concept is encountered but the solution to the task only involves the use of familiar skills.
	(viii)	A minor adaption of the method outlined in the examples has to be made in order to get the required solution. The skills required are familiar but the use or application of such skill is slightly modified.
Not Novel	(vi)	The presentation, context and concepts of the task are familiar.
	(vii)	The solution to the exercise or problem has been modelled in preceding exercises or has been encountered earlier in the same chapter.
	(viii)	The skills required are very familiar to the user and the method of solution is clear due to the similarity between the exercise and preceding examples and exercises.

A.2 Definitions of Terms Used in Coding and on Coding Forms:

Unit of Data Collection: Each textbook exercise or parts that make up such an exercise set like (a), (b), (c) or (i), (ii), (iii).

Task number: For identification purposes, each task or unit of data is given a unique number.

Exercise ID: Each task is identified by its original exercise number from the textbook.

Classification: Indication of whether the task is novel, somewhat novel or not novel including an indication of which criterion or criteria led to the decision such as '(i)' in the Novel column, or '(ii) and (iii)' in the Not Novel column.

The textbooks being coded are Ordinary Level and Higher Level versions of Active Maths, Text and Tests and Concise Maths, which have been labelled Texts A, B and C in some order.

Examples of coding undertaken using this framework:

Expository material and worked examples from Text C (H)

Sequences

A **sequence** is a set of numbers, separated by commas, in which each number after the first is formed by some definite rule.

Each number in a sequence is a **term** of that sequence. The first number is the **first term** and is denoted by T_1. Similarly, the second term is denoted by T_2 and so on.

T_n represents the nth term.

- 3, 7, 11, 15, . . .
  Each number after the first is obtained by adding 4 to the previous number.
  In this example, 3 is called the **first term**, 7 the **second term** and so on.
- 1, 3, 9, 27, . . .
  Each number after the first is obtained by multiplying the previous number by 3.
  In this example, 1 is called the **first term**, 3 the **second term** and so on.

Alternative notation

In the chapter on indices in Book 4, recurrence equations were described.

The equation $u_{n+1} = 2u_n + 4^n$ was given as an example.

In this case, u_n represented the nth term.

While u_n and T_n are usually interchangeable, it is not unusual to see a reference to u_0, as under some circumstances, the expression for u_n can be more easily understood when zero may be substituted to generate the first term. Consider this sequence:

$$1, x, x^2, x^3, \ldots$$

Expressing its rule as $u_n = x^n$ for $n \geq 0$ is neater than $T_n = x^{n-1}$ for $n \geq 1$.

The T_n notation should always begin with T_1. Both notations will be used in this chapter.

EXAMPLE 1

A sequence is given by $T_n = n^2 - 3n$, where $n \in \mathbb{N}$.

(i) Find T_{10}. **(ii)** For what value of $n \in \mathbb{N}$ is $u_n = 40$?

Solution:

(i)
$$\begin{aligned} T_n &= n^2 - 3n \\ T_{10} &= (10)^2 - 3(10) \\ &= 100 - 30 \\ &= 70 \end{aligned}$$

(ii) Given: $T_n = 40$
$$\begin{aligned} \therefore \quad n^2 - 3n &= 40 \\ n^2 - 3n - 40 &= 0 \\ (n+5)(n-8) &= 0 \\ n = -5 \text{ or } n &= 8 \end{aligned}$$

Thus, $n = 8$, as $n \in \mathbb{N}$.

EXAMPLE 2

If $T_n = \dfrac{n}{n+1}$, show that $T_{n+1} > T_n$.

Solution:

$$T_n = \frac{n}{n+1} \quad \Rightarrow \quad T_{n+1} = \frac{n+1}{(n+1)+1} = \frac{n+1}{n+2}$$

$$\begin{aligned} T_{n+1} &> T_n \\ \frac{n+1}{n+2} &> \frac{n}{n+1} \\ (n+1)(n+1) &= n(n+2) && \left(\begin{array}{l}\text{multiply both sides by } (n+2) \text{ and } (n+1); \\ (n+2) \text{ and } (n+1) \text{ are both positive as } n \in \mathbb{N}\end{array}\right) \\ n^2 + 2n + 1 &> n^2 + 2n \\ 1 &> 0 \quad \text{true} && (\text{subtract } n^2 \text{ and } 2n \text{ from both sides}) \\ \therefore T_{n+1} &> T_n \end{aligned}$$

Exercise Set 4.2 from Text C (H)

Exercise 4.2

1. (i) If $u_n = 3n + 2$, find u_1 and u_2. (ii) Show that $u_{n+1} - u_n = 3$.
2. (i) If $u_n = n^2 - 3$, find u_1, u_2 and u_{n+1}.

Sample Task 1:

2. (i) If $u_n = n^2 - 3$, find u_1, u_2 and u_{n+1}.

Solution: $u_1 = (1)^2 - 3 = -2,\ u_2 = (2)^2 - 3 = 1$

$u_{n+1} = (n+1)^2 - 3 = n^2 + 2n + 1 - 3 = n^2 + 2n - 2$

Classification:

Not novel (ii) and (iii), the solution has been modelled in example 1.

The skills required are very familiar from this example.

Sample Task 2:

1. (i) If $u_n = 3n + 2$, find u_1 and u_2.

Solution: $u_1 = 3(1) + 2 = 5,\ u_2 = 3(2) + 2 = 8$

Not Novel (i), (ii) and (iii), solution familiar from task 1.

The presentation, context and concepts of the task are familiar.

The skills required and the solution method are similar to example 1 part (i).

Sample Task 3:

(ii) Show that $u_{n+1} - u_n = 3$.

Solution:

$u_{n+1} = 3(n+1) + 2 = 3n + 3 + 2 = 3n + 5$

$u_n = 3n + 2$

$u_{n+1} - u_n = (3n+5) - (3n+2) = 3n + 5 - 3n - 2 = 3$

Classification:

Somewhat Novel (i), (iv) and (iii)– The presentation of the task makes the question appear unfamiliar.

A minor adaption of the method outlined in the examples has to be made in order

to get the required solution.

However its solution requires the use of familiar skills, encountered in example 2.

5. **(i)** If $T_n = 2^n + 1$, find T_1, T_2 and T_{n+1}. **(ii)** Show that $T_{n+1} > T_n$.

Sample Task 4:

5 (ii) If $T_n = 2^n + 1$ show that $T_{n+1} > T_n$

Solution:

$T_{n+1} = 2^{n+1} + 1 = 2.2^n + 1$

$T_n = 2^n + 1$

$2.2^n + 1 > 2^n + 1$

$2.2^n > 2^n$

$2 > 1$ true $\therefore$ $T_{n+1} > T_n$

Classification:

Somewhat novel (iii) and (i), a new aspect of the concept is encountered here due to the presence of powers. The solution is based on familiar skills from example 2.

4. $T_n = an^2 + bn$, where $a, b \in \mathbb{R}$. If $T_1 = 7$ and $T_2 = 20$, find the following.

(i) The value of a and the value of b

Sample Task 5:

Ex. 4 (i) $T_n = an^2 + bn$, where $a, b \in R$. If $T_1 = 7$ and $T_2 = 20$, find the value of a and b.

Solution:

$T_1 = a(1)^2 + b(1) = 7 \Rightarrow a + b = 7$

$T_2 = a(2)^2 + b(2) = 20 \Rightarrow 4a + 2b = 20$

Simultaneous equations solved to get $a = 3$ and $b = 4$

Classification:

Novel (i), (ii) and (iii), the skills involved in finding the solution are not familiar from the preceding exercises or the two examples due to the presence of the unknown values and the simultaneous equations.

Significant adaption of the method outlined in examples and exercises must be made.

A.3 Coding Form

Sample Task	Exercise Number	Novel	Somewhat Novel	Not Novel
1	4.2-2(i)			(ii), (iii)
2	4.2-1(i)			(i), (ii), (iii)
3	4.2-1(ii)		(i), (iii),(iv)	
4	4.2-5(ii)		(i), (iii)	
5	4.2-4(i)	(i), (ii), (iii)		

A.4 Practice Coding Material

Example Text C (O)

Arithmetic sequences 2

In some questions we are given two terms of an arithmetic sequence. In this case, we use the method of simultaneous equations to find a and d.

EXAMPLE

In an arithmetic sequence, the fifth term, T_5, is 19 and the eighth term, T_8, is 31.
Find the first term, a, and the common difference, d.

Solution:

We are given two equations in disguise and we use these to find a and d.

$$T_n = a + (n-1)d$$

Given: $T_5 = 19$

$\therefore a + 4d = 19$ ①

Given: $T_8 = 31$

$\therefore a + 7d = 31$ ②

Now solve the simultaneous equations ① and ② to find a and d.

$$\begin{aligned} a + 7d &= 31 \quad ② \\ \underline{-a - 4d} &\underline{= -19} \quad ① \times -1 \\ 3d &= 12 \\ d &= 4 \end{aligned}$$

Put $d = 4$ into ① or ②

$$\begin{aligned} a + 4d &= 19 \quad ① \\ a + 4(4) &= 19 \\ a &= 3 \end{aligned}$$

Thus, $a = 3$ and $d = 4$.

Practice Task 1

Ex. 3 In an arithmetic sequence, the third term, T_3, is 7 and the fifth term, T_5, is 11.
Find the first term, a, and the common difference, d.

Practice Task 2

Ex. 7 In an arithmetic sequence, the sum of the third term T_3 and the seventh term T_7, is 38.
The sixth term, T_6, is 23
Find the first term, a, and the commond difference, d.

Practice Task 3

Ex. 8 In an arithmetic sequence, the sum of the third term T_3 and the seventh term T_7, is 22.
The fifth term, T_5, is 11
Find the first term, a, and the commond difference, d.

Practice Task 4

Ex. 10 p, q and r are three numbers in an arithmetic sequence.
Prove that $p^2 + r^2 \geq 2q^2$

Coding Form

Practice Task Number	Exercise Number	Novel	Somewhat Novel	Not Novel
1	3			
2	7 (i)			
3	8			
4	10			

A.5 Coding Material

Expository Material from Text C (O)

CHAPTER 9 PATTERNS, SEQUENCES AND SERIES

Patterns 1

Much of mathematics is about patterns. Some are simple and numeric:

$$2, 4, 6, 8, 10, \ldots \qquad \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots \qquad 10, 21, 32, 43, 54, \ldots$$

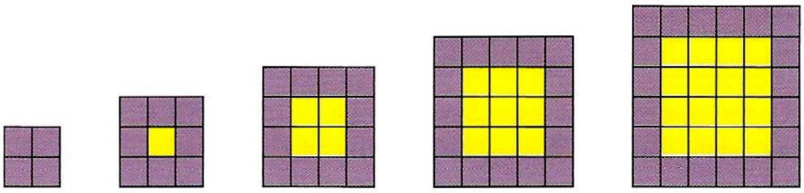
You should be able to write down the next three numbers in the list; you may even be able to predict the 10th number in the list *without* having to write out all the terms.

Other numeric patterns are more complicated:

$$1, 2, 4, 8, 16, \ldots \qquad 1, 0, 2, 0, 0, 3, 0, 0, 0, 4, \ldots \qquad 1, 2, 6, 24, 120, 720, \ldots$$

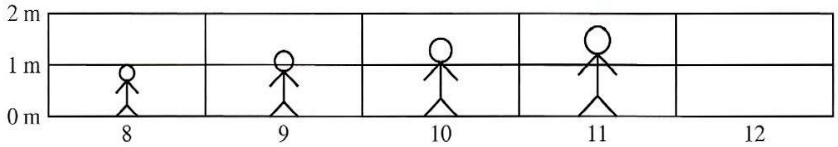
Even if you see a pattern and can write out the next number, it is much more difficult to predict what the 20th number or the 100th number in the list would be. If those numbers represented the population of the planet or the number of cancerous cells in a patient, then it would be very important to be able to predict future values.

Not all patterns are numeric. For example:



Is it possible to predict the number of squares in the 10th diagram? What about the number of yellow squares in the 10th diagram? What about the purple squares?

Many young children like to watch how tall they are growing and use some simple measuring techniques to record their growth. Is it possible to predict a child's height as each year passes?



As an eight-year-old, Freddie was 1 m tall. By nine he was 1·25 m tall and by 10 he had reached 1·5 m. Could you use this information to calculate his height when he is 12 years old? What about when Freddie is 21? Do you see a problem?

Sequences

A **sequence** is a set of numbers, separated by commas, in which each number after the first is formed by some definite rule.

Each number in a sequence is a **term** of that sequence. The first number is the **first term** and is denoted by T_1. Similarly, the second term is denoted by T_2 and so on.

3, 7, 11, 15, . . .

Each number after the first is obtained by adding 4 to the previous number. In this example, 3 is called the **first term**, 7 is the **second term** and so on.

1, 3, 9, 27, . . .

Each number after the first is obtained by multiplying the previous number by 3. In this example, 1 is called the **first term**, 3 is the **second term** and so on.

The general term, T_n

Very often a sequence is given by a **rule** which defines the **general term**. We use T_n to denote the general term of the sequence. T_n may be used to obtain any term of a sequence. T_1 will represent the first term, T_2 the second term and so on.

Notes: **1.** The general term, T_n, is often called the nth term.

2. n used with this meaning must always be a positive whole number. It can never be fractional or negative.

3. A sequence is often called a progression.

Consider the sequence whose general term is $T_n = 3n + 2$.

We can find the value of any term of the sequence by putting in the appropriate value for n on both sides:

$$T_n = 3n + 2$$

$T_1 = 3(1) + 2 = 3 + 2 = 5$ (first term, put in 1 for n)

$T_2 = 3(2) + 2 = 6 + 2 = 8$ (second term, put in 2 for n)

$T_5 = 3(5) + 2 = 15 + 2 = 17$ (fifth term, put in 5 for n)

In each case, n is replaced with the same number on both sides.

The notation $T_n = 3n + 2$ is very similar to function notation when n is the input and T_n is the output, i.e. (input, output) = (n, T_n).

EXAMPLE

The nth term of a sequence is given by $T_n = n^2 + 3$.

(i) Write down the first three terms of the sequence.

(ii) Show that: **(a)** $\frac{T_5}{T_2} = T_1$ **(b)** $2T_4 = T_6 - 1$

Solution:

(i) $T_n = n^2 + 3$

$T_1 = 1^2 + 3 = 1 + 3 = 4$ (put in 1 for n)

$T_2 = 2^2 + 3 = 4 + 3 = 7$ (put in 2 for n)

$T_3 = 3^2 + 3 = 9 + 3 = 12$ (put in 3 for n)

Thus, the first three terms are 4, 7, 12.

(ii) **(a)** From **(i)**, $T_1 = 4$ and $T_2 = 7$.

$T_5 = 5^2 + 3 = 25 + 3 = 28$

$\frac{T_5}{T_2} = \frac{28}{7} = 4$

$T_1 = 4$

$\therefore \frac{T_5}{T_2} = T_1$

(b) $T_4 = 4^2 + 3 = 16 + 3 = 19$

$T_6 = 6^2 + 3 = 36 + 3 = 39$

$2T_4 = 2(19) = 38$

$T_6 - 1 = 39 - 1 = 38$

$\therefore 2T_4 = T_6 - 1$

Exercise 9.1

In questions 1–8, write down the next four terms.

1. 1, 5, 9, 13, . . .

2. 40, 35, 30, 25, . . .

3. −11, −9, −7, −5, . . .

4. 13, 10, 7, 4, . . .

5. 2·5, 2·9, 3·3, 3·7, . . .

6. 2·8, 2·2, 1·6, 1, . . .

7. 1, 2, 4, 8, . . .

8. 2, 6, 18, 54, . . .

In questions 9–20, write down the first four terms of the sequence defined by the given nth term.

9. $T_n = 2n + 3$

10. $T_n = 3n + 1$

11. $T_n = 4n - 1$

12. $T_n = 5n - 3$

13. $T_n = 1 - 2n$

14. $T_n = 3 - 4n$

15. $T_n = n^2 + 5$

16. $T_n = n^2 + 2n$

17. $T_n = \frac{n+1}{n}$

18. $T_n = \frac{2n}{n+1}$

19. $T_n = 2^n$

20. $T_n = 3^n$

Expository material from Text B (H)

Differential Calculus

chapter 2

Key words

average rate of change **instantaneous rate of change** **derived function** **differentiation from first principles** **product rule** **quotient rule** **chain rule** **trigonometric function** **inverse trigonometric function** **exponential function** **logarithmic function**

Introduction to calculus

In this chapter, we begin the study of a very important branch of mathematics called *calculus*. Differential calculus is mainly concerned with measuring the rate of change of one quantity with respect to another. For example, the speed of a car is the rate at which the distance it travels changes with respect to time. However, we know that a car is unlikely to travel at a constant speed, even for a short time. If a car is accelerating, it is changing speed by the second. If 60 km/hr is registered on the speedometer, this tells us the **instantaneous** speed. Calculus is the mathematical tool that will enable us to find **instantaneous rates of change**.

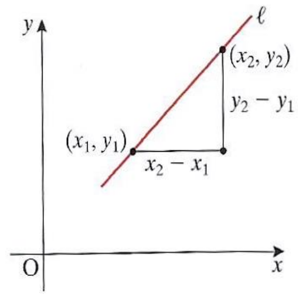
Section 2.1 Average rate of change

We have already learned how to find the slope of a line if we are given two points on the line.

The slope, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

We will now refer to this slope as the **rate of change** of y with respect to x.

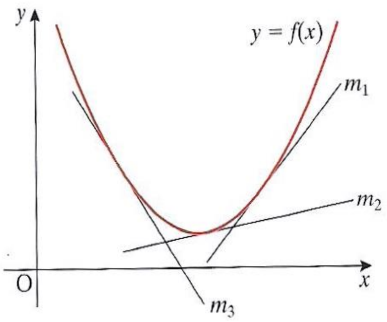
The slope of a line will always be a fixed number as the slope is constant all along the line.



The curve on the right is the graph of a function $y = f(x)$.

How do we find the slope of a curve?

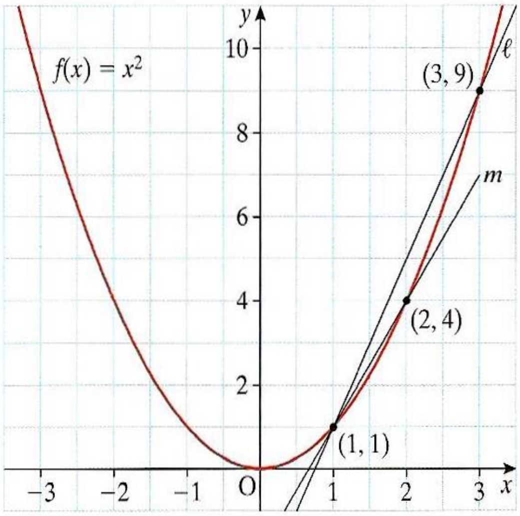
The slope of a curve at any point is defined as the slope of the tangent to the curve at that point.



The diagram above also shows tangents drawn at three different points on the curve. The three tangents have different slopes – m_1, m_2 and m_3.

In the next section of this chapter, we will show how calculus can be used to find the slope of the tangent to a curve at any point on the curve.

Average rate of change



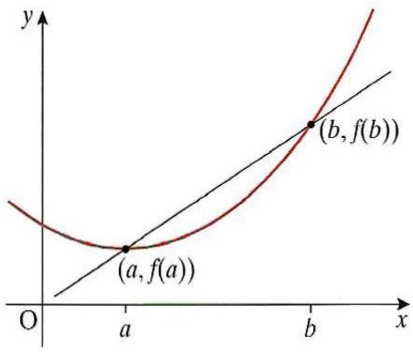
The curve on the right is the graph of $f(x) = x^2$.
The points (1, 1) (2, 4) and (3, 9) are shown on the curve.
Lines are drawn through (1, 1) and (3, 9) and also through (1, 1) and (2, 4).
These lines are marked l and m.

Slope of $l = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{9-1}{3-1} = \dfrac{8}{2} = 4$

Slope of $m = \dfrac{4-1}{2-1} = \dfrac{3}{1} = 3$

The slope of the line l joining (1, 1) and (3, 9) is generally referred to as the **average rate of change**.

The average rate of change of the line m = slope of m = 3.



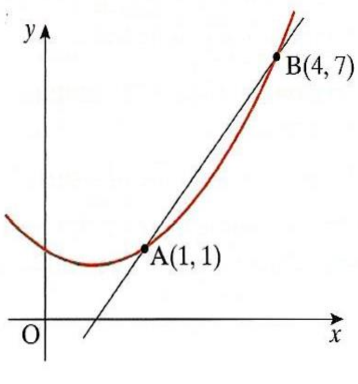
In general, for any function $y = f(x)$, the average rate of change of y with respect to x over the interval $[a, b]$ is the slope of the line joining $(a, f(a))$ to $(b, f(b))$.

$$\text{Average rate of change} = \frac{f(b) - f(a)}{b - a}$$

The interval $[a, b]$ represents $a \leqslant x \leqslant b$.

Example 1

Find the average rate of change of y with respect to x for the function $y = f(x)$ over the interval [1, 4] as shown.



The average rate of change = slope_{AB}

$$= \frac{7-1}{4-1} = \frac{6}{3} = 2$$

Example 2

The temperature T (°C) in a classroom on a particular day can be modelled by the equation

$$T = \frac{200}{t^2 + 2t + 20}, \text{ where } t \text{ is the time after 6.00 p.m..}$$

Find (i) the temperature in the room at 6.00 p.m.
(ii) the temperature in the room at midnight
(iii) the average rate of change of temperature from 6.00 p.m. to midnight.

(i) At 6.00 p.m., $t = 0$

$$\Rightarrow T = \frac{200}{(0)^2 + 2(0) + 20} = 10°\text{C}$$

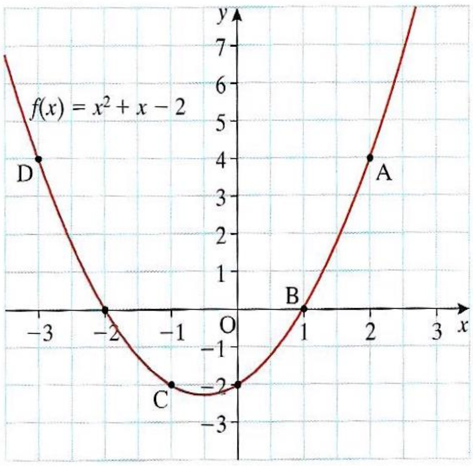
(ii) At midnight, $t = 6$

$$\Rightarrow T = \frac{200}{(6)^2 + 2(6) + 20} = 2.94°\text{C}$$

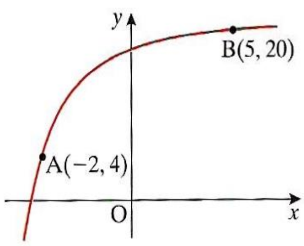
(iii) The average rate of change $= \dfrac{10 - 2.94}{6 - 0} = 1.18°\text{C/hour}$

Exercise 2.1

1. The curve on the right is the graph of the function $f(x) = x^2 + x - 2$.
The points A, B, C and D are shown.
Find the average rate of change of y with respect to x of the line through
(i) A and B
(ii) B and C
(iii) C and D.

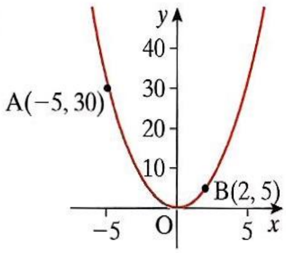


2. Find the average rate of change of the function depicted in the graph shown for the interval $[-2, 5]$.

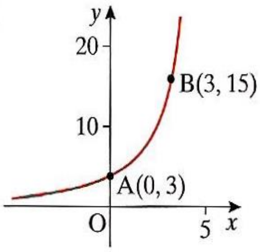


3. Find the average rate of change of y with respect to x from point A to point B for each of the following graphs.

(i)



(ii)



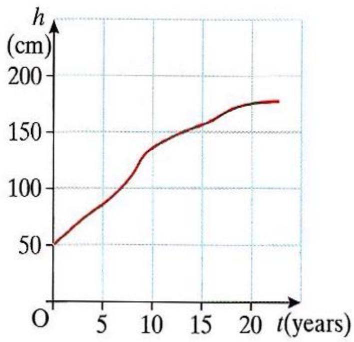
4. The depth, d cm, of water in a bath tub t minutes after the tap is turned on is modelled by the function $d(t) = \dfrac{-300}{(t+6)} + 50, t \geqslant 0$.

Find the average rate of change of the depth of the water in the tub over the first 10 minutes after the tap is turned on.

5. The graph of a person's height h (cm) versus t (years) from some time after birth to age 20 is shown.

(i) When is the growth rate greatest?

(ii) Estimate the average rate of growth between the ages of 5 and 10 years.



6. A cube has edge of length x cm.

(i) Find an expression for the surface area, $S(x)$, of the cube.

(ii) Find the average rate at which the surface area changes with respect to x as x increases from $x = 2$ cm to $x = 5$ cm.

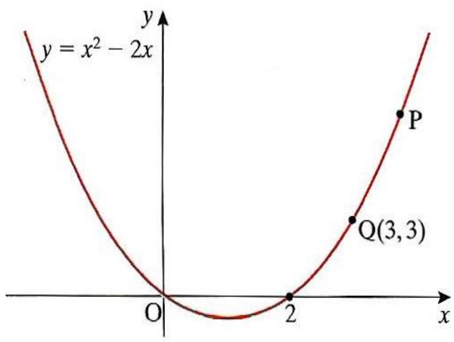
7. The curve on the right is the graph of the function $y = x^2 - 2x$.
Q is the point (3, 3).
P is any other point on the curve.

(i) If P is the point (4, 8), find the slope of PQ.

(ii) If P is the point (3.5, 5.25), find the slope of PQ.

(iii) If P is the point (3.1, 3.41), find the slope of PQ.

(iv) What do the results in parts (i) to (iii) suggest for the slope of the tangent to the curve at Q?



A.6 Patterns, Sequences and Series - Coding Form

Coder ID____________________

Task Number	Exercise Number	Novel	Somewhat Novel	Not Novel
1	9.1-1			
2	9.1-2			
3	9.1-3			
4	9.1-4			
5	9.1-5			
6	9.1-6			
7	9.1-7			
8	9.1-8			
9	9.1-9			
10	9.1-10			
11	9.1-11			
12	9.1-12			
13	9.1-13			
14	9.1-14			
15	9.1-15			

A.7 Differential Calculus - Coding Form

Coder ID____________________

Task Number	Exercise Number	Novel	Somewhat Novel	Not Novel
16	2.1-1(i)			
17	2.1-1(ii)			
18	2.1-1(iii)			
19	2.1-2			
20	2.1-3(i)			
21	2.1-3(ii)			
22	2.1-4			
23	2.1-5(i)			
24	2.1-5(ii)			
25	2.1-6(i)			
26	2.1-6(ii)			
27	2.1-7(i)			
28	2.1-7(ii)			
29	2.1-7(iii)			
30	2.1-7(iv)			

A.8 Questionnaire

1. Did you find the Novelty framework easy to use? Please outline any difficulties you encountered.

2. Do you agree with the descriptions (novel, somewhat novel, not novel) used in this framework?

3. Any other comments?

Appendix B Chi-square Tests Tables

The following are the Chi-square test tables that are referred to in Chapter 5. Any significant p values are highlighted in bold.

Subset of Data	**Variables for independence test**		**N**	**Degree of freedom**	**x^2**	**p**
Active Maths Higher Level	Textbook era	Topics	1743	1	0.07	0.79
Active Maths Higher Level	Textbook era	LCD [Higher Level Demand, Lower Level Demand]	1743	1	66.12	**<0.001**
Active Maths Ordinary Level	Textbook era	Topic	1183	1	2.85	0.91
Active Maths Ordinary Level	Textbook era	LCD [Higher Level Demand, Lower Level Demand]	1183	1	44.62	**<0.001**
Active Maths Post-Project Maths	Syllabus level	LCD [Higher Level Demand, Lower Level Demand]	1538	1	6.51	**0.01**
Active Maths Pre-Project Maths	Syllabus level	LCD [Higher Level Demand, Lower Level Demand]	1388	1	7.71	**0.01**
Active Maths Post-Project Maths Higher Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	871	1	37	**<0.001**
Active Maths Post-Project Maths Ordinary Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	647	1	3.6	0.06
Active Maths Pre-Project Maths Higher Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	852	1	5.98	**0.01**
Active Maths Pre-Project Maths Ordinary Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	536	1	10.87	**<0.001**

Table 50: Chi-square test results for Active Maths textbook series and LCD

Subset of Data	Variables for independence test		N	Degree of freedom	x^2	p
Text and Tests Higher Level	Textbook era	Topic	1275	1	0.004	0.95
Text and Tests Higher Level	Textbook era	LCD [Higher Level Demand, Lower Level Demand]	1275	1	98.7	**<0.001**
Text and Tests Ordinary Level	Textbook era	Topic	857	1	1.93	0.16
Text and Tests Ordinary Level	Textbook era	LCD [Higher Level Demand, Lower Level Demand]	857	1	14.26	**<0.001**
Text and Tests Post-Project Maths	Syllabus level	LCD [Higher Level Demand, Lower Level Demand]	1102	1	46.23	**<0.001**
Text and Tests Pre-Project Maths	Syllabus level	LCD [Higher Level Demand, Lower Level Demand]	1030	1	3.3	0.07
Text and Tests Post-Project Maths Higher Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	631	1	37	**<0.001**
TT Post –Project Maths Ordinary Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	471	1	3.6	0.06
Text and Tests Post-Project Maths Higher Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	644	1	5.79	**0.02**
Text and Tests pre-Project Maths Ordinary Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	386	1	9.85	**0.01**

Table 51: Chi-square test results for Text and Tests textbook series and LCD

Subset of Data	Variables for independence test		N	Degree of freedom	χ^2	p
Concise Maths Higher Level	Textbook era	Topic	1516	1	0.01	0.94
Concise Maths Higher Level	Textbook era	LCD [Higher Level Demand, Lower Level Demand]	1516	1	44.12	**<0.001**
Concise Maths Ordinary Level	Textbook era	Topic	1061	1	8.39	**0.01**
Concise Maths Ordinary Level	Textbook era	LCD [Higher Level Demand, Lower Level Demand]	1061	1	17.32	**<0.001**
Concise Maths Post-Project Maths	Syllabus level	LCD [Higher Level Demand, Lower Level Demand]	1411	1	0.94	0.33
Concise Maths Pre-Project Maths	Syllabus level	LCD [Higher Level Demand, Lower Level Demand]	1166	1	0.62	0.43
Concise Maths Post-Project Maths Higher Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	831	1	5.58	**0.02**
Concise Maths Post-Project Maths Ordinary Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	580	1	32.12	**<0.001**
Concise Maths Pre-Project Maths Higher Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	685	1	6.34	**0.01**
Concise Maths Pre-Project Maths Ordinary Level	Topic	LCD [Higher Level Demand, Lower Level Demand]	481	1	1.7	0.19

Table 52: Chi-square test results for Concise Maths textbook series and LCD

Subset of Data	Variables for independence test		N	Degree of freedom	χ^2	p
Post-Project Maths Higher Level	LCD [Higher Level Demand, Lower Level Demand]	Textbook series	2353	2	28.97	**<0.001**
Post-Project Maths Ordinary Level	LCD [Higher Level Demand, Lower Level Demand]	Textbook series	1698	2	20.62	**<0.001**
Pre-Project Maths Higher Level	LCD [Higher Level Demand, Lower Level Demand]	Textbook series	2181	2	17.55	**<0.001**
Pre-Project Maths Ordinary Level	LCD [Higher Level Demand, Lower Level Demand]	Textbook series	1403	2	4.31	0.12

Table 53: Chi-square test results for Textbook Series and LCD

Subset of Data	**Variables for independence test**		**N**	**Degree of freedom**	x^2	**p**
Active Maths Higher Level	Textbook era	Reasoning [Creative Reasoning, Imitative Reasoning]	1743	1	32.85	**<0.001**
Active Maths Ordinary Level	Textbook era	Reasoning [Creative Reasoning, Imitative Reasoning]	1183	1	21.25	**<0.001**
Active Maths Post-Project Maths	Syllabus level	Reasoning [Creative Reasoning, Imitative Reasoning]	1538	1	3.67	0.06
Active Maths Pre-Project Maths	Syllabus level	Reasoning [Creative Reasoning, Imitative Reasoning]	1388	1	3.19	0.07
Active Maths Post-Project Maths Higher Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	891	1	0.29	0.59
Active Maths Post-Project Maths Ordinary Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	647	1	0.85	0.36
Active Maths Pre-Project Maths Higher Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	852	1	0.12	0.72
Active Maths Pre-Project Maths Ordinary Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	536	1	10.87	**<0.001**

Table 54: Chi-square test results for Active Maths textbook series and Reasoning

Subset of Data	**Variables for independence test**		**N**	**Degree of freedom**	**x^2**	**p**
Text and Tests Higher Level	Textbook era	Reasoning [Creative Reasoning, Imitative Reasoning]	1275	1	16.7	**<0.001**
Text and Tests Ordinary Level	Textbook era	Reasoning [Creative Reasoning, Imitative Reasoning]	857	1	4.81	**0.03**
Text and Tests Post-Project Maths	Syllabus level	Reasoning [Creative Reasoning, Imitative Reasoning]	1102	1	10.77	**0.01**
Text and Tests Pre-Project Maths	Syllabus level	Reasoning [Creative Reasoning, Imitative Reasoning]	1030	1	3.7	0.06
Text and Tests Post-Project Maths Higher Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	631	1	39.04	**<0.001**
Text and Tests Post-Project Maths Ordinary Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	471	1	3.74	0.06
Text and Tests Pre-Project Maths Higher Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	644	1	7.05	**0.01**
Text and Tests Pre-Project Maths Ordinary Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	386	1	9.85	**0.01**

Table 55: Chi-square test results for Text and Tests textbook series and Reasoning

Subset of Data	Variables for independence test		N	Degree of freedom	x^2	p
Concise Maths Higher Level	Textbook era	Reasoning [Creative Reasoning, Imitative Reasoning]	1516	1	23.47	**<0.001**
Concise Maths Ordinary Level	Textbook era	Reasoning [Creative Reasoning, Imitative Reasoning]	1061	1	13.49	**<0.001**
Concise Maths Post-Project Maths	Syllabus level	Reasoning [Creative Reasoning, Imitative Reasoning]	1411	1	0.12	0.72
Concise Maths Pre-Project Maths	Syllabus level	Reasoning [Creative Reasoning, Imitative Reasoning]	1166	1	0.62	0.43
Concise Maths Post-Project Maths Higher Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	831	1	8.28	**0.01**
Concise Maths Post-Project Maths Ordinary Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	580	1	23.41	**<0.001**
Concise Maths Pre-Project Maths Higher Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	685	1	6.34	**0.01**
Concise Maths Pre-Project Maths Ordinary Level	Topic	Reasoning [Creative Reasoning, Imitative Reasoning]	481	1	1.7	0.19

Table 56: Chi-square test results for Concise Maths textbook series and Reasoning

Subset of Data	Variables for independence test		N	Degree of freedom	x^2	p
Post-Project Maths Higher Level	Reasoning	Textbook series	2353	2	12.85	**0.01**
Post-Project Maths Ordinary Level	Reasoning	Textbook series	1698	2	16.48	**<0.001**
Pre-Project Maths Higher Level	Reasoning	Textbook series	2181	2	7.65	**0.02**
Pre-Project Maths Ordinary Level	Reasoning	Textbook series	1403	2	4.31	0.12

Table 57: Chi-square test results for Textbook series and Reasoning

Subset of Data	**Variables for independence test**		**N**	**Degree of freedom**	x^2	**p**
Active Maths Higher Level	Textbook era	Novelty	1743	2	20.89	**<0.001**
Active Maths Ordinary Level	Textbook era	Novelty	1183	2	20.3	**<0.001**
Active Maths Post-Project Maths	Syllabus level	Novelty	1538	2	15.41	**<0.001**
Active Maths Pre-Project Maths	Syllabus level	Novelty	1388	2	26.69	**<0.001**
Active Maths Post-Project Maths Higher Level	Topic	Novelty	891	2	4.92	0.09
Active Maths Post-Project Maths Ordinary Level	Topic	Novelty	647	2	16.36	**<0.001**
Active Maths Pre-Project Maths Higher Level	Topic	Novelty	852	2	0.74	0.69
Active Maths Pre-Project Maths Ordinary Level	Topic	Novelty	536	2	8.25	**0.02**

Table 58: Chi-square test results for Active Maths textbook series and Novelty

Subset of Data	**Variables for independence test**		**N**	**Degree of freedom**	x^2	**p**
Text and Tests Higher Level	Textbook era	Novelty	1275	2	22.23	**<0.001**
Text and Tests Ordinary Level	Textbook era	Novelty	857	2	10.11	**0.01**
Text and Tests Post-Project Maths	Syllabus level	Novelty	1102	2	36.25	**<0.001**
Text and Tests Pre-Project Maths	Syllabus level	Novelty	1030	2	16.43	**<0.001**
Text and Tests Post-Project Maths Higher Level	Topic	Novelty	631	2	0.13	0.94
Text and Tests Post-Project Maths Ordinary Level	Topic	Novelty	471	2	16.33	**<0.001**
Text and Tests Pre-Project Maths Higher Level	Topic	Novelty	644	2	6.31	**0.04**
Text and Tests Pre-Project Maths Ordinary Level	Topic	Novelty	386	2	15.48	**<0.001**

Table 59: Chi-square test results for Text and Tests textbook series and Novelty

Subset of Data	**Variables for independence test**		**N**	**Degree of freedom**	**x^2**	**p**
Concise Maths Higher Level	Textbook era	Novelty	1516	2	17.93	**<0.001**
Concise Maths Ordinary Level	Textbook era	Novelty	1061	2	13.18	**0.01**
Concise Maths Post-Project Maths	Syllabus level	Novelty	1411	2	2.79	0.25
Concise Maths Pre-Project Maths	Syllabus level	Novelty	1166	2	1.54	0.46
Concise Maths Post-Project Maths Higher Level	Topic	Novelty	831	2	2.19	0.34
Concise Maths Post-Project Maths Ordinary Level	Topic	Novelty	580	2	8.07	**0.02**
Concise Maths Pre-Project Maths Higher Level	Topic	Novelty	685	2	4.4	0.11
Concise Maths Post-Project Maths Ordinary Level	Topic	Novelty	481	2	1.75	0.42

Table 60: Chi-square test results for Concise Maths textbook series and Novelty

Subset of Data	**Variables for independence test**		**N**	**Degree of freedom**	**x^2**	**p**
Post-Project Maths Higher Level	Novelty	Textbook Series	2353	4	30.81	**<0.001**
Post-Project Maths Ordinary Level	Novelty	Textbook Series	1698	4	15.92	**0.01**
Pre-Project Maths Higher Level	Novelty	Textbook Series	2181	4	2.7	0.61
Pre-Project Maths Ordinary Level	Novelty	Textbook Series	1403	4	15.01	**0.01**

Table 61: Chi-square test results for Textbook series and Novelty

Appendix C Task Design Material

Code Book

You will be classifying tasks using two frameworks: the first is the Level of Cognitive Demand Framework and the second is a framework based on the objectives of the Project Maths syllabus.

Level of Cognitive Demand framework: There are four possible classifications here. Lower-level: Memorization, Lower-level: procedures without connections to meaning, Higher-level: procedures with connections to meaning, Higher-level: Doing Mathematics.

Outline of the Levels of Cognitive Demand Framework

Levels of Demands
Lower-level demands (memorization) • Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • Have no connections to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced. Lower-level demands (procedures without connections to meaning) • Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience or placement of the task. • Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlies the procedure being used. • Are focused on producing correct answers instead of developing mathematical understanding. • Require no explanations or explanations that focus solely on describing the procedure that was used.

List of criteria for lower-level classifications using the LCD framework

Higher-level demands (Procedures with connections to meaning)

- Focus students' attention on the use of procedures for the purposes of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and develop understanding.

Higher-level demands (Doing Mathematics)

- Require complex and nonalgorithmic thinking – a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instruction, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experience and make appropriate use of them in working through the task.
- Require students to analyse the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

List of criteria for higher-level classifications using the LCD framework

LCD Sample Task 1 Find Tn, the nth term of the following arithmetic sequence.

8, 13, 18, 23, …

Likely method of solution: Let the first term a = 8, the common difference (13-8) d = 5. Then

Tn = a+(n-1)d = 8+(n-1)5 = 8+5n-5 = 5n+3

Commentary: The student has been shown a formula a + (n-1)d to use in this situation. Finding the nth term in this way does not require any great thought or consideration.

Classification LCD: Lower Level: Procedure without connection to meaning.

Algorithmic. Limited cognitive demand. No explanation needed. Focused on getting correct answer.

LCD Sample Task 2 Anna saves money each week to buy a printer which costs €190. Her plan is to start with €10 and to put aside €2 more each week (ie €12, €14, etc.) until she has enough money to buy the printer. At this rate, how many weeks will it take Anna to save for the printer?

Likely method of solution: Note there are two ways to do this:

10+12+14+16+18+20+22+24+26+28=190 (10 weeks)

OR

$$\frac{n}{2}\{2a+(n-1)d\}=190$$

$\frac{n}{2}\{2(10)+(n-1)(2)\}=190$ which leads to n = 10

Commentary: The student has more than one method of solution so must consider approach that they think best.

Classification LCD: Higher Level: Procedure **with** connection to meaning.

Not narrow algorithm, some cognitive effort required as the student must consider how to approach the task. A procedure is being used but it is not being used mindlessly.

LCD sample task 3 Show that $S_n = \frac{n(a+l)}{2}$ is the sum to n terms of an arithmetic sequence where *l* is the last term.

Likely method of solution: Student will have to experiment in order to determine how to deal with the last term l and incorporate it into the sum to n terms.

Commentary: The student has the formula $\frac{n}{2}\{2a+(n-1)d\}$ to use for Sn but this task is not procedural. Student has to consider the terms a, a+d, a+2d, …, l-d, l in order to get the required sum.

Classification LCD: Higher Level: 'Doing Mathematics'.

It requires complex and nonalgorithmic thinking. Student must analyse task and its constraints.

LCD sample task 4 Find the coordinates of the turning point of the given function and determine if the turning point is a local maximum or local minimum.

$$y = x^2 - 2x + 5$$

Likely method of solution: Differentiate to get the derivative. Solve it by letting equal to zero. This gives x =1, this is subbed into the function to get y = 4. So turning point is (1,4). Second derivative test is most likely used to show that it is a local minimum.

Commentary: The student has been shown a procedure to use in this situation. It does not involve any great thought or consideration.

Classification LCD: Lower Level: Procedure without connection to meaning. Following an algorithm, can be completed mindlessly.

LCD sample task 5 Given that the curve $y = ax^2 + 12x + 1$ has a turning point at x =2, calculate the value of a.

Likely method of solution: To get the turning point, the familiar procedure involves differentiating and letting the derivative equal to zero. In this case, the student would find the derivative to be 2ax+12. Given x =2, this is subbed into 2ax+12=0.

2a(2)+12=0

4a+12=0

4a=-12

a=-3

Commentary: A procedure is used but the presence of a means that it cannot be followed blindly. They must consider how to find the value of a.

Classification LCD: Higher Level: Procedure with connection to meaning.

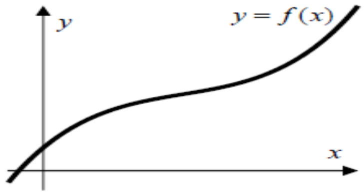
The use of the procedure is for the purposes of developing deeper levels of understanding of mathematical concepts. It requires some use of cognitive effort.

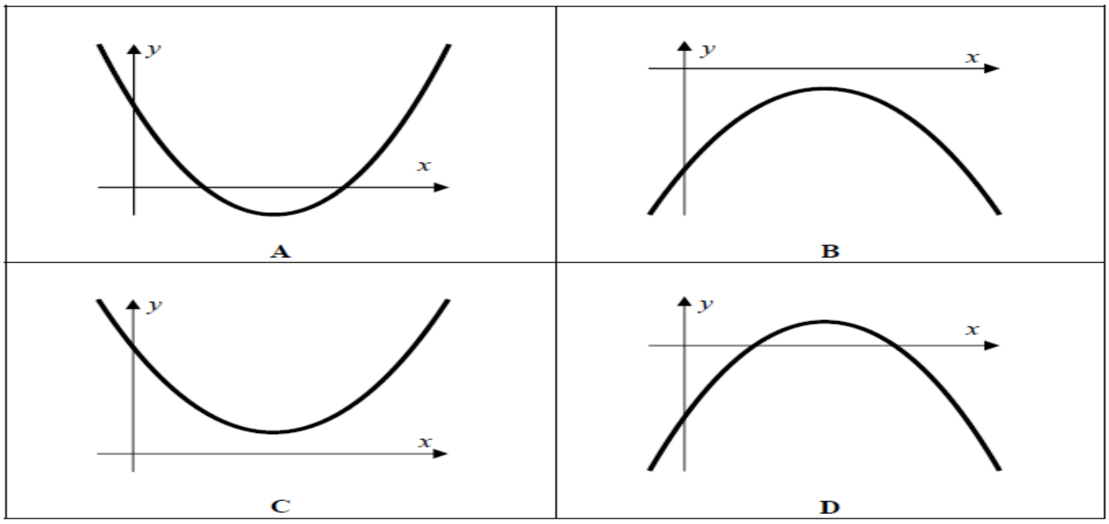
LCD sample task 6

The graph of a cubic function f is shown on the right.

One of the four diagrams **A**, **B**, **C**, **D** below shows the graph of the derivative of f.

State which one it is, and justify your answer.





Likely method of solution: It is necessary here to look at the behaviour of the original graph. Where does it cross the x-axis? Where does it cross the y-axis. Which derivative graph would result from a graph like this?

Commentary: The student must use mathematical thinking to analyse the graphs and corresponding constraints. It is necessary to use some cognitive effort to come up with the graph that represents the derivative.

Classification LCD: Higher Level – 'Doing Mathematics'.

The student does not use an algorithm here and is instead considering the behaviour of the functions and their graphs.

Project Maths Objectives Framework:

The development of synthesis and problem-solving skills is a key goal of the Project Maths syllabus. The second framework that you will use is an amended list of Project Maths objectives in relation to synthesis and problem-solving. Amending it in this way allows one to classify what kind of learning is experienced by the student when completing tasks. It is possible for a task to display more than one of these objectives and it gives a clear picture of whether tasks are meeting the Project Maths curricular goals in relation to synthesis and problem-solving.

- O1 Explore patterns and formulate conjectures.
- O2 Explain findings.
- O3 Justify conclusions.
- O4 Communicate mathematics in written form.
- O5 Apply their knowledge and skills to solve problems in familiar/unfamiliar contexts.
- O6 Analyse information presented in written form/words and translate it into mathematical form.
- O7 (a) Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.
- O7 (b) Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.
- O7 (c) Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Amended synthesis and problem-solving objectives in the Leaving Certificate syllabus

PMO Sample task 1 Find Tn, the nth term of the following arithmetic sequence.

8, 13, 18, 23, …

Likely method of solution: Let the first term a = 8, the common difference (13-8) d = 5. Then

Tn = a+(n-1)d = 8+(n-1)5 = 8+5n-5 = 5n+3

Commentary: The student has been shown a formula a + (n-1)d to use in this situation. Finding the nth term in this way does not require any great thought or consideration.

Classification PMO: Use Model – student has a familiar formula to work with.

PMO Sample Task 2 Anna saves money each week to buy a printer which costs €190. Her plan is to start with €10 and to put aside €2 more each week (ie €12, €14, etc.) until she has enough money to buy the printer. At this rate, how many weeks will it take Anna to save for the printer?

Likely method of solution: Recall that there are two ways to do this:

10+12+14+16+18+20+22+24+26+28=190 (10 weeks)

OR

$$\frac{n}{2}\{2a+(n-1)d\}=190$$

$\frac{n}{2}\{2(10)+(n-1)(2)\}=190$ which leads to n = 10

Commentary: You will recall that the student is likely to have more than one method in order to solve this task. The student has to select which one to complete the task with.

Classification PMO: **Select Model**

Classification PMO: Analyse information presented in written form and translate it into mathematical form.

PMO Sample Task 3 Show that $S_n = \frac{n(a+l)}{2}$ is the sum to n terms of an arithmetic sequence where *l* is the last term.

Likely method of solution: Student will have to experiment in order to determine how to deal with the last term l and incorporate it into the sum to n terms.

Commentary: The student has the formula $\frac{n}{2}\{2a+(n-1)d\}$ to use for Sn but this task is not procedural. Student has to consider the terms a, a+d, a+2d, …, l-d, l in order to get the required sum.

Classification PMO: Devise Model: The student has not been shown a method for doing this, they must find their own way of solving it. The formula for Sn is not sufficient here as a model to use.

Classification PMO: Apply knowledge and skills to solve problems in familiar/unfamiliar contexts: It is important here to realise that the student has experience of working with finding the sum of the arithmetic sequence but the task is a problem in the sense that it is novel, meaning that it is not as straight forward as might have been encountered previously.

PMO Sample Task 4 Find the coordinates of the turning point of the given function and determine if the turning point is a local maximum or local minimum.

$$y = x^2 - 2x + 5$$

Likely method of solution: Differentiate to get the derivative. Solve it by letting equal to zero. This gives x =1, this is subbed into the function to get y = 4. So turning point is (1,4). Second derivative test is most likely used to show that it is a local minimum.

Commentary: The student has been shown a procedure to use in this situation.

Classification PMO: Use Model: As seen earlier, the student uses a very familiar model to get the required answer.

PMO Sample Task 5 Given that the curve $y = ax^2 + 12x + 1$ has a turning point at x =2, calculate the value of a.

Likely method of solution: To get the turning point, the familiar procedure involves differentiating and letting the derivative equal to zero. In this case, the student would find the derivative to be 2ax+12. Given x =2, this is subbed into 2ax+12=0.

2a(2)+12=0

4a+12=0

4a=-12

a=-3

Commentary: A procedure is used but the presence of a means that it cannot be followed blindly. They must consider how to find the value of a but this does not involve devising a completely new model.

Classification PMO: Use Model: The student has a model for finding a turning point, the unknown a is a new feature which must be incorporated with the known procedure.

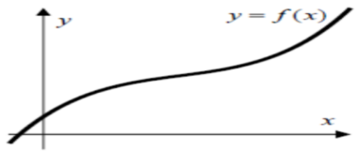
Classification PMO: Apply knowledge and skills to solve problems in familiar/unfamiliar contexts. Again the context is familiar but it is a problem because the student is not overly familiar with how to approach the task in order to solve it.

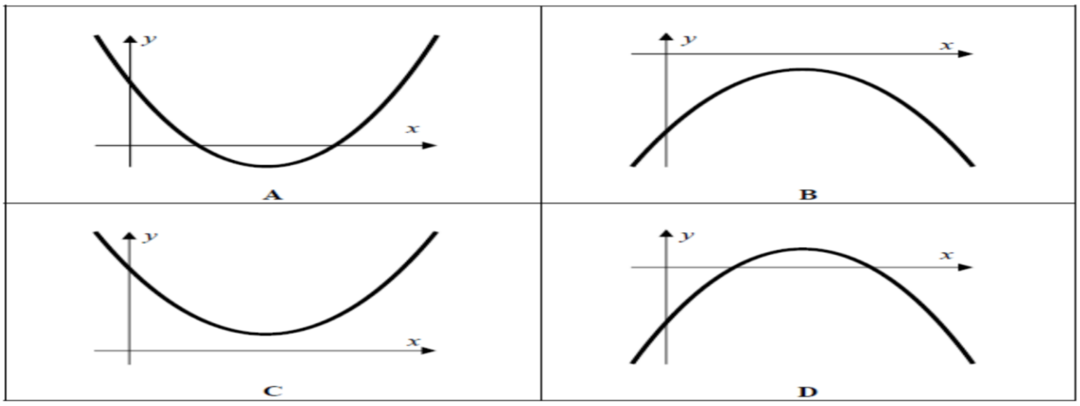
PMO Sample Task 6

The graph of a cubic function f is shown on the right.

One of the four diagrams **A**, **B**, **C**, **D** below shows the graph of the derivative of f.

State which one it is, and justify your answer.





Likely method of solution: It is necessary here to look at the behaviour of the original graph. Where does it cross the x-axis? Where does it cross the y-axis? Which derivative graph would result from a graph like this?

Commentary: The student must analyse the graphs and corresponding constraints. It is necessary to use some mathematical thinking to come up with the graph that represents the derivative.

Classification PMO: Devise Model – student must look at the graphs and consider their patterns and constraints. It is also necessary to link their knowledge of the derivative with the graphs.

Classification PMO: Explore patterns and formulate conjectures. Student must look at the behaviour of the graphs.

Classification PMO: Apply knowledge and skills to solve problems in familiar/unfamiliar contexts.

Classification PMO: Justify conclusions -the student must make a case for excluding certain graphs and give a valid reason behind the choice of graph that is eventually chosen.

Sequences and Series

Students' material exposure for Practice Task 1

1. Definition of a number sequence.
2. Finding the next term or finding the next two terms of a sequence e.g 2, 5, 8, 11, …

Students' material exposure for Practice Tasks 2, 3 and 4

1. Definition of a number sequence.
2. Finding the next term or finding the next two terms of a sequence e.g 2, 5, 8, 11, …
3. Writing out the first few terms of a sequence from a given Tn

e.g. Tn = 3n-2 gives the sequence 1, 4, 7, 10 (when asked for the first four terms).

Differential Calculus

Students' material exposure for Practice Tasks 1, 2 and 3

1. Differentiation from First Principles
2. Differentiation by Rule $f(x) = x^n$ $f'(x) = nx^{n-1}$
3. Product Rule $y = uv$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
4. Quotient Rule $y = \frac{u}{v}$ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
5. The Chain Rule
6. A curve is increasing where $\frac{dy}{dx} > 0$. A curve is decreasing where $\frac{dy}{dx} < 0$.
7. Stationary Points: a stationary point of a curve is a point on the curve at which the curve is differentiable, and the tangent is horizontal / the tangent has a slope of zero / $\frac{dy}{dx} = 0$.

To find the stationary points of the curve y = f(x):

(i) Find $\frac{dy}{dx}$

(ii) Put $\frac{dy}{dx} = 0$ and solve for x.

(iii) Find the y co-ordinates of the stationary points.

8. There are several types of stationary points. Two of these are called turning points, as the curve turns around at these points. The third is not a turning point, but is a point at which the tangent has slope 0 but the sign of the derivative does not change.

A Local Maximum Point: This is the highest point on the curve in a specified locality. It may not be the highest point overall. At a local ma point, the curve goes from increasing on the left to decreasing on the right. In other words, $\frac{dy}{dx}$ goes from positive, through zero, to negative.

B Local Minimum Point: This is the lowest point on the curve in a specified locality. It may not be the lowest point overall. At a local min, the curve goes from decreasing on the left to increasing on the right. In other words $\frac{dy}{dx}$ goes from negative, through zero, to positive.

9. The sign of the second derivative tells us about the curvature of the curve. (Which way the curve is facing). If $\frac{d^2y}{dx^2} > 0$ then the curve is concave upwards. If $\frac{d^2y}{dx^2} < 0$ then the curve is concave downwards.

10. Second Derivative Test for Turning Points

If $\frac{d^2y}{dx^2} < 0$ at a stationary point, then the point is a local max

If $\frac{d^2y}{dx^2} > 0$ at a stationary point, then the point is a local min

11. Point of Inflection

A point of inflection is a point on a curve at which the curve goes from being concave upwards to concave downwards, or the other way round.

The curve $y = f(x)$ has a point of inflection at $C(x_1, y_1)$ if (i) $\frac{d^2y}{dx^2} = 0$ at C and (ii) $\frac{d^2y}{dx^2}$ changes sign at C.

Note: Can assume that students **do not** have a procedure for showing a function is injective, surjective or bijective.

Students' material exposure for Workshop Tasks 1, 2 and 3

1. Definition of a number sequence.
2. Finding the next term or finding the next two terms of a sequence e.g 2, 5, 8, 11, …
3. Writing out the first few terms of a sequence from a given Tn

e.g. Tn = 3n-2 gives the sequence 1, 4, 7, 10 (when asked for the first four terms).

4. Finding the nth term of an arithmetic sequence. Model used Tn = a + (n-1)d
5. Finding the sum of arithmetic series manually ie The terms of 1+3+5+7 are added manually to get 16.
6. Finding the sum of an arithmetic series using a formula $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Note: No models or formulas beyond this. Student has no experience of working with general cases or justifying things mathematically.

Students' material exposure for tasks 4, 5 and 6

1. Differentiation from First Principles
2. Differentiation by Rule $f(x) = x^n \ f'(x) = nx^{n-1}$
3. Product Rule $y = uv \ \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
4. Quotient Rule $y = \frac{u}{v} \ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
5. The Chain Rule
6. A curve is increasing where $\frac{dy}{dx} > 0$. A curve is decreasing where $\frac{dy}{dx} < 0$.
7. Stationary Points: a stationary point of a curve is a point on the curve at which the curve is differentiable, and the tangent is horizontal / the tangent has a slope of zero / $\frac{dy}{dx} = 0$.

To find the stationary points of the curve y = f(x):

(i) Find $\frac{dy}{dx}$

(ii) Put $\frac{dy}{dx} = 0$ and solve for x.

(iii) Find the y co-ordinates of the stationary points.

(iv) There are several types of stationary points. Two of these are called turning points, as the curve turns around at these points. The third is not a turning point, but is a point at which the tangent has slope 0 but the sign of the derivative does not change.

A Local Maximum Point: This is the highest point on the curve in a specified locality. It may not be the highest point overall. At a local ma point, the curve goes from increasing on the left to decreasing on the right. In other words, $\frac{dy}{dx}$ goes from positive, through zero, to negative.

B Local Minimum Point: This is the lowest point on the curve in a specified locality. It may not be the lowest point overall. At a local min, the curve goes from decreasing on the left to increasing on the right. In other words $\frac{dy}{dx}$ goes from negative, through zero, to positive.

(v) The sign of the second derivative tells us about the curvature of the curve. (Which way the curve is facing). If $\frac{d^2y}{dx^2} > 0$ then the curve is concave upwards. If $\frac{d^2y}{dx^2} < 0$ then the curve is concave downwards.

(vi) Second Derivative Test for Turning Points

If $\frac{d^2y}{dx^2} < 0$ at a stationary point, then the point is a local max

If $\frac{d^2y}{dx^2} > 0$ at a stationary point, then the point is a local min

(vii) Point of Inflection

A point of inflection is a point on a curve at which the curve goes from being concave upwards to concave downwards, or the other way round.

The curve $y = f(x)$ has a point of inflection at $C(x_1, y_1)$ if (i) $\frac{d^2y}{dx^2} = 0$ at C and (ii) $\frac{d^2y}{dx^2}$ changes sign at C.

Workshop Tasks

W1. A display of cans on a supermarket shelf consists of 15 cans on the bottom, 13 cans in the next row, and so on in an arithmetic sequence until the top row has 9 cans.

The manager would like to see a different number of rows in the same display using the same number of cans.

(iii)	<u>CLASSIFY</u> Can you suggest two other arithmetic sequences for arranging the cans on the shelf?

(iv)	<u>CLASSIFY</u> Explain how you found the sequence.

W2. The first terms of the Fibonacci sequence are given below.

0, 1, 1, 2, 3, 5, 8, 13, 21, … .

(i) Describe in words how the sequence is formed.
Hence write out the next four terms in the sequence.

(ii) Choose any four consecutive Fibonacci numbers. Add the first and last terms from your selection, then divide by 2. Repeat the process again with four other consecutive Fibonacci numbers, and then another four.

(iii)	<u>CLASSIFY</u> What do you notice in part (ii)?

(iv)	<u>CLASSIFY</u> Can you justify your observation mathematically?

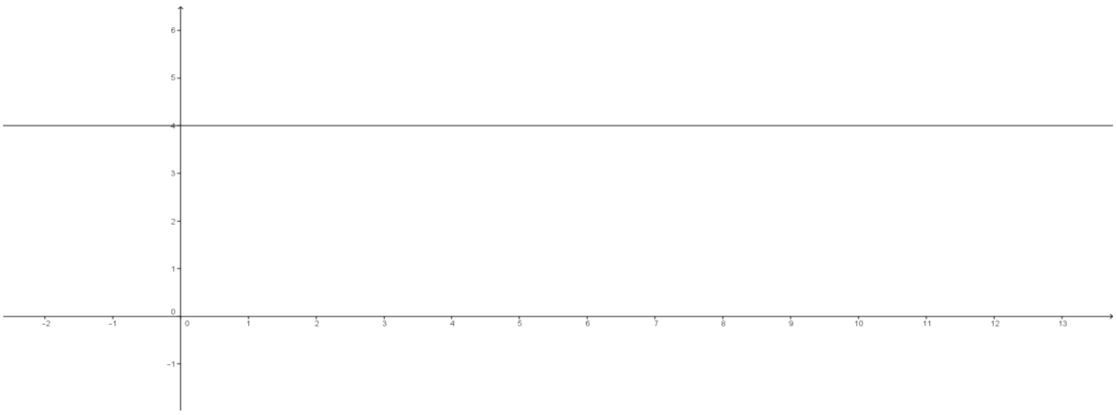
W3. A ball is thrown upwards from ground level and rises to a height of 15 metres. Once it reaches this height, it falls and strikes the ground and bounces to 60% of its previous height. It repeats the process, each time bouncing to 60% of the previous height.

(iv)	CLASSIFY Find the total distance travelled by the ball by the time it bounces for the fifth time. Give your answer to two decimal places.

(v)	CLASSIFY Rewrite the question above so that the ball's height is INCREASING each time instead of decreasing. What would be the total height after five bounces in this situation? (Give your answer to two decimal places)

(vi)	CLASSIFY If the ball from part (i) was caught on the third bounce, how high would the ball have to rise to on the first bounce in order for the total distance travelled to be 98 metres after three bounces?

W4.



(iv)	CLASSIFY You are given the graph of $f''(x)$ above, use your knowledge of derivatives to draw three possible graphs of $f'(x)$.

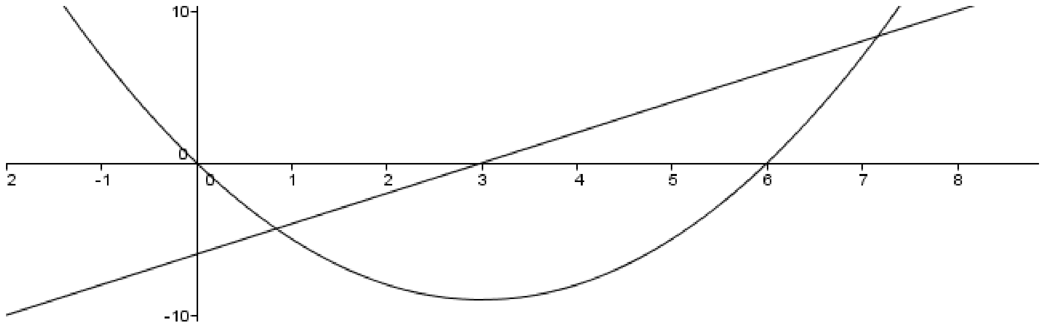
(v)	CLASSIFY Use your knowledge of derivatives to draw three possible graphs of $f(x)$.

(vi)	CLASSIFY Justify why you have drawn the graphs in this way.

W5.

The function $f(x)$ is defined for all $x \in R$.

Graphs of $f'(x)$ [the curve] and $f''(x)$ [the line] are shown.



(i)	<u>CLASSIFY</u> Using the diagram above, find the stationary points of *f (x)*

(ii)	<u>CLASSIFY</u> Identify them as maximum or minimum points.

(iii)	<u>CLASSIFY</u> Justify your answer.

W6.

(i) Show that $f(x) = x + \dfrac{1}{x}$ does not have any points of inflection.

(ii)	<u>CLASSIFY</u> What can be altered in the given function in order to ensure that the function has a point or points of inflection?

(iii)	<u>CLASSIFY</u> Give an example of another function which does not have any points of inflection.

(iv)	<u>CLASSIFY</u> Examine the following statement: 'If $(x, f(x))$ is a point of inflection, then $f''(x) = 0$ or $f''(x)$ does not exist.'

Write down the converse of this statement. Is it true? Justify your conclusion using a relevant example showing that points of inflection either exist or do not exist.

Questionnaire

Coder ID:

Frameworks

1. Do you feel that the Levels of Cognitive Demand framework is useful to teachers in their work? Please comment.

2. Did you have any difficulties applying the Levels of Cognitive Demand framework? Please comment.

3. Do you feel it is important to classify tasks using the Project Maths syllabus framework? Please comment.

4. Did you have any difficulties applying the Project Maths syllabus framework? Please comment.

Tasks

Please consider the tasks you classified or coded in the final part of the workshop:

5. Was each task clearly described? Please comment.

6. Was it easy to identify the level of cognitive demand of each task? Please comment.

7. Was it easy to identify Project Maths objectives addressed by each task? Please comment.

8. Do you think these tasks are suitable for use in the classroom? If so, please comment on how you would use them.

9. Do you think these tasks are suitable for assignment as homework? Please comment.

10. Have you any further comments to make on this set of tasks?

Participants were asked to complete the same table as below for each part of each workshop task.

Coding Form

Coder ID:

Workshop Task

Levels of Cognitive Demand	Tick **ONE** clearly if present
Higher Level Demand: Doing Mathematics	
Higher Level Demand: Procedures with connections to meaning	
Lower Level Demand: Procedures without connections to meaning	
Lower Level Demand: Memorization	

Workshop Task

Project Maths Syllabus Objective	Tick **as many** as you feel are applicable.
O1 Explore patterns and formulate conjectures	
02 Explain findings	
O3 Justify conclusions	
O4 Communicate mathematics in written form	
O5 Apply knowledge and skills to solve problems in familiar/unfamiliar contexts	
O6 Analyse information presented in written form and translate it into mathematical form	
O7a Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.	
O7b Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.	
O7c Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.	

Guide - DM: Doing Mathematics, HP: Procedures with connections to meaning, LP: Procedures without connections to meaning, LM: Memorization

ID	W1 (i)	W1 (ii)	W2 (iii)	W2 (iv)	W3 (i)	W3 (ii)	W3 (iii)	W4 (i)	W4 (ii)	W4 (iii)	W5 (i)	W5 (ii)	W5 (iii)	W6 (ii)	W6 (iii)	W6 (iv)
1	HP	HP	HP	DM	LP	HP	DM	HP	HP	HP	HP	LP	LM	HP	LM	HP
3	DM	HP	DM	DM	HP	HP	DM	HP	HP	HP	LP	LM	HP	HP	HP	DM
4	HP	HP	LP	DM	HP	HP	DM	HP	HP	HP	LP	LP	HP	HP	LP	DM
5	LP	HP	HP	HP	LP	HP	DM	HP	HP	HP	LP	LP	HP	HP	HP	HP
6	HP	HP	HP	HP	LP	LP	HP	HP	LP	HP	LP	LP	HP	HP	HP	HP
7	HP	HP	HP	HP	LP	LP	HP	HP	LP	LP	LP	LP	HP	HP	HP	HP
8	DM	HP	DM	DM	HP	DM	DM	HP	HP	HP	LP	LP	HP	DM	DM	HP
9	HP	DM	HP	DM	HP	DM	DM	HP	HP	DM	LP	LM	DM	HP	LM	DM
10	DM	HP	HP	HP	DM	DM	DM	HP	HP	HP	LP	HP	HP	HP	HP	HP
11	HP	HP	HP	HP	HP	HP	DM	HP	HP	HP	LP	LP	HP	DM	HP	DM
12	HP	HP	HP	DM	HP	HP	DM	LP	LP	HP	HP	DM	DM	DM	HP	DM
13	HP	HP	HP	DM	LP	LP	DM	HP	DM	HP	LP	LP	HP	HP	LM	DM
14	HP	HP	LP	DM	LP	LP	DM	HP	HP	DM	HP	HP	DM	HP	DM	DM
15	DM	DM	HP	DM	HP	DM	DM	DM	DM	DM	LP	LP	DM	LP	HP	DM
16	HP	HP	HP	DM	DM	HP	DM	LP	HP	HP	DM	LP	HP	LM	DM	HP
17	HP	DM	LP	HP	LP	HP	HP	HP	HP	LP	LM	DM	HP	HP	HP	DM
18	HP	HP	HP	HP	HP	DM	DM	DM	HP	DM	LP	LP	HP	HP	HP	DM
19	HP	HP	HP	DM	HP	HP	DM	HP	HP	HP	HP	HP	HP	LP	LM	DM

Table 62: Individual Coder Classifications for LCD framework

ID	W1 (i)	W1 (ii)	W2 (iii)	W2 (iv)	W3 (i)	W3 (ii)	W3 (iii)	W4 (i)	W4 (ii)	W4 (iii)	W5 (i)	W5 (ii)	W5 (iii)	W6 (ii)	W6 (iii)	W6 (iv)
1	O1 O4 O5 O6 O7b	O1 O2 O3 O4 O5	O1 O2 O4	O1 O3 O5 O7a	O1 O5 O7c	O1 O5 O7c	O1 O5 O6 O7a	O1 O5 O6 O7b	O1 O5 O6 O7b	O2 O3	O1 O7c	O5 O7c	O5 O7c	O5 O6 O7c	O5	O6 O7c
3	O1 O5 O6 O7a	O2 O5	O2 O5	O5 O6 O7a	O1 O5 O6 O7b	O5 O6 O7b	O5 O6 O7a	O5	O5	O2 O3 O5	O5 O7b	O5	O2 O3	O5	O5	O3 O5 O6 O7b
4	O1 O5 O6	O1 O2 O4	O1 O2 O6	O1 O3 O5 O6	O5 O6 O7a	O4 O5 O6 O7a	O5 O6 O7a	O5 O6 O7b	O5 O6 O7b	O3 O4	O6 O7c	O6 O7c	O3	O5 O6 O7a	O6 O7c	O3 O5 O6 O7a
5	O1 O6	O2 O3	O1 O2	O5 O6 O7a	O6 O7c	O5 O6 O7a	O5 O6 O7a	O7c	O5 O7c	O2 O3	O7c	O7c	O2 O3	O2 O3 O5	O7a	O2 O3 O5 O7c

Table 63: Individual Coder Classifications for PMO – Coders 1 – 5

ID	W1 (i)	W1 (ii)	W2 (iii)	W2 (iv)	W3 (i)	W3 (ii)	W3 (iii)	W4 (i)	W4 (ii)	W4 (iii)	W5 (i)	W5 (ii)	W5 (iii)	W6 (ii)	W6 (iii)	W6 (iv)
6	O1 O5 O6 O7c	O1 O2 O3 O6	O1 O2 O3 O5 O6 O7c	O1 O2 O3 O4	O1 O5 O6 O7b	O1 O6 O7b	O1 O5 O6 O7b	O5 O7c	O5 O7b	O3 O7c	O5 O7b	O3 O5	O3 O5	O3 O5	O3 O5	O3 O5 O7c
7	O1 O5 O6 O7c	O1 O2 O3 O6	O1 O2 O3 O5 O6 O7c	O1 O2 O3	O1 O5 O6 O7a	O1 O6	O1 O5 O6 O7b	O5 O7b	O5 O7b	O5 O6 O7b	O5 O7b	O3 O5	O3 O5	O4 O6	O3 O5	O3 O5 O7c
8	O1	O2 O4	O1 O2	O2 O3 O6 O7a	O1 O6 O7a	O6 O7a	O1 O5 O7b	O5 O7b	O7a	O3 O4	O5 O7c	O5 O7c	O2 O3 O5	O5 O7a	O5 O7a	O1 O5 O7a

Table 64: Individual Coder Classifications for PMO Coders 6 – 8

ID	W1 (i)	W1 (ii)	W2 (iii)	W2 (iv)	W3 (i)	W3 (ii)	W3 (iii)	W4 (i)	W4 (ii)	W4 (iii)	W5 (i)	W5 (ii)	W5 (iii)	W6 (ii)	W6 (iii)	W6 (iv)
9	O1 O6 O7c	O2 O4	O1 O4 O6 O7c	O1 O3 O6 O7c	O1 O5 O6 O7b	O1 O4 O5 O6 O7b	O5 O6 O7a	O5 O7b	O5 O7b	O2 O3 O5 O7a	O7b	O7b	O2 O3 O7b	O5 O7b	O5	O3 O5 O6 O7a
10	O1 O4 O5 O6	O2 O4	O1 O2 O4	O1O O7a	O1 O5 O6 O7b	O1 O5 O6 O7b	O1 O5 O6 O7a	O5 O7c	O5 O7c	O3 O4	O6 O7c	O4 O7b	O3 O4 O7b	O2 O4 O7c	O2 O5	O2 O3 O4 O7c
11	O1 O5 O6 O7b	O2	O1 O5 O7c	O3	O5 O6 O7c	O5 O6 O7c	O5 O6 O7a	O5 O7b	O5 O7c	O3	O5 O7c	O5	O2	O1 O4 O5	O7c	O3 O4 O5 O7a
12	O1 O5 O6	O2 O4	O1	O3 O5 O7a	O1 O6 O7b	O5 O6 O7a	O2 O5 O7a	O5 O6 O7c	O5 O7c	O3 O4 O7b	O5 O6 O7b	O2 O5 O7a	O5 O6 O7a	O1 O4 O5 O7a	O5 O7b	O4 O5 O7a

Table 65: Individual Coder Classifications for PMO Coders 9-12

ID	W1 (i)	W1 (ii)	W2 (iii)	W2 (iv)	W3 (i)	W3 (ii)	W3 (iii)	W4 (i)	W4 (ii)	W4 (iii)	W5 (i)	W5 (ii)	W5 (iii)	W6 (ii)	W6 (iii)	W6 (iv)
13	O1 O5 O6 O7a	O1 O2	O1 O2 O5	O3 O4 O7a	O5 O6 O7c	O5 O6 O7c	O1 O5 O6 O7a	O5 O7c	O5 O7c	O2 O3 O4	O5 O7c	O5 O7c	O3 O7b	O5 O7c	O4 O5	O1 O2 O3 O4 O5 O7a
14	O1 O5 O6 O7b	O1 O2	O1 O4 O7c	O1 O3 O4 O7a	O1 O6 O7b	O1 O4 O6 O7b	O1 O5 O6 O7c	O7c	O7c	O2 O4	O5 O7c	O5 O7c	O3 O4 O7b	O5 O7b	O5 O7a	O3 O4 O5 O7a
15	O1 O6 O7a	O1 O2 O4	O1 O4	O2 O3 O4 O6	O1 O6 O7b	O1 O4 O6	O1 O5 O6 O7b	O5 O7b	O5 O7b	O2 O3	O7c	O7c	O3	O7c	O1 O5	O3 O5
16	O1 O6 O7a	O2 O3 O4	O1 O2 O3 O4	O3 O4 O7a	O5 O6 O7a	O1 O5 O6 O7a	O1 O5 O6 O7a	O7c	O7a O7c	O2 O3 O4	O5 O7a	O7b	O2 O3 O4	O5 O7c	O1 O5 O7a	O2 O3 O5 O6 O7a
17	O1 O6 O7b	O1 O2 O4 O7c	O1 O2 O4 O6 O7c	O1 O3 O7c	O1 O6 O7c	O1 O6 O7c	O1 O5 O6 O7a	O5 O6 O7c	O5 O6 O7c	O2 O3 O4 O5 O7b	O6 O7b	O6 O7c	O2 O3 O4 O7a	O3 O6 O7a	O2 O6 O7a	O2 O5 O6 O7a

Table 66: Individual Coder Classifications for PMO Coders 13-17

ID	W1 (i)	W1 (ii)	W2 (iii)	W2 (iv)	W3 (i)	W3 (ii)	W3 (iii)	W4 (i)	W4 (ii)	W4 (iii)	W5 (i)	W5 (ii)	W5 (iii)	W6 (ii)	W6 (iii)	W6 (iv)
18	O1 O4 O5 O7a	O2 O6	O1	O1 O2 O4 O5 O6 O7b	O4 O5 O6	O4 O5 O6	O5 O6 O7a	O5 O7a	O5	O2 O3 O4 O6	O6	O5 O6	O5 O7a	O5	O5 O6	O5 O6 O7a
19	O1 O5 O7b	O1 O5 O7c	O3 O4	O2 O3 O6 O7b	O1 O5 O6	O1 O2 O4 O5 O7a	O1 O2 O5 O6 O7a	O1 O5 O6 O7c	O1 O5 O6 O7b	O2 O3 O4 O5	O1 O5 O7b	O1 O5 O7c	O3 O4	O1 O4 O5 O7b	O5 O6 O7c	O1 O3 O4 O5 O6

Table 67: Individual Coder Classifications for PMO Coders 18-19