LLULL’S WRITINGS
ON ELECTORAL SYSTEMS*

1. Introduction

We present three of Llull’s works dealing with electoral systems, to be cited with the abbreviations given in the parentheses:

Artifitium electionis personarum (AEP)
En qual manera Natanafo eleta a abadessa (B24)
De arte electionis (DAE)

B24 is the twenty-fourth chapter of the novel Blaquerna. For a long time this was the only source on Llull’s electoral systems until, in 1937, Honecker [10] rediscovered DAE. In his commentary, Honecker [10, p. 307, n. 11a] doubted whether the available bibliographical evidence would imply the existence of yet another work of similar title and content. In 1959, finally, Pérez Martínez [28] announced the discovery of the manuscript of AEP, and later included a description of it in his catalogue of Roman libraries [29, p. 39]. While Pérez Martínez was referenced by Platzeck [30, vol. II, p. 7] and Llinarès [12, p. 177, n. 25], neither of the three authors mentioned Honecker’s discovery of DAE. Conversely, in vol. 17 (1989) of the authoritative Raimundi Lulli Opera Latina [20, p.

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the editors claimed that the *Ars electionis* is preserved only in a manuscript in Kues, that is DAE, thus neglecting AEP.

The oldest list of Llull's works is that in his autobiographical *Vita coetanea* [19, p. 307], reproduced with annotations in *Hillgarth* [9, p. 341]. The items of interest to us are:

86. *Ars notandi*
87. *Ars eleccionis*
88. *Alia ars eleccionis*

We believe AEP to be identical with number 87 and DAE to coincide with number 88, on the grounds that in Cod. Vat. lat. 9332 the text of AEP is immediately preceded by that of the *Ars notandi*, and that AEP is longer and more detailed than DAE. DAE comprises about 650 words, while AEP is about 100 words longer. Furthermore, Honecker [10, p. 304, n. 2a] quotes two titles, *De arte eligendorum prelatorum magna* and *De arte electionis compendiosa*, from a Renaissance bibliography drawn up by Arias de Loyola. Honecker identifies DAE with the second title. This assignment seems persuasive to us, in that it leaves the first title for AEP, which indeed mentions repeatedly that its electoral system is intended for the election of prelates.

The recent literature, surveyed by Lohr/Bonner [22], concentrates on Llull's philosophy and does not embrace his electoral systems. Moreover, the electoral systems proposed both by Llull and by Cusanus have long been ignored in the political science literature. This is beginning to change, thanks chiefly to the research of McLean/London [24, 25], Meuthen [27], and Felsenthal/Machover [5], as evidenced in the recent monographs of McLean/Urken [26] and Colomer [3].

Our paper is organized as follows. In Section 2 we comment on the chronological sequence of AEP, B24, and DAE. For a particular work, we refer to the number and the date of composition given in *A Chronological Catalogue of Ramon Llull's Works* of Bonner [1, vol. II, pp. 1257-1304], quoted as «Bo». Sections 3-5 contain transcriptions and translations of the three texts. In Section 6 we describe and compare the electoral systems in these writings. The tract AEP, edited here for the first time, contributes at least three details leading beyond B24 and DAE:

1. multiple records are kept during the election session,
2. *unus punctus* is recorded for the winner of each pairwise comparison,
3. lots are thrown if candidates are tied with the same number of winning points.

In Section 7 we discuss alternative interpretations. Section 8 concludes the paper with a final résumé.
The present paper is accompanied by a web edition at www.uni-augsburg.de/llull/ which offers a linked triple text consisting of facsimile, transcription and translation, for each of the three works. For details of this web edition, see Drton et al. [4]. A narrative on the somewhat accidental circumstances that led to the present research can be found in Pukelsheim [34].

2. The sources

The dating of the first text, AEP (Bo II.A.10: 1274-83), is least certain. Platzeck [30, vol. II, p. 7] places it in the period 1273-5, without providing any detailed evidence. The recent study of Santanach i Suñol [39] reconfirms the period 1274-83 from Bonner’s catalogue. The expert opinion is that, in all probability, Llull completed the novel, Blaquerna, in 1283 in Montpellier; see Soler i Llopart [42]. We shall argue in Section 6 that the electoral systems in AEP and in B24 are distinct from each other. In fact, they are so different that it would seem to us most unlikely that Llull wrote the two works back to back. Therefore we would tend to believe that AEP was written quite some time before 1283.

The AEP is preserved in a single manuscript, Codex Vaticanus latinus 9332 of the Biblioteca Apostolica Vaticana, as reported by Pérez Martínez [28]. The call number 9322 quoted in Pérez Martínez [29, p. 38] is a misprint. The bibliographic information given by Glorieux [8, p. 301], that AEP appears in codex Clm. 9332 in Munich, confuses libraries; at no time did the Bayerische Staatsbibliothek own a codex with this call number.

Cod. Vat. lat. 9332 is an undecorated collection of fascicles in quarto (22.5 x 15 cm), consisting of several distinct parts and totaling 326 folios (paper). It begins with Llull’s Ars notatoria (Bo II.A.2: 1274-6), immediately followed by AEP on folios 11r-12v. Both texts date from the second half of the fifteenth century, and are written by the same hand. The first text starts with a heading giving the title. The second text, AEP, has no heading; instead the title has to be inferred from the colophon: Finis artifitii electionis personarum. The appearance of AEP arouses sympathetic feelings as to how difficult life was for a medieval copyist: the pen blunted, the ink blotted, the writing blurred. Gayà [13, p. 23] tactfully calls the impression that AEP conveys as being de caligrafía más oscura.

AEP appears to have been written by a skilled hand, as a quick, cursory excerpt. The small cursive, with many abbreviations, is in parts hard to read. The number of lines per page changes repeatedly, the rather complicated figure on folio 11r is drawn free-hand. There are no references to the author of the text, nor to the copyist of the manuscript. However, on folios 13r-15v of the codex appears a third Llull text, Liber de accidente et substantia (Bo IV.75: 1313). This text, though by a different hand, says that it is a Raimundo compilatus.
Formerly, Cod. Vat. lat. 9332 was owned by Pier Leoni (d. 1492), court physician of Lorenzo de’ Medici. It is most likely that it was Leoni himself who copied the *Ars notatoria* and the *Artifitium electionis personarum*. This is the conclusion arrived at by Ruysschaert [38, p. 57], on the basis of a comparison of the hands in Cod. Vat. lat. 11585, 9414, 9425 and in Reginensis 1773, all from Leoni’s library. Note that Ruysschaert’s singular, *le premier texte*, comprises a plural, the *Ars notatoria* and the *Artifitium electionis personarum*. For more on Leoni and his relations to Llull’s work see Rotzoll [37].

For B24, Chapter 24 of the novel *Blaquerna* (Bo II.A.17: 1283), we rely on the Catalan manuscript, Codex Hispanicus 67 of the Bayerische Staatsbibliothek, Munich. The volume consists of 268 folios (paper), and dates from the end of the fourteenth or the beginning of the fifteenth century. Authors occasionally quote the obsolete call number Cod. Hisp. 610, from p. 91 in volume 7 of the 1858 (reprinted in 1971) *Catalogus codicum manu scriptorum Bibliothecae Regiae Monacensis*. The transcription in Section 4 below was kindly provided by Soler i Llopart [15]. We also consulted the transcription of Galmés/Ferrà [17], the English translations of Peers [14] and McLean/London [24, 25], and the French translation of Llinarès [18].

DAE (Bo III.38: 1299) is dated rather exactly by Llull himself: it was written on 1 July 1299, in Paris. The *codex unicus* preserving this text is the Codex Cusanus 83 in the Library of the Sankt Nikolaus-Hospital/Cusanusstift in Bernkastel-Kues. Honecker [11, p. 571, n. 23] considers it almost certain that it was written by Nicholas Cusanus. This view is reconfirmed by the more recent findings that Cusanus spent the spring of 1428 in Paris, copying parts of Llull’s works available in the Paris libraries. Roth [36, Sections A.I and A.V] reviews the current state of research on the relation of Cusanus to Llull, and also gives a description of Cod. Cus. 83. For the transcription and the translations we consulted Honecker [10, pp. 308-309] and McLean/London [24, 25].

In our transcriptions, numbers in small print are the line counts in the manuscripts; most of the paragraph breaks are ours. In the translations, we follow the transcriptions rather closely except for adding punctuation. A few additions and omissions that we considered helpful are indicated by brackets [ ]. In the main text, bracketed numbers refer to reference items at the end of the paper. Llull’s electoral systems rely on his Third Figure, the quaternary 16 candidate version being part of AEP and the ternary 9 candidate version heading DAE. These tables are built up from entities which we call *cells*, following today’s statistical terminology. While Llull’s notion of *ars* carries a rather broad meaning, see Bonner [1, vol. I, p. 62], we translate *ars eleccionis* simply by *electoral system*, for the purposes of the present paper.
3. Artifitium electionis personarum
(Biblioteca Apostolica Vaticana, Cod. Vat. lat. 9332, f. 11r-12v)

This is a figure consisting of 16 letters and composed of 120 cells, by which it is explained how an election is carried out. These are the letters: b c d e f g h i k l m n o p q r. And these letters are mixed so as to combine them in pairs. Through the mixture the following figure is composed and formed, as is shown in the Ars compendiosa inveniendi veritatem as well as in the Principia theologiae, philosophiae, and juris. This is the figure:

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bc cd de ef fg gh hi ik kl lm mn no op pq qr
bd ce df eg fh gi hk il km ln mn no op pq qr
be cf dg eh fi gi hk il km ln mn no op pq qr
bf cg dh ei fk gl hm in ko lp mq nr
bg ch di ek fl gm hn io kp lq mr
b[ ] ci dk el fn gn ho ip kq lr
bh ck dl em fn go hp iq kr
bk cl dm en fo gp hq ir
bl cm dn eo fp gq hr
bm cn do ep fq gr
bn co dp eq fr
bo cp dq er
bp cq dr
bq cr
br
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Hec quidem figura signat artem per quam semper poterunt eligi homines conuenientes ad dignitatem prelati- cam et per quam quod in electione ipsa non fiat fraus nec simonia poterit evitari. [folio xlv] Littere enim xvi ex quibus composita est ipsa figura signant 16m personas habentes duocem ad eligendum prelatum. Si uero in capitulo fuerint plures aut pauciores persone habentes uocem ad faciendum ipsam electionem multiplicari debent vel diminui littere ac etiam figura proportionaliter secundum quod persone plures uel pauciores existent. Si autem essent tot persone quot littere alphabetai non sufficere loco ipsarum ponantur alia signa velud numerus vel unus duo tres uel ut primus secundus tertius quartus et sic multiplicando quousque quelibet persona habet signum suum. Et postea ex dictionibus ipsis debet componi figura sicut ex litteris consistit. Composita enim predicta figura non sufficit nisi pro personis xvim.

Prius quidem ordinari oportet quod secundum quod alique persone fuerint in capitulo primus alteris in sedibus preponatur ac quod eis attribuatur priores littere uel priora signia ipsius figure atque quod primo uocenetur et prius de ipsis investigentur.

Post autem hoc oportet statui quod in electione considerentur tria quorum

This figure signifies the system by which one can always elect men worthy of the dignity of a prelate, and by which fraud does not occur and by which simony can be avoided. Now the 16 letters of which this figure is composed signify 16 persons having a vote in the election of the prelate. If in the chapter there are more or fewer persons having a vote to conduct this election, the letters and also the figure must be augmented or diminished in accordance with the greater or lesser number of persons. If, however, there are so many persons that the letters of the alphabet do not suffice, other signs are introduced, such as one, two, three, or first, second, third, fourth, and so on, continuing until each person has his own sign. And thereafter the figure must be composed from these signs as if it consisted of letters. The figure composed as described above, however, suffices only for 16 persons.

Initially it is necessary to establish that, since several persons are in the chapter, the first is seated before the others, and that the prior letters or the prior signs of the figure are assigned to them, and that they should be the first to be called upon and examined in these letters.

After this it is necessary to ascertain that in the election three things should

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(AEP f.11v, 1.12): We are uncertain whether *primus* means the most senior person, the person highest in rank, or the foremost person.
Primum est honestas et sanctitas vitae. Secundum est scientia et sapientia. Tertium est conveniens dispositio cordis. Que quidem tria quilibet persona habens uocem in capitulo ad sacra dei evangelia\(^a\) iuret considerare ac semper preeligere personam in qua ipsa tria melius fuerint.

Post uero ordinacionem predictam oportet quod habeantur tres figure similes figure predicte que teneantur in locis diversis. Et \(^b\) imponantur nomina videlicet quod vna personarum uocetur \(b\), alia \(c\), alia \(d\) et sic de singulis quoque quelibet persona habet litteram [folio 12r] \(sibi\) appropriatam. Si autem figura composita fuerit ex aliis signis \(^c\) attribuantur ipsa signia personis ipsis secundum quod dixi de litteris. Quo quidem ordinato ponant se in domo et incipient facere electionem suam tali modo.

Primo enim oportet quod exeant duo persone quibus litterae vel signia prime camere attribute fuerint et postea querat omnibus aliis per sacramentum que ipsarum duarum melius conueniens et dignia fuerit secundum tria predicta ad dignitatem ipsam habendam et etc. Et omnes responderint et eligerint \(sibi\) pro prout eis uidebatur fiat unus punctus in littera attribuata illi persone que plures uoces be considered, of which the first is honesty and holiness of life, the second is knowledge and wisdom, and the third is a suitable disposition of the heart. Each person having a vote in the chapter should take an oath by the holy gospels of God to consider these three things and to always elect the person in whom they are best (embodied).

After this ruling it is necessary that three figures identical to the figure described above be held ready at distinct locations. And thereafter the names should be inserted in such a way that one of the persons is denoted by \(b\), another by \(c\), another by \(d\), and so on for each, until every person has an appropriate letter. If, however, the figure is composed of other signs, these signs are assigned to the persons according to what I described for the letters. With this matter taken care of, [the electors] should betake themselves into a hall and begin to conduct the election in the following way.

Firstly, the two persons whose letters or signs appear in the first cell should leave the hall. And afterwards [somebody] inquires of all the others on oath which of the two is better suited and worthier according to the three things described above to be bestowed this dignity, etc. And all shall respond, and shall elect as it appears [fit] to them. [Then] a dot is placed by the letter assigned to the person who has the

\(^a\) (AEP f.11v, 1.18): The ms. has evangelia, a Tuscan variant of evangelia.

\(^b\) (AEP f.11v, 1.22): postmodum corrected from posmodum.
habuerit. Qui punctus fiat ipsi littere in qualibet figurarum existentium in locis diuersis. Si uero vna habuerit tot uoces ut altera fiat in qualibet littera ipsius camere punctus vnus et hoc in qualibet figurarum.

Cum autem investigatum fuerit taliter de camera prima videlicet quod de camera ipsius b c fiat illud idem de camera 2a videlicet ipsorum b d et c debet ad locum suum redire et facere illud quod altere persone facient quousque uocabuntur camere in quibus ipse erit. Et hec talis investigatio semper fiat per ordinem videlicet quod primo fiat investigatio de omnibus caerimernis in quibus erit b et postea de illis in quibus erit c postremo de illis in quibus erit d et sic de omnibus per ordinem.

Facta autem inuestigatione camerarum omnium debent numerari puncta cuiuslibet littere et si in aliqua litterarum sive signiorum ex quibus composita fuit figura ipsa reperiantur plura puncta quam in aliqua litterarum persona illa pro qua illa littera seu signum posita fuerit eligatur ad dignitatem propter quam facta fuerit inuestigatio supradicta [folio I2v] quecucumque dignitas sit.

Si autem contiguerit duas personas aut plures habuisse numerum uocum equalem oportet ipsas exire domum

most votes. Such a dot is marked in each of the figures existing at the distinct locations. If now one [person] has as many votes as another, then a dot is placed by both letters of this cell, and this [is done] in each of the figures.

After in this way [the pairwise comparison] has been examined in the first cell, namely in the cell with b and c, the same is done in the 2nd cell, namely that with b and d: and c must go back to his place and do what the other persons do, until again cells will be called in which his letter occurs. And this examination continues in the following order, namely that the examination is first extended over all cells in which b appears, and afterwards over those in which c appears, and then over those in which d appears, and this way over all [cells], in order.

Once the examination of all cells has been completed, the dots of each letter must be counted. And if in any of the letters or signs from which this figure is composed more dots are determined than in any of the other letters, the person for whom this letter or sign stands is elected to the dignity for which the examination that is described above has been conducted, whatever the dignity.

If, however, it happens that two persons or more have an equal number of votes, it is necessary that they leave

\[\text{(AEP f.12r, l.17): ipse corrected from ipsum.}\]
et quod relique persone remanentes qui nequaquam tot uoces habuerint ibi de nouo iurent obseruando tria superius posita preheligere digniorem et magis conuenientem ad ipsam dignitatem habendam et quod illa in qua plures uoces erunt concordes eligatur. Si autem contingat tot uoces concordari in una sicut in altera miscentur sortes super illas que in ista electione ultima equalem numerum uocum habebunt et illa cui sors euenerit eligatur.

Si autem in principio inuestigationis predicte absens fuerit aliqua persona habens uocem in ipso capitulo oportet quod attribuitur ei vna predictarum litterarum ipsius figure uel signiorum si fuerint alia signia quam littere. Si uero plures absentes fuerint attribuat cuilibet littera vna uel signum. Et de ipsa uel de ipsis quemadmodum de presentibus inuestigetur secundum ordinationem predictam.

Finis artifitii electionis personarum.

\( ^{6} \) (AEP I.12v, 1.7): miscentur corrected from mictantur.
4. En qual manera Natana fo eleta a abadessa
(Bayerische Staatsbibliothek, Cod. Hisp. 67, f. 32v-34r)

[folio 32v] Nathana e totes les dones qui ha-vien veu a elecció foren en lo capitol per eleger abadesa. Natana dix a totes les dones que necessitat es molt gran haver bo major, car en la bonitat del major dona Deus virtut al menor. «On, con nostre major sia passat d'esta vida en l'altra segons que natura e rahó requer, neçessària cosa es a nosaltres que encerquem enfre nos qual de nos es major en sànta vida e en la amor de Deu; cor, aquel es digné de esser nostre pastor, segons l'ordenament e la voluntat de Deu.

Totes les dones volgueren elegir abadessa segons la manera en la qual havien acustumat a eleger, mas Nathana dix que ella havia enesa novella manera de elecció, la qual está en art e en figures, la qual art segueix les condicions del Libre del gentil e dels iij savis, lo qual segueix la Art de trobar veritat. «Per aquella manera— dix Natana, —es trobada veritat per la qual veritat podem trobar aquella dona qui es pus cuvinent e millor a esser abadessa.

Pregada fo Natana per totes les dones que digne la manera segons la qual per art poguesen trobar e eleger la dona qui es millor a abadessa. Nathana respòs dient estes paraules: «De
la art de elec-ció vos diré breument los començamens. Aquella art es departida en dues parts: la primera part es de evelopar los elegedors qui elgen lur pastor; segona part es en qual manera dejen eleger lur major; e, per 11açó, primerament vos vull recomptar de la primera part e puys de la segona.

Natana dix: —Nos som 13xx dones en est capitol qui havem veu a eleger pas14tor. Segons art se cové que ele-jam, d’estes xx dones, 15nombre senar qui sia en v o en vij, cor aquest nom16bre es pus cuvinent a elecció que altre; e lo vij nom17bre es pus cuvinent quel v. On, primerament, sia 18 fet sagrament per totes les dones a dir veritat e sia 19 demanat secretament a la primera dona qual de les 20xix dones son pus cuvinents a esser vij qui elegen 21major; e, aprés, sia demanada la segona dona, he 22 puxes la terça e axí per orde tro a la derrera. E cas23 cun-
a vegada, scriva hom ço que diu cas-cuna de les 24 dones; a la fi, sia vist quals son aquelles dones qui han 25 haudes mes veus; e aquelles que hauran haudes 26 mes de veus, aquelles sien les vij dones qui degen 27 eleger abadessa.

«Segona part de elecció es con los 28 vij elegedors elgen pastor. On, pri-merament, [folio 33v] cové que los vij elegedors se covenguen al eleger de 29 cert nombre e de certes persones, segons quel[s] serà vijares, 3e que cas-

with these words: «I will tell you briefly the principles of the electoral system. The system is divided into two parts. The first part is to elect the electors who then elect their pastor; the second part concerns the method by which they should elect their superior. So I will first tell you about the first part, and then about the second.»

Natana said: «In this chapter we are 20 sisters who have a vote in electing our pastor. According to the system we must elect from these 20 sisters an odd number, say 5 or 7; for such a [odd] number is more convenient for the election than another [even number], and the number 7 is more convenient than 5. First let an oath be taken by all the sisters to tell the truth. Then let the first sister be asked in secrecy which of the other 19 sisters are most suitable to be among the 7 who elect the superior. Afterwards, let the second be asked, and then the third, and so on, in order, until the last. And on each occasion, let it be written down what each sister says. In the end, let it be seen who are the sisters who have received the most votes. And those who have the most votes shall be the 7 sisters to elect the abbess.»

«The second part of the electoral system is how the 7 electors elect the pastor. Firstly, it is necessary that the 7 electors agree upon a certain number and upon certain persons, according to what seems to them good, and that
cuna persona comparen ab l’altra, segons quatre condicions, ço es, a saber: qual ama e coneix mes Deu, ni qual ama e coneix mes virtuts, ni qual coneix e desa mas forment vicis; quarta es qual ha pus cu-vinent persona.

Cascú dels vij elegedors pot eleger una persona a esser en lo nombre del qual deu esser elet major; e cascú dels vij elegedors cové esser en aquel nombre on pusca esser elet major. E, per ço que pus planament puschats entendre la art, sotsposem que lo nombre sert sia en ix persones de les quals sia triat elet nostre pastor; on, primerament, cové que los vij sien devesits en dues parts: ij a la una part e v a l’altra e cové que los encerquen quals dels ij deu esser elet e scrivas secretament aquell qui ha mes de veus. Apres cové que ab la qui ha mes de veus haudas sia comparat altre d’aquells e que sia mes en lo loch de aquel qui es estat vençut per menors veus; e aquel vençut sia mes en lo loch d’aquel qui es comparat ab lo primer o ab lo segon. E açò mateix, per orde, sia en tots los altres, e sien en est nombre mes los vij el ix qui no son dels elegedors. On siguient aquest nombre, se-ran multiplicades xxxvij cambres en les quals apparran les veus de cas, e sia elet aquel qui haurá mes veus en mes 27 cambres.

they compare each person with every other according to four conditions, namely: which of them best loves and knows God, which of them best loves and knows the virtues, which of them knows and most strongly hates the vices and, fourthly, which is the most suitable person.»

«Each of the 7 electors can select a person to be among those from whom the superior will be elected, and each of the 7 electors necessarily is among this number from whom the superior is elected. And so that you can understand the system more plainly, let us suppose that this number consists of 9 persons from whom our pastor is to be elected. Firstly, it is necessary that the 7 are divided into two groups: 2 on the one side and 5 on the other; and then the 5 must decide which of the 2 should be elected; and in secrecy let it be written down who has the most votes. Afterwards, the 1 who has the most votes should be compared to another one of the 5, and that this sister be set in the place of the one who is defeated by fewer votes; and the one who is defeated is put in the place of the sister who is now compared with the first or with the second. And let this be done, in order, with all the others; and to this number are added the 8th and 9th persons who are not among the electors. Following this numbering, 36 cells are generated in which appear the votes of each; and let her be elected who has the most votes in the most cells.»
Con Nathana hac mostrada la art de elecció, una dona demana a Nathana: -Si s’esdeve que en les cambres haja qui hagen equals veus, quen mana fer la art? Respós Nathana: -La art mana que s’ia sobre aquells i o iij o mes encerchat per art tant solament; e sia encerchat qual d’aquells se cové mils ab les iiij condicions demunt dites; e aquel qui mils se cové ab les condicions, aquel es digne de esser elet.

Molt plach la art e la manera de elecció a totes les dones, e totes digueren que segons aquella art no pudia hom errar en elecció; e totes feeren establiment que per tots temps elegisen per aquella art e manera que Natana recomptà; e trameteren encerchar la art e aprengueren aquella. Après pochs de dies feeren elecció segons la art e atrobaren per art que Natana devia esser abadesa.

Eleta fo Nathana a abadesa; gran desplaer hac Natana de son honra-ment. Deus beney qui la volch honrar sobre totes les altres. Duptá que les dones haguessen errat en la art e volch veer les xxxvj cambres en que està la art, per tal que si havien errada la art e ella no devia esser abadesa, que elegisen aquella ab qui l’endreçament de la art se covengués. Natana e les altres dones qui no eren stades de les vij dones qui havien ele-git veeren la manera que havien

When Natana had explained the electoral system, one of the sisters asked her: «If it turns out that in the cells there are some who have equal votes, in which manner does the system proceed?» Natana replied: «The system demands that among these 2 or 3 or more one decides solely by means of the system; and that [in this way] one decides who of them combines best the 4 conditions mentioned above. And she who combines the conditions best is worthy to be elected.»

All the sisters were very pleased with the system and the electoral method; and all said that according to this system an electoral error could not occur. They all established a rule that at all times they would elect using the system and the method which Natana had described; and they sent away to find out about the system and they learned it. After a few days they held an election according to the system, and by the system it was found that Natana was to be abbess.

Natana was elected abbess; [but] Natana felt great displeasure at being thus honored. She blessed God for intending to honor her above all the others, [yet] feared that the sisters had erred in the system. And she wanted to see the 36 cells of which the system consists so that, if they had made an error in the system and she should not be abbess, they should elect another one in keeping with the proper working of the system. Natana and the other sisters who had not been among the 7 sisters who
had participated in the election, reviewed the method which had been followed in the election, according to the system, and found that the system had been followed as stipulated. Natana began to think deeply on how she could and should rule herself and the sisters, and contemplated every day how she could direct the convent so as to encourage good ways.

5. De arte eleccionis
(Sankt Nikolaus-Hospital/Cusanusstift, Cod. Cus. 83, f. 47v-48r)

Evidently a good electoral procedure in the Holy Church is very necessary for electing office holders, since by them the church is governed and they fight against the church’s enemies who commit sins like the infidels and schismatics. And the elected office holders...
electe facere non possunt nisi bone\textsuperscript{6} sint et bene proporcionate matri \textsuperscript{7}ea- rum que bona est et alta. Quamur ma-
ter est sacrosancta roma\textsubscript{8}na ecclesia que magnas habet passiones per illos qui se fingunt \textsuperscript{9}bonos esse filios qui boni non sunt sed mali matri derogan-
tes \textsuperscript{10}et bona eius iniuste occupantes. Unde propter hoc doctrinam\textsuperscript{11} dare v0\textsubscript{11}lumus ad elygendum communem personam secundum modum tercie fi-
gure artis \textsubscript{12}generalis ita quod per ar-
tem suam sequendo processum pote-
runt si velint eligentes \textsubscript{13}meliorem eli-
gere personam et hoc palam et si me-
liorem non eligant et 14omnibus in

capitulo existentibus manifestum quod peyorem eligunt et ipsi sunt perjuri sine \textsubscript{15}aliquo colore excusacionis. Modus autem eligendi est iste.

Primo ut dispositio\textsubscript{16} ne apparet\textsuperscript{b} aut-
tem primam personam significat que in ecclesia vacante prius fuit \textsubscript{17}recepta

c\textsuperscript{'} signifies secundam personam et sic
deinceps usque ad \textsubscript{k} ita quod prima

ersa vacantis ecclesie vocabatur \textsubscript{b}
et secunda persona vocetur \textsubscript{c}\textsuperscript{'} et sic de
aliis \textsubscript{s} suo modo. Si vero in ecclesia
plures sint quam \textsubscript{i}x persone multipli-
centur eamere \textsubscript{l} in figura suprascripta
addendo \textsubscript{l}. Et si sint undecim addatur \textsubscript{m}. Si autem \textsubscript{21}plures sint quam littere
sint in alhabeto apponatur alius nu-
cannot do this unless they are good
and in good relations with their mother
who is good and noble. Their mother
is the Holy Roman Church who
endures great pains from those who
pretend to be good sons, yet who are
not good but evil, and who disparage
their mother and unjustly take her
goods. Therefore we want to give this
explanation on how to elect an office
holder, based on the Third Figure of
the Ars generalis, so that in following
this system the electors can, if they
wish, elect the better person, and do so
in public. And if they do not elect the
better one, it will be manifest to all
who exist in the chapter that they are
electing an inferior one and they are
perjuring themselves without any
shadow of an excuse. The electoral
method is the following.

Firstly, as appears in the [above] dis-
play, let \textsubscript{b} signify the person who was
received first into the vacant church, \textsubscript{c}
signifies the second person and so on
to \textsubscript{k}, so that the first person of
the vacant church is called \textsubscript{b}, the second
person is called \textsubscript{c}, and so on with the
others in the same way. If, however,
there are more than 9 persons in the
church, the cells in the figure shown
above are increased by adding \textsubscript{l}. And
if there are eleven, \textsubscript{m} is added. Now if
there are more [persons] than there are

\textsuperscript{6} (DAE f.47v, 1.6): bone corrected from boni.
\textsuperscript{7} (DAE f.47v, 1.10): doctrinam duplicated.
\textsuperscript{8} (DAE f.47v, 1.16): Honecker [10, p. 308, n. 5] reads: Primo ut discretioni vestre apparet ...
\textsuperscript{9} (DAE f.47v, 1.17): c corrected from s.
\textsuperscript{10} (DAE f.47v, 1.18): c corrected from s.
merus scilicet unus vocetur primus frater alius secundus etcetera et omnes sequuntur modum predictum.

In principio omnes facient iuramentum quod personam eligent meliorem et magis ydoneam. Deinde qui eligere debeant sedeant et b c sint stantes ad partem ita prope quod audiant verba eligencium et ut omnes ipsos videant. Postmodum d petat ab unoquoque sedente quem de b c plus voluerit pro abbate priore aut episcopo et sic de alius et tunc computentur voces singulorum eligencium et si b plures voces habeat quam c sedeat c in locum suum et b pedes remaneat vel e converso. Et mittatur ad et tunc surgat c qui ab unoquoque petat quem de b d voluerit pro prelato et si b plures voces habeat sedeat d et c vadat e ad b et tunc petat c vel d aut alius ab unoquoque correom omnibus quem plus voluerit pro domino scilicet b aut e et si e vincat b tunc vadat ad e aut contrario et si b sit devictus c petat ab unoquoque quem de e f plus voluerit pro domino et sic per ordinem usque ad k ita quod [ ] in [ ] k fiat determinatio eleccionis sive k devincat vel devincatur et sic per ordinem predictum

\[1\] (DAE f.47v, 1.23f): deinde duplicated.
\[2\] (DAE f.47v, 1.27): eligencium corrected from elegencium.
\[3\] (DAE f.47v, 1.28): e corrected from s.
\[4\] (DAE f.47v, 1.28): c corrected from s.
\[5\] (DAE f.48r, 1.1): Erasure [ ].
\[6\] (DAE f.48r, 1.1): Erasure [ ].
or is defeated, and this in the order described above. That is, if there are ten in the chapter then the end is reached with $l$, and in the order that has just been described.

This electoral method is very useful and secure, since it is far from the secret $\textit{scrutinium}$ and the election $\textit{per compromissum}$ wherein more fraud can be committed than in the above method. Those who elect publicly face great disgrace by their colleagues if they elect badly; those who elect secretly do not.

This new electoral method is also good in that it is as general as [no other] can be. And this is so because in the election a comparison is made between all persons who exist in the chapter. Each of them elects according to his will in each cell of the above figure, and thus is more content with the election.

Moreover, with this electoral method each person in the chapter presumably would strive to be good and honorable, if this method were used in the election of prelates; and [each would] procure friends in the church, bring about peace, and avoid animosity, so that at the time of the [next] election he might be elected. And the brethren would

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$^a$ (DAE f.48r, 1.8): \textit{etiam autem} corrected from \textit{ecciam ante}.

$^b$ (DAE f.48r, 1.9): Honecker [10, p. 309, n. 15] inserts \textit{quantum alius} before \textit{esse non potest}.

$^c$ (DAE f.48r, 1.13f): \textit{essencia} expunged.

$^d$ (DAE f.48r, 1.15): Erasure [ ].
cione unus\textsuperscript{v} staret, pro alio et sic exaltatum esset capitulum scilicet per exaltacionem quam fratres haberent per caritatem iusticiam prudenciam et per alias virtutes. Si autem eleccio sit facienda de personis absentibus fiat eleccio secundum predictum etc.

Factus est iste modus eleccionis parysius anno incarnacionis domini nostri Jhesu Christi Mo ducentesimo primo die Julii. Deo\textsuperscript{w} gracias.

show mutual esteem, so that in an election one would stand for the other. And so the chapter would be enhanced, that is, [it would gain] through the enhancement which the brethren would enjoy through charity, justice, prudence, and other virtues. If now an election of absent persons is to be made, the election should be carried out as described above, etc.

This electoral method was devised in Paris, on the first day of July in the year 1299 of the Incarnation of our Lord Jesus Christ. Thanks be to God.

6. Llull’s electoral systems

6.1. The trail to truth

Llull’s electoral systems in AEP, B24 and DAE have in common that they are intended for the election of church officials in clerical communities. In AEP there are 16 persons or a few more who elect a prelate, without any further indication as to who they are or how they are selected. In B24 it is 20 nuns of a convent who elect a new abbess. DAE has 9 persons or a few more electing an abbot, prior, or bishop.

In each of the three writings Llull quotes some of his prior work. AEP and B24 both mention the \textit{Ars compendiosa inveniendi veritatem} (Bo II.A.1: c. 1274), see [32]. In addition, AEP refers in a somewhat cursory fashion to the \textit{Liber principiorum theologiae} (Bo II.A.6: 1274-8), the \textit{Liber principiorum philosophiae} (Bo II.A.7: 1274-8), and the \textit{Liber principiorum juris} (Bo II.A.8: 1274-8). For an edition of these along with the \textit{Liber principiorum medicinae} (Bo II.A.9: 1274-8) see [21]. B24 quotes the famous \textit{Llibre del gentil e dels tres savis} (Bo II.A.4: 1274-6), see [16].

\textsuperscript{v}(DAE f.48r, l.15): unus corrected from unius.

\textsuperscript{w}(DAE f.48r, l.20): Deo corrected from De.
In DAE, Llull says that the Third Figure is taken from the *Ars generalis*. Platzeck [30, vol. II, p. 34, n. 85] identifies this reference with the *Ars compendiosa* (Bo III.36: 1299). Bonner [1, vol. I, p. 22, n. 88] points out, however, that from 1294 on Llull used the term *Ars generalis* as a generic reference to his system, the final version of which appeared in the *Ars brevis* (Bo III.77: 1308) and the *Ars generalis ultima* (Bo III.80: 1305-8). A version of the Third Figure is reproduced on plate XVII in Bonner [1, vol. I, after p. 582].

Self-references of authors to their own works have been popular from the middle ages till today and, besides advertising the author's oeuvre, serve to claim an advanced level of competence. It would seem to us that, in the present case, Llull may want to send a more pointed message to his readers, emphasizing the ultimate function that he attributes to an electoral system. In our time we view an election as a fundamental building block of mass democracies, a means to collect the opinions of individual voters and to aggregate them across the entire society. This integrating function of popular elections in democratic societies would have appeared totally alien to Llull.

As a devoted Christian in his time, Llull's concern was not the aggregation of many individual truths, but the discovery of the one and only truth existing, the truth of God. We understand Llull's electoral systems as manifestations of his *Ars generalis*, and as such they are means to set mankind on a trail leading to the unique, divine truth: «By this method,» said Natana, «is found the truth: by this truth we will be able to find the sister who is most suitable and best to be our abbess.» (B24, f. 32v, l. 17 - f. 33r, l. 3).

6.2. The bookkeeping of sincerity

The Third Figure of the *Ars generalis* is displayed right at the beginning of DAE. It is an arrangement of the 36 pairs that can be formed from 9 candidates named \( b, c, \ldots, k \). A larger version of the figure, for the 120 pairs from 16 candidates \( b, c, \ldots, r \), is included on the first folio of AEP. The use of letters to indicate candidates serves a double purpose, of exhibiting the universal applicability of the system, and of producing a figure small enough to fit the limited space available.

The question arises, however, whether the sequence of letters is meant to indicate a ranking of the candidates. If so, the candidate with the first letter would hold the highest rank, and the candidate who is assigned the last letter would be ranked lowest. The other candidates would be ranked in-between, as determined by the order of the letters. If the letters were to indicate a ranking, then it is conceivable that Llull might have wished to design a biased electoral
system favoring candidates of higher merits at the expense of candidates of lower merits. We maintain that such hypothesis, that the assignment of letters presupposes a ranking by merits, is inconsistent with Llull’s way of thinking.

Llull felt himself called upon to devote his life to spreading the Christian truth to the Muslim world. Pindl [16, p. 264] recalls a tragic incident from Llull’s autobiography, the *Vita coetanea*, as an episode of lasting effect, see [19, p. 279] or [1, vol. I, p. 21]: Llull and his Saracen teacher of Arabic enter into a fierce argument. The teacher draws the sword against his master. Llull successfully defends himself, and imprisons the teacher. While Llull contemplates what to do next, the discouraged Muslim hangs himself. Thus powerful demonstration of authority has led to a double defeat: the infidel is lost to everlasting damnation, and Llull failed to convert the disbelieving soul to his Christian God. Henceforth Llull builds on the force of arguments, not on the power of authority. He wishes to convert his opponents to Christianity through dialogue, not to dictate.

To this end the combinatorial aids of the *Ars generalis* serve as a warranty of fairness, so that no contraposition of opinions is omitted. For the electoral systems, the Third Figure is used to implement a bookkeeping of sincerity, that is, to secure a complete comparison of all arguments. Furthermore, Llull uses combinatorial structures quite generally to organize his philosophical view. But nowhere does Llull imply that the assignment of letters presupposes a rank order of arguments, see Platzeck [30], Pring-Mill [33], Hillgarth [9], or Bonner [1]. Such a clandestine outmaneuvering of opponents is simply incompatible with a protagonist of the cross-cultural dialogue, as Llull is hailed by Pindl [16, p. 259]. We believe that it was entirely outside Llull’s way of thinking that the assignment of the letters might bias his electoral systems in one way or the other.

Not surprisingly, the instructions of how to assign letters to persons turn out to be cursory and incidental. In addition, the relevant section in AEP (f. 11v, l. 11-14) is somewhat hard to decipher. We understand that somebody, a primus of whatever sort, takes the lead to randomly attribute letters to persons. A brief examination then makes sure that all electors remember their letters. B24 makes mention neither of the letters nor of the assignment process. We take this as an indication of how unimportant this step is, in Llull’s view. In DAE (f. 48r, l. 9-10) Llull praises his electoral system because in the election a comparison is made between all persons who exist in the chapter, that is, because all chapter members are treated equally.

In the sequel our analysis is based on the assumption that all pairwise comparisons are of equal value, so that the decision between the first and the second candidates, *b* and *c*, enters the final evaluation with exactly the same weight as the decision between the first and the last candidates, *b* and *k* (or *b* and *r*).
6.3. The evaluation of a single pairwise comparison

All three variants of Llull's electoral system build on pairwise comparisons of two candidates at a time. They differ in whether an exhaustive, or a partial series of pairwise comparisons is stipulated. In the present subsection we discuss the evaluation of a single pairwise comparison.

The instruction in AEP (f. 12r, l. 9-10) is clearest: [Then] a dot is placed by the letter assigned to the person who has the most votes. Thus the victorious candidate scores one point, and what is recorded is this winning point and not the number of votes that led to the victory. We are reminded of an ordeal by judicial duel, in which the truth emerges on the side of the victor no matter how easy or how hard he had to fight. This way of arriving at a decision is concordant with Llull's belief in the uniqueness of the divine truth: among two candidates, one must be more worthy than the other.

The electors are humans tempted by sin, however, and it is not guaranteed that they recognize the truth in any case and at any time. Therefore Llull includes a rule for the situation when the electors fail to recognize the truth and two candidates are tied with the same number of votes (AEP, f. 12r, l. 11-12): If now one person has as many votes as another, then a dot is placed by both letters of this cell. By awarding a winning point to each of the two candidates, they remain equal between each other and yet gain an advantage over the other candidates.

The corresponding passage in B24 (f. 33v, l. 16-17) is briefer: and in secrecy let it be written down who has the most votes. We think that the meaning of the phrase is identical to the ruling in AEP. For at the beginning of B24 (f. 32v, l. 14-15), Llull claims to propose a new electoral method based on a system and figures. In the middle (B24, f. 33v, l. 3), he states that the electors compare each person with every other. Moreover (B24, f. 33v, l. 24-25), a total of 36 cells are generated. Clearly this points to the Third Figure, even more so since in the other two texts the figure is reproduced in its entirety. From this we conclude that 'written down' means 'marked': 'and in secrecy let the letter of the person be marked who has the most votes'. With this interpretation, B24 defines the same rule as does AEP. We cannot imagine that the quoted passage would support an extended meaning, to write down the number of votes of the victorious candidate, let alone two numbers, one for each candidate.

B24 does not contain a rule on how to break a tie. Llull may have hoped that his insistence on an odd number of electors (B24, f. 33r, l. 15) makes a tie-breaking rule superfluous. This is true for the majority of comparisons, when two of the seven electors are compared by the remaining five, or when the two additional candidates are compared by all seven electors. However, the other
cases are left dangling in the air. When one of the electors and one of the additional candidates are compared, then there are six electors and a tie with three votes for each candidate becomes possible. Such ties are not accounted for in B24.

Nor does the last text of the electoral trilogy, DAE, feature a tie-handling rule even though here it is needed most. DAE promotes a partial system of pairwise comparisons, based on successive eliminations. Therefore each pairwise comparison must leave a well-defined victor, who then proceeds to the next round. Other than in B24, no mention is made that an odd number of electors would be advantageous. Nor would the AEP rule, that tied candidates are treated equally, be of any help since just one candidate can proceed to the next round. In our view, the absence of a tie-breaking rule in DAE constitutes a fatal omission. If DAE was practiced as is, then the divine truth must have shone rather brightly in order to keep the electoral business on the road of decidability.

6.4. The overall evaluation of the election

The global evaluation of all pairwise comparisons is settled most clearly again in AEP (f. 12r, l. 20-21): Once the examination of all cells has been completed, the dots of each letter must be counted. The overall winner of the election is the one who scores the greatest number of winning points, that is, who wins most of the pairwise comparisons. Two extra rules take care of the possibility that two or more candidates are tied with the same number of winning points. Firstly, another election is to be conducted among those tied. Secondly, if yet again a tie emerges, lots are thrown.

The last rule sounds surprising since the Christian Church condemned the throwing of lots ever since the soldiers threw lots at the feet of the crucified Jesus to divide his garments among them (St John 19,24). Maleczek [23, p. 129] reports that the Church banned lots from electoral decisions in 1223. Did Llull realize that the inclusion of lots was detrimental to promoting his system to a clerical clientele? In any case, in B24 and DAE he does not follow up on the idea. Remarkably, in the political world of today, every electoral law we know of includes a final clause to break eventual ties - by drawing lots.

In B24 (f. 33v, l. 26-27) the winner of the election is the candidate who has the most votes in the most cells. We think that this ambiguous instruction is meant to set the same rule as in AEP, that the candidate with the largest number of winning points across all pairwise comparisons is the winner of the election. For the case of a tie between two or more candidates, only the first tie-handling rule of AEP is repeated, to conduct a further election among those tied.
In contrast to the exhaustive schemes in AEP and B24, of considering all pairwise comparisons, DAE advocates a partial system of successive eliminations. A candidate who loses in a pairwise comparison is eliminated from further consideration. Only the victorious candidate enters into the next round, and the last round's victor is the overall winner of the entire election. We are reminded of the devout Christian knight who, in Chapter 64 of Blaquerna, proves the Christian truth to the Saracen king by fighting duels with all the knights of the king's court, one after the other, see [31, p. 82] and [1, vol. II, p. 754, n. 44].

Today's understanding of electoral systems focuses on the freedom with which individuals cast their votes; then the exhaustive system of AEP and B24, and the DAE system of successive eliminations may lead to diverging results. However, Llull coped with devising a decision procedure that would unerringly converge to the one and only divine will. He may have hoped that his goal would not be hampered by changing some electoral details here and there. In fact, it is easy to propose some hypotheses under which he could rightly hope so.

6.5. A hypothetical convergence of systems

Formally, the exhaustive comparisons of AEP and B24 and the partial comparisons of DAE are different systems, possibly leading to different results. Under mild hypotheses, however, the differences vanish and the final conclusions coincide. For an illustration, let us agree on the following two hypotheses,

(i) that the truth set by God is unique, and
(ii) that, in any decision instance, it discloses itself to at least half of the electorate.

Llull would have emphatically supported our assumptions, or so we believe. Under hypotheses (i) and (ii), the electoral systems of AEP, B24, and DAE lead to the same result. We forgo a stringent proof of this claim, instead illustrating it by example.

Suppose there are nine electors a, b, c, d, e, f, g, h, i, k, each of whom also stands as a candidate. By hypothesis (i) we may assume that there is a unique true ranking (*) in which God sees the candidates. In this example, we suppose it is given as follows:

(*)  e < g < k < b < d < i < h < c < f
That is, $e$ is the weakest candidate, $g$ is the second weakest candidate, and so on up to $c$, the second most worthy candidate, and finally $f$, the worthiest candidate.

We assume that the six electors $b, c, d, e, f, g$ wisely see the candidates in the same order that is determined by the true ranking (*). The other three electors, $h, i, k$, deviate from the divine will (*) and base their decisions on the following individual rankings:

- Elector $h$: $d < i < h < k < e < f < g < b < c$
- Elector $i$: $h < d < g < k < b < f < c < e < i$
- Elector $k$: $h < e < k < d < i < f < c < g < b$

When two of the six wise electors are up for comparison, they are not permitted to vote. Therefore the least favorable decision instance still has four wise electors as opposed to three individualists, so that our assumptions satisfy hypothesis (ii).

Now the election begins. Round 1 features the pairwise comparison between $b$ and $c$. The electors $d, e, f, g$ check the ranking (*) to find that $b$ ranks below $c$, and hence vote for $c$. So do electors $h$ and $i$. Only elector $k$, upon checking his or her individual preference scheme, ranks $b$ ahead of $c$, whence this vote goes to $b$. In total, $c$ beats $b$ by 6 to 1 votes. Candidate $c$ records a winning point, and proceeds to the next round.

Following the rules of DAE, round 2 compares $c$ and $d$. Everybody ranking $c$ above $d$, candidate $c$ wins with 7 to 0 votes. Thus $d$ is eliminated. Candidate $c$ enters round 3, together with $e$. It transpires that $c$ beats $e$ by 6 to 1 votes, whence $e$ is out.

In round 4 the margin of victory turns out to be as narrow as possible. For, according to the electoral rules, candidates $c$ and $f$ are excluded from voting. This leaves just four electors blessed with the insight into the true ranking (*); they vote for $f$. When electors $h, i, k$ check their individual preferences, each finds $c$ ahead of $f$. Hence $f$ beats $c$ by 4 to 3 votes, and the victor of round 4 is $f$.

Moreover, $f$ also wins rounds 5 through 8, against $g, h, i, k$. Thus eight pairwise comparisons, out of a total of 36 possible pairings, establish the end result: the overall winner is $f$, in agreement with the divine ranking (*).

Following the rules of AEP and B24, another 28 rounds are called for until all pairwise comparisons are exhausted. In each case the two candidates standing for comparison are banned from voting, and the remaining electors check the preferences peculiar to them before they cast their votes. Then unius punctus is marked near the letter of the winning candidate. In the end, the electoral record looks as follows:
The final step is to count the number of winning points. Candidate $f$ gains the largest number of winning points, 8, and hence is the winner of the election. Moreover, we may count the winning points for all other candidates, so as to arrange them in winning order: $e$ is lowest with 0 winning points, $g$ is second-lowest with 1 point, etc., thereby actually reproducing the true ranking (*). Thus Natana's promise is fulfilled that, by this method, is found the truth.

From our modern viewpoint it may seem overly simplistic to postulate hypotheses under which Llull's electoral systems cannot but find the truth. And evidently our two hypotheses are not exhaustive of Llull's thinking, because then he would not have added additional rules to handle ties. On the other hand it would seem anachronistic to accuse Llull, who lived seven centuries ago, of not providing a stringent argument that we find pleasing by today's standards. Llull may have well satisfied himself by checking a few sample situations, like the one we have discussed just now.

6.6. The reduction of complexity

For all of his life Llull endeavored to improve the structure of his *Ars generalis*, so as to make it more easily accessible to his audience. Llull's contemporaries were intrigued by the complexity of his philosophy, and presumably also felt that his electoral systems were fairly complicated and complex. We think that the changes from AEP over B24 to DAE may well have been motivated by Llull's desire to increase acceptability by decreasing complexity.

It is plainly evident that basing the electoral record on the Third Figure was overly demanding. None of its versions mentioned in the present paper is drawn
without error. In AEP the cells $bh$ and $bi$ are labeled incorrectly, in DAE the cell $ei$ is misnamed, and so is the cell $EK$ in plate XVII of Bonner [1, vol. I, after p. 582]. Perhaps this is why Llull requires multiple copies of the control figures (AEP, f. 11v, l. 20-21), and perhaps this is why Natana hopes that an electoral error spares her the burden of becoming abbess (B24, f. 34r, l. 16-24).

The first electoral system, from AEP, needs a considerable amount of time to be carried through. For every pairwise comparison

1. two candidates leave the electoral hall,
2. the remaining 14 electors submit their votes,
3. the votes are totaled,
4. a winning point is marked in the electoral records, and
5. the candidates are called in again.

Assuming that the electors declare their votes with appropriate deliberation, a single round may well have taken from 5 to 10 minutes. With 120 pairwise comparisons, the election then lasts from 10 to 20 hours. An adjournment overnight, however, counteracts Llull’s concern to prevent fraud and simony, as expressed in AEP (f. 11r, l. 9) and DAE (f. 48r, l. 5-6).

B24 presents the accelerating idea of reducing the electoral community, by selecting a smaller electoral college. A college size of seven is taken to be optimum. In the least favorable case, two of the electors are up for comparison and hence cannot cast a vote themselves. This leaves five voters, and a winning majority must gather at least three votes. To claim legitimacy, a vote of three may have seemed sufficient to Llull, for where there are two or three gathered together in my name, there am I in the midst of them (Matt. 18,20), see also Ganzer [6, p. 76]. With an electoral college of seven, plus two additional candidates as in B24, the duration of the electoral business is shortened to last between 3 and 6 hours.

The third system, in DAE, does not decrease the number of electors. Instead it diminishes the number of pairwise comparisons. With a touch of ingenuity Llull turns away from an exhaustive scheme of pairwise comparisons, and replaces it by a partial system of successive eliminations. For $n$ candidates, an exhaustive system needs $(n-1)n/2$ rounds, while a system of successive eliminations makes do with $n-1$ rounds. With 16 candidates, this is a spectacular reduction from 120 comparisons down to 15, with 9 candidates, from 36 down to 8. Since a system of successive eliminations does not require keeping a detailed electoral record, the election may need no more than one hour.

Curiously, though, Llull neglects the hazards of a successive eliminations system, which may suffer from a severe bias in favor of those who enter late at
the expense of those who enter early. In summary, we conclude that the changes from AEP via B24 to DAE entail a remarkable acceleration of the electoral business.

6.7. The abandonment of saniority

In medieval electoral systems it was neither time nor number that in the end constituted the binding effect of the electoral outcome. The crucial requirement is saniority, that is, the claim of the majority that in the face of God they are more sound of mind and more pleasing than the minority. *Maior et sanior pars* is the motto, see Ganzer [6] and Schimmelpfennig [40], [41]. The majority is well advised not to just celebrate their numerical victory, but to also claim a level of superior quality. If the minority succeeds in occupying this position, they may insist on having won the election on the grounds of better insight, even though they fall short numerically. With the result of the election contested, the way out is to take recourse to an arbitrator. Thus the notion of saniority necessitates an authority outside the electorate.

Llull never makes any mention of saniority, yet he shows a great concern that the more worthy and more suitable candidate be elected. Above and beyond the vow binding the electors as nuns (B24) or brethren (DAE), he makes them take an oath to rank the candidates according to pertinent properties. AEP (f. 11v, l. 15-19) stipulates that three things should be considered: honesty and holiness of life, knowledge and wisdom, and a suitable disposition of the heart. B24 (f. 33v, l. 4-6) defines four conditions: love and knowledge of God, love and knowledge of the virtues, knowledge of and opposition to vices, and greatest suitability. These attributes are similar to those that are characteristic of saniority, see Maleczek [23, p. 122]. We mention in passing that AEP, with three attributes, is from the quaternary phase of Llull’s writings, while B24, with four attributes, is from the ternary phase.

In DAE (f. 47v, l. 23) the oath plainly demands to *elect the better and more suitable person* (personam meliorem et magis ydoneam). Thus Llull shifts the emphasis from worthiness of the electors (saniority), to worthiness of the candidates (idoneity). We take it that in Llull’s writings the absence of the notion of saniority is a deliberate abandonment of the concept, not an accidental omission. Secret voting would have been another option to deal with the problem, and indeed was under consideration at that time. The 1283 Montpellier General Chapter of the Dominican Order introduced secrecy of the votes, see Maleczek [23, p. 129] and Gaudemet et al. [7, p. 332]. Secrecy does away with saniority, because a secret vote cannot be traced back to the voter and his or her state of
soundness. Llull remains somewhat undecided on whether he favors secret voting or not. In DAE (f. 48r, l. 6-8) he finally advocates open voting, welcoming the social control that comes with it.

We believe that Llull, designing his system so as to lead to the more worthy candidate, wants to shield it from saniority and outside intervention. The trail to truth envisioned by Llull needs no recourse to a higher worldly authority. When asked why in his electoral systems he mentions the notion of saniority not even once, Llull might have coined the phrase, half a millennium before Laplace: Je n'ai pas besoin de cette hypothèse-là.

7. Discussion

The credit for the discovery of DAE in Cusanus' library in Kues lies with Honecker. Regrettably, his comments on the electoral details are unreliable and have obscured the subject to date. We would like to correct his misjudgement of the overall evaluation of the election, his grossly misleading implication that all electoral systems lead to identical results, and his false statement on the transitivity of pairwise comparisons.

Honecker [11, p. 567] summarizes the global evaluation of a B24 election as follows: Wer nitn dabei in allen Wahlgängen zusammen die ineisten Stinunen erhalten, das heißt in den meisten Einzelwahlen gesiegt hat, gilt als gewählt. [«The one who among all pairwise comparisons has totaled the most votes, that is, has won most of the individual comparisons, wins the election.»] The conjunction das heißt [that is] is fatally misplaced, for the electoral system in the initial section of the sentence, building on vote totals, is different from the electoral system in the final section, counting victories. In our contemporary terminology the electoral system of adding votes is called the system of Borda, while the system of counting victories is named after Condorcet, see McLean/London [24, p. 106; 25, p. 34].

With his past as a courtier, Llull may have seen the electoral process as a voting tournament. His prescription that unus punctus is awarded to a duel's victor is most clearly formulated in the tract AEP edited here for the first time and not accessible to Honecker. Yet the texts that Honecker did have available should have prevented him from going astray. The evaluation of a single pairwise comparison is summarized correctly [11, p. 567]: und es wird jedesmal geheim aufgeschrieben, wer bei jeder einzelnen Abstimmung die meisten Stimmen bekommt. [«and each time it is recorded in secrecy who in each pairwise comparison gains the most votes.»] If in every round the name of the winner is recorded rather than the score of votes, then it is impossible to finally total those nonrecorded votes.
Honecker's misjudgment is presumably caused by the fact that in B24 the word *veus* stands for both the votes of the electors as well as the winning points of the candidates. Of course, it is irritating when one and the same word is used with distinct meanings. There are more examples, though, where Llull passes over a shift of meaning as he exposes his electoral systems in more and more detail: *persona* is a member of the convent as well as a member of the electoral college as well as a candidate, and *nombre* is the number of electors as well as the number of candidates as well as their numbering as well as the number of pairwise comparisons.

Honecker [11, p. 568] also erroneously equates the results from the electoral system in B24 with those from the system in DAE: *Im Vergleich zur Wahl im Libre de Blancerna stellt der Wahlmodus des Traktats De arte electionis einen abgekürzten Prozeß dar. Die Zuverlässigkeit des Verfahrens wird davon aber nicht berührt.* [«Compared to the election in Biaquerna, the electoral system of the tract De arte electionis presents a shortened process. The reliability of the procedure, however, remains untouched.»] Everything is lumped together, including the contributions of Cusanus. Peculiar to Cusanus is [11, p. 574] die *'Punktwertung', die das Verfahren, wenn man von der ars electionis ausgeht, nicht nur erheblich vereinfacht, sondern zugleich auch eine zuverlässige zahlenmäßige Auswertung der Abstimmung gestattet.* [«the ‘scoring scheme’ that not only simplifies the procedure considerably, as compared with the ars electionis, but at the same time permits a reliable numerical evaluation of the voting system.»] We cannot understand these quotations other than that, to Honecker, these systems always lead to the same result and differ only in how to get there. This is grossly misleading. As a matter of fact, the electoral systems of Llull/Condorcet and of Cusanus/Borda occasionally lead to different results, though, of course, not always.

Finally, Honecker [11, S. 574, n. 34] makes a false or, at best, confusing statement on the transitivity of pairwise comparisons. Transitivity means that the following logical conclusion holds: if $b$ beats $c$ and $c$ beats $d$, then $b$ beats $d$. For a system of exhaustive pairwise comparisons, as in AEP and B24, this conclusion is false, as may be illustrated with an example of three electors $X$, $Y$, $Z$, and three candidates $b$, $c$, $d$, see McLean/London [25, p. 23]:

Elector $X$: $d < c < b$
Elector $Y$: $b < d < c$
Elector $Z$: $c < b < d$

Here $b$ beats $c$ by 2 to 1 votes and, with the same score, $c$ wins against $d$. Yet the voting duel between $b$ and $d$ sees $b$ not winning, but losing by 1 to 2 votes.
Thus the Llull/Condorcet system suffers from occasional intransitivity. In contrast, the Cusanus/Borda system is free from this defect.

Blinded by the similarity of the various electoral systems, Honecker creates the impression - though he never explicitly says so - that the systems differ only in procedural details, but in the end lead to identical results. As we have pointed out, this is not true and, in particular, it inappropriately belittles the originality of Cusanus' contributions. A more detailed discussion of Cusanus' electoral system will be presented elsewhere [35].

McLean/London [25, p. 27], upon referring to the inauguration of an electoral college in B24, propose the following view: «Llull seems to wish to compromise between democracy and giving a more decisive voice to better qualified electors. This may represent Llull’s attempt to compromise between the rival principles of maior pars and sanior pars.» We disagree and believe, as detailed in Section 6.7, that idoneity of the winning candidate is in Llull’s eyes superior to saniority of the supportive electorate. Moreover, a democracy-Llull who devises an electoral system in order to mediate between the equal rights of the many and the specific talents of the few is, in our opinion, light years remote from Llull’s Llull whom Bonner [1, vol. I, p. xii] tries to present.

One question remains open, namely whether Llull’s electoral systems are as free of bias as depicted in our Section 6.1. After all, Llull lived in a time when hierarchies were omnipresent in all parts of society, and honoring them might have gone without saying. The relevant paragraph in AEP (f. 11v, l. 11-14) remains somewhat enigmatic to us.

Nor does DAE (f. 47v, l. 16-17) provide a definite answer. The key passage says that the letter b signifes primam personam ... que in ecclesia vacante prius fuit recepta. What does this mean? Honecker [11, p. 568], McLean/London [25, p. 29, n. 4], and Meuthen [27, p. 5] all consider, with varying degrees of confidence, that the phrase implies a ranking by seniority. They rely on the words in ecclesia vacante, a technical term from canon law, designating the church community who have lost their leader; note that the crucial word vacante is missing from n. 4 in McLean/London [25, p. 29]. Then prius fuit recepta might mean that the letters are assigned in the sequence in which the electors became members of that church community. Cusanus, in 1428 a canon lawyer recently graduated from university, may well have understood the phrase in this way. Llull, however, was neither a canon lawyer nor did he enjoy a university education.

What else could Llull have had in mind? Honecker [10, p. 308, n. 6] suspects that the DAE version copied by Cusanus stems from a Catalan original. His evidence consists of four substitutions of a long whip-s for the letter c, though it is not clear to us why this would point to the Catalan language. Nevertheless, the expunged essencia (DAE, f. 48r, l. 13-14) is more easily misread from a Catalan
esglesia, than from a Latin ecclesia on its sixth occurrence. Did indeed Llull draft a Catalan original, 1299 in Paris, which somehow or other mutated into the 1428 Latin copy of Cusanus?

Perhaps an original Catalan wording would be supportive of an alternative interpretation proposed by McLean/London [25, p. 29, n. 4]. Before starting the election, the electors celebrate of course a mass, see Viollet [43, p. 70]. Might the quote from DAE then be interpreted so that after the service the electors leave the church, and that the letters are assigned as they re-enter the empty building in random procession? The location where the election is to be held is never specified by Llull. Only in AEP (f. 12r, l. 3-4) is a domus mentioned although it remains open what exactly is meant by it. According to Schimmel-pfennig [41, p. 191], the election of a bishop normally took place in the canon chancel of the cathedral, or in the chapter house. A linguistic analysis of Llull’s œuvre may shed more light on the problem. For instance in Felix, Dame Reynard appeals to a church building [1, vol. II, p. 783]: «An election was once being held in a cathedral church, and in the chapter there was disagreement over the election of a bishop.» Later, though, the hermit refers to the church community [1, vol. II, p. 1067]: «Two clerics came before a bishop asking for a church position which was vacant.»

A more detailed semantic study may be also profitable in other ways. For example, apart from early years, Llull reserved the notion of dignitates for the divine virtues attributed to God, see Bonner [2]. The changing usage of the word is also noticeable in the electoral trilogy under discussion. When searching the transcriptions in www.uni-augsburg.de/llull/ for the occurrence of the string dign, AEP scores seven hits, B24 two, and DAE none. Clearly, AEP was written before dignitas was restricted to its narrower meaning.

8. Résumé

Llull’s electoral systems have in common that they are exclusively based on pairwise comparisons between two candidates at a time, but they differ in detail. AEP and B24 describe systems of exhaustive pairwise comparisons, whereby AEP accepts everybody in the community as an elector, while B24 comprises a preliminary step of first selecting a smaller electoral college. In both systems the winner of the election is the candidate who is victorious in most of the pairwise comparisons. In contrast, DAE proposes a system of successive eliminations; the victor in the last pairwise contest is the winner of the whole election.

The newly edited manuscript AEP precedes B24 and DAE. It is longer than the two later texts, and contains more detailed instructions. All three writings
reflect Llull’s fundamental view that every person will be rewarded with the divine truth provided he or she trusts in God and researches honestly. Llull developed his systems for clerical communities and, having fought vehemently throughout his life for founding new monasteries specializing in missionary work, he may have aimed at such institutions. However, whether the electoral systems have actually ever been used is not known.

Nor is it known what triggered Llull’s interest in the subject, and which sources he built on. In 1283 the Dominican Order held a General Chapter in Montpellier, and electoral matters were on the agenda, see Ganzer [6, p. 76]. In 1285 Guilelmus de Mandagoto (d. 1321), archdeacon in Nimes, published his influential Tractatus de electionibus novorum praetoriorum; see Schimmelpfenning [40, p. 476]. Viollet [43, p. 88] points out that Mandagoto treats three systems, the elections per scrutinium, per compromissum, and per inspirationem, while the next generation canon Ioannis Andreae (1270-1348) recognizes only the first two methods. Llull fits in-between and anticipates Andreae, as DAE (f. 48r, l. 5) lists only the secret scrutinium and the election per compromissum.

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[Bo = Bonner’s Catalogue]


SUMMARY

We present three writings of Ramon Llull on electoral systems: Artifitium electionis personarum (before 1283), Chapter 24 En qual manera Natana fo eleta a abadessa from the novel Blaquerna (about 1283), and De arte electionis (1299). The tract Artifitium electionis personarum, edited here for the first time, proves to be an independent work shedding new light on the two later sources. In all three cases the electoral system is based exclusively on pairwise comparisons between two candidates at a time. The first two texts describe exhaustive schemes: the candidate winning most of the pairwise comparisons is the overall winner. The last text proposes a system of successive eliminations: the winner of the last comparison also wins the election.
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