



University of Pisa

Ph.D. Program in Mathematics for Economic Decisions

*Leonardo Fibonacci School*

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Ph.D. Thesis

*Data envelopment analysis:  
uncertainty, undesirable outputs and  
an application to world cement industry*

Roberta Toninelli

Ph.D. Supervisors:

**Prof. Riccardo Cambini**

**Dr. Rossana Riccardi**

SSD: SECS-S/06





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**Dr. Roberta Toninelli**



*A Filippo e Luna*



*“If I would be a young man again  
and had to decide how to make my living,  
I would not try to become a scientist  
or scholar or teacher.  
I would rather choose  
to be a plumber...”*

*Albert Einstein*





# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Data Envelopment Analysis: an overview</b>	<b>5</b>
2.1	DEA models	5
2.1.1	CCR DEA Model	6
2.1.2	BCC DEA Models	12
2.2	Chance-constrained DEA models	16
2.2.1	Stochastic dominance	16
2.2.2	LLT model	17
2.2.3	OP model	19
2.2.4	Sueyoshi models	19
2.2.5	Bruni model	21
2.3	DEA models with undesirable factors	24
2.3.1	INP model: undesirable factors treated as inputs	25
2.3.2	Korhonen-Luptacik DEA model	26
2.3.3	TR $\beta$ model: a linear transformation approach	27
2.3.4	A directional distance function approach	29
<b>3</b>	<b>DEA with outputs uncertainty</b>	<b>31</b>
3.1	DEA models with outputs uncertainty	32
3.1.1	The VRS1 model	33
3.1.2	The VRS2 model	34
3.2	A deterministic approach for VRS1 and VRS2	35
3.2.1	Deterministic VRS1	35
3.2.2	Deterministic VRS2	37
3.3	A particular case: Constant Returns to Scale	39
3.4	Models effectiveness and computational results	41
3.4.1	Further deterministic models: VRSqEV and VRSEV	42
3.4.2	Comparing VRS1, VRS2, VRSqEV and VRSEV	43
3.4.3	Comparing CRS1, CRS2, CRSqEV and CRSEV	49
3.5	Unifying model for returns to scale	51
3.6	Conclusions	54
<b>4</b>	<b>Eco-efficiency of the cement industry</b>	<b>55</b>
4.1	Introduction	55
4.2	Modeling Eco-efficiency	58
4.2.1	Pollutants as inputs	60
4.2.2	Pollutant as undesirable outputs	63

4.3	Database description . . . . .	66
4.4	Empirical Results . . . . .	68
4.4.1	Eco-efficiency measure as input and pollutant contraction . . . . .	69
4.4.2	Eco-efficiency measure as pollutant contraction . . . . .	72
4.4.3	Eco-efficiency as input contraction . . . . .	73
4.4.4	Eco-efficiency as undesirable output contraction and desirable output expansion . . . . .	75
4.5	Conclusions . . . . .	77
<b>5</b>	<b>Evaluating the efficiency of the cement</b>	<b>79</b>
5.1	Introduction . . . . .	79
5.2	Model specification . . . . .	80
5.2.1	A first model comparison: standard BCC DEA model and undesirable factors treated as inputs . . . . .	81
5.2.2	A second model comparison: the directional distance function approach and undesirable factors treated as output . . . . .	83
5.2.3	Construction of the production frontier . . . . .	84
5.3	Empirical Results . . . . .	85
5.3.1	Efficiency evaluation: standard BCC DEA model and undesirable factors treated as inputs . . . . .	86
5.3.2	Efficiency evaluation: the directional distance function approach and undesirable factors treated as output . . . . .	92
5.3.3	Overall comments . . . . .	98
5.4	Conclusions . . . . .	99
<b>6</b>	<b>Conclusions</b>	<b>101</b>
<b>A</b>	<b>Simulations</b>	<b>105</b>
A.1	Cement: first instance . . . . .	106
A.1.1	Contemporaneous frontier for representative plants: European and non-European countries . . . . .	107
A.1.2	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	108
A.1.3	Sequential frontier for representative plants: European and non-European countries . . . . .	109
A.1.4	Sequential frontier with aggregated data: European and non-European countries . . . . .	110
A.2	Cement: second instance . . . . .	111
A.2.1	Contemporaneous frontier for representative plants: European and non-European countries . . . . .	112
A.2.2	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	113
A.2.3	Sequential frontier for representative plants: European and non-European countries . . . . .	114
A.2.4	Sequential frontier with aggregated data: European and non-European countries . . . . .	115
A.3	Cement: third instance . . . . .	116
A.3.1	Contemporaneous frontier for representative plants: European and non-European countries . . . . .	117
A.3.2	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	118
A.3.3	Sequential frontier for representative plants: European and non-European countries . . . . .	119

A.3.4	Sequential frontier with aggregated data: European and non-European countries	120
A.4	Clinker instance . . . . .	121
A.4.1	Contemporaneous frontier for representative plants: European and non-European countries . . . . .	122
A.4.2	Contemporaneous frontier for representative plants: European countries . . . . .	123
A.4.3	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	124
A.4.4	Contemporaneous frontier with aggregated data: European countries . . . . .	125
A.4.5	Sequential frontier for representative plants: European and non-European countries . . . . .	126
A.4.6	Sequential frontier for representative plants: European countries . . . . .	127
A.4.7	Sequential frontier with aggregated data: European and non-European countries	128
A.4.8	Sequential frontier with aggregated data: European countries . . . . .	129
A.5	Cement without undesirable factor: first instance . . . . .	130
A.5.1	Contemporaneous frontier for representative plants: European and non-European countries . . . . .	130
A.5.2	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	130
A.5.3	Sequential frontier for representative plants: European and non-European countries . . . . .	131
A.5.4	Sequential frontier with aggregated data: European and non-European countries	131
A.6	Cement without undesirable factor: second instance . . . . .	132
A.6.1	Contemporaneous frontier for representative plants: European and non-European countries . . . . .	132
A.6.2	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	132
A.6.3	Sequential frontier for representative plants: European and non-European countries . . . . .	133
A.6.4	Sequential frontier with aggregated data: European and non-European countries	133
A.7	Cement without undesirable factor: third instance . . . . .	134
A.7.1	Contemporaneous frontier for representative plants: European and non-European countries . . . . .	134
A.7.2	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	134
A.7.3	Sequential frontier for representative plants: European and non-European countries . . . . .	135
A.7.4	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	135
A.8	Clinker without undesirable factor instance . . . . .	136
A.8.1	Contemporaneous frontier for representative plants: European and non-European countries . . . . .	136
A.8.2	Contemporaneous frontier for representative plants: European countries . . . . .	136
A.8.3	Contemporaneous frontier with aggregated data: European and non-European countries . . . . .	137
A.8.4	Contemporaneous frontier with aggregated data: European countries . . . . .	137
A.8.5	Sequential frontier for representative plants: European and non-European countries . . . . .	138
A.8.6	Sequential frontier for representative plants: European countries . . . . .	138
A.8.7	Sequential frontier with aggregated data: European and non-European countries	139
A.8.8	Sequential frontier with aggregated data: European countries . . . . .	139

**B Database Sources**

**141**

**Bibliografy**

**146**

# Chapter 1

## Introduction

Data Envelopment Analysis (DEA) is a data-oriented approach for performance evaluation and improvement of a set of peer entities called Decision Making Units (DMUs) which convert multiple inputs into multiple outputs. The definition of a DMU is generic and flexible. The interest in DEA techniques and their applications is highly increased in the recent literature. In this light, basic DEA models and techniques have been well documented in [16, 19, 20, 48]. Literature shows a great variety of applications of DEA in evaluating performances of entities in different countries (see [17, 23, 66] for an extensive survey of DEA research covering theoretical developments as well as "real-world" applications). One reason is that DEA can be used for use in cases which have been resistant to other approaches because of the complex (often unknown) nature of the relations the multiple inputs and multiple outputs involved (e.g. in the case of non-commeasurable units).

Starting from the pioneering papers by Charnes, Cooper and Rhodes (CCR model) and Banker, Charnes and Cooper (BCC model) (see [5, 14]) and their original formulation of data envelopment analysis, in this Ph.D thesis alternative DEA model which consider uncertain and undesirable outputs are taken into account. Classical models assume a deterministic framework with no uncertainty and this seems unsuitable for concrete applications, due to the presence of errors and noise in the estimation of inputs and outputs values. For this reason, the first goal of this work is to propose, starting from the generalized input-oriented (BCC) model, two different models with uncertain outputs and deterministic inputs. Various applications, in fact, are affected by random perturbations in output values estimation (see, for instance, [9, 56, 58]). Random perturbations can be motivated by a concrete difficulty in estimating the right output value (for instance, in the case of energy companies, electricity production has to take care of different and uncertain energy dispersion factors according to the employed technologies) or to obtain good output provisions (for instance, in the case of DEA applied to health care problems, early screening efficiency measures are related to the estimation of true positive and false positive screens which are indeed outputs with a stochastic nature). In particular, two different models are proposed where uncertainty is managed with a scenario generation approach. For the sake of completeness, these models are compared with two further ones based on an expected value approach, that is to say that the uncertainty is managed by means of the expected values of random factors both in the objective function and in the constraints. Deeply speaking, the main difference between the two proposed models and the expected value approaches lies in their mathematical formulation. In the models based on the scenario generation approach, the constraints concerning efficiency level are expressed for each scenario, while in the expected value models they are satisfied in expected value. As a consequence, the first kind of models result to be more selective in finding a ranking of efficiency, thus becoming useful strategic management tools aimed to determine a restrictive efficiency score ranking. The main research results are collected in [50], accepted for publication in the Journal of Information &

Optimization Sciences.

In a second part of this study, we focus on the environmental policy and the concept of eco-efficiency. One of the most intensively discussed concepts in the international political debate today is, in fact, the concept of sustainability and the need for eco-efficient solutions that enable the production of goods and services with less energy and resources and with less waste and emissions. In particular, we consider the environmental impact of CO<sub>2</sub> in cement and clinker production processes. Cement industry is, in fact, responsible for approximately 5% of the current worldwide CO<sub>2</sub> emissions. DEA models can provide an appropriate methodological approach for developing eco-efficiency indicators. With eco-efficiency, we indicate the possibility of producing goods (or services) by reducing the quantity of energy and resources employed and/or the amount of waste and emissions generated. We provide different eco-efficiency measures by applying variants of DEA approaches, where emissions can be either considered as inputs or undesirable outputs. The standard DEA models, in fact, rely on the assumption that inputs are minimized and outputs are maximized. However, when some outputs are undesirable factors (e.g., pollutants or wastes), these outputs should be reduced to improve efficiency. For this reason, in DEA approach, emissions can be considered either as inputs or as undesirable outputs. This leads to different eco-efficiency measures. In the first part of the analysis, we provide an eco-efficiency measure for 21 prototypes of cement industries operating in many countries by applying both a data envelopment analysis and a directional distance function approach, which are particularly suitable for models where several production inputs and desirable and undesirable outputs are taken into account. Several studies have been carried out to monitor CO<sub>2</sub> emission performance trends in different countries and sectors (see, for instance, [57, 64, 66]). However, to the best of our knowledge, few papers treat undesirable outputs of cement sector as a DEA model and in all of them only interstate analyzes have been developed (Bandyopadhyay [3], Mandal and Madheswaran [40] and Sadjadi and Atefeh [54]). This work differs from previous literature because it compares 21 countries covering 90% of the world's cement production. To understand whether this eco-efficiency is due to a rational utilization of inputs or to a real carbon dioxide reduction as a consequence of environmental regulation, we analyze the cases where CO<sub>2</sub> emissions can either be considered as an input or as an undesirable output. The obtained results are collected in [51], accepted for publication in journal *Energy Policy*. In the second part of the eco-efficiency analysis, we try to answer to the following questions: do undesirable factors modify the efficiency levels of cement industry? Is it reasonable to omit CO<sub>2</sub> emissions in evaluating the performances of the cement sector in different countries? In order to answer to these questions, alternative formulations of standard Data Envelopment Analysis model and directional distance function are compared both in presence and in absence of undesirable factors. The obtained results are collected in [53] and submitted to journal *Resource and Energy Economics*. Above mentioned studies have been developed taking into account a specific assumption, namely that the production possibility set can be expanded each year, and no technological regress is admitted. In the formulation of DEA models, this assumption can be incorporated through the construction of so-called sequential frontier. Results on eco-efficiency measure through the standard Contemporaneous Frontier, where the frontier in each year is constructed with only the observations of the year under consideration, are collected in [52].

The thesis is divided into the following chapters. Chapter 2 contains a survey of data envelopment analysis literature. In Chapter 3 two new models for Data Envelopment Analysis with uncertain outputs are proposed, with the aim to manage uncertainty through a scenario generation approach. Chapters 4 and 5 focus on the study of DEA models, extending the study of efficiency in case of undesirable factors arising from the production process. In particular, Chapter 4 provide an eco-efficiency measure for twenty-one prototypes of cement industries operating in many countries by applying both a Data Envelopment Analysis and a directional distance function approach, which are particularly suitable for models where several production inputs and desirable and undesirable outputs are taken into account. In Chapter 5 alternative formulations of standard Data Envelopment

Analysis model and directional distance function are compared both in presence and in absence of undesirable factors to understand if undesirable factors can modify the efficiency levels of cement industry. Finally, for the sake of completeness, two appendixes are provided. Appendix A contains the tables collecting all of the results of the deep preliminary tests that lead to obtain the ones exposed in Chapter 4 and 5. Finally, in Appendix B the main web sources for our database construction are collected. The original results of this Ph.D. thesis have been collected in the following research papers:

- Riccardi R. and R. Toninelli. *Data Envelopment Analysis with outputs uncertainty*. Journal of Information & Optimization Sciences, to appear.
- Riccardi R., Oggioni G. and R. Toninelli. *The cement industry: eco-efficiency country comparison using Data Envelopment Analysis*. Journal of Statistics & Management Systems, accepted for publication.
- Riccardi R., Oggioni G. and R. Toninelli. *Eco-efficiency of the world cement industry: A Data Envelopment Analysis*. Energy Policy, Vol. 39, Issue 5, p. 2842-2854, 2011, available online at: <http://dx.doi.org/10.1016/j.enpol.2011.02.057>
- Riccardi R., Oggioni G. and R. Toninelli. *Evaluating the efficiency of the cement sector in presence of undesirable output: a world based Data Envelopment Analysis*. Technical Report n. 344, Department of Statistics and Applied Mathematics, University of Pisa, 2011, submitted to Resource and Energy Economics.

The research topic considered in this thesis shows many different lines for future developments. In particular, from a theoretical point of view, starting from the models proposed in [50] we are studying for a bi-objective like DEA formulation where both uncertainty desirable and undesirable factor are taken into account. As regards the applicative aspects, we are also studying and applying bootstrap techniques to manage uncertainty and generate empirical distributions of efficiency scores, in order to capture and analyze the sensitivity of samples with respect to changes in the estimated frontier.





## Chapter 2

# Data Envelopment Analysis: an overview

This chapter contains a brief and absolutely not exhaustive survey of data envelopment analysis, in particular of DEA with uncertainty and undesirable data. Some known properties of data envelopment analysis, especially the ones more related to this Ph.D. thesis, are recalled. An outline of the more recent developments in the field will be also given. This chapter is primarily based on [16, 19, 20, 48], which are among the most popular books on data envelopment analysis. Some of the results recalled in this chapter are given without the proof, please refer to the corresponding bibliographic source for the complete theorem. In section 2.1 the classic DEA models are presented. The section 2.2 takes in account uncertainty data and describes some of the most popular chance-constrained models. In section 2.3 the main DEA models with undesirable outputs are provided.

### 2.1 DEA models

Data Envelopment Analysis (DEA) is a data oriented approach method for evaluating the performance of a set of entities called Decision Making Units (DMUs) which convert multiple inputs into multiple outputs. The definition of a DMU is generic and flexible. Recent years have seen a great variety of applications of DEA for use in evaluating the performances of many different kinds of entities engaged in many different activities in many different contexts in many different countries. DEA applications have used DMUs of various forms to evaluate the performance of entities, such as hospitals, universities, cities, courts, business firms, and others, including the performance of countries, regions, etc. Because it requires very few assumptions, DEA has also opened up possibilities for use in cases which have been resistant to other approaches because of the complex (often unknown) nature of the relations between the multiple inputs and multiple outputs involved in DMUs. DEA's empirical orientation and the absence of a need for the numerous a priori assumptions that accompany other approaches (such as standard forms of statistical regression analysis) have resulted in its use in a number of studies involving efficient frontier estimation in the governmental and non-profit sector, in the regulated sector, and in the private sector. These kinds of applications extend to evaluating the performances of cities, regions and countries with many different kinds of inputs and outputs that include "social" and "safety-net" expenditures as inputs and various "quality-of-life" dimensions as outputs. See [17, 23, 66] for an extensive survey of DEA research covering theoretical developments as well as "real-world" applications.

In their originating study, Charnes, Cooper and Rhodes [14] described DEA as a 'mathematical programming model applied to observational data [that] provides a new way of obtaining

empirical estimates of relations - such as the production functions and/or efficient production possibility surfaces - that are cornerstones of modern economies'. In fact, DEA proves particularly adept at uncovering relationships that remain hidden from other methodologies. For instance, consider what one wants to mean by "efficiency", or more generally, what one wants to mean by saying that one DMU is more efficient than another DMU. This is accomplished in a straightforward manner by DEA without requiring explicitly formulated assumptions and variations with various types of models such as in linear and nonlinear regression models.

We assume that there are  $n$  DMUs to be evaluated. Each DMU consumes varying amounts of  $m$  different inputs to produce  $q$  different outputs. Specifically, DMU  $j$  consumes amount  $x_{ij}$  of input  $i$  and produces amount  $y_{rj}$  of output  $r$ . We assume that  $x_{ij} \geq 0$ ,  $i = 1, \dots, m$ ,  $j = 1, \dots, n$  and  $y_{rj} \geq 0$ ,  $r = 1, \dots, q$ ,  $j = 1, \dots, n$  and further assume that each DMU has at least one positive input and one positive output value.

### 2.1.1 CCR DEA Model

A fractional programming model known as the CCR model was developed by Charnes, Cooper and Rhodes [14] to determine the efficiency score of each of the DMUs in a data set of comparable units. In this form, the ratio of outputs to inputs is used to measure the relative efficiency of the DMU  $j_0$  to be evaluated relative to the ratios of all of DMU  $j$ ,  $j = 1, 2, \dots, n$ . We can interpret the CCR construction as the reduction of the multiple-output/multiple-input situation (for each DMU) to that of a single 'virtual' output and 'virtual' input, through the choice of appropriate multipliers, as weighted sum of inputs and weighted sum of outputs. For a particular DMU the ratio of this single virtual output to single virtual input provides a measure of efficiency that is a function of the multipliers. The weights are chosen in a manner that assigns a best set of weights to each DMU. The term "best" is used here to mean that the resulting input to output ratio for each DMU is maximized relative to all other DMU when these weights are assigned to these inputs and outputs for every DMU. The objective function maximizes the efficiency of the DMU using the weights  $\mu_r$  and  $\nu_i$  for each outputs  $r$  and each inputs  $i$  respectively. The mathematical formulation is provided below.

$$\max_{\mu, \nu} \frac{\sum_{r=1}^q \mu_r y_{rj_0}}{\sum_{i=1}^m \nu_i x_{ij_0}}, \quad (2.1)$$

where it should be noted that the variables are the  $\mu = (\mu_1, \dots, \mu_r, \dots, \mu_q)$  and the  $\nu = (\nu_1, \dots, \nu_i, \dots, \nu_m)$  and the  $y_{j_0} = (y_{1j_0}, \dots, y_{rj_0}, \dots, y_{qj_0})$  and  $x_{j_0} = (x_{1j_0}, \dots, x_{ij_0}, \dots, x_{mj_0})$  are the observed output and input values, respectively, of DMU  $j_0$ , the DMU to be evaluated. Of course, without further additional constraints (developed below) problem (2.1) is unbounded. A set of normalizing constraints (one for each DMU) reflects the condition that the virtual output to virtual input ratio of every DMU, including DMU  $j_0$ , must be less than or equal to unity. The

mathematical programming problem may thus be stated as

$$\begin{aligned}
 & (\widetilde{P}_C) \\
 \max_{\mu, \nu} z = & \frac{\sum_{r=1}^q \mu_r y_{rj_0}}{m} \\
 \text{s.t.} & \frac{\sum_{r=1}^q \mu_r y_{rj}}{\sum_{i=1}^m \nu_i x_{ij}} \leq 1, \quad j = 1, \dots, n, \\
 & \mu_r \geq 0, \quad r = 1, \dots, q, \\
 & \nu_i \geq 0, \quad i = 1, \dots, m,
 \end{aligned} \tag{2.2}$$

where  $\mu_r \geq 0$ ,  $r = 1, \dots, q$ , and  $\nu_i \geq 0$ ,  $i = 1, \dots, m$  have at least one positive value. The above ratio form yields an infinite number of solutions; if  $(\mu^*, \nu^*)$  is an optimal solution of  $(\widetilde{P}_C)$ , then  $(\alpha\mu^*, \alpha\nu^*)$  is also optimal for  $\alpha > 0$ . However, this fractional problem can be converted into a linear program through the transformation developed by Charnes and Cooper [13] selecting a representative solution [i.e., the solution  $(\mu, \nu)$  for which  $\sum_{i=1}^m \nu_i x_{ij_0} = 1$ ]. The equivalent linear programming problem in which the change of variables from  $(\mu_r, \nu_i)$  to  $(u_r, v_i)$  is a result of the Charnes-Cooper transformation,

$$\begin{aligned}
 & (P_C) \\
 \max_{u, v} z = & \sum_{r=1}^q u_r y_{rj_0}
 \end{aligned} \tag{2.4}$$

$$\text{s.t.} \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \tag{2.5}$$

$$\sum_{r=1}^q u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \tag{2.6}$$

$$u_r \geq 0, \quad r = 1, \dots, q, \tag{2.7}$$

$$v_i \geq 0, \quad i = 1, \dots, m. \tag{2.8}$$

This primal formulation, whose objective is to maximize outputs while using at least the give inputs levels, resulting to imposing the condition that  $\sum_{i=1}^m \nu_i x_{ij_0} = 1$ , is known as input-oriented CCR. There is another type of model that attempts to minimize inputs while using no more than the observed amount of outputs. This is referred to as the output-oriented model.

**Theorem 2.1.1** *The fractional program  $(P_C)$  is equivalent to  $(\widetilde{P}_C)$ .*

*Proof* Under the nonzero assumption of  $v_i$  and  $x_{ij}$ , the denominator of the constraint (2.3) is positive for every  $j$ , and hence we obtain (2.6) by multiplying both sides of (2.3) by the denominator. Next, we

note that a fractional number is invariant under multiplication of both numerator and denominator by the same nonzero number. After making this multiplication, we set the denominator of (2.2) equal to 1, move it to a constraint, as is done in (2.5), and maximize the numerator, resulting in  $(P_C)$ . Let an optimal solution of  $(P_C)$  be  $(v^*, u^*)$  and the optimal objective value  $z^*$ . The solution  $(\mu^*, \nu^*)$  is also optimal for  $(\widetilde{P}_C)$ , since the above transformation is reversible under the assumptions above.  $(\widetilde{P}_C)$  and  $(P_C)$  therefore have the same optimal objective value  $z^*$ .  $\square$

We also note that the measures of efficiency we have presented are "units invariant" i.e., they are independent of the units of measurement used in the sense that multiplication of each input by a constant  $t_i > 0$ ,  $i = 1, \dots, m$  and each output by a constant  $p_r > 0$ ,  $r = 1, \dots, q$  does not change the obtained solution. Stated in precise form we have

**Theorem 2.1.2 (Units Invariance Theorem)** *The optimal values of  $\max z = z^*$  in  $(P_C)$  and  $(\widetilde{P}_C)$  are independent of the units in which the inputs and outputs are measured provided these units are the same for every DMU.*

*Proof* Let  $(z^*, u^*, v^*)$  be optimal for  $(\widetilde{P}_C)$ . Now replace the original  $y_{rj}$  and  $x_{ij}$  by  $p_r y_{rj}$  and  $t_i x_{ij}$  for some choices of  $p_r, t_i > 0$ . But then choosing  $u' = u^*/p$  and  $v' = v^*/t$  we have a solution to the transformed problem with  $z' = z^*$ . An optimal value for the transformed problem must therefore have  $z' \geq z^*$ . Now suppose we could have  $z' > z^*$ . Then, however,  $u = u'p$  and  $v = v't$  satisfy the original constraints so the assumption  $z' \geq z^*$  contradicts the optimality assumed for  $z^*$  under these constraints. The only remaining possibility is  $z' = z^*$ . This proves the invariance claimed for (2.2). Theorem 2.1.1 demonstrated the equivalence of  $(P_C)$  to  $(\widetilde{P}_C)$  and thus the same result must hold and the theorem is therefore proved.  $\square$

Thus, one person can measure outputs in miles and inputs in gallons of gasoline and quarts of oil while another measures these same outputs and inputs in kilometers and liters. They will nevertheless obtain the same efficiency value from  $(P_C)$  and  $(\widetilde{P}_C)$  when evaluating the same collection of DMU using different units of measurement.

Let us suppose we have an optimal solution of  $(P_C)$  for DMU  $j_0$  which we represent by  $(z^*, u^*, v^*)$  where  $u^*$  and  $v^*$  are values with constraints given in (2.7) and (2.8). We can then identify whether CCR-efficiency has been achieved as follows:

**Definition 2.1.1 (CCR-Efficiency)** *For a CCR model,*

1. *DMU  $j_0$  is CCR-efficient if  $z^* = 1$  and there exists at least one optimal  $(u^*, v^*)$ , with  $u^* > 0$  and  $v^* > 0$ ;*
2. *Otherwise, DMU  $j_0$  is CCR-inefficient.*

For input-oriented problem  $(P_C)$ , a dual equivalent formulation can be obtained as follows:

$$(D_C)$$

$$\min_{\theta, \lambda} \theta$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj} \geq y_{rj_0}, \quad r = 1, \dots, q, \quad (2.9)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (2.10)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n,$$

where  $\theta$  and  $\lambda = (\lambda_1, \dots, \lambda_j, \dots, \lambda_n)$  are the dual variables corresponding to the primal constraints. This last model,  $(D_C)$ , is sometimes referred to as the "Farrell model" because it is the one used in Farrell [30]. By virtue of the dual theorem of linear programming we have  $z^* = \theta^*$ . Hence either problem may be used. One can solve say  $(D_C)$ , to obtain an efficiency score. Because we can set  $\theta = 1$  and  $\lambda_{j_0} = 1$  and all other  $\lambda_j = 0, j = 1, \dots, n, j \neq j_0$ , a solution of  $(D_C)$  always exists. Moreover this solution implies  $\theta \leq 1$ . The optimal solution,  $\theta$  yields an efficiency score for a particular DMU  $j_0$ . The process is repeated for each DMU and DMUs for which  $\theta < 1$  are inefficient, while DMUs for which  $\theta = 1$  are boundary points. In the dual input-oriented formulation, the variable  $\theta$  represents the reduction inputs factor, which states how much the inputs consumed by DMU  $j_0$  can be reduced in order to improve its efficiency.

The other type of dual model formulation, that attempts to maximize outputs while using no more than the observed amount of any input, referred to as the output-oriented model, is formulated as:

$$(D_{C_o})$$

$$\max_{\eta, \tau} \quad \eta$$

$$s.t. \quad \sum_{j=1}^n \tau_j y_{rj} \geq \eta y_{rj_0}, \quad r = 1, \dots, q, \quad (2.11)$$

$$\sum_{j=1}^n \tau_j x_{ij} \leq x_{ij_0}, \quad i = 1, \dots, m. \quad (2.12)$$

$$\tau_j \geq 0, \quad j = 1, \dots, n.$$

**Theorem 2.1.3** *Let  $(\theta^*, \lambda^*)$  an optimal solution for the input oriented model  $(D_C)$ . Then  $(\frac{1}{\theta^*}, \frac{\theta^*}{\lambda^*}) = (\eta^*, \tau^*)$  is an optimal solution for the corresponding output oriented  $(D_{C_o})$ . Similarly if  $(\eta^*, \tau^*)$  is optimal for the output oriented model then  $(\frac{1}{\eta^*}, \frac{\eta^*}{\tau^*}) = (\theta^*, \lambda^*)$  is optimal for the input oriented model.*

From the above relations, we can conclude that an input-oriented CCR model will be efficient for any DMU if and only if it is also efficient when the output-oriented CCR model is used to evaluate its performance.

The concept of efficiency in production has received a precise meaning when Koopmans and Debreau introduced in 1951 the production set notion, also known as production technology set, in the theory of production. A production technology set is a collection  $T$  of pairs  $(x, y)$  that have the property of being feasible ones. By feasible it means that the quantities are such that the output  $y$  can physically be produced by making use of the input  $x$ . In formal terms:

$$T = \{(x, y) : x \in R_+, y \in R_+; (x, y) \text{ is feasible}\}.$$

In DEA we construct a benchmark technology from the observed input-output bundles of the firms in the sample. For this, we make the following general assumptions about the production technology without specifying any functional form. These are fairly weak assumption and holds for all technologies represented by a quasi-concave and weakly monotonic production function:

**Assumptions 2.1.1** *Properties of the Production Possibility Set  $T$ .*

*Let  $x_j = (x_{1j}, x_{2j}, \dots, x_{nj})$  and  $y_j = (y_{1j}, y_{2j}, \dots, y_{qj})$  for  $j = 1, \dots, n$  the input-output bundle for each DMU.*

- (A1) The input-output bundle  $(x_j, y_j)$   $j = 1, \dots, n$  belong to  $T$ .
- (A2) The production possibility set is convex. Consider two feasible input-output bundle  $(x_{j_1}, y_{j_1})$  and  $(x_{j_2}, y_{j_2})$ . Then, the weighted average input-output bundle  $(\bar{x}, \bar{y})$ , obtained as  $\bar{x} = \lambda x_{j_1} + (1 - \lambda)x_{j_2}$  and  $\bar{y} = \lambda y_{j_1} + (1 - \lambda)y_{j_2}$  for some  $\lambda$  satisfying  $0 \leq \lambda \leq 1$  is also belong to  $T$ .
- (A3) Inputs are freely disposable. If  $(x_{j_0}, y_{j_0})$   $j = 1, \dots, n$  belong to  $T$ , then for any  $x \geq x_{j_0}$ ,  $(x, y_{j_0})$  is also feasible.
- (A4) Outputs are freely disposable. If  $(x_{j_0}, y_{j_0})$  for  $j = 1, \dots, n$  belong to  $T$ , then for any  $y \leq y_{j_0}$ ,  $(x_{j_0}, y)$  is also feasible.
- (A5) If, additionally, we assume CRS holds:  
Let  $x = \sum_{j=1}^n x_j$  and  $y = \sum_{j=1}^n y_j$ . If  $(x, y)$  is feasible, then for any  $k \geq 0$ ,  $(kx, ky)$  is feasible.

Under the hypothesis of constant return to scale, we can define the production possibility set  $T_C$  satisfying (A1) through (A5) by

$$T_C = \left\{ (x, y) : x \geq \sum_{j=1}^n \lambda_j x_j; y \leq \sum_{j=1}^n \lambda_j y_j; \lambda_j \geq 0, j = 1, \dots, n \right\}. \quad (2.13)$$

The constraints of  $(D_C)$  require the bundle  $(\theta x_{j_0}, y_{j_0})$  to belong to  $T_C$ , while the objective seeks the minimum  $\theta$  that reduces the input vector  $x_{j_0}$  radially to  $\theta x_{j_0}$  while remaining in  $T_C$ . In  $(D_C)$ , we are looking for an activity in  $T_C$  that guarantees at least the output level  $y_{j_0}$  of DMU  $j_0$  in all components while reducing the input vector  $x_{j_0}$  proportionally (radially) to a value as small as possible. Under the assumptions of the preceding section, it can be said that  $(\sum_{j=1}^n \lambda_j x_j, \sum_{j=1}^n \lambda_j y_j)$  outperforms  $(\theta x_{j_0}, y_{j_0})$  when  $\theta^* < 1$ . With regard to this property, we define the input excesses  $s^- = (s_1^-, \dots, s_i^-, \dots, s_m^-)$  and the output shortfalls  $s^+ = (s_1^+, \dots, s_r^+, \dots, s_q^+)$  and identify them as "slack" vectors by:

$$s^- = \theta x_{j_0} - \sum_{j=1}^n \lambda_j x_j, \quad s^+ = \sum_{j=1}^n \lambda_j y_j - y_{j_0}, \quad (2.14)$$

with  $s^- \geq 0$ ,  $s^+ \geq 0$  for any feasible solution  $(\theta, \lambda)$  of  $(D_C)$ .

Under this consideration, problem  $(D_C)$  can be rewritten as:

$$\begin{aligned} & \widehat{(D_C)} \\ & \min_{\theta, \lambda} \quad \theta \\ & \text{s.t.} \quad \sum_{j=1}^n \lambda_j y_{rj} = y_{rj_0} + s_r^+, \quad r = 1, \dots, q, \end{aligned} \quad (2.15)$$

$$\sum_{j=1}^n \lambda_j x_{ij} = \theta x_{ij_0} + s_i^-, \quad i = 1, \dots, m, \quad (2.16)$$

$$\begin{aligned} & \lambda_j \geq 0, \quad j = 1, \dots, n, \\ & s_r^+ \geq 0, \quad r = 1, \dots, q, \\ & s_i^- \geq 0, \quad i = 1, \dots, m. \end{aligned}$$

**Definition 2.1.2** *If an optimal solution  $(\theta^*, \lambda^*, s^{+*}, s^{-*})$  satisfies  $\theta^* = 1$  and is zero-slack ( $s^{+*} = 0, s^{-*} = 0$ ), then the DMU  $j_0$  is called CCR-efficient. Otherwise, the DMU is called CCR-inefficient, because*

i.  $\theta^* = 1$ ;

ii. All slacks are zero;

*must both be satisfied if full efficiency is to be attained.*

The first of these two conditions is referred to as "radial efficiency." It is also referred to as "technical efficiency" because a value of  $\theta^* < 1$  means that all inputs can be simultaneously reduced without altering the mix (=proportions) in which they are utilized. Because  $(1 - \theta^*)$  is the maximal proportionate reduction allowed by the production possibility set, any further reductions associated with nonzero slacks will necessarily change the input proportions. Hence the inefficiency is associated with any nonzero slack variables. When attention is restricted to condition (i) in Definition 2.1.2 the term "weak efficiency" is sometime used to characterize this inefficiency. The conditions (i) and (ii) taken together describe what is also called "Pareto-Koopmans" or "strong" efficiency, which can be verbalized as follows

**Definition 2.1.3 (Pareto-Koopmans Efficiency)** *A DMU is fully efficient if and only if it is not possible to improve any input or output without worsening some other input or output.*

We have already given a definition of CCR-efficiency for the model in the primal formulation. We now prove that the CCR-efficiency above, gives the same efficiency characterization as is obtained from Definition 2.1.1. This is formalized by:

**Theorem 2.1.4** *The CCR-efficiency given in Definition 2.1.2 is equivalent to that given by Definition 2.1.1.*

*Proof* First, notice that the vectors  $u$  and  $v$  of  $(P_C)$  are dual multipliers corresponding to the constraints (2.9) and (2.10) of  $(D_C)$ , respectively. Now the following "complementary conditions" hold between any optimal solutions  $(u^*, v^*)$  of  $(P_C)$  and  $(\lambda_j^*, s^{+*}, s^{-*})$  of  $(\widehat{D_C})$ .

$$u^* s^{+*} = 0 \quad \text{and} \quad v^* s^{-*} = 0. \quad (2.17)$$

This means that if any component of  $u^*$  or  $v^*$  is positive then the corresponding component of  $s^{+*}$  or  $s^{-*}$  must be zero, and conversely, with the possibility also allowed in which both components may be zero simultaneously. Now we demonstrate that Definition 2.1.2 implies Definition 2.1.1.

- i. If  $\theta^* < 1$ , then DMU  $j_0$  is CCR-inefficient by Definition 2.1.1, since  $(P_C)$  and  $(D_C)$  have the same optimal objective value by virtue of the dual theorem of linear programming.
- ii. If  $\theta^* = 1$  and is not zero-slack ( $s^{+*} \neq 0, s^{-*} \neq 0$ ), then, by the complementary conditions above, the elements of  $u^*$  or  $v^*$  corresponding to the positive slacks must be zero. Thus, DMU  $j_0$  is CCR- inefficient by Definition 2.1.1.
- iii. Lastly if  $\theta^* = 1$  and zero-slack, then, by the "strong theorem of complementarity,  $(P_C)$  is assured of a positive optimal solution  $(u^*, v^*)$  and hence DMU  $j_0$  is CCR-efficient by Definition 2.1.1.

The reverse is also true by the complementary relation and the strong complementarity theorem between  $(u^*, v^*)$  and  $(s^{+*}, s^{-*})$ .  $\square$

Up to this point, we have been dealing models known as CCR (Charnes, Cooper, Rhodes, [14]) models. If the convexity condition  $\sum_{j=1}^n \lambda_j = 1$  is adjoined, they are known as BCC (Banker, Charnes, Cooper, [5]) models. This added constraint introduces an additional variable,  $u_0$ , into the multiplier problems ( $P_C$ ). As will be seen in the next subsection, this extra variable makes it possible to effect returns-to-scale evaluations (increasing, constant and decreasing). So the BCC model is also referred to as the VRS (Variable Returns to Scale) model and distinguished from the CCR model which is referred to as the CRS (Constant Returns to Scale) model.

### 2.1.2 BCC DEA Models

In the literature of classical economics, Returns To Scale (RTS) have typically been defined only for single output situations. RTS are considered to be increasing if a proportional increase in all the inputs results in a more than proportional increase in the single output. Let  $\alpha$  represent the proportional input increase and  $\beta$  represent the resulting proportional increase of the single output. Increasing returns to scale prevail if  $\beta > \alpha$  and decreasing returns to scale prevail if  $\beta < \alpha$ . Banker [4], Banker, Charnes and Cooper [5] (BCC model) and Banker and Thrall [6] extend the RTS concept from the single output case to multiple output cases using DEA.

The efficiency of a specific DMU  $j_0$  can be evaluated by the BCC model of DEA as follows,

$$(P_V) \quad \max_{u, v, u_0} \sum_{r=1}^q u_r y_{rj_0} + u_0 \quad (2.18)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (2.19)$$

$$\sum_{r=1}^q u_r y_{rj} + u_0 - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad (2.20)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m.$$

The dual form of the BCC model represented in  $(P_V)$  is obtained from the same data which are then used in the following form,

$$(D_V) \quad \min_{\theta, \lambda} \quad \theta \quad (2.21)$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj} \geq y_{rj_0}, \quad r = 1, \dots, q,$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (2.22)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (2.23)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$



We can define the production possibility set  $T_V$  for DEA model under the hypothesis of variable return to scale, taking into account assumptions (A1)-(A4) presented in the previous subsection, by

$$T_V = \left\{ (x, y) : x \geq \sum_{j=1}^n \lambda_j x_j; y \leq \sum_{j=1}^n \lambda_j y_j; \sum_{j=1}^n \lambda_j = 1 \right\} \quad (2.24)$$

It is clear that a difference between the CCR and BCC models is present in constraint  $\sum_{j=1}^n \lambda_j = 1$  in the dual formulation, which is the constraint associated with the free variable  $u_0$  in primal formulation that also does not appear in the CCR model.

As concern problems  $(P_V)$  and  $(D_V)$ , notice that we confine attention to input-oriented versions of these efficiency measure models. This specification is necessary because, if under the hypothesis of constant return to scale, input-oriented CCR model will be efficient for any DMU if and only if it is also efficient when the output-oriented CCR model, in the case of variables returns to scale, to a different approach corresponds a different efficiency score.

Analogously to the CCR model, we can rewrite problem  $(D_V)$  as:

$$\begin{aligned} & (\widehat{D}_V) \\ & \min_{\theta, \lambda} \quad \theta \\ & \text{s.t.} \quad \sum_{j=1}^n \lambda_j y_{rj} = y_{rj_0} + s_r^+, \quad r = 1, \dots, q, \end{aligned} \quad (2.25)$$

$$\sum_{j=1}^n \lambda_j x_{ij} = \theta x_{ij_0} - s_i^-, \quad i = 1, \dots, m, \quad (2.26)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (2.27)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n,$$

$$s_r^+ \geq 0, \quad r = 1, \dots, q,$$

$$s_i^- \geq 0, \quad i = 1, \dots, m.$$

Let  $(\theta_V^*, \lambda^*, s^{-*}, s^{+*})$  an optimal solution for  $(\widehat{D}_V)$ . Notice that  $\theta_V^*$  is not less than the optimal objective value  $\theta$  of  $(D_C)$  model, since  $(BCC)$  imposes one additional constraint,  $\sum_{j=1}^n \lambda_j = 1$ , so its feasible region is a subset of feasible region for the CCR model.

It is possible to define efficiency concept under variable returns to scale, as

**Definition 2.1.4** (*BCC-Efficiency*) *An optimal solution  $(\theta_V^*, \lambda^*, s^{-*}, s^{+*})$  satisfies  $\theta_V^* = 1$  and has no slack ( $s^{-*} = 0, s^{+*} = 0$ ), then the DMU  $j_0$  is called BCC-efficient, otherwise it is BCC-inefficient.*

All the models assume that  $y_{rj} \geq 0$  and  $x_{ij} \geq 0$ ,  $r = 1, \dots, q$ ,  $i = 1, \dots, m$ ,  $j = 1, \dots, n$ . Also in  $(P_V)$  all variables are constrained to be non-negative, except for  $u_0$  which may be positive, negative or zero with consequences that make it possible to use optimal values of this variable to identify RTS.

When a DMU  $j_0$  is efficient with the Definition 2.1.1, the optimal value of  $u_0$ , i.e.  $u_0^*$ , in  $(P_V)$ , can be used to characterize the situation for Returns to Scale.

RTS generally has an unambiguous meaning only if DMU  $j_0$  is on the efficiency frontier, since it is only in this state that a tradeoff between inputs and outputs is required to improve one or the other of these elements. However, there is no need to be concerned about the efficiency status in our analyses because efficiency can always be achieved as follows. If a DMU  $j_0$  is not BCC efficient,

we can use optimal values from  $(D_V)$  to project this DMU onto the BCC efficiency frontier via the following formulas,

$$\begin{cases} \hat{y}_{rj_0} = \sum_{j=1}^n \lambda_j^* y_{rj} + s_r^{+*}, & r = 1, \dots, q, \\ \theta \hat{x}_{ij_0} = \sum_{j=1}^n \lambda_j^* x_{ij} - s_i^{-*}, & i = 1, \dots, m, \end{cases} \quad (2.28)$$

where  $s_r^{+*}$  and  $s_i^{-*}$  are the optimum slack variable associated to the inequality constraints (2.25) and (2.26) respectively. These are sometimes referred to as the "CCR Projection Formulas" because Charnes, Cooper and Rhodes (1978) showed that the resulting  $\hat{x}_{ij_0} \leq x_{ij_0}$  and  $\hat{y}_{rj_0} \geq y_{rj_0}$  correspond to the coordinates of a point on the efficiency frontier. They are, in fact, coordinates of the point used to evaluate DMU  $j_0$  when  $(D_V)$  is employed.

The following theorem for returns to scale, as obtained from Banker and Thrall [6], identifies RTS with the sign of  $u_0^*$  in  $(P_V)$  as follows:

**Theorem 2.1.5** *The following conditions identify the situation for RTS for the BCC model given in  $(P_V)$ .*

1. *Increasing RTS prevail at  $(\hat{x}_{j_0}, \hat{y}_{j_0})$  if and only if  $u_0^* < 0$  for all optimal solutions.*
2. *Decreasing RTS prevail at  $(\hat{x}_{j_0}, \hat{y}_{j_0})$  if and only if  $u_0^* > 0$  for all optimal solutions.*
3. *Constant RTS prevail at  $(\hat{x}_{j_0}, \hat{y}_{j_0})$  if and only if  $u_0^* = 0$  for at least one optimal solution.*

Here, it may be noted,  $(\hat{x}_{j_0}, \hat{y}_{j_0})$  are the coordinates of the point on the efficiency frontier which is obtained from (2.28) in the evaluation of DMU  $j_0$  via the solution to  $(\widehat{D}_V)$ . Note, therefore, that a use of the projection makes it unnecessary to assume that the points to be analyzed are all on the BCC efficient frontier - as was assumed in Banker and Thrall [6].

Now consider again model  $(D_C)$ ; this model is the same as the "envelopment form" of the BBC model in  $(D_V)$  except for the fact that the condition  $\sum_{j=1}^n \lambda_j = 1$  is omitted. The projection formulas expressed in (2.28) are the same for both models. We can therefore use these same projections to move all points onto the efficient frontier for  $(D_C)$  and proceed directly to returns to scale characterizations for  $(D_C)$  which are supplied by the following theorem from Banker and Thrall [6].

**Theorem 2.1.6** *The following conditions identify the situation for RTS for the CCR model given in  $(D_C)$ :*

1. *Constant returns to scale prevail at  $(\hat{x}_{j_0}, \hat{y}_{j_0})$  if  $\sum_{j=1}^n \lambda_j^* = 1$  in any alternate optimum;*
2. *Decreasing returns to scale prevail at  $(\hat{x}_{j_0}, \hat{y}_{j_0})$  if  $\sum_{j=1}^n \lambda_j^* > 1$  for all alternate optima;*
3. *Increasing returns to scale prevail at  $(\hat{x}_{j_0}, \hat{y}_{j_0})$  if  $\sum_{j=1}^n \lambda_j^* < 1$  for all alternate optima.*

For BBC model it is possible establish (see Ali and Seiford [1]) that the solution of the above linear programming problem with translated data is exactly the same as the solution of the linear programming problem with the original data. Let  $\bar{y}_{rj} = y_{rj} + \beta_r$ ,  $r = 1, \dots, q$  and  $\bar{x}_{ij} = x_{ij} + z_i$ ,  $i = 1, \dots, m$ , where  $\beta$  and  $z$  are proper translation vectors; without loss of generality it is assumed that  $\beta_r > 0$ ,  $r = 1, \dots, q$  and  $z_i > 0$ ,  $i = 1, \dots, m$ . Based upon the above linear transformation, the standard BCC DEA model can be modified as the following linear program:

$$\begin{aligned}
& (\widetilde{D}_V) \\
& \min_{\theta, \lambda} \theta \\
& \text{s.t.} \quad \sum_{j=1}^n \lambda_j \bar{y}_{rj} \geq \bar{y}_{rj_0}, \quad r = 1, \dots, q,
\end{aligned} \tag{2.29}$$

$$\sum_{j=1}^n \lambda_j \bar{x}_{ij} \leq \theta \bar{x}_{ij_0}, \quad i = 1, \dots, m, \tag{2.30}$$

$$\begin{aligned}
& \sum_{j=1}^n \lambda_j = 1, \\
& \lambda_j \geq 0, \quad j = 1, \dots, n.
\end{aligned} \tag{2.31}$$

**Theorem 2.1.7 (Translation Invariance Theorem)** *For the BBC model:*

1. *DMU  $j_0$  is efficient in  $(D_V)$  iff DMU  $j_0$  is efficient for  $(\widetilde{D}_V)$ ;*
2. *DMU  $j_0$  is inefficient in  $(D_V)$  iff DMU  $j_0$  is inefficient for  $(\widetilde{D}_V)$ .*

*Proof*

1. When  $\theta^* = 1$ , since  $\sum_{j=1}^n \lambda_j = 1$  and  $\bar{y}_{rj} = y_{rj} + \beta_r$ ,  $r = 1, \dots, q$  and  $\bar{x}_{ij} = x_{ij} + z_i$ ,  $i = 1, \dots, m$ , constrains (2.29) and (2.30) become  $\sum_{j=1}^n \lambda_j y_{rj} \geq y_{rj_0}$   $r = 1, \dots, q$  and  $\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}$   $i = 1, \dots, m$  and so, problem  $(D_V)$  and problem  $(\widetilde{D}_V)$  are equivalent.
2. Statement 2. is logically equivalent to statement 1.

□

## 2.2 Chance-constrained DEA models

Data envelopment analysis (DEA) is a technique for estimating the efficiency of production decisions made in private industry or in the not-for-profit sector. Observations located at the production frontier are assigned an efficiency value equal to unity; those behind the frontier are assigned a value less than unity. In other words, production efficiency is no longer taken for granted as a matter of assumption but is determined and tested empirically. As the literature of data envelopment analysis has grown many researchers have felt the need to incorporate stochastic considerations into the model to accommodate the presence of measurement and specification errors. Production relationships are often stochastic in nature. In agriculture the unpredictability of weather makes the input-output relationship stochastic. In manufacturing there may be considerable variability in the quality of output obtained, as attested by the need for statistical quality control. In product development there is uncertainty whether new designs will be technically viable and about the prospective market.

Chance-constrained programming is the most used technique to include noise variations in data and to solve data envelopment analysis problems with uncertainty in data. This kind of approach makes it possible to replace deterministic characterizations in DEA, such as "efficient" and "not efficient," with characterizations such as "probably efficient" and "probably not efficient." Indeed, it is possible to go still further into characterizations such as "sufficiently efficient," with associated probabilities of not being correct in making inferences about the performance of a DMU. Recent contributions to this approach are due to Land et al. [36], Olesen [44], Sueyoshi [56], Talluri et al. [58], stating a deterministic equivalent formulation in the case of normally distributed data.

The purpose of this section, however, is to provide a systematic presentation of major developments of chance constrained DEA models that have appeared in the literature. Our analysis will be restricted to what is referred to as the "E-model", so named because its objective is stated in terms of optimizing "expected values". Most of the other DEA literature on this topic has utilized the "P-model" of chance constrained programming to obtain the "most probable" occurrences, where also the objective function is represented as a probabilistically condition.

### 2.2.1 Stochastic dominance

In this subsection, we present basic concepts of efficiency and efficiency dominance, first in deterministic contest and after in stochastic one. Consider, for example, variable return to scale and so, BCC model. As we have stressed in Section 2.1.2, we can define the production possibility set  $T_V$  by

$$T_V = \left\{ (x, y) : x \geq \sum_{j=1}^n \lambda_j x_j; y \leq \sum_{j=1}^n \lambda_j y_j; \sum_{j=1}^n \lambda_j = 1 \right\}.$$

In this case we can able to formulate the following definition:

**Definition 2.2.1 (General efficiency dominance)** *Let  $(x', y') \in T_V$  and  $(x'', y'') \in T_V$ . We say that  $(x', y')$  dominates  $(x'', y'')$  with respect to the production possibility set  $T_V$  if and only if  $x' < x''$  and  $y' > y''$  with strict inequality holding for at least one of the components in the input or the output vector.*

Thus, a point in  $T_V$  is not dominated if and only if there is no other point in  $T_V$  which satisfies the definition. This leads to the following definition of efficiency:

**Definition 2.2.2 (Efficiency)** *DMU  $j_0$  is efficient with respect to  $T_V$  if and only if there is no  $(x, y) \in T_V$  such that  $(x_{j_0}, y_{j_0})$  is dominated by  $(x, y)$ .*

In other word,

**Definition 2.2.3 (Efficiency 2)** *DMU  $j_0$  is efficient if it is impossible to find a feasible solution for the following problem:*

$$\left( \begin{array}{l} \sum_{j=1}^n \lambda_j y_{rj} \geq y_{rj_0}, \quad r = 1, \dots, q, \\ \sum_{j=1}^n \lambda_j x_{ij} \leq x_{ij_0}, \quad i = 1, \dots, m, \end{array} \right)$$

with  $\lambda_j \geq 0 \quad j = 1, \dots, n$  satisfying  $\sum_{j=1}^n \lambda_j = 1$

In order to define stochastic efficiency concept, the following parameters are introduced.:

$y_{rj}(\xi) \in \mathbb{R}_+$ :  $r^{th}$  output quantity produced by the  $j^{th}$  DMU  
depending on the random factor  $\xi$ ,  
 $r = 1, \dots, q, \quad j = 1, \dots, n$ ;

$x_{ij}(\xi) \in \mathbb{R}_+$ :  $i^{th}$  input quantity used by the  $j^{th}$  DMU  
depending on the random factor  $\xi$ ,  
 $i = 1, \dots, m, \quad j = 1, \dots, n$ .

The concept of dominance can be extended to stochastic efficiency dominance by jointly comparing the outputs and inputs of the DMU under investigation, with every other observed DMU. Informally, it possible to say that DMU  $j_0$  is stochastically not dominated in its efficiency if it is stochastically impossible to find a feasible alternative which is no worse in all criteria and better for at least one criterion. Since the above definition of stochastic efficiency dominance is very strong, Huang and Li [31] suggested to modify the definition as follows. For a given scalar  $\alpha \in [0, 1]$ , DMU  $j_0$  is not stochastically dominated in its efficiency if and only if there is a joint probability less than or equal to  $\alpha$  that some other observed DMU displays efficiency dominance relative to DMU  $j_0$ . It can be report the exact mathematical characterizations to this concept of stochastic efficiency dominance in the following definition (see [9]).

**Definition 2.2.4 ( $\alpha$ -stochastic efficiency)** *DMU  $j_0$  is  $\alpha$ -stochastic efficient if and only if for any  $\lambda_j \geq 0$  satisfying  $\sum_{j=1}^n \lambda_j = 1$  we have*

$$\mathbb{P} \left( \begin{array}{l} \sum_{j=1}^n \lambda_j y_{rj}(\xi) \geq y_{rj_0}(\xi), \quad r = 1, \dots, q, \\ \sum_{j=1}^n \lambda_j x_{ij}(\xi) \leq x_{ij_0}(\xi), \quad i = 1, \dots, m. \end{array} \right) \geq (1 - \alpha),$$

where "P" means "probability", so they restrict definition of efficiency to the probability of the existence of dominating DMU to be less than  $\alpha$ .

### 2.2.2 LLT model

Land, Lovell and Thore [36] modified the standard DEA model to measure technical efficiency in the presence of random variation in the output produced from a give input bundle. Their chance-constrained DEA model (LLT model) builds on the method of chance-constrained programming

(CCP) developed by Charnes and Cooper [13]. The essence of a CCP model is that it allows a positive (although low) probability that one or more inequality restrictions will be violated at the optimal solution of the problem. They developed a chance constrained model in which inputs are assumed to be deterministic and outputs are jointly normal distributed. In this setting, the efficient frontier is a soft margin which may be crossed by a few DMUs. Extending the concept of efficiency in a chance constrained setting the authors stated the equivalence between chance constrained efficiency and Pareto Koopmans efficiency.

The Land, Lovell and Thore chance-constrained input-oriented BCC DEA model can be specified as follows:

$$\begin{aligned} \min_{\theta, \lambda} \quad & \theta \\ \text{s.t.} \quad & \mathbb{P} \left( \sum_{j=1}^n \lambda_j y_{rj}(\xi) \geq y_{rj_0}(\xi) \right) \geq (1 - \alpha), \quad r = 1, \dots, q, \end{aligned} \quad (2.32)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (2.33)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (2.34)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

The meaning of the chance constraints is that all the constraints should not be violated with probability at most  $\alpha$ . If we assume also random distribution for outputs, we obtain the follows model:

$$\begin{aligned} \min_{\theta, \lambda} \quad & \theta \\ \text{s.t.} \quad & \mathbb{P} \left( \sum_{j=1}^n \lambda_j y_{rj}(\xi) \geq y_{rj_0}(\xi) \right) \geq (1 - \alpha), \quad r = 1, \dots, q, \end{aligned} \quad (2.35)$$

$$\mathbb{P} \left( \sum_{j=1}^n \lambda_j x_{ij}(\xi) \leq \theta x_{ij_0}(\xi) \right) \geq (1 - \alpha), \quad i = 1, \dots, m, \quad (2.36)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (2.37)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

**Definition 2.2.5 (Chance Constrained Efficiency)** *DMU  $j_0$  is stochastic efficient if and only if the following two conditions are both satisfied:*

*i.  $\theta^* = 1$ ;*

*ii. Slack values are all zero for all optimal solutions.*

Although the model has the advantage of making the estimated frontier less sensitive to extreme observations, it introduces a bias due to the normality assumption. Moreover, the probabilistic constraints are individually imposed on stochastic inputs/outputs. Thus, the model enables to handle dependencies inter-DMUs, but ignores the correlation intra-DMUs.

### 2.2.3 OP model

Olesen and Petersen [45] developed a chance constrained DEA model (OP model) imposing chance constraints on a DEA formulation in multiplier form. They assumed that the inefficiency term of the considered DMU can be decomposed into true inefficiency and disturbance term. In the chance constrained OP model, only observations belonging to confidence region shave to be included in the empirical production possibility set (PPS). Since chance constraints are imposed individually on each DMU, the OP model accounts for correlation among outputs, inputs and inputs/outputs.

$$\max_{u,v} \sum_{r=1}^q u_r y_{rj}(\xi) \quad (2.38)$$

$$s.t. \sum_{i=1}^m v_i x_{ij}(\xi) = 1, \quad (2.39)$$

$$\mathbb{P} \left( \sum_{r=1}^q u_r y_{rj}(\xi) \leq \sum_{i=1}^m v_i x_{ij}(\xi) \right) \geq (1 - \alpha), \quad j = 1, \dots, n, \quad (2.40)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m.$$

### 2.2.4 Sueyoshi models

Sueyoshi [56] proposed a stochastic DEA model able to incorporate future information. The author referred to this approach as "DEA future analysis "and applied it to plan the restructure strategy of a Japanese petroleum. In this study is assumed that it can be control the quantity of inputs as the decision variables, whilst being unable to control outputs, because these quantities depend upon external factors such as an economic condition, a demographic change, and other socio-economic factors that influence the magnitude of outputs. Hence, the inputs are considered as deterministic variables and the outputs are considered as stochastic variables. Sueyoshi model, proposed by the author in the primal formulation, is defined as follows:

$$\max_{u,v} \mathbb{E}_\xi \left[ \sum_{r=1}^q u_r y_{rj_0}(\xi) \right] \quad (2.41)$$

$$s.t. \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (2.42)$$

$$\mathbb{P} \left( \frac{\sum_{r=1}^q u_r y_{rj}(\xi)}{\sum_{i=1}^m v_i x_{ij}} \leq \beta_j \right) \geq (1 - \alpha_j), \quad j = 1, \dots, n, \quad (2.43)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

where:

- $\alpha_j \in [0, 1]$ : specifies the allowable likelihood of falling the  $j^{th}$  constraint,  
 $j = 1, \dots, n$ ;  
 $\beta_j \in [0, 1]$ : it is referred to as an "aspiration level",  
specifies the desired efficiency level for the  $j_0^{th}$  DMU,  
 $j = 1, \dots, n$ .

In other words, the symbol  $\alpha_j$  stands for a probability that the output/input ratio becomes more than  $\beta_j$  with a choice of weight multipliers. Thus,  $\alpha_j$  is considered as a risk criterion representing the utility of a decision maker. On the other hand,  $1 - \alpha_j$  indicates the probability of attaining the requirement. Like  $\beta_j$ , the risk criterion ( $\alpha_j$ ) is also a prescribed value that is measured on the range between 0 and 1. When  $\alpha_j = 0$  in constraints (2.43), it is certainly required that the output/input ratio becomes less than or equal to  $\beta_j$ . Conversely,  $\alpha_j = 1$  omits the requirement under any selection of weight multipliers.

It can be easily thought that the above DEA future model needs to be reformulated to obtain its computational feasibility. In particular, through the transformation developed by Charnes and Cooper [13], constraints (2.43) including the stochastic process, can be rewritten as follows:

$$\mathbb{P} \left( \sum_{r=1}^q u_r \tilde{y}_{rj} \leq \beta_j \left( \sum_{i=1}^m v_i x_{ij} \right) \right) \geq (1 - \alpha_j), \quad j = 1, \dots, n, \quad (2.44)$$

where, for the sake of completeness, the stochastic outputs is expressed as  $\tilde{y}_{rj} \equiv y_{rj}(\xi)$ . Conditions (2.44) is equivalent to:

$$\mathbb{P} \left( \frac{\sum_{r=1}^q u_r (\tilde{y}_{rj} - \bar{y}_{rj})}{\sqrt{V_j}} \leq \frac{\beta_j \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^q u_r \bar{y}_{rj}}{\sqrt{V_j}} \right) \geq (1 - \alpha_j), \quad j = 1, \dots, n, \quad (2.45)$$

where  $\bar{y}_{rj}$  is the expected value of stochastic outputs  $\tilde{y}_{rj}$  and  $V_j$  indicates the variance-covariance matrix of the  $j^{th}$  DMU.

These reformulated conditions introduce the following new variable  $\tilde{z}_j$ :

$$\tilde{z}_j = \frac{\sum_{r=1}^q u_r (\tilde{y}_{rj} - \bar{y}_{rj})}{\sqrt{V_j}}, \quad j = 1, \dots, n, \quad (2.46)$$

which follows the standard normal distribution with zero mean and unit variance. Substitution of condition (2.46) in constraints (2.45) produces

$$\mathbb{P} \left( \tilde{z}_j \leq \frac{\beta_j \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^q u_r \bar{y}_{rj}}{\sqrt{V_j}} \right) \geq (1 - \alpha_j), \quad j = 1, \dots, n. \quad (2.47)$$

Since  $\tilde{z}_j$  follows the standard normal distribution, the invertibility of above condition is executed as follows:



$$\frac{\beta_j \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^q u_r \bar{y}_{rj}}{\sqrt{V_j}} \geq F^{-1}(1 - \alpha_j), \quad j = 1, \dots, n. \quad (2.48)$$

Here,  $F$  stands for a cumulative distribution function of the normal distribution and  $F^{-1}$  indicates its inverse function.

Sueyoshi's determinist equivalent formulation is then defined as follows:

$$\max_{u,v} \sum_{r=1}^q u_r \bar{y}_{rj_0} \quad (2.49)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (2.50)$$

$$\begin{aligned} \beta_j \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^q u_r \bar{y}_{rj} &\geq \sqrt{V_j} F^{-1}(1 - \alpha_j), \quad j = 1, \dots, n, \\ u_r &\geq 0, \quad r = 1, \dots, q, \\ v_i &\geq 0, \quad i = 1, \dots, m. \end{aligned} \quad (2.51)$$

Notice that, this kind of resolution method is also applicable to the other chance-constrained model, above mentioned.

### 2.2.5 Bruni model

The main research field in Stochastic DEA considers a chance-constrained approach which permits constraint violations up to specified probability limits. Bruni et al. [9] introduce a less restrictive formulation than classical chance-constrained approach. The authors take further developments in order to derive a sufficient criterion for efficiency, complementing the Huang and Li approach [32]. They shall assume that the random variables follow a discrete distribution and each realization can be represented by a scenario  $s$ . Let us denote by  $T_S$  the production possibility set for scenario  $s$ :

$$T_S = \left\{ (x^s, y^s) : \begin{aligned} &\sum_{j=1}^n \lambda_j y_{rj}^s \geq y_{rj_0}^s, \quad r = 1, \dots, q, \quad s = 1, \dots, S, \\ &\sum_{j=1}^n \lambda_j x_{ij}^s \leq \theta x_{ij_0}^s, \quad i = 1, \dots, m, \quad s = 1, \dots, S, \\ &\sum_{j=1}^n \lambda_j = 1, \\ &\lambda_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \right\} \quad (2.52)$$

**Definition 2.2.6 (Scenario efficiency)** *DMU  $j_0$  is efficient with respect to the scenario  $s$  if it is*

impossible to find a feasible solution for the following problem:

$$\sum_{j=1}^n \lambda_j y_{rj}^s \geq y_{rj_0}^s, \quad r = 1, \dots, q, \quad (2.53)$$

$$\sum_{j=1}^n \lambda_j x_{ij}^s \leq x_{ij_0}^s, \quad i = 1, \dots, m, \quad (2.54)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (2.55)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n, \quad (2.56)$$

and strict inequality holding for at least one constraint. Only if for DMU  $j_0$  problem (2.53)-(2.56) is infeasible with respect to the full scenario set, DMU  $j_0$  is 100% stochastically efficient.

Bruni et al. propose a model which removes the hypothesis of normal data distribution and uses a scenario generation approach to linearize the problem formulated through the chance-constrained approach. Starting from the LLT model and by adopting a standard technique used in disjunctive programming (see Balas [2]) Bruni et al. defined the deterministic equivalent formulation model for joint probabilistic constraints as:

$$\begin{aligned} \min_{\theta, \lambda, \delta} \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j y_{rj}^s + M^s \delta^s \geq y_{rj_0}^s, \quad r = 1, \dots, q, \quad s = 1, \dots, S, \end{aligned} \quad (2.57)$$

$$\sum_{j=1}^n \lambda_j x_{ij}^s \leq \theta x_{ij_0}^s + M^s \delta^s, \quad i = 1, \dots, m, \quad s = 1, \dots, S, \quad (2.58)$$

$$\sum_{s=1}^S p^s \delta^s \leq (1 - \alpha), \quad (2.59)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n,$$

$$\delta^s \in \{0, 1\}, \quad s = 1, \dots, S,$$

where:

- $y_{rj}^s \in \mathbb{R}_+$ :  $r^{\text{th}}$  output quantity produced by the  $j^{\text{th}}$  DMU for each scenario  $s$ ,  $r = 1, \dots, q$ ,  $j = 1, \dots, n$ ,  $s = 1, \dots, S$ ;
- $x_{ij}^s \in \mathbb{R}_+$ :  $i^{\text{th}}$  input quantity produced by the  $j^{\text{th}}$  DMU for each scenario  $s$ ,  $i = 1, \dots, m$ ,  $j = 1, \dots, n$ ,  $s = 1, \dots, S$ ;
- $p^s \in [0, 1]$ : realization probability of any scenario  $s$ ,  $s = 1, \dots, S$ ;
- $\delta^s \in \{0, 1\}$ : binary variable for each scenario  $s$ ,  $s = 1, \dots, S$ .

Bruni's Model, according to chance-constrained approach, permits constraint violations up to a fixed probability level; in particular, constraint (2.59) defines a binary knapsack constraint which guarantees the violation of the stochastic constraints for a subset of scenarios whose cumulative

probability is less than the complement of the imposed reliability level  $1 - \alpha$ . Stating  $\alpha = 1$  and  $p^s > 0$ , the constraint (2.59) forces the binary variables  $\delta^s$  to assume the value zero for all scenarios. This is the situation where it is impossible to violate any constraints so that all the scenarios have to be considered.

### 2.3 DEA models with undesirable factors

Data envelopment analysis (DEA) uses linear programming problems to evaluate the relative efficiencies and inefficiencies of peer decision-making units (DMUs) which produce multiple outputs by using multiple inputs. Once DEA identifies the efficient frontier, DEA improves the performance of inefficient DMUs by either increasing the current output levels or decreasing the current input levels. However, both desirable (good) and undesirable (bad) output and input factors may be present. Consider production process where outputs are produced with undesirable outputs of pollutants such as biochemical oxygen demand, suspended solids, particulates and sulfur oxides. If inefficiency exists in the production, the undesirable pollutants should be reduced to improve the inefficiency, i.e., the undesirable and desirable outputs should be treated differently when we evaluate the production performance of production process. However, in the standard DEA model, decreases in outputs are not allowed and only inputs are allowed to decrease. When undesirable outputs are taken into consideration, the choice between two alternative disposable technologies (improved technologies or reference technologies) has an important impact on DMUs efficiencies. Technology disposability can be also read in terms of strong and weak disposability of undesirable outputs. A production process is said to exhibit strong disposability of undesirable outputs, e.g., heavy metals, CO<sub>2</sub>, etc., if the undesirable outputs are freely disposable, i.e. they do not have limits. The case of weak disposability refers to situations when a reduction in waste or emissions forces a lower production of desirable outputs, i.e., in order to meet some pollutant emission limits (regulations), reducing undesirable outputs may not be possible without assuming certain costs (see Zofio and Prieto, [67]). In order to include undesirable outputs in DEA models, different approaches have been introduced. In the next subsections, a brief review of existing linear models is presented. For the sake of convenience, the list of common variables and parameters used in the different models is provided below.

**Parameters:**

$x_{ij} \in \mathbb{R}_+$ :  $i^{th}$  input quantity used by the  $j^{th}$  decision making unit,  
 $i = 1, \dots, m, \quad j = 1, \dots, n$ ;

$y_{rj}^g \in \mathbb{R}_+$ :  $r^{th}$  “good” output quantity produced by the  $j^{th}$   
 decision making unit,  $r = 1, \dots, q, \quad j = 1, \dots, n$ ;

$y_{kj}^b \in \mathbb{R}_+$ :  $k^{th}$  “bad” output quantity produced by  
 the  $j^{th}$  decision making unit,  $k = 1, \dots, l, \quad j = 1, \dots, n$ .

**Variables:**

$v_i \in \mathbb{R}_+$ : weight multipliers related to the  $i^{th}$  input,  
 $j = 1, \dots, n$ ;

$u_r \in \mathbb{R}_+$ : weight multipliers related to the  $r^{th}$  “good” output,  
 $r = 1, \dots, q$ ;

$w_k \in \mathbb{R}_+$ : weight multipliers related to the  $k^{th}$  “bad” output,  
 $k = 1, \dots, l$ ;

$u_0 \in \mathbb{R}$ : scale factor variable;

$\theta \in \mathbb{R}_+$ : dual variable related to the first constraint;

$\lambda_j \in \mathbb{R}_+$ : dual variables related to the second set of constraints,  
 $j = 1, \dots, n$ .

By means of the previous notations, when the undesirable outputs are strong disposable, the production possibility set can be expressed as

$$T^S = \left\{ (x, y^g, y^b) : x \geq \sum_{j=1}^n \lambda_j x_j; y^g \leq \sum_{j=1}^n \lambda_j y_j^g; y^b \leq \sum_{j=1}^n \lambda_j y_j^b; \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0 \right\}. \quad (2.60)$$

When the undesirable outputs are weakly disposable, the production possibility set may be written as

$$T^W = \left\{ (x, y^g, y^b) : x \geq \sum_{j=1}^n \lambda_j x_j; y^g \leq \sum_{j=1}^n \lambda_j y_j^g; y^b = \sum_{j=1}^n \lambda_j y_j^b; \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0 \right\}. \quad (2.61)$$

### 2.3.1 INP model: undesirable factors treated as inputs

A first class of DEA models with undesirable data suggests to include undesirable inputs as desirable outputs, or undesirable outputs as desirable inputs in the production process (see [38]). Its starting point is that efficient DMUs wish to minimize desirable inputs and undesirable outputs, and to maximize desirable outputs and undesirable inputs. If one only wishes to investigate operational efficiency from this point of view, there is no need to distinguish between inputs and outputs, but only minimum and maximum. In our perspective we focus our attention on desirable inputs and outputs and undesirable outputs only. The mathematical formulation of the model, in case of strong output disposability and input oriented DEA, is as follows:

$$(P_{INP}) \quad \max_{u,v,w,u_0} \sum_{r=1}^q u_r y_{rj_0}^g + u_0 \quad (2.62)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} + \sum_{k=1}^l w_k y_{kj_0}^b = 1, \quad (2.63)$$

$$\sum_{r=1}^q u_r y_{rj}^g + u_0 - \sum_{i=1}^m v_i x_{ij} - \sum_{k=1}^l w_k y_{kj}^b \leq 0, \quad j = 1, \dots, n, \quad (2.64)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$w_k \geq 0, \quad k = 1, \dots, l,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0 \in \mathbb{R},$$

and the corresponding dual formulation is:

$$(D_{INP}) \quad \min_{\theta, \lambda} \quad \theta \quad (2.65)$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \quad (2.66)$$

$$\sum_{j=1}^n \lambda_j y_{kj}^b \leq \theta y_{kj_0}^b, \quad k = 1, \dots, l, \quad (2.67)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (2.68)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (2.69)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

The primal formulation corresponds to a standard input-oriented primal BCC-model where undesirable outputs behave like inputs. The objective function of the primal formulation maximizes the weighted sum of desirable outputs, under the condition that the weighted sum of inputs and undesirable outputs for the considered DMU is equal to one (as it results in constraint (4.2)). From a dual point of view, this means that a DMU can simultaneously reduce all inputs and undesirable outputs by the same proportion  $\theta$  in order to increase its eco-efficiency. An efficient DMU will have as optimal solution  $\theta^* = 1$  implying that no equiproportional reduction in inputs and undesirable outputs is possible.

Note that models ( $P_{INP}$ ) and ( $D_{INP}$ ) can be used with the assumption of weak disposability by respectively considering variables  $w_k$  as unconstrained in sign in the primal formulation and by assuming that constraint (4.6) holds with equality in the corresponding dual formulation. For an exhaustive discussion on strong and weak disposability in this class of models see Liu et al. [37]. Note that considering the undesirable outputs as inputs, the resulting DEA model does not reflect the true production process. This is the main drawback of this formulation.

### 2.3.2 Korhonen-Luptacik DEA model

The class of DEA models we present in this subsection permits to measure efficiency as the ability to reduce pollutants while maintaining the same inputs and desirable outputs levels. The efficiency measure, as formally defined by Korhonen and Luptacik in [34], is then the ratio between the weighted sum of the desirable outputs minus the weighted sum of the inputs and the weighted sum of the undesirable outputs. The Primal-Dual linearized version of this class of DEA models is as follows:

$$(P_{KL})$$

$$\max_{u,v,w,u_0} \sum_{r=1}^q u_r y_{rj_0}^g - \sum_{i=1}^m v_i x_{ij_0} + u_0 \quad (2.70)$$

$$s.t. \quad \sum_{k=1}^l w_k y_{kj_0}^b = 1, \quad (2.71)$$

$$\sum_{r=1}^q u_r y_{rj}^g - \sum_{i=1}^m v_i x_{ij} + u_0 - \sum_{k=1}^l w_k y_{kj}^b \leq 0, \quad j = 1, \dots, n, \quad (2.72)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$w_k \geq 0, \quad k = 1, \dots, l,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0 \in \mathbb{R}.$$

(D<sub>KL</sub>)

$$\min_{\theta, \lambda} \quad \theta$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj}^g \geq \theta y_{rj_0}^g, \quad r = 1, \dots, q, \quad (2.73)$$

$$\sum_{j=1}^n \lambda_j y_{kj}^b \leq \theta y_{kj_0}^b, \quad k = 1, \dots, l, \quad (2.74)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq x_{ij_0}, \quad i = 1, \dots, m, \quad (2.75)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

The objective function in the dual formulation gives information on the pollutants contraction to the largest extent possible. If  $\theta^* < 1$ , that is DMU is inefficient, the firm can still reduce undesirable outputs without increasing the corresponding inputs level or reducing desirable outputs. Model (P<sub>KL</sub>)-(D<sub>KL</sub>) under the hypothesis of weak disposability can be obtained considering variable  $w_k$  as unconstrained in sign in the primal formulation and assuming that constraints (4.13) hold with equality in dual formulation.

### 2.3.3 TR $\beta$ model: a linear transformation approach

The analysis on efficiency can be deepened on by considering a third efficiency measure which concentrates on the efficiency improvement by reducing inputs while maintaining the same fixed outputs levels both for undesirable and desirable ones. In this light, we present the approach proposed by Seiford and Zhu [55]. Under the context of the BCC model (Banker et al., [5]), Seiford and Zhu developed an alternative method to deal with desirable and undesirable factors in DEA. In order to increase the desirable outputs and to decrease the undesirable outputs, they transform

the values of the undesirable outputs by a monotone decreasing function. The transformed data can then be included as desirable outputs in the problem and maximized. In fact, after the decreasing transformation, maximizing these values means minimizing the original undesirable outputs. In particular, for the purpose of preserving linearity and convexity relations, Seiford and Zhu [55] suggested a linear monotone decreasing transformation,  $\bar{y}_{kj}^b = -y_{kj}^b + \beta_k > 0$ , where  $\beta$  is a proper translation vector that makes  $\bar{y}_{kj}^b > 0$ . Based upon the above linear transformation, the standard BCC DEA model can be modified as the following pair of linear programs:

$$(P_{TR\beta}) \quad \max_{u,v,w,u_0} \sum_{r=1}^q u_r y_{rj_0}^g + \sum_{k=1}^l w_k \bar{y}_{kj_0}^b + u_0 \quad (2.76)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (2.77)$$

$$\sum_{r=1}^q u_r y_{rj}^g + \sum_{k=1}^l w_k \bar{y}_{kj}^b + u_0 - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad (2.78)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$w_k \geq 0, \quad k = 1, \dots, l,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0 \in \mathbb{R}.$$

$$(D_{TR\beta}) \quad \min_{\theta, \lambda} \quad \theta$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \quad (2.79)$$

$$\sum_{j=1}^n \lambda_j \bar{y}_{kj}^b \geq \bar{y}_{kj_0}^b, \quad k = 1, \dots, l, \quad (2.80)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (2.81)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

In model  $(D_{TR\beta})$ , like the classical BCC models, the efficiency is measured taking into account the possible input reductions while the outputs are kept at their current levels. According to this model a DMU  $j_0$  can improve the eco-efficiency by reduce the inputs, while the values of the desirable and undesirable outputs of the DMU  $j_0$  are taken as lower bounds for a linear combination of the other desirable and undesirable outputs.

From a theoretical point of view, notice that, by assuming Variable Return to Scale (VRS), the model is invariant with respect to the linear translation. It has been proved by Ali and Seiford in [1] that affine translation of data values does not alter the efficient frontier. Thus the classification of DMUs as efficient or inefficient is translation invariant. We recall that the same models can be used



with the assumption of weak disposability by respectively considering variables  $w_k$  as unconstrained in sign in the primal formulation ( $P_{TR\beta}$ ) and by assuming that constraints (4.20) hold with equality in the corresponding dual formulation ( $D_{TR\beta}$ ).

### 2.3.4 A directional distance function approach

The directional output distance function, in its original formulations by Färe et al. [28], is an alternative approach to evaluate efficiency. This approach expands desirable outputs and contracts undesirable outputs along a path that varies according to the direction vector adopted, in order to increase efficiency. Extensions of this methodology (see for all [25, 26, 46]) obtain a measure of efficiency from the potential for increasing outputs while reducing inputs and undesirable outputs simultaneously.

In order to describe this approach, let us define the following sets. Let  $T$  be the technology set, such that:

$$T = [(x, y^g, y^b) : x \text{ can produce } (y^g, y^b)]. \quad (2.82)$$

In presence of undesirable outputs, the output set  $\mathcal{P}(x)$  represents all the feasible output vectors  $(y^g, y^b)$  for a given input vector  $x$ , that is:

$$\mathcal{P}(x) = [(y^g, y^b) : (x, y^g, y^b) \in T]. \quad (2.83)$$

The directional technology distance function generalizes both input and output Shephard's distance functions, providing a complete representation of the production technology.

Let  $d = (-d^x, d^g, -d^b)$ , the function is formally defined as:

$$\vec{D}_T(x, y^g, y^b; d) = \sup [\delta : (y^g + \delta d^g, y^b - \delta d^b) \in \mathcal{P}(x - \delta d^x)]. \quad (2.84)$$

Expression (4.23) seeks for the maximum attainable expansion of desirable outputs in the  $d^g$  direction and the largest feasible contraction of undesirable outputs and inputs in  $d^b$  and  $d^x$  directions. Under the assumptions made on the technology of reference, the directional technology distance function of expression (4.23) can be computed for firm  $j_0$  by solving the following programming problem:

$$\begin{aligned} & (P_{DDF}) \\ & \max_{\delta, \lambda} \quad \delta \\ & \text{s.t.} \quad \sum_{j=1}^n \lambda_j y_{rj}^g - \delta d_{rj_0}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \end{aligned} \quad (2.85)$$

$$\sum_{j=1}^n \lambda_j y_{kj}^b + \delta d_{kj_0}^b \leq y_{kj_0}^b, \quad k = 1, \dots, l, \quad (2.86)$$

$$\sum_{j=1}^n \lambda_j x_{ij} + \delta d_{ij_0}^x \leq x_{ij_0}, \quad i = 1, \dots, m, \quad (2.87)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

The choice of a direction vector  $d = (-x, y^g, -y^b)$  permits to evaluate a global technology and ecological efficiency by reducing inputs and undesirable outputs and simultaneously expanding desirable outputs. A different direction vector can be used in order to restrict the analysis on output factors,

by considering, for instance, a direction vector  $d = (0, y^g, -y^b)$ . In this case Mandal and Madheswaran [40] focus their attention on expansion of desirable factors and contraction of undesirable ones without increasing the inputs.

Notice that in the directional distance function model, efficiency is reached when  $\delta = 0$ , corresponding to the case of  $\theta = 1$  in the standard DEA formulations.

Let us recall that this model can be also considered under the assumption of weak disposability by assuming that constraints (2.86) hold with equalities instead of inequalities.

## Chapter 3

# Data Envelopment Analysis with outputs uncertainty

The aim of this chapter is to present two new different models for Data Envelopment Analysis with uncertain outputs studied in this Ph.D. thesis. These results, collected in [50], are aimed to generalize and improve the ones already published in the literature (see, for instance, [9, 18, 20, 33, 36, 44, 48, 56, 58]).

As streded in the previous chapter, Data Envelopment Analysis (DEA) has been first proposed in the pioneering paper by Charnes, Cooper and Rhodes (CCR) [14]. It is a nonparametric method for estimating the efficiency of decision-making units (DMUs), such as firms or public sector agencies. In the classic DEA model there are  $n$  DMUs to be evaluated. Each DMU consumes various inputs to produce different outputs. No production function needs to be specified.

The pioneer model (CCR) measures technical efficiency of a DMU which exhibits Constant Returns to Scale (CRS) everywhere on the production frontier. In an important extension of this approach, Banker, Charnes and Cooper [5] generalized the original DEA approach formulating a model (BCC) for exhibiting Variable Returns to Scale (VRS) at different points on the production frontier.

The classical models assume a deterministic framework with no uncertainty and this seems not suitable for concrete applications, due to the presence of errors and noise in the estimation of inputs and outputs values.

In this chapter, starting from the generalized input-oriented (BCC) model, two different models with uncertain outputs and deterministic inputs are proposed. Various applications, in fact, are affected by random perturbations in output values estimation (see, for instance, [9, 56, 58]). Random perturbations can be addicted to a concrete difficulty in estimating the right output value (for instance, in the case of energy companies, electricity production has to take care of different and uncertain energy dispersion factors according to the employed technologies) or to obtain good output provisions (for instance, in the case of DEA applied to health care problems, early screening efficiency measures are related to the estimation of true positive and false positive screens which are indeed outputs with a stochastic nature).

A large number of papers, based on different approaches, can be found in the literature concerning DEA with outputs uncertainty. In particular, chance-constrained programming is the most used technique to include noise variations in data and to solve data envelopment analysis problems with uncertainty in data. Chance-constrained programming admits random data variations and permits constraint violations up to specified probability limits, allowing linear deterministic equivalent formulations in the case a normal distribution of the data uncertainty is assumed (see

for all [18, 20, 33, 36, 44, 48]). The formulations proposed in this chapter move away the classical chance-constrained method with the aim to obtain a more accurate DMU ranking whatever situation occurs. In particular, two different models are proposed where uncertainty is managed with a scenario generation approach. For the sake of completeness, these models are compared with two further ones based on an expected value approach, that is to say that the uncertainty is managed by means of the expected values of random factors both in the objective function and in the constraints.

Deeply speaking, the main difference between the two proposed models and the expected value approaches lies in their mathematical formulation. In the models based on the scenario generation approach the constraints concerning efficiency level are expressed for each scenario, while in the expected value models they are satisfied in expected value. As a consequence, the first kind of models result to be more selective in finding a ranking of efficiency, thus becoming useful strategic management tools aimed to determine a restrictive efficiency score ranking.

In Sections 3.1 and 3.2 two different models with uncertain data and assuming Variable Returns to Scale (VRS) are presented as a pair of Primal-Dual problems. Their corresponding deterministic formulations are obtained via a scenario approach. In the first model formulation (VRS1), the specific optimal weight composition is searched for each realization of the random factor  $\xi$ . On the other hand, the second model (VRS2) is formulated with the aim of optimizing an unique vector of weights whatever scenario occurs. Two further models based on the expected value approach are considered in order to be compared with the previously introduced ones. In Section 3.3 a constant returns to scale version of the proposed models is given and some theoretical results are presented. Finally, in Section 3.4 the results of a complete computational test are collected in order to compare the scenario generation models with the expected value approaches.

### 3.1 DEA models with outputs uncertainty

In this section two DEA models with variable returns to scale and uncertain data are presented (see Section 3.3 for corresponding constant returns to scale models). Both a primal and a dual versions are developed for each formulation and some properties are remarked. The output parameters for both formulations are assumed to be uncertain depending on a random factor  $\xi$ . Inputs parameter are deterministic. In the following subsections these models are described and compared.

It is worth noticing that many models in the literature are introduced only in dual form. Actually, the dual form does not provide an intuitive interpretation of them. For this very reason, in this chapter the models are presented as a Primal-Dual formulation: the primal formulation is suitable for the model interpretation, while the dual one allows a direct comparison with the results in the literature.

Finally, notice that chance-constrained programming is the most used technique to include noise variations in data and to solve Data Envelopment Analysis problems with uncertainty in data. Chance-constrained programming permits constraint violations up to specified probability limits and generally assumes normal data distribution. The choice of assuming that random data follow a normal distribution, motivated by the simplifications provided from a computational standpoint, is not well founded in real settings. This work moves a step towards this direction, proposing two different models that can be move away from a specific form of distribution functions. The key assumption of these models is that the random variables, representative of the uncertain data, follow a discrete distribution or that a discrete approximation of continuous distribution is available, in order to manage uncertainty through the generation of confident realizations (or scenarios). In this work we provide, therefore, two models based on scenario generation approach to include data perturbations. In DEA literature, as far as we know, there are no similar scenario approaches, except the one of Bruni et al. [9] where scenarios generation is used to linearize the problem formulated through the chance-constrained approach.

### 3.1.1 The VRS1 model

In this model both weights and outputs depend on the random factor  $\xi$ . In other words, a different efficiency level is computed for each realization of the uncertain factor  $\xi$ . In order to describe the Primal-Dual couple of model VRS1, the following parameters are introduced:

$$\begin{aligned} y_{rj}(\xi) \in \mathbb{R}_+ : & \quad r^{th} \text{ output quantity produced by the } j^{th} \text{ DMU} \\ & \quad \text{depending on the random factor } \xi, \\ & \quad r = 1, \dots, q, \quad j = 1, \dots, n; \\ x_{ij} \in \mathbb{R}_+ : & \quad i^{th} \text{ input quantity used by the } j^{th} \text{ decision making unit,} \\ & \quad i = 1, \dots, m, \quad j = 1, \dots, n. \end{aligned}$$

By means of the previous notations, the following problem is defined:

$$\begin{aligned} & (\widehat{P1}_V) \\ \max_{u(\xi), u_0(\xi), v} & \quad \mathbb{E}_\xi \left[ \sum_{r=1}^q u_r(\xi) y_{rj_0}(\xi) + u_0(\xi) \right] \end{aligned} \quad (3.1)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.2)$$

$$\sum_{r=1}^q u_r(\xi) y_{rj}(\xi) + u_0(\xi) - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad (3.3)$$

$$u_r(\xi) \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0(\xi) \in \mathbb{R},$$

where the following further notations are used:

$$u_r(\xi) \in \mathbb{R}_+ : \quad \text{weight variable related to the } r^{th} \text{ output depending on the random factor } \xi, \quad r = 1, \dots, q;$$

$$v_i \in \mathbb{R}_+ : \quad \text{weight variable related to the } i^{th} \text{ input,} \quad i = 1, \dots, m;$$

$$u_0(\xi) \in \mathbb{R} : \quad \text{scale variable depending on the uncertain factor } \xi.$$

The corresponding dual formulation is as follows:

$$\begin{aligned} & (\widehat{D1}_V) \\ \min_{\theta, \lambda(\xi)} & \quad \theta \\ s.t. & \quad \sum_{j=1}^n \lambda_j(\xi) y_{rj}(\xi) \geq y_{rj_0}(\xi), \quad r = 1, \dots, q, \end{aligned} \quad (3.4)$$

$$\mathbb{E}_\xi \left[ \sum_{j=1}^n \lambda_j(\xi) x_{ij} \right] \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.5)$$

$$\sum_{j=1}^n \lambda_j(\xi) = 1, \quad (3.6)$$

$$\lambda_j(\xi) \geq 0, \quad j = 1, \dots, n,$$

where  $\theta \in \mathbb{R}$ ,  $\lambda_j(\xi) \in \mathbb{R}_+$  are the dual variables corresponding to the primal constraints.

Problem  $(\widehat{P1}_V)$  maximizes the expected efficiency for each DMU considering a different vector of weights for each realization of the uncertain factor  $\xi$ . Notice that a corresponding constant return to scale version of VRS1 can be obtained by deleting variable  $u_0(\xi)$  in the primal formulation and by deleting constraints (3.6) in the dual one, as it will be shown in Section 3.3.

Notice also that in this model there is no need to make assumptions on the random variable distribution, thanks to the forthcoming use of scenarios for stating a corresponding deterministic formulation.

### 3.1.2 The VRS2 model

The model introduced in this section has the aim of finding an unique weights composition for the primal problem not depending on the realizations of the random factor  $\xi$ . At the same time, the corresponding dual problem presents less tightening constraints. The primal formulation of VRS2 is as follows:

$$(\widehat{P2}_V) \quad \max_{u, u_0(\xi), v} \mathbb{E}_\xi \left[ \sum_{r=1}^q u_r y_{rj_0}(\xi) + u_0(\xi) \right] \quad (3.7)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.8)$$

$$\sum_{r=1}^q u_r y_{rj}(\xi) + u_0(\xi) - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad (3.9)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0(\xi) \in \mathbb{R},$$

where:

$u_r \in \mathbb{R}_+$ : is the weight variable related to the  $r^{th}$  output.

The corresponding dual formulations results to be:

$$\begin{aligned}
& \widehat{(D2_V)} \\
& \min_{\theta, \lambda(\xi)} \quad \theta \\
& s.t. \quad \mathbb{E}_\xi \left[ \sum_{j=1}^n \lambda_j(\xi) y_{rj}(\xi) - y_{rj_0}(\xi) \right] \geq 0, \quad r = 1, \dots, q, \tag{3.10}
\end{aligned}$$

$$\mathbb{E}_\xi \left[ \sum_{j=1}^n \lambda_j(\xi) x_{ij} \right] \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \tag{3.11}$$

$$\sum_{j=1}^n \lambda_j(\xi) = 1,$$

$$\lambda_j(\xi) \geq 0, \quad j = 1, \dots, n.$$

As it was pointed out for VRS1, a corresponding CRS version for VRS2 can be easily obtained for both the primal and the dual formulations.

VRS2 model differs from VRS1 one in the task of finding an unique weights composition for uncertain outputs. In particular, in problem  $(\widehat{P1_V})$  weights variables  $u_r(\xi)$  depend on the random factor  $\xi$ , while in problem  $(\widehat{P2_V})$  the weights are independent from its realizations. In the corresponding dual problems  $(\widehat{D1_V})$  and  $(\widehat{D2_V})$  the constraints are different. In problem  $(\widehat{D2_V})$ , in fact, constraints (3.10) have to be verified in expected value. These differences will be clarified in the next sections, where corresponding deterministic formulations for the proposed models are given.

## 3.2 A deterministic approach for VRS1 and VRS2

In order to manage the uncertainty of outputs and weights, a corresponding deterministic formulation for both VRS1 and VRS2 is obtained. Assuming that the random factor  $\xi$  is induced by a known probability distribution, this distribution can be discretized as follows. Let each scenario  $s$  represent a realization of the uncertain parameter  $\xi$ . In order to provide the corresponding deterministic formulation, the following notations are introduced:

$$\begin{aligned}
y_{rj}^s \in \mathbb{R}_+ &: \quad r^{th} \text{ output quantity produced by the } j^{th} \text{ DMU for each} \\
& \quad \text{scenario } s, \quad r = 1, \dots, q, \quad j = 1, \dots, n, \quad s = 1, \dots, S; \\
p^s \in [0, 1] &: \quad \text{realization probability of any scenario } s, \quad s = 1, \dots, S.
\end{aligned}$$

### 3.2.1 Deterministic VRS1

The deterministic model corresponding to  $(\widehat{P1_V})$  is as follows:

$$(P1_V) \quad \max_{u, v, u_0} \sum_{s=1}^S p^s \left[ \sum_{r=1}^q u_r^s y_{rj_0}^s + u_0^s \right] \quad (3.12)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.13)$$

$$\begin{aligned} \sum_{r=1}^q u_r^s y_{rj}^s + u_0^s - \sum_{i=1}^m v_i x_{ij} &\leq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S, \\ u_r^s &\geq 0, \quad r = 1, \dots, q, \quad s = 1, \dots, S, \\ v_i &\geq 0, \quad i = 1, \dots, m, \\ u_0^s &\in \mathbb{R}, \quad s = 1, \dots, S, \end{aligned} \quad (3.14)$$

where:

- $u_r^s \in \mathbb{R}_+$ : is the weight variable related to the  $r^{th}$  output for each scenario  $s$ ,  $r = 1, \dots, q$ ,  $s = 1, \dots, S$ ;
- $u_0^s \in \mathbb{R}$ : is the scale variable for each scenario  $s$ ,  $s = 1, \dots, S$ .

The objective function (3.12) of the primal deterministic model ( $P1_V$ ) represents the expected value of the efficiency of DMU  $j_0$  with respect to all the possible realizations of the random factor  $\xi$ . In this model, all the scenarios are considered and for each scenario an optimal weight composition is established. In this light, by solving the model for each DMU, a global efficiency index is obtained taking into account the different reaction of each DMU to extreme scenarios. The scenario based method is very flexible, it does not require the use of a specific probability distribution and it allows concrete applications.

In particular, taking into account constraints (3.14), with simple calculations it results:

$$u_0^s \leq \min_j \left\{ \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^q u_r^s y_{rj}^s \right\}, \quad s = 1, \dots, S,$$

so that, since problem ( $P1_V$ ) is a maximization problem, the optimal solution  $(u_0^s)^*$  assumes the following values:

$$(u_0^s)^* = \min_j \left\{ \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^q u_r^s y_{rj}^s \right\}, \quad s = 1, \dots, S.$$

This result implies that, for each scenario  $s$  and for each DMU  $j$ , variable  $(u_0^s)^* < 0$  can be interpreted as the maximum surplus (gap between weighted outputs and inputs for the most efficient unit), while in the case  $(u_0^s)^* > 0$  it represents the minimum deficit (gap between weighted inputs and outputs for the most efficient unit). In other words, in each scenario, variable  $(u_0^s)^*$  gives informations about the most efficient DMU in maximizing surplus or minimizing deficit.

As in the case of model ( $P1_V$ ), the corresponding dual deterministic model can be obtained



as follows:

$$\begin{aligned} \min_{\theta, \tilde{\lambda}} \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^n \tilde{\lambda}_j^s y_{rj}^s \geq p^s y_{rj_0}^s, \quad r = 1, \dots, q, \quad s = 1, \dots, S, \end{aligned} \quad (3.15)$$

$$\sum_{s=1}^S \sum_{j=1}^n \tilde{\lambda}_j^s x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.16)$$

$$\sum_{j=1}^n \tilde{\lambda}_j^s = p^s, \quad s = 1, \dots, S,$$

$$\tilde{\lambda}_j^s \geq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S.$$

In order to improve the dual formulation, with a simple variable substitution, we have the following equivalent version:

(D1<sub>V</sub>)

$$\begin{aligned} \min_{\theta, \lambda} \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j^s y_{rj}^s \geq y_{rj_0}^s, \quad r = 1, \dots, q, \quad s = 1, \dots, S, \end{aligned} \quad (3.17)$$

$$\sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.18)$$

$$\sum_{j=1}^n \lambda_j^s = 1, \quad s = 1, \dots, S,$$

$$\lambda_j^s \geq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S,$$

where  $\tilde{\lambda}_j^s = p^s \lambda_j^s$ .

Problem (D1<sub>V</sub>) optimizes the values of  $\lambda_j^s$  in order to compare each DMU  $j_0$  with a reference efficient unit, obtained as an optimal combination between available inputs and outputs. Constraints (3.17) impose that the reference unit produces, for each kind of outputs and in each scenario, a quantity of outputs greater or equal to the one produced by the considered unit  $j_0$ . Constraints (3.18) mean that the reference unit uses, in expected value for each input, a total amount lower or equal to a fraction  $\theta$  of inputs consumed by the considered unit  $j_0$ . The variable  $\theta$  represents the reduction inputs factor, which states how much the inputs consumed by DMU  $j_0$  can be reduced in order to improve its efficiency.

### 3.2.2 Deterministic VRS2

The problem formulation corresponding to  $(\widehat{P2}_V)$  is given below:

$$(P2_V) \quad \max_{u,v,u_0} \sum_{s=1}^S p^s \left[ \sum_{r=1}^q u_r y_{rj_0}^s + u_0^s \right] \quad (3.19)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.20)$$

$$\sum_{r=1}^q u_r y_{rj}^s + u_0^s - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S, \quad (3.21)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0^s \in \mathbb{R}, \quad s = 1, \dots, S,$$

where:

$u_r \in \mathbb{R}_+$ : weight variable related to the  $r^{th}$  output,  $r = 1, \dots, q$ .

The dual corresponding model, after the same transformation stated in  $(D1_V)$ , results to be:

$$(D2_V) \quad \min_{\theta, \lambda} \quad \theta$$

$$s.t. \quad \sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s y_{rj}^s \geq \sum_{s=1}^S p^s y_{rj_0}^s, \quad r = 1, \dots, q, \quad (3.22)$$

$$\sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.23)$$

$$\sum_{j=1}^n \lambda_j^s = 1, \quad s = 1, \dots, S,$$

$$\lambda_j^s \geq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S.$$

The difference between  $(P1_V)$  and  $(P2_V)$ , as mentioned before, concerns the choice of weights for the uncertain outputs. In  $(P1_V)$  we find out for each scenario the optimal weight composition  $u_r^s$  maximizing the efficiency of the DMU denoted with  $j_0$ . The DMU  $j_0$  global efficiency index is the expected value of each scenario efficiency index. In model  $(P2_V)$  we find out a unique weight for each output that can handle the uncertainty, whatever scenario will occur. In the dual versions  $(D1_V)$  and  $(D2_V)$ , the models differ in constraints (3.17) and (3.22). In model  $(D1_V)$  constraints (3.17) hold for each scenario  $s = 1 \dots S$ , that is to say for each output in each scenario; in model  $(D2_V)$  constraints (3.22) for each output hold in expected value.

Notice that models VRS1 and VRS2 differs from the classical DEA formulations that can be found in the literature. In particular, the choice of considering constraints (3.14) and (3.21) scenario by scenario answers the need of guaranteeing the feasibility of the optimal solution in every single considered scenario, in order to avoid "solutions" that cannot be concretely taken into account in all of the used scenario. In other words, whatever scenario will occur the optimal weights of problem VRS1 and VRS2 remain a feasible solution of the ex post deterministic problem. This property can not be verified if we consider constraints (3.14) and (3.21) in expected value, as it will be shown in details in Subsection 3.4.2 (Table 3.3).

### 3.3 A particular case: Constant Returns to Scale

The pioneer model of Data Envelopment Analysis by Charnes et al. [14] determine the most efficient DMU under the assumption of constant returns to scale.

Constant returns to scale means that the producers are able to linearly scale the inputs and outputs without increasing or decreasing efficiency. In other words, if we assume that CRS holds, we assume that if  $(x, y)$  is couple of input-output feasible, then for any  $k \geq 0$ ,  $(kx, ky)$  is also feasible. This is a very tightening assumption. For this reason, CRS tends to lower the efficiency scores while VRS tends to raise efficiency scores.

In this section the corresponding couples of VRS1 and VRS2 models arising from the assumption of constant returns to scale are presented and a theoretical result on the optimal solution for the corresponding CRS2 is proved. The deterministic CRS1 formulation corresponding to VRS1 is as follows:

$$(P1_C) \quad \max_{u,v} \quad \sum_{s=1}^S p^s \sum_{r=1}^q u_r^s y_{rj_0}^s \quad (3.24)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.25)$$

$$\sum_{r=1}^q u_r^s y_{rj}^s - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S, \quad (3.26)$$

$$u_r^s \geq 0, \quad r = 1, \dots, q, \quad s = 1, \dots, S,$$

$$v_i \geq 0, \quad i = 1, \dots, m.$$

(D1<sub>C</sub>)

$$\min_{\theta, \lambda} \quad \theta$$

$$s.t. \quad \sum_{j=1}^n \lambda_j^s y_{rj}^s \geq y_{rj_0}^s, \quad r = 1, \dots, q, \quad s = 1, \dots, S, \quad (3.27)$$

$$\sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.28)$$

$$\lambda_j^s \geq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S.$$

Notice that the difference between constant and variable returns to scale is in the absence of variables  $u_0^s$  in the primal formulation and in the absence of the constraints  $\sum_{j=1}^n \lambda_j^s = 1$ ,  $s = 1, \dots, S$ , in the corresponding dual model. These differences hold also for the couple CRS2 of primal and dual of VRS2 which is defined as follows:

(P2<sub>C</sub>)

$$\max_{u,v} \sum_{s=1}^S p^s \sum_{r=1}^q u_r y_{rj_0}^s \quad (3.29)$$

$$\text{s.t.} \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.30)$$

$$\sum_{r=1}^q u_r y_{rj}^s - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S, \quad (3.31)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m.$$

(D2<sub>C</sub>)

$$\min_{\theta, \lambda} \quad \theta$$

$$\text{s.t.} \quad \sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s y_{rj}^s \geq \sum_{s=1}^S p^s y_{rj_0}^s, \quad r = 1, \dots, q, \quad (3.32)$$

$$\sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.33)$$

$$\lambda_j^s \geq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S.$$

The following theorem shows that, in the case of CRS2, the most efficient units are characterized by an efficiency equal to one if and only if their efficiency is equal to one for each single scenario.

**Theorem 3.3.1** Consider problem (P2<sub>C</sub>) and assume  $\sum_{i=1}^S p^s = 1$ ,

$p^s > 0 \forall s$ . Then,

$$\sum_{s=1}^S p^s \sum_{r=1}^q u_r y_{rj_0}^s = 1 \iff \sum_{r=1}^q u_r y_{rj_0}^s = 1, \quad \forall s = 1, \dots, S$$

.

*Proof* ( $\Leftarrow$ ) Taking into account that  $\sum_{r=1}^q u_r y_{rj_0}^s = 1 \quad \forall s = 1, \dots, S$  the thesis follows trivially.

( $\Rightarrow$ ) From the model (P2<sub>C</sub>), taking into account constraints (3.30)-(3.31), we obtain  $\sum_{r=1}^q u_r y_{rj_0}^s \leq \sum_{i=1}^m v_i x_{ij_0} = 1$ . Suppose, by contradiction, that there exist a scenario  $\bar{s}$  and a value  $\epsilon > 0$  such that

$$\sum_{r=1}^q u_r y_{rj_0}^{\bar{s}} \leq 1 - \epsilon.$$

This implies that

$$\begin{aligned} \sum_{s=1}^S p^s \sum_{r=1}^q u_r y_{rj_0}^s &= \sum_{s \neq \bar{s}} p^s \sum_{r=1}^q u_r y_{rj_0}^s + p^{\bar{s}} \sum_{r=1}^q u_r y_{rj_0}^{\bar{s}} = \\ &= 1 - p^{\bar{s}} + p^{\bar{s}}(1 - \epsilon) = 1 - p^{\bar{s}}\epsilon < 1. \end{aligned}$$

□

In the light of Theorem 3.3.1, the following simple example proves that there exist cases in which no DMU reaches the level of efficiency equal to one.

**Example 3.3.1** We suppose to have a simple instance in which it is:

$$n = 2, \quad m = 1, \quad q = 1, \quad S = 3, \quad p^s = \frac{1}{3}, \quad \forall s.$$

Suppose also that  $y_{11}^1 > y_{11}^2 > y_{11}^3 > 0$ .

The corresponding Problem ( $P2_C$ ) for DMU1 is:

$$\max_{u_1} \quad \frac{1}{3} u_1 \sum_{s=1}^3 y_{11}^s \tag{3.34}$$

$$s.t. \quad v_1 x_{11} = 1 \tag{3.35}$$

$$u_1 y_{11}^s \leq v_1 x_{11}, \quad s = 1, 2, 3,$$

$$u_1 y_{12}^s \leq v_1 x_{12}, \quad s = 1, 2, 3,$$

$$u_1 \geq 0,$$

$$v_1 \geq 0.$$

Taking into account constraints (3.34)-(3.35), we obtain  $u_1 y_{11}^s \leq 1, \forall s$  which yields

$$u_1 \leq \min \left\{ \frac{1}{y_{11}^1}, \frac{1}{y_{11}^2}, \frac{1}{y_{11}^3} \right\}.$$

Since  $y_{11}^1 > y_{11}^2 > y_{11}^3 > 0$  we have that  $u_1 \leq \frac{1}{y_{11}^1}$  and hence

$$\frac{1}{3} u_1 \sum_{s=1}^3 y_{11}^s \leq \frac{1}{3} \frac{1}{y_{11}^1} (y_{11}^1 + y_{11}^2 + y_{11}^3) = \frac{1}{3} \left( 1 + \frac{y_{11}^2}{y_{11}^1} + \frac{y_{11}^3}{y_{11}^1} \right) < 1.$$

In the light of Theorem 3.3.1, the efficiency equal to one is not reached since it is impossible to have  $u_1 = \frac{1}{y_{1j}^s}$  for all  $s = 1, 2, 3$ .

The same simple calculations hold for DMU2, so that no DMU reaches the level of efficiency equal to one.

### 3.4 Models effectiveness and computational results

The aim of this section is to analyze the behaviour of the proposed models in order to point out their effectiveness. This will be done by means of computational tests which compare the proposed models with two further deterministic models based on the expected value approach.

### 3.4.1 Further deterministic models: VRSqEV and VRSEV

The basic idea of the proposed models VRS1 and VRS2 is that the efficiency inequality constraints (3.14) and (3.21) have to be verified for each scenario  $s = 1, \dots, S$ , that is to say that the efficiency index has to be smaller or equal to one whatever scenario will occur. Actually, in the literature, this hypothesis is often relaxed by considering efficiency constraints in expected value. In this light, it is worth comparing the proposed models VRS1 and VRS2 with another model, named VRSqEV, having the following Primal-Dual formulation:

$$(PqEV_V) \quad \max_{u,v,u_0} \quad \sum_{s=1}^S p^s \sum_{r=1}^q u_r^s y_{rj_0}^s + u_0 \quad (3.36)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.37)$$

$$\sum_{s=1}^S p^s \sum_{r=1}^q u_r^s y_{rj}^s + u_0 - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad (3.38)$$

$$u_r^s \geq 0, \quad r = 1, \dots, q, \quad s = 1, \dots, S$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0 \in \mathbb{R}.$$

$$(DqEV_V) \quad \min_{\theta, \lambda} \quad \theta$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj}^s \geq y_{rj_0}^s, \quad r = 1, \dots, q, \quad s = 1, \dots, S, \quad (3.39)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.40)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

In Problems  $(P1_V)$  and  $(P2_V)$ , constraints (3.14) and (3.21) define, for each scenario  $s$ , an optimal compositions of weights feasible for all DMUs and so that the maximum level of efficiency is one for each DMU and for each scenario  $s$ . In  $(PqEV_V)$  the linear constraints (3.38) are satisfied in expected value on all possible scenarios. This means, for example, that in some scenarios the efficiency level can be greater than one, but it is smaller or equal to one in expected value. As a consequence, the dual formulations of VRS1 and VRS2 differ from  $(DqEV_V)$  in the corresponding dual variables  $\lambda$ : in  $(D1_V)$  and  $(D2_V)$  models variables  $\lambda$  depend on the selected scenario  $s$ .

For the sake of completeness, it is finally worth comparing the proposed models VRS1 and VRS2 with the classical expected value model VRSEV, obtained by substituting the random factors with their expected values:

(PEV<sub>V</sub>)

$$\max_{u,v,u_0} \sum_{r=1}^q u_r \bar{y}_{rj_0} + u_0 \quad (3.41)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.42)$$

$$\sum_{r=1}^q u_r \bar{y}_{rj} + u_0 - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad (3.43)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0 \in \mathbb{R}.$$

(DEV<sub>V</sub>)

$$\min_{\theta, \lambda} \quad \theta$$

$$s.t. \quad \sum_{j=1}^n \lambda_j \bar{y}_{rj} \geq \bar{y}_{rj_0}, \quad r = 1, \dots, q, \quad (3.44)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.45)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n,$$

where  $\bar{y}_{rj} = \sum_{s=1}^S p^s y_{rj}^s$ .

From a computational point of view, as it will be pointed out in the following subsections, since constraints (3.14) and (3.21) have to be satisfied for all units and for all scenarios, VRS1 and VRS2 models provide a ranking of the units efficiency more accurate than VRSqEV and VRSEV.

Notice that, as stressed in Section 3.1, the couples of VRSqEV and VRSEV models can be modified by assuming constant returns to scale, thus obtaining further models named CRSqEV and CRSEV, respectively. In particular, these constant returns to scale models can be obtained by removing variables  $u_0^s$  in the primal formulation and constraints  $\sum_{j=1}^n \lambda_j^s = 1$ ,  $s = 1, \dots, S$ , in the corresponding dual model.

### 3.4.2 Comparing VRS1, VRS2, VRSqEV and VRSEV

Models VRS1 and VRS2 have been implemented in order to test them in comparison with the models based on the expected value approach. In all the simulations, different outputs distributions have been considered. In particular, the results concerning normal and beta distribution are presented. The data generation procedure has been implemented in MatLab 2008b by using low discrepancy sequences generated by the method of Halton, to properly distribute the data in the intervals considered. The optimal solution of the models is obtained by solving the linear formulation

with AMPL+CPLEX v.11. Different instances have been tested varying the number of DMUs, the number of inputs and outputs. For each instance the input data have been generated in the interval  $[0,1500]$  by using the “rand” MatLab function. Output uncertain parameters have been generated taking into account different distributions, different mean and standard deviation values and using Halton sequence.

A first computational test compares the optimal solutions of VRS1, VRS2, VRSqEV and VRSEV models. In particular, 40 different classes of problems have been considered varying the data distribution function and the number of inputs, outputs and DMUs. In particular, in order to cover more real situations as possible, we looked at different combinations of input and output (2 inputs and 2 outputs, 4 inputs and 5 outputs, 4 inputs and 10 outputs, 2 inputs and 5 outputs and 2 inputs and 10 outputs) and we varied the number of DMUs considered, from a minimum of 8 units to a maximum of 30 units. For each class of problems we fix the number of scenarios equal to 500 and we generate 1000 different random instances, in order to highlight the general trend of the models. In particular, we focused on evaluating the number of efficient units found for each instance. In Table 3.4.2 the results concerning the mean, minimum and maximum number of efficient units for each class of problems are collected. We can note that models VRS1 and VRS2 results to be very selective in identifying the most efficient units. On the other hand, model VRSqEV results to be unable to provide a valuable ranking of the units, as it is pointed out by the average number of efficient units which is close to the half of the number of units themselves.



I/O	DMU	VRS1						VRS2						VRSqEV						VRSEV								
		$\bar{m}$	m	M	$\bar{m}$	m	M	$\bar{m}$	m	M	$\bar{m}$	m	M	$\bar{m}$	m	M	$\bar{m}$	m	M	$\bar{m}$	m	M	$\bar{m}$	m	M			
2 I 2 O	8	1.35	1	3	1.43	1	6	1.27	1	3	1.36	1	5	5.28	2	8	5.37	2	8	2.43	1	5	2.43	1	5	2.43	1	5
	10	1.31	1	4	1.41	1	4	1.24	1	3	1.36	1	5	6.26	3	10	6.24	2	10	2.48	1	6	2.48	1	6	2.48	1	6
	15	1.31	1	3	1.44	1	3	1.24	1	3	1.37	1	3	8.50	4	13	8.29	3	13	2.78	1	9	2.78	1	9	2.78	1	9
4 I 5 O	30	1.38	1	3	1.53	1	6	1.31	1	3	1.47	1	6	13.97	4	21	12.31	2	20	3.23	1	7	3.23	1	7	3.23	1	7
	8	1.34	1	3	1.38	1	5	1.26	1	3	1.51	1	5	5.21	1	8	5.35	2	8	2.38	1	5	2.38	1	5	2.38	1	5
	10	1.34	1	4	1.62	1	4	1.27	1	4	1.55	1	4	6.15	3	10	6.23	3	10	2.52	1	5	2.52	1	5	2.52	1	5
4 I 10 O	15	1.31	1	3	1.60	1	5	1.25	1	3	1.54	1	5	8.52	3	13	8.39	4	13	2.79	1	6	2.80	1	6	2.80	1	6
	30	1.33	1	3	1.67	1	5	1.25	1	3	1.61	1	5	14.18	4	25	12.62	2	24	3.21	1	8	3.31	1	8	3.31	1	8
	8	1.34	1	3	1.76	1	4	1.26	1	3	1.70	1	4	5.25	2	8	5.37	2	8	2.36	1	5	2.36	1	5	2.36	1	5
2 I 5 O	10	1.32	1	3	1.78	1	5	1.23	1	3	1.70	1	5	6.24	2	10	6.31	2	10	2.52	1	6	2.51	1	6	2.51	1	6
	15	1.32	1	3	1.84	1	5	1.25	1	3	1.77	1	5	8.52	4	14	8.38	4	13	2.85	1	7	2.85	1	7	2.85	1	7
	30	1.35	1	3	1.90	1	6	1.28	1	3	1.81	1	6	14.08	4	23	12.45	2	20	3.15	1	7	3.28	1	7	3.28	1	7
2 I 5 O	8	1.30	1	4	1.56	1	4	1.24	1	3	1.50	1	4	5.24	2	8	5.36	2	8	2.35	1	5	2.36	1	5	2.36	1	5
	10	1.34	1	3	1.62	1	6	1.24	1	3	1.56	1	4	6.22	2	10	6.30	3	10	2.50	1	7	2.50	1	7	2.50	1	7
	15	1.31	1	3	1.63	1	5	1.26	1	3	1.57	1	5	8.49	4	14	8.35	4	14	2.79	1	7	2.80	1	7	2.80	1	7
2 I 10 O	30	1.34	1	3	1.67	1	5	1.27	1	3	1.60	1	5	14.06	3	22	12.48	2	20	3.20	1	7	3.26	1	7	3.26	1	7
	8	1.34	1	3	1.78	1	4	1.28	1	3	1.71	1	4	5.28	2	8	5.41	2	8	2.39	1	5	2.39	1	5	2.39	1	5
	10	1.29	1	3	1.77	1	5	1.22	1	3	1.71	1	5	6.24	2	10	6.34	2	10	2.49	1	5	2.49	1	5	2.49	1	5
15	15	1.32	1	3	1.80	1	5	1.26	1	3	1.74	1	5	8.44	4	14	8.34	4	13	2.81	1	6	2.81	1	6	2.81	1	6
	30	1.34	1	4	1.93	1	6	1.26	1	3	1.86	1	6	13.91	6	21	12.23	2	20	3.25	1	7	3.39	1	7	3.39	1	7

Table 3.1: Test on DMU efficiency: 500 scenarios, 1000 instances, Mean ( $\bar{m}$ ), minimum (m) and maximum (M) number of efficient DMUs

Moreover, by considering the minimum and maximum number of efficient DMUs the results show that models VRS1 and VRS2, both in the case of normal and beta distribution, have a minimum number of efficient DMUs equal to 1 and have a maximum number of efficient DMUs smaller or equal than the expected values based models. This implies that the results are stable in terms of efficiency and confirms the selectiveness of the proposed models VRS1 and VRS2. As regards to the two models based on the expected value approach, in VRSqEV the maximum number of efficient DMUs often reaches the total number of considered DMUs. Generally speaking, the behaviour of model VRSqEV suggests a high overestimation of efficient DMUs.

In order to deeply analyze the behaviour of the proposed models, the results concerning a single instance are fully reported in Table 3.4.2 by varying the number of scenarios. We consider 4 inputs, 5 outputs and 8 DMUs; the number of considered scenarios varies between 5 and 500 scenarios. Taking into account Table 3.4.2, it can be observed that models VRS1 and VRS2 are very selective in identifying the most efficient units and the hierarchy between the different DMUs. In particular, the most efficient unit DMU4 with score one is the most efficient for all the four models. Due to the high selectiveness of our models, even the smallest difference between DMUs can be recognized. Specifically speaking, it is worth noticing that both VRS1 and VRS2 models are able to classify DMU5, DMU7 and DMU1 as second, third and fourth, respectively, while for both VRSEV and VRSqEV models these three DMUs have the same maximum score. Notice also that, for normal distribution, this accuracy can be observed even in the case of a small number of scenario. It can be observed also that VRSqEV model seems even less selective in identifying efficient units than VRSEV; in fact, in VRSqEV model the DMUs reaching the maximum score are the ones having the maximum score in VRSEV model plus the DMU2 unit. Another aspect to point out is that the inefficient DMUs for expected value based models are also inefficient in the scenario based models.

DMU2 results to be one of the most efficient DMUs in VRSqEV. Nevertheless, it reaches in VRS1 and VRS2 formulations a low position in ranking score. In order to understand this result, Table 3.3 shows, in the case of DMU2, the constraints of the primal formulation of  $(PqEV_V)$  model corresponding to constraints (3.14) for  $(P1_V)$  and (3.21) for  $(P2_V)$ . In the first four columns of Table 3.3 there are the constraints (3.38) of  $(PqEV_V)$  model in the case of 5 scenarios and it results that the constraints are satisfied according to their expected value based formulation. On the other hand, if the constraints are considered scenario by scenario it comes out that some of the constraints are not satisfied in all scenarios, that is to say that an efficiency level greater than one holds in some scenario  $s$ . This paradoxical situation can not be accepted in VRS1 and VRS2. It can then be concluded that VRS1 and VRS2 are more selective in identifying the best efficient units.

FVal	DMU1		DMU2		DMU3		DMU4		DMU5		DMU6		DMU7		DMU8	
	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta
5 scen	VRS1	0.5617	0.9106	0.4822	0.5582	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	0.5138	0.8692	0.0211	0.0211
	VRS2	0.4505	0.7876	0.2637	0.4313	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	0.3369	0.8553	0.0211	0.0211
	VRSqEV	1.0000	1.0000	1.0000	1.0000	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	1.0000	1.0000	0.0211	0.0211
	VRSEV	1.0000	1.0000	0.5863	0.6020	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	1.0000	1.0000	0.0211	0.0211
10 scen	VRS1	0.6534	0.9158	0.5000	0.5641	0.0188	0.0190	1.0000	1.0000	1.0000	0.0939	0.0948	0.7569	0.9346	0.0211	0.0212
	VRS2	0.5401	0.8271	0.3497	0.4673	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	0.6559	0.9277	0.0211	0.0211
	VRSqEV	1.0000	1.0000	1.0000	1.0000	0.0188	0.0213	1.0000	1.0000	1.0000	0.1185	0.1279	1.0000	1.0000	0.0222	0.0222
	VRSEV	1.0000	1.0000	0.6476	0.6267	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	1.0000	1.0000	0.0211	0.0211
20 scen	VRS1	0.6694	0.9202	0.5238	0.5718	0.0197	0.0198	1.0000	1.0000	1.0000	0.0925	0.0930	0.7845	0.9673	0.0211	0.0211
	VRS2	0.5663	0.8360	0.3243	0.4491	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	0.6200	0.9501	0.0211	0.0211
	VRSqEV	1.0000	1.0000	1.0000	1.0000	0.0314	0.0300	1.0000	1.0000	1.0000	0.1185	0.1279	1.0000	1.0000	0.0222	0.0222
	VRSEV	1.0000	1.0000	0.5797	0.5779	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	1.0000	1.0000	0.0211	0.0211
50 scen	VRS1	0.7194	0.9083	0.4920	0.5546	0.0194	0.0196	1.0000	1.0000	1.0000	0.0953	0.0958	0.8421	0.9869	0.0213	0.0215
	VRS2	0.5840	0.8529	0.3145	0.4426	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	0.7194	0.9727	0.0211	0.0211
	VRSqEV	1.0000	1.0000	1.0000	1.0000	0.0314	0.0300	1.0000	1.0000	1.0000	0.1613	0.1468	1.0000	1.0000	0.0291	0.0317
	VRSEV	1.0000	1.0000	0.5481	0.5482	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	1.0000	1.0000	0.0211	0.0211
100 scen	VRS1	0.7443	0.9144	0.4878	0.5603	0.0192	0.0195	1.0000	1.0000	1.0000	0.0976	0.0972	0.8592	0.9865	0.0214	0.0216
	VRS2	0.6074	0.8675	0.3206	0.4450	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	0.7244	0.9692	0.0211	0.0211
	VRSqEV	1.0000	1.0000	1.0000	1.0000	0.0314	0.0361	1.0000	1.0000	1.0000	0.2587	0.2716	1.0000	1.0000	0.0374	0.0317
	VRSEV	1.0000	1.0000	0.5438	0.5414	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	1.0000	1.0000	0.0211	0.0211
200 scen	VRS1	0.7444	0.9112	0.4932	0.5712	0.0193	0.0194	1.0000	1.0000	1.0000	0.0967	0.0964	0.8565	0.9821	0.0214	0.0215
	VRS2	0.6021	0.8642	0.3166	0.4417	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	0.7154	0.9659	0.0211	0.0211
	VRSqEV	1.0000	1.0000	1.0000	1.0000	0.0314	0.0388	1.0000	1.0000	1.0000	0.2387	0.2981	1.0000	1.0000	0.0374	0.0321
	VRSEV	1.0000	1.0000	0.5421	0.5401	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	1.0000	1.0000	0.0211	0.0211
500 scen	VRS1	0.7515	0.9053	0.4885	0.5659	0.0193	0.0194	1.0000	1.0000	1.0000	0.0968	0.0970	0.8534	0.9859	0.0214	0.0215
	VRS2	0.6024	0.8632	0.3136	0.4407	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	0.7136	0.9687	0.0211	0.0211
	VRSqEV	1.0000	1.0000	1.0000	1.0000	0.0716	0.0538	1.0000	1.0000	1.0000	0.3684	0.3809	1.0000	1.0000	0.0399	0.0444
	VRSEV	1.0000	1.0000	0.5392	0.5389	0.0188	0.0188	1.0000	1.0000	1.0000	0.0911	0.0911	1.0000	1.0000	0.0211	0.0211

Table 3.2: Variable returns to scale: 4 inputs, 5 outputs, 8 DMUs

DMU	$\sum_{s=1}^S p^s \sum_{r=1}^q u_r^s y_{rj}^s$	$\leq$	$\sum_{i=1}^m v_i x_{ij} - u_0 - v_j$	Scen	$\sum_{r=1}^q u_r^s y_{rj}^s$	$\leq$	$\sum_{i=1}^m v_i x_{ij} - u_0 - v_j$	$\forall$	$\sum_{i=1}^m v_i x_{ij} - u_0 - v_j \forall s$
1				1	16.5049			V	8.0283
1	8.0283	$\leq$	8.0283	2	0			V	8.0283
1				3	13.8874			V	8.0283
1				4	9.7491			V	8.0283
1				5	0			V	8.0283
2				1	24.0499			V	8.3635
2	8.3635	$\leq$	8.3635	2	0			V	8.3635
2				3	9.29076			V	8.3635
2				4	8.4769			V	8.3635
2				5	0			V	8.3635
3				1	3.7220			V	9.6888
3				2	0			V	9.6888
3	2.3174	$\leq$	9.6888	3	5.1008			V	9.6888
3				4	2.7644			V	9.6888
3				5	0			V	9.6888
4				1	8.1538			V	7.4071
4				2	0			V	7.4071
4	4.5331	$\leq$	7.4071	3	11.1219			V	7.4071
4				4	3.3899			V	7.4071
4				5	0			V	7.4071
5				1	17.1815			V	7.4853
5				2	0			V	7.4853
5	7.4853	$\leq$	7.4853	3	15.7576			V	7.4853
5				4	4.4875			V	7.4853
5				5	0			V	7.4853
6				1	7.5644			V	7.8420
6				2	0			V	7.8420
6	3.7388	$\leq$	7.8420	3	7.8153			V	7.8420
6				4	3.3141			V	7.8420
6				5	0			V	7.8420
7				1	24.3288			V	9.2015
7				2	0			V	9.2015
7	9.2015	$\leq$	9.2015	3	15.0977			V	9.2015
7				4	6.5811			V	9.2015
7				5	0			V	9.2015
8				1	5.7368			V	9.4354
8				2	0			V	9.4354
8	2.7788	$\leq$	9.4354	3	6.1825			V	9.4354
8				4	1.9749			V	9.4354
8				5	0			V	9.4354

Table 3.3: Constraints comparison

### 3.4.3 Comparing CRS1, CRS2, CRSqEV and CRSEV

The same computational tests have been done for constant return to scale models, collected in Table 3.4. As it is shown in Theorem 3.3.1, the efficiency level equal to one can be reached in extreme cases. From a computational point of view, this result is confirmed in our tests. The most efficient unit for CRS1 and CRS2, even if it does not reach the unitary value, is the same than in models CRSEV and CRSqEV. A ranking of efficiency in the scenario based models can be stated with more accuracy than the one obtained with the expected value approach. As in the case of corresponding models under the assumption of variable returns to scale, it can be observed that the models CRSqEV appears to be less selective in identifying the most efficient units.

FVal	DMU1		DMU2		DMU3		DMU4		DMU5		DMU6		DMU7		DMU8	
	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta	Norm	Beta
5 scen																
CRS1	0.1069	0.1183	0.1199	0.1407	0.3548	0.4185	0.1146	0.1191	0.9553	0.9921	0.1164	0.1266	0.5188	0.5017	0.7873	0.7541
CRS2	0.0851	0.0974	0.1198	0.1158	0.2813	0.3446	0.0932	0.0980	0.7843	0.8579	0.0951	0.1042	0.4192	0.4129	0.6368	0.6263
CRSQEV	0.1532	0.3022	0.1754	0.3710	0.4393	1.0000	0.1423	0.3178	1.0000	1.0000	0.1629	0.3254	0.6673	1.0000	1.0000	1.0000
CRSEV	0.1085	0.1136	0.1220	0.1350	0.3587	0.4017	0.1189	0.1143	1.0000	1.0000	0.1213	0.1214	0.5345	0.4815	0.8119	0.7302
10 scen																
CRS1	0.1115	0.1161	0.1349	0.1389	0.3811	0.3992	0.1221	0.1234	0.9514	0.9833	0.1231	0.1254	0.5043	0.5238	0.7731	0.7835
CRS2	0.0853	0.0971	0.1165	0.1165	0.2897	0.3338	0.0936	0.1031	0.7528	0.8553	0.0957	0.1058	0.3893	0.4414	0.5937	0.6581
CRSQEV	0.1589	0.1580	0.2107	0.2163	0.5314	0.5658	0.1824	0.1765	1.0000	1.0000	0.1812	0.1865	0.6981	0.7268	1.0000	1.0000
CRSEV	0.1133	0.1136	0.1369	0.1373	0.3849	0.3903	0.1244	0.1238	1.0000	1.0000	0.1271	0.1248	0.5171	0.5342	0.7887	0.7899
20 scen																
CRS1	0.1171	0.1213	0.1381	0.1443	0.4083	0.4296	0.1166	0.1203	0.9757	0.9917	0.1240	0.1261	0.5051	0.5227	0.7455	0.7662
CRS2	0.0862	0.0984	0.1160	0.1160	0.2960	0.3425	0.0865	0.0990	0.7442	0.8557	0.0927	0.1051	0.3718	0.4231	0.5529	0.6349
CRSQEV	0.1945	0.2291	0.2715	0.2909	0.6295	0.7574	0.1852	0.1992	1.0000	1.0000	0.1924	0.1949	0.6983	1.0000	1.0000	1.0000
CRSEV	0.1158	0.1158	0.1358	0.1358	0.3977	0.4002	0.1162	0.1164	1.0000	1.0000	0.1245	0.1234	0.4995	0.5014	0.7429	0.7484
50 scen																
CRS1	0.1129	0.1172	0.1340	0.1392	0.3998	0.4163	0.1144	0.1180	0.9779	0.9949	0.1203	0.1239	0.4750	0.4946	0.7344	0.7548
CRS2	0.0766	0.0975	0.1153	0.1153	0.2698	0.3438	0.0775	0.0984	0.6830	0.8645	0.0821	0.1043	0.3226	0.4115	0.4976	0.6318
CRSQEV	0.1945	0.2296	0.2715	0.2909	0.6601	0.7631	0.1937	0.2005	1.0000	1.0000	0.1925	0.1988	0.8863	1.0000	1.0000	1.0000
CRSEV	0.1121	0.1128	0.1325	0.1334	0.3951	0.3977	0.1135	0.1139	1.0000	1.0000	0.1202	0.1207	0.4724	0.4759	0.7286	0.7315
100 scen																
CRS1	0.1134	0.1172	0.1358	0.1404	0.4039	0.4182	0.1148	0.1184	0.9738	0.9908	0.1214	0.1246	0.4798	0.4972	0.7373	0.7566
CRS2	0.0766	0.0975	0.0911	0.1160	0.2712	0.3446	0.0772	0.0982	0.6771	0.8591	0.0820	0.1042	0.3216	0.4107	0.4972	0.6296
CRSQEV	0.2025	0.2421	0.2824	0.2999	0.7608	0.7841	0.2142	0.2112	1.0000	1.0000	0.2239	0.2246	1.0000	1.0000	1.0000	1.0000
CRSEV	0.1131	0.1134	0.1345	0.1350	0.4005	0.4011	0.1139	0.1143	1.0000	1.0000	0.1211	0.1213	0.4750	0.4780	0.7342	0.7345
200 scen																
CRS1	0.1148	0.1180	0.1369	0.1409	0.4075	0.4198	0.1160	0.1193	0.9785	0.9916	0.1227	0.1258	0.4857	0.5011	0.7384	0.7558
CRS2	0.0759	0.0975	0.0902	0.1158	0.2686	0.3447	0.0764	0.0981	0.6686	0.8578	0.0812	0.1042	0.3209	0.4130	0.4900	0.6275
CRSQEV	0.2381	0.2531	0.3276	0.3329	0.7747	0.8304	0.2481	0.2643	1.0000	1.0000	0.2846	0.2830	1.0000	1.0000	1.0000	1.0000
CRSEV	0.1135	0.1137	0.1349	0.1350	0.4017	0.4018	0.1143	0.1145	1.0000	1.0000	0.1215	0.1214	0.4799	0.4818	0.7329	0.7326
500 scen																
CRS1	0.1151	0.1183	0.1369	0.1407	0.4070	0.4185	0.1159	0.1191	0.9794	0.9921	0.1231	0.1266	0.4873	0.5017	0.7368	0.7541
CRS2	0.0734	0.0974	0.0873	0.1158	0.2597	0.3446	0.0739	0.0980	0.6465	0.8579	0.0785	0.1042	0.3110	0.4129	0.4724	0.6263
CRSQEV	0.2592	0.3022	0.3801	0.3710	0.9393	1.0000	0.3071	0.3178	1.0000	1.0000	0.2893	0.3254	1.0000	1.0000	1.0000	1.0000
CRSEV	0.1136	0.1136	0.1350	0.1350	0.4018	0.4017	0.1143	0.1143	1.0000	1.0000	0.1215	0.1214	0.4810	0.4815	0.7307	0.7302

Table 3.4: Constant returns to scale: 4 inputs, 5 outputs, 8 DMUs

### 3.5 Unifying model for returns to scale

The aim of this section is to propose a new model which covers, in an unifying framework, both previously described models based on constant returns to scale and variable returns to scale. Two new parameters  $\alpha^s, \beta^s \in \mathbb{R}_+$  will be introduced for each scenario  $s = 1, \dots, S$ , with  $\alpha^s \leq \beta^s \quad \forall s$ . Varying the values for  $\alpha^s$  and  $\beta^s$  it is possible to describe both the model under variable returns to scale and under constant returns to scale. As stressed in previous chapter, Section 2.1.2, the kind of returns to scale can be increasing or decreasing according to the proportional input and output increase. As regards to VRS, by controlling the values of  $\alpha^s$  and  $\beta^s$  it's also possible describe increasing or decreasing returns to scale and control how the inputs increase affects the outputs increase. In order to understand the role of the parameters  $\alpha^s$  and  $\beta^s$  in models, models  $(P1_V) - (P1_C)$  can be unifying as follows:

$$(P1_U) \quad \max_{u, v, u_0} \sum_{s=1}^S p^s \left[ \sum_{r=1}^q u_r^s y_{rj_0}^s + \alpha u_{0\alpha}^s - \beta u_{0\beta}^s \right] \quad (3.46)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \quad (3.47)$$

$$\begin{aligned} \sum_{r=1}^q u_r^s y_{rj}^s + u_{0\alpha}^s - u_{0\beta}^s - \sum_{i=1}^m v_i x_{ij} &\leq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S, \\ u_r^s &\geq 0, \quad r = 1, \dots, q, \quad s = 1, \dots, S, \\ v_i &\geq 0, \quad i = 1, \dots, m, \\ u_{0\alpha}^s &\geq 0, \quad s = 1, \dots, S, \\ u_{0\beta}^s &\geq 0, \quad s = 1, \dots, S, \end{aligned} \quad (3.48)$$

where  $u_{0\alpha} \in \mathbb{R}_+$  and  $u_{0\beta} \in \mathbb{R}_+$  are the scale variables for each scenario  $s, s = 1, \dots, S$ , associated to parameters  $\alpha^s$  and  $\beta^s$  respectively.

Similarly to problem  $(P1_V)$ , taking into account constraints (3.48), with simple calculations it results:

$$\begin{aligned} u_{0\alpha}^s &\leq \max \left\{ \min_j \left\{ \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^q u_r^s y_{rj}^s + u_{0\beta}^s \right\}, 0 \right\}, \quad s = 1, \dots, S, \\ u_{0\beta}^s &\geq \max \left\{ \max_j \left\{ \sum_{r=1}^q u_r^s y_{rj}^s - \sum_{i=1}^m v_i x_{ij} + u_{0\alpha}^s \right\}, 0 \right\}, \quad s = 1, \dots, S, \end{aligned}$$

so that, since problem  $(P1_U)$  is a maximization problem, the optimal solution  $(u_0^s)^*$  assumes the following values:

$$\begin{aligned} u_{0\alpha}^s &= \max \left\{ \min_j \left\{ \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^q u_r^s y_{rj}^s + u_{0\beta}^s \right\}, 0 \right\}, \quad s = 1, \dots, S, \\ u_{0\beta}^s &= \max \left\{ \max_j \left\{ \sum_{r=1}^q u_r^s y_{rj}^s - \sum_{i=1}^m v_i x_{ij} + u_{0\alpha}^s \right\}, 0 \right\}, \quad s = 1, \dots, S. \end{aligned}$$

In order to improve the Dual formulation, with a simple variable substitution, we have the following unifying version for problem  $(D1_V) - (D1_C)$ :

$$\begin{aligned}
& (D1_U) \\
& \min_{\theta, \lambda} \quad \theta \\
& \text{s.t.} \quad \sum_{j=1}^n \lambda_j^s y_{rj}^s \geq y_{rj_0}^s, \quad r = 1, \dots, q, \quad s = 1, \dots, S,
\end{aligned} \tag{3.49}$$

$$\sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \tag{3.50}$$

$$\alpha^s \leq \sum_{j=1}^n \lambda_j^s \leq \beta^s, \quad s = 1, \dots, S,$$

$$\lambda_j^s \geq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S,$$

where  $\tilde{\lambda}_j^s = p^s \lambda_j^s$ .

Notice that:

i. for  $\alpha^s = \beta^s = 1 \quad \forall s = 1, \dots, S$ , we obtain model  $(\widehat{P1}_V)$ , where, in the primal formulation, it's possible to define  $u_0^s = u_{0\alpha}^s - u_{0\beta}^s$ ;

ii. for  $\alpha^s = 0$  and  $\beta^s \rightarrow +\infty \quad \forall s = 1, \dots, S$ , we obtain model  $(\widehat{P1}_C)$ ;

and for the other values of  $\alpha^s$  and  $\beta^s$ , we can able to underline how much returns are variable.

Similarly, takes into account models  $(P2_V) - (P2_C)$  and  $(D2_V) - (D2_C)$ , the unifying model results as follows:

$$\begin{aligned}
& (P2_U) \\
& \max_{u, v, u_0} \quad \sum_{s=1}^S p^s \left[ \sum_{r=1}^q u_r y_{rj_0}^s + \alpha u_{0\alpha}^s - \beta u_{0\beta}^s \right]
\end{aligned} \tag{3.51}$$

$$\text{s.t.} \quad \sum_{i=1}^m v_i x_{ij_0} = 1, \tag{3.52}$$

$$\sum_{r=1}^q u_r y_{rj}^s + u_{0\alpha}^s - u_{0\beta}^s - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S, \tag{3.53}$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_{0\alpha}^s \geq 0, \quad s = 1, \dots, S,$$

$$u_{0\beta}^s \geq 0, \quad s = 1, \dots, S.$$



The Dual corresponding model is defined as follows:

$$(D2_U)$$

$$\min_{\theta, \lambda} \quad \theta$$

$$s.t. \quad \sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s y_{rj}^s \geq \sum_{s=1}^S p^s y_{rj_0}^s, \quad r = 1, \dots, q, \quad (3.54)$$

$$\sum_{s=1}^S p^s \sum_{j=1}^n \lambda_j^s x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (3.55)$$

$$\alpha^s \leq \sum_{j=1}^n \lambda_j^s \leq \beta^s, \quad s = 1, \dots, S,$$

$$\lambda_j^s \geq 0, \quad j = 1, \dots, n, \quad s = 1, \dots, S.$$

**Theorem 3.5.1** *Let us consider Problem (P2<sub>U</sub>). Let  $(u_{0\alpha}^s)^*$  and  $(u_{0\beta}^s)^*$  the optimal solutions for the variables  $u_{0\alpha}^s$  and  $u_{0\beta}^s$  for all  $s = 1, \dots, S$ , then, if  $\beta^s > \alpha^s \geq 0$ ,*

$$(u_{0\alpha}^s)^* (u_{0\beta}^s)^* = 0.$$

*Proof* Suppose, by contradiction, that there exists  $\gamma > 0$ , such that  $(u_{0\alpha}^s)^* \geq \gamma$  and  $(u_{0\beta}^s)^* \geq \gamma$ . Let us consider:

$$(\bar{u}_{0\alpha}^s)^* = (u_{0\alpha}^s)^* - \gamma,$$

$$(\bar{u}_{0\beta}^s)^* = (u_{0\beta}^s)^* - \gamma.$$

Let us consider the objective function (3.51)  $FvalP2_U$  evaluated in  $(\bar{u}_{0\alpha}^s)^*$  and  $(\bar{u}_{0\beta}^s)^*$ . We obtain:

$$\begin{aligned} FvalP2_U|_{((\bar{u}_{0\alpha}^s)^*, (\bar{u}_{0\beta}^s)^*)} &= \sum_{s=1}^S p^s \left[ \sum_{r=1}^q u_r y_{rj_0}^s + \alpha^s ((u_{0\alpha}^s)^* - \gamma) - \beta^s ((u_{0\beta}^s)^* - \gamma) \right] = \\ &= \sum_{s=1}^S p^s \left[ \sum_{r=1}^q u_r y_{rj_0}^s + \alpha^s (u_{0\alpha}^s)^* - \beta^s (u_{0\beta}^s)^* \right] + \sum_{s=1}^S p^s \gamma (\beta^s - \alpha^s) = \\ &= FvalP2_U|_{((u_{0\alpha}^s)^*, (u_{0\beta}^s)^*)} + \sum_{s=1}^S p^s \gamma (\beta^s - \alpha^s) > FvalP2_U|_{((u_{0\alpha}^s)^*, (u_{0\beta}^s)^*)}. \end{aligned}$$

In fact for hypothesis  $\beta^s > \alpha^s \geq 0$ ,  $p^s > 0$  and  $\gamma > 0$ . □

Also for this formulation, it's possible to notice that:

- i. for  $\alpha^s = \beta^s = 1 \quad \forall s = 1, \dots, S$ , we obtain model (P2<sub>V</sub>), where, in the primal formulation, it's possible to define  $u_0^s = u_{0\alpha}^s - u_{0\beta}^s$ ;
- ii. for  $\alpha^s = 0$  and  $\beta^s \rightarrow +\infty \quad \forall s = 1, \dots, S$ , we obtain model (P2<sub>C</sub>);

and for the other values of  $\alpha^s$  and  $\beta^s$ , we can able to underline how much returns are variable.

## 3.6 Conclusions

This chapter presents a scenario approach to handle uncertainty in a Data Envelopment Analysis model. Two different formulations are defined both with Variable Returns to Scale (VRS1 and VRS2) and with Constant Returns to Scale (CRS1 and CRS2). Both VRS1 and VRS2 models show a different approach, with respect to the current literature, to manage uncertainty in data. The main research field in DEA considers a chance-constrained approach which permits constraint violations up to specified probability limits and which generally assume normal data distribution. The proposed models remove the hypothesis of normal data distribution and uses a scenario generation approach to include data perturbations. The results of a full computational test are collected by comparing VRS1 and VRS2 formulations with two alternative formulations, a first one differing in constraints formulation (VRSqEV) and the other one considering a classical expected value model (VRSEV). It can be observed that the rank between DMUs efficiency scores is different, some efficient DMUs in the models VRSqEV and VRSEV are not efficient neither in VRS1 nor in VRS2. A simple example remarks the reason of such a different behaviour. The proposed models VRS1 and VRS2 can be a useful strategic management tool aimed to determine a restrictive efficiency score ranking. A natural future validation of these models could be their application to concrete problems discussing a more accurate scenario selection and parameter estimation.

Finally a unifying formulation able to describe both variable and constant returns to scale is proposed by controlling the values of exogenous parameters. The model allows to describe increasing or decreasing returns to scale and to control how the increase in inputs affects the increase in outputs.

## Chapter 4

# Eco-efficiency of the world cement industry: A Data Envelopment Analysis

The aim of this chapter is to study eco-efficiency measures for twenty-one prototypes of cement industries operating in many countries by applying both a Data Envelopment Analysis (DEA) and a directional distance function approach, which are particularly suitable for models where several production inputs and desirable and undesirable outputs are taken into account. To understand whether this eco-efficiency is due to a rational utilization of inputs or to a real carbon dioxide reduction as a consequence of environmental regulation, we analyze the cases where CO<sub>2</sub> emissions can either be considered as an input or as an undesirable output. The results, collected in [52, 51], show that countries where cement industries invest in technologically advanced kilns and adopt alternative fuels and raw materials in their production processes are eco-efficient. This gives a comparative advantage to emerging countries, such as India and China, which are incentivized to modernize their production processes.

### 4.1 Introduction

Cement is essential for the economic development of a country, but its production is highly energetic and emission intensive. Among the non-metallic mineral production processes, cement manufacturing is the most expensive in terms of energy consumption. According to the European Cement Association (Cembureau), “each ton of cement produced requires 60 to 130 kg of fuel oil or its equivalent, depending on the cement variety and the process used, and approximately 105 KWh of electricity”<sup>1</sup>. Energy costs, in the form of fuel and electricity, represent 40% of the total production costs for one ton of cement (see European Commission [24]). In addition, the cement industry is responsible for approximately 5% of the current worldwide CO<sub>2</sub> emissions (see IEA [35]). These data are worrisome because the worldwide production of cement has more than quadrupled over the last twenty-five years, reaching 3 million tons in 2009 (see Cembureau [10]). Production is expected to further increase because of the exponential growth rates in developing countries, such as China and India, which are the major cement producers in the world. Clinker production is primarily responsible for CO<sub>2</sub> emissions. Clinker is a cement sub-product that is produced by burning a mixture of limestone, silicon oxides, aluminum oxides and iron oxides in kilns and differs according to

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<sup>1</sup>See <http://www.cembureau.be/about-cement/cement-industry-main-characteristics>

the process adopted<sup>2</sup> at an average temperature of approximately 1,450 degrees Celsius. This high temperature, which is usually reached by burning highly emitting fuels (such as coal and pet-coke), leads to chemical reactions that transform raw materials into clinker and also generate CO<sub>2</sub> and other greenhouse gas emissions as undesirable outputs.

CO<sub>2</sub> emissions become a problem for industries that operate in countries where environmental regulations apply. This is the situation that European cement industries have faced since 2005, when the European Emission Trading Scheme (EU-ETS) was developed. Introduced by Directive 2003/87/EC, the EU-ETS is the widest cap-and-trade system applied in the world and regulates CO<sub>2</sub> emissions generated from specific installations<sup>3</sup>. The cap-and-trade system implies the imposition of a CO<sub>2</sub> emission ceiling for all covered installations in the different countries, the National Allocation Plans (NAPs<sup>4</sup>), and the creation of an emission permit market where players can buy or sell CO<sub>2</sub> allowances at a certain price defined by the market. The EU-ETS was originally organized in two phases: the first has already concluded and was conducted from 2005 to 2007, and the second covers the period 2008-2012<sup>5</sup>. A third phase has been announced for the period 2013-2020. This will be regulated by the new Directive 2009/29/EC, which enlarges the number of sectors and greenhouse gases subject to regulation. The European energy intensive industries (and also cement producers) complain about EU-ETS because it imposes additional costs from emissions abatement and the purchase of allowances, which put their European plants at a competitive disadvantage with respect to those operating in countries where emissions constraints are more lenient or even absent (see Business Europe [12] and Cefic [15]). In fact, in many countries, emission trading schemes are not mandatory but are organized on a voluntary basis. Voluntary emission regulation systems apply in some of the countries included in our study.

In Europe, a voluntary emission trading scheme is applied in Switzerland. This system began in January 2008 after the approval of the Swiss Parliament and the Federal Government in 2007. It is voluntary, but it becomes legally binding once accepted. As with the current EU-ETS program, the Swiss ETS covers the CO<sub>2</sub> emissions generated by the heating process and energy intensive industries, such as the cement, paper and pulp and glass and ceramics sectors during the 2008-2012 period. Allowance are freely allocated taking into account the emission reduction potential of each company<sup>6</sup>.

In Japan, a voluntary emission trading scheme was introduced in 2005 to cover CO<sub>2</sub> emissions (see [49]). This is organized as follows: the Japanese Ministry of the Environment distributes allowances to all companies participating in this project and concedes subsidies for investments in new technologies. On the other hand, each company has to accomplish a specific target by the end of each period (each of which has a duration of approximately one year). The sectors involved in the Japanese ETS are the following: industries (steel, chemicals, paper, cement, glass, automobiles and other manufacturing), energy conservation (power generation, oil refining), business (corporations and banks) and transportation (aviation and freight) (see [63]).

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<sup>2</sup>Cement can be produced with four different processes: dry, wet, semi-dry and semi-wet. Dry and semi-dry processes are generally more productive and require a lower amount of energy than the other two. Cement production is subdivided into two main steps: first, clinker is produced from raw materials in kilns, whose efficiency varies according to the process adopted, and then cement results from the mixture of clinker with other additives.

<sup>3</sup>The sectors currently involved are the following: energy and refining, iron and steel, pulp and paper and cement.

<sup>4</sup>See [http://ec.europa.eu/clima/policies/ets/allocation\\_2005\\_en.htm](http://ec.europa.eu/clima/policies/ets/allocation_2005_en.htm) for the National Allocation Plans of the two phases.

<sup>5</sup>European Commissions list the countries involved in the EU-ETS and provides information about the two phases at [http://ec.europa.eu/clima/policies/ets/index\\_en.htm](http://ec.europa.eu/clima/policies/ets/index_en.htm). A special case is represented by Norway. This EU country started a domestic emission trading scheme in 2005. The organization of the original Norwegian ETS was identical to that of the EU-ETS. Thanks to an agreement signed between the EU and the members of the European Economic Association (Iceland, Liechtenstein and Norway) in October 2007, Norway officially entered into the EU-ETS in 2008 (see Reinaud [49]). The same happened to the United Kingdom, which, after some problems in determining its NAP, was included in the EU-ETS in 2008.

<sup>6</sup>See [49] and <http://www.bafu.admin.ch/emissionshandel/05538/05540/index.html?lang=en> for more details.

In the USA, some states (and also some Canadian states) participate in the Western Climate Initiative<sup>7</sup> and the Regional Greenhouse Gas Initiative<sup>8</sup> on a voluntary base, and only California regulates emissions, as a result of the “Global Warming Solution Act” (or AB 32), signed into law on September 27, 2006. This program will take effect by 2012<sup>9</sup> and covers six in-state greenhouse gas emissions<sup>10</sup> generated by several industrial sectors. It aims to reduce the emission level by 25%, compared to the baseline level, by 2020.

Another example of emission regulation is Alberta’s Climate Change and Emission Management Act, applied in Canada. Starting from July 2007, the Canadian facilities in the Alberta region, whose greenhouse emissions are equivalent to or higher than 100,000 tons, are subject to this regulation (see [49]). Different from the EU-ETS, the Alberta’s Act uses an emission intensity approach<sup>11</sup>. This system forces the involved facilities to improve their performance either by reducing their greenhouse gases emissions or by buying credits from the Climate Change and Emission Management Fund at a price of 15 Canadian dollars per ton of reduced emission. Parallel to this system, in 2006, the Canadian government issued a regulatory framework for industrial greenhouse gas emissions that sets the basis for an emission trading scheme<sup>12</sup>. This scheme covers several sectors (power generation, oil and gas, pulp and paper, iron and steel, smelting and refining of metals, cement, lime, potash, and chemicals and fertilizers) and should have begun in 2010<sup>13</sup>. As with Alberta’s Act, the Canadian ETS aims to reduce the carbon intensity of industrial activities using specific intensity-based targets for each sector<sup>14</sup>. This should globally induce an absolute emission reduction of 20%, compared to 2006 levels, by 2020. This emission cut should reach 50%-60% by 2050.

Finally, not all announced programs have a positive outcome. This is the case of the Australian ETS program. Even though Australia is one of the highest CO<sub>2</sub> emitters among the developed countries, the proposal for an emission trading scheme advanced by Prime Minister Kevin Rudd has been blocked in the senate<sup>15</sup>. It has been shelved at least until 2013, because the government prefers to wait for the expiration of the Kyoto Protocol before imposing an emission regulatory program. The proposed ETS was intended to reduce CO<sub>2</sub> emission by 25%, with respect to 2000 levels, by 2020.

However, one can notice a growing awareness of greenhouse gas emissions and of the environment in general in developing countries. China is the world’s largest CO<sub>2</sub> emitter but has shown a determination to curb its greenhouse gas emissions. The application of China’s national Climate Change Program has been the first step in a modernization process whereby China intends “*to address climate change and promote sustainable development*” through “*policies and measures, such as economic restructuring, energy efficiency improvement, development and utilization of hydropower and other renewable energy*”<sup>16</sup>. This means that China will not impose a cap on CO<sub>2</sub> emissions but will reduce emissions by setting binding energy intensive reduction targets, stringent fuel efficiency standards and investments in more efficient technologies. Several key sectors are involved in this program, and cement is one of them. Note that the effectiveness of the targets imposed by the Chinese climate program on the cement sector<sup>17</sup> is confirmed by a study conducted by the Inter-

<sup>7</sup>See <http://www.westernclimateinitiative.org/>

<sup>8</sup>See <http://www.rggi.org/home>

<sup>9</sup><http://arb.ca.gov/cc/ab32/ab32.htm>

<sup>10</sup>Those that are also covered by the Kyoto Protocol.

<sup>11</sup>The emission intensity measures the amount greenhouse gases generated per unit of economic output.

<sup>12</sup>Available at [http://www.ec.gc.ca/doc/virage-corner/2008-03/pdf/COM-541\\_Framework.pdf](http://www.ec.gc.ca/doc/virage-corner/2008-03/pdf/COM-541_Framework.pdf)

<sup>13</sup>Both in [49] and [http://www.ec.gc.ca/doc/virage-corner/2008-03/pdf/COM-541\\_Framework.pdf](http://www.ec.gc.ca/doc/virage-corner/2008-03/pdf/COM-541_Framework.pdf), but we have not found any document announcing its launch.

<sup>14</sup>See <http://www.aph.gov.au/library/pubs/climatechange/governance/foreign/canadian.htm>

<sup>15</sup>See <http://news.bbc.co.uk/2/hi/asia-pacific/8645767.stm> and

<http://www.rsc.org/chemistryworld/News/2010/April/29041002.asp>

<sup>16</sup>Directly taken from <http://www.ccchina.gov.cn/WebSite/CCChina/UpFile/File188.pdf>

<sup>17</sup>At point (4) of China’s national Climate Change Program, one can read that “*new dry process kiln with precalci-*

national Energy Agency, which suggested that cement industries dispose of four tools suitable for reducing their CO<sub>2</sub> emissions, namely, thermal and electric efficiency, the utilization of alternative fuels, clinker substitution and the adoption of a carbon capture and storage process that captures CO<sub>2</sub> before being released into the atmosphere (see IEA [35]).

Similar policies are also in force in India, where an Energy Conservation Act<sup>18</sup> was introduced in 2001, and in Brazil, where the National Climate Change Plan has been effective since 2008<sup>19</sup>. Also, in Turkey, there are some signals for policies in this direction. Turkey's candidacy to become a European Member State induced the Turkish government to ratify the Kyoto Protocol in May 2009<sup>20</sup> and to introduce eco-innovation policies to reduce their emissions (see OECD [43]).

Considering this framework, the aim of this chapter is to study the eco-efficiency level of cement industries operating in different countries. In this light, there are several environmental performance indicators to choose from (see Tyteca [60] for a complete review). We have chosen to apply a Data Envelopment Analysis (DEA) approach which has the advantage of simultaneously considering multiple inputs (with their respective measures) and both the desirable (produced good) and undesirable (waste and pollutants) outputs that characterize a certain production process. This allows DMUs to have immediate information on their global efficiency (or inefficiency) status and, depending on the DEA approach adopted, on which input or output should be examined to improve their production.

This study is different from other studies on the cement sector already existing in the literature (Bandyopadhyay [3], Mandal and Madheswaran [40] and Sadjadi and Atefeh [54]) where the analysis is conducted at the interstate level. In our study, the DMUs are prototypes of cement plants located in twenty-one countries. We measured their eco-efficiency by including CO<sub>2</sub> emissions as an undesirable production factor. According to the DEA literature, undesirable factors can be modeled either as an input or as an undesirable output. We apply both of these existing approaches in addition to a directional distance function model. With these models, the eco-efficiency of cement DMUs can be measured either as a contraction of CO<sub>2</sub> emissions or as an increased utilization of alternative fuels and raw materials. Our analysis shows that the units' efficiency levels are affected by the tendency of different DMUs to invest in technologically advanced kilns and adopt alternative fuels and raw materials in their cement production processes. Surprisingly, emerging countries, such as India and China, which are the largest cement producers in the world, appear efficient. As we will explain in Section 4.4, their recent economic booms and the energy efficiency targets imposed by their authorities have forced their cement companies to invest in the most advanced technologies.

The remainder of the chapter is organized as follows. Section 4.2 presents the model used in our analysis, while Sections 4.3 and 4.4 illustrate the dataset used in our simulations and the obtained results, respectively. Final remarks are reported in Section 4.5.

## 4.2 Modeling Eco-efficiency

The notion of eco-efficiency comes with different meanings and definitions. We define *eco-efficiency*, in an operational way, as the ability to produce goods or services by saving energy and resources and/or by reducing waste and emissions. Different instruments for measuring eco-efficiency are in-

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*nator technology should be developed; promote energy efficient grinding equipment and power generating technology by using waste heat recovered from cement kiln; improve the performance of existing large-and medium-size rotary kiln, mills and drying machines for the purpose of energy conservation; gradually phase out mechanized vertical kiln, wet process kiln and long dry process kiln and other backward cement production technologies". Taken directly from <http://www.ccchina.gov.cn/WebSite/CCChina/UpFile/File188.pdf>*

<sup>18</sup>See [http://www.powermin.nic.in/acts\\_notification/pdf/ecact2001.pdf](http://www.powermin.nic.in/acts_notification/pdf/ecact2001.pdf)

<sup>19</sup>See [http://www.eoearth.org/article/Greenhouse\\_Gas\\_Control\\_Policies\\_in\\_Brazil](http://www.eoearth.org/article/Greenhouse_Gas_Control_Policies_in_Brazil)

<sup>20</sup>See [http://en.wikipedia.org/wiki/List\\_of\\_Kyoto\\_Protocol\\_signatories#cite\\_note-13](http://en.wikipedia.org/wiki/List_of_Kyoto_Protocol_signatories#cite_note-13)

produced in the literature (see Tyteca [60]), but most of them are simple indicators<sup>21</sup> that approach eco-efficiency from a very limited perspective because they only consider a few factors in the production process. One should aggregate all these indicators to synthesize information on the overall impact of certain production processes on the environment. Moreover, measuring eco-efficiency at a worldwide scale, as we do in this study, creates a problem of information availability because environmental policies are not uniformly applied around the world.

Some of these difficulties can be overcome by using DEA to determine eco-efficiency. Data Envelopment Analysis was first proposed in the pioneering paper by Charnes, Cooper and Rhodes (CCR) [14]. It is a nonparametric method for estimating the efficiency of  $n$  DMUs. Each DMU consumes various inputs to produce different outputs. No production function needs to be specified.

The classic DEA model (see [14]) is a linear fractional problem that measures the efficiency of the  $j^{\text{th}}$  DMU through a ratio between the weighted sum of outputs and the weighted sum of inputs that are aggregated into a composite input and a composite output for all inputs and all outputs. The CCR pioneer model estimates the technical efficiency of a DMU with Constant Returns to Scale (CRS) over the entire production frontier. The extension proposed by Banker, Charnes and Cooper [5] generalized this assumption and formulated the so-called BCC model, which exhibits Variable Returns to Scale (VRS) at different points in the production frontier.

In general, DEA evaluates the efficiency of each DMU through a better system of weights (or shadow prices) for the considered DMU and identifies the best one. Stating the benchmark, DEA classifies the remaining DMUs from most to least efficient. Efficiency is evaluated by taking into account both the desirable (good) and undesirable (bad) output and input factors. Note that the undesirable and desirable outputs should be treated in different ways: if production is inefficient, the undesirable pollutants should be reduced. However, standard DEA models do not allow decreases in outputs; only inputs can be reduced.

When undesirable outputs are considered, the choice between improved technologies or reference technologies has an important impact on DMU efficiencies. Technology disposability can also be applied in terms of strong and weak disposability of undesirable outputs. A production process is said to exhibit strong disposability of undesirable outputs if the undesirable outputs are freely disposable, i.e., they do not have limits. The case of weak disposability refers to situations where a reduction in the undesirable output forces a lower production of desirable outputs. This means that the reduction of undesirable outputs imposed by an external regulation may not be possible without assuming certain costs (see Zoffo and Prieto, [67]).

As stressed in the Section 2.3 of this thesis, different variants of DEA models can be used for estimating of eco-efficiency. According to the literature, two different approaches can be used to model undesirable factors: one group of DEA models treats them as inputs, whereas a second group considers them as undesirable outputs. In light of this distinction, we now consider the four alternative DEA models presented in overview, in order to measure eco-efficiency. At this time, we analyze the construction of the production frontier. In the DEA literature, three types of frontiers have been proposed to evaluate efficiency in a panel-data framework. The first is the standard Contemporaneous Frontier, where the frontier in each year is constructed with only the observations of the year under consideration. The second type of frontier is the Intertemporal Frontier, and it is based on observations from all of the considered time periods. The third frontier is the Sequential Frontier, where each observation for a given year is compared to all other observations in the same year and to observations in the previous years (see Tulkens and Eeckaut [62] for a detailed discussion about different DEA frontiers). This last methodology is based on the assumption that the production possibility set can expand each year, and no technological regress is admitted. For the aim of this work, the Sequential Frontier seems to be the most suitable for analysis of the

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<sup>21</sup>For instance, economic output per unit of waste ratios



world cement sector in the years 2005-2008<sup>22</sup>. In these years, the world cement industry faced rapid expansion and technological improvement (especially in developing countries). For DEA application with sequential frontier, the production possibility set is define as:

$$T_V = \left\{ (x, y) : x \geq \sum_{j=1}^n \sum_{t=1}^T \lambda_j x_j^t; \quad y \leq \sum_{j=1}^n \sum_{t=1}^T \lambda_j y_j^t; \quad \sum_{j=1}^n \lambda_j = 1 \right\},$$

where there are  $n$  units observed and  $t$  corresponds to the time period at which the DMU is being evaluate. The constranints for DEA models discussed in Section 2.3 are revised accordingly and provided in the next section. For the sake of convenience, a list of the variables and parameters used in the different models is provided below again.

**Parameters:**

- $x_{ij}^t \in \mathbb{R}_+$ :  $i^{th}$  input quantity used by the  $j^{th}$  decision making unit at time  $t$ ,  
 $i = 1, \dots, m, \quad j = 1, \dots, n, \quad t = 1, \dots, T$ ;
- $y_{rj}^{gt} \in \mathbb{R}_+$ :  $r^{th}$  “good” output quantity produced by the  $j^{th}$  decision making unit at time  $t$ ,  
 $r = 1, \dots, q, \quad j = 1, \dots, n, \quad t = 1, \dots, T$ ;
- $y_{kj}^{bt} \in \mathbb{R}_+$ :  $k^{th}$  “bad” output quantity produced by the  $j^{th}$  decision making unit at time  $t$ ,  
 $k = 1, \dots, l, \quad j = 1, \dots, n, \quad t = 1, \dots, T$ .

**Variables:**

- $v_i \in \mathbb{R}_+$ : weight multipliers related to the  $i^{th}$  input,  
 $j = 1, \dots, n$ ;
- $u_r \in \mathbb{R}_+$ : weight multipliers related to the  $r^{th}$  “good” output,  
 $r = 1, \dots, q$ ;
- $w_k \in \mathbb{R}_+$ : weight multipliers related to the  $k^{th}$  “bad” output,  
 $k = 1, \dots, l$ ;
- $u_0 \in \mathbb{R}$ : scale factor variable;
- $\theta \in \mathbb{R}_+$ : dual variable related to the first constraint;
- $\lambda_j \in \mathbb{R}_+$ : dual variables related to the second set of constraints,  
 $j = 1, \dots, n$ .

### 4.2.1 Pollutants as inputs

#### Eco-efficiency measure as input and pollutant contraction

A first approach to treat both desirable and undesirable factors following the standard linear BCC model suggests that we include the undesirable outputs as desirable inputs in the production process (see [38]). Efficient DMUs wish to minimize the desirable inputs and undesirable outputs. The mathematical formulation of the model, for the case of strong output disposability and input-oriented DEA, for each time  $\bar{t} \in (1, T)$  in the considered period, is as follows:

<sup>22</sup>However, tests were carried out also considering Contemporaneous Frontier. See Appendix A for the complete collection of all simulations



$$(P_{INP}) \quad \max_{u,v,w,u_0} \sum_{r=1}^q u_r y_{rj_0}^{g\bar{t}} + u_0 \quad (4.1)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0}^{\bar{t}} + \sum_{k=1}^l w_k y_{kj_0}^{b\bar{t}} = 1, \quad (4.2)$$

$$\sum_{r=1}^q u_r y_{rj}^{gt} + u_0 - \sum_{i=1}^m v_i x_{ij}^t - \sum_{k=1}^l w_k y_{kj}^{bt} \leq 0, \quad j = 1, \dots, n, \quad t = 1, \dots, T, \quad (4.3)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$w_k \geq 0, \quad k = 1, \dots, l,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0 \in \mathbb{R}.$$

The corresponding dual formulation is as follows:

$$(D_{INP}) \quad \min_{\theta, \lambda} \theta \quad (4.4)$$

$$s.t. \quad \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{rj}^{gt} \geq y_{rj_0}^{g\bar{t}}, \quad r = 1, \dots, q, \quad (4.5)$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{kj}^{bt} \leq \theta y_{kj_0}^{b\bar{t}}, \quad k = 1, \dots, l, \quad (4.6)$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t x_{ij}^t \leq \theta x_{ij_0}^{\bar{t}}, \quad i = 1, \dots, m, \quad (4.7)$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t = 1, \quad (4.8)$$

$$\lambda_j^t \geq 0, \quad j = 1, \dots, n, \quad t = \dots, T.$$

The primal formulation corresponds to a standard input-oriented primal BCC model, where undesirable outputs behave like inputs. The objective function of the primal formulation maximizes the weighted sum of the desirable outputs under the condition that the weighted sum of the inputs and undesirable outputs for the considered DMU is equal to one (as it results in a constraint (4.2)). From a dual point of view, this means that a DMU can simultaneously reduce all inputs and undesirable outputs by the same proportion  $\theta$  to increase its eco-efficiency. Constraints (4.5) ensure that optimal good outputs are no lower than what is actually being produced. According to inequalities (4.6) and (4.7), eco-efficiency in the cement sector can be interpreted as the ability to reduce undesirable outputs by introducing more efficient technologies (bad output contraction) and reducing inputs by substituting them with alternative raw materials while maintaining the same output levels. In particular, inequalities (4.6) are related to pollutants emissions which are modeled as a sort of production input in this framework; constraints (4.7) measure the admissible contraction of classical

input factors (namely, in the cement industry, materials, capital, energy and labor). Finally, note that inequality (4.8) indicates that production set is convex and allows for variable return to scale. An efficient DMU will have  $\theta^* = 1$  as an optimal solution, which implies that no equiproportional reduction in inputs and undesirable outputs is possible.

Note that models ( $P_{INP}$ ) and ( $D_{INP}$ ) can be used with the assumption of weak disposability by considering variables  $w_k$  as unconstrained in sign in the primal formulation and by assuming that the constraint (4.6) holds with equality in the corresponding dual formulation. For an exhaustive discussion on strong and weak disposability in this class of models see Liu et al. [37]. Note that considering the undesirable outputs as inputs, the resulting DEA model does not reflect the true production process. This is the main drawback of this formulation.

### Eco-efficiency measure as pollutant contraction

The class of DEA models we present in this subsection allows us to measure eco-efficiency as the ability to reduce pollutants, while maintaining the same inputs and desirable output levels. The eco-efficiency measure, as formally defined by Korhonen and Luptacik in [34], is the ratio between the weighted sum of the desirable outputs minus the weighted sum of the inputs and the weighted sum of the undesirable outputs. The Primal-Dual linearized version of this class of DEA models, for each  $\bar{t}$  period, is as follows:

$$(P_{KL}) \quad \max_{u,v,w,u_0} \sum_{r=1}^q u_r y_{rj_0}^{g\bar{t}} - \sum_{i=1}^m v_i x_{ij_0}^{i\bar{t}} + u_0 \quad (4.9)$$

$$s.t. \quad \sum_{k=1}^l w_k y_{kj_0}^{b\bar{t}} = 1, \quad (4.10)$$

$$\sum_{r=1}^q u_r y_{rj}^{gt} - \sum_{i=1}^m v_i x_{ij}^{it} + u_0 - \sum_{k=1}^l w_k y_{kj}^{bt} \leq 0, \quad j = 1, \dots, n, \quad t = 1, \dots, T, \quad (4.11)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$w_k \geq 0, \quad k = 1, \dots, l,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0 \in \mathbb{R}.$$

$$\begin{aligned}
& (D_{KL}) \\
& \min_{\theta, \lambda} \quad \theta \\
& \text{s.t.} \quad \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{rj}^{gt} \geq y_{rj_0}^{g\bar{t}}, \quad r = 1, \dots, q, \tag{4.12}
\end{aligned}$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{kj}^{bt} \leq \theta y_{kj_0}^{b\bar{t}}, \quad k = 1, \dots, l, \tag{4.13}$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t x_{ij}^t \leq x_{ij_0}, \quad i = 1, \dots, m, \tag{4.14}$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t = 1,$$

$$\lambda_j^t \geq 0, \quad j = 1, \dots, n, \quad t = 1, \dots, T.$$

The objective function in the dual formulation provides information on the pollutant contraction to the largest extent possible. If  $\theta^* < 1$ , e.g. DMU is inefficient, the firm can still reduce undesirable outputs without increasing the corresponding input levels or reducing desirable outputs. This can be guaranteed by both constraints (4.14) which state that desirable inputs will not be increased at the optimal solution and constraints (4.12) which impose that optimal desirable output level is not lower than the actual produced one. Model  $(P_{KL})$ - $(D_{KL})$ , under the hypothesis of weak disposability, can be obtained considering variable  $w_k$  as unconstrained in sign in the primal formulation and assuming that constraints (4.13) hold with equality in the dual formulation.

Notice that, in the case of a single undesirable factor, weak and strong eco-efficiency scores coincide. In fact, taking into account constraints (4.13), with simple calculations it results:

$$\theta \geq \frac{1}{y_{kj_0}^{b\bar{t}}} \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{kj}^{bt} \quad k = 1, \dots, l,$$

so that, since problem  $D_{KL}$  is a minimization problem, the optimal solution  $\theta^*$  assumes the following values:

$$\theta^* = \max_k \left\{ \frac{1}{y_{kj_0}^{b\bar{t}}} \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{kj}^{bt} \right\}.$$

As a consequence, when a single undesirable factor is considered ( $k = 1$ ) there is no difference in using constraint (4.13) in inequality form or in using it just as an equality constraint.

## 4.2.2 Pollutant as undesirable outputs

### Eco-efficiency as input contraction

The analysis of eco-efficiency can further be examined by considering a third efficiency measure. This measure focuses on efficiency improvement by reducing the inputs and maintaining fixed levels for undesirable and desirable outputs. Taking this into consideration, we present the approach proposed by Seiford and Zhu [55]. Under the context of the BCC model (Banker et al., [5]), Seiford and Zhu developed an alternative method to deal with desirable and undesirable factors in DEA. To increase

the desirable outputs and decrease the undesirable outputs, they transformed the values of the undesirable outputs by a monotone decreasing function. The transformed data were then included as desirable outputs in the problem and maximized. In fact, after the decreasing transformation, maximizing these values means minimizing the original undesirable outputs. In particular, for the purpose of preserving linearity and convexity relations, Seiford and Zhu [55] suggested a linear monotone decreasing transformation,  $\bar{y}_{kj}^b = -y_{kj}^b + \beta_k > 0$ , where  $\beta$  is a proper translation vector that makes  $\bar{y}_{kj}^b > 0$ . Based upon the above linear transformation, the standard BCC DEA model can be modified as the following pair of linear programs:

$$(P_{TR\beta})$$

$$\max_{u,v,w,u_0} \sum_{r=1}^q u_r y_{rj_0}^{g\bar{t}} + \sum_{k=1}^l w_k \bar{y}_{kj_0}^{b\bar{t}} + u_0 \quad (4.15)$$

$$s.t. \quad \sum_{i=1}^m v_i x_{ij_0}^{\bar{t}} = 1, \quad (4.16)$$

$$\sum_{r=1}^q u_r y_{rj}^{gt} + \sum_{k=1}^l w_k \bar{y}_{kj}^{bt} + u_0 - \sum_{i=1}^m v_i x_{ij}^t \leq 0, \quad j = 1, \dots, n, \quad t = 1, \dots, T \quad (4.17)$$

$$u_r \geq 0, \quad r = 1, \dots, q,$$

$$w_k \geq 0, \quad k = 1, \dots, l,$$

$$v_i \geq 0, \quad i = 1, \dots, m,$$

$$u_0 \in \mathbb{R}.$$

$$(D_{TR\beta})$$

$$\min_{\theta, \lambda} \quad \theta$$

$$s.t. \quad \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{rj}^{gt} \geq y_{rj_0}^{g\bar{t}}, \quad r = 1, \dots, q, \quad (4.18)$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t \bar{y}_{kj}^{bt} \geq \bar{y}_{kj_0}^{b\bar{t}}, \quad k = 1, \dots, l, \quad (4.19)$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t x_{ij}^t \leq \theta x_{ij_0}^{\bar{t}}, \quad i = 1, \dots, m, \quad (4.20)$$

$$\sum_{j=1}^n \sum_{t=1}^T \lambda_j^t = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n, \quad t = 1, \dots, T.$$

In model  $(D_{TR\beta})$ , like the classical BCC models, the efficiency is measured by the possible input reductions, whereas the outputs are kept at their current levels. According to this model, a DMU  $j_0$  can improve the eco-efficiency by reducing the inputs, whereas the values of the desirable and undesirable outputs of the DMU  $j_0$  are taken as lower bounds for a linear combination of the other desirable and undesirable outputs (constraints (4.18) and (4.20)). Notice that constraints (4.20) assume a peculiar meaning after the translation. The worst performing DMU in terms of CO<sub>2</sub>

emissions results to produce the lowest amount of translated undesirable output. In this light,  $\bar{y}_{kj}^b$  can be interpreted as the emission saving amount corresponding to the good output production level. From a theoretical point of view, if we assume Variable Return to Scale (VRS), the model is invariant with respect to the linear translation. It has been proven by Ali and Seiford [1] that affine translation of data values does not alter the efficient frontier. Therefore, the classification of DMUs as efficient or inefficient is translation invariant. We recall that the same models can be used with the assumption of weak disposability by considering variables  $w_k$  as unconstrained in sign in the primal formulation ( $P_{TR\beta}$ ) and by assuming that the constraints (4.20) hold with equality in the corresponding dual formulation ( $D_{TR\beta}$ ).

### Eco-efficiency as undesirable output contraction and desirable output expansion

The directional output distance function, in its original formulations by Färe et al. [28], is an alternative approach to evaluate eco-efficiency. This approach expands the desirable outputs and contracts undesirable outputs along a path that varies according to the direction vector adopted to increase eco-efficiency. Extensions of this methodology (see for all [25, 26, 46]) obtain a measure of eco-efficiency from the potential for increasing outputs while reducing inputs and undesirable outputs simultaneously.

To describe this approach, let us define the following sets. Let  $T$  be the technology set, such that:

$$T = [(x, y^g, y^b) : x \text{ can produce } (y^g, y^b)]. \quad (4.21)$$

In presence of undesirable outputs, the output set  $\mathcal{P}(x)$  represents all the feasible output vectors  $(y^g, y^b)$  for a given input vector  $x$ , that is:

$$\mathcal{P}(x) = [(y^g, y^b) : (x, y^g, y^b) \in T]. \quad (4.22)$$

The directional technology distance function generalizes both the input and output Shephard's distance functions and provides a complete representation of the production technology.

Let  $d = (-d^x, d^g, -d^b)$ , where the function is formally defined as the following:

$$\vec{D}_T(x, y^g, y^b; d) = \sup [\delta : (y^g + \delta d^g, y^b - \delta d^b) \in \mathcal{P}(x - \delta d^x)] \quad (4.23)$$

The expression (4.23) seeks the maximum attainable expansion of desirable outputs in the  $d^g$  direction and the largest feasible contraction of undesirable outputs and inputs in the  $d^b$  and  $d^x$  directions. Under the assumptions made on the technology of reference, the directional technology distance function for expression (4.23) can be computed for firm  $j_0$ , at time  $\bar{t}$ , by solving the following programming problem:

$$\begin{aligned}
& (P_{DDF}) \\
& \max_{\delta, \lambda} \quad \delta \\
& s.t. \quad \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{rj}^{gt} - \delta d_{rj_0}^{g\bar{t}} \geq y_{rj_0}^{g\bar{t}}, \quad r = 1, \dots, q, \tag{4.24} \\
& \quad \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t y_{kj}^{bt} + \delta d_{kj_0}^{b\bar{t}} \leq y_{kj_0}^{b\bar{t}}, \quad k = 1, \dots, l, \tag{4.25} \\
& \quad \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t x_{ij}^t + \delta d_{ij_0}^{x\bar{t}} \leq x_{ij_0}^{x\bar{t}}, \quad i = 1, \dots, m, \tag{4.26} \\
& \quad \sum_{j=1}^n \sum_{t=1}^T \lambda_j^t = 1, \\
& \quad \lambda_j^t \geq 0, \quad j = 1, \dots, n, \quad t = 1, \dots, T.
\end{aligned}$$

The choice of a direction vector  $d = (-x, y^g, -y^b)$  allows us to evaluate a global technology and its ecological efficiency by reducing the inputs and undesirable outputs and simultaneously expanding the desirable outputs. A different direction vector can be used to restrict the analysis on output factors by considering, for instance, a direction vector  $d = (0, y^g, -y^b)$ . In this case, Mandal and Madheswaran [40] focused their attention on the expansion of desirable factors and the contraction of undesirable ones without increasing the inputs.

Notice that in the directional distance function model, efficiency is reached when  $\delta = 0$ , which corresponds to the case of  $\theta = 1$  in the standard DEA formulations.

Let us recall that this model can be also considered under the assumption of weak disposability by assuming that constraints (4.25) hold with equality.

### 4.3 Database description

A database with twenty-one regulated and non-regulated cement-producing countries was collected. The dataset was divided into European (EU) and non-European (non-EU) countries according to the geographic and regulation emission aspects. The thirteen European countries included in the database (Austria, Belgium, Czech Republic, Denmark, Estonia, France, Germany, Italy, Norway, Poland, Spain, Switzerland and United Kingdom) account for more than 80% of the total EU cement production. To compare the eco-efficiency of these EU countries with non-EU countries, data for eight major non-EU countries (Australia, Brazil, Canada, China, India, Japan, USA and Turkey) were added.

For the purpose of our analysis, the choice of input and output factors for the DEA models accounted for the cement and clinker production processes. Four input data were considered: installed capacity, energy, labor and materials. The desirable output was represented by cement production, while CO<sub>2</sub> emissions were the undesirable by-product.

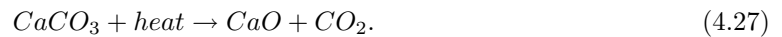
The inputs for all twenty-one countries (DMUs in DEA) were as follows: capital in the form of installed capacity (a similar approach can be found in Fare et al. [29], Tyteca [61]), energy as the sum of electricity and thermal energy, labor as the number of employees and materials as raw materials (slug, limestone, etc), in addition to clinker imports and production. Notice that alternative fuels and raw materials were not included in the input factors because we assumed that

the use of alternative fuels and alternative raw materials was costly and contributed to the reduction in emission factors. Consequently, countries that decide to use alternative fuels and materials can improve their eco-efficiency.

The desirable output was Portland cement production. The undesirable by-product was measured as the value of carbon dioxide emissions ( $CO_2$ ) resulting from the clinker production process without considering those related to raw materials, fuels and clinker transportation.  $CO_2$  can be interpreted as an input or undesirable output according to the different DEA approaches.

The data sources for the EU countries were as follows: the European association of cement industries (Cembureau), the national cement associations for each country (see Cembureau website for the link to members' national associations and Appendix B), OECD (especially for labor data), Eurostat and ComTrade (for clinker import/export data), and the European Pollutant Emission Registry (EPER for  $CO_2$  emission data). National cement associations also provided data for non-EU countries (see the detailed list of the references in Appendix B).

Missing data on emission factors were estimated according to the Intergovernmental Panel on Climate Change (IPCC)<sup>23</sup>. Carbon dioxide is released during the production of clinker, in which calcium carbonate ( $CaCO_3$ ) is heated in a rotary kiln to induce a series of complex chemical reactions. Specifically,  $CO_2$  is released as a by-product during calcination, which occurs in the upper, cooler end of the kiln, or a precalciner, at temperatures of 600-900C, and this reaction results in the conversion of carbonates to oxides. The simplified stoichiometric relationship is as follows:



At higher temperatures, in the lower end of the kiln, the lime (CaO) reacts with silica, aluminum and iron-containing materials to produce minerals in the clinker. The clinker is then removed from the kiln to cool, ground to a fine powder, and mixed with a small fraction (approximately five percent) of gypsum to create the most common form of cement, known as Portland cement.

The formula to calculate  $CO_2$  emissions has been defined according to the following steps:

1. Data on clinker production (in tons) were collected. In the case of missing data, the clinker production was estimated as a fixed proportion of cement production (estimated coefficient varying between 75% and 95% according to the cement blending).
2. Ton of raw material (T) per ton of clinker (*RM/clinker ratio*) was estimated in the case of data missing with a fixed coefficient of 1.54 according to IPCC guidelines.
3. The  $CaCO_3$  Equivalent to Raw Material Ratio (%) is fixed at 78.5%.
4. The  $CO_2$  to  $CaCO_3$  Stoichiometric Ratio is fixed at 0.44.

The total  $CO_2$  emissions expressed in tons (T) were estimated as follows:

$$CO_2 = clinker(T) \cdot (RM/clinker\ ratio) \cdot CaCO_3\% \cdot 0.44. \quad (4.28)$$

In Table 4.1, we report the means and standard deviations of the input and output parameters used in the models for the 2005-2008 period. Note that the high standard deviation values depend on the inclusion of China in the dataset. The cement industry in China accounts for more than 40% of world cement production.

The time-varying analysis of the mean values in Table 4.1 shows that worldwide cement production has grown since 2005, with a peak value in 2007 and a stable situation in 2008. This growth

<sup>23</sup>see <http://www.ghgprotocol.org/calculation-tools/cement-sector> for the clinker-based tool suitable when the amount of clinker consumed is known. We also recall that only direct emissions have been considered in this work.

Variable	2005		2006		2007		2008	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Capacity (MI t)	139.74	320.05	170.75	437.79	169.94	430.95	169.52	424.16
Energy (TWh)	89.17	273.83	98.96	312.62	99.33	310.03	95.30	294.43
Labour (x1000)	77.64	297.01	89.10	347.41	97.02	381.06	99.73	392.24
Materials (MI t)	178.24	502.92	200.58	586.21	214.34	637.25	213.71	636.01
Clinker	63.42	176.74	70.67	204.15	74.67	217.90	71.21	205.11
Import clinker	0.70	1.71	1.03	2.12	0.98	2.34	0.62	1.21
Raw materials	114.12	324.43	128.88	379.91	138.68	417.01	141.88	429.68
Cement (MI t)	79.68	222.05	89.91	260.04	96.54	285.40	97.92	294.14
CO2 (MI t)	68.13	205.65	76.32	237.12	79.88	251.35	80.23	254.60
<b>Ratio</b>	<b>Ratio</b>		<b>Ratio</b>		<b>Ratio</b>		<b>Ratio</b>	
Clinker/Cement	0.7959		0.7859		0.7735		0.7272	
Energy/Cement	1.1191		1.1006		1.0289		0.9733	
CO2/Cement	0.8550		0.8488		0.8275		0.8194	

Table 4.1: Cross Sectional Mean and Standard Deviation of the variables and relevant ratios.

can be attributed to Turkey and other non-OECD countries that have developed their economies with relatively lenient environmental limits. Moreover, looking at the cement and clinker mean values over the years, one can see a progressive reduction in the average clinker-to-cement ratio from 0.79 in 2005 to 0.72 in 2008. This is due to major use of alternative raw materials in the clinker production process and to the increased production of blended cement, especially in developing countries, which requires a lower proportion of clinker. A similar behavior is found in energy consumption, where the increased use of alternative fuels, such as waste or biomass, justifies the corresponding decrease in the energy/cement ratio. The combination of these two effects leads to a reduction in CO<sub>2</sub> emissions over time.

Since the aim of this work was evaluate the eco-efficiency of different countries, all inputs and outputs were divided by the total number of plants in each country to evaluate the eco-efficiency of a representative plant within each country (a similar approach can be found in Mandal [39], Mandal and Madheswaran [40], Mukherjee [41, 42]).

## 4.4 Empirical Results

The three DEA models and the directional distance function described in Section 4.2 were implemented in MatLab 2010a to capture the various measures of eco-efficiency in the cement industry. To avoid imbalances caused by different magnitudes, the input and output parameters were normalized with respect to their mean value (see Table 4.1). We recall that the study of eco-efficiency using model  $(P_{INP})-(D_{INP})$  provided information on eco-efficiency measured by both input and CO<sub>2</sub> contraction; the analysis was expanded by separately measuring the impact of pollutant contraction (as resulting from model  $(P_{KL})-(D_{KL})$ ) and the impact of good inputs contraction, namely, capacity, energy, labor and materials (model  $(P_{TR\beta})-(D_{TR\beta})$ ). The directional distance approach (model  $(P_{DDF})$ ) was also tested to verify the possibility of expanding the desirable output while reducing the undesirable output. Finally, to illustrate the impact of regulation, all models were tested under both a strong and weak disposability assumption. As noted in Section 4.2.1, model  $(P_{KL})-(D_{KL})$ , under both strong and weak disposability assumptions, resulted in the same optimal efficiency scores when a single undesirable factor was used.

The methodology used to evaluate these eco-efficiency measures was based on a sequential frontier approach. In this way, we avoid the possibility of "technical regress" because the sequential frontier assumes all current and past observations as feasible. Starting, then, with a reference sample



of 21 observations for the year 2005, we successively accumulated the observations from one more year to create the frontier of each subsequent period.

The results of our tests are reported in Tables 4.2, 4.3, 4.4 and 4.5.

#### 4.4.1 Eco-efficiency measure as input and pollutant contraction

Table 4.2 presents the results of model  $(P_{INP})-(D_{INP})$  measuring eco-efficiency in terms of input and pollutant reduction. We consider both weak and strong disposability assumptions. The strong disposability assumption corresponds to a situation where undesirable outputs are freely disposable and a reduction in emission factors does not produce a corresponding contraction in good outputs. Under the weak disposability assumption, a reduction in undesirable factors is not possible without assuming a certain loss in terms of good output production (for instance, a regulation that could imply an emission control).

The mean values for world eco-efficiency in the cement sector under the hypothesis of strong and weak disposability were 0.91358 and 0.93458, respectively. This means that input utilization and emission factors can still be reduced by a proportion of 8.6% in the case of strong disposability and by 9.3% in the case of weak disposability. It is well known that efficiency levels under the weak disposability assumption are higher than the ones obtained using the strong disposability hypothesis. The difference in eco-efficiency between the strong and weak models is equal to 2%. This means that without regulation, an additional contraction of 2% of input factors (good input and pollutant) can be reached without reducing the corresponding good output. A Wilcoxon signed rank test was used to compare the eco-efficiency mean results under these two hypotheses. This methodology is a non-parametric test used to verify the hypothesis of no difference between the eco-efficiency score under the strong disposability assumption and the eco-efficiency score under weak disposability. The test was performed with R software . The value of the Wilcoxon statistic was 3.52 with a two-tailed p-value equal to 0.00015. The hypothesis of no difference among efficiency scores under strong and weak disposability assumptions was rejected at the 1% confidence level. In other words, the assumption of weak or strong disposability significantly influenced the efficiency measure.

Looking at the annual means, a progressive decrease in eco-efficiency was measured in the case of strong disposability. Countries without strong or mandatory environmental regulation (like USA, Turkey, Brazil, Canada) were the worst performing during the considered period and had a negative trend, except for China and India, which will be analyzed in more detail in the following section. European countries under the EU-ETS regulation maintained an average efficiency level that was nearly constant during the four years. The best performing countries were China, Denmark, India, Japan and Switzerland, and five other countries had an eco-efficiency value greater than 96% (Austria, Belgium, Canada, Italy, Spain).

Considering the European countries, Switzerland's efficiency can be attributed to a massive use of alternative fuels that, on average, amounts to 45% of total fuel consumption. The combination of these policies leads to a lower emission factor per ton of cement produced. Denmark made significant investments in environmental improvements and energy optimization between 2005-2008, which lead to a progressive decrease in the CO<sub>2</sub> emission factor and energy utilization (see Aalborg Portland Environmental Report 2009).

The Japanese cement industry has been involved in the Voluntary Emissions Trading Scheme since 2005. The technology used is dry in 90% of plants since 2000, and a progressive substitution of traditional fuels with alternative ones has been operated by the Japan Cement Association. In addition, the proportion of alternative raw materials used in cement production has constantly increased.

Among the remaining efficient countries, India performed well because of the progressive abandonment of wet technologies in favor of less energy expensive dry processes based on five and six stages with pre-heating and pre-calcination kilns. Note also that the major Indian companies

agreed with the Cement Sustainability Initiative (CSI) launched by the World Business Council for Sustainable Development (WBCSD).

The case of the China cement industry is more controversial. On one hand, the recent development of the Chinese economy has led to huge investments in new plants with the best available technologies and has focused industry production on low quality cement, which requires a lower amount of clinker than Portland cement and reduces energy consumption. For these reasons, the emission factor for CO<sub>2</sub> emissions and cement production is one of the best performing among the considered countries. On the other hand, the analysis of the Chinese cement sector suffers because of difficulties with data finding. Only 5% of Chinese Cement companies agree with the CSI of the WBCSD, and the data available for the National Cement Association only refers to the larger companies. It is very difficult to take an exact look at the sector; therefore, our results may be affected by data uncertainty.

Table 4.2: Eco-efficiency measure as input and pollutant contraction

Country	2005		2006		2007		2008		Mean	
	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak
Australia	1	1	0.82478	0.82554	0.83482	0.83482	0.84703	0.84703	0.87666	0.87685
Austria	1	1	0.93766	0.95959	0.96267	0.99578	0.97907	1	0.96985	0.98884
Belgium	1	1	1	1	0.96747	0.98484	0.97881	0.97881	0.98657	0.99091
Brazil	0.88562	0.88562	0.83374	0.83374	0.82356	0.82356	0.82694	0.82694	0.84247	0.84247
Canada	0.99654	1	1	1	0.98914	0.9915	0.93381	0.93501	0.97987	0.98163
China	1	1	1	1	1	1	1	1	1	1
Czech Republic	1	1	0.84239	0.84239	0.86684	0.90213	0.87742	0.88935	0.89666	0.90847
Denmark	1	1	1	1	1	1	0.97882	0.98152	0.99471	0.99538
Estonia	0.82765	1	0.85362	1	0.7624	1	0.65617	1	0.77496	1
France	0.918	0.918	0.8896	0.8896	0.88939	0.88939	0.88509	0.88509	0.89552	0.89552
Germany	0.8887	0.8887	0.88121	0.88121	0.8606	0.86077	0.86754	0.86754	0.87451	0.87456
India	1	1	1	1	1	1	0.98695	1	0.99674	1
Italy	1	1	1	1	0.9923	0.99669	0.95976	0.96175	0.98802	0.98961
Japan	1	1	0.99611	0.99645	1	1	0.97442	0.97725	0.99263	0.99343
Norway	0.85751	0.87328	0.81787	0.83404	0.85053	0.86664	0.86139	0.87147	0.84683	0.86136
Poland	0.85297	0.85297	0.74777	0.80857	0.7489	0.82494	0.81705	0.81871	0.79167	0.82630
Spain	1	1	1	1	1	1	0.94827	0.94827	0.98707	0.98707
Switzerland	1	1	1	1	0.99965	0.99965	1	1	0.99991	0.99991
Turkey	0.80229	0.84793	0.78781	0.82635	0.7701	0.7701	0.75885	0.75885	0.77976	0.80081
U.S.A	0.9169	1	0.88714	0.96493	0.88856	0.96407	0.85515	0.90059	0.88694	0.95740
United Kingdom	0.86022	0.87471	0.82553	0.84497	0.8322	0.8475	0.77723	0.85536	0.82380	0.85564
Mean	0.94316	0.95911	0.91073	0.92892	0.90663	0.93107	0.89380	0.91922	0.91358	0.93458
Improvement	0.01594		0.01820		0.02444		0.02542		0.0210	

#### 4.4.2 Eco-efficiency measure as pollutant contraction

In the previous section, cement industries improved their efficiencies by either moderating the use of traditional inputs (fuel and raw materials) or minimizing their CO<sub>2</sub> emission level. With model  $(P_{KL})-(D_{KL})$ , eco-efficiency is measured in terms of pollutant contraction only and the results are collected in Table 4.3. In this case, the means of eco-efficiency taken by year were lower and varied between 0.69 (in 2005) and 0.60 (in 2008). This implies that focusing only on emissions results in greater potential to improve the current status of cement technologies in the different countries.

Considering eco-efficiency mean values by country, Table 4.3 shows that China, India, Japan, Denmark and Switzerland remain the most efficient countries, followed by Spain, Italy and Belgium, with an efficiency rate greater than 90%. This confirms the results of the previous eco-efficiency analysis. In particular, the Italian and Spanish cement industries show a similar behavior. In the period 2005-2007, which coincided with the first phase of the EU-ETS, their efficiency levels were equal to or slightly lower than one, but these levels fall in 2008 with the inception of the second and more stringent EU-ETS phase. Belgium, apart from 2007, was efficient as a result of its effort to progressively reduce CO<sub>2</sub> emissions and improve energy efficiency since 2005.<sup>24</sup>

Table 4.3: Eco-efficiency measure as pollutant contraction

Country	2005	2006	2007	2008	Mean
Australia	1	0.33302	0.40802	0.46142	0.55062
Austria	1	0.56387	0.61639	0.72356	0.72596
Belgium	1	1	0.8761	0.92221	0.94958
Brazil	0.31298	0.3277	0.38556	0.53459	0.39021
Canada	0.72471	1	0.86006	0.57942	0.79105
China	1	1	1	1	1.00000
Czech Republic	1	0.26051	0.29932	0.33776	0.47440
Denmark	1	1	1	0.95593	0.98898
Estonia	0.15611	0.28768	0.30787	0.14854	0.22505
France	0.45618	0.26915	0.26384	0.28034	0.31738
Germany	0.23397	0.37931	0.38406	0.41761	0.35374
India	1	1	1	0.90682	0.97671
Italy	1	1	0.96277	0.8378	0.95014
Japan	1	0.99344	1	0.96423	0.98942
Norway	0.20207	0.1936	0.31251	0.28671	0.24872
Poland	0.59322	0.44637	0.55098	0.80581	0.59910
Spain	1	1	1	0.67496	0.91874
Switzerland	1	1	0.98755	1	0.99689
Turkey	0.37663	0.46219	0.5165	0.55245	0.47694
USA	0.24495	0.19857	0.1713	0.19025	0.20127
United Kingdom	0.20817	0.2036	0.23474	0.19657	0.21077
Mean	0.69090	0.61519	0.62560	0.60843	0.63503

<sup>24</sup>See the IEE (*indice d'amélioration de l'efficacité énergétique*) and IGES (*indice de réduction des émissions de CO<sub>2</sub> énergétique (combustibles)*) indices in the Report Febelcem 2009 at page 20, available at <http://www.febelcem.be/index.php?id=rappports-annuels>.

### 4.4.3 Eco-efficiency as input contraction

In the  $(P_{TR\beta})-(D_{TR\beta})$  DEA approach introduced in Section 4.2.2, we measured eco-efficiency as the ability of cement industries to reduce input factors (traditional raw materials and fuels) without increasing CO<sub>2</sub> emissions or cement production curtailment. Traditional raw materials can be reduced by substituting them with alternative materials in the cement production process. This implies a reduction in the clinker to cement ratio. Results of this model are reported in Table 4.4. The mean eco-efficiency levels were equal to 0.91342 in the case of a strong disposability assumption and 0.95240 in the case of a weak disposability assumption. When emission factors are freely disposable, an additional 3.8% in the input contraction can be obtained without reducing the corresponding outputs. The eco-efficiency mean values significantly differed when comparing the results under these two assumptions. The value of the Wilcoxon statistic, equalled 3.92 with a two-tailed p-value equal to 0.000096. The hypothesis of no difference among efficiency scores under the strong and weak disposability assumptions was rejected at a confidence level of 1%.

In the case of a strong disposability assumption, eco-efficiency mean levels were in line with those of Table 4.2. China, India, Japan, Denmark and Switzerland remained the top five in terms of efficiency and Austria, Belgium, Canada, Italy and Spain reached an efficiency level greater than 98%. By comparing the results of Tables 4.2, 4.3 and 4.4, we can argue that Austria and Canada's efficiency levels reported in Table 4.2 are mainly related to the utilization of alternative raw materials because the eco-efficiency levels based only on emission contraction (Table 4.3) were significantly lower. Concerning the Spanish cement industry, it has doubled the utilization of alternative fuels and raw materials in the last decades. In 2008, alternative fuels accounted for 15% of the total, while alternative raw materials were 10% of total use. This environmental policy was reflected in both eco-efficiency measures provided in Tables 4.3 and 4.4. A similar reasoning can be applied to Belgium and Italy, as discussed in the previous section.

Table 4.4: Eco-efficiency as input contraction

Country	2005		2006		2007		2008		Mean	
	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak
Australia	1	1	0.82478	0.82829	0.83463	0.83546	0.84389	0.98398	0.87583	0.91193
Austria	1	1	0.96449	0.96449	1	1	0.97907	1	0.98589	0.99112
Belgium	1	1	1	1	0.96747	0.96803	0.97095	1	0.984605	0.992008
Brazil	0.87641	0.89829	0.82956	0.83939	0.82303	0.82303	0.826	0.94549	0.83875	0.87655
Canada	0.99654	1	1	1	0.99617	0.99617	0.93381	1	0.98163	0.99904
China	1	1	1	1	1	1	1	1	1	1
Czech Republic	1	1	0.84111	0.84111	0.86705	0.86705	0.87742	0.98347	0.89639	0.92291
Denmark	1	1	1	1	1	1	0.97882	1	0.99471	1
Estonia	0.82765	1	0.85362	0.97389	0.7624	1	0.65617	1	0.77496	0.99347
France	0.91359	0.92332	0.89679	0.89679	0.9048	0.9048	0.88217	0.99852	0.89934	0.93086
Germany	0.88137	0.89653	0.87763	0.87763	0.8645	0.8645	0.86288	0.95544	0.87159	0.89853
India	1	1	1	1	1	1	0.98695	1	0.99674	1
Italy	1	1	1	1	1	1	0.95976	1	0.98994	1
Japan	1	1	1	1	1	1	0.97442	1	0.99361	1
Norway	0.85751	0.9113	0.81787	0.83015	0.85053	0.85188	0.86139	0.98102	0.84683	0.89359
Poland	0.84992	0.9139	0.74777	0.84933	0.7489	0.83451	0.81705	0.96038	0.79091	0.88953
Spain	1	1	1	1	1	1	0.94795	1	0.98699	1
Switzerland	1	1	1	1	1	1	0.9304	0.93147	0.9826	0.98287
Turkey	0.80229	0.92308	0.78781	0.84155	0.7701	0.78395	0.75867	0.91883	0.77972	0.86685
U.S.A	0.9169	1	0.88714	0.92442	0.88856	0.89547	0.85515	1	0.88694	0.95497
United Kingdom	0.86022	0.90948	0.82553	0.84786	0.83227	0.83227	0.77723	0.99537	0.82381	0.89624
Mean	0.94202	0.97029	0.9121	0.92928	0.91002	0.92653	0.88953	0.98352	0.91342	0.95240
Improvement	0.02826		0.01718		0.01651		0.09399		0.03899	

#### 4.4.4 Eco-efficiency as undesirable output contraction and desirable output expansion

The results of models  $(P_{INP})-(D_{INP})$  and  $(P_{KL})-(D_{KL})$  presented in Sections 4.4.1 and 4.4.2 were based on the assumption that undesirable factors (CO<sub>2</sub> emissions) were treated as an input to the production process, and the eco-efficiency measured the reduction of CO<sub>2</sub> emissions without changing the desirable output levels (cement production). The results of the directional distance function approach (model  $(P_{DDF})$ ) are presented in Table 4.5 and provides an alternative eco-efficiency measure. This approach allows us to measure the potential reduction of the undesirable emission output and the potential expansion of the desirable output. We considered both weak and strong disposability assumptions. Strong disposability corresponds to a situation where good outputs can be arbitrarily expanded, while the weak disposability assumption limits their expansion according to a certain regulation (that could imply an emission control). The difference between efficiency levels under weak and strong disposability in the directional distance approach can be interpreted as the cost of regulation with respect to the emission factors (see [40]), which we denote as “Normative Price” in Table 4.5.

In Table 4.5, the mean eco-efficiency under the hypothesis of strong disposability is equal to 0.09983, which means that desirable outputs could still be increased by approximately 10%. In the case of a weak disposability assumption, this percentage is 9%. This means that, in the presence of normative constraints, cement industries have a more limited production expansion capability. In the cases of both weak and strong disposability assumptions, the eco-efficiency slightly decreased over time. Considering the mean eco-efficiency levels in 2008, the values for the strong and weak disposability approaches were very similar, while in 2005-2007, their discrepancy was more evident. These results are justified as follows. In 2008, only India showed a significant difference between strong and weak disposability eco-efficiencies, whereas in the other countries, the two directional distance function approaches provided identical results. This can be explained by the fact that the cement market (apart from developing countries, especially India, whose consumption level increased 7.5% in 2008 with respect to the previous year) was suffering from a decrease in cement consumption due to the global financial crisis. However, globally, the differences in eco-efficiency measures under the hypothesis of weak and strong disposability assumptions were not statistically significant according to this model formulation (Wilcoxon test statistics equal to 1.83). The difference between weak and strong disposability assumptions in the Direction Distance formulation was almost nil. This can be attributed to the effects of existing environmental regulations during the period 2005-2008 that were adopted to reduce greenhouse gas emissions through the abatement of old technologies and investment in more-efficient ones. Belgium reached eco-efficiency levels comparable to the top five countries shown in all models as most efficient, namely, China, Denmark, India, Japan and Switzerland. Apart from these countries, Austria, in model  $(P_{DDF})$ , lost the efficiencies presented in models  $(P_{INP})-(D_{INP})$  and  $(P_{TR\beta})-(D_{TR\beta})$ . A deeper analysis of the cement industries operating in this country revealed that the CO<sub>2</sub> emissions rate (CO<sub>2</sub>/cement) was higher with respect to the national average in 2006 and 2007. This is strictly related to the increase in the clinker to cement ratio in the same years (see the references for Austria in Appendix B). The Spanish, Italian and Canadian cement industries confirm the results of the previous models.

Table 4.5: Eco-efficiency as undesirable output contraction and desirable output expansion

Country	2005		2006		2007		2008		Mean	
	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak
Australia	0	0	0.20043	0.20043	0.18364	0.18364	0.16194	0.16194	0.13650	0.13650
Austria	0	0	0.21615	0.21615	0.14425	0.14425	0.10284	0.10284	0.11581	0.11581
Belgium	0	0	0	0	0.02746	0.02741	0.00857	0.00857	0.00901	0.00900
Brazil	0.12000	0.12000	0.17361	0.17361	0.18885	0.18885	0.18107	0.18107	0.16588	0.16588
Canada	0.00471	0	0	0	0.01145	0.01145	0.08243	0.08243	0.02465	0.02347
China	0	0	0	0	0	0	0	0	0	0
Czech Republic	0	0	0.16462	0.16462	0.15227	0.15227	0.13286	0.13286	0.11244	0.11244
Denmark	0	0	0	0	0	0	0.01315	0.01315	0.00329	0.00329
Estonia	0.21799	0	0.18071	0	0.21929	0	0.39763	0.39763	0.25391	0.09941
France	0.11212	0.11212	0.16398	0.16398	0.17336	0.17336	0.16619	0.16619	0.15391	0.15391
Germany	0.11468	0.11468	0.12500	0.12500	0.15117	0.15117	0.13200	0.13200	0.13071	0.13071
India	0	0	0	0	0	0	0.01560	0	0.00390	0
Italy	0	0	0	0	0.00861	0.00828	0.04127	0.04169	0.01247	0.01249
Japan	0	0	0.00241	0.00241	0	0	0.01344	0.01344	0.00396	0.00396
Norway	0.15773	0.15773	0.23148	0.23148	0.21332	0.21332	0.18372	0.18372	0.19656	0.19656
Poland	0.14250	0.14250	0.24280	0.24280	0.19207	0.19207	0.06618	0.06618	0.16089	0.16089
Spain	0	0	0	0	0	0	0.05349	0.05349	0.01337	0.01337
Switzerland	0	0	0	0	0.00051	0.00051	0	0	0.00013	0.00013
Turkey	0.23533	0.23533	0.22851	0.22851	0.20310	0.20310	0.18780	0.18780	0.21369	0.21369
USA	0.12461	0	0.18233	0.14516	0.18405	0.14510	0.24317	0.24317	0.18354	0.13336
United Kingdom	0.15262	0.15262	0.19205	0.19205	0.18757	0.18757	0.27518	0.27518	0.20186	0.20186
Mean	0.06582	0.04928	0.10972	0.09934	0.10671	0.09440	0.11707	0.11635	0.09983	0.08984
Normative Price	0.01654		0.01038		0.01232		0.00072		0.00999	



## 4.5 Conclusions

In this chapter, we present a cross-country comparison of the *eco-efficiency* level of the worldwide cement industry. By adopting a DEA approach, this paper attempts to evaluate the impact of environmental regulations on cement industry efficiency by considering a joint production framework of both desirable and undesirable outputs. This work differs from previous literature because it compares 21 countries covering 90% of the world's cement production. Traditional industrialized countries are compared with emerging producers like India and China.

With *eco-efficiency*, we indicate the possibility of producing goods (or services) by reducing the quantity of energy and resources employed and/or the amount of waste and emissions generated. We provide different eco-efficiency measures by applying three DEA approaches, where emissions can be either considered as inputs or undesirable outputs. Moreover, a directional distance function approach has been used to evaluate the ability of a country to simultaneously expand the desirable output and contract the CO<sub>2</sub> emissions by the same proportion without increasing the inputs. Strong and weak disposability assumptions are analyzed in order to evaluate the impact of environmental regulations interpreted as the cost of regulation.

Countries without strong or mandatory environmental regulations (like the USA, Turkey, Brazil, and Canada) were the worst-performing during the considered period and had a negative trend, except for China and India. European countries under the EU-ETS regulation kept a nearly constant efficiency level during the four years. Our analysis has shown that the efficiency level mainly depends on decisions to invest in alternative raw materials and alternative fuels, both in the case of regulated countries and in the case of voluntary emission-trading schemes.

Emerging countries that have been increasing their cement production in recent years, like China and India, show high efficiency levels. This feature can be attributed to two different factors: investments in more-efficient technologies (progressive substitution of small wet process plants with bigger and dry technology based ones) and the production of low-quality cements, which require less proportion of clinker, component which is the main responsible for CO<sub>2</sub> emissions.

India, particularly, performs well because of the progressive abandonment of wet technologies in favor of less energy expensive dry processes based on five and six stages with pre-heating and pre-calcination kilns. Note also that the major Indian companies agree with the Cement Sustainability Initiative (CSI) launched by the World Business Council for Sustainable Development (WBCSD).

However, in the case of the Chinese cement industry, the partial availability of data (mainly concerning big plants with efficient technology processes) may have affected the analysis. In addition, the construction of CO<sub>2</sub> emissions indirectly may not be a true representation of actual CO<sub>2</sub> emission.

This study highlights, both at national and international levels, the possibility of reducing CO<sub>2</sub> emissions and expanding cement production. The use of alternative raw materials, alternative fuels and the possibility of producing blended cements, which require less energy consumption and reduce pollutant emissions, seem to be appropriate means. Environmental regulations can provide incentives in terms of tax exemption benefits or more restrictive pollutant limits.

On one hand the comparison among 21 cement producing countries based on several DEA models and a directional distance function approach provides a comprehensive outlook on world cement industry efficiency. On the other hand it is worth pointing out that DEA is a tool that measures the efficiency scores of DMUs operating in similar business environments using similar technologies. In our paper, we have compared both European and non-European countries that differ from a geographical and economic point of view. Moreover, the countries included in our dataset have also shown a dissimilar attitude with respect to environmental issues. Despite these limitations, the study of prototypes of cement production plants highlights the potential for reducing CO<sub>2</sub> emissions based on the comparison of performance results and environmental policies among the 21 countries.

A further step of our analysis will be the enlargement of the actual dataset with the inclusion of

additional undesirable greenhouse gases, like  $\text{NO}_x$  and  $\text{SO}_2$ , as production factors and the extension of the number of considered countries.

## Chapter 5

# Evaluating the efficiency of the cement sector in presence of undesirable output: a world based Data Envelopment Analysis

The aim of this chapter is to evaluate the importance of emission regulation on efficiency for twenty-one worldwide countries within a production framework of both desirable and undesirable factors. In order to expand the study of the previous chapter, we try to answer to the following questions: do undesirable factors modify the efficiency levels of cement industry? Is it reasonable to omit CO<sub>2</sub> emissions in evaluating the performances of the cement sector in different countries? In order to answer to these questions, in this chapter alternative formulations of standard Data Envelopment Analysis model and directional distance function are compared both in presence and in absence of undesirable factors. The results, collected in [53] show that efficiency levels vary if we include or omit undesirable factors and that efficiency levels are influenced by investments in best available technologies and by the utilization of alternative fuels and raw materials in cement and clinker production processes.

### 5.1 Introduction

Used in building and in civil engineering constructions, cement is at the basis of the economic development of a country. Starting from a figure of 594 million tons in 1970, the worldwide production of cement is quadruplicated in the last twenty-five years, reaching an amount of 2,284 million tons in 2005 (see [59]). In 2009, despite the global economic crisis, the worldwide production of cement increased by 6.4% towards 2008 up to 3 million tons (see Cembureau [10]). This growth was mainly due to China and India, the two largest cement producers in the world<sup>1</sup>. Other leading countries in cement production are the USA, Japan and Turkey. Even European countries, like Italy, Spain, Germany, France have a significant cement production that contributes to the cement global demand. Cement is the result of a long production process beginning with the extraction of specific raw materials from quarries, continuing with the intermediate production of clinker in specific kilns and concluding with the grinding of clinker with additives whose quantity varies according to the

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<sup>1</sup>These countries respectively produced 1,637 and 193 million tons of cement in 2009 (see Cembureau [10])

type of cement produced<sup>2</sup>. The efficiency of the cement manufacturing process depends on clinker production. Clinker production is the core of cement manufacturing, but it is also a very energy and intensive production phase in terms of ( $\text{NO}_x$ ,  $\text{SO}_2$ ,  $\text{CO}_2$ ) emissions.

Considering that  $\text{CO}_2$  and the other greenhouse gas emissions are the undesirable but unavoidable outputs of the cement production process, these can be curbed only by reducing the ratio clinker to cement<sup>3</sup>, increasing the utilization of alternative fuel and raw materials and improving technology energy efficiency. The application of these policies could have a positive impact on environment. This has also an economic effect for those companies operating in countries that price  $\text{CO}_2$  through a market of emission permits.

Considering the environmental and the economic roles assumed by  $\text{CO}_2$  emissions we intend to measure the efficiency of cement production at worldwide level, defying efficiency as the ability of producing a certain good by saving energy and resources and/or reducing waste and emissions.

In this chapter, we compare prototypes of cement industries operating around the world. In particular, we evaluate the efficiency level of twenty-one world countries in terms of cement and clinker production. This set of countries includes the world major producers (China, India, the USA, Japan and Turkey), almost all European producers in addition to Canada, Australia and Brazil. Taking  $\text{CO}_2$  emissions as undesirable factor, we compare the results of a standard DEA model and a directional distance function approach. In order to evaluate the importance of emission regulation on efficiency we analyze these two classes of models with and without undesirable factors. The results show that the average efficiency measures obtained by the models including both desirable and undesirable factors significantly differ from those obtained omitting  $\text{CO}_2$  emissions. When incorporating undesirable factors, the efficiency of both the whole cement production process and the clinker sub-process are explored. Finally, the models have been tested under the hypothesis of weak and strong disposability in order to evaluate the cost arising from environmental regulations.

This chapter is organized as follows. Section 5.2 presents a literature review and illustrate the model that we adopt in our analysis. Section 5.3 illustrates the simulation results. Final remarks are reported in Section 5.4.

## 5.2 Model specification

In this section we try to answer to the following questions: do undesirable factors modify the efficiency levels of cement industry? Is it reasonable to omit  $\text{CO}_2$  emissions in evaluating the performances of the cement sector in different countries? In order to evaluate these sentences, the standard DEA formulation is compared with alternative formulations which include undesirable factors. Two different approaches can be followed: including undesirable factors as inputs or, according to the production process, considering undesirable factors as undesirable output.

When undesirable factors are taken into consideration, the choice between two alternative disposable technologies (improved technologies or reference technologies) has an important impact on DMUs efficiencies. Technology disposability can also be read in terms of strong and weak disposability of undesirable outputs. A production process is said to exhibit strong disposability of undesirable outputs (such as heavy metals,  $\text{CO}_2$ , etc.), if the undesirable outputs are freely disposable, i.e. they do not have limits. The case of weak disposability refers to situations where a reduction in waste or emissions forces a lower production of desirable outputs. In other words, in order to meet some emission limits (for instance because of regulatory constraints), a reduction of undesirable outputs may not be possible without assuming certain costs (see Zofío and Prieto, [67]).

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<sup>2</sup>According to Ponnasard and Walker [47] one ton of cement is typically composed for the 80% of clinker and for the remaining 20% of other materials. For more details see [24].

<sup>3</sup>Namely, the amount of clinker needed to produce one ton of cement.

There are different approaches to incorporate undesirable pollutants in DEA models. Considering the models presented in the previous chapter, in this section, a classical DEA model with undesirable factor treated as inputs and a directional distance function model where CO<sub>2</sub> emission are undesirable output have been considered. For the sake of convenience, the considered models are provided below.

### 5.2.1 A first model comparison: standard BCC DEA model and undesirable factors treated as inputs

The first DEA model we consider is the standard BCC model following the lines of Banker et al. [5]. The original formulation of this model does not comply the presence of undesirable output. According to this model, in the input oriented version, DMU efficiency is defined as the ability to contract the amount of inputs without reducing the corresponding output volumes. The mathematical formulation of the model *M1* is as follows:

$$(M1) \quad \min_{\theta, \lambda} \theta \quad (5.1)$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \quad (5.2)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (5.3)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (5.4)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n,$$

where variables  $\theta$  and  $\lambda_j$  are respectively defined as follows:

$\theta \in \mathbb{R}_+$ : radial efficiency measure;

$\lambda_j \in \mathbb{R}_+$ : intensity factor associated to each DMU,  
 $j = 1, \dots, n$ .

A first approach, where both desirable and undesirable factors are considered, suggests to include undesirable outputs as desirable inputs in the production process (see [38]). Its starting point is that efficient DMUs wish to minimize desirable inputs and undesirable outputs while maximizing desirable outputs and undesirable inputs. The mathematical formulation of the model, in case of strong disposability and input oriented DEA, is as follows:

$$(M2) \quad \min_{\theta, \lambda} \theta \quad (5.5)$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \quad (5.6)$$

$$\sum_{j=1}^n \lambda_j y_{kj}^b \leq \theta y_{kj_0}^b, \quad k = 1, \dots, l, \quad (5.7)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (5.8)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (5.9)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

Considering our analysis where CO<sub>2</sub> emissions are the undesirable factor of cement/clinker production, the comparison between models *M1* and *M2* provides information on the effects of considering CO<sub>2</sub> emissions as an input of the production process. Model *M2*, in facts, attempts to proportionately contract both desirable inputs and the CO<sub>2</sub> undesirable output. The differences in efficiency between models *M1* and *M2* can be interpreted as a greater or lower efficiency in absence of environmental regulation.

Since the prominent interest of national authorities is to curb CO<sub>2</sub> emissions by environmental regulation, constraints (5.7) in model *M2* can be slightly modified in order to assume weak disposability of undesirable input. In this light the efficiency measure can evaluate the ability to contract all inputs of the production process without increasing the amount of CO<sub>2</sub> emitted.

Under weak disposability assumption model *M2* is then modified as follows:

$$(M2weak) \quad \min_{\theta, \lambda} \theta \quad (5.10)$$

$$s.t. \quad \sum_{j=1}^n \lambda_j y_{rj}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \quad (5.11)$$

$$\sum_{j=1}^n \lambda_j y_{kj}^b = \theta y_{kj_0}^b, \quad k = 1, \dots, l, \quad (5.12)$$

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_0}, \quad i = 1, \dots, m, \quad (5.13)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (5.14)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

The comparison between models *M1* and *M2weak* can highlight the importance of considering CO<sub>2</sub> emissions in efficiency assessment in presence of environmental regulations which curb CO<sub>2</sub> emissions. The analysis of weak and strong disposability assumptions gives information on the use

of alternative fuels and raw materials in order to reduce CO<sub>2</sub> levels maintaining the same efficiency levels and output quantities. For an exhaustive discussion on strong and weak disposability in these models see Liu et al. [37].

### 5.2.2 A second model comparison: the directional distance function approach and undesirable factors treated as output

The directional output distance function, in its original formulations by Färe et al. [28], is an alternative approach to evaluate efficiency. This approach evaluates efficiency as the ability of simultaneously expanding desirable outputs and contracting inputs.

Let  $T$  be the technology set, such that:

$$T = \{(x, y^g) : x \text{ can produce } y^g\}, \quad (5.15)$$

the directional technology distance function generalizes both input and output Shephard's distance functions, providing a complete representation of the production technology.

Let  $d = (-d^x, d^g)$  be a direction vector, the function is formally defined as:

$$\vec{D}_T(x, y^g; d) = \sup \{\delta : (x - \delta d^x, y^g + \delta d^g) \in T\}. \quad (5.16)$$

Expression (5.16) seeks for the maximum attainable expansion of desirable outputs in the  $d^g$  direction and the largest feasible contraction of inputs in  $d^x$  directions. Under the assumptions made on the reference technology, the directional technology distance function of expression (5.16) can be computed for firm  $j_0$  by solving the following linear programming problem:

(M3)

$$\begin{aligned} \max_{\delta, \lambda} \quad & \delta \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j y_{rj}^g - \delta d_{rj_0}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \end{aligned} \quad (5.17)$$

$$\sum_{j=1}^n \lambda_j x_{ij} + \delta d_{ij_0}^x \leq x_{ij_0}, \quad i = 1, \dots, m, \quad (5.18)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

Notice that in the directional distance function model, efficiency is reached when  $\delta = 0$ , corresponding to the case of  $\theta = 1$  in the standard DEA formulations.

The choice of a direction vector  $d = (-x, y^g)$  permits to evaluate a global technology by reducing inputs and simultaneously expanding desirable outputs. A different direction vector can be used in order to restrict the analysis on output factors, by considering, for instance, a direction vector  $d = (0, y^g)$ .

Extensions of this methodology in presence of undesirable outputs (see for all [25, 26, 46]) leads to a measure of technical efficiency from the potential for increasing outputs while reducing inputs and undesirable outputs simultaneously. The corresponding model, according to Section 4.2.2, results to be:

(M4)

$$\begin{aligned} & \max_{\delta, \lambda} \quad \delta \\ & \text{s.t.} \quad \sum_{j=1}^n \lambda_j y_{rj}^g - \delta d_{rj_0}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \end{aligned} \quad (5.19)$$

$$\sum_{j=1}^n \lambda_j y_{kj}^b + \delta d_{kj_0}^b \leq y_{kj_0}^b, \quad k = 1, \dots, l, \quad (5.20)$$

$$\sum_{j=1}^n \lambda_j x_{ij} + \delta d_{ij_0}^x \leq x_{ij_0}, \quad i = 1, \dots, m, \quad (5.21)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

Similarly to model *M3*, the choice of a direction vector  $d = (0, y^g, -y^b)$  focus the attention on expansion of desirable factors and contraction of undesirable ones without increasing the inputs (see Mandal and Madheswaran [40] for a such approach).

Let us finally recall that this model can be also considered under the assumption of weak disposability by assuming that constraints (5.20) hold with equality. The corresponding weak disposability formulation is as follows:

(M4weak)

$$\begin{aligned} & \max_{\delta, \lambda} \quad \delta \\ & \text{s.t.} \quad \sum_{j=1}^n \lambda_j y_{rj}^g - \delta d_{rj_0}^g \geq y_{rj_0}^g, \quad r = 1, \dots, q, \end{aligned} \quad (5.22)$$

$$\sum_{j=1}^n \lambda_j y_{kj}^b + \delta d_{kj_0}^b = y_{kj_0}^b, \quad k = 1, \dots, l, \quad (5.23)$$

$$\sum_{j=1}^n \lambda_j x_{ij} + \delta d_{ij_0}^x \leq x_{ij_0}, \quad i = 1, \dots, m, \quad (5.24)$$

$$\sum_{j=1}^n \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n.$$

### 5.2.3 Construction of the production frontier

It is well known that, in the DEA literature, three types of frontiers have been proposed to evaluate efficiency in a panel-data framework: Contemporaneous Frontier, Intertemporal Frontier and Sequential Frontier. According to previous chapter, for the aim of this study, the Sequential Frontier seems to be the most suitable for the analysis of the world cement sector in the years 2005-2008<sup>4</sup>.

<sup>4</sup>However, tests were carried out also considering Contemporaneous Frontier. See Appendix A for the complete collection of all simulations.



In these years, in facts, the world cement industry has faced a rapid expansion and technological improvement (especially in developing countries). Let us recall that, for classical DEA applications with sequential frontier, the production possibility set is defined as

$$T_V = \left\{ (x, y) : x \geq \sum_{j=1}^n \sum_{t=1}^T \lambda_j x_j^t; \quad y \leq \sum_{j=1}^n \sum_{t=1}^T \lambda_j y_j^t; \quad \sum_{j=1}^n \lambda_j = 1 \right\},$$

where there are  $n$  units observed and  $t$  corresponds to the time period at which the DMU is being evaluated<sup>5</sup>.

### 5.3 Empirical Results

For the purpose of our analysis, the database concerning the cement production of twenty-one world countries presented in Section 4.3 has been used. However, in this particular study, the choice of input and output factors of DEA models has been done taking into account the cement and clinker production processes. In this light, for all the models presented in Section 5.2 two different instances have been considered: in the first instance (CEM) we focus on the entire cement production process, in the second one (CLK) we limit our attention on clinker. For this reason, in CEM we take clinker, installed cement capacity, energy, labour and raw materials as inputs. Desirable output is represented by cement production while CO<sub>2</sub> emissions are the undesirable by-product. More specifically, clinker includes both imports and local production, capital corresponds to installed cement capacity (a similar approach can be found in Fare et al. [29], Tyteca [61]), energy results from the sum of electricity and thermal energy, labour is the number of employees and finally, raw materials is defined as the sum of slug, limestone and all those additives needed in cement production.

Desirable output is Portland cement production. The undesirable by-product is measured by the value of carbon dioxide emissions (CO<sub>2</sub>) mainly resulting from the clinker production process without considering those related to raw material, fuels and clinker transportation. CO<sub>2</sub> can be interpreted as input or undesirable output according to the different DEA approaches.

The second set of computational tests (CLK) gives information on efficiency of the clinker production process which is the main responsible of CO<sub>2</sub> emissions. In order to evaluate the clinker process efficiency in the twenty-one countries, three input factors have been taken into account, namely energy and raw materials consumed in the clinker production and labour. Clinker production is the desirable output while CO<sub>2</sub> emissions are the undesirable one.

Since the aim of this work is to evaluate the efficiency of different countries, all input and output are divided by the total number of plants for each country in order to determine the efficiency of a representative plant within each country (a similar approach can be found in Mandal [39], Mandal and Madheswaran [40], Mukherjee [41], [42]).

Both the DEA model and the directional distance function described in Section 5.2 have been implemented in MatLab 2010a in order to evaluate the efficiency of the cement sector in presence or absence of undesirable factors.

Both CEM and CLK instances have been analyzed with and without undesirable factors. As a consequence, the relevance of including CO<sub>2</sub> emissions in the study of efficiency in countries involved in environmental regulations can be tested.

Tables 5.1, 5.2, 5.3 and 5.4 respectively report the results of computational test from models  $M1$ ,  $M2$  and  $M2weak$  applied to both CEM and CLK instances. We recall that model  $M1$  values the efficiency through input reductions and do not include undesirable by-products. The study of efficiency using  $M2$  and  $M2weak$  models gives information on the efficiency taking into account both input and CO<sub>2</sub> reductions in absence or presence of environmental regulations.

<sup>5</sup>The constraints for the DEA models presented in Section 5.2 are revised accordingly.

Tables 5.5, 5.6, 5.7 and 5.8 list the results obtained from models  $M3$ ,  $M4$  and  $M4weak$  based on the directional distance approach applied to both CEM and CLK instances. Model  $M3$  evaluates the expansion of desirable output without considering undesirable factors; models  $M4$  and  $M4weak$  simultaneously measure the undesirable output contraction and desirable output expansion in absence or presence of environmental regulations respectively.

### 5.3.1 Efficiency evaluation: standard BCC DEA model and undesirable factors treated as inputs

In this paragraph the results of instances CEM and CLK investigated with DEA models of Section 5.2.1 are presented. Tables 5.1 and 5.2 are related to CEM instance that considers the whole cement production process. Table 5.1 shows the cement production efficiency values based on the classical DEA model  $M1$  which considers only desirable outputs. The average world efficiency is equal to 0.93775, this implies that it would be possible to reduce input factors by a maximum amount of 6.23% and still produce the given level of output. However, the efficiency level varies among the considered countries. While Brazil and China remain efficient during the whole period under analysis, Denmark, India, and Spain reach an efficiency level close to 100%. The worst performing countries are Estonia, Turkey, U.S.A. Norway and United Kingdom. As concerning the country average, a progressive decline in efficiency can be observed: the average level of efficiency 0.96 in 2005 drops to 0.9273 in 2008 (3.27%).

Table 5.1: CEM instance: efficiency scores based on Standard BCC DEA model ( $M1$ )

Country	2005	2006	2007	2008	Annual average <sup>b</sup>
Australia	1	0.82653	0.83476	0.84843	0.87743
Austria	1	0.93766	0.96267	0.97907	0.96985
Belgium	1	1	0.96747	0.97095	0.98461
Brazil	1	1	1	1	1
Canada	0.99654	1	0.99125	0.94537	0.98329
China	1	1	1	1	1
Czech Republic	1	0.91053	0.92137	0.9233	0.9388
Denmark	1	1	1	0.99336	0.99834
Estonia	0.85171	0.85815	0.76254	0.65691	0.78233
France	0.91878	0.92693	0.90822	0.92649	0.92011
Germany	0.94205	0.97557	0.91795	0.96376	0.94983
India	1	1	1	0.99244	0.99811
Italy	1	1	0.99326	0.96179	0.98876
Japan	1	0.99769	1	0.98759	0.99632
Norway	0.87686	0.84397	0.86662	0.88002	0.86687
Poland	0.97027	0.80569	0.84129	0.99987	0.90428
Spain	1	1	1	0.97643	0.99411
Switzerland	0.94282	0.93559	0.93518	0.93602	0.9374
Turkey	0.87014	0.88991	0.86754	0.85285	0.87011
U.S.A.	0.93147	0.89994	0.89371	0.85589	0.89525
United Kingdom	0.86053	0.83083	0.83243	0.82379	0.8369
<b>Country average<sup>a</sup></b>	0.96006	0.93519	0.92839	0.92735	0.93775

<sup>a</sup> Country average is the average efficiency of the 21 countries for a given year.

<sup>b</sup> Annual average is the average efficiency for a given country over 4 years.

Table 5.2 presents the results of models  $M2$  and  $M2weak$  that include both desirable and

undesirable outputs. In particular, model *M2* assumes that no environmental regulation is applied while model *M2weak* assumes that all the 21 countries are subject to any environmental normative. While comparing the efficiency scores provided by model *M1* (Table 5.1) and model *M2weak* (Table 5.1), it can be seen that the average efficiency measure obtained by model *M2weak* is substantially higher than the one obtained from model *M1*. In order to verify whether omitting undesirable output can affect efficiency estimations, the Wilcoxon Signed Rank Test has been conducted. The null hypothesis is that the efficiency scores obtained from the two models belong to the same population of relative frequency distribution, whereas alternative hypothesis is that the mean efficiency value obtained with model *M1* significantly differs from the one obtained with *M2weak* model. The value of Wilcoxon statistics is 3.82 and the value of two tailed “*p*” statistic is lower than 0.0001. Then, the null hypothesis can be rejected at 1% level, implying that omitting CO<sub>2</sub> emissions (undesirable output) results in biased efficiency estimates.

In Table 5.1 efficiency scores of models *M2* and *M2weak* are directly compared. It is worth noticing that mean efficiency scores under the weak disposability assumption are strictly higher than the ones related to the strong disposability assumption and this difference increases over years. Four countries show efficiency scores equal to one, namely China, Brazil, Estonia and India. Switzerland, Japan and Denmark are very close to 100% efficiency. This implies that average efficiency in presence of environmental regulation is higher than that obtained in absence of it. In other words without regulation an additional contraction of 2% of input factors (good input and pollutant) can be reached without reducing the corresponding good output. The Wilcoxon Signed Rank Test has been used to compare the efficiency mean results under these two hypothesis. The value of Wilcoxon statistic is 3.18 with a two tailed p-value equal to 0.0016. The hypothesis of no difference among efficiency scores under strong and weak disposability assumption can be rejected at 1% confidence level, namely the assumption of weak or strong disposability significantly influences the efficiency measure. In this case environmental regulations have a double effect: to curb CO<sub>2</sub> emissions and to improve the good input contraction by the use of alternative fuels and raw materials.

Table 5.2: CEM instance: efficiency scores based on models  $M2$  and  $M2weak$ 

Country	2005		2006		2007		2008		Annual average	
	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak
Australia	1	1	0.8271	0.8271	0.83493	0.83493	0.85088	0.85088	0.87823	0.87823
Austria	1	1	0.93766	0.96493	0.96267	1	0.97907	1	0.96985	0.99123
Belgium	1	1	1	1	0.96747	0.98484	0.97881	0.97881	0.98657	0.99091
Brazil	1	1	1	1	1	1	1	1	1	1
Canada	0.99654	1	1	1	0.99125	0.99784	0.94537	0.98463	0.98329	0.99562
China	1	1	1	1	1	1	1	1	1	1
Czech Republic	1	1	0.91053	0.91245	0.92137	0.94315	0.9233	0.93387	0.9388	0.94737
Denmark	1	1	1	1	1	1	0.99402	0.99402	0.9985	0.9985
Estonia	0.85171	1	0.85815	1	0.76254	1	0.65691	1	0.78233	1
France	0.92246	0.92246	0.9314	0.9314	0.91293	0.91293	0.93162	0.93162	0.9246	0.9246
Germany	0.94556	0.94556	0.98204	0.98204	0.92096	0.92096	0.97159	0.97159	0.95504	0.95504
India	1	1	1	1	1	1	0.99244	1	0.99811	1
Italy	1	1	1	1	0.99326	1	0.96179	0.96179	0.98876	0.99045
Japan	1	1	0.99769	1	1	1	0.98759	0.98845	0.99632	0.99711
Norway	0.87686	0.89016	0.84397	0.85725	0.86662	0.87923	0.88002	0.88543	0.86687	0.87802
Poland	0.97027	0.97845	0.80569	0.85011	0.84129	0.86738	1	1	0.90431	0.92398
Spain	1	1	1	1	1	1	0.97979	0.97979	0.99495	0.99495
Switzerland	1	1	1	1	0.99965	0.99965	1	1	0.99991	0.99991
Turkey	0.87014	0.93516	0.88991	0.9201	0.86754	0.86936	0.85285	0.85441	0.87011	0.89476
U.S.A.	0.93147	1	0.89994	0.9695	0.89371	0.96737	0.85589	0.94683	0.89525	0.97093
United Kingdom	0.86053	0.87859	0.83083	0.84594	0.83243	0.85625	0.82379	0.87551	0.8369	0.86407
Country average	0.96312	0.97859	0.93881	0.95528	0.93184	0.954	0.9317	0.95893	0.94137	0.9617
Improvement	0.01547		0.01647		0.02216		0.02723		0.02033	

Tables 5.3 and 5.4 present the results concerning CLK instance. Taking into account that clinker production process is the main responsible of CO<sub>2</sub> direct emissions, CLK instance is focused on this production sub-process. We recall that in these computational tests three inputs are taken into account (raw materials and energy related to the clinker production process, labour) and one desirable output, namely clinker production. As in the previous case, we analyze whether the efficiency scores can vary in presence or absence of undesirable factor. The study of clinker production efficiency provides informations on the ability of substituting classical raw materials and fuels with alternative ones that produce less emissions and it avoids imbalances in efficiency scores due to the different composition of Portland Cement. The varieties of Portland Cement, in facts, can contain different proportions of clinker and raw materials. Blended cements are the cement varieties with the lower clinker percentage and they are mainly produced in developing countries. A third aspect which has to be taken into account is the possibility of importing clinker. Countries involved in environmental regulations can decide to import clinker from unregulated countries in order to curb CO<sub>2</sub> emissions and produce higher cement quantities without incurring in additional costs (acquisition of additional emission permits or emission penalties).

Tables 5.3 collects the results of *M1* applied to CLK instance. The average world efficiency is equal to 0.9225; this implies that it would be possible to reduce input factors by a maximum amount of 7.75% and still produce the given level of output. With respect to the corresponding results of Table 5.1, clinker production process seems to be less efficient than the cement one. Let us, however, recall that a direct comparison can not be stated since the DEA models provide a relative efficiency measure that depends on the peer units of the considered instance.

It can be easily seen that the efficiency level varies among the considered countries. The benchmark unit is Austria during the whole period under analysis and China, Belgium and Switzerland are very close to efficiency. The worst performing countries are Brazil, Turkey, Australia, and Italy. As concerning the country average, a progressive decline in efficiency can be observed: the average level of efficiency 0.94 in 2005 drops to 0.89 in 2008. It can be also noticed that countries involved in environmental regulation seem to perform better in clinker production process. This can be a positive effect of the regulation which force these countries to adopt efficient technologies and to increase the use of alternative materials and fuels.

These results are confirmed if we refer to efficiency scores collected in Table 5.4. Model *M2weak* shows that Austria, China, India and Switzerland are efficient followed by Belgium, Canada, Japan and the U.S.A. Let us notice that Brazil when considering only the clinker production has a significant drop in efficiency. This can be explained by taking into account that Brazil mainly produces blended cement that requires a lower proportion of clinker and that the use of alternative fuels in this country is irrelevant. As concerning the country average efficiency among years, starting from the level of 0.95375 in 2005 it decreases to the level of 0.90733 in 2008. This trend slightly increases in 2007 and significantly decreases in 2008. In terms of environmental regulation let us recall that the EU-ETS normative in Europe faces a new phase in 2008 with more restrictive constraints on CO<sub>2</sub> emissions. The Statistical Wilcoxon Signed Rank Test has been conducted also for the CLK instance by firstly comparing the efficiency scores obtained with model *M1* and model *M2weak*. In this case we test the null hypothesis of equal mean efficiency scores between the two models with respect to the alternative hypothesis of different efficiency scores. The value of Wilcoxon statistics is 3.92 and the value of two tailed “*p*” statistic is lower than 0.00001. Then, the null hypothesis can be rejected at 1% level, implying that omitting CO<sub>2</sub> emissions (undesirable output) results in biased efficiency estimates. Table 5.4 shows the comparison between efficiency scores in presence or absence of environmental regulation in the CLK instance. As in the previous case The Statistical Wilcoxon Signed Rank Test has been conducted in order to test the differences in efficiency values under the strong or weak disposability assumption. The value of Wilcoxon statistics is 3.41 and the value of two tailed “*p*” statistic is lower than 0.0008. By considering strong and weak disposability assumptions, the main differences in efficiency values are related to India,

Table 5.3: CLK instance: efficiency scores based on Standard DEA model without CO<sub>2</sub> emissions

Country	2005	2006	2007	2008	Annual average
Australia	0.81311	0.81521	0.80488	0.78313	0.80408
Austria	1	1	1	1	1
Belgium	1	1	1	0.97642	0.99411
Brazil	0.77828	0.7646	0.78051	0.76188	0.77132
Canada	1	1	0.99575	0.94323	0.98475
China	1	1	0.99973	1	0.99993
Czech Republic	1	0.87418	0.93571	0.93979	0.93742
Denmark	1	0.98809	1	0.75093	0.93476
Estonia	0.91725	0.85088	0.8959	0.84416	0.87705
France	0.88141	0.85399	0.85436	0.84112	0.85772
Germany	0.91564	0.86108	0.89887	0.85143	0.88175
India	0.99856	0.94732	0.95253	0.9368	0.9588
Italy	0.82899	0.82412	0.82544	0.8172	0.82394
Japan	1	1	0.9536	0.94823	0.97546
Norway	1	0.93335	0.94148	0.94286	0.95442
Poland	0.87271	0.95857	0.9416	0.83787	0.90269
Spain	0.97513	0.96948	0.95831	0.90201	0.95123
Switzerland	1	0.979	1	0.98954	0.99214
Turkey	0.86896	0.8243	0.8297	0.85999	0.84574
U.S.A.	1	1	1	0.93394	0.98348
United Kingdom	1	0.96542	0.96309	0.84298	0.94287
Country average	0.94524	0.92427	0.93007	0.89064	0.92255

which differs of an amount of 5% between the two models and Estonia that loses 10% of efficiency in the case of no environmental regulation. The input average contraction, including the undesirable CO<sub>2</sub> emissions, in the case of absence of regulation can be 1.1% greater than in the case of weak disposability assumption. This difference is substantially constant among the different years.

Table 5.4: CLK instance: efficiency scores based on Models  $M2$  and  $M2weak$ 

Country	2005		2006		2007		2008		Annual average	
	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak
Australia	0.81311	0.83168	0.81521	0.81798	0.80488	0.80677	0.78813	0.78403	0.80408	0.81011
Austria	1	1	1	1	1	1	1	1	1	1
Belgium	1	1	1	1	1	1	0.99545	0.99545	0.99886	0.99886
Brazil	0.77828	0.7821	0.77211	0.77211	0.78751	0.78751	0.77002	0.77002	0.77698	0.77794
Canada	1	1	1	1	0.99698	0.99698	0.94633	0.94633	0.98583	0.98583
China	1	1	1	1	0.99973	1	1	1	0.99993	1
Czech Republic	1	1	0.8763	0.8763	0.93571	0.97303	0.93979	0.95985	0.93795	0.9523
Denmark	1	1	0.98809	1	1	1	0.75119	0.75119	0.93482	0.9378
Estonia	0.91725	1	0.85088	0.98369	0.8959	1	0.84416	0.92172	0.87705	0.97635
France	0.88141	0.89612	0.85399	0.85451	0.85436	0.85472	0.84607	0.84607	0.85896	0.86285
Germany	0.91564	0.93349	0.8663	0.8663	0.90257	0.90257	0.85877	0.85877	0.88582	0.89028
India	0.99856	1	0.94732	1	0.95253	1	0.9368	1	0.9588	1
Italy	0.82899	0.83407	0.82692	0.82692	0.82544	0.82667	0.82294	0.82294	0.82607	0.82765
Japan	1	1	1	1	0.97391	0.97391	0.96663	0.96663	0.98513	0.98513
Norway	1	1	0.93335	0.9443	0.94148	0.94215	0.94286	0.95347	0.95442	0.95998
Poland	0.87271	0.87799	0.95857	0.97648	0.9416	0.95085	0.84935	0.84935	0.90556	0.91367
Spain	0.97513	1	0.96948	0.99924	0.95831	0.97204	0.90201	0.90251	0.95123	0.96845
Switzerland	1	1	1	1	1	1	1	1	1	1
Turkey	0.86896	0.87326	0.82513	0.82513	0.84615	0.84615	0.88012	0.88012	0.85509	0.85616
U.S.A.	1	1	1	1	1	1	0.93465	0.93465	0.98366	0.98366
United Kingdom	1	1	0.96542	0.98067	0.96309	0.98235	0.84298	0.91074	0.94287	0.96844
Country average	0.94524	0.95375	0.92615	0.93922	0.93239	0.9436	0.89587	0.90733	0.92491	0.93597
Improvement	0.00851		0.01307		0.01122		0.01146		0.01106	



### 5.3.2 Efficiency evaluation: the directional distance function approach and undesirable factors treated as output

The results of models  $M2$  and  $M2weak$  presented in Section 5.3.1 are based on the assumption that undesirable factors (CO<sub>2</sub> emissions) are treated as an input of the production process and the efficiency measures evaluate the reduction of CO<sub>2</sub> emissions without changing the desirable output levels (cement production in CEM instance and clinker production in CLK instance). In this section the computational results concerning the directional distance function both in presence and in absence of desirable factors are shown. Let us recall that in the case of Directional distance function the maximum efficiency level is reached when the score is equal to zero.

Table 5.5 and 5.7 lists the results of model  $M3$  that measures efficiency as the ability to expand desirable output maintaining fixed the input proportions referring to CEM instance and CLK instance, respectively. Directional distance function approach including undesirable factors (models  $M4$  and  $M4weak$ ), whose results are presented in Tables 5.6 and 5.8 of this section, provides an alternative efficiency measure. It allows to measure the potential reduction of undesirable emission output and the potential expansion of desirable output. We consider both weak and strong disposability assumptions. Strong disposability implies that good outputs can be arbitrary expanded, while the weak disposability assumption limits their expansion according to existing regulation (that could imply an emission control). The difference between efficiency levels under weak and strong disposability in the directional distance approach can be interpreted as the cost of regulation with respect to the emission factors (see [40]) that we denote as “Normative Price” in Tables 5.6 and 5.8.

Let us focus on CEM instance whose results are presented in Tables 5.5 and 5.6. Table 5.5 shows the efficiency results without considering undesirable output. The average efficiency is 0.07868, meaning that cement production can be further expanded by 7.87% with the same input levels. In 2008 the maximum expansion level, that coincides with the minimum average efficiency, is reached as equal to 9.23%. The inefficiency slightly decreases in 2007 when the world cement demand achieved its maximum level over the period studied. It is reasonable to suppose that all countries have used all their available plants (efficient and non efficient) at full capacity in order to fulfill high demand levels.

The most efficient countries, according to this model, are Brazil and China, while Denmark, India, Japan and Spain have efficiency scores very close to zero. With respect to the same instance evaluated with DEA model (see Table 5.1 in the previous section), the most efficient countries are the same, while some differences can be accounted for countries whose efficiency is close to the peer units. Imbalances in efficiency scores can be found among years, efficient countries like Denmark, Spain, Belgium and Italy lose their efficiency in 2008.

In order to analyze more in details the reasons of this drop in efficiency, in Table 5.6 we compare the results of the directional distance function approach including undesirable output in the case of weak and strong disposability. We want to test if the second phase of EU-ETS normative, that imposes, starting from 2008, more restrictive limits on CO<sub>2</sub> emissions, can highlight the cause of the lower efficiency levels in these european countries. More stringent CO<sub>2</sub> limits induce implicit reductions of cement production.

The efficiency values based on the Direction Distance model in the case of regulation constraints (weak disposability) significantly differ from the corresponding values of Table 5.5. The application of the Wilcoxon test between efficiency scores of models  $M3$  and  $M4weak$  is still significant with a two tailed p statistic equal to 0.002 (value of Wilcoxon statistics 3.59).

As already explained at the beginning of the section, the difference between efficiency levels under strong and weak disposability assumptions gives the Normative Price. In CEM instance, reported in Table 5.6, we see an average normative price of 0.01. More in detail, the highest value of normative price is reached in 2005, corresponding to the year of the inception of the EU-ETS. During the period under analysis several non-european countries adopted environmental policies in



Table 5.5: CEM instance: efficiency scores based on the Direction Distance model without undesirable outputs

Country	2005	2006	2007	2008	Annual average
<b>Australia</b>	0	0.22784	0.22346	0.21002	0.16533
<b>Austria</b>	0	0.28959	0.18743	0.13195	0.15224
<b>Belgium</b>	0	0	0.02746	0.01332	0.0102
<b>Brazil</b>	0	0	0	0	0
<b>Canada</b>	0.00471	0	0.00878	0.05829	0.01794
<b>China</b>	0	0	0	0	0
<b>Czech Republic</b>	0	0.10073	0.0869	0.08332	0.06774
<b>Denmark</b>	0	0	0	0.00602	0.0015
<b>Estonia</b>	0.17768	0.16848	0.21929	0.4226	0.24701
<b>France</b>	0.10207	0.09017	0.11559	0.09114	0.09975
<b>Germany</b>	0.06471	0.02634	0	0.0391	0.03254
<b>India</b>	0	0	0	0.00941	0.00235
<b>Italy</b>	0	0	0.00855	0.04221	0.01269
<b>Japan</b>	0	0.00148	0	0.01028	0.00294
<b>Norway</b>	0.14757	0.20308	0.16812	0.15097	0.16743
<b>Poland</b>	0	0.20821	0.16491	0.00011	0.09331
<b>Spain</b>	0	0	0	0.02489	0.00622
<b>Switzerland</b>	0.06621	0.07565	0.07832	0.07146	0.07291
<b>Turkey</b>	0.13132	0.1139	0.14005	0.15835	0.1359
<b>U.S.A.</b>	0.10956	0.17357	0.17808	0.19452	0.16393
<b>United Kingdom</b>	0.16541	0.20789	0.20893	0.21948	0.20043
<b>Country average</b>	0.04615	0.08985	0.08647	0.09226	0.07868

order to mitigate CO<sub>2</sub> effects. This is confirmed by the 2008 value of normative price. However, globally considered, the differences in efficiency measures under the hypothesis of weak and strong disposability assumptions are statistically significant according to this model formulation (Wilcoxon test statistics equal to 3.06).

Table 5.6: CEM instance: efficiency measure as undesirable output contraction and desirable output expansion

Country	2005		2006		2007		2008		Annual average	
	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak
Australia	0	0	0.18953	0.18953	0.17537	0.17537	0.15829	0.15829	0.1308	0.1308
Austria	0	0	0.20852	0.20852	0.14425	0.14425	0.10284	0.10284	0.1139	0.1139
Belgium	0	0	0	0	0.02746	0.02741	0.00857	0.00857	0.00901	0.009
Brazil	0	0	0	0	0	0	0	0	0	0
Canada	0.00471	0	0	0	0.00878	0.00278	0.05829	0.02097	0.01794	0.00594
China	0	0	0	0	0	0	0	0	0	0
Czech Republic	0	0	0.09104	0.09104	0.0869	0.0852	0.07924	0.07924	0.06429	0.06387
Denmark	0	0	0	0	0	0	0.00475	0.00475	0.00119	0.00119
Estonia	0.17768	0	0.16848	0	0.21929	0	0.39763	0.39763	0.24077	0.09941
France	0.08982	0.08982	0.07778	0.07778	0.10051	0.10051	0.07798	0.07798	0.08652	0.08652
Germany	0.05622	0.05622	0.01762	0.01762	0.07992	0.07992	0.02678	0.02678	0.04514	0.04514
India	0	0	0	0	0	0	0.00941	0	0.00235	0
Italy	0	0	0	0	0.00855	0	0.03428	0.03428	0.01071	0.00857
Japan	0	0	0.00148	0	0	0	0.0096	0.0096	0.00277	0.0024
Norway	0.13813	0.13813	0.18697	0.18697	0.15549	0.15549	0.1402	0.1402	0.1552	0.1552
Poland	0.02539	0.02116	0.20289	0.20289	0.15989	0.15989	0	0	0.09704	0.09598
Spain	0	0	0	0	0	0	0.01957	0.01957	0.00489	0.00489
Switzerland	0	0	0	0	0.00051	0.00051	0	0	0.00013	0.00013
Turkey	0.13132	0.12841	0.1139	0.11058	0.13549	0.13549	0.14508	0.14508	0.13145	0.12989
U.S.A.	0.10956	0	0.17357	0.12148	0.17808	0.1436	0.19016	0.19016	0.16284	0.11381
United Kingdom	0.15156	0.15156	0.18919	0.18919	0.18473	0.18473	0.21031	0.21031	0.18395	0.18395
Country average	0.04211	0.02787	0.07719	0.06646	0.0793	0.06643	0.07967	0.07744	0.06957	0.05955
Normative Price	0.01424		0.01073		0.01286		0.00223		0.01002	

Tables 5.7 and 5.8 present the results concerning CLK instance. Following the same lines of CEM instance, in Table 5.7 we analyze the clinker production efficiency without including CO<sub>2</sub> undesirable output. The results confirm the tendency of the corresponding instance studied with DEA models in Table 5.3. The average efficiency level is 0.08717, lower than the one in Table 5.5. In addition, countries like Brazil that are efficient in the CEM instance, results to be one of the most inefficient when referring to the clinker production process only, followed by Australia, Turkey and Italy. Among years, 2008 confirms to be the year with the lowest efficiency score that amounts to 11.595% while in the 2007 is 3.75% higher.

Table 5.8 presents the efficiency scores of models  $M4$  and  $M4weak$ , directional distance function approaches under strong and weak disposability assumptions respectively. The CO<sub>2</sub> undesirable output is included in the efficiency estimations and the models highlight how the environmental regulation can limit desirable output expansion. The annual average under the hypothesis of strong disposability is equal 0.07993: this means that desirable output could be still increased by an amount of about 8%. In the case of weak disposability assumption, this percentage amounts to 7.5%. This difference in efficiency scores between model  $M4$  and  $M4weak$  is supported by the Wilcoxon Test with a statistic equal to 3.06 and a p-value equal to 0.0025. This means that in presence of normative constraints, clinker production has a more limited production expansion capability. The most efficient countries, according to this model, are Austria, China, Switzerland and Belgium. With respect to the same instance evaluated with DEA model (see Table 5.4 in the previous section), the most efficient countries do not vary, except for India which reaches levels of efficiency quite close to the peer units. In fact, according to the explanation expressed in the previous section, we assist again to an efficiency collapse of Brazil. Finally, in order to capture the importance of including undesirable outputs in the clinker process efficiency study, we have compared the annual average of models  $M3$  and  $M4weak$  for the CLK instance. The Wilcoxon test (value of statistical 3.92 and p-value 0.00009) highlights a significant difference between the two formulations at the 1% significance level.

Table 5.7: CLK instance: efficiency scores based on Standard Directional Distance model

Country	2005	2006	2007	2008	Annual average
Australia	0.21811	0.23107	0.24954	0.2882	0.24673
Austria	0	0	0	0	0
Belgium	0	0	0	0.01549	0.00387
Brazil	0.29815	0.30316	0.27332	0.29125	0.29147
Canada	0	0	0.00456	0.06277	0.01683
China	0	0	0.00043	0	0.00011
Czech Republic	0	0.15048	0.0714	0.06664	0.07213
Denmark	0	0.00259	0	0.107	0.0274
Estonia	0.09557	0.13479	0.09704	0.15185	0.11981
France	0.14126	0.18027	0.18026	0.19794	0.17493
Germany	0.09356	0.15464	0.10116	0.16728	0.12916
India	0.00193	0.06061	0.05434	0.07323	0.04752
Italy	0.2217	0.22568	0.22336	0.23409	0.22621
Japan	0	0	0.03698	0.04869	0.02142
Norway	0	0.0651	0.06353	0.05614	0.04619
Poland	0.13993	0.0404	0.05828	0.18043	0.10476
Spain	0.02447	0.03198	0.04443	0.09849	0.04984
Switzerland	0	0.02265	0	0.01111	0.00844
Turkey	0.14283	0.22102	0.16968	0.13391	0.16686
U.S.A.	0	0	0	0.07434	0.01859
United Kingdom	0	0.02772	0.0293	0.17609	0.05828
Country average	0.0656	0.0882	0.07893	0.11595	0.08717

Table 5.8: CLK instance: efficiency measure as undesirable output contraction and desirable output expansion

Country	2005		2006		2007		2008		Annual average	
	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong	Weak
Australia	0.21811	0.2125	0.21624	0.21624	0.23333	0.23333	0.27354	0.27354	0.23531	0.2339
Austria	0	0	0	0	0	0	0	0	0	0
Belgium	0	0	0	0	0	0	0.00307	0.00307	0.00077	0.00077
Brazil	0.29815	0.29755	0.27928	0.27928	0.24833	0.24833	0.22649	0.22649	0.26306	0.26291
Canada	0	0	0	0	0.00285	0.00285	0.05601	0.05601	0.01471	0.01471
China	0	0	0	0	0.00043	0	0	0	0.00011	0
Czech Republic	0	0	0.13899	0.13899	0.07107	0.07107	0.06494	0.06494	0.06875	0.06875
Denmark	0	0	0.00259	0	0	0	0.09132	0.09132	0.02348	0.02283
Estonia	0.09557	0	0.13479	0.09511	0.09704	0	0.15185	0.1298	0.11981	0.05623
France	0.14126	0.12554	0.17374	0.17374	0.17564	0.17564	0.1795	0.1795	0.16754	0.16361
Germany	0.09356	0.07523	0.14129	0.14129	0.09199	0.09199	0.14898	0.14898	0.11896	0.11438
India	0.00193	0	0.06038	0.06038	0.05434	0.02951	0.07307	0.07307	0.04743	0.04074
Italy	0.2217	0.21972	0.2177	0.2177	0.22336	0.22309	0.21066	0.21066	0.21836	0.21779
Japan	0	0	0	0	0.01712	0.01712	0.02235	0.02235	0.00987	0.00987
Norway	0	0	0.06317	0.06317	0.06042	0.06042	0.05423	0.05423	0.04446	0.04446
Poland	0.13993	0.13951	0.04009	0.04009	0.05597	0.05597	0.10377	0.10377	0.08494	0.08484
Spain	0.02447	0	0.03198	0.00146	0.04384	0.04384	0.09242	0.09242	0.04818	0.03443
Switzerland	0	0	0	0	0	0	0	0	0	0
Turkey	0.13596	0.13596	0.19063	0.19063	0.12859	0.12859	0.0952	0.0952	0.13759	0.13759
U.S.A.	0	0	0	0	0	0	0.07197	0.07197	0.01799	0.01799
United Kingdom	0	0	0.02772	0.01809	0.0293	0.01689	0.17222	0.17222	0.05731	0.0518
Country average	0.06527	0.05743	0.08184	0.07791	0.07303	0.0666	0.0996	0.09855	0.07993	0.07512
Normative Price	0.00784		0.00392		0.00643		0.00105		0.00481	

### 5.3.3 Overall comments

In this section the determinants of efficiency and inefficiency are investigated by country and main differences are reported below. The presence or absence of environmental regulations strongly affects country efficiency scores. According to the computational results of Sections 5.3.1 and 5.3.2, European countries that are generally efficient are Austria, Switzerland, Spain, Belgium and Denmark while among the non-Europeans Japan, Canada, China and India results to be almost efficient.

Austria Cement industry in recent years has continuously reduced its specific CO<sub>2</sub> emissions that are very low compared to the other countries thanks to a massive use of alternative fuels (more than 50%). In addition, it has developed a research project in order to reduce the clinker proportion in cement manufacturing. The project aims to study whether and how the optimization of ultra-fine particles (particle size distribution, particle shape and roughness) in cement can substitute clinker content. The expected emissions contractions vary between 5 % and 15%.

Switzerland efficiency can be attributed to a massive use of alternative fuels that, on average, amounts to 45% of total fuel consumption and of alternative raw material. The combination of these policies leads to a lower emission factor per ton of cement. In Spain, cement industry has doubled the utilization of alternative fuels and raw materials in the last decades. In 2008, in fact, alternative fuels accounted for the 15% of the total, while alternative raw materials were the 10% of total use. In the Annual Belgian Cement Association Report 2008, the IEE and IGES indexes show a progressive effort in reducing CO<sub>2</sub> emission and in improving energy efficiency since 2005<sup>6</sup>. Danish cement industry has replaced by at least 40% the fuel energy used in the production of grey cement by alternative fuel. In addition, thanks to its participation to the FUTURECEM project, supported by the Danish National Advanced Technology Foundation, the cement sector has performed full-scale trials on a new type of clinker based on nanotechnology. The clinker will be used in the cement of the future and produce less CO<sub>2</sub> emission. This project was completed in 2010. Again, the combination of all these factors has made Danish cement industry efficient.

As concerning non European countries, different mandatory or voluntary environmental regulations are applied. Japanese Cement Industry is involved in the Voluntary Emissions Trading Scheme, in particular cement industry outperforms the CO<sub>2</sub> emission target imposed by regulation. Among the other efficient countries, India benefits from the progressive abandon of wet technologies in favour of less energy expensive dry processes based on five and six stages pre-heating and pre-calcination kilns. Note also that main Indian companies agree with the Cement Sustainability Initiative (CSI) launched by the World Business Council for Sustainable Development (WBCSD). The case of China cement industry is more controversial. On one hand, the recent fast growth of Chinese economy led to huge investments in best available technologies new plant. This development of Chinese cement sector is in part due to foreign investors that operate in emission regulated countries and transfer their production maintaining high efficiency levels. On the other hand it can also be explained by the increase in production of blended cement which requires a lower clinker to cement ratio and reduces energy consumption and CO<sub>2</sub> emissions. As to environmental policies, China has implemented a national Climate Change Program and it is involved in the Asia-Pacific Partnership on Clean Development and Climate Partners with cement industries operating in India, Japan and U.S.A. For all these reasons, the emission factor that is ratio between CO<sub>2</sub> emission and cement production is one of the best performing among the considered countries.

However, the analysis of Chinese cement sector is difficult because of data lacks. Only 5% of Chinese Cement companies agrees with the CSI of WBCSD and data available on National Cement Association only refer to the larger operating companies. It is very difficult to have a full snapshot of the sector, so our results may be affected by data uncertainty.

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<sup>6</sup>IEE stands for *indice d'amélioration de l'efficacité énergétique* and IGES stands for *indice de réduction des émissions de CO<sub>2</sub> énergétique (combustibles)*. See Report Febelcem 2009 at page 20, available at <http://www.febelcem.be/index.php?id=rappports-annuels>

In Canada the development of cement industry is similar to the one of the U.S.A. cement industry. Regional regulations (like Alberta's Climate Change and Emission Management Act) and voluntary compliance to international environmental programs have forced cement industry to increase their level of efficiency. Differently from the EU-ETS, the Alberta's Act uses an emission intensity approach<sup>7</sup>. This system forces the involved facilities to improve their performance either by reducing their greenhouse gases emissions or by buying credits from the Climate Change and Emission Management Fund at a price of 15 Canadian dollars per each ton of reduced emission.

The ranking of inefficient countries is different with respect to the instance we consider. However, Australia and Turkey show to be inefficient in almost all cases. We first recall that these two countries are not subject to any environmental regulation. The proposal for an emission trading scheme in Australia has been blocked and its possible implementation will be postponed after 2013. Turkey only recently has shown an environmental awareness by ratifying the Kyoto Protocol in 2009.

Finally Brazil shows an hybrid behaviour. Depending on the instance under consideration Brazil is either efficient or inefficient. More specifically, the Brazilian cement industry is efficient in the CEM instance both with and without CO<sub>2</sub> undesirable output. The overall cement production process appears to be efficient because the clinker/cement ratio (0.68%-0.70%) is relatively low compared to standard ratio (0.76%-0.80%). These efficiency levels fall down when considering the CLK instance. In fact, the proportion of the alternative fuels is minimal compared to the total use and moreover only starting 2009 a voluntary environmental program has been introduced.

## 5.4 Conclusions

In this chapter we analyze the impact of CO<sub>2</sub> emissions on the efficiency of the world cement industry using Data Envelopment Analysis (DEA). This work differs from literature since it analyzes 21 countries covering the 90% of the world cement production during the period 2005-2008. Traditional industrialized countries are compared with emerging producers like India, China, Turkey and Brazil.

There are different approaches to incorporate undesirable pollutants in DEA models. In this work a classical DEA model with undesirable factor treated as inputs and a directional distance function model where CO<sub>2</sub> emission are undesirable output have been considered. The same kind of models have been implemented without considering undesirable factors. The differences resulting from the comparison of these models with and without undesirable factors are statistically significant (as highlighted by a Wilcoxon Rank Sum Test). From these statistical tests we can deduce that CO<sub>2</sub> emissions modify the efficiency levels and they can not be excluded in the efficiency evaluation of the worldwide cement industry.

Two different instances have been formulated in order to understand efficiency and inefficiency causes. The first instance (CEM) describes efficiency of the whole cement production process taking energy, raw materials, clinker, capital and labour as inputs, cement production as desirable output and CO<sub>2</sub> emissions as undesirable factor (assumed to be input or output according to the model studied). The second instance (CLK) studies the clinker production sub-process. We choose to also analyze clinker production because it is most critical phase of the entire cement production process: CO<sub>2</sub> emissions are mainly generated by chemical reactions in the calcination of raw materials. Focusing on this phase we avoid imbalances in efficiency scores due to plant relocation or blended cement production. Countries in which environmental regulations curb CO<sub>2</sub> emissions can decide to transfer their facilities in unregulated regions in order to limit the emission costs without changing their production volumes. In the case of the cement production, companies can move their clinker plants.

Our analysis has shown that the efficiency levels mainly depend on decisions to invest in alternative raw materials and alternative fuels both in the case of mandatory and voluntary emission

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<sup>7</sup>The emission intensity measures the amount greenhouse gases generated per unit of economic output.

regulated countries. Among European countries, compulsory involved in the EU-ETS, Belgium, Austria, Denmark and Spain appear to be efficient both in the CEM and CLK instances. Substitution levels in raw materials and fuels, significant investments in advanced technology and research and development programs (like in Austria and Denmark) are the determinants of this success. A similar reasoning applies to Switzerland and Japan, where the use of alternative raw materials and fuels is about the 50% of the total quantities. Among emerging countries that face fast growth of cement production in recent years, China and India show high efficiency levels. This feature can be explained by two different factors: plants with more efficient technologies (progressive substitution of small wet process plants with bigger and dry technology ones), investments in the production of blended quality cements requiring less proportion of clinker. The case of China cement industry, however, requires careful attention because of lack or fragmentary data. Brazil efficiency results confirm the importance of considering both CEM and CLK instances: being inefficient in the CLK case it becomes a peer unit in CEM instance. Finally, countries without any environmental regulation have no incentive to improve their ecological performance. This is the case of Australia and Turkey.

The two classes of models have been tested under strong and weak disposability assumptions. Strong disposability corresponds to an unregulated framework while weak disposability assumption imposes normative constraints. The differences in efficiency scores is denoted as Normative Price in the directional distance approach. Our results show that this difference is statistically significant and the Normative Price tend to decrease over time. This means that countries are able to progressively adapt their technologies to environmental targets.

Finally it is worth pointing out that Data Envelopment Analysis is a tool that measures the efficiency scores of DMUs operating in similar business environments using similar technologies. In our study, we have compared both European and non-European countries that differ from a geographical and economic point of view. Moreover, the countries included in our dataset have also shown a dissimilar attitude with respect to environmental issues. Despite these limitations, the study of prototypes of cement/clinker production plants highlights the potential for reducing CO<sub>2</sub> emissions based on the comparison of performance results and environmental policies among different countries.

Further developments of this work are in the direction of enlarging the actual dataset by including more cement producing countries, useful to increase the discrimination power of the DEA models and to modify input and output data of the instances. In this light a further analysis will consider more than one undesirable factor. National authorities in Europe has recently shown the intention to extend environmental regulation to a wider class of greenhouse gases like NO<sub>x</sub>, SO<sub>2</sub> emissions.



# Chapter 6

## Conclusions

In this Ph.D thesis alternative DEA models which consider uncertain and undesirable outputs are studied.

First of all, starting from the generalized input-oriented (BCC) model, two different models with uncertain outputs and deterministic inputs are proposed. Various applications, in fact, are affected by random perturbations in output values estimation (see, for instance, [9, 56, 58]). Random perturbations can be addicted to a concrete difficulty in estimating the right output value (for instance, in the case of energy companies, electricity production has to take care of different and uncertain energy dispersion factors according to the employed technologies) or to obtain good output provisions (for instance, in the case of DEA applied to health care problems, early screening efficiency measures are related to the estimation of true positive and false positive screens, which are indeed outputs with a stochastic nature). A large number of papers, based on different approaches, can be found in the literature concerning DEA with outputs uncertainty. In particular, chance-constrained programming is the most used technique to include noise variations in data and to solve data envelopment analysis problems with uncertainty in data. Chance-constrained programming admits random data variations and permits constraint violations up to specified probability limits, allowing linear deterministic equivalent formulations, in case a normal distribution of the data uncertainty is assumed (see for all [18, 20, 33, 36, 44, 48]). The formulations proposed in the first part of this thesis move away the classical chance-constrained method with the aim to obtain a more accurate DMU ranking whatever situation occurs. In particular, the proposed models (VRS1 and VRS2 model under the assumption of variable return to scale and CRS1 and CRS2 under constant return to scale) remove the hypothesis of normal data distribution and uses a scenario generation approach to include data perturbations. Models VRS1 and VRS2 have been implemented with the aim to analyze their behaviour in order to point out their effectiveness. This has been done by means of computational tests which compare the proposed models with two further deterministic ones based on the expected value approach. Deeply speaking, the main difference, between the two proposed models and the expected value approaches, lies in their mathematical formulation. In the models based on the scenario generation approach, the constraints concerning efficiency level are expressed for each scenario, while in the expected value models, they are satisfied in expected value. As a consequence, the first kind of models result to be more selective in finding a ranking of efficiency, thus becoming useful strategic management tools aimed to determine a restrictive efficiency score ranking. To witness that the models proposed here are more selective than those based on the expected value, different classes of problems have been considered varying the data distribution function and the number of inputs, outputs, DMUs and scenarios.

The obtained results are collected in [50], accepted for publication in the Journal of Information & Optimization Sciences.

In the second part of this study, we focus on the environmental policy and the concept of eco-efficiency. A cross-country comparison of the eco-efficiency level of the worldwide cement industry are presented. This analysis differs from previous literature because it compares 21 countries, European (EU) and non-European (non-EU) countries, covering 90% of the world's cement production. The thirteen European countries included in the database (Austria, Belgium, Czech Republic, Denmark, Estonia, France, Germany, Italy, Norway, Poland, Spain, Switzerland and United Kingdom) account for more than 80% of the total EU cement production. To compare the eco-efficiency of these EU countries with non-EU countries, data for eight major non-EU countries (Australia, Brazil, Canada, China, India, Japan, USA and Turkey) were added. By adopting a DEA approach, where emissions can be either considered as inputs or undesirable outputs, the impact of environmental regulations on cement industry efficiency, by considering a joint production framework of both desirable and undesirable outputs, has been studied. Moreover, a directional distance function approach has been used to evaluate the ability of a country to simultaneously expand the desirable output and contract the CO<sub>2</sub> emissions by the same proportion without increasing the inputs. Strong and weak disposability assumptions are analyzed in order to evaluate the impact of environmental regulations interpreted as the cost of regulation. For the purpose of this analysis, different instances have been considered by considering different inputs and outputs.

First, we consider the following inputs: capital in the form of installed capacity, energy as the sum of electricity and thermal energy, labor as the number of employees and materials as raw materials (slug, limestone, etc), in addition to clinker imports and production. The desirable output was Portland cement production. The undesirable by-product was measured as the value of carbon dioxide emissions (CO<sub>2</sub>) resulting from the clinker production process without considering those related to raw materials, fuels and clinker transportation. From the analysis results that countries without strong or mandatory environmental regulations (like the USA, Turkey, Brazil and Canada) were the worst-performing during the considered period and had a negative trend, except for China and India. European countries under the EU-ETS regulation kept a nearly constant efficiency level during the four years. Our analysis shows that the efficiency level mainly depends on decisions to invest in alternative raw materials and alternative fuels, both in the case of regulated countries and in the case of voluntary emission-trading schemes. This study highlights, both at national and international levels, the possibility of reducing CO<sub>2</sub> emissions and expanding cement production. The use of alternative raw materials, alternative fuels and the possibility of producing blended cements, which require less energy consumption and reduce pollutant emissions, seem to be appropriate means. Environmental regulations can provide incentives in terms of tax exemption benefits or more restrictive pollutant limits.

The obtained results, collected in [51], are available online at: <http://dx.doi.org/10.1016/j.enpol.2011.02.057>

In the second part of the eco-efficiency analysis, we try to answer to the following questions: do undesirable factors modify the efficiency levels of cement industry? Is it reasonable to omit CO<sub>2</sub> emissions in evaluating the performances of the cement sector in different countries? In order to answer to these questions, alternative formulations of standard Data Envelopment Analysis model and directional distance function are compared both in presence and in absence of undesirable factors. For the purpose of this second analysis, the choice of input and output factors of DEA models has been done taking into account the cement and clinker production processes. In this light, two different instances have been considered: in the first instance (CEM) we focus on the entire cement production process, in the second one (CLK) we limit our attention on clinker. For this reason, in CEM we take again clinker, installed cement capacity, energy, labour and raw materials as inputs. Desirable output is represented by cement production while CO<sub>2</sub> emissions are the undesirable by-product. The undesirable by-product is measured by the value of carbon dioxide emissions mainly resulting from the clinker production process. The second set of computational tests (CLK) gives information on efficiency of the clinker production process which is the main responsible of CO<sub>2</sub>

emissions. In order to evaluate the clinker process efficiency in the twenty-one countries, three input factors have been taken into account, namely energy and raw materials consumed in the clinker production and labour. Clinker production is the desirable output while CO<sub>2</sub> emissions are the undesirable one. This analysis has shown that the efficiency levels mainly depend on decisions to invest in alternative raw materials and alternative fuels both in the case of mandatory and voluntary emission regulated countries. Among European countries, compulsory involved in the EU-ETS, Belgium, Austria, Denmark and Spain appear to be efficient both in the CEM and CLK instances. Substitution levels in raw materials and fuels, significant investments in advanced technology and research and development programs (like in Austria and Denmark) are the determinants of this success. A similar reasoning applies to Switzerland and Japan, where the use of alternative raw materials and fuels is about the 50% of the total quantities. Among emerging countries that face fast growth of cement production in recent years, China and India show high efficiency levels. This feature can be explained by two different factors: plants with more efficient technologies (progressive substitution of small wet process plants with bigger and dry technology ones), investments in the production of blended quality cements requiring less proportion of clinker. The case of China cement industry, however, requires careful attention because of lack or fragmentary data. Brazil efficiency results confirm the importance of considering both CEM and CLK instances: being inefficient in the CLK case it becomes a peer unit in CEM instance. Finally, countries without any environmental regulation have no incentive to improve their ecological performance. This is the case of Australia and Turkey. The two classes of models have been tested under strong and weak disposability assumptions. Strong disposability corresponds to an unregulated framework while weak disposability assumption imposes normative constraints. The differences in efficiency scores is denoted as Normative Price in the directional distance approach. Our results show that this difference is statistically significant and the Normative Price tend to decrease over time. This means that countries are able to progressively adapt their technologies to environmental targets.

The obtained results are collected in [53] and submitted to Resource and Energy Economics.

Above mentioned studies have been developed taking into account a specific assumption, namely that the production possibility set can be expanded each year, and no technological regress is admitted. In the formulation of DEA models, this assumption can be incorporated through the construction of so-called sequential frontier. Results on eco-efficiency measure through the standard Contemporaneous Frontier, where the frontier in each year is constructed with only the observations of the year under consideration, are collected in [52] and accepted for publication in Journal of Statistics & Management Systems.



# Appendix A

## Simulations

For sake of completeness, this Appendix provides all computational results obtained in the deep preliminary computational tests that lead to obtain the final results shown in Chapter 4 and 5. The three DEA models and the directional distance function described in the overview and in Chapters 4 and 5 (INP,  $TR_\beta$ , KL, DDF models) have been implemented in MatLab 2010a in order to capture the various aspects of environmental and production efficiency in the cement industry. For the purpose of our analysis, the choice of input and output factors of DEA models has been done taking into account the cement and clinker production processes. Specifically speaking, for all 21 countries (DMUs in DEA) the collected data are: clinker and cement production, considering also clinker import, the consumption of raw materials, electricity and thermal energy, the number of employees (labour) and  $CO_2$  emissions. Different instances have been implemented in order to capture various aspects of eco-efficiency in the manufacturing of cement and clinker. In particular, different combinations of input and output have been taken into account, as specified in the following sections. For each different application, both contemporaneous and sequential frontier has been built. In addition, each instance has been implemented considering, for each input and output, data at the aggregate level (e.g. all the cement produced or all  $CO_2$  emitted by all plants of a given country) or the data divided by the total number of plants in each country to evaluate the eco-efficiency of a representative plant within each country (Sections A.1, A.2, A.3, A.4).

Finally, all instance have been considered excluding undesirable factor in order to evaluate if eco-efficiency levels of countries are affect by emission levels (Sections A.5, A.6, A.7, A.8).

## A.1 Cement: first instance

In the first instance, we have considered:

Inputs:

- Energy
- Materials (Raw materials + Clinker production + Clinker import)
- Labour
- Capacity

Output:

- Cement production

Undesirable factor:

- CO<sub>2</sub> emissions

### A.1.1 Contemporaneous frontier for representative plants: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	1	1	1	1	1	1	0	0
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8856	0.8856	0.8764	0.8983	0.3130	0.3130	0.1200	0.1200
Canada	0.9965	1	0.9965	1	0.7247	0.7247	0.0047	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8277	1	0.8277	1	0.1561	0.1561	0.2180	0
France	0.9180	0.9180	0.9136	0.9233	0.4562	0.4562	0.1121	0.1121
Germany	0.8887	0.8887	0.8814	0.8965	0.2340	0.2340	0.1147	0.1147
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8575	0.8733	0.8575	0.9113	0.2021	0.2021	0.1577	0.1577
Poland	0.8530	0.8530	0.8499	0.9139	0.5932	0.5932	0.1425	0.1425
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8023	0.8479	0.8023	0.9231	0.3766	0.3766	0.2353	0.2353
U.S.A.	0.9169	1	0.9169	1	0.2450	0.2450	0.1246	0
United Kingdom	0.8602	0.8747	0.8602	0.9095	0.2082	0.2082	0.1526	0.1526
<b>2006</b>								
Australia	0.8439	0.8542	0.8439	0.8968	0.3417	0.3417	0.1925	0.1925
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8345	0.8345	0.8299	0.8838	0.3295	0.3295	0.1698	0.1698
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.8431	0.8431	0.8416	0.8950	0.2748	0.2748	0.1610	0.1610
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8648	1	0.8648	1	0.3077	0.3077	0.1707	0
France	0.9482	0.9891	0.9482	0.9899	0.2869	0.2869	0.1058	0.0355
Germany	0.8948	0.9001	0.8948	0.9067	0.3909	0.3909	0.1212	0.1212
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8647	0.8932	0.8647	0.9141	0.2057	0.2057	0.2277	0.2277
Poland	0.7479	0.8140	0.7479	0.9105	0.4471	0.4471	0.2427	0.2427
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8176	0.8463	0.8176	0.9302	0.4630	0.4630	0.2284	0.2284
U.S.A.	0.9427	1	0.9427	1	0.2000	0.2000	0.1071	0
United Kingdom	0.8259	0.8466	0.8259	0.8959	0.2162	0.2162	0.1892	0.1892
<b>2007</b>								
Australia	0.8555	0.8626	0.8555	0.8943	0.4275	0.4275	0.1700	0.1700
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8252	0.8448	0.8252	0.9182	0.3948	0.3948	0.1851	0.1851
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.8694	0.9550	0.8694	0.9796	0.3663	0.3663	0.1510	0.1510
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8401	1	0.8401	1	0.3335	0.3335	0.1514	0
France	0.9629	1	0.9629	1	0.2953	0.2953	0.0887	0
Germany	0.8872	0.8938	0.8872	0.9147	0.4069	0.4069	0.1453	0.1453
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8868	0.9079	0.8868	0.9371	0.3220	0.3220	0.2095	0.2095
Poland	0.7507	0.8892	0.7507	0.9509	0.5526	0.5526	0.1921	0.1921
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7846	0.7915	0.7846	0.8795	0.5213	0.5213	0.2025	0.2025
U.S.A.	0.9500	1	0.9500	1	0.1882	0.1882	0.0943	0
United Kingdom	0.8346	0.8899	0.8346	0.9376	0.2424	0.2424	0.1849	0.1849
<b>2008</b>								
Australia	1	1	1	1	1	1	0	0
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.9353	0.9353	0.9321	0.9455	0.6744	0.6744	0.0502	0.0502
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9266	0.9689	0.9266	0.9835	0.4823	0.4823	0.0803	0.0803
Denmark	1	1	1	1	1	1	0	0
Estonia	0.7937	1	0.7937	1	0.1684	0.1684	0.1958	0
France	0.9689	1	0.9689	1	0.2976	0.2976	0.0542	0
Germany	0.9658	0.9658	0.9625	0.9625	0.6789	0.6789	0.0332	0.0332
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9427	0.9665	0.9427	0.9810	0.3050	0.3050	0.0753	0.0657
Poland	1	1	1	1	1	1	0	0
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8256	0.8331	0.8256	0.9188	0.6287	0.6287	0.1065	0.1065
U.S.A.	0.9947	1	0.9947	1	0.1912	0.1912	0.1155	0
United Kingdom	0.8363	0.9890	0.8363	0.9954	0.2083	0.2083	0.1995	0.1995

Table A.1: Cement 2005-2008 comparison for plants with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

## A.1.2 Contemporaneous frontier with aggregated data: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	1	1	1	1	1	1	0	0
Austria	0.9864	0.9864	0.9854	0.9854	0.8082	0.8082	0.0142	0.0142
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8639	0.8639	0.8584	0.8997	0.8639	0.8639	0.0701	0.0701
Canada	0.9860	1	0.9860	1	0.7070	0.7070	0.0149	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.8501	1	0.8501	1	0.4101	0.4101	0.1771	0.1771
Estonia	1	1	1	1	1	1	0	0
France	0.8566	0.8566	0.8557	0.8964	0.7447	0.7447	0.1180	0.1180
Germany	0.8721	0.8721	0.8691	0.9007	0.8559	0.8559	0.0738	0.0738
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9825	0.9825	0.9812	0.9812	0.7566	0.7566	0.0215	0.0215
Poland	0.8643	0.8643	0.8626	0.9028	0.6457	0.6457	0.1465	0.1465
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7950	0.8655	0.7950	0.9227	0.6797	0.6797	0.1828	0.1828
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8559	0.8767	0.8559	0.9214	0.5960	0.5960	0.1576	0.1576
<b>2006</b>								
Australia	0.8101	0.8101	0.8060	0.8932	0.5061	0.5061	0.1882	0.1882
Austria	0.6804	0.6804	0.6532	0.7435	0.2051	0.2051	0.3711	0.3711
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8167	0.8167	0.8167	0.8921	0.7996	0.7996	0.1076	0.1076
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.8810	0.8810	0.8736	0.8985	0.2289	0.2289	0.1334	0.1334
Denmark	0.8403	1	0.8403	1	0.4364	0.4364	0.1859	0.1859
Estonia	1	1	1	1	1	1	0	0
France	0.7977	0.7977	0.7962	0.8821	0.7219	0.7219	0.1512	0.1512
Germany	0.8576	0.8576	0.8509	0.8921	0.8569	0.8569	0.0736	0.0736
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.7515	0.8154	0.7515	0.9215	0.5107	0.5107	0.2907	0.2907
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7658	0.8478	0.7658	0.9331	0.6751	0.6751	0.1754	0.1754
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8199	0.8526	0.8199	0.9257	0.5694	0.5694	0.1967	0.1967
<b>2007</b>								
Australia	0.8360	0.8360	0.8273	0.8899	0.5139	0.5139	0.1613	0.1613
Austria	0.7344	0.7344	0.7029	0.7622	0.2040	0.2040	0.2930	0.2930
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8143	0.8425	0.8143	0.9097	0.7542	0.7542	0.1362	0.1362
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9308	0.9957	0.9308	0.9974	0.3510	0.3510	0.0827	0.0167
Denmark	0.9487	1	0.9487	1	0.6675	0.6675	0.0672	0
Estonia	1	1	1	1	1	1	0	0
France	0.7881	0.8045	0.7881	0.8902	0.6742	0.6742	0.1833	0.1833
Germany	0.8319	0.8492	0.8319	0.9090	0.7593	0.7593	0.1313	0.1313
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.7672	0.8561	0.7672	0.9333	0.5207	0.5207	0.2899	0.2899
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7602	0.7904	0.7602	0.8742	0.7068	0.7068	0.1535	0.1535
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8444	0.8917	0.8444	0.9379	0.5676	0.5676	0.1824	0.1824
<b>2008</b>								
Australia	0.8394	0.8394	0.8977	0.8977	0.5311	0.5311	0.1622	0.1622
Austria	0.6983	0.6983	0.7420	0.7420	0.1794	0.1794	0.3445	0.3445
Belgium	0.9834	0.9834	1	1	0.9295	0.9295	0.0151	0.0151
Brazil	0.8195	0.8195	0.9204	0.9204	0.7823	0.7823	0.1028	0.1028
Canada	0.9187	0.9233	0.9818	0.9818	0.5568	0.5568	0.0827	0.0827
China	1	1	1	1	1	1	0	0
Czech Republic	0.9122	0.9122	0.9429	0.9429	0.2981	0.2981	0.0948	0.0948
Denmark	0.8619	0.8892	0.9395	0.9395	0.4188	0.4188	0.1783	0.1783
Estonia	1	1	1	1	0.6638	0.6638	0.1208	0.1208
France	0.7997	0.7997	0.8231	0.8231	0.6831	0.6831	0.1763	0.1763
Germany	0.8453	0.8453	0.9589	0.9589	0.8158	0.8158	0.0971	0.0971
India	1	1	1	1	1	1	0	0
Italy	0.9469	0.9536	1	1	0.8320	0.8320	0.0524	0.0524
Japan	0.9491	0.9536	1	1	0.8697	0.8697	0.0267	0.0267
Norway	1	1	1	1	1	1	0	0
Poland	0.8121	0.8121	0.8383	0.8383	0.6692	0.6692	0.1818	0.1818
Spain	0.9374	0.9374	1	1	0.8927	0.8927	0.0547	0.0547
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7544	0.7566	0.7799	0.7799	0.7086	0.7086	0.1376	0.1376
U.S.A.	0.9294	0.9294	1	1	0.9242	0.9242	0.0288	0.0288
United Kingdom	0.7694	0.8597	0.7832	0.7832	0.4346	0.4346	0.2835	0.2835

Table A.2: Cement 2005-2008 comparison with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)



### A.1.3 Sequential frontier for representative plants: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	1	1	1	1	1	1	0	0
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8856	0.8856	0.8764	0.8983	0.3130	0.3130	0.1200	0.1200
Canada	0.9965	1	0.9965	1	0.7247	0.7247	0.0047	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8277	1	0.8277	1	0.1561	0.1561	0.2180	0
France	0.9180	0.9180	0.9136	0.9233	0.4562	0.4562	0.1121	0.1121
Germany	0.8887	0.8887	0.8814	0.8965	0.2340	0.2340	0.1147	0.1147
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8575	0.8733	0.8575	0.9113	0.2021	0.2021	0.1577	0.1577
Poland	0.8530	0.8530	0.8499	0.9139	0.5932	0.5932	0.1425	0.1425
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8023	0.8479	0.8023	0.9231	0.3766	0.3766	0.2353	0.2353
U.S.A.	0.9169	1	0.9169	1	0.2449	0.2449	0.1246	0
United Kingdom	0.8602	0.8747	0.8602	0.9095	0.2082	0.2082	0.1526	0.1526
<b>2006</b>								
Australia	0.8248	0.8255	0.8248	0.8283	0.3330	0.3330	0.2004	0.2004
Austria	0.9377	0.9596	0.9645	0.9645	0.5639	0.5639	0.2161	0.2161
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8337	0.8337	0.8296	0.8394	0.3277	0.3277	0.1736	0.1736
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.8424	0.8424	0.8411	0.8411	0.2605	0.2605	0.1646	0.1646
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8536	1	0.8536	0.9739	0.2877	0.2877	0.1807	0
France	0.8896	0.8896	0.8968	0.8968	0.2692	0.2692	0.1640	0.1640
Germany	0.8812	0.8812	0.8776	0.8776	0.3793	0.3793	0.1250	0.1250
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	0.9961	0.9965	1	1	0.9934	0.9934	0.0024	0.0024
Norway	0.8179	0.8340	0.8179	0.8301	0.1936	0.1936	0.2315	0.2315
Poland	0.7478	0.8086	0.7478	0.8493	0.4464	0.4464	0.2428	0.2428
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7878	0.8263	0.7878	0.8415	0.4622	0.4622	0.2285	0.2285
U.S.A.	0.8871	0.9649	0.8871	0.9244	0.1986	0.1986	0.1823	0.1452
United Kingdom	0.8255	0.8450	0.8255	0.8479	0.2036	0.2036	0.1921	0.1921
<b>2007</b>								
Australia	0.8348	0.8348	0.8346	0.8355	0.4080	0.4080	0.1836	0.1836
Austria	0.9627	0.9958	1	1	0.6164	0.6164	0.1442	0.1442
Belgium	0.9675	0.9848	0.9675	0.9680	0.8761	0.8761	0.0275	0.0274
Brazil	0.8236	0.8236	0.8230	0.8230	0.3856	0.3856	0.1888	0.1888
Canada	0.9891	0.9915	0.9962	0.9962	0.8601	0.8601	0.0114	0.0114
China	1	1	1	1	1	1	0	0
Czech Republic	0.8668	0.9021	0.8670	0.8670	0.2993	0.2993	0.1523	0.1523
Denmark	1	1	1	1	1	1	0	0
Estonia	0.7624	1	0.7624	1	0.3079	0.3079	0.2193	0
France	0.8894	0.8894	0.9048	0.9048	0.2638	0.2638	0.1734	0.1734
Germany	0.8606	0.8608	0.8645	0.8645	0.3841	0.3841	0.1512	0.1512
India	1	1	1	1	1	1	0	0
Italy	0.9923	0.9967	1	1	0.9628	0.9628	0.0086	0.0083
Japan	1	1	1	1	1	1	0	0
Norway	0.8505	0.8666	0.8505	0.8519	0.3125	0.3125	0.2133	0.2133
Poland	0.7489	0.8249	0.7489	0.8345	0.5510	0.5510	0.1921	0.1921
Spain	1	1	1	1	1	1	0	0
Switzerland	0.9997	0.9997	1	1	0.9875	0.9875	0.0005	0.0005
Turkey	0.7701	0.7701	0.7701	0.7839	0.5165	0.5165	0.2031	0.2031
U.S.A.	0.8886	0.9641	0.8886	0.8955	0.1713	0.1713	0.1841	0.1451
United Kingdom	0.8322	0.8475	0.8323	0.8323	0.2347	0.2347	0.1876	0.1876
<b>2008</b>								
Australia	0.8470	0.8470	0.8439	0.9840	0.4614	0.4614	0.1619	0.1619
Austria	0.9791	1	0.9791	1	0.7236	0.7236	0.1028	0.1028
Belgium	0.9788	0.9788	0.9710	1	0.9222	0.9222	0.0086	0.0086
Brazil	0.8269	0.8269	0.8260	0.9455	0.5346	0.5346	0.1811	0.1811
Canada	0.9338	0.9350	0.9338	1	0.5794	0.5794	0.0824	0.0824
China	1	1	1	1	1	1	0	0
Czech Republic	0.8774	0.8893	0.8774	0.9835	0.3378	0.3378	0.1329	0.1329
Denmark	0.9788	0.9815	0.9788	1	0.9559	0.9559	0.0132	0.0132
Estonia	0.6562	1	0.6562	1	0.1485	0.1485	0.3976	0.3976
France	0.8851	0.8851	0.8822	0.9985	0.2803	0.2803	0.1662	0.1662
Germany	0.8675	0.8675	0.8629	0.9554	0.4176	0.4176	0.1320	0.1320
India	0.9870	1	0.9870	1	0.9068	0.9068	0.0156	0
Italy	0.9598	0.9618	0.9598	1	0.8378	0.8378	0.0413	0.0413
Japan	0.9744	0.9772	0.9744	1	0.9642	0.9642	0.0134	0.0134
Norway	0.8614	0.8715	0.8614	0.9810	0.2867	0.2867	0.1837	0.1837
Poland	0.8170	0.8187	0.8170	0.9604	0.8058	0.8058	0.0662	0.0662
Spain	0.9483	0.9483	0.9479	1	0.6750	0.6750	0.0535	0.0535
Switzerland	1	1	0.9304	0.9315	1	1	0	0
Turkey	0.7588	0.7588	0.7587	0.9188	0.5524	0.5524	0.1878	0.1878
U.S.A.	0.8552	0.9006	0.8552	1	0.1902	0.1902	0.2432	0.2432
United Kingdom	0.7772	0.8554	0.7772	0.9954	0.1966	0.1966	0.2752	0.2752

Table A.3: Cement 2005-2008 comparison for plants with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

### A.1.4 Sequential frontier with aggregated data: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	1	1	1	1	1	1	0	0
Austria	0.9864	0.9864	0.9854	0.9854	0.8082	0.8082	0.0142	0.0142
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8639	0.8639	0.8584	0.8997	0.8639	0.8639	0.0701	0.0701
Canada	0.9860	1	0.9860	1	0.7070	0.7070	0.0149	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.8501	1	0.8501	1	0.4101	0.4101	0.1771	0.1771
Estonia	1	1	1	1	1	1	0	0
France	0.8566	0.8566	0.8557	0.8964	0.7447	0.7447	0.1180	0.1180
Germany	0.8721	0.8721	0.8691	0.9007	0.8559	0.8559	0.0738	0.0738
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9825	0.9825	0.9812	0.9812	0.7566	0.7566	0.0215	0.0215
Poland	0.8643	0.8643	0.8626	0.9028	0.6457	0.6457	0.1465	0.1465
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7950	0.8655	0.7950	0.9227	0.6797	0.6797	0.1828	0.1828
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8559	0.8767	0.8559	0.9214	0.5960	0.5960	0.1576	0.1576
<b>2006</b>								
Australia	0.8020	0.8020	0.8057	0.8057	0.5033	0.5033	0.2008	0.2008
Austria	0.6714	0.6714	0.6528	0.6528	0.1931	0.1931	0.4008	0.4008
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8167	0.8167	0.8167	0.8167	0.7994	0.7994	0.1076	0.1076
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.8760	0.8760	0.8735	0.8735	0.2146	0.2146	0.1405	0.1405
Denmark	0.8342	1	0.8397	0.8397	0.3982	0.3982	0.2033	0.2033
Estonia	1	1	1	1	1	1	0	0
France	0.7974	0.7974	0.7962	0.7962	0.7211	0.7211	0.1517	0.1517
Germany	0.8568	0.8568	0.8509	0.8509	0.8565	0.8565	0.0738	0.0738
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	0.9746	0.9746	1	1	0.9480	0.9480	0.0127	0.0127
Norway	0.9533	0.9533	1	1	0.5949	0.5949	0.0681	0.0681
Poland	0.7513	0.8084	0.7515	0.7515	0.5090	0.5090	0.2916	0.2916
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7658	0.8279	0.7658	0.7658	0.6750	0.6750	0.1754	0.1754
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8196	0.8489	0.8199	0.8199	0.5673	0.5673	0.1995	0.1995
<b>2007</b>								
Australia	0.8143	0.8143	0.8270	0.8270	0.5059	0.5059	0.1846	0.1846
Austria	0.6969	0.6969	0.7026	0.7026	0.1786	0.1786	0.3465	0.3465
Belgium	0.9831	0.9897	1	1	0.8825	0.8825	0.0192	0.0192
Brazil	0.8119	0.8160	0.8139	0.8139	0.7539	0.7539	0.1362	0.1362
Canada	0.9882	0.9910	1	1	0.8576	0.8576	0.0115	0.0115
China	1	1	1	1	1	1	0	0
Czech Republic	0.9003	0.9079	0.9306	0.9306	0.2699	0.2699	0.1119	0.1119
Denmark	0.8836	1	0.9474	0.9474	0.5228	0.5228	0.1524	0
Estonia	1	1	1	1	1	1	0	0
France	0.7808	0.7808	0.7877	0.7877	0.6717	0.6717	0.1845	0.1845
Germany	0.8304	0.8340	0.8316	0.8316	0.7582	0.7582	0.1318	0.1318
India	1	1	1	1	1	1	0	0
Italy	0.9928	1	1	1	0.9711	0.9711	0.0070	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.7547	0.8130	0.7666	0.7666	0.5168	0.5168	0.2920	0.2920
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7582	0.7626	0.7597	0.7597	0.7066	0.7066	0.1535	0.1535
U.S.A.	0.9944	0.9944	1	1	0.9941	0.9941	0.0023	0.0023
United Kingdom	0.8271	0.8493	0.8440	0.8440	0.5617	0.5617	0.1929	0.1929
<b>2008</b>								
Australia	0.8394	0.8394	0.8977	0.8977	0.5311	0.5311	0.1622	0.1622
Austria	0.6983	0.6983	0.7420	0.7420	0.1794	0.1794	0.3445	0.3445
Belgium	0.9834	0.9834	1	1	0.9295	0.9295	0.0151	0.0151
Brazil	0.8195	0.8195	0.9204	0.9204	0.7823	0.7823	0.1028	0.1028
Canada	0.9187	0.9233	0.9818	0.9818	0.5568	0.5568	0.0827	0.0827
China	1	1	1	1	1	1	0	0
Czech Republic	0.9122	0.9122	0.9429	0.9429	0.2981	0.2981	0.0948	0.0948
Denmark	0.8619	0.8892	0.9395	0.9395	0.4188	0.4188	0.1783	0.1783
Estonia	1	1	1	1	0.6638	0.6638	0.1208	0.1208
France	0.7997	0.7997	0.8231	0.8231	0.6831	0.6831	0.1763	0.1763
Germany	0.8453	0.8453	0.9589	0.9589	0.8158	0.8158	0.0971	0.0971
India	1	1	1	1	1	1	0	0
Italy	0.9469	0.9536	1	1	0.8320	0.8320	0.0524	0.0524
Japan	0.9491	0.9536	1	1	0.8697	0.8697	0.0267	0.0267
Norway	1	1	1	1	1	1	0	0
Poland	0.8121	0.8121	0.8383	0.8383	0.6692	0.6692	0.1818	0.1818
Spain	0.9374	0.9374	1	1	0.8927	0.8927	0.0547	0.0547
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.7544	0.7566	0.7799	0.7799	0.7086	0.7086	0.1376	0.1376
U.S.A.	0.9294	0.9294	1	1	0.9242	0.9242	0.0288	0.0288
United Kingdom	0.7694	0.8597	0.7832	0.7832	0.4346	0.4346	0.2835	0.2835

Table A.4: Cement 2005-2008 comparison with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

## A.2 Cement: second instance

In the second instance, we have considered:

Input

- Energy
- Clinker production plus Clinker import
- Raw materials
- Labour
- Capacity

Output

- Cement production

Undesirable factor

- CO<sub>2</sub> emissions

## A.2.1 Contemporaneous frontier for representative plants: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	1	1	1	1	1	1	0	0
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	0.9965	1	0.9965	1	0.7247	0.7247	0.0047	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8517	1	0.8517	1	0.1561	0.1561	0.1777	0
France	0.9225	0.9225	0.9188	0.9268	0.4562	0.4562	0.0898	0.0898
Germany	0.9456	0.9456	0.9420	0.9452	0.4152	0.4152	0.0562	0.0562
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8769	0.8902	0.8769	0.9183	0.2021	0.2021	0.1381	0.1381
Poland	0.9703	0.9784	0.9703	0.9812	0.7579	0.7579	0.0254	0.0212
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8701	0.9352	0.8701	0.9622	0.3766	0.3766	0.1313	0.1284
U.S.A.	0.9315	1	0.9315	1	0.2449	0.2449	0.1096	0
United Kingdom	0.8605	0.8786	0.8605	0.9192	0.2087	0.2087	0.1516	0.1516
<b>2006</b>								
Australia	0.8447	0.8559	0.8447	0.8975	0.3417	0.3417	0.1853	0.1853
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9163	0.9170	0.9163	0.9286	0.3614	0.3614	0.0835	0.0835
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8665	1	0.8665	1	0.3100	0.3100	0.1560	0
France	0.9703	0.9911	0.9703	0.9917	0.3831	0.3831	0.0463	0.0241
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8715	0.8973	0.8715	0.9279	0.2057	0.2057	0.1799	0.1799
Poland	0.8106	0.8501	0.8106	0.9198	0.4471	0.4471	0.2026	0.2026
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.9043	0.9406	0.9043	0.9628	0.4630	0.4630	0.1061	0.1016
U.S.A.	0.9573	1	0.9573	1	0.2000	0.2000	0.0884	0
United Kingdom	0.8312	0.8467	0.8312	0.9091	0.3051	0.3051	0.1886	0.1886
<b>2007</b>								
Australia	0.8570	0.8627	0.8570	0.8950	0.4275	0.4275	0.1700	0.1700
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9379	0.9732	0.9379	0.9839	0.4247	0.4247	0.0670	0.0456
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8418	1	0.8418	1	0.3335	0.3335	0.1499	0
France	0.9813	1	0.9813	1	0.2953	0.2953	0.0408	0
Germany	0.9516	0.9520	0.9516	0.9663	0.4566	0.4566	0.0518	0.0518
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9026	0.9297	0.9026	0.9532	0.3220	0.3220	0.1260	0.1260
Poland	0.8502	0.8938	0.8502	0.9575	0.5526	0.5526	0.1478	0.1478
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8943	0.9129	0.8943	0.9340	0.5213	0.5213	0.1199	0.1149
U.S.A.	0.9641	1	0.9641	1	0.1882	0.1882	0.0807	0
United Kingdom	0.8407	0.9042	0.8407	0.9557	0.3537	0.3537	0.1804	0.1804
<b>2008</b>								
Australia	1	1	1	1	1	1	0	0
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9450	0.9701	0.9450	0.9861	0.4840	0.4840	0.0583	0.0535
Denmark	1	1	1	1	1	1	0	0
Estonia	0.7937	1	0.7937	1	0.1684	0.1684	0.1958	0
France	0.9919	1	0.9919	1	0.2983	0.2983	0.0125	0
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9547	0.9923	0.9547	0.9953	0.3348	0.3348	0.0658	0.0453
Poland	1	1	1	1	1	1	0	0
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8854	0.8913	0.8854	0.9254	0.6287	0.6287	0.0960	0.0960
U.S.A.	0.9949	1	0.9949	1	0.1912	0.1912	0.1153	0
United Kingdom	0.8448	1	0.8448	1	0.2084	0.2084	0.1839	0.1839

Table A.5: Cement 2005-2008 comparison for plants with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

## A.2.2 Contemporaneous frontier with aggregated data: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	1	1	1	1	1	1	0	0
Austria	0.9881	0.9881	0.9870	0.9870	0.8580	0.8580	0.0125	0.0125
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	0.9860	1	0.9860	1	0.7070	0.7070	0.0149	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.9241	1	0.9241	1	0.4595	0.4595	0.0882	0
Estonia	1	1	1	1	1	1	0	0
France	0.9003	0.9003	0.8987	0.9211	0.7447	0.7447	0.0942	0.0942
Germany	0.9347	0.9347	0.9335	0.9454	0.8559	0.8559	0.0510	0.0510
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9866	0.9866	0.9855	0.9855	0.8264	0.8264	0.0167	0.0167
Poland	0.9551	0.9551	0.9548	0.9604	0.6796	0.6796	0.0458	0.0458
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8695	0.9412	0.8695	0.9763	0.6797	0.6797	0.1385	0.1385
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8619	0.8785	0.8619	0.9245	0.5960	0.5960	0.1543	0.1543
<b>2006</b>								
Australia	0.8386	0.8386	0.8381	0.8934	0.5061	0.5061	0.1801	0.1801
Austria	0.6966	0.6966	0.6819	0.8077	0.2051	0.2051	0.3456	0.3456
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9500	0.9500	0.9459	0.9459	0.3811	0.3811	0.0517	0.0517
Denmark	0.9669	1	0.9669	1	0.5328	0.5328	0.0362	0
Estonia	1	1	1	1	1	1	0	0
France	0.9222	0.9222	0.9218	0.9440	0.7219	0.7219	0.0816	0.0816
Germany	0.9938	0.9938	0.9936	0.9936	0.9395	0.9395	0.0060	0.0060
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.8175	0.8417	0.8175	0.9253	0.5107	0.5107	0.2142	0.2142
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.9040	1	0.9040	1	0.6751	0.6751	0.1039	0
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8336	0.8566	0.8336	0.9348	0.5694	0.5694	0.1897	0.1897
<b>2007</b>								
Australia	0.8534	0.8534	0.8530	0.8919	0.5139	0.5139	0.1611	0.1611
Austria	0.7460	0.7460	0.7293	0.8291	0.2040	0.2040	0.2786	0.2786
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9891	1	0.9891	1	0.6673	0.6673	0.0119	0
Denmark	1	1	1	1	1	1	0	0
Estonia	1	1	1	1	1	1	0	0
France	0.9186	0.9233	0.9186	0.9441	0.6742	0.6742	0.0897	0.0893
Germany	0.9405	0.9428	0.9405	0.9606	0.7593	0.7593	0.0618	0.0618
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.8463	0.8889	0.8463	0.9357	0.5207	0.5207	0.1831	0.1831
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8929	0.9086	0.8929	0.9387	0.7068	0.7068	0.1106	0.1075
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8536	0.8979	0.8536	0.9536	0.5676	0.5676	0.1730	0.1730
<b>2008</b>								
Australia	0.9909	0.9926	0.9909	0.9926	0.6871	0.6871	0.0094	0.0077
Austria	0.8310	0.8451	0.8310	0.9042	0.1893	0.1893	0.1985	0.1985
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9929	1	0.9929	1	0.7800	0.7800	0.0077	0
Denmark	1	1	1	1	1	1	0	0
Estonia	1	1	1	1	1	1	0	0
France	0.9305	0.9338	0.9305	0.9543	0.7628	0.7628	0.0754	0.0752
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9660	0.9660	0.9651	0.9654	0.7885	0.7885	0.0346	0.0346
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.9140	0.9140	0.9035	0.9322	0.9091	0.9091	0.0353	0.0353
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8561	1	0.8561	1	0.4807	0.4807	0.1715	0.1702

Table A.6: Cement 2005-2008 comparison with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

### A.2.3 Sequential frontier for representative plants: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	1	1	1	1	1	1	0	0
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	0.9965	1	0.9965	1	0.7247	0.7247	0.0047	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8517	1	0.8517	1	0.1561	0.1561	0.1777	0
France	0.9225	0.9225	0.9188	0.9268	0.4562	0.4562	0.0898	0.0898
Germany	0.9456	0.9456	0.9420	0.9452	0.4152	0.4152	0.0562	0.0562
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8769	0.8902	0.8769	0.9183	0.2021	0.2021	0.1381	0.1381
Poland	0.9703	0.9784	0.9703	0.9812	0.7579	0.7579	0.0254	0.0212
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8701	0.9352	0.8701	0.9622	0.3766	0.3766	0.1313	0.1284
U.S.A.	0.9315	1	0.9315	1	0.2449	0.2449	0.1096	0
United Kingdom	0.8605	0.8786	0.8605	0.9192	0.2087	0.2087	0.1516	0.1516
<b>2006</b>								
Australia	0.8271	0.8271	0.8265	0.8292	0.3330	0.3330	0.1895	0.1895
Austria	0.9377	0.9649	0.9645	0.9645	0.5639	0.5639	0.2085	0.2085
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9105	0.9125	0.9107	0.9107	0.3497	0.3497	0.0910	0.0910
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8581	1	0.8581	0.9761	0.2882	0.2882	0.1685	0
France	0.9314	0.9314	0.9405	0.9405	0.3618	0.3618	0.0778	0.0778
Germany	0.9820	0.9820	0.9959	0.9959	0.8315	0.8315	0.0176	0.0176
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	0.9977	1	1	1	0.9934	0.9934	0.0015	0
Norway	0.8440	0.8572	0.8440	0.8502	0.1936	0.1936	0.1870	0.1870
Poland	0.8057	0.8501	0.8057	0.8671	0.4464	0.4464	0.2029	0.2029
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8899	0.9201	0.8899	0.9093	0.4622	0.4622	0.1139	0.1106
U.S.A.	0.8999	0.9695	0.8999	0.9365	0.1986	0.1986	0.1736	0.1215
United Kingdom	0.8308	0.8459	0.8308	0.8500	0.2167	0.2167	0.1892	0.1892
<b>2007</b>								
Australia	0.8349	0.8349	0.8348	0.8373	0.4080	0.4080	0.1754	0.1754
Austria	0.9627	1	1	1	0.6164	0.6164	0.1442	0.1442
Belgium	0.9675	0.9848	0.9675	0.9680	0.8761	0.8761	0.0275	0.0274
Brazil	1	1	1	1	1	1	0	0
Canada	0.9912	0.9978	0.9962	0.9962	0.8601	0.8601	0.0088	0.0028
China	1	1	1	1	1	1	0	0
Czech Republic	0.9214	0.9432	0.9214	0.9214	0.3672	0.3672	0.0869	0.0852
Denmark	1	1	1	1	1	1	0	0
Estonia	0.7625	1	0.7625	1	0.3079	0.3079	0.2193	0
France	0.9129	0.9129	0.9293	0.9293	0.2638	0.2638	0.1005	0.1005
Germany	0.9210	0.9210	0.9332	0.9332	0.4012	0.4012	0.0799	0.0799
India	1	1	1	1	1	1	0	0
Italy	0.9933	1	1	1	0.9628	0.9628	0.0086	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8666	0.8792	0.8699	0.8699	0.3125	0.3125	0.1555	0.1555
Poland	0.8413	0.8674	0.8413	0.8624	0.5510	0.5510	0.1599	0.1599
Spain	1	1	1	1	1	1	0	0
Switzerland	0.9997	0.9997	1	1	0.9875	0.9875	0.0005	0.0005
Turkey	0.8675	0.8694	0.8675	0.8682	0.5165	0.5165	0.1355	0.1355
U.S.A.	0.8937	0.9674	0.8937	0.9077	0.1713	0.1713	0.1781	0.1436
United Kingdom	0.8324	0.8562	0.8325	0.8325	0.2688	0.2688	0.1847	0.1847
<b>2008</b>								
Australia	0.8509	0.8509	0.8484	0.9840	0.4614	0.4614	0.1583	0.1583
Austria	0.9791	1	0.9791	1	0.7236	0.7236	0.1028	0.1028
Belgium	0.9788	0.9788	0.9710	1	0.9222	0.9222	0.0086	0.0086
Brazil	1	1	1	1	1	1	0	0
Canada	0.9454	0.9846	0.9454	1	0.5794	0.5794	0.0583	0.0210
China	1	1	1	1	1	1	0	0
Czech Republic	0.9233	0.9339	0.9233	0.9861	0.4004	0.4004	0.0792	0.0792
Denmark	0.9940	0.9940	0.9934	1	0.9759	0.9759	0.0047	0.0047
Estonia	0.6569	1	0.6569	1	0.1485	0.1485	0.3976	0.3976
France	0.9316	0.9316	0.9265	0.9986	0.2968	0.2968	0.0780	0.0780
Germany	0.9716	0.9716	0.9638	0.9989	0.7518	0.7518	0.0268	0.0268
India	0.9924	1	0.9924	1	0.9148	0.9148	0.0094	0
Italy	0.9618	0.9618	0.9618	1	0.8418	0.8418	0.0343	0.0343
Japan	0.9876	0.9885	0.9876	1	0.9642	0.9642	0.0096	0.0096
Norway	0.8800	0.8854	0.8800	0.9953	0.2867	0.2867	0.1402	0.1402
Poland	1	1	0.9999	1	1	1	0	0
Spain	0.9798	0.9798	0.9764	1	0.7946	0.7946	0.0196	0.0196
Switzerland	1	1	0.9406	0.9406	1	1	0	0
Turkey	0.8528	0.8544	0.8528	0.9254	0.5524	0.5524	0.1451	0.1451
U.S.A.	0.8559	0.9468	0.8559	1	0.1902	0.1902	0.1902	0.1902
United Kingdom	0.8238	0.8755	0.8238	1	0.1969	0.1969	0.2103	0.2103

Table A.7: Cement 2005-2008 comparison for plants with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

## A.2.4 Sequential frontier with aggregated data: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	1	1	1	1	1	1	0	0
Austria	0.9881	0.9881	0.9870	0.9870	0.8580	0.8580	0.0125	0.0125
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	0.9860	1	0.9860	1	0.7070	0.7070	0.0149	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.9241	1	0.9241	1	0.4595	0.4595	0.0882	0
Estonia	1	1	1	1	1	1	0	0
France	0.9003	0.9003	0.8987	0.9211	0.7447	0.7447	0.0942	0.0942
Germany	0.9347	0.9347	0.9335	0.9454	0.8559	0.8559	0.0510	0.0510
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9866	0.9866	0.9855	0.9855	0.8264	0.8264	0.0167	0.0167
Poland	0.9551	0.9551	0.9548	0.9604	0.6796	0.6796	0.0458	0.0458
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8695	0.9412	0.8695	0.9763	0.6797	0.6797	0.1385	0.1385
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8619	0.8785	0.8619	0.9245	0.5960	0.5960	0.1543	0.1543
<b>2006</b>								
Australia	0.8263	0.8263	0.8367	0.8367	0.5033	0.5033	0.1859	0.1859
Austria	0.6941	0.6941	0.6810	0.6810	0.1931	0.1931	0.3539	0.3539
Belgium	1	1	1	1	1	1	0	0
Brazil	0.9977	0.9977	1	1	0.9871	0.9871	0.0017	0.0017
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9441	0.9441	0.9459	0.9459	0.3613	0.3613	0.0581	0.0581
Denmark	0.9410	1	0.9631	0.9631	0.4569	0.4569	0.0672	0
Estonia	1	1	1	1	1	1	0	0
France	0.9058	0.9058	0.9212	0.9212	0.7211	0.7211	0.0926	0.0926
Germany	0.9806	0.9806	0.9936	0.9936	0.9203	0.9203	0.0157	0.0157
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	0.9796	0.9846	1	1	0.9480	0.9480	0.0127	0.0127
Norway	0.9533	0.9533	1	1	0.5949	0.5949	0.0652	0.0652
Poland	0.8132	0.8408	0.8168	0.8168	0.5090	0.5090	0.2203	0.2203
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8855	0.9458	0.9009	0.9009	0.6750	0.6750	0.1115	0.1081
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8328	0.8497	0.8330	0.8330	0.5673	0.5673	0.1906	0.1906
<b>2007</b>								
Australia	0.8309	0.8309	0.8520	0.8520	0.5059	0.5059	0.1726	0.1726
Austria	0.7170	0.7170	0.7289	0.7289	0.1786	0.1786	0.3099	0.3099
Belgium	0.9831	0.9897	1	1	0.8825	0.8825	0.0192	0.0192
Brazil	1	1	1	1	1	1	0	0
Canada	0.9914	0.9985	1	1	0.8576	0.8576	0.0086	0.0020
China	1	1	1	1	1	1	0	0
Czech Republic	0.9506	0.9535	0.9886	0.9886	0.3870	0.3870	0.0522	0.0522
Denmark	0.9459	1	1	1	0.5952	0.5952	0.0614	0
Estonia	1	1	1	1	1	1	0	0
France	0.8843	0.8843	0.9172	0.9172	0.6717	0.6717	0.1139	0.1139
Germany	0.9159	0.9159	0.9399	0.9399	0.7582	0.7582	0.0746	0.0746
India	1	1	1	1	1	1	0	0
Italy	0.9954	1	1	1	0.9711	0.9711	0.0045	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.8250	0.8576	0.8449	0.8449	0.5168	0.5168	0.2041	0.2041
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8574	0.8574	0.8911	0.8911	0.7066	0.7066	0.1243	0.1243
U.S.A.	0.9944	0.9944	1	1	0.9941	0.9941	0.0023	0.0023
United Kingdom	0.8324	0.8527	0.8525	0.8525	0.5617	0.5617	0.1889	0.1889
<b>2008</b>								
Australia	0.8449	0.8449	0.8984	0.8984	0.5311	0.5311	0.1572	0.1572
Austria	0.7189	0.7189	0.7673	0.7673	0.1794	0.1794	0.3075	0.3075
Belgium	0.9834	0.9834	1	1	0.9295	0.9295	0.0151	0.0151
Brazil	1	1	1	1	1	1	0	0
Canada	0.9442	0.9890	0.9818	0.9818	0.5568	0.5568	0.0592	0.0151
China	1	1	1	1	1	1	0	0
Czech Republic	0.9580	0.9580	0.9858	0.9858	0.4729	0.4729	0.0427	0.0427
Denmark	0.9472	0.9772	1	1	0.5315	0.5315	0.0604	0.0432
Estonia	1	1	1	1	0.6638	0.6638	0.1208	0.1208
France	0.9013	0.9013	0.9235	0.9235	0.6831	0.6831	0.1002	0.1002
Germany	0.9711	0.9711	1	1	0.8953	0.8953	0.0249	0.0249
India	1	1	1	1	1	1	0	0
Italy	0.9525	0.9543	1	1	0.8360	0.8360	0.0407	0.0407
Japan	0.9800	0.9907	1	1	0.8697	0.8697	0.0157	0.0133
Norway	1	1	1	1	1	1	0	0
Poland	0.9506	0.9506	0.9595	0.9595	0.7556	0.7556	0.0485	0.0485
Spain	0.9760	0.9760	1	1	0.9181	0.9181	0.0193	0.0193
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8501	0.8513	0.8721	0.8721	0.7086	0.7086	0.1268	0.1268
U.S.A.	0.9619	0.9636	1	1	0.9242	0.9242	0.0288	0.0288
United Kingdom	0.8295	0.8962	0.8451	0.8451	0.4346	0.4346	0.2031	0.2031

Table A.8: Cement 2005-2008 comparison with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

### A.3 Cement: third instance

In the third instance, we have considered:

Input

- Energy
- Clinker production plus Clinker import
- Raw materials
- Labour

Output

- Cement production

Undesirable factor

- CO<sub>2</sub> emissions



### A.3.1 Contemporaneous frontier for representative plants: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	0.8410	0.8410	0.8371	0.8650	0.2180	0.2180	0.1816	0.1816
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	0.9839	1	0.9839	1	0.7136	0.7136	0.0159	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8498	1	0.8498	1	0.1557	0.1557	0.1808	0
France	0.9207	0.9207	0.9164	0.9222	0.2801	0.2801	0.0898	0.0898
Germany	0.9456	0.9456	0.9420	0.9452	0.4152	0.4152	0.0562	0.0562
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8769	0.8902	0.8769	0.9183	0.2021	0.2021	0.1381	0.1381
Poland	0.9703	0.9784	0.9703	0.9812	0.7579	0.7579	0.0254	0.0212
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8701	0.9352	0.8701	0.9622	0.3766	0.3766	0.1313	0.1284
U.S.A.	0.8745	1	0.8745	1	0.2165	0.2165	0.1566	0
United Kingdom	0.8605	0.8786	0.8605	0.9192	0.2087	0.2087	0.1516	0.1516
<b>2006</b>								
Australia	0.8447	0.8559	0.8447	0.8932	0.2763	0.2763	0.1947	0.1947
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9163	0.9170	0.9163	0.9286	0.3614	0.3614	0.0835	0.0835
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8550	1	0.8550	1	0.1568	0.1568	0.1727	0
France	0.9703	0.9911	0.9703	0.9917	0.3734	0.3734	0.0463	0.0241
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8715	0.8973	0.8715	0.9279	0.2057	0.2057	0.1799	0.1799
Poland	0.8106	0.8501	0.8106	0.9198	0.4471	0.4471	0.2026	0.2026
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.9043	0.9406	0.9043	0.9628	0.4630	0.4630	0.1061	0.1016
U.S.A.	0.9501	1	0.9501	1	0.2000	0.2000	0.1063	0
United Kingdom	0.8312	0.8467	0.8312	0.9091	0.3051	0.3051	0.1886	0.1886
<b>2007</b>								
Australia	0.8509	0.8621	0.8509	0.8893	0.3282	0.3282	0.1875	0.1875
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9379	0.9732	0.9379	0.9839	0.4247	0.4247	0.0670	0.0456
Denmark	1	1	1	1	1	1	0	0
Estonia	0.6720	1	0.6720	1	0.1802	0.1802	0.4590	0.4590
France	0.9813	1	0.9813	1	0.2953	0.2953	0.0408	0
Germany	0.9516	0.9520	0.9516	0.9663	0.3784	0.3784	0.0518	0.0518
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9026	0.9297	0.9026	0.9532	0.3220	0.3220	0.1260	0.1260
Poland	0.8502	0.8938	0.8502	0.9575	0.5526	0.5526	0.1478	0.1478
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8943	0.9129	0.8943	0.9340	0.5213	0.5213	0.1199	0.1149
U.S.A.	0.9641	1	0.9641	1	0.1882	0.1882	0.0807	0
United Kingdom	0.8407	0.9042	0.8407	0.9557	0.3537	0.3537	0.1804	0.1804
<b>2008</b>								
Australia	0.8536	0.8814	0.8536	0.8999	0.3839	0.3839	0.1733	0.1733
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9450	0.9701	0.9450	0.9861	0.4840	0.4840	0.0583	0.0535
Denmark	1	1	1	1	1	1	0	0
Estonia	0.5794	1	0.5794	1	0.1035	0.1035	0.5269	0.5269
France	0.9919	1	0.9919	1	0.2983	0.2983	0.0125	0
Germany	1	1	1	1	1	1	0	0
India	0.9927	1	0.9927	1	0.8363	0.8363	0.0076	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9388	0.9593	0.9388	0.9763	0.3348	0.3348	0.1077	0.0884
Poland	1	1	1	1	1	1	0	0
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8854	0.8913	0.8854	0.9254	0.6287	0.6287	0.0960	0.0960
U.S.A.	0.9949	1	0.9949	1	0.1912	0.1912	0.1153	0
United Kingdom	0.8426	1	0.8426	1	0.2084	0.2084	0.1908	0.1908

Table A.9: Cement 2005-2008 comparison for plants with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

### A.3.2 Contemporaneous frontier with aggregated data: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	0.8442	0.8442	0.8408	0.8682	0.5360	0.5360	0.1775	0.1775
Austria	0.9881	0.9881	0.9870	0.9870	0.8580	0.8580	0.0125	0.0125
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	0.9850	1	0.9850	1	0.7070	0.7070	0.0151	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.9223	1	0.9223	1	0.3427	0.3427	0.0907	0
Estonia	1	1	1	1	1	1	0	0
France	0.8956	0.8956	0.8946	0.9211	0.7447	0.7447	0.1029	0.1029
Germany	0.9342	0.9342	0.9330	0.9454	0.8559	0.8559	0.0510	0.0510
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9866	0.9866	0.9855	0.9855	0.8264	0.8264	0.0167	0.0167
Poland	0.9551	0.9551	0.9548	0.9604	0.6796	0.6796	0.0458	0.0458
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8695	0.9412	0.8695	0.9763	0.6797	0.6797	0.1385	0.1385
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8619	0.8785	0.8619	0.9245	0.5960	0.5960	0.1543	0.1543
<b>2006</b>								
Australia	0.8370	0.8370	0.8354	0.8800	0.5061	0.5061	0.1894	0.1894
Austria	0.6845	0.6845	0.6692	0.8077	0.2051	0.2051	0.3771	0.3771
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9500	0.9500	0.9459	0.9459	0.3811	0.3811	0.0517	0.0517
Denmark	0.9331	1	0.9331	1	0.4243	0.4243	0.0774	0
Estonia	1	1	1	1	1	1	0	0
France	0.9212	0.9212	0.9211	0.9433	0.7219	0.7219	0.0829	0.0829
Germany	0.9930	0.9930	0.9928	0.9928	0.9329	0.9329	0.0069	0.0069
India	1	1	1	1	1	1	0	0
Italy	0.9989	1	0.9989	1	0.8500	0.8500	0.0011	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.8175	0.8417	0.8175	0.9253	0.5107	0.5107	0.2142	0.2142
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.9040	1	0.9040	1	0.6751	0.6751	0.1039	0
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8336	0.8566	0.8336	0.9348	0.5694	0.5694	0.1897	0.1897
<b>2007</b>								
Australia	0.8507	0.8525	0.8507	0.8852	0.5139	0.5139	0.1797	0.1797
Austria	0.7249	0.7256	0.7249	0.8281	0.2040	0.2040	0.3592	0.3592
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9891	1	0.9891	1	0.6673	0.6673	0.0119	0
Denmark	1	1	1	1	1	1	0	0
Estonia	1	1	1	1	1	1	0	0
France	0.9186	0.9233	0.9186	0.9439	0.6742	0.6742	0.0897	0.0893
Germany	0.9394	0.9428	0.9394	0.9596	0.7593	0.7593	0.0649	0.0649
India	1	1	1	1	1	1	0	0
Italy	0.9932	1	0.9932	1	0.7979	0.7979	0.0069	0
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.8463	0.8889	0.8463	0.9357	0.5207	0.5207	0.1831	0.1831
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8929	0.9086	0.8929	0.9387	0.7068	0.7068	0.1106	0.1075
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8536	0.8979	0.8536	0.9536	0.5676	0.5676	0.1730	0.1730
<b>2008</b>								
Australia	0.8207	0.8207	0.8489	0.8489	0.5059	0.5059	0.2085	0.2085
Austria	0.6717	0.6717	0.7243	0.7243	0.1786	0.1786	0.4027	0.4027
Belgium	0.9792	0.9818	1	1	0.8825	0.8825	0.0206	0.0206
Brazil	1	1	1	1	1	1	0	0
Canada	0.9914	0.9985	1	1	0.8576	0.8576	0.0086	0.0020
China	1	1	1	1	1	1	0	0
Czech Republic	0.9506	0.9535	0.9886	0.9886	0.3870	0.3870	0.0522	0.0522
Denmark	0.9187	1	1	1	0.3926	0.3926	0.0958	0
Estonia	0.8705	1	1	1	0.6168	0.6168	0.3133	0.3133
France	0.8843	0.8843	0.9172	0.9172	0.6717	0.6717	0.1164	0.1164
Germany	0.9123	0.9123	0.9386	0.9386	0.7582	0.7582	0.0796	0.0796
India	1	1	1	1	1	1	0	0
Italy	0.9465	0.9467	0.9917	0.9917	0.7812	0.7812	0.0441	0.0441
Japan	1	1	1	1	1	1	0	0
Norway	0.9924	0.9924	1	1	0.9367	0.9367	0.0104	0.0104
Poland	0.8237	0.8576	0.8449	0.8449	0.5168	0.5168	0.2061	0.2061
Spain	1	1	1	1	1	1	0	0
Switzerland	0.9987	0.9987	1	1	0.9644	0.9644	0.0015	0.0015
Turkey	0.8574	0.8574	0.8911	0.8911	0.7066	0.7066	0.1243	0.1243
U.S.A.	0.9944	0.9944	1	1	0.9941	0.9941	0.0023	0.0023
United Kingdom	0.8324	0.8527	0.8525	0.8525	0.5617	0.5617	0.1889	0.1889

Table A.10: Cement 2005-2008 comparison with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

### A.3.3 Sequential frontier for representative plants: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	0.8410	0.8410	0.8371	0.8650	0.2180	0.2180	0.1816	0.1816
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	0.9839	1	0.9839	1	0.7136	0.7136	0.0159	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8498	1	0.8498	1	0.1557	0.1557	0.1808	0
France	0.9207	0.9207	0.9164	0.9222	0.2801	0.2801	0.0898	0.0898
Germany	0.9456	0.9456	0.9420	0.9452	0.4152	0.4152	0.0562	0.0562
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.8769	0.8902	0.8769	0.9183	0.2021	0.2021	0.1381	0.1381
Poland	0.9703	0.9784	0.9703	0.9812	0.7579	0.7579	0.0254	0.0212
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8701	0.9352	0.8701	0.9622	0.3766	0.3766	0.1313	0.1284
U.S.A.	0.8745	1	0.8745	1	0.2165	0.2165	0.1566	0
United Kingdom	0.8605	0.8786	0.8605	0.9192	0.2087	0.2087	0.1516	0.1516
<b>2006</b>								
Australia	0.8230	0.8230	0.8218	0.8243	0.2747	0.2747	0.2026	0.2026
Austria	0.8558	0.8558	0.8941	0.8941	0.5187	0.5187	0.3064	0.3064
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9104	0.9125	0.9104	0.9104	0.3497	0.3497	0.0911	0.0911
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8519	1	0.8519	0.9461	0.1554	0.1554	0.1773	0
France	0.9314	0.9314	0.9405	0.9405	0.3529	0.3529	0.0778	0.0778
Germany	0.9820	0.9820	0.9959	0.9959	0.8166	0.8166	0.0176	0.0176
India	1	1	1	1	1	1	0	0
Italy	0.9861	0.9861	1	1	0.8662	0.8662	0.0151	0.0151
Japan	0.9977	1	1	1	0.9934	0.9934	0.0015	0
Norway	0.8440	0.8572	0.8440	0.8502	0.1936	0.1936	0.1870	0.1870
Poland	0.8057	0.8501	0.8057	0.8671	0.4464	0.4464	0.2029	0.2029
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8899	0.9201	0.8899	0.9093	0.4622	0.4622	0.1139	0.1106
U.S.A.	0.8398	0.9503	0.8398	0.8997	0.1986	0.1986	0.2099	0.2099
United Kingdom	0.8308	0.8459	0.8308	0.8500	0.2167	0.2167	0.1892	0.1892
<b>2007</b>								
Australia	0.8179	0.8179	0.8202	0.8202	0.3186	0.3186	0.2075	0.2075
Austria	0.8369	0.8369	0.9213	0.9213	0.4851	0.4851	0.3127	0.3127
Belgium	0.9675	0.9848	0.9675	0.9680	0.8761	0.8761	0.0275	0.0274
Brazil	0.9996	0.9996	1	1	0.9688	0.9688	0.0004	0.0004
Canada	0.9912	0.9978	0.9962	0.9962	0.8601	0.8601	0.0088	0.0028
China	1	1	1	1	1	1	0	0
Czech Republic	0.9214	0.9432	0.9214	0.9214	0.3672	0.3672	0.0869	0.0852
Denmark	1	1	1	1	1	1	0	0
Estonia	0.6535	1	0.6535	0.9121	0.1750	0.1750	0.4633	0.4633
France	0.9129	0.9129	0.9293	0.9293	0.2638	0.2638	0.1005	0.1005
Germany	0.9210	0.9210	0.9332	0.9332	0.3458	0.3458	0.0822	0.0822
India	0.9798	0.9859	0.9838	0.9838	0.8701	0.8701	0.0239	0.0239
Italy	0.9627	0.9627	0.9860	0.9860	0.6502	0.6502	0.0412	0.0412
Japan	1	1	1	1	1	1	0	0
Norway	0.8666	0.8792	0.8699	0.8699	0.3125	0.3125	0.1555	0.1555
Poland	0.8413	0.8674	0.8413	0.8624	0.5510	0.5510	0.1599	0.1599
Spain	1	1	1	1	1	1	0	0
Switzerland	0.9973	0.9973	1	1	0.9586	0.9586	0.0034	0.0034
Turkey	0.8675	0.8694	0.8675	0.8682	0.5165	0.5165	0.1355	0.1355
U.S.A.	0.8595	0.9660	0.8595	0.8683	0.1713	0.1713	0.1822	0.1694
United Kingdom	0.8324	0.8562	0.8325	0.8325	0.2688	0.2688	0.1847	0.1847
<b>2008</b>								
Australia	0.8161	0.8161	0.8101	0.8964	0.3539	0.3539	0.2103	0.2103
Austria	0.8506	0.8506	0.8341	1	0.5340	0.5340	0.3010	0.3010
Belgium	0.9788	0.9788	0.9593	1	0.9222	0.9222	0.0086	0.0086
Brazil	1	1	1	1	1	1	0	0
Canada	0.9454	0.9836	0.9454	1	0.5794	0.5794	0.0583	0.0211
China	1	1	1	1	1	1	0	0
Czech Republic	0.9233	0.9339	0.9233	0.9861	0.4004	0.4004	0.0792	0.0792
Denmark	0.9940	0.9940	0.9934	1	0.9759	0.9759	0.0047	0.0047
Estonia	0.5630	0.7674	0.5630	1	0.0981	0.0981	0.5658	0.5658
France	0.9316	0.9316	0.9265	0.9986	0.2968	0.2968	0.0780	0.0780
Germany	0.9715	0.9715	0.9638	0.9989	0.7130	0.7130	0.0280	0.0280
India	0.9649	1	0.9649	1	0.7511	0.7511	0.0373	0.0279
Italy	0.9509	0.9509	0.9485	1	0.5937	0.5937	0.0524	0.0524
Japan	0.9876	0.9885	0.9876	1	0.9642	0.9642	0.0096	0.0096
Norway	0.8800	0.8854	0.8800	0.9763	0.2867	0.2867	0.1402	0.1402
Poland	1	1	0.9999	1	1	1	0	0
Spain	0.9798	0.9798	0.9764	1	0.7946	0.7946	0.0196	0.0196
Switzerland	1	1	0.9406	1	1	1	0	0
Turkey	0.8528	0.8544	0.8528	0.9254	0.5524	0.5524	0.1451	0.1451
U.S.A.	0.8485	0.9409	0.8485	1	0.1902	0.1902	0.1902	0.1902
United Kingdom	0.8238	0.8755	0.8238	1	0.1969	0.1969	0.2112	0.2112

Table A.11: Cement 2005-2008 comparison for plants with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

### A.3.4 Sequential frontier with aggregated data: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	0.8442	0.8442	0.8408	0.8682	0.5360	0.5360	0.1775	0.1775
Austria	0.9881	0.9881	0.9870	0.9870	0.8580	0.8580	0.0125	0.0125
Belgium	1	1	1	1	1	1	0	0
Brazil	1	1	1	1	1	1	0	0
Canada	0.9850	1	0.9850	1	0.7070	0.7070	0.0151	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.9223	1	0.9223	1	0.3427	0.3427	0.0907	0
Estonia	1	1	1	1	1	1	0	0
France	0.8956	0.8956	0.8946	0.9211	0.7447	0.7447	0.1029	0.1029
Germany	0.9342	0.9342	0.9330	0.9454	0.8559	0.8559	0.0510	0.0510
India	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Japan	1	1	1	1	1	1	0	0
Norway	0.9866	0.9866	0.9855	0.9855	0.8264	0.8264	0.0167	0.0167
Poland	0.9551	0.9551	0.9548	0.9604	0.6796	0.6796	0.0458	0.0458
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8695	0.9412	0.8695	0.9763	0.6797	0.6797	0.1385	0.1385
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8619	0.8785	0.8619	0.9245	0.5960	0.5960	0.1543	0.1543
<b>2006</b>								
Australia	0.8261	0.8261	0.8345	0.8345	0.5033	0.5033	0.1995	0.1995
Austria	0.6814	0.6814	0.6692	0.6692	0.1931	0.1931	0.3941	0.3941
Belgium	1	1	1	1	1	1	0	0
Brazil	0.9977	0.9977	1	1	0.9871	0.9871	0.0017	0.0017
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9441	0.9441	0.9459	0.9459	0.3613	0.3613	0.0581	0.0581
Denmark	0.9182	1	0.9319	0.9319	0.3989	0.3989	0.0966	0
Estonia	1	1	1	1	1	1	0	0
France	0.9058	0.9058	0.9204	0.9204	0.7211	0.7211	0.0945	0.0945
Germany	0.9797	0.9797	0.9928	0.9928	0.9168	0.9168	0.0175	0.0175
India	1	1	1	1	1	1	0	0
Italy	0.9697	0.9813	0.9975	0.9975	0.8398	0.8398	0.0260	0.0260
Japan	0.9796	0.9846	1	1	0.9480	0.9480	0.0127	0.0127
Norway	0.9529	0.9529	1	1	0.5949	0.5949	0.0652	0.0652
Poland	0.8132	0.8408	0.8168	0.8168	0.5090	0.5090	0.2203	0.2203
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8855	0.9458	0.9009	0.9009	0.6750	0.6750	0.1115	0.1081
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8328	0.8497	0.8330	0.8330	0.5673	0.5673	0.1906	0.1906
<b>2007</b>								
Australia	0.8207	0.8207	0.8489	0.8489	0.5059	0.5059	0.2085	0.2085
Austria	0.6717	0.6717	0.7243	0.7243	0.1786	0.1786	0.4027	0.4027
Belgium	0.9792	0.9818	1	1	0.8825	0.8825	0.0206	0.0206
Brazil	1	1	1	1	1	1	0	0
Canada	0.9914	0.9985	1	1	0.8576	0.8576	0.0086	0.0020
China	1	1	1	1	1	1	0	0
Czech Republic	0.9506	0.9535	0.9886	0.9886	0.3870	0.3870	0.0522	0.0522
Denmark	0.9187	1	1	1	0.3926	0.3926	0.0958	0
Estonia	0.8705	1	1	1	0.6168	0.6168	0.3133	0.3133
France	0.8843	0.8843	0.9172	0.9172	0.6717	0.6717	0.1164	0.1164
Germany	0.9123	0.9123	0.9386	0.9386	0.7582	0.7582	0.0796	0.0796
India	1	1	1	1	1	1	0	0
Italy	0.9465	0.9467	0.9917	0.9917	0.7812	0.7812	0.0441	0.0441
Japan	1	1	1	1	1	1	0	0
Norway	0.9924	0.9924	1	1	0.9367	0.9367	0.0104	0.0104
Poland	0.8237	0.8576	0.8449	0.8449	0.5168	0.5168	0.2061	0.2061
Spain	1	1	1	1	1	1	0	0
Switzerland	0.9987	0.9987	1	1	0.9644	0.9644	0.0015	0.0015
Turkey	0.8574	0.8574	0.8911	0.8911	0.7066	0.7066	0.1243	0.1243
U.S.A.	0.9944	0.9944	1	1	0.9941	0.9941	0.0023	0.0023
United Kingdom	0.8324	0.8527	0.8525	0.8525	0.5617	0.5617	0.1889	0.1889
<b>2008</b>								
Australia	0.8190	0.8190	0.8418	0.8418	0.5311	0.5311	0.2128	0.2128
Austria	0.6716	0.6716	0.7021	0.7021	0.1794	0.1794	0.4036	0.4036
Belgium	0.9729	0.9729	1	1	0.9295	0.9295	0.0201	0.0201
Brazil	1	1	1	1	1	1	0	0
Canada	0.9442	0.9890	0.9818	0.9818	0.5568	0.5568	0.0592	0.0151
China	1	1	1	1	1	1	0	0
Czech Republic	0.9580	0.9580	0.9858	0.9858	0.4729	0.4729	0.0427	0.0427
Denmark	0.9322	0.9753	1	1	0.4818	0.4818	0.0813	0.0526
Estonia	0.7415	0.7953	1	1	0.6121	0.6121	0.3879	0.3879
France	0.9013	0.9013	0.9235	0.9235	0.6831	0.6831	0.1005	0.1005
Germany	0.9704	0.9704	0.9914	0.9914	0.8953	0.8953	0.0258	0.0258
India	1	1	1	1	1	1	0	0
Italy	0.9361	0.9381	0.9671	0.9671	0.7671	0.7671	0.0587	0.0587
Japan	0.9800	0.9907	1	1	0.8697	0.8697	0.0157	0.0133
Norway	1	1	1	1	1	1	0	0
Poland	0.9481	0.9481	0.9549	0.9549	0.7556	0.7556	0.0511	0.0511
Spain	0.9760	0.9760	1	1	0.9181	0.9181	0.0193	0.0193
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8501	0.8513	0.8721	0.8721	0.7086	0.7086	0.1268	0.1268
U.S.A.	0.9619	0.9636	1	1	0.9242	0.9242	0.0288	0.0288
United Kingdom	0.8291	0.8962	0.8451	0.8451	0.4346	0.4346	0.2037	0.2037

Table A.12: Cement 2008 comparison with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

## A.4 Clinker instance

In the fourth instance, we have considered:

Input

- Energy
- Raw materials
- Labour

Output

- Clinker production

Undesirable factor

- CO<sub>2</sub> emissions

This particular instance was also implemented considering only the European countries.

### A.4.1 Contemporaneous frontier for representative plants: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	0.8131	0.8316	1	1	0.2032	0.2031	0.2181	0.2125
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.7783	0.7821	0.7783	0.7819	0.2292	0.2292	0.2981	0.297
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.9172	1	0.9172	1	0.1555	0.1555	0.0955	0
France	0.8814	0.8961	0.8814	0.8953	0.2743	0.2743	0.1412	0.1255
Germany	0.9156	0.9334	0.9156	0.9160	0.2337	0.2337	0.0935	0.0752
India	0.9985	1	0.9985	1	0.9493	0.9493	0.0019	0
Italy	0.8289	0.8340	0.8289	0.8645	0.2282	0.2282	0.2216	0.2197
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.8727	0.8779	0.8729	0.8729	0.4391	0.4391	0.1399	0.1395
Spain	0.9751	1	0.9763	0.9763	0.4425	0.4425	0.0245	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8689	0.8732	0.8697	0.8697	0.3982	0.3982	0.1359	0.1359
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2006</b>								
Australia	0.8278	0.8278	0.8272	0.8528	0.2587	0.2587	0.1914	0.1914
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.7971	0.7971	0.7864	0.8054	0.2153	0.2153	0.2297	0.2297
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.8936	0.8936	0.8908	0.9120	0.2732	0.2732	0.1139	0.1139
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8509	1	0.8509	1	0.2343	0.2343	0.1348	0
France	0.8684	0.8684	0.8645	0.8724	0.2818	0.2818	0.1454	0.1454
Germany	0.8887	0.8887	0.8815	0.8815	0.2421	0.2421	0.1084	0.1084
India	0.9753	1	0.9753	1	0.7680	0.7680	0.0272	0
Italy	0.8410	0.8410	0.8396	0.8396	0.2396	0.2396	0.1939	0.1939
Japan	1	1	1	1	1	1	0	0
Norway	0.9402	0.9464	0.9402	0.9641	0.6034	0.6034	0.0570	0.0570
Poland	0.9739	0.9841	0.9739	0.9879	0.6049	0.6049	0.0256	0.0227
Spain	0.9805	1	0.9805	1	0.6521	0.6521	0.0198	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8347	0.8347	0.8337	0.8745	0.4555	0.4555	0.1792	0.1792
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.9654	0.9859	0.9654	0.9888	0.7735	0.7735	0.0277	0.0140
<b>2007</b>								
Australia	0.8100	0.8211	0.8100	0.8414	0.2637	0.2637	0.2078	0.2078
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8062	0.8062	0.7997	0.8251	0.2141	0.2141	0.2188	0.2188
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9510	0.9976	0.9510	0.9986	0.2452	0.2452	0.0533	0.0091
Denmark	1	1	1	1	1	1	0	0
Estonia	0.9505	1	0.9505	1	0.4986	0.4986	0.0415	0
France	0.8616	0.8616	0.8572	0.8749	0.2952	0.2952	0.1586	0.1586
Germany	0.9193	0.9193	0.9168	0.9205	0.2693	0.2693	0.0730	0.0730
India	0.9779	1	0.9779	1	0.7608	0.7608	0.0245	0
Italy	0.8289	0.8306	0.8289	0.8416	0.2529	0.2529	0.2047	0.2047
Japan	1	1	1	1	1	1	0	0
Norway	0.9599	0.9694	0.9599	0.9762	0.5610	0.5610	0.0387	0.0387
Poland	0.9791	1	0.9791	1	0.8033	0.8033	0.0187	0
Spain	0.9806	1	0.9806	1	0.6552	0.6552	0.0175	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8854	0.8854	0.8752	0.8802	0.5997	0.5997	0.0892	0.0892
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.9931	1	0.9931	1	0.8494	0.8494	0.0049	0
<b>2008</b>								
Australia	0.7931	0.8145	0.7931	0.8502	0.2277	0.2277	0.2397	0.2397
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.7973	0.7973	0.7913	0.8331	0.3002	0.3002	0.2141	0.2141
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9619	1	0.9619	1	0.3358	0.3358	0.0412	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.9014	1	0.9014	1	0.4829	0.4829	0.0851	0
France	0.8647	0.8647	0.8612	0.8932	0.2968	0.2968	0.1540	0.1540
Germany	0.8791	0.8791	0.8733	0.8788	0.2418	0.2418	0.1221	0.1221
India	0.9791	1	0.9791	1	0.6927	0.6927	0.0231	0
Italy	0.8493	0.8493	0.8457	0.8883	0.2617	0.2617	0.1732	0.1732
Japan	1	1	1	1	1	1	0	0
Norway	0.9626	1	0.9626	1	0.6684	0.6684	0.0340	0
Poland	0.8880	0.8880	0.8810	0.8811	0.7146	0.7146	0.0861	0.0861
Spain	0.9257	0.9256	0.9257	0.9761	0.2939	0.2939	0.0678	0.0678
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.9312	0.9312	0.9184	0.9216	0.6999	0.6999	0.0496	0.0496
U.S.A.	0.9760	1	0.9760	1	0.1912	0.1912	0.0288	0
United Kingdom	0.8565	1	0.8565	1	0.2081	0.2081	0.1533	0.1533

Table A.13: Clinker 2005-2008 comparison for plants with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

## A.4.2 Contemporaneous frontier for representative plants: European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.9252	1	0.9252	1	0.1808	0.1808	0.0893	0
France	0.8965	0.9038	0.9517	0.9517	0.4178	0.4178	0.1368	0.1215
Germany	0.9170	0.9335	0.9709	0.9709	0.2338	0.2338	0.0910	0.0752
Italy	0.8844	0.8844	1	1	0.3271	0.3271	0.1761	0.1761
Norway	1	1	1	1	1	1	0	0
Poland	0.8802	0.8802	0.8804	0.8804	0.5968	0.5968	0.0839	0.0839
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2006</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	1	1	1	1	1	1	0	0
France	1	1	1	1	1	1	0	0
Germany	0.9226	0.9226	0.9200	0.9206	0.2746	0.2746	0.0659	0.0659
Italy	1	1	1	1	1	1	0	0
Norway	0.9873	0.9873	0.9795	0.9893	0.8598	0.8598	0.0085	0.0085
Poland	1	1	1	1	1	1	0	0
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2007</b>								
Austria	0.8888	0.8888	0.8271	0.8687	0.8873	0.8873	0.0442	0.0442
Belgium	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.8522	0.8522	0.7997	0.8272	0.8188	0.8188	0.0690	0.0690
Estonia	1	1	1	1	1	1	0	0
France	1	1	1	1	1	1	0	0
Germany	0.9510	1	0.9510	1	0.8200	0.8200	0.0533	0
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.8932	0.8932	0.8573	0.8795	0.7750	0.7750	0.0875	0.0875
Spain	1	1	1	1	1	1	0	0
Switzerland	0.9779	1	0.9779	1	0.8922	0.8922	0.0245	0
United Kingdom	0.8741	0.8741	0.8367	0.8506	0.6984	0.6984	0.1021	0.1021
<b>2008</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	1	1	1	1	1	1	0	0
France	1	1	1	1	1	1	0	0
Germany	0.8992	0.8992	0.8940	0.8940	0.2418	0.2418	0.1114	0.1114
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9954	0.9954	0.9949	0.9949	0.9864	0.9864	0.0032	0.0032
Spain	0.9406	0.9544	0.9406	0.9866	0.3540	0.3540	0.0441	0.0441
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	0.8735	1	0.8735	1	0.2810	0.2810	0.1363	0.1363

Table A.14: Clinker 2005-2008 comparison for plants with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU countries)

### A.4.3 Contemporaneous frontier with aggregated data: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	0.8302	0.8363	0.8302	0.8426	0.5975	0.5975	0.1874	0.1874
Austria	0.9705	0.9940	0.9705	0.9951	0.2123	0.2123	0.0303	0.0076
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8848	0.8848	0.8134	0.8134	0.8698	0.8698	0.0647	0.0647
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.7135	0.7213	0.7135	0.8745	0.2326	0.2326	0.4160	0.4028
Estonia	1	1	1	1	1	1	0	0
France	0.9200	0.9200	0.8971	0.8990	0.9048	0.9048	0.0446	0.0446
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	0.9185	0.9185	0.8941	0.9150	0.8877	0.8877	0.0526	0.0526
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9034	0.9034	0.8858	0.8877	0.7297	0.7297	0.0735	0.0735
Spain	0.9999	1	0.9999	1	0.9830	0.9830	0.0001	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.9137	0.9137	0.8878	0.8951	0.8822	0.8822	0.0581	0.0581
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2006</b>								
Australia	0.8297	0.8297	0.8231	0.8573	0.6099	0.6099	0.1749	0.1749
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8981	0.8981	0.8463	0.8463	0.8827	0.8827	0.0582	0.0582
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9053	0.9053	0.9032	0.9245	0.2280	0.2280	0.1047	0.1047
Denmark	0.7179	0.7385	0.7179	0.8819	0.2381	0.2381	0.3965	0.3965
Estonia	1	1	1	1	1	1	0	0
France	0.9296	0.9296	0.8989	0.8989	0.9083	0.9083	0.0393	0.0393
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	0.9360	0.9360	0.9154	0.9174	0.8876	0.8876	0.0488	0.0488
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9727	0.9745	0.9727	0.9845	0.7863	0.7863	0.0248	0.0248
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8933	0.8933	0.8475	0.8765	0.8840	0.8840	0.0585	0.0585
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.9834	0.9834	0.9785	0.9826	0.9635	0.9635	0.0105	0.0105
<b>2007</b>								
Australia	0.8181	0.8181	0.8180	0.8424	0.6084	0.6084	0.1829	0.1829
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.9155	0.9155	0.8767	0.8767	0.8761	0.8761	0.0581	0.0581
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9614	1	0.9614	1	0.2695	0.2695	0.0423	0
Denmark	0.7140	0.7368	0.7140	0.9146	0.2851	0.2851	0.4186	0.4186
Estonia	1	1	1	1	1	1	0	0
France	0.9228	0.9228	0.9006	0.9006	0.8834	0.8834	0.0549	0.0549
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	0.9322	0.9322	0.9127	0.9145	0.8763	0.8763	0.0496	0.0496
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9535	0.9546	0.9535	0.9708	0.7722	0.7722	0.0442	0.0442
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	1	1	1	1	1	1	0	0
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.9772	0.9772	0.9720	0.9776	0.9503	0.9503	0.0146	0.0146
<b>2008</b>								
Australia	0.8000	0.8021	0.8000	0.8511	0.5830	0.5830	0.1911	0.1911
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.9162	0.9162	0.8726	0.8726	0.8820	0.8820	0.0512	0.0512
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.9688	0.9948	0.9688	0.9962	0.2866	0.2866	0.0335	0.0091
Denmark	0.7531	0.8048	0.7531	0.9409	0.4044	0.4044	0.3347	0.3347
Estonia	1	1	1	1	1	1	0	0
France	0.9276	0.9276	0.9076	0.9082	0.8644	0.8644	0.0501	0.0501
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	0.9274	0.9274	0.9006	0.9019	0.8743	0.8743	0.0497	0.0497
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9175	0.9175	0.8942	0.8942	0.7940	0.7940	0.0647	0.0647
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	1	1	1	1	1	1	0	0
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.8848	1	0.8848	1	0.6012	0.6012	0.1201	0.1193

Table A.15: Clinker 2005-2008 comparison with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)



#### A.4.4 Contemporaneous frontier with aggregated data: European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Austria	0.7547	0.7547	0.7344	0.7524	0.2088	0.2088	0.3028	0.3028
Belgium	1	1	1	1	1	1	0	0
Czech Republic	0.7794	0.7794	0.7712	0.7925	0.2266	0.2266	0.2904	0.2904
Denmark	0.8770	1	0.8770	1	0.3083	0.3083	0.1579	0.1579
Estonia	1	1	1	1	1	1	0	0
France	0.9376	0.9376	0.8318	0.8318	0.9178	0.9178	0.0340	0.0340
Germany	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.7405	0.7405	0.6203	0.6203	0.7144	0.7144	0.1422	0.1422
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	0.9796	0.9796	0.9771	0.9771	0.8524	0.8524	0.0190	0.0190
<b>2006</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	0.9053	0.9053	0.9032	0.9249	0.2280	0.2280	0.1047	0.1047
Denmark	0.7179	0.7386	0.7179	0.9465	0.2381	0.2381	0.3965	0.3965
Estonia	1	1	1	1	1	1	0	0
France	0.9399	0.9399	0.9064	0.9064	0.9083	0.9083	0.0362	0.0362
Germany	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9727	0.9753	0.9727	0.9857	0.8289	0.8289	0.0248	0.0248
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2007</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	0.9614	1	0.9614	1	0.2722	0.2722	0.0423	0
Denmark	0.7140	0.7382	0.7140	0.9587	0.2851	0.2851	0.4186	0.4186
Estonia	1	1	1	1	1	1	0	0
France	0.9300	0.9300	0.9037	0.9037	0.8834	0.8834	0.0535	0.0535
Germany	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9535	0.9685	0.9535	0.9982	0.7790	0.7790	0.0442	0.0442
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2008</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	0.9688	1	0.9688	1	0.2888	0.2888	0.0335	0
Denmark	0.7531	0.8180	0.7531	0.9711	0.4044	0.4044	0.3347	0.3347
Estonia	1	1	1	1	1	1	0	0
France	0.9354	0.9354	0.9181	0.9181	0.8690	0.8690	0.0500	0.0500
Germany	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9464	0.9464	0.9230	0.9230	0.8237	0.8237	0.0401	0.0401
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	0.8932	1	0.8932	1	0.6332	0.6332	0.1122	0

Table A.16: Clinker 2005-2008 comparison with contemporaneous frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU countries)

### A.4.5 Sequential frontier for representative plants: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	0.8131	0.8317	1	1	0.2032	0.2032	0.2181	0.2125
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.7783	0.7821	0.7783	0.7820	0.2293	0.2293	0.2982	0.2975
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.9173	1	0.9173	1	0.1556	0.1556	0.0956	0
France	0.8814	0.8961	0.8814	0.8953	0.2744	0.2744	0.1413	0.1255
Germany	0.9156	0.9335	0.9156	0.9161	0.2338	0.2338	0.0936	0.0752
India	0.9986	1	0.9986	1	0.9494	0.9494	0.0019	0
Italy	0.8290	0.8341	0.8290	0.8645	0.2283	0.2283	0.2217	0.2197
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.8727	0.8780	0.8729	0.8729	0.4391	0.4391	0.1399	0.1395
Spain	0.9751	1	0.9763	0.9763	0.4425	0.4425	0.0245	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8690	0.8733	0.8697	0.8697	0.3983	0.3983	0.1360	0.1360
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2006</b>								
Australia	0.8152	0.8180	0.8152	0.8153	0.2338	0.2338	0.2162	0.2162
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.7721	0.7721	0.7646	0.7769	0.2027	0.2027	0.2793	0.2793
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.8763	0.8763	0.8742	0.8838	0.2602	0.2602	0.1390	0.1390
Denmark	0.9881	1	0.9881	1	0.9362	0.9362	0.0026	0
Estonia	0.8509	0.9837	0.8509	0.8639	0.1784	0.1784	0.1348	0.0951
France	0.8540	0.8545	0.8540	0.8541	0.2677	0.2677	0.1737	0.1737
Germany	0.8663	0.8663	0.8611	0.8700	0.2280	0.2280	0.1413	0.1413
India	0.9473	1	0.9473	0.9704	0.7286	0.7286	0.0604	0.0604
Italy	0.8269	0.8269	0.8241	0.8284	0.2275	0.2275	0.2177	0.2177
Japan	1	1	1	1	1	1	0	0
Norway	0.9333	0.9443	0.9333	0.9378	0.4583	0.4583	0.0632	0.0632
Poland	0.9586	0.9765	0.9586	0.9666	0.5871	0.5871	0.0401	0.0401
Spain	0.9695	0.9992	0.9695	0.9735	0.4524	0.4524	0.0320	0.0015
Switzerland	1	1	0.9790	0.9810	1	1	0	0
Turkey	0.8251	0.8251	0.8243	0.8294	0.4391	0.4391	0.1906	0.1906
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.9654	0.9807	0.9654	0.9684	0.7185	0.7185	0.0277	0.0181
<b>2007</b>								
Australia	0.8049	0.8068	0.8049	0.8062	0.2612	0.2612	0.2333	0.2333
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.7875	0.7875	0.7805	0.7929	0.1930	0.1930	0.2483	0.2483
Canada	0.9970	0.9970	0.9957	0.9990	0.9756	0.9756	0.0028	0.0028
China	0.9997	1	0.9997	0.9997	0.9785	0.9785	0.0004	0
Czech Republic	0.9357	0.9730	0.9357	0.9458	0.2167	0.2167	0.0711	0.0711
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8959	1	0.8959	0.9065	0.4616	0.4616	0.0970	0
France	0.8544	0.8547	0.8544	0.8549	0.2621	0.2621	0.1756	0.1756
Germany	0.9026	0.9026	0.8989	0.9067	0.2682	0.2682	0.0920	0.0920
India	0.9525	1	0.9525	0.9681	0.7145	0.7145	0.0543	0.0295
Italy	0.8254	0.8267	0.8254	0.8265	0.2242	0.2242	0.2234	0.2231
Japan	0.9739	0.9739	0.9536	0.9730	0.9590	0.9590	0.0171	0.0171
Norway	0.9415	0.9422	0.9415	0.9454	0.4139	0.4139	0.0604	0.0604
Poland	0.9416	0.9508	0.9416	0.9490	0.6907	0.6907	0.0560	0.0560
Spain	0.9583	0.9720	0.9583	0.9630	0.4724	0.4724	0.0438	0.0438
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8461	0.8461	0.8297	0.8313	0.5805	0.5805	0.1286	0.1286
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.9631	0.9823	0.9631	0.9663	0.7196	0.7196	0.0293	0.0169
<b>2008</b>								
Australia	0.7831	0.7840	0.7831	0.7848	0.2254	0.2254	0.2735	0.2735
Austria	1	1	1	1	1	1	0	0
Belgium	0.9954	0.9954	0.9764	1	0.9850	0.9850	0.0031	0.0031
Brazil	0.7700	0.7700	0.7619	0.7809	0.2915	0.2915	0.2265	0.2265
Canada	0.9463	0.9463	0.9432	0.9603	0.6990	0.6990	0.0560	0.0560
China	1	1	1	1	1	1	0	0
Czech Republic	0.9398	0.9598	0.9398	0.9549	0.2316	0.2316	0.0649	0.0649
Denmark	0.7512	0.7512	0.7509	1	0.7342	0.7342	0.0913	0.0913
Estonia	0.8442	0.9217	0.8442	0.8688	0.4554	0.4554	0.1519	0.1298
France	0.8461	0.8461	0.8411	0.8551	0.2795	0.2795	0.1795	0.1795
Germany	0.8588	0.8588	0.8514	0.8652	0.2291	0.2291	0.1490	0.1490
India	0.9368	1	0.9368	0.9713	0.6498	0.6498	0.0731	0.0731
Italy	0.8229	0.8229	0.8172	0.8367	0.2466	0.2466	0.2107	0.2107
Japan	0.9666	0.9666	0.9482	1	0.9334	0.9334	0.0223	0.0223
Norway	0.9429	0.9535	0.9429	0.9496	0.5010	0.5010	0.0542	0.0542
Poland	0.8494	0.8494	0.8379	0.8581	0.6721	0.6721	0.1038	0.1038
Spain	0.9020	0.9025	0.9020	0.9147	0.2872	0.2872	0.0924	0.0924
Switzerland	1	1	0.9895	0.9924	1	1	0	0
Turkey	0.8801	0.8801	0.8600	0.8677	0.6601	0.6601	0.0952	0.0952
U.S.A.	0.9347	0.9347	0.9339	0.9452	0.1903	0.1903	0.0720	0.0720
United Kingdom	0.8430	0.9107	0.8430	0.8543	0.1961	0.1961	0.1722	0.1722

Table A.17: Clinker 2005-2008 comparison for plants with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

## A.4.6 Sequential frontier for representative plants: European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	1	1	1	1	1	1	0	0
Estonia	0.9252	1	0.9252	1	0.1808	0.1808	0.0893	0
France	0.8965	0.9038	0.9517	0.9517	0.4178	0.4178	0.1368	0.1215
Germany	0.9170	0.9335	0.9709	0.9709	0.2338	0.2338	0.0910	0.0752
Italy	0.8844	0.8844	1	1	0.3271	0.3271	0.1761	0.1761
Norway	1	1	1	1	1	1	0	0
Poland	0.8802	0.8802	0.8804	0.8804	0.5968	0.5968	0.0839	0.0839
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2006</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	0.8995	0.8995	0.8957	0.9026	0.4093	0.4093	0.1247	0.1247
Denmark	1	1	1	1	1	1	0	0
Estonia	0.8797	1	0.8797	0.8853	0.1888	0.1888	0.0909	0
France	0.8875	0.8875	0.8839	0.8858	0.3871	0.3871	0.1602	0.1602
Germany	0.8717	0.8717	0.8675	0.8745	0.2280	0.2280	0.1341	0.1341
Italy	0.8840	0.8840	0.8632	0.8632	0.3282	0.3282	0.1789	0.1789
Norway	0.9819	0.9819	0.9793	0.9793	0.7867	0.7867	0.0124	0.0124
Poland	1	1	1	1	1	1	0	0
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	0.9958	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2007</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	0.9428	0.9855	0.9428	0.9525	0.2984	0.2984	0.0702	0.0702
Denmark	1	1	1	1	1	1	0	0
Estonia	1	1	1	1	1	1	0	0
France	0.8875	0.8875	0.8839	0.8851	0.3614	0.3614	0.1597	0.1597
Germany	0.9094	0.9094	0.9056	0.9119	0.2754	0.2754	0.0737	0.0737
Italy	0.8729	0.8729	0.8589	0.8592	0.3047	0.3047	0.1931	0.1931
Norway	0.9790	0.9790	0.9781	0.9796	0.7885	0.7885	0.0151	0.0151
Poland	1	1	1	1	1	1	0	0
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2008</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	0.9860	1	1	1	0	0
Czech Republic	0.9493	0.9674	0.9493	0.9667	0.3357	0.3357	0.0602	0.0602
Denmark	0.9950	0.9990	0.9950	1	0.9731	0.9731	0.0036	0.0009
Estonia	0.9438	0.9439	0.9438	0.9884	0.6240	0.6240	0.0387	0.0387
France	0.8888	0.8888	0.8851	0.8905	0.4396	0.4396	0.1601	0.1601
Germany	0.8618	0.8618	0.8543	0.8701	0.2291	0.2291	0.1394	0.1394
Italy	0.8744	0.8744	0.8663	0.8737	0.3975	0.3975	0.1831	0.1831
Norway	0.9807	0.9807	0.9800	0.9931	0.8415	0.8415	0.0120	0.0120
Poland	0.9785	0.9785	0.8724	0.9167	0.9609	0.9609	0.0096	0.0096
Spain	0.9133	0.9133	0.9125	0.9244	0.3141	0.3141	0.0677	0.0677
Switzerland	1	1	0.9969	1	1	1	0	0
United Kingdom	0.8442	0.9335	0.8442	0.8590	0.2008	0.2008	0.1589	0.1589

Table A.18: Clinker 2005-2008 comparison for plants with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU countries)

### A.4.7 Sequential frontier with aggregated data: European and non-European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Australia	0.8302	0.8363	0.8302	0.8426	0.5975	0.5975	0.1874	0.1874
Austria	0.9705	0.9940	0.9705	0.9951	0.2123	0.2123	0.0303	0.0076
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8848	0.8848	0.8134	0.8134	0.8698	0.8698	0.0647	0.0647
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.7135	0.7213	0.7135	0.8745	0.2326	0.2326	0.4160	0.4028
Estonia	1	1	1	1	1	1	0	0
France	0.9200	0.9200	0.8971	0.8990	0.9048	0.9048	0.0446	0.0446
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	0.9185	0.9185	0.8941	0.9105	0.8877	0.8877	0.0527	0.0527
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9034	0.9034	0.8858	0.8877	0.7297	0.7297	0.0735	0.0735
Spain	0.9999	1	0.9999	1	0.9830	0.9830	0.0001	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.9138	0.9138	0.8878	0.8952	0.8822	0.8822	0.0581	0.0581
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2006</b>								
Australia	0.8155	0.8155	0.8226	0.8226	0.6008	0.6008	0.1862	0.1862
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.8934	0.8934	0.8463	0.8463	0.8827	0.8827	0.0582	0.0582
Canada	1	1	1	1	1	1	0	0
China	1	1	1	1	1	1	0	0
Czech Republic	0.8848	0.8848	0.9031	0.9031	0.2146	0.2146	0.1328	0.1328
Denmark	0.7009	0.7138	0.7174	0.7174	0.2220	0.2220	0.4403	0.4403
Estonia	0.9649	0.9649	1	1	0.9236	0.9236	0.0368	0.0368
France	0.9223	0.9223	0.8989	0.8989	0.9067	0.9067	0.0433	0.0433
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	0.9222	0.9222	0.9152	0.9152	0.8876	0.8876	0.0523	0.0523
Japan	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9636	0.9659	0.9727	0.9727	0.7731	0.7731	0.0357	0.0357
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	0.8897	0.8897	0.8474	0.8474	0.8840	0.8840	0.0585	0.0585
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.9829	0.9829	0.9785	0.9785	0.9538	0.9538	0.0116	0.0116
<b>2007</b>								
Australia	0.8019	0.8019	0.8143	0.8143	0.6076	0.6076	0.1878	0.1878
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Brazil	0.9068	0.9068	0.8767	0.8767	0.8747	0.8747	0.0599	0.0599
Canada	0.9921	0.9921	1	1	0.9627	0.9627	0.0072	0.0072
China	1	1	1	1	1	1	0	0
Czech Republic	0.9442	0.9795	0.9613	0.9613	0.2652	0.2652	0.0619	0.0619
Denmark	0.6970	0.7048	0.7137	0.7137	0.2180	0.2180	0.4471	0.4471
Estonia	0.9663	1	1	1	0.7108	0.7108	0.0590	0
France	0.9138	0.9138	0.9006	0.9006	0.8832	0.8832	0.0554	0.0554
Germany	1	1	1	1	1	1	0	0
India	1	1	1	1	1	1	0	0
Italy	0.9199	0.9199	0.9126	0.9126	0.8738	0.8738	0.0546	0.0546
Japan	0.9915	0.9915	1	1	0.9909	0.9909	0.0043	0.0043
Norway	1	1	1	1	1	1	0	0
Poland	0.9417	0.9419	0.9533	0.9533	0.7696	0.7696	0.0555	0.0555
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
Turkey	1	1	1	1	1	1	0	0
U.S.A.	1	1	1	1	1	1	0	0
United Kingdom	0.9770	0.9770	0.9719	0.9719	0.9477	0.9477	0.0150	0.0150
<b>2008</b>								
Australia	0.7763	0.7763	0.7945	0.7945	0.5830	0.5830	0.2007	0.2007
Austria	1	1	1	1	1	1	0	0
Belgium	0.9934	0.9934	1	1	0.9768	0.9768	0.0047	0.0047
Brazil	0.9064	0.9064	0.8726	0.8726	0.8733	0.8733	0.0568	0.0568
Canada	0.9364	0.9364	1	1	0.7876	0.7876	0.0589	0.0589
China	1	1	1	1	1	1	0	0
Czech Republic	0.9485	0.9653	0.9682	0.9682	0.2568	0.2568	0.0562	0.0562
Denmark	0.7400	0.7438	0.7514	0.7514	0.2849	0.2849	0.3658	0.3658
Estonia	0.8697	0.8900	1	1	0.6054	0.6054	0.1785	0.1785
France	0.9018	0.9018	0.9069	0.9069	0.8616	0.8616	0.0641	0.0641
Germany	0.9956	0.9956	1	1	0.9952	0.9952	0.0022	0.0022
India	1	1	1	1	1	1	0	0
Italy	0.9155	0.9155	0.9002	0.9002	0.8630	0.8630	0.0565	0.0565
Japan	0.9894	0.9894	1	1	0.9851	0.9851	0.0065	0.0065
Norway	1	1	1	1	1	1	0	0
Poland	0.9046	0.9046	0.8942	0.8942	0.7781	0.7781	0.0744	0.0744
Spain	0.9734	0.9734	1	1	0.9360	0.9360	0.0189	0.0189
Switzerland	1	1	1	1	1	1	0	0
Turkey	1	1	1	1	1	1	0	0
U.S.A.	0.9777	0.9777	1	1	0.9777	0.9777	0.0100	0.0100
United Kingdom	0.8682	0.9098	0.8773	0.8773	0.6012	0.6012	0.1388	0.1388

Table A.19: Clinker 20052008 comparison with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU + other countries)

## A.4.8 Sequential frontier with aggregated data: European countries

Country	INP		TR $\beta$		KL		DDF	
	Weak	Strong	Weak	Strong	Weak	Strong	Weak	Strong
<b>2005</b>								
Austria	0.9705	1	0.9705	1	0.2123	0.2123	0.0303	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	1	1	1	1	1	1	0	0
Denmark	0.7135	0.7219	0.7135	0.9178	0.2326	0.2326	0.4160	0.4023
Estonia	1	1	1	1	1	1	0	0
France	0.9442	0.9442	0.9162	0.9162	0.9105	0.9105	0.0389	0.0389
Germany	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9315	0.9315	0.8972	0.8972	0.7951	0.7951	0.0494	0.0494
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2006</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	0.8848	0.8848	0.9031	0.9031	0.2146	0.2146	0.1328	0.1328
Denmark	0.7009	0.7138	0.7174	0.7174	0.2220	0.2220	0.4403	0.4403
Estonia	0.9649	0.9649	1	1	0.9236	0.9236	0.0368	0.0368
France	0.9366	0.9366	0.9064	0.9064	0.9067	0.9067	0.0396	0.0396
Germany	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9636	0.9668	0.9727	0.9727	0.8121	0.8121	0.0331	0.0331
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2007</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	1	1	1	1	1	1	0	0
Czech Republic	0.9442	0.9842	0.9613	0.9613	0.2657	0.2657	0.0619	0.0619
Denmark	0.6970	0.7048	0.7137	0.7137	0.2180	0.2180	0.4471	0.4471
Estonia	0.9663	1	1	1	0.7108	0.7108	0.0590	0
France	0.9298	0.9298	0.9037	0.9037	0.8832	0.8832	0.0535	0.0535
Germany	1	1	1	1	1	1	0	0
Italy	1	1	1	1	1	1	0	0
Norway	1	1	1	1	1	1	0	0
Poland	0.9434	0.9434	0.9534	0.9534	0.7779	0.7779	0.0514	0.0514
Spain	1	1	1	1	1	1	0	0
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	1	1	1	1	1	1	0	0
<b>2008</b>								
Austria	1	1	1	1	1	1	0	0
Belgium	0.9957	0.9957	1	1	0.9860	0.9860	0.0029	0.0029
Czech Republic	0.9485	0.9665	0.9682	0.9682	0.2571	0.2571	0.0562	0.0562
Denmark	0.7400	0.7438	0.7514	0.7514	0.2849	0.2849	0.3658	0.3658
Estonia	0.8697	0.8970	1	1	0.6054	0.6054	0.1785	0.1785
France	0.9191	0.9191	0.9181	0.9181	0.8667	0.8667	0.0631	0.0631
Germany	0.9972	0.9972	1	1	0.9972	0.9972	0.0013	0.0013
Italy	0.9457	0.9457	1	1	0.9069	0.9069	0.0367	0.0367
Norway	1	1	1	1	1	1	0	0
Poland	0.9290	0.9290	0.9230	0.9230	0.7782	0.7782	0.0535	0.0535
Spain	0.9818	0.9818	1	1	0.9500	0.9500	0.0136	0.0136
Switzerland	1	1	1	1	1	1	0	0
United Kingdom	0.8686	1	0.8781	0.8781	0.6089	0.6089	0.1347	0.1347

Table A.20: Clinker 2005-2008 comparison with sequential frontier: INP, TR $\beta$ , KL, DDF models, Weak and Strong disposability (EU countries)

## A.5 Cement without undesirable factor: first instance

### A.5.1 Contemporaneous frontier for representative plants: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	1	0	0.8439	0.2547	0.8555	0.2334	0.9986	0.0012
Austria	1	0	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	1	0
Brazil	0.8764	0.1527	0.8299	0.2147	0.8252	0.2214	0.9321	0.0618
Canada	0.9965	0.0047	1	0	1	0	1	0
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.8416	0.2000	0.8694	0.1584	0.9266	0.0819
Denmark	1	0	1	0	1	0	1	0
Estonia	0.8277	0.2180	0.8648	0.1707	0.8401	0.1514	0.7937	0.1958
France	0.9136	0.1366	0.9482	0.1058	0.9629	0.0887	0.9689	0.0542
Germany	0.8814	0.1402	0.8948	0.1641	0.8872	0.1842	0.9604	0.0424
India	1	0	1	0	1	0	1	0
Italy	1	0	1	0	1	0	1	0
Japan	1	0	1	0	1	0	1	0
Norway	0.8575	0.1759	0.8647	0.2652	0.8868	0.2191	0.9427	0.0753
Poland	0.8499	0.1854	0.7479	0.3310	0.7507	0.2308	0.9840	0.0124
Spain	1	0	1	0	1	0	1	0
Switzerland	0.9371	0.0924	1	0	1	0	1	0
Turkey	0.8023	0.2559	0.8176	0.3059	0.7846	0.3045	0.8256	0.1453
U.S.A.	0.9169	0.1246	0.9427	0.1071	0.9500	0.0943	0.9947	0.1155
United Kingdom	0.8602	0.1694	0.8259	0.2219	0.8346	0.2083	0.8363	0.2025

Table A.21: Cement 2005-2008 comparison for plants with contemporaneous frontier: BCC and DDF models (EU + other countries)

### A.5.2 Contemporaneous frontier with aggregated data: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	1	0	0.8060	0.2451	0.8273	0.2146	0.9909	0.0094
Austria	0.9838	0.0182	0.6532	0.5547	0.7029	0.4502	0.7631	0.3403
Belgium	1	0	1	0	1	0	1	0
Brazil	0.8584	0.1657	0.8167	0.2257	0.8143	0.2184	0.9233	0.0668
Canada	0.9860	0.0149	1	0	1	0	1	0
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.8736	0.1640	0.9308	0.0827	0.9743	0.0286
Denmark	0.8501	0.1893	0.8403	0.2009	0.9487	0.0672	0.9618	0.0494
Estonia	1	0	1	0	1	0	1	0
France	0.8557	0.1701	0.7962	0.2585	0.7881	0.2751	0.8894	0.1254
Germany	0.8691	0.1515	0.8509	0.1763	0.8319	0.2039	0.9602	0.0416
India	1	0	1	0	1	0	1	0
Italy	1	0	1	0	1	0	1	0
Japan	1	0	1	0	1	0	1	0
Norway	0.9713	0.0380	1	0	1	0	1	0
Poland	0.8626	0.1655	0.7515	0.3366	0.7672	0.3141	0.9121	0.0978
Spain	1	0	1	0	1	0	1	0
Switzerland	0.9480	0.0644	1	0	0.9695	0.0390	1	0
Turkey	0.7950	0.2401	0.7658	0.2442	0.7602	0.2429	0.8758	0.1037
U.S.A.	1	0	1	0	1	0	1	0
United Kingdom	0.8559	0.1709	0.8199	0.2239	0.8444	0.1922	0.8384	0.1961

Table A.22: Cement 2005-2008 comparison with contemporaneous frontier: BCC and DDF models (EU + other countries)

### A.5.3 Sequential frontier for representative plants: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	1	0	0.8248	0.2605	0.8346	0.2449	0.8439	0.2284
Austria	1	0	0.9377	0.4151	0.9627	0.2400	0.9791	0.1319
Belgium	1	0	1	0	0.9675	0.0275	0.9710	0.0133
Brazil	0.8764	0.1527	0.8296	0.2150	0.8230	0.2245	0.8260	0.2182
Canada	0.9965	0.0047	1	0	0.9891	0.0120	0.9338	0.0885
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.8411	0.2006	0.8668	0.1618	0.8774	0.1467
Denmark	1	0	1	0	1	0	0.9788	0.0152
Estonia	0.8277	0.2180	0.8536	0.1807	0.7624	0.2193	0.6562	0.4288
France	0.9136	0.1366	0.8866	0.1992	0.8869	0.2023	0.8822	0.1967
Germany	0.8814	0.1402	0.8736	0.1644	0.8606	0.1916	0.8629	0.1760
India	1	0	1	0	1	0	0.9870	0.0156
Italy	1	0	1	0	0.9923	0.0086	0.9598	0.0459
Japan	1	0	0.9961	0.0027	1	0	0.9744	0.0172
Norway	0.8575	0.1759	0.8179	0.2733	0.8505	0.2443	0.8614	0.2024
Poland	0.8499	0.1854	0.7478	0.3310	0.7489	0.2326	0.8170	0.1122
Spain	1	0	1	0	1	0	0.9479	0.0629
Switzerland	0.9371	0.0924	0.9342	0.1018	0.9326	0.1059	0.9304	0.1097
Turkey	0.8023	0.2559	0.7878	0.3060	0.7701	0.3047	0.7587	0.2825
U.S.A.	0.9169	0.1246	0.8871	0.1823	0.8886	0.1841	0.8552	0.2570
United Kingdom	0.8602	0.1694	0.8255	0.2222	0.8322	0.2117	0.7772	0.3031

Table A.23: Cement 2005-2008 comparison for plants with sequential frontier: BCC and DDF models (EU + other countries)

### A.5.4 Sequential frontier with aggregated data: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	1	0	0.7995	0.2555	0.8130	0.2400	0.8365	0.2043
Austria	0.9838	0.0182	0.6474	0.5698	0.6762	0.4998	0.6763	0.4997
Belgium	1	0	1	0	0.9831	0.0195	0.9747	0.0279
Brazil	0.8584	0.1657	0.8166	0.2257	0.8119	0.2184	0.8192	0.1836
Canada	0.9860	0.0149	1	0	0.9882	0.0120	0.9187	0.0885
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.8689	0.1700	0.9003	0.1229	0.9107	0.1089
Denmark	0.8501	0.1893	0.8342	0.2196	0.8836	0.1524	0.8619	0.1929
Estonia	1	0	1	0	1	0	1	0.1614
France	0.8557	0.1701	0.7961	0.2586	0.7805	0.2840	0.7993	0.2536
Germany	0.8691	0.1515	0.8509	0.1764	0.8304	0.2053	0.8424	0.1883
India	1	0	1	0	1	0	1	0
Italy	1	0	1	0	0.9928	0.0070	0.9469	0.0556
Japan	1	0	0.9735	0.0154	1	0	0.9491	0.0282
Norway	0.9713	0.0380	0.9449	0.0896	1	0	1	0
Poland	0.8626	0.1655	0.7513	0.3369	0.7547	0.3297	0.8095	0.2426
Spain	1	0	1	0	1	0	0.9365	0.0681
Switzerland	0.9480	0.0644	0.9532	0.0590	0.9374	0.0802	0.9391	0.0779
Turkey	0.7950	0.2401	0.7658	0.2494	0.7582	0.2480	0.7544	0.2387
U.S.A.	1	0	0.9827	0.0117	0.9749	0.0205	0.8998	0.0734
United Kingdom	0.8559	0.1709	0.8196	0.2242	0.8271	0.2128	0.7694	0.3061

Table A.24: Cement 2005-2008 comparison with sequential frontier: BCC and DDF models (EU + other countries)

## A.6 Cement without undesirable factor: second instance

### A.6.1 Contemporaneous frontier for representative plants: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	1	0	0.8447	0.2104	0.8570	0.1979	0.9986	0.0012
Austria	1	0	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	1	0
Brazil	1	0	1	0	1	0	1	0
Canada	0.9965	0.0047	1	0	1	0	1	0
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9163	0.0934	0.9379	0.0670	0.9450	0.0583
Denmark	1	0	1	0	1	0	1	0
Estonia	0.8517	0.1777	0.8665	0.1560	0.8418	0.1499	0.7937	0.1958
France	0.9188	0.1021	0.9703	0.0463	0.9813	0.0408	0.9919	0.0125
Germany	0.9420	0.0647	0.9991	0.0009	0.9516	0.0549	1	0
India	1	0	1	0	1	0	1	0
Italy	1	0	1	0	1	0	1	0
Japan	1	0	1	0	1	0	1	0
Norway	0.8769	0.1476	0.8715	0.1891	0.9026	0.1285	0.9547	0.0658
Poland	0.9703	0	0.8106	0.2082	0.8502	0.1539	1	0
Spain	1	0	1	0	1	0	1	0
Switzerland	0.9428	0.0662	1	0	1	0	1	0
Turkey	0.8701	0.1313	0.9043	0.1061	0.8943	0.1199	0.8854	0.1124
U.S.A.	0.9315	0.1096	0.9573	0.0884	0.9641	0.0807	0.9949	0.1153
United Kingdom	0.8605	0.1654	0.8312	0.2068	0.8407	0.1917	0.8448	0.1849

Table A.25: Cement 2005-2008 comparison for plants with contemporaneous frontier: BCC and DDF models (EU + other countries)

### A.6.2 Contemporaneous frontier with aggregated data: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	1	0	0.8381	0.1971	0.8530	0.1773	0.9909	0.0094
Austria	0.9854	0.0166	0.6819	0.4882	0.7293	0.3958	0.8310	0.2114
Belgium	1	0	1	0	1	0	1	0
Brazil	1	0	1	0	1	0	1	0
Canada	0.9860	0.0149	1	0	1	0	1	0
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9457	0.0606	0.9891	0.0119	0.9929	0.0077
Denmark	0.9241	0.0882	0.9669	0.0362	1	0	1	0
Estonia	1	0	1	0	1	0	1	0
France	0.8987	0.1134	0.9218	0.0855	0.9186	0.0897	0.9305	0.0754
Germany	0.9335	0.0716	0.9930	0.0071	0.9405	0.0637	1	0
India	1	0	1	0	1	0	1	0
Italy	1	0	1	0	1	0	1	0
Japan	1	0	1	0	1	0	1	0
Norway	0.9746	0.0350	1	0	1	0	1	0
Poland	0.9548	0.0485	0.8175	0.2260	0.8463	0.1854	0.9651	0.0367
Spain	1	0	1	0	1	0	1	0
Switzerland	0.9658	0.0380	1	0	0.9958	0.0044	1	0
Turkey	0.8695	0.1443	0.9040	0.1039	0.8929	0.1106	0.9035	0.0859
U.S.A.	1	0	1	0	1	0	1	0
United Kingdom	0.8619	0.1622	0.8336	0.2023	0.8536	0.1759	0.8561	0.1715

Table A.26: Cement 2005-2008 comparison with contemporaneous frontier: BCC and DDF models (EU + other countries)



### A.6.3 Sequential frontier for representative plants: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	1	0	0.8265	0.2278	0.8348	0.2235	0.8484	0.2100
Austria	1	0	0.9377	0.2896	0.9627	0.1874	0.9791	0.1319
Belgium	1	0	1	0	0.9675	0.0275	0.9710	0.0133
Brazil	1	0	1	0	1	0	1	0
Canada	0.9965	0.0047	1	0	0.9912	0.0088	0.9454	0.0583
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9105	0.1007	0.9214	0.0869	0.9233	0.0833
Denmark	1	0	1	0	1	0	0.9934	0.0060
Estonia	0.8517	0.1777	0.8581	0.1685	0.7625	0.2193	0.6569	0.4226
France	0.9188	0.1021	0.9269	0.0902	0.9082	0.1156	0.9265	0.0911
Germany	0.9420	0.0647	0.9756	0.0263	0.9180	0	0.9638	0.0391
India	1	0	1	0	1	0	0.9924	0.0094
Italy	1	0	1	0	0.9933	0.0086	0.9618	0.0422
Japan	1	0	0.9977	0.0015	1	0	0.9876	0.0103
Norway	0.8769	0.1476	0.8440	0.2031	0.8666	0.1681	0.8800	0.1510
Poland	0.9703	0	0.8057	0.2082	0.8413	0.1649	0.9999	0.0001
Spain	1	0	1	0	1	0	0.9764	0.0249
Switzerland	0.9428	0.0662	0.9356	0.0757	0.9352	0.0783	0.9360	0.0715
Turkey	0.8701	0.1313	0.8899	0.1139	0.8675	0.1400	0.8528	0.1583
U.S.A.	0.9315	0.1096	0.8999	0.1736	0.8937	0.1781	0.8559	0.1945
United Kingdom	0.8605	0.1654	0.8308	0.2079	0.8324	0.2089	0.8238	0.2195

Table A.27: Cement 2005-2008 comparison for plants with sequential frontier: BCC and DDF models (EU + other countries)

### A.6.4 Sequential frontier with aggregated data: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	1	0	0.8240	0.2176	0.8287	0.2142	0.8424	0.1954
Austria	0.9854	0.0166	0.6796	0.4961	0.7093	0.4306	0.7098	0.4293
Belgium	1	0	1	0	0.9831	0.0195	0.9747	0.0279
Brazil	1	0	0.9955	0.0040	0.9948	0.0047	1	0
Canada	0.9860	0.0149	1	0	0.9914	0.0086	0.9442	0.0592
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9399	0.0677	0.9506	0.0545	0.9565	0.0478
Denmark	0.9241	0.0882	0.9410	0.0672	0.9459	0.0614	0.9472	0.0604
Estonia	1	0	1	0	1	0	1	0.1586
France	0.8987	0.1134	0.9046	0.1062	0.8831	0.1334	0.9004	0.1115
Germany	0.9335	0.0716	0.9732	0.0277	0.9144	0.0941	0.9625	0.0392
India	1	0	1	0	1	0	1	0
Italy	1	0	1	0	0.9954	0.0045	0.9525	0.0499
Japan	1	0	0.9796	0.0148	1	0	0.9800	0.0157
Norway	0.9746	0.0350	0.9449	0.0896	1	0	1	0
Poland	0.9548	0.0485	0.8132	0.2326	0.8250	0.2143	0.9470	0.0568
Spain	1	0	1	0	1	0	0.9739	0.0269
Switzerland	0.9658	0.0380	0.9633	0.0405	0.9613	0.0429	0.9668	0.0366
Turkey	0.8695	0.1443	0.8855	0.1115	0.8564	0.1264	0.8501	0.1318
U.S.A.	1	0	0.9827	0.0117	0.9877	0.0104	0.9619	0.0324
United Kingdom	0.8619	0.1622	0.8328	0.2036	0.8324	0.2039	0.8295	0.2087

Table A.28: Cement 2005-2008 comparison with sequential frontier: BCC and DDF models (EU + other countries)

## A.7 Cement without undesirable factor: third instance

### A.7.1 Contemporaneous frontier for representative plants: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	0.8371	0	0.8447	0.2131	0.8509	0.1996	0.8536	0.1885
Austria	1	0	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	1	0
Brazil	1	0	1	0	1	0	1	0
Canada	0.9839	0.0159	1	0	1	0	1	0
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9163	0.0934	0.9379	0.0670	0.9450	0.0583
Denmark	1	0	1	0	1	0	1	0
Estonia	0.8498	0.1808	0.8550	0.1727	0.6720	0.4871	0.5794	0.6755
France	0.9164	0.1021	0.9703	0.0463	0.9813	0.0408	0.9919	0.0125
Germany	0.9420	0.0647	0.9991	0.0009	0.9516	0.0549	1	0
India	1	0	1	0	1	0	0.9927	0.0076
Italy	1	0	1	0	1	0	1	0
Japan	1	0	1	0	1	0	1	0
Norway	0.8769	0.1476	0.8715	0.1891	0.9026	0.1285	0.9388	0.1077
Poland	0.9703	0	0.8106	0.2082	0.8502	0.1539	1	0
Spain	1	0	1	0	1	0	1	0
Switzerland	0.9428	0.0662	1	0	1	0	1	0
Turkey	0.8701	0.1313	0.9043	0.1061	0.8943	0.1199	0.8854	0.1124
U.S.A.	0.8745	0.1566	0.9501	0.1063	0.9641	0.0807	0.9949	0.1153
United Kingdom	0.8605	0.1654	0.8312	0.2068	0.8407	0.1917	0.8426	0.1931

Table A.29: Cement 2005-2008 comparison for plants with contemporaneous frontier: BCC and DDF models (EU + other countries)

### A.7.2 Contemporaneous frontier with aggregated data: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	0.8408	0.1928	0.8354	0.2011	0.8507	0.1805	0.8482	0.1815
Austria	0.9854	0.0166	0.6692	0.5339	0.7249	0.4502	0.7473	0.3644
Belgium	1	0	1	0	1	0	1	0
Brazil	1	0	1	0	1	0	1	0
Canada	0.9850	0.0151	1	0	1	0	1	0
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9457	0.0606	0.9891	0.0119	0.9929	0.0077
Denmark	0.9223	0.0907	0.9331	0.0774	1	0	1	0
Estonia	1	0	1	0	1	0	1	0
France	0.8946	0.1186	0.9211	0.0861	0.9186	0.0897	0.9279	0.0785
Germany	0.9330	0.0722	0.9928	0.0073	0.9394	0.0650	0.9925	0.0076
India	1	0	1	0	1	0	1	0
Italy	1	0	0.9989	0.0011	0.9932	0.0069	0.9711	0.0299
Japan	1	0	1	0	1	0	1	0
Norway	0.9746	0.0350	1	0	1	0	1	0
Poland	0.9548	0.0485	0.8175	0.2260	0.8463	0.1854	0.9572	0.0455
Spain	1	0	1	0	1	0	1	0
Switzerland	0.9658	0.0380	1	0	0.9958	0.0044	1	0
Turkey	0.8695	0.1443	0.9040	0.1039	0.8929	0.1106	0.9035	0.0859
U.S.A.	1	0	1	0	1	0	1	0
United Kingdom	0.8619	0.1622	0.8336	0.2023	0.8536	0.1759	0.8538	0.1755

Table A.30: Cement 2005-2008 comparison with contemporaneous frontier: BCC and DDF models (EU + other countries)

### A.7.3 Sequential frontier for representative plants: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	0.8371	0	0.8218	0.2310	0.8145	0.2387	0.8101	0.2446
Austria	1	0	0.8417	0.4243	0.8227	0.4242	0.8341	0.4135
Belgium	1	0	1	0	0.9675	0	0.9593	0.0187
Brazil	1	0	1	0	0.9996	0.0004	1	0
Canada	0.9839	0.0159	1	0	0.9912	0.0088	0.9454	0.0583
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9104	0.1009	0.9214	0.0869	0.9233	0.0833
Denmark	1	0	1	0	1	0	0.9934	0.0060
Estonia	0.8498	0.1808	0.8519	0.1773	0.6535	0.5037	0.5630	0.7562
France	0.9164	0.1021	0.9269	0.0902	0.9082	0.1156	0.9265	0.0911
Germany	0.9420	0.0647	0.9756	0.0263	0.9180	0.0940	0.9638	0.0391
India	1	0	1	0	0.9798	0.0247	0.9649	0.0373
Italy	1	0	0.9787	0.0253	0.9573	0.0522	0.9485	0.0619
Japan	1	0	0.9977	0.0015	1	0	0.9876	0.0103
Norway	0.8769	0.1476	0.8440	0.2031	0.8666	0.1681	0.8800	0.1510
Poland	0.9703	0	0.8057	0.2082	0.8413	0.1649	0.9999	0.0001
Spain	1	0	1	0	1	0	0.9764	0.0249
Switzerland	0.9428	0.0662	0.9348	0.0757	0.9327	0.0783	0.9360	0.0715
Turkey	0.8701	0.1313	0.8899	0.1139	0.8675	0.1400	0.8528	0.1583
U.S.A.	0.8745	0.1566	0.8398	0.2121	0.8595	0.1822	0.8485	0.1945
United Kingdom	0.8605	0.1654	0.8308	0.2079	0.8324	0.2089	0.8238	0.2201

Table A.31: Cement 2005-2008 comparison for plants with sequential frontier: BCC and DDF models (EU + other countries)

### A.7.4 Contemporaneous frontier with aggregated data: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	0.8408	0.1928	0.8239	0.2178	0.8178	0.2280	0.8150	0.2322
Austria	0.9854	0.0166	0.6674	0.5366	0.6615	0.5495	0.6602	0.5526
Belgium	1	0	1	0	0.9792	0.0228	0.9549	0.0489
Brazil	1	0	0.9955	0.0040	0.9948	0.0047	1	0
Canada	0.9850	0.0151	1	0	0.9914	0.0086	0.9442	0.0592
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9399	0.0677	0.9506	0.0545	0.9565	0.0478
Denmark	0.9223	0.0907	0.9182	0.0966	0.9187	0.0958	0.9322	0.0813
Estonia	1	0	1	0	0.8705	0.3531	0.7415	0.5912
France	0.8946	0.1186	0.9046	0.1062	0.8831	0.1334	0.9004	0.1115
Germany	0.9330	0.0722	0.9732	0.0277	0.9114	0.0977	0.9620	0.0397
India	1	0	1	0	1	0	1	0
Italy	1	0	0.9697	0.0271	0.9465	0.0466	0.9361	0.0685
Japan	1	0	0.9796	0.0148	1	0	0.9800	0.0157
Norway	0.9746	0.0350	0.9393	0.0965	0.9762	0.0355	1	0
Poland	0.9548	0.0485	0.8132	0.2326	0.8237	0.2163	0.9448	0.0593
Spain	1	0	1	0	1	0	0.9739	0.0269
Switzerland	0.9658	0.0380	0.9584	0.0458	0.9556	0.0489	0.9605	0.0434
Turkey	0.8695	0.1443	0.8855	0.1115	0.8564	0.1264	0.8501	0.1318
U.S.A.	1	0	0.9827	0.0117	0.9877	0.0104	0.9619	0.0324
United Kingdom	0.8619	0.1622	0.8328	0.2036	0.8324	0.2039	0.8291	0.2094

Table A.32: Cement 2005-2008 comparison with contemporaneous frontier: BCC and DDF models (EU + other countries)

## A.8 Clinker without undesirable factor instance

### A.8.1 Contemporaneous frontier for representative plants: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	0.8131	0.2181	0.8272	0.2113	0.8100	0.2198	0.7931	0.2502
Austria	1	0	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	1	0
Brazil	0.7783	0.2982	0.7864	0.2605	0.7997	0.2350	0.7913	0.2367
Canada	1	0	1	0	1	0	1	0
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.8908	0.1271	0.9510	0.0533	0.9619	0.0412
Denmark	1	0	1	0	1	0	1	0
Estonia	0.9173	0.0956	0.8509	0.1348	0.9505	0.0415	0.9014	0.0851
France	0.8814	0.1413	0.8645	0.1647	0.8572	0.1737	0.8612	0.1688
Germany	0.9156	0.0936	0.8804	0.1257	0.9168	0.0782	0.8733	0.1354
India	0.9986	0.0019	0.9753	0.0272	0.9779	0.0245	0.9791	0.0231
Italy	0.8290	0.2217	0.8394	0.2006	0.8289	0.2189	0.8457	0.1908
Japan	1	0	1	0	1	0	1	0
Norway	1	0	0.9402	0.0595	0.9599	0.0389	0.9626	0
Poland	0.8727	0.1399	0.9739	0.0256	0.9791	0	0.8810	0.1234
Spain	0.9751	0.0245	0.9805	0	0.9806	0.0175	0.9257	0.0697
Switzerland	1	0	0.9923	0.0081	1	0	1	0
Turkey	0.8690	0.1428	0.8337	0.2111	0.8752	0.1111	0.9184	0.0692
U.S.A.	1	0	1	0	1	0	0.9760	0
United Kingdom	1	0	0.9654	0.0277	0.9931	0.0049	0.8565	0.1538

Table A.33: Clinker 2005-2008 comparison for plants with contemporaneous frontier: BCC and DDF models (EU + other countries)

### A.8.2 Contemporaneous frontier for representative plants: European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Austria	1	0	1	0	0.8271	0.1342	1	0
Belgium	1	0	1	0	1	0	1	0
Czech Republic	1	0	1	0	1	0	1	0
Denmark	1	0	1	0	0.7997	0.2031	1	0
Estonia	0.9252	0.0893	1	0	1	0	1	0
France	0.8965	0.1368	1	0	1	0	1	0
Germany	0.9170	0.0910	0.9200	0.0706	0.9510	0.0533	0.8939	0.1229
Italy	0.8717	0.2029	1	0	1	0	1	0
Norway	1	0	0.9795	0.0177	1	0	1	0
Poland	0.8796	0.1073	1	0	0.8573	0.1737	0.9775	0.0169
Spain	1	0	1	0	0.9423	0.0387	0.9406	0.0462
Switzerland	1	0	1	0	0.9779	0.0245	1	0
United Kingdom	1	0	1	0	0.8367	0.2107	0.8735	0.1370

Table A.34: Clinker 2005-2008 comparison for plants with contemporaneous frontier: BCC and DDF models (EU countries)

### A.8.3 Contemporaneous frontier with aggregated data: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	0.8302	0.2000	0.8231	0.2085	0.8180	0.2184	0.8000	0.2472
Austria	0.9705	0.0303	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	1	0
Brazil	0.8041	0.2301	0.8283	0.1971	0.8633	0.1541	0.8551	0.1628
Canada	1	0	1	0	1	0	1	0
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.9032	0.1136	0.9614	0.0423	0.9688	0.0335
Denmark	0.7135	0.4160	0.7179	0.4269	0.7140	0.4360	0.7531	0.3527
Estonia	1	0	1	0	1	0	1	0
France	0.8971	0.1137	0.8903	0.1215	0.8952	0.1147	0.9076	0.0974
Germany	0.9477	0.0549	0.9203	0.0849	0.9618	0.0392	0.9419	0.0596
India	1	0	1	0	1	0	1	0
Italy	0.8941	0.1131	0.9154	0.0881	0.9127	0.0914	0.9006	0.1078
Japan	1	0	1	0	1	0	1	0
Norway	1	0	1	0	1	0	1	0
Poland	0.8858	0.1272	0.9727	0.0274	0.9535	0.0474	0.8868	0.1229
Spain	0.9999	0.0001	1	0	1	0	1	0
Switzerland	1	0	1	0	1	0	1	0
Turkey	0.8878	0.1255	0.8475	0.1767	0.8812	0.1247	0.9564	0.0443
U.S.A.	1	0	1	0	1	0	1	0
United Kingdom	1	0	0.9785	0.0203	0.9720	0.0265	0.8848	0.1201

Table A.35: Clinker 2005-2008 comparison with contemporaneous frontier: BCC and DDF models (EU + other countries)

### A.8.4 Contemporaneous frontier with aggregated data: European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Austria	0.7344	0.3823	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	1	0
Czech Republic	0.7712	0.3287	0.9032	0.1136	0.9614	0.0423	0.9688	0.0335
Denmark	0.8770	0.1594	0.7179	0.4269	0.7140	0.4360	0.7531	0.3527
Estonia	1	0	1	0	1	0	1	0
France	0.8276	0.2120	0.8903	0.1215	0.8952	0.1147	0.9180	0.0865
Germany	0.7941	0.2473	0.9371	0.0624	0.9618	0.0378	0.9662	0.0325
Italy	1	0	1	0	1	0	1	0
Norway	1	0	1	0	1	0	1	0
Poland	0.6191	0.5896	0.9727	0.0274	0.9535	0.0474	0.9182	0.0762
Spain	1	0	1	0	1	0	1	0
Switzerland	1	0	1	0	1	0	1	0
United Kingdom	0.9677	0.0340	1	0	1	0	0.8932	0.1122

Table A.36: Clinker 2005-2008 comparison with contemporaneous frontier: BCC and DDF models (EU countries)

### A.8.5 Sequential frontier for representative plants: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	0.8131	0.2181	0.8152	0.2311	0.8049	0.2495	0.7831	0.2882
Austria	1	0	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	0.9764	0.0155
Brazil	0.7783	0.2982	0.7646	0.3032	0.7805	0.2733	0.7619	0.2913
Canada	1	0	1	0	0.9957	0.0046	0.9432	0.0628
China	1	0	1	0	0.9997	0.0004	1	0
Czech Republic	1	0	0.8742	0.1505	0.9357	0.0714	0.9398	0.0666
Denmark	1	0	0.9881	0.0026	1	0	0.7509	0.1070
Estonia	0.9173	0.0956	0.8509	0.1348	0.8959	0.0970	0.8442	0.1519
France	0.8814	0.1413	0.8540	0.1803	0.8544	0.1803	0.8411	0.1979
Germany	0.9156	0.0936	0.8611	0.1546	0.8989	0.1012	0.8514	0.1673
India	0.9986	0.0019	0.9473	0.0606	0.9525	0.0543	0.9368	0.0732
Italy	0.8290	0.2217	0.8241	0.2257	0.8254	0.2234	0.8172	0.2341
Japan	1	0	1	0	0.9536	0.0370	0.9482	0.0487
Norway	1	0	0.9333	0.0651	0.9415	0.0635	0.9429	0.0561
Poland	0.8727	0.1399	0.9586	0.0404	0.9416	0.0583	0.8379	0.1804
Spain	0.9751	0.0245	0.9695	0.0320	0.9583	0.0444	0.9020	0.0985
Switzerland	1	0	0.9790	0.0227	1	0	0.9895	0.0111
Turkey	0.8690	0.1428	0.8243	0.2210	0.8297	0.1697	0.8600	0.1339
U.S.A.	1	0	1	0	1	0	0.9339	0.0743
United Kingdom	1	0	0.9654	0.0277	0.9631	0.0293	0.8430	0.1761

Table A.37: Clinker 2005-2008 comparison for plants with sequential frontier: BCC and DDF models (EU + other countries)

### A.8.6 Sequential frontier for representative plants: European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Austria	1	0	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	0.9860	0.0057
Czech Republic	1	0	0.8957	0.1387	0.9428	0.0706	0.9493	0.0623
Denmark	1	0	1	0	1	0	0.9950	0.0036
Estonia	0.9252	0.0893	0.8797	0.0909	1	0	0.9438	0.0410
France	0.8965	0.1368	0.8839	0.1716	0.8839	0.1701	0.8851	0.1715
Germany	0.9170	0.0910	0.8675	0.1435	0.9056	0.0811	0.8543	0.1556
Italy	0.8717	0.2029	0.8621	0.2172	0.8589	0.2151	0.8663	0.2038
Norway	1	0	0.9793	0.0152	0.9781	0.0179	0.9800	0.0135
Poland	0.8796	0.1073	1	0	1	0	0.8724	0.0831
Spain	1	0	1	0	1	0	0.9125	0.0734
Switzerland	1	0	0.9958	0.0039	1	0	0.9969	0.0024
United Kingdom	1	0	1	0	1	0	0.8442	0.1619

Table A.38: Clinker 2005-2008 comparison with sequential frontier: BCC and DDF models (EU countries)

### A.8.7 Sequential frontier with aggregated data: European and non-European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Australia	0.8302	0.2000	0.8125	0.2237	0.8007	0.2473	0.7760	0.2917
Austria	0.9705	0.0303	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	0.9730	0.0231
Brazil	0.8041	0.2301	0.8172	0.2131	0.8476	0.1717	0.8410	0.1811
Canada	1	0	1	0	0.9906	0.0097	0.9328	0.0731
China	1	0	1	0	1	0	1	0
Czech Republic	1	0	0.8834	0.1395	0.9442	0.0620	0.9485	0.0570
Denmark	0.7135	0.4160	0.7009	0.4618	0.6970	0.4701	0.7400	0.3844
Estonia	1	0	0.9622	0.0536	0.9663	0.0590	0.8697	0.1954
France	0.8971	0.1137	0.8849	0.1274	0.8839	0.1286	0.8777	0.1360
Germany	0.9477	0.0549	0.9117	0.0953	0.9526	0.0490	0.9050	0.1033
India	1	0	1	0	1	0	1	0
Italy	0.8941	0.1131	0.8994	0.1068	0.8941	0.1132	0.8799	0.1299
Japan	1	0	0.9970	0.0029	0.9776	0.0223	0.9777	0.0222
Norway	1	0	1	0	1	0	1	0
Poland	0.8858	0.1272	0.9636	0.0365	0.9417	0.0601	0.8520	0.1676
Spain	0.9999	0.0001	1	0	0.9975	0.0025	0.9556	0.0456
Switzerland	1	0	0.9855	0.0156	1	0	0.9915	0.0092
Turkey	0.8878	0.1255	0.8462	0.1783	0.8746	0.1389	0.8948	0.1143
U.S.A.	1	0	1	0	0.9999	0.0001	0.9293	0.0660
United Kingdom	1	0	0.9772	0.0216	0.9711	0.0274	0.8682	0.1449

Table A.39: Clinker 2005-2008 comparison with sequential frontier: BCC and DDF models (EU + other countries)

### A.8.8 Sequential frontier with aggregated data: European countries

Country	2005		2006		2007		2008	
	BCC	DDF	BCC	DDF	BCC	DDF	BCC	DDF
Austria	0.9705	0.0303	1	0	1	0	1	0
Belgium	1	0	1	0	1	0	0.9957	0.0029
Czech Republic	1	0	0.8834	0.1395	0.9442	0.0620	0.9485	0.0562
Denmark	0.7135	0.4160	0.7009	0.4618	0.6970	0.4701	0.7400	0.3658
Estonia	1	0	0.9622	0.0536	0.9663	0.0590	0.8697	0.1785
France	0.9016	0.1082	0.8849	0.1274	0.8842	0	0.9191	0.0631
Germany	0.9490	0.0535	0.9370	0.0625	0.9527	0.0490	0.9972	0.0013
Italy	1	0	1	0	1	0	0.9457	0.0367
Norway	1	0	1	0	1	0	1	0
Poland	0.8894	0.1135	0.9636	0.0365	0.9420	0.0598	0.9290	0.0535
Spain	1	0	1	0	1	0	0.9818	0.0136
Switzerland	1	0	0.9888	0.0111	1	0	1	0
United Kingdom	1	0	1	0	1	0	0.8686	0.1347

Table A.40: Clinker 2005-2008 comparison with sequential frontier: BCC and DDF models (EU countries)





# Appendix B

## Database Sources

In this appendix, the main web sources for our database construction are collected. These are provided by country and general information on the cement industry are also indicated.

### Cement Industry

The European Cement Association (CEMBUREAU) <http://www.cembureau.be/>

World Business Council for Sustainable Development (WBCSD). *Cement Sustainability Initiative (CSI)* <http://www.wbcscement.org/>

United Nations Commodity Trade Statistics Database (UN Comtrade).  
<http://comtrade.un.org/db/default.aspx>

Eurostat Database.  
<http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home>

OECD employment database.  
[http://www.oecd.org/document/34/0,3343,en\\_2649\\_39023495\\_40917154\\_1\\_1\\_1\\_1,00.html](http://www.oecd.org/document/34/0,3343,en_2649_39023495_40917154_1_1_1_1,00.html)

European Pollutant Emission Register.  
<http://ec.europa.eu/environment/ets/welcome.do>

### Australia

Australian Cement Federation. *Australian cement industry sustainability Report*. 2009. Available at: <http://cement.org.au/publications/environment-sustainability-reports>

Australian Cement Federation. *CIF Technical Reports. FastFacts*. 2009-2005. Available at: <http://cement.org.au/publications/cif-technical-reports>

Australian Cement Federation. *CIF Technical Reports. Review of the Technology Pathway for the Australian Cement Industry 2005 - 2030*. 2007.  
Available at: <http://cement.org.au/publications/cif-technical-reports>

### Austria

Vereinigung der Osterreichischen Zementindustrie (VOZ). *Nachhaltigkeitsbericht 2008/2009 der sterreichischen Zementindustrie*. 2008.  
Available at: [http://www.zementindustrie.at/file\\_upl/voez\\_nhb0809.pdf](http://www.zementindustrie.at/file_upl/voez_nhb0809.pdf)

Mauschitz G. *Emissionen aus Anlagen der sterreichischen Zementindustrie Berichtsjahr 2007*. 2007.

### **Belgium**

Febelcem. *Standpunten. De Belgische cementindustrie*. 2006.

Available at: [http://www.febelcem.be/fileadmin/user\\_upload/rapports\\_annuels/nl/Jaarverslag-cementindustrie-2006-nl.pdf](http://www.febelcem.be/fileadmin/user_upload/rapports_annuels/nl/Jaarverslag-cementindustrie-2006-nl.pdf)

Febelcem. *Standpunten. De Belgische cementindustrie*. 2008.

Available at: [http://www.febelcem.be/fileadmin/user\\_upload/rapports\\_annuels/nl/Jaarverslag-cementindustrie-2008-nl.pdf](http://www.febelcem.be/fileadmin/user_upload/rapports_annuels/nl/Jaarverslag-cementindustrie-2008-nl.pdf)

Febelcem. *Milieurapport van de Belgische cementnijverheid*. 2006.

Available at: <http://www.febelcem.be/index.php?id=rapports-environnementaux&L=2>

Febelcem. *Rapport annuel de l'industrie cimentière belge*. 2008-2009.

Available at: <http://www.febelcem.be/index.php?id=101&L=1>

### **Brazil**

Sindicato Nacional da Indústria do Cimento. *Relatórios Anuais*. 2008.

Available at: <http://www.snic.org.br/>

### **Canada**

Natural Resources Canada. Office of Energy Efficiency. *Energy Consumption Benchmark Guide: Cement Clinker Production*. 2001.

Available at: [http://oee.nrcan.gc.ca/publications/industrial/BenchmCement\\_e.pdf](http://oee.nrcan.gc.ca/publications/industrial/BenchmCement_e.pdf)

Cement Association of Canada. *Canadian Cement Industry. Sustainability Report*. 2008.

Available at: <http://www.uaecement.com/articles/Canadiancement2008.pdf>

Cement Association of Canada. *Canadian Cement Industry. Sustainability Report*. 2010.

Available at: <http://www.cement.ca/>

### **China**

Tsinghua University of China. *Assisting Developing Country Climate Negotiators through Analysis and Dialogue: Report of Energy Saving and CO<sub>2</sub> Emission Reduction Analysis in China Cement Industry*. 2008. Available at:

[http://www.ccap.org/docs/resources/694/China\\_Cement\\_Sector\\_Case\\_Study.pdf](http://www.ccap.org/docs/resources/694/China_Cement_Sector_Case_Study.pdf)

Price, L. *Prospects for Efficiency Improvements in China's Cement Sector*. 2006. Presentation at the "Cement Energy Efficiency Workshop". Available at:

<http://www.iea.org/work/2006/cement/Price.pdf>

WWF. *A blueprint for a climate friendly cement industry*. Available at:

[http://assets.panda.org/downloads/englishsummary\\_\\_1r\\_pdf.pdf](http://assets.panda.org/downloads/englishsummary__1r_pdf.pdf)

Tongbo, S. *A brief on China Cement Status Towards A Sustainable Industry*. 2010. Presentation at the "IEA-BEE International Workshop on Industrial Energy Efficiency". Available at:

<http://www.iea.org/work/2006/cement/Price.pdf>

Taylor, M., C. Tam and D. Gielen. *Energy Efficiency and CO<sub>2</sub> Emissions from the Global Cement Industry*. 2006. Available at:

[http://www.iea.org/work/2006/cement/taylor\\_background.pdf](http://www.iea.org/work/2006/cement/taylor_background.pdf)

## **Czech Republic**

Data and several publications are available at <http://www.svcement.cz/>

## **Denmark**

AalborgPortland (Cementir Holding). *Environmental Report*. 2009.

Available at: [http://www.aalborgportland.com/media/annual\\_report/environmental\\_report\\_2009.pdf](http://www.aalborgportland.com/media/annual_report/environmental_report_2009.pdf)

AalborgPortland (Cementir Holding). *Annual Report*. 2009.

Available at: [http://www.aalborgportland.com/media/annual\\_report/annual\\_reporta\\_2009.pdf](http://www.aalborgportland.com/media/annual_report/annual_reporta_2009.pdf)

## **Estonia**

Kunda Nordic (HeidelbergCement Group). *Sustainability Report. Continuous development is the basis of sustainability*. 2007.

Available at: [http://www.heidelbergcement.com/NR/rdonlyres/7C8311B6-51F6-418A-BCBA-A0787B9923CB/0/Sust\\_Kunda\\_ENG\\_2007.pdf](http://www.heidelbergcement.com/NR/rdonlyres/7C8311B6-51F6-418A-BCBA-A0787B9923CB/0/Sust_Kunda_ENG_2007.pdf)

Further information are available at:

[http://www.heidelbergcement.com/ee/en/kunda/keskkond/sustainability\\_report.htm](http://www.heidelbergcement.com/ee/en/kunda/keskkond/sustainability_report.htm)

## **France**

Cimbeton. *Infociments. Rapport Annuel*. 2008.

Available at: <http://www.infociments.fr/publications/industrie-cimentiere/rapports-activite/ra-g03-2008>

Further information are available at: <http://www.infociments.fr/publications>

## **Germany**

BDZ Deutsche Zementindustrie. *Zement-Jahresbericht. Bundesverband der Deutschen Zementindustrie e.V.* 2009-2010.

Available at:

[http://www.bdzement.de/fileadmin/gruppen/bdz/1Presse\\_Veranstaltung/Jahresberichte/BDZ-Jahresbericht\\_08\\_09.pdf](http://www.bdzement.de/fileadmin/gruppen/bdz/1Presse_Veranstaltung/Jahresberichte/BDZ-Jahresbericht_08_09.pdf)

VDZ Deutsche Zementindustrie. *Umweltdaten der deutschen Zementindustrie*. 2008.

Available at:

[http://www.bdzement.de/fileadmin/gruppen/bdz/Themen/Umwelt/Umweltdaten\\_2008.pdf](http://www.bdzement.de/fileadmin/gruppen/bdz/Themen/Umwelt/Umweltdaten_2008.pdf)

Bundesverband der Deutschen Zementindustrie e.V. and Verein Deutscher Zementwerke e.V. *Zementrohstoffe in Deutschland*. 2002.

VDZ Deutsche Zementindustrie. *Monitoring-Bericht 2004-2007. Verminderung der CO<sub>2</sub>-Emissionen*. 2008.

Further information are available at: <http://www.bdzement.de/167.html>

## **India**

Cement Manufacturers' Association. *Annual Report*. 2008-2009.

Ghosh, A., M. Sabyasachi, I. Rohit, A. Gupta. *Indian Cement Industry. Profitability to come under pressure as new capacities take concrete shape.* 2010.

Saxena, A. *Best Practices & Technologies for energy efficiency in Indian Cement Sector.* Presentation.

De Vries, H.J.M., A. Revi, G.K. Bhat, H. Hilderink, P. Lucas. *India 2050: scenarios for an uncertain future.* Netherlands Environmental Assessment Agency, n. 550033002, 2007.

Ghosh S.P. *Energy Efficiency Initiatives, Estimation of CO<sub>2</sub> Emission and Benchmarking Energy and Environmental Performance in Indian Cement Industry.* Presentation at the “Workshop on CO<sub>2</sub> Benchmarking and Monitoring and CDM Benchmarking in Cement Industry”, 2007.

Singhi, M.K., R. Bhargava. *Sustainable Indian Cement Industry.* Presentation at the “Workshop on International Comparison of Industrial Energy efficiency”, 2010.

Chattopadhyay, S. *The Cement Sustainability Initiative.* Presentation at the “IEA-BEE workshop on energy efficiency”, 2010.

### **Italy**

Aitec. *Relazione Annuale.* 2005-2009. Available at: <http://www.aitecweb.com/>

### **Japan**

Data are available at [www.jcassoc.or.jp/cement/2eng/ea.html](http://www.jcassoc.or.jp/cement/2eng/ea.html)

### **Norway**

Norcem (HeidelbergCement Group). *Rapport om Baerekrafting Utvikling. Vart ansvar a bygge for framtiden.* 2007.

Available at: <http://www.heidelbergcement.com/no/no/norcem/sustainability/Rapporter/index.htm>

Further data are available at

<http://www.heidelbergcement.com/no/no/norcem/sustainability/Rapporter/index.htm>

### **Poland**

Data available at <http://www.polskicement.pl/> for several years.

Dejaa, J., A. Uliasz-Bochenczykb, E. Mokrzyckib. *CO<sub>2</sub> emissions from Polish cement industry.* International Journal of Greenhouse Gas Control Vol. 4, p. 583588, 2010.

### **Spain**

Annual reports are available at [http://www.oficemen.com/reportajePag.asp?id\\_rep=634](http://www.oficemen.com/reportajePag.asp?id_rep=634) for several years.

### **Switzerland**

CemSuisse. *Jahresbericht.* 2010. Available at: <http://www.cemsuisse.ch/cemsuisse/index.html>

CemSuisse. *Kennzahlen.* 2010. Available at: <http://www.cemsuisse.ch/cemsuisse/index.html>

## **Turkey**

Data are available at: <http://www.tcma.org.tr/index.php?page=icerikgoster\&menuID=1>

## **USA**

Portland Cement Association. *Report on sustainable manufacturing*. 2009. Available at: [www.cement.org/smreport09](http://www.cement.org/smreport09)

USGS. Science for a changing world. *Minerals Yearbook. Cement (Advance Release)*. 2007. Available at: <http://minerals.usgs.gov/minerals/pubs/commodity/lime/myb1-2007-lime.pdf>

Further data available at:

<http://www.cement.org/index.asp>,

<http://minerals.usgs.gov/minerals/pubs/commodity/cement/>

## **United Kingdom**

British Cement Association (BCA). *Performance reports*. 2003-2008. Available at: [http://cement.mineralproducts.org/downloads/performance\\_reports.php](http://cement.mineralproducts.org/downloads/performance_reports.php)

Quarry Products Association (QPA). *Sustainable Development Report Summary*. 2008. Available at: [http://www.mineralproducts.org/documents/QPA\\_SD\\_08\\_Rep.pdf](http://www.mineralproducts.org/documents/QPA_SD_08_Rep.pdf)

British Geological Survey. Mineral Profile. *Cement Raw Materials*. 2005. Available at: <http://www.bgs.ac.uk/downloads/start.cfm?id=1408>

Other data and information are available at <http://cement.mineralproducts.org/downloads/>



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