Adaptive STATCOM Control for a Multi-machine Power System
A.H.M.A. Rahim and M. Baber Abbas
King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia; Saudi Electricity Company, Dammam, Saudi Arabia.

Abstract — Synchronous static compensator (STATCOM) can be used to improve the dynamic performance of a power system. This article presents an online adaptive pole-shift method for stabilization of a multi-machine power system. Considering the speed deviation of the generator to which the STATCOM is connected as the output and the magnitude of the converter voltage as the input, an adaptive linear plant model is estimated. The stabilizing control is derived from a variable pole-shift strategy which moves the poles of the identified model to the center of unit circle in z-domain. The controller performance has been tested for various disturbances from a number of simulation studies on a multi-machine power system. It was observed that the adaptive algorithm converges very quickly and also provides robust damping profiles to the under damped power system.

Index Terms — Adaptive control, Variable pole-shift control, Power system damping control.

I. INTRODUCTION

The static synchronous compensator (STATCOM) is a power electronic based synchronous voltage generator that generates a three-phase voltage from a DC capacitor. By controlling the magnitude of the STATCOM voltage, the reactive power exchange between the STATCOM and the transmission line and hence the amount of shunt compensation in the power system can be controlled [1]. The reactive power is provided by the ac inductors, dc bus capacitor and solid state commutating devices in the STATCOM. In addition to reactive power exchange, properly controlled STATCOM can also provide damping to a power system [2, 3].

The modeling, operation and control fundamentals of the STATCOM have been extensively discussed in the literature [1, 4]. While most of the control designs are carried out with linearized models, nonlinear control strategies for STATCOM have also been reported recently. STATCOM controls for stabilization have been attempted through complex Lyapunov procedures for simple power system models [5]. Applications of fuzzy logic and neural network based controls have also been reported [6]. Stabilizers based on conventional linear control theory with fixed parameters can be very well tuned to an operating condition and provide excellent damping under that condition. But they cannot provide effective control over a wide operating range for systems that are nonlinear, time varying and subject to uncertainty. In order to yield satisfactory control performance, it is desirable to develop a controller which has the ability to adjust its parameters from on-line determination of system structure or model according to the environment in which it works. Application of adaptive control methods to power system excitation control systems and static VAR systems have been reported [7]; however, its application to FACTS devices has been very limited.

This article investigates the stability enhancement problem of a multi-machine power system installed with STATCOM through adaptive online control of converter voltage magnitude. The adaptive control strategy developed has been tested for its robustness over wide ranges of operation.

II. THE SYSTEM MODEL WITH STATCOM

A 4-machine power system with STATCOMs located at the middle of the transmission lines connecting each generator to the rest of the grid is shown in Fig.1. The synchronous generator is represented by a two-axis model for the internal voltages and the swing equations, and its excitation system is assumed to be equipped with IEEE type-ST model. The STATCOM is represented by a first order differential equation relating the STATCOM DC capacitor voltage and current.

![Fig. 1 A 4-machine power system with STATCOM](image-url)
The dynamic model for the $i^{th}$ machine in the power system including the exciter and the STATCOM is expressed through the following differential equations:

\[
\begin{align*}
\dot{e}_{d_i} &= \left[ -e_{d_i} + (x_{q_i} - x_{d_i})I_{q_i} \right] \frac{1}{T_{qoi}} \\
\dot{e}_{q_i} &= [E_{fd_i} - e_{q_i} - (x_{d_i} - x_{q_i})I_{di}] \frac{1}{T_{doi}} \\
\dot{\omega}_i &= -\frac{1}{2H_i} [P_{nm} - P_{ei} - K_{D_i} \omega_i] \\
\dot{\delta}_i &= \omega_i - \omega \\
\dot{E}_{fd_i} &= \frac{1}{T_{Ai}} E_{fd_i} - \frac{K_{Ai}}{T_{Ai}} (V_{ti} - V_{fi}) \\
V_{DCi} &= \frac{m_i}{C_{DC}} \left[ I_{sd_i} \cos \psi_i + I_{sdq_i} \sin \psi_i \right]
\end{align*}
\]

(1)

A list of the symbols is given in the Nomenclature.

The generator–STATCOM systems are connected to the network consisting of the transmission lines, transformer, loads, etc. The voltages and currents of the network are considered to be related through algebraic equations. The block diagram in Fig. 2 shows the conversion of the variable from the synchronously rotating network reference frame [D-Q] to individual machine frames [d-q].

[Diagram showing transformation between the network (D-Q) and machine frames (d-q)]

The [D-Q] quantities relate to [d-q] variables by the transformation matrix $T_i = \text{diag} \left[ e^{j \phi_i} \right]$. In this article, the network quantities were transformed to the generator frames for relative ease in controller design. The loads are represented as constant impedances and all the load buses have been eliminated to arrive at a closed-loop representation. The non-state variables in (1) are expressed in terms of the state variables using the reduced network equations. The dynamic equations for the multi-machine system are then written as,

\[
\dot{x} = f(x, u)
\]

where, the state vector $x$ contains variables $[e'_{d_i}, e'_{q_i}, \omega_i, \delta_i, E_{fd_i}, V_{DC}]^T$ of each machine, and $u$ is the vector of controls in the STATCOM circuit.

### III. ADAPTIVE CONTROL SYSTEM

Self-tuning control is one form of adaptive control which has the ability of self-adjusting its control parameters according to system conditions. Fig.3 shows the structure of an adaptive regulator. The self-tuning strategy is composed of two processes - the system identifier and the controller. The identifier determines the parameters of the mathematical model of the system from input-output measurement of the plant. The parameters of the identifier are updated at each sampling instant so that it can track the changes in the controlled plant. The control for the plant is then calculated based on this recursively updated system model.

[Diagram showing structure of the adaptive controller]

The plant model is assumed to be of the form,

\[
A(z^{-1})y(t) = B(z^{-1})u(t) + e(t)
\]

(3)

where, $y(t), u(t)$ and $e(t)$ are system output, input and the white noise, respectively; $z^{-1}$ is the delay operator. The polynomial $A$ and $B$ are defined as:

\[
A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + \ldots \ldots \ldots (4)
\]

\[
B(z^{-1}) = 1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + \ldots \ldots \ldots (5)
\]

The vector of parameters $\theta(t) = [a_1, a_2, \ldots, b_1, b_2, \ldots \ldots ]^T$ are calculated recursively on-line through the recursive least square [11] technique using.

\[
\theta(t+1) = \theta(t) + K(t) [y(t) - \theta^T(t) \phi(t)]
\]

(6)

The measurement vector, modifying gain vector, and the covariance matrix, respectively are:

\[
\phi(t) = [-y(t-1), y(t-2), \ldots, y(t-n_y), u(t-1), u(t-2), \ldots, u(t-n_u)]^T
\]

where, $n_y, n_u$ are the order of the plant model and controller, respectively.
\[ K(t) = \frac{P(t)\phi(t)}{\lambda(t) + \phi^T(t)P(t)\phi(t)} \]  
(7)

\[ P(t+1) = \frac{1}{\lambda(t)}[P(t) - K(t)P(t)\phi(t)] \]

\( \lambda(t) \) is the forgetting factor; \( n_a \) and \( n_b \) denote the order of the polynomials A and B, respectively. The identified parameters in (6) depend on all the past records of input and output samples.

IV. THE POLE-SHIFT CONTROL

Using the parameters obtained from the real time parameter identification method, a self-tuning controller based on pole assignment is computed online and fed to the plant. Under the pole shifting control strategy, the poles of the closed loop system are shifted radially towards the centre of the unit circle in the z-domain by a factor \( \alpha \), which is less than one. The procedure for deriving the pole–shifting algorithm is given below.

Assume that the input and output of the identified plant model are related as,

\[ u(t) = \frac{G(z^{-1})}{F(z^{-1})}y(t) \]  
(8)

where,

\[ F(z^{-1}) = 1 + f_1z^{-1} + f_2z^{-2} + f_3z^{-3} + \ldots \ldots + f_\text{nf}z^{-\text{nf}} \]

\[ G(z^{-1}) = g_0 + g_1z^{-1} + g_2z^{-2} + g_3z^{-3} + \ldots \ldots + g_\text{ng}z^{-\text{ng}} \]

\( n_f = n_b-1, n_g = n_a-1 \)

From (3) and (8) the characteristic polynomial can be derived as,

\[ T(z^{-1}) = A(z^{-1})F(z^{-1}) + B(z^{-1})G(z^{-1}) \]  
(9)

The pole-shifting algorithm makes \( T(z^{-1}) \) take the form of \( A(z^{-1}) \) but the pole locations are shifted by a factor \( \alpha \), i.e.

\[ A(z^{-1})F(z^{-1}) + B(z^{-1})G(z^{-1}) = A(\alpha z^{-1}) \]  
(10)

Expanding both sides of (10) and comparing the coefficients give,

\[
\begin{bmatrix}
1 & 0 & 0 & b_1 & 0 & 0 & f_1 \\
a_1 & 1 & 0 & b_2 & b_1 & 0 & 0 \\
\ldots \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
a_{n_a} & \ldots & 1 & b_{n_b} & \ldots & b_n & f_{\text{nf}} \\
0 & a_{n_a} & a_1 & 0 & b_{n_b} & b_2 & g_0 \\
\ldots \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & a_{n_a} & 0 & 0 & b_{n_b} & g_{\text{ng}} \\
f_0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{bmatrix}
= 
\begin{bmatrix}
a_1(\alpha - 1) \\
a_2(\alpha^2 - 1) \\
\ldots \ldots \\
a_{n_a}(\alpha^{n_a} - 1) \\
0 \\
\ldots \ldots \\
0 \\
\end{bmatrix}
\]

The above is written in the form,

\[ MZ(\alpha) = L(\alpha) \]  
(11)

If parameters \([\{a_i\}, \{b_i\}]\) are identified at every sampling period and pole-shift factor \( \alpha \) is known, the control parameters \( Z = [\{f_i\}, \{g_i\}] \) solved from (11) when substituted in (8) will give,

\[ u(t, \alpha) = X^2(t)Z = X^T(t)M^{-1}L(\alpha) \]  
(12)

Here, \( X(t) = [-u(t-1) -u(t-2)\ldots u(t-n_1) -y(t) -y(t-1) -y(t-2)\ldots y(t-n_2)] \).

The controller objective is to force the system output \( y(t) \) to follow the reference output \( y_r(t) \). The objective function can then be expressed as:

\[ J = \min_{\alpha} [y(t) - y_r(t)]^2 \]  
(13)

It can be shown that the change in \( \alpha \) which minimizes \( J \) can be expressed as,

\[ \Delta \alpha = \frac{\varepsilon_1 - f_1f_2}{\varepsilon_1 + \frac{1}{2}[f_1f_1 + 2f_1f_2 + f_2f_2]} \]  
(14)

In the above,

\[ f_1 = \frac{\partial J}{\partial u}; f_2 = \frac{\partial J}{\partial \alpha}; f_3 = \frac{\partial^2 J}{\partial \alpha^2} \]

The partial derivatives are evaluated from (12) and (13), and the updates of control is obtained considering first few significant terms of the Taylor series expansion of \( u(t, \alpha) \). The algorithm can be started by selecting an initial value of \( \alpha \) and updating it at every sample through the relationship,

\[ \alpha(t) = \alpha(t-1) + \Delta \alpha \]  
(15)

The control function is limited by the upper and lower limits and the pole shift factor should be such that it should be bounded by the reciprocal of the largest value of characteristic root of \( A(z^{-1}) \). The latter requirement is satisfied by constraining the magnitude of \( \alpha \) to unity.

V. RESULTS

The variable pole-shift adaptive control strategy was tested on the 4-machine power system given in Fig.1. The output was considered to be the change in power of generator #2 which is connected to the system through bus 9. The input signal is the modulation index of the STATCOM converter voltage. In order to excite the plant, a sequence of torque pulses was applied on the shaft of generator #2. The diagonal elements of the initial covariance matrix \( P \) is assumed to be \( 2 \times 10^5 \); the initial pole shift factor \( \alpha \) and the forgetting factor of \( 1 \) were used. The starting values of all the parameters were considered to be 0.001 in all the simulations. The model order to be estimated was assumed to be 3.

Fig.5 shows the relative speed deviations of the generators with no control when excited by a sequence of torque steps of +5%, -5%, +5% and -5%. Fig.6 shows the variation of the generator speed with the
adaptive variable pole-shift control applied to the identified process.

Fig. 4 Generator speed deviation when excited with alternate torque steps

Fig. 5 Speed deviation of the generator with adaptive pole-shift control

From Figs 4 and 5, it is apparent that the electromechanical transients are damped very well by the adaptive controller. The plant parameters are unknown at the start of the estimation process giving rise to large overshoots. However, as the identification process progresses, the plant parameters are estimated more and more accurately to yield better updates of the pole shift factor, and hence providing better damping profiles.

Fig. 6 shows the convergence of the pole shift factor as the estimation process progresses. The convergence of the \{a\} and \{b\} parameters of the estimated plant function are shown in Figs. 7 and 8, respectively. The control generated by the adaptive pole-shift algorithm is shown in Fig. 9. It can be observed that the estimation algorithm converges to the desired values rapidly with minimum subsequent control expenditure. The convergence of the algorithm is independent of the initial choice of the pole shift factor \( \alpha \). Fig. 10 shows the rotor angle variations recorded for 2 operating for a 50% input torque pulse on generator \#2 for 0.1s

The loadings are quite apart from each other. It can be observed that the damping properties are very good for the whole range of operation considered. It is to be noted that without control the system is very much under damped, and for this disturbance is on the verge of instability.
VI. CONCLUSION

An adaptive control technique has been used to enhance the dynamic performance of a multi-machine power system installed with a synchronous static compensator. The control employed is in the modulation index of one of the converter voltages. Since the controller design employs linear plant models, the control design can be carried out considering only one input and one output at a time. For power output variation of a particular generator, the algorithm has been shown to converge to estimated parameter model rapidly. The on-line adaptive controller has demonstrated to provide very good damping to the electromechanical modes. The robustness of the controller has been tested considering widely different loading conditions. Coordinated application of controllers at different sites requires exploration.

ACKNOWLEDGEMENT

The authors wish to acknowledge the facilities provided at the King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia and The Saudi Electricity Company-East, Dammam, Saudi Arabia towards this research.

NOMENCLATURE

- $\delta$: Generator rotor angle
- $\omega$: Rotor speed
- $\omega_o$: Base (synchronous) speed
- $P_m$: Mechanical power input
- $P_e$: Electrical power output
- $D$: Damping coefficient of generator
- $e_q$: Quadrature (q) axis internal voltage
- $E_{fd}$: Field voltage
- $x_d, x_q$: Synchronous, transient direct (d) axis reactance
- $I_d$: d-component of armature current
- $K_A, T_A$: Exciter gain, time constant
- $V_i$: Generator terminal voltage
- $E_{fd0}, V_{to}$: Nominal field, terminal voltage
- $V_{dc}$: dc capacitor voltage of STATCOM
- $C_{dc}$: Capacitance of dc capacitor
- $m, \psi$: Modulation index, phase of STATCOM voltage

REFERENCES