In this paper we extend the benchmark model of Diamond-Mortensen-Pissarides in a two-sector general equilibrium framework by introducing a frictionless segment of the labour market where all jobs are filled and all workers are employed. Match friction is the root cause of unemployment in the other sector of the economy. Here, both wages are flexible. Non-frictional wage is determined by the no-arbitrage condition and the frictional wage is determined by the Nash-bargaining solution. We also examine the effects of reforms policies on equilibrium rate of frictional unemployment and wage-gap in our friction oriented small open economy. We find that trade reforms via the reduction in the price of the product produced in the frictional sector reduce both the equilibrium rate of frictional unemployment and wage-gap. However, labour market reforms dig out a trade-off between wage-gap and unemployment rate in a small open economy having frictional labour market. These results may provide a strong theoretical basis for trade reform over the labour market reforms in a small open frictional economy.
ECONOMIC REFORMS, FRICTIONAL UNEMPLOYMENT AND WAGE INEQUALITY: A GENERAL EQUILIBRIUM ANALYSIS

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Abstract: In this paper we extend the benchmark model of Diamond-Mortensen-Pissarides in a two-sector general equilibrium framework by introducing a frictionless segment of the labour market where all jobs are filled and all workers are employed. Match friction is the root cause of unemployment in the other sector of the economy. Here, both wages are flexible. Non-frictional wage is determined by the no-arbitrage condition and the frictional wage is determined by the Nash-bargaining solution. We also examine the effects of reforms policies on equilibrium rate of frictional unemployment and wage-gap in our friction oriented small open economy. We find that trade reforms via the reduction in the price of the product produced in the frictional sector reduce both the equilibrium rate of frictional unemployment and wage-gap. However, labour market reforms dig out a trade-off between wage-gap and unemployment rate in a small open economy having frictional labour market. These results may provide a strong theoretical basis for trade reform over the labour market reforms in a small open frictional economy.

Key words: Economic reforms, frictional unemployment, wage inequality, job-searching, job-matching, general equilibrium.

JEL Classification Number: F16

1. INTRODUCTION

The striking feature of the labour market is that both jobs and workers are heterogeneous. Workers are specialized with respect to their skills and all jobs are not suitable as well as available to all workers. Workers seek high-paid jobs which ensure good working condition, less exploitation and less harassment. At the same time, firms also...
seek good workers who are suitable to the existing jobs. Thus, job-searching and job-matching are the two key features in the complicated, versatile and vast labour market.

In the labour market we find flows of jobs, flows of workers, old jobs are destroyed, both firms and workers search each other to match together. All these ideas have been captured in the search and matching models of the labour market. The path-breaking work in the line is the Diamond-Mortensen-Pissarides (called DMP hereafter) model. Others notable works on this front are Diamond (1982a, 1982b, 1984), Pissarides (1979, 1984, 1985a, 1985b, 1986, 2000), Mortensen (1987, 2011), Mortensen and Pissarides (1994, 1998, 1999). The job-matching models generally explain the existence of frictional unemployment where matching plays the central role to dictate unemployment. The use of the matching function has been first observed in Hall (1979), Pissarides (1979), Diamond and Maskin (1979), Bowden (1980).

The traditional literature shows that matching is a function of unemployment rate and vacancy rate and is subject to the constant return to scale. In the matching framework production starts only when workers and firm are matched. However, matching is a costly and time-consuming process. Once match is formed, cost of searching on both sides are reduced and this generates surplus which is distributed between workers and forms. In the existing literature the most commonly used surplus-sharing rule is the Nash-bargaining solution.

The job-matching models have been extended by introducing cyclicality, efficiency wage etc. These have been found in the models of Albrecht et al. (1984), Cole and Rogerson (1989), Andolfatto (1996), Shimer (2005). Further, the DMP model has also been extended in two-sector general equilibrium set-up by Davidson et al. (1988, 1989), Hosios (1990), Dutta, Mitra and Ranjan (2009).

It is worth noting that the theoretical literature on search and match-induced unemployment has not been adequately dealt with the dualistic structure of the labour market where non-frictional segment co-exists along with the frictional segment of the labour market.

The development economists are very much interested regarding the effects of economic reforms on the world economy. However, these reforms have not been explained elaborately in the search and matching models. The liberalized economic policies have affected the working conditions and welfare of the labour force. Khan (1998) and Tendulkar et al. (1996) have found that the incidence of poverty has increased in India in the post-reform period. Liete et al. (2006) have shown that a significant decrease in average real wage for informal workers in the South Africa during 200–2004. ILO (2006) has shown that the current pattern of globalization continues to have an uneven social impact with some experiencing rising living standards and others left behind.

Wood (1997) argues that diversity in the amount of wage-gap between the East-Asia and the Latin America is probably due to the entry of China into the world economy. Feenstra and Hanson (1996), Marjit et al. (2000, 2004), Chaudhuri and Yabuuchi (2007) and Yabuuchi and Chaudhuri (2007) explain theoretically the deteriorating wage inequality in the developing economies. Empirical studies also show that in the post-reform period the informal sectors have expanded in developing countries. But it could
not mitigate the problem of unemployment as vast pool of workers from the formal sector was not absorbed in the expanding informal sector.

Another economic reform is the reform in the labour market. Many developing countries are now thinking to implement such reform so that the rigid labour laws can be relaxed to attract the large foreign as well as domestic investment in the developing countries (Chaudhuri, 2006). It is generally believed that labour market reforms would lead to a rise in wage inequality and unemployment in developing economies. However, Chaudhuri (2006) has shown that labour market reforms may raise social welfare and soften the problem of unemployment. We also hardly find any work on such reforms in the search and match theoretic models.

In this paper, we develop a two-sector job-search and job-match model in general equilibrium framework along the line of DMP. Here, one sector has match oriented labour market and frictional. The other sector does not have any formal matching in the labour market. In this sector all vacancies are filled and all workers are employed. So, unemployment exists only in the match oriented sector and this is due to the match frictions. We also assume that in both sectors a single firm is attached with single labour which produces one unit of output using only one unit of capital. It has two implications: employment of labour and capital in each sector is identical to total production in that sector and factor intensities in production are equal in the two sectors. In our model, along with match friction there is also distortion in the labour market emanating from different wage rates. Though labour productivities are equal in the two sectors, wage rates differ due to match frictions. Thus match frictions leads to frictional unemployment and friction induced wage-gap.

Our model differs from the DMP (2000) model as follows: firstly, the DMP model uses partial equilibrium framework but our model is fabricated in a two sector general equilibrium set-up. Secondly, the DMP model is a closed one but our model is an open economy model which is handy to investigate the efficacy of trade policies. Thirdly, The DMP model did not consider destruction of capital and they assume that capital can be re-used for new match when a job is destroyed. However, we assume that capital is also destroyed along with job and this is applicable only to the frictional sector where job is destroyed at an exogenous rate. We have tried to synchronise the DMP (2000) model and the Jones (1965) model to analyze the effects of development as well as labour policies in a frictional small open economy. Our results show that trade reforms through the reduction of price of the product produced in the frictional sector reduce both wage-gap and frictional rate of unemployment but in the case of labour market reforms there is a trade-off between wage-gap and the mitigation of the problem of unemployment. These finding may shed light on labour economist to prescribe appropriate policies for the frictional labour market in a small open economy.
2. THE MODEL

We consider a two-sector small open economy. The two sectors are sector 1 and sector 2. Sector 1 is informal, non-frictional and export sector which produces commodity, $X_1$. The other sector 2 is formal, frictional and import-competing sector which produces commodity, $X_2$. The prices of the two commodities $P_1$, $P_2$ are given due to the small country assumption. The price of commodity 1 is chosen as numeraire. The two sectors use both labour and capital in production. The production functions are subject to CRS and fixed coefficient technology. Capital is mobile between the two sectors and this gives a unique rate of return on capital.

Labour is also mobile across the sectors but labour market is segmented. Both workers and the firms search in sector 2 and workers are employed only when they are matched with the firms. In this sector there is match frictions. But in sector 1, workers are matched with the firms instantaneously. In this sector all workers get jobs and all vacancies are filled and so there is no unemployment in this sector. Unemployment exists only in sector 2 and this is due to the matching frictions. Workers in sector 1 are paid according to the no-arbitrage rule. However, in the other sector wage rate is determined by the Nash-bargaining solution. The DMP (2000) model assumes that firm hires capital once a worker is matched with him and there exists a perfect second hand market for capital goods which implies that capital can be used at every instance of time. Thus, in the DMP model destruction of jobs does not lead to the destruction of capital. However, in a frictional labour market where each match (job) needs capital along with labour the assumption of continuous use of capital is irrational and unrealistic. In our two-sector search model we also assume that initially, a firm purchases capital once he is matched with labour. However, when job is destroyed both labour and capital become idle. We assume that both factors are perfectly mobile across the sectors. So, a part of the idle factors move to the non-frictional sector and get employed there. The remaining parts wait for new match in the frictional sector. The frictions in matches may create long duration of vacancies as well as long duration of unemployment which causes wastes of resources. Thus, unlike DMP (2000) we assume that capital is also destroyed along with jobs. For simplicity, we assume a constant rate of destruction of capital in sector 2 where job is destroyed. On the other hand in friction-less sector 1, there is no job destruction and so no idle factors and so no capital destruction. This is the fundamental difference between the DMP model and ours.

In the frictional labour market job-search is an ongoing process. Jobs are offered to the workers and the workers arrive at the jobs offered. So, there exists job-matching between workers and firm in sector 2. Following DMP we may consider the matching function as $m = m(u, v)$, where $m$ stands for matching, $u$ is the rate of unemployment and $v$ is the vacancy rate and $m_1, m_2 > 0, m_{11}, m_{22} < 0, m_{12} = m_{21} = 0$. Total flow of matches is $m = au$ and total flow of jobs is $m = vq$. So, $\frac{m}{u} = a$ is the job arrival rate

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1 Grinols (1991), Chandra and Khan (1993) and Gupta (1997) have assumed in their models that the products of the urban informal sector are export goods. This is also empirically justified in developing countries like India where many hand-made products are exported to the foreign countries.
and \( \frac{m}{v} = q \) is the job offer rate. Matching function is assumed to possess CRS property and so we may write \( q = q(\theta), a \equiv \frac{m}{u} = \frac{m}{v} = \theta q(\theta) \) where \( \theta = \frac{v}{u} \) is the labor market tightness and \( q'(\theta) < 0 \), \(|e'_q| < 1 \).

The general equilibrium structure of the model is as follows:

For the frictional sector the Bellman equations for the values of unemployment \((U)\), employment \((W)\), vacancy \((V)\) and jobs filled in \((J)\) are

\[
\begin{align*}
ru &= b + eq(e)(w - u) \\
rW &= u - \lambda W \\
rV &= -C + q(e)(J - V) \\
rJ &= P_2 - w_2 - (r + \delta) - \lambda J
\end{align*}
\]

Equation (1) states that unemployment gives option of a discrete change in the valuation from \( U \) to \( W \). This equation holds at steady state where discount rate, transaction rate and income flows are constant. Equation (2) embraces that the asset value of employment is the wage rate in sector 2 \( (w_2) \) less employment gain when negative shock arises, where \( \lambda \) the job destruction rate which is given exogenously. Equation (3) shows that the asset value of vacancy yields, at the rate \( q(\theta) \), a discrete change in its valuation from \( V \) to \( J \) less a given flow cost \( C \) to maintain vacancy. Finally, Equation (4) states that the value of a job filled in is the flow of profit \((P_2 - w_2 - (r + \delta))\) to the firm less the jobs destroyed where \( P_2 \) is the international price of the product produced in sector 2, \( \delta \) is the rate of destruction of capital in sector 2.

A firm creates jobs up to the point where \( V = 0 \). Putting this condition into Equation (3) one gets

\[ J = \frac{C}{q(\theta)} \]

This is the job-creation condition in the frictional sector 2.

Substituting (5) into (4) we can write

\[ P_2 = w_2 + (r + \delta) + \frac{(r + \lambda)C}{q(\theta)} \]

This is the price equation for sector 2. This shows that unit price of the product produced in this sector is equal to the wage cost plus rental cost plus recruitment cost of labour. This price equation is different from Jones (1965). In Jones (1965) model destruction of capital and recruitment costs are absent because there does not exist job-matching and job-destructions in the labour market.

The Bellman equations for the non-frictional sector 1 are

\[
\begin{align*}
rU_1 &= 0 \text{ (as there is no unemployment of labour in the informal sector)} \\
rW_1 &= w_1 \text{ (} \lambda = 0 \text{ as there is no job destruction in this sector)} \\
rV_1 &= q(\theta_1)(J_1 - V_1) \quad \because \ C = 0 \text{ here as job is filled instantaneously in this sector}
\end{align*}
\]

\(^2\) Note that in steady state, \( \frac{1}{e'_q(\theta)} \) is the expected duration of vacancy and \( \frac{1}{e'_q(\theta)} \) is the expected duration of unemployment (Pissarides, 2000).
\( rJ_1 = P_1 - w_1 - r \) (\( \therefore \) there is no destructions of job and capital in sector 1) (10)

In sector 1 a firm also creates job up to the point where \( V = 0 \). Putting this condition into Equation (9) one gets

\[ J = 0. \] (11)

This is the job-creation condition in sector 1.

Substituting (11) into (10) we can write

\[ w_1 + r = P_1 = 1 \] (Commodity 1 is taken as numeraire)

(12)

This is the price equation of the non-frictional sector. Note that in the frictional sector job-creation condition is \( J = \frac{C}{q(0)} \). However, in the frictionless sector this condition boils down to \( J = 0 \) as in this sector all vacancies are filled instantaneously and so the cost of maintaining vacancies is zero here. Further, zero profit condition is the competitive equilibrium condition for the non-frictional sector but the competitive equilibrium condition is changed in the case of frictional sector. In this sector the flow of profit is equal to the value of job where job is valued at extended rate of discount.

We assume that in both sectors each firm is matched with only one worker and they together produce only one unit of output by using one unit of capital.\(^3\) Production starts after matching. This implies that all jobs are created at the full productivity of 1 in both sectors.\(^4\) This one-to-one pair of matching between firm, worker and capital in both sectors result in same factor intensities in production across the sectors.\(^5\)

In the search and matching model, production begins when firm and workers are matched. If the match is broken both of them again search and can produce after new match. But the search is expensive which can be saved by staying together. So, match generates surplus. This surplus can be shared by both the matched workers and firms. The most commonly used surplus sharing rule is the Nash-bargaining solution. The Nash-bargaining solution allocates surplus according to the returns from search on both sides.

The Nash-bargaining solution can be obtained from the following exercise:

\[
\max_{\Omega} \Omega = (W - U)^{\beta} (J - V)^{1 - \beta}
\] (13)

Where \( \beta \) is the bargaining strength of the workers and \( 1 > \beta > 0 \).

Assuming interior solutions exist, the first order condition is

\[
(W - U) = \beta(W - U + J - V)
\] (14)

This is the surplus sharing rule in search equilibrium. This rule states that at steady state the net gain to the workers \((W - U)\) is equal to the fixed proportion, \( \beta \) of the total

\(^3\) We find this type of production structure in Davidson et al. (1988, 1989), Hosios (1990), DMP (2000), Priya Ranjan (2000).

\(^4\) Apparently, the equal productivities in both sectors seem to be unrealistic. However, in reality we find many informal jobs are more productive and many formal jobs are less productive. So, the assumption of uniform job productivities between the two sectors is not a vague.

\(^5\) In the DMP (2000) model factor intensity in production differs from factor intensity in sector due to frictional unemployment. This is true in the frictional sector where search unemployment exists. However, in the friction-less sector without any search unemployment the two factor intensities are same.
surplus, \((W - U + J - V)\).

Using Equations (1), (2), (4) and the zero-profit condition for the firm, \(V = 0\), from (14) we can get\(^6\)

\[
w_2 = b + \frac{\beta C \theta}{(1 - \beta)} + \frac{\beta}{1 - \beta}(r + \lambda) \frac{C}{q(\theta)}
\]  

(15)

This is the wage equation for the frictional sector. This shows that wage in frictional sector depends positively on the unemployment benefit, market tightness, discount rate, job destruction rate, worker's bargaining strength and on vacancy costs.

An unemployed worker in sector 2 either searches job in this sector or get employed in sector 1. As job-seeker he gets unemployment income \(rU\) and as worker in sector 1 he gets wage, \(w_1\). Thus no-arbitrage condition implies that in equilibrium,

\[rU = w_1\]  

(16)

Using (1) and (16) one gets

\[
w_1 = b + \frac{\beta C \theta}{(1 - \beta)}
\]  

(17)

Let \(\xi = \frac{\beta C \theta}{(1 - \beta)q(\theta)}\) = Frictional cost of labour in sector 2. We may write \(\xi = \xi(\beta, \theta)\) with \(\xi_\beta, \xi_\theta > 0\) \((\xi_i\) is the elasticity of frictional cost of labour with respect to \(i\) where \(i = \beta, \theta\).

Using this into (15) and (17) one can write

\[
w_1 = b + \xi \theta q(\theta)
\]  

(18)

\[
w_2 = b + \xi(r + \lambda + \theta q(\theta))
\]  

(19)

Thus, both wage rates can be expressed in terms of the matching frictions. This is a great difference between our model and Jones (1965).

Note that \((w_2 - w_1) = \xi(r + \lambda) > 0\). This shows that in this matching model even though the labour productivities are equal in the two sectors wage rates are different and absolute wage-gap is equal to the extended value of the frictional cost of the labour.

The conventional labour force is fixed. Following Pissarides (2000) we may derive the rate of unemployment in the following way:

Suppose, at time \(t\) unemployment is \(u_t\) and employment is \((1 - u_t)\). In short time interval \(\alpha t\), \(\theta_t q(\theta_t) u_t \alpha t\) workers are matched and \(\lambda(1 - u_t) \alpha t\) lose their jobs. So, unemployment in this interval is

\[
u_{t+\alpha t} - u_t = -\theta_t q(\theta_t) u_t \alpha t + \lambda(1 - u_t) \alpha t
\]  

(20)

\[
\therefore \frac{\nu_{t+\alpha t} - u_t}{\alpha t} \to 0 \text{ as } \alpha t \to 0
\]

\[
\therefore \dot{u} = -\theta q(\theta) u + \lambda(1 - u)
\]  

(21)

At steady state, \(\dot{u} = 0\).

\(^6\) See Appendix A.
This is the equilibrium rate of frictional unemployment. This is also known as the Beveridge curve which shows an inverse relation between $u, \theta$, given $\lambda$.

We assume that a single labour and one unit of capital produce one unit of output. So, both labour and capital employment can be expressed in terms of output. Labour is not fully employed but capital is fully employed and a part of capital is destroyed along with jobs. Thus, the two factor endowment equations are

$$X_1 + X_2 = (1 - u)L \quad (23)$$
$$X_1 + (1 + \delta)X_2 = K \quad (24)$$

Where $L, K$ are the fixed supply of labour and capital. Using (1) and (16) into (6) and (12) one gets $b + \xi q(\theta) + r = 1 \quad (25)$
$$\frac{b + \xi q(\theta)}{\beta} + r + \delta \frac{r + \lambda}{q(\theta)} = P_2 \quad (26)$$

Now we can determine the equilibrium values of seven endogenous variables: $w_1, w_2, r, \theta, u, X_1, X_2$ from seven Equations (18), (19), (22), (23), (24), (25) and (26). Solving (25) and (26) we get equilibrium values of $\theta, r$. Then, from (18) and (19) we obtain $w_1, w_2$. From (22) we find $u$. Finally, Equations (23) and (24) determine $X_1, X_2$.

3. COMPARATIVE STATIC EXERCISES

Taking total differentials of Equations (18), (19), (25), (26) and after simplification the following results can be obtained:

$$\frac{d(w_2 - w_1)}{dP_2} > 0 \quad (27)$$
$$\frac{d(w_2 - w_1)}{db} < 0 \quad \text{if } r \geq \theta \quad (28)$$
$$\frac{d(w_2 - w_1)}{dB} > 0 \quad \text{if } \beta \leq e^\xi_\beta \quad (29)$$

These results lead to the following proposition:

**PROPOSITION 1.** In the presence of search frictions in the labour market a decrease in the price of the product produced in the frictional sector reduces absolute wage-gap. A fall in the unemployment benefit may raise such gap. Further, labour market reforms through the reduction in the bargaining power of the labour may reduce the wage gap in a small open economy where labour market is frictional.

We may give an intuitive explanation of proposition 1. Trade liberalization reduces $P_2$. It can be verified from Equations (25) and (26) that a fall in $P_2$ leads to an increase

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7 See Appendix A.
8 See Appendix B, Appendix C.
in $\theta$ and a decrease in $r$. When $\theta$ the average recruitment rises cost, $C\theta$ rises. As a result value of unemployment, $rU$ rises, given $\beta$. This, under the no-arbitrage condition, implies that $w_1$ also rises. From (18) it can be observed that $w_2$ also rises if $\xi < 1$. Now, $w_2$ rises less than $w_1$ if $\xi \theta q(\theta) > (r + \lambda)e_0^\xi$. Therefore, trade liberalization via reduction in commodity price reduces wage-gap in our small open economy where labour market is frictional. On the contrary, a fall in the bargaining power of the labour raises both $\theta$, $r$. From (16) it can be verified that $rU$ falls and so also $w_1$. From (18) it can be observed that $w_2$ also fall. In this case, $w_2$ falls more than $w_1$ if $\xi \theta q(\theta) > (r + \lambda)e_0^\xi$ and $\beta \leq e_0^\xi$. So, wage inequality decreases. Again, a fall in the unemployment benefit raises both wage rates. Here, $w_2$ rises more than $w_1$ if $r > \theta$ and so wage-gap increases.

Taking total differentials of (22) and using (25), (26) and after simplification one gets

$$\left(\frac{\hat{u}}{\hat{\beta}}\right) > 0, \quad \left(\frac{\hat{u}}{\hat{\theta}}\right) > 0, \quad \left(\frac{\hat{u}}{\hat{\beta}}\right) < 0$$

These results give the following proposition:

**PROPOSITION 2.** A fall in the price of the product produced in the frictional sector and / a reduction in the unemployment benefit reduces the equilibrium rate of frictional unemployment in a small open economy. However, a fall in the bargaining strength of the labour union raises frictional unemployment rate. Proposition 2 can be explained as follows. From (25) and (26) it can be verified that a fall in $P_2$ and / $b$ raises $\theta$. From the Beveridge curve (Equation, 22) it is evident that $u$ must fall when $\theta$ rises.

Taking total differentials of (23) and (24) and after simplification one gets

$$\left(\frac{\hat{X}_2 - \hat{X}_1}{L}\right) < 0, \quad \left(\frac{\hat{X}_2 - \hat{X}_1}{K}\right) > 0$$

These results give the following proposition:

**PROPOSITION 3.** In the absence of factor intensity ranking in production, a rise in labour (capital) endowment lowers (raises) relative output in sector 2 if capital is also destroyed along with job in the frictional sector.

4. CONCLUDING REMARKS

In this paper we extend the DMP model in a two-sector general equilibrium framework where one sector is frictional and the other sector is frictionless. Like the DMP model we also assume determination of wage rate in the frictional sector through the Nash-bargaining solution. However, the no-arbitrage rule is applied to determine wage rate in the non-frictional sector where all workers are employed and all jobs are filled. The one-to-one pair of matching between vacancy, worker and capital in both frictional and non-frictional sector leads to same factor intensities in production across the sectors. Like the DMP model we also assume that job is destroyed at an exogenous rate.

9 See Appendix B, Appendix C.
However, unlike the DMP model we assume a part of idle capital is destroyed in the frictional sector. This particular assumption is necessary in our model to get feasible output in both sectors.

Our theoretical analysis shows that trade reforms through the reduction in price of the product produced in the frictional sector reduces both wage-gap and the equilibrium rate of frictional unemployment unambiguously. A decrease in unemployment benefit lowers the rate of frictional unemployment but raises wage-gap. On the other hand, a reduction of the bargaining power of labour in the frictional sector reduces wage-gap but raises frictional unemployment rate. Thus, we may conclude that trade reforms are better policy than the labour market reforms in a frictional small open economy.

**APPENDIX**

**A. DERIVATION OF SOME USEFUL EXPRESSIONS**

The Nash-bargaining problem is

$$\max_{\omega_2} \Omega = (W - U)^{\beta} (J - V)^{(1-\beta)}$$

The first order condition for maximization is

$$\beta (J - V) \frac{\partial}{\partial \omega_2} (W - U) + (1 - \beta) (W - U) \frac{\partial}{\partial \omega_2} (J - V) = 0$$

(32)

Using (2), (4) and the zero-profit condition $V = 0$ into (32) one gets

$$(1 - \beta)(w_2 - rU) = \beta(P_2 - w_2 - (r + \delta))$$

Or

$$w_2 = (1 - \beta)rU + \beta(P_2 - (r + \delta))$$

(33)

Using (2), (5), (32) and $V=0$ from (1) we can write

$$rU = b + \frac{BC}{1 - \beta}$$

(34)

Using (34) into (33) one gets

$$w_2 = b + \frac{\beta C}{1 - \beta} + \frac{\beta}{1 - \beta} (r + \lambda) \frac{C}{q(\theta)}$$

**B. EFFECTS OF A CHANGE IN $P_2$, $b$, $\beta$ ON $\theta$, $r$, $w_1$, $w_2$, $u$**

Using (34) from (26) we get

$$w_1 = b + \frac{\beta C}{1 - \beta} = b + \xi q(\theta)$$

Using (34) into (25) we get

$$w_2 = b + \frac{\beta}{1 - \beta} \frac{C}{q(\theta)} (r + \lambda + \theta q(\theta))$$

$$= b + \xi (r + \lambda + \theta q(\theta))$$

Taking total differentials of Equations (25) and (26) and after simplifications we get
\[
\begin{align*}
\theta &= 1 - \frac{k}{\Delta} L (1 - \beta) \left( \theta + \xi \theta q(\theta) \right) + \frac{C(r + \lambda) C \xi \theta q(\theta)}{(1 - \beta) q(\theta)} r \hat{\theta} \\
&= P_2 \hat{P}_2 - b \hat{b} - \xi (r + \lambda + \theta q(\theta)) e \hat{\beta} 
\end{align*}
\]

Solving (35) and (36) we get
\[
\begin{align*}
\hat{\theta} &= \frac{1}{\Delta} \left[ -b \hat{b} \cdot \frac{C r}{(1 - \beta) q(\theta)} - r P_2 \hat{P}_2 + (r + \lambda) \xi e \hat{\beta} \right] \\
\hat{r} &= \frac{1}{\Delta} \left[ \xi \theta q(\theta) P_2 \hat{P}_2 + \frac{b(r + \lambda) C}{(1 - \beta) q(\theta)} e \hat{\beta} - \frac{\xi \theta (r + \lambda) C}{(1 - \beta)} (\beta - e \hat{\beta}) \right]
\end{align*}
\]

where
\[
\Delta = \frac{C r}{(1 - \beta) q(\theta)} \left[ \xi \theta q(\theta) - (r + \lambda) e \hat{\beta} \right] > 0 \quad \text{if} \quad (\xi \theta q(\theta) > (r + \lambda) e \hat{\beta})
\]

From (37)–(39) we get
\[
\begin{align*}
\frac{\hat{\theta}}{P_2} &= \frac{r P_2}{\Delta} < 0, \quad \hat{\theta} = -\frac{b}{\Delta} \cdot \frac{C r}{(1 - \beta) q(\theta)} < 0, \quad \hat{\theta} = (r + \lambda) \xi e \hat{\beta} > 0 \\
\frac{\hat{r}}{P_2} &= \frac{1}{\Delta} \xi \theta q(\theta) P_2 > 0, \quad \hat{r} = \frac{b(r + \lambda) C e \hat{\beta}}{\Delta(1 - \beta) q(\theta)} > 0, \\
\frac{\hat{r}}{\hat{\beta}} &= -\frac{1}{\Delta} \xi \theta (r + \lambda) C \Delta \beta - e \hat{\beta} > 0 \quad \text{if} \quad \beta < e \hat{\beta}
\end{align*}
\]

Again from (18) and (19) we get
\[
\hat{w}_1 = \frac{1}{W_1} \left[ b \hat{b} + \xi \theta q(\theta) e \hat{\beta} + \xi \theta q(\theta) \hat{\theta} \right]
\]

and
\[
\hat{w}_2 = \frac{1}{W_2} \left[ b \hat{b} + \xi (r + \lambda + \theta q(\theta)) e \hat{\beta} + \xi (r + \lambda) e \hat{\beta} \right] \hat{\theta} + \xi r \hat{r}
\]

Using (37), (38) from (42) and (43) we get
\[
\begin{align*}
\frac{d(w_2 - w_1)}{dP_2} &= \frac{\xi}{\Delta} [\xi \theta q(\theta) - (r + \lambda) e \hat{\beta}] = \beta \\
\frac{d(w_2 - w_1)}{db} &= -\frac{(r + \lambda) b \xi \theta}{\Delta(1 - \beta) q(\theta)} \left[ C(r - \theta) + (r + \delta)(1 - \beta) q(\theta) \right] \\
\frac{d(w_2 - w_1)}{d\beta} &= \frac{\xi (r + \lambda)}{\beta(1 - \beta) \Delta} \left[ (1 - \beta) \Delta \cdot e \hat{\beta} + (r + \lambda)(1 - \beta) \xi \cdot e \hat{\beta} - r \xi \theta C \Delta \beta - e \hat{\beta} \right]
\end{align*}
\]

Taking total differentials of (22) and after simplifying one gets
\[
\hat{u} = -\frac{u}{\lambda} \theta q(\theta) (1 + e \hat{\beta}) \hat{\theta}
\]

Using (37) from (47) we get
Taking total differentials of Equations (23) and (24) and after simplification we get

\[ \lambda_{L1} \dot{X}_1 + \lambda_{L2} \dot{X}_2 = \dot{L} - \lambda_U \dot{u} \]  
(51)

\[ \lambda_{K1} \dot{X}_1 + \lambda_{K2} (1 + \delta) \dot{X}_2 = \dot{K} \]  
(52)

where \( \lambda_{ij} \) is the proportion of the \( i \)th factor used in the \( j \)th sector, \( \forall i = L, K; j = 1, 2 \).

Solving (51) and (52) by Cramer's rule one gets

\[ \dot{X}_1 = \frac{1}{|\lambda|} \left[ \lambda_{K2} (1 + \delta) (\dot{L} - \lambda_U \dot{u}) - \lambda_{L2} \dot{K} \right] \]  
(53)

\[ \dot{X}_2 = \frac{1}{|\lambda|} \left[ \lambda_{L1} \dot{K} - \lambda_{K1} (\dot{L} - \lambda_U \dot{u}) \right] \]  
(54)

where

\[ |\lambda| = (\lambda_{L1} \lambda_{K2} (1 + \delta) - \lambda_{K1} \lambda_{L2}) = \frac{L_1}{L} \cdot \frac{K_2}{K} (1 + \delta) - \frac{K_1}{K} \cdot \frac{L_2}{L} \]

\[ = \frac{1}{LK} \left[ L_1 K_2 (1 + \delta) - K_1 L_2 \right] \]

\[ = \frac{X_1 X_2}{LK} \left[ L_1 K_2 (1 + \delta) - K_1 L_2 \right] \frac{X_1 X_2}{X_1 X_2} \]

\[ = \frac{X_1 X_2}{LK} \cdot (1 + \delta - 1) \]

\[ = \frac{X_1 X_2}{LK} \cdot \delta > 0 \]  
(55)

Using (48)–(50) from (53), (54) one gets

\[ \frac{\dot{X}_1}{P_2} = \frac{1}{|\lambda|} \left[ \lambda_{K2} (1 + \delta) \lambda_U \left( \frac{\dot{u}}{P_2} \right) \right] < 0 \]  
(56)

\[ (+) \hspace{1cm} (+) \]

\[ \frac{\dot{X}_1}{b} = \frac{1}{|\lambda|} \left[ \lambda_{K2} (1 + \delta) \lambda_U \left( \frac{\dot{u}}{b} \right) \right] < 0 \]  
(57)
\[ \frac{\dot{X}_1}{\dot{\beta}} = \frac{1}{|\lambda|} \left[ \lambda_{K2}(1 + \delta)\lambda U \left( \frac{\dot{u}}{\dot{\beta}} \right) \right] > 0 \]  
\[ \frac{\dot{X}_1}{\dot{L}} = \frac{1}{|\lambda|} \left[ \lambda_{K2}(1 + \delta) \right] > 0 \]  
\[ \frac{\dot{X}_1}{\dot{K}} = \frac{\lambda_{L2}}{|\lambda|} < 0 \]  
\[ \frac{\dot{X}_2}{\dot{P}_2} = \frac{1}{|\lambda|} \left[ \lambda_{K1}\lambda U \left( \frac{\dot{u}}{\dot{P}_2} \right) \right] > 0 \]  
\[ \frac{\dot{X}_2}{\dot{b}} = \frac{1}{|\lambda|} \left[ \lambda_{K1}\lambda U \left( \frac{\dot{u}}{\dot{b}} \right) \right] > 0 \]  
\[ \frac{\dot{X}_2}{\dot{b}} = \frac{1}{|\lambda|} \left[ \lambda_{K1}\lambda U \left( \frac{\dot{u}}{\dot{b}} \right) \right] < 0 \]  
\[ \frac{\dot{X}_2}{\dot{L}} = \frac{-\lambda_{K1}}{|\lambda|} < 0 \]  
\[ \frac{\dot{X}_2}{\dot{K}} = \frac{\lambda_{L1}}{|\lambda|} > 0 \]  

REFERENCES


