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WATER-HAMMER IN A TAPERED PIPE LINE

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ABSTRACT

An analytical solution which neglects the viscous effects is obtained for the pressure rise occuring in a tapered pipe when the flow is stopped by the quick closure of a valve. It is shown that the pressure rise is given by a function of the radius ratio; (the radius at the valve end/the radius at the opposite end).

Moreover the effect of viscosity on the peak pressure value is investigated numerically by the method of characteristics.

Nomenclature

$A(x)$	cross sectional area of a tapered pipe $[=\pi R^2(x)]$
$A^*(x^*)$	dimensionless cross sectional area of a tapered pipe $[=A(x)/\{\pi R^2(0)\}]$
Dn	dissipation number $[=4\nu L/\{aR^2(0)\}]$
L	pipe length
$R(x)$	pipe radius
$R^*(x^*)$	dimensionless pipe radius $[=R(x)/R(0)]$
a	wave velocity
m	radius ratio $[=R(L)/R(0)]$
p	pressure
p^*	dimensionless pressure $[=p/\{\rho_0 a q_0/A(0)\}]$
q	volume flow rate
q^*	dimensionless volume flow rate $[=q/q_0]$
q_0	initial volume flow rate
t	time
t^*	dimensionless time $[=t/(L/a)]$
x	axial coordinate
x^*	dimensionless axial coordinate $[=x/L]$
ν	kinematic viscosity of fluid

ρ_0 initial density
τ valve closing time $[0 \leqq \tau < 2L/a]$
τ^* dimensionless valve closing time $[= \tau/(L/a)]$

1. Introduction

Recently, TARANTINE and ROULEAU (1969) numerically investigated the effectiveness of a tapered section in attenuating the pressure surges (water hammer) which are caused by the quick valve closure of a tapered pipe line.

The step-line impedance method employed by them has the practical advantage; i.e. it can be easily applied to a complex tapered pipe line. However, this method requires a tremendous amount of computational time to predict the following.

(1) The dependence of the magnitude of the initial pressure rise at the valve before the appearance of the reflected wave from the opposite end of a tapered pipe line upon the radius ratio.

(2) The dependence of the average pressure value as later defined in Section 4-[b] upon the radius ratio.

Moreover the numerical results which neglect the effect of viscosity indicate that the peak pressure occurs after a long time, which brings out the following third point:

(3) Can the effect of viscosity on the peak pressure value be neglected or not?

The initial pressure rise and the average pressure value are analytically derived in this paper. The effect of viscosity on the peak pressure value is numerically examined by the method of characteristics.

Furthermore the numerical results derived by the method of characteristics are compared with the analytical one for the nonviscous case.

2. Basic Equations

A physical model considered here for the water-hammer problem is obtained by placing a valve at the location $x=L$ (Fig. 1). The following assumptions are made for this model;

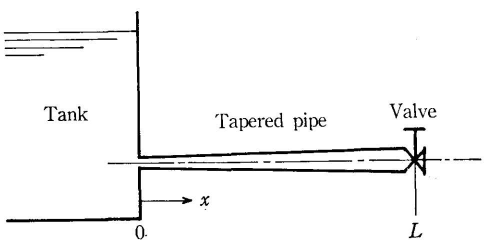

Fig. 1. Tapered pipe line

(1) The pipe radius is small compared with its length ($R \ll L$).

(2) The pipe radius varies slowly along its length ($dR/dx \ll 1$).

(3) The elasticity of the pipe walls can be neglected when compared with the compressibility of the fluid.

(4) The initial velocity is sufficiently small when compared with the wave velocity. ($q_0/A(0) \ll a$).

(5) The effect of viscosity is included in the form of pressure loss due to friction (see ZIELKE, 1968).

Then the following two differential equations describe completely the volume flow rate and the pressure

$$\frac{1}{a^2}\frac{\partial p}{\partial t}+\frac{\rho_0}{A}\frac{\partial q}{\partial x}=0, \tag{2.1}$$

$$\frac{1}{A}\frac{\partial q}{\partial t}+\frac{1}{\rho_0}\frac{\partial p}{\partial x}=-\frac{4\nu}{AR^2}\left\{2q+\int_0^t \frac{\partial q(u)}{\partial t}W(t-u)du\right\}, \tag{2.2}$$

By Zielke (1968), the weighting function W can be expressed as follows;

$$W(\xi)=\begin{cases} e^{-26.3744\xi}+e^{-70.8493\xi}+e^{-135.0198\xi}+e^{-218.9126\xi}+e^{-322.5544\xi}, (\xi>0.02), \\ 0.282095\xi^{-1/2}-1.25+1.057855\xi^{1/2}+0.9375\xi+0.396696\xi^{3/2} \\ -0.351563\xi^2. (\xi<0.02) \end{cases}$$

Moreover, under steady flow conditions, it is assumed that the volume flow rate in the line is q_0 (constant) and that the pressure along the line is uniform. The latter assumption means that the Bernoulli pressure variations in the tapered pipe line, $\rho_0\{q_0/A(0)\}^2/2$, and the pressure loss due to friction, $\rho_0\nu L\{q_0/A(0)\}/R^2(0)$, are both negligible in comparison with the pressure rise $\rho_0 a q_0/A(0)$. These initial conditions are written as follows;

$$p(x,0)=0, \ (0\leqq x\leqq L), \tag{2.3}$$

$$q(x,0)=q_0, (0\leqq x\leqq L). \tag{2.4}$$

It is assumed that the tapered pipe is terminated by an infinite tank at $x=0$ and the valve located at $x=L$ is closed quickly; these boundary conditions can be expressed as follows;

$$p(0,t)=0, (t\geqq 0), \tag{2.5}$$

$$q(L,t)=q_0 f(t), \tag{2.6}$$

where $f(t)$ is any function of t and equals 1 at $t=0$, 0 at $t>\tau(0\leqq\tau<2L/a)$.

Now we introduce the dimensionless variables:

$$p^*=\frac{p}{\rho_0 a q_0/A(0)}, q^*=\frac{q}{q_0}, x^*=\frac{x}{L}, t^*=\frac{t}{L/a}, R^*(x^*)=\frac{R(x)}{R(0)}.$$

Then the following dimensionless equations are obtained from Eqs. (2.1) through (2.6):

$$\frac{\partial p^*}{\partial t^*}+\frac{1}{R^{*2}}\frac{\partial q^*}{\partial x^*}=0. \tag{2.7}$$

(2.8) $$\frac{\partial q^*}{\partial t^*}+R^{*2}\frac{\partial p^*}{\partial t^*}=\frac{-Dn}{R^{*2}}\left\{2q^*+\int_0^{t^*}\frac{\partial q^*(u)}{\partial t^*}W(t^*-u)\,du\right\},$$

(2.9), (2.10) $$p^*(x^*,0)=0,\ q^*(x^*,0)=0,\ (0\leqq x^*\leqq 1),$$

(2.11) $$p^*(0,t^*)=1,\ (t^*\geqq 0),$$

(2.12) $$q^*(1,t^*)=f(t^*),\ (t^*\geqq 0).$$

The dimensionless number Dn is called Dissipation Number $[=4\nu L/\{aR^2(0)\}]$. The value of this number is indicative of the attenuation and the distorsion along the line.

3. Methods of Solutions

(a) Analytical Method for the case of $Dn=0$.

From Eqs. (2.7) through (2.12), we obtain the equations for $p^*(x^*,t^*)$ and define $\Phi(x^*,t^*)$ as follows;

(3.1) $$p^*(x^*,t^*)=\Phi(x^*,t^*)/R^*(x^*).$$

Then the equations for $\Phi(x^*,t^*)$ are obtained:

(3.2) $$\frac{\partial^2\Phi}{\partial t^{*2}}=\frac{\partial^2\Phi}{\partial x^{*2}}-\frac{1}{R^*}\frac{d^2R^*}{dx^{*2}}\Phi,$$

(3.3) $$\Phi(x^*,0)=0,\ \frac{\partial\Phi}{\partial t^*}(x^*,0)=0,\ (0\leqq x^*\leqq 1),$$

(3.4) $$\Phi(0,t^*)=0,\ (t^*\geqq 0),$$

(3.5) $$\frac{\partial\Phi}{\partial x^*}(1,t^*)-\frac{1}{R^*(1)}\frac{dR^*(1)}{dx^*}\Phi(1,t^*)=-\frac{1}{R^*(1)}\frac{df}{dt^*},\ (t^*\geqq 0).$$

Now we consider the case of the linear tapered pipe, of which the dimensionless radius is defined as

(3.6) $$R^*(x^*)=1+(m-1)x^*,\ m=R(L)/R(0).$$

Then the solution for the Laplace transform of the pressure is obtained;

(3.7) $$\hat{p}^*(x^*)=\frac{1}{mR^*(x^*)}\frac{(f(0)-s\hat{f})\sinh sx^*}{s\cosh s-((m-1)/m)\sinh s},\ (0\leqq x^*\leqq 1),$$

by using Eqs. (3.1) through (3.6).

For the instantaneous valve closure, $f(t^*)$ in Eq. (3.7) is expressed as follows;

(3.8) $$f(t^*)=\{1(t^*=0),\ 0(t^*>0)\},$$

and taking the Laplace transform of f gives

(3.9) $$\hat{f}=0.$$

Then expanding Eq. (3.9) with the power series of e^{-2s} and taking the inverse Laplace transform lead to the expression for the pressure $p^*(x^*, t^*)$ in the following form,

$$(3.10)\qquad p^*(x^*,t^*)=\begin{cases}\dfrac{1}{mR^*(x^*)}e^{((m-1)/m)(t^*+x^*-1)},\ (-x^*+1<t^*<x^*+1),\\ \dfrac{1}{mR^*(x^*)}\Big[e^{((m-1)/m)(t^*+x^*-1)}-e^{((m-)1/m)(t^*-x^*-1)}\\ +\Big\{-\dfrac{2(m-1)}{m}(t^*+x^*)-\dfrac{6(m-1)}{m}\Big\}e^{((m-1)/m)(t^*+x^*-3)}\Big],\\ (-x^*+3<t^*<x^*+3).\end{cases}$$

Also taking the inverse Laplace transform Eq. (3.9) directly, the pressure $p^*(x^*, t^*)$ is expressed by

$$(3.11)\qquad \begin{aligned}&p^*(x^*,t^*)=\frac{2}{mR^*(x^*)}\sum_{n=1}^{\infty}\frac{\sin k_n}{2k_n-\sin 2k_n}\{\cos k_n(t^*-x^*)-\cos k_n(t^*+x^*)\},\\ &\tan k_n=mk_n/(m-1).\end{aligned}$$

Defining $\Psi(x^*,t^*)=p^*(x^*,t^*)/m/R^*(x^*)$ where $p^*(x^*,t^*)$ is expressed as Eq. (3.10), $p^*(x^*,t^*)$ of Eq. (3.11) is written by

$$(3.12)\qquad p^*(x^*,t^*)=\frac{1}{mR^*(x^*)}\sum_{n=1}^{\infty}\frac{\int_0^1\Psi(\xi,t^*)\sin k_n\xi d\xi}{\int_0^1\sin^2 k_n\xi d\xi}\sin k_nx^*,\ \tan k_n=mk_n/(m-1).$$

$p^*(x^*,t^*)$ expressed by Eq. (3.10) is convenient to explain the initial profile of the pressure wave, and $p^*(x^*,t^*)$ expressed by Eq. (3.11) is convenient to calculate the wave profile past a long time.

For the case of finite-time valve closure, the pressure $p^*(x^*,t^*)$ is expressed by

$$(3.13)\qquad p^*(x^*,t^*)=-\frac{1}{mR^*(x^*)}\int_0^{t^*}\frac{\partial f(u)}{\partial t^*}e^{((m-1)/m)(t^*-u+x^*-1)}du,\ (0<t^*<x^*+1),$$

where f is any function of t^*, i.e. $f=f(t^*)$.

(b) Numerical Method for the case of $Dn\neq 0$.

From Eqs. (2.7) and (2.8), we obtain the following characteristic differential equations along the characteristic curves $dx^*=\pm dt^*$ in the x^*-t^* plane (see TANAHASHI; 1974);

for $dx^*=dt^*$,

$$(3.14)\qquad dp^*+\frac{1}{R^{*2}}dq^*=-Dn\frac{H}{R^{*4}}dx^*,$$

for $dx^*=-dt^*$,

$$(3.15)\qquad dp^*-\frac{1}{R^{*2}}dq=-Dn\frac{H}{R^{*4}}dx^*,$$

where

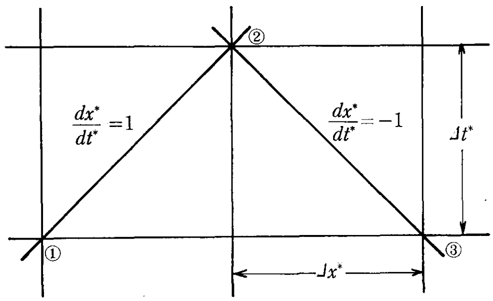

Fig. 2. Characteristic curves in a physical plane

(3.16) $$H(x^*, t^*)=2q^*(x^*, t^*)+\int_0^{t^*} \frac{\partial q^*(x^*, u)}{\partial t^*} W(t^*-u)du.$$

When Eq. (3.14) is integrated to ① and ② with reference to Fig. 2 and when Eq. (3.15) is integrated to ③ and ② approximately, the following equations are obtained,

(3.17) $$p_2^*-p_1^*-\frac{1}{R_1^{*2}}(q_2^*-q_1^*)=-Dn\frac{H_1}{R_1^{*4}}\Delta x^*,$$

(3.18) $$p_2^*-p_3^*-\frac{1}{R_3^{*2}}(q_2^*-q_3^*)=Dn\frac{H_3}{R_3^{*4}}\Delta x^*.$$

Therefore, determining the division number ($1/\Delta x^*$ in $0\leqq x^*\leqq 1$), we can obtain the pressure $p^*(x^*, t^*)$ numerically from the initial conditions, Eqs. (2.4) and (2.5), the boundary conditions, Eqs. (2.6) and (2.7), and simultaneous equations, Eqs. (3.17) and (3.18).

4. Results

(a) The initial pressure rise

We can explain analytically the initial pressure rise, i.e. the pressure $p^*(x^*, t^*)$ before being affected by the reflected wave from the tank, especially the pressure at the valve $p^*(1, t^*)$.

In the case of $m=1+\delta\,(|\delta|\ll 1)$, the pressure at the valve is given by

(4.1) $$p^*(1, t^*)=1+(t^*-2)\delta, \quad (0<t^*<2).$$

From Eq. (4.1), we see that for $\delta>0$, $p^*(1, t^*)$ increases with the time, and for $\delta<0$, decreases with the time. The maximum and the minimum pressures at the valve, during $0<t^*<2$, are 1 and $1-2\delta$ regardless of the sign of δ. Here $t^*=0$ means the time of valve closure and $t^*=2$ means the time when the reflected pressure wave arrives at the valve.

For any finite $m, p^*(1, t^*)$ expressed by Eq. (3.10) is shown for $0<t^*<2$ in Fig. 3 and Fig. 4,. The maximum and the minimum pressures of $p^*(1, t^*)$ during $0<t^*<2$, which occur at $t^*=0$ and $t^*=2$, are shown in Fig. 5. We notice that $p^*(1, 1_+)=1/m^2$ shown in Fig. 5 coincides with the result of TARANTINE and ROULEAU (1969).

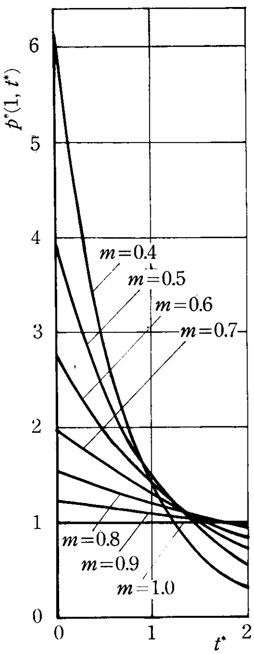

Fig. 3. Initial pressure rise at the valve with a linear tapered pipe; $m<1.0$

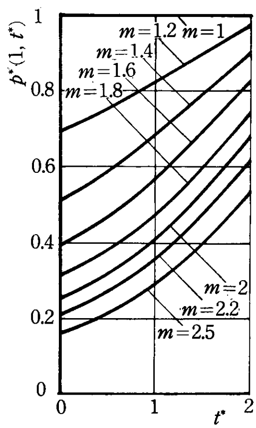

Fig. 4. Initial pressure rise at the valve with a linear tapered pipe; $m>1.0$

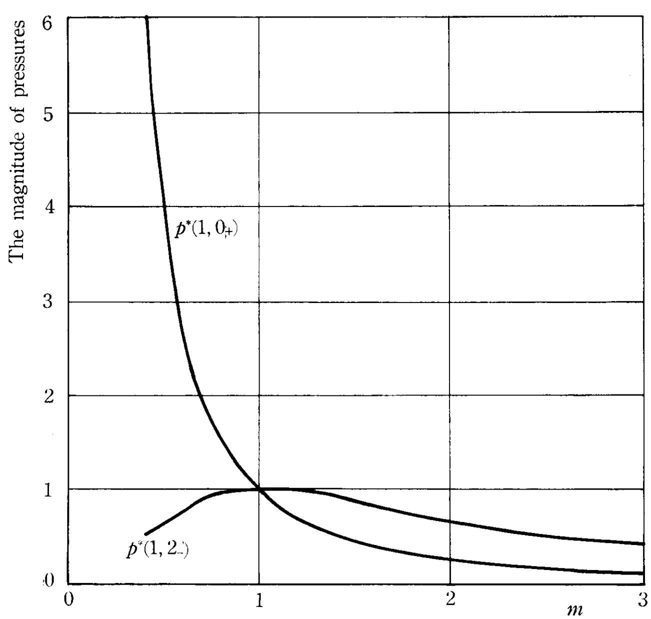

Fig. 5. Maximum and minimum pressures at the valve

As the basic example which accounts for the finite valve-closure time τ^*, $(0<\tau^*<2)$, we consider the case that $f(t^*)$ is written by

$$f(t^*)=\begin{cases}1 & ,(t^*=0)\\ 1-t^*/\tau^*, & (0<t^*<\tau^*)\\ 0 & ,(\tau^*\leqq t^*).\end{cases} \tag{4.2}$$

Then the pressure is expressed as follows;

$$p^*(x^*,t^*)=\begin{cases}\dfrac{1}{\tau^*R(x^*)(m-1)}\{e^{((m-1)/m)(t^*+x^*-1)}-1\}, & (1-x^*<t<1-x^*+\tau^*),\\ \dfrac{1}{\tau^*R(x^*)(m-1)}e^{((m-1)/m)(t^*+x^*-1)}(1-e^{((m-1)/m)\tau}), & (1-x^*+\tau^*<t^*<1+x^*).\end{cases} \tag{4.3}$$

In the case of $m=1+\delta(|\delta|\ll 1)$, the pressure at the valve is given by

$$p^*(1,t^*)=\begin{cases}\dfrac{t^*}{\tau^*}-\left(\dfrac{2t^*}{\tau^*}-\dfrac{t^{*2}}{2\tau^*}\right)\delta, & (t^*<\tau^*)\\ 1+\left(t^*-2-\dfrac{\tau^*}{2}\right)\delta, & (\tau^*\leqq t^*<2).\end{cases} \tag{4.4}$$

After the valve is completely closed $(t^*\geqq\tau^*)$, $p^*(1,t^*)$ is differed by $-\tau^*\delta/2$ from the one in the case of the instantaneous valve closure.

For a finite m, the effect of τ^* on the pressure $p^*(1,t^*)$ is shown in Fig. 6.

(b) The average pressure value

Figure 8 shows the pressure $p^*(1,t^*)$, calculated from Eq. (3.14). $p^*(1,t^*)$ changes periodically with the time, deforming its wave profile.

As the parameter indicating the intensity of the water hammer, we define the average pressure value during $a<t^*<b$;

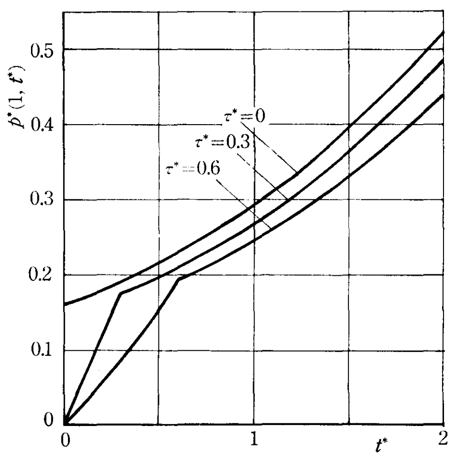

Fig. 6. Initial pressure rises at the valve for various linear closure rates with a linear tapered pipe; $m=2.5$

(4.5) $$\bar{p}^*_{ab}=\frac{1}{b-a}\int_a^b |p^*(1,t^*)|dt^*.$$

The average pressure value, during $0<t^*<2$, becomes

(4.6) $$\bar{p}^*_{02}=\frac{1}{2m(m-1)}\{e^{2(m-1)/m}-1\}$$

by using Eqs. (3.10) and (4.5).

And the average pressure value, during $0<t^*<4$ which is the one period of pressure wave, becomes

(4.7) $$\bar{p}^*_{04}=\begin{cases}\dfrac{1}{4m(m-1)}\left\{-e^{4(m-1)/m}+\dfrac{2(3m-2)}{m}-1\right\}, (0.722<m<1.53),\\ \dfrac{1}{4m(m-1)}\left\{-e^{4(m-1)/m}+\dfrac{4(m-1)}{m}e^{2(m-1)/m}+4e^{((m-1)/m)(t_0^*-2)}-1\right\}, (m<0.722),\\ \dfrac{1}{4m(m-1)}\left\{e^{4(m-1)/m}-\dfrac{2(m-2)}{m}e^{2(m-1)/m}-4e^{((m-1)/m)(t_0^*-2)}-1\right\}, (1.53<m),\end{cases}$$

by using Eqs. (3.10) and (4.5), where t_0^* is defined as

(4.8) $$t_0^*=\frac{m}{2(m-1)}\left\{e^{2(m-1)/m}+\frac{2(m-2)}{m}\right\}.$$

In the case of $m=1+\delta\,(|\delta|\ll 1)$, Eqs. (4.6) and (4.7) give

(4.9) $$\bar{p}^*_{02},\ \bar{p}^*_{04}=1-\delta.$$

The above equation implies that the dimensionless average pressure values are $1-\delta$.

For the case of any finite m, $\bar{p}^*_{02}$ and $\bar{p}^*_{04}$ are shown in Fig. 7.

The magnitude of the pressure is estimated from the momentum equation and the initial conditions;

$$\frac{q_0}{L/a}\sim\frac{A}{\rho_0}\frac{\Delta p}{L}.$$

(4.10) $$\Delta p\sim\rho_0 aU,\ (U\equiv q_0/A).$$

From Eq. (4.10), the magnitude of the onset pressure $p^*(1,0+)$ at the valve is estimated as follows;

(4.11) $$\frac{\Delta p}{\rho_0 a q_0/A(0)}\sim\frac{1}{m^2}$$

by using $U\sim q_0/A(L)$. This result coincides with Eq. (3.10). The average pressure values, $\bar{p}^*_{02}$ and $\bar{p}^*_{04}$, are estimated similarly;

(4.12) $$\frac{\Delta p}{\rho_0 a q_0/A(0)}\sim\frac{1}{m}$$

by using

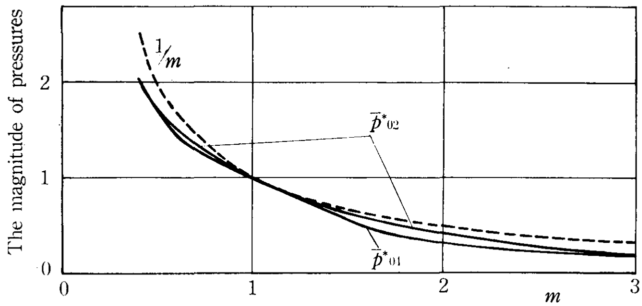

Fig. 7. Average pressure values at the valve with a linear tapered pipe and a tank

$$U \sim \frac{1}{L}\int_0^L \frac{q_0}{A(x)}dx \sim \frac{q_0}{A(0)m}.$$

In the case of $m=1+\delta\,(|\delta|\ll 1)$, these estimated values of $\bar{p}^*_{02}$, $\bar{p}^*_{04}$ coincide with Eq. (4.9). On the other hand, in the case of any finite m, they are larger than the results of Eqs. (4.6) and (4.7) as shown in Fig. 7.

Consequently the pressure rise which occurs by the quick valve closure, increases or decreases depending on the initial velocity as seen in Eq. (4.11). However, the average pressure values $\bar{p}^*_{02}$, and $\bar{p}^*_{04}$ are also influenced by the reflected pressure wave from the valve and the tank.

(c) The peak pressure value

The peak pressure shown in Fig. 10, occurs after a long time in the case of $Dn=0$, but the effect of viscosity on the peak pressure value can not be neglected. Its effect is investigated numerically by the method of characteristics, since it can not be solved analytically.

(1) Considerations on the method of characteristics

Before we investigate the effect of viscosity numerically, we compare the solutions calculated by the method of characteristics with the analytical solutions for the two examples; (i) a linear tapered pipe and (ii) an exponential tapered pipe, for which the radius is written as

$$R^*(x^*)=e^{x^*\log m} \tag{4.13}$$

in the case of $Dn=0$.

For the case of the exponential tapered pipe, the pressure $p^*(x^*, t^*)$ is expressed analytically;

$$p^*(x^*,t^*)=\frac{2}{mR^*(x^*)}\sum_{n=1}^{\infty}\frac{k_n\sin k_n}{w_n(2k_n-\sin 2k_n)}\left\{\cos w_n\left(t^*-\frac{k_n}{w_n}x^*\right)\right. \tag{4.14}$$

$$\left.-\cos w_n\left(t^*+\frac{k_n}{w_n}x^*\right)\right\},\quad \tan k_n=k_n/\log m,\quad w_n=\sqrt{k_n^2+(\log m)^2}$$

by using Eqs. (3.1)~(3.5) and (4.13).

Figures 8 and 9 show the numerical results for the pressure $p^*(1, t^*)$, which are calculated by the analytical method and the method of characteristics. As shown in Fig. 8, the numerical solution derived by the method of characteristics

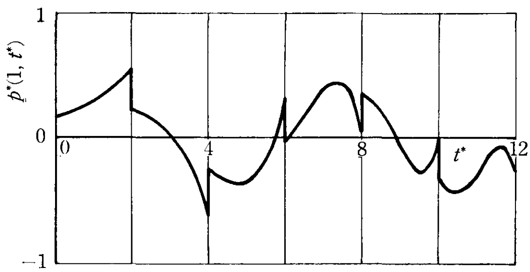

Fig. 8. Pressure rise at the valve with a linear tapered pipe and a tank; $m=2.5$

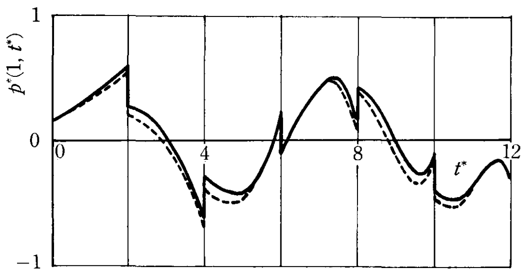

Fig. 9. Pressure rise at the valve with an exponential tapered pipe and a tank; $m=2.5$

coincides with the analytical one in the case of the linear tapered pipe. But Fig. 9 shows that the both solutions are different in the case of the exponential tapered pipe.

These results can be explained as follows. The pressure wave $p^*(x^*-t^*)$ at (x_0^*+h, t_0^*+h) which propagates to the valve is expressed as follows;

$$(4.15) \qquad p^*\{(x_0^*+h),\ (t_0^*+h)\}=\left\{1+\frac{R^{*2}(x_0^*)-R^{*2}(x_0^*+h)}{R^{*2}(x_0^*)+R^{*2}(x_0^*+h)}\right\}p^*(t_0^*-x_0^*)$$

$$=\left\{1-\frac{R^{*\prime}(x_0^*)}{R^*(x_0^*)}h+O(h^2)\right\}p^*(t_0^*-x_0^*)$$

by the method of characteristics.

On the other hand, this pressure wave in the case of the linear tapered pipe is expressed analytically as follows;

$$(4.16) \qquad p^*\{(x_0^*+h),\ (t_0^*+h)\}=\frac{1}{R^*(x_0^*+h)}\sum_{n=1}^{\infty}\Phi_n(t_0^*-x_0^*)$$

$$=\left\{1-\frac{R^{*\prime}(x_0^*)}{R^*(x_0^*)}h+O(h^2)\right\}p^*(t_0^*-x_0^*).$$

Therefore the difference between Eqs. (4.15) and (4.16) is of the order h^2.

The pressure wave in the case of the exponential tapered pipe is written by

$$(4.17)\qquad p^*\{(x_0^*+h),\ (t_0^*+h)\}=\frac{1}{R^*(x_0^*+h)}\sum_{n=1}^{\infty}\Phi_n\{(t_0^*+h)-\gamma_n(x_0^*+h)\}$$

$$=\left\{1-\frac{R^{*\prime}(x_0^*)}{R^*(x_0^*)}h+O(h^2)\right\}p^*(t_0^*-x_0^*)$$

$$+\frac{h}{R^*(x_0^*)}\sum_{n=1}^{\infty}(1-\gamma_n)\Phi_n'(t_0^*-\gamma_n x_0^*).$$

Then, in this case, the difference between Eqs. (4.15) and (4.18) is of the order h and does not become smaller even if the number of divisions is increased.

(2) The effect of viscosity on the peak pressure value

The pressure at the valve, which occurs by the quick valve closure, is obtained numerically in the case of $Dn\neq 0$ as indicated in Fig. 10.

To consider the effect of viscosity on the peak pressure value, we regard the average pressure value during $8<t^*<12$ as

$$(4.18)\qquad \bar{p}_{8.12}^*=\frac{1}{4}\int_8^{12}|p^*(1,t^*)|dt^*$$

and the peak pressure value as $p^*(1,10_-)$. And we define the ratio γ as follows;

$$(4.19)\qquad \gamma=\frac{p^*_{\text{viscous}}(1,10_-)}{p^*_{\text{nonviscous}}(1,10_-)}\bigg/\frac{\bar{p}^*_{8.12\text{viscous}}}{\bar{p}^*_{8.12\text{nonviscous}}}$$

where the suffixes "viscous" and "nonviscous" mean the cases of $Dn\neq 0$ and $Dn=0$ respectively.

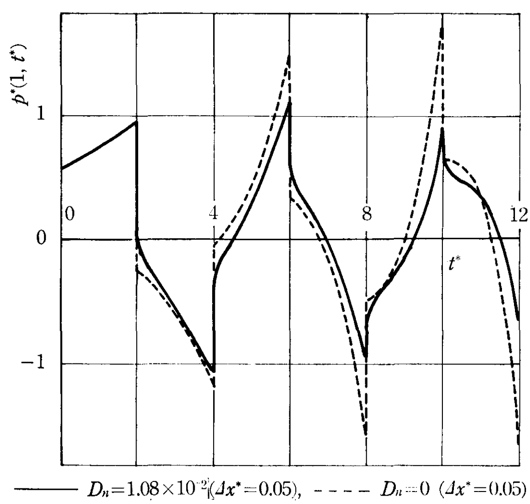

Fig. 10. Pressure rise at the valve with a linear tapered pipe and a tank; $m=1.3$

For the numerical example ($m=1.3, Dn=1.08\times10^{-2}$) in the case of the linear tapered pipe shown in Fig. 10, γ is calculated as

$$\frac{p^*_{\text{viscous}}(1,10_-)}{p^*_{\text{nonviscous}}(1,10_-)} \doteqdot 0.48 \qquad \frac{\bar{p}^*_{8.12\text{viscous}}}{\bar{p}^*_{8.12\text{nonviscous}}} \doteqdot 0.5 \qquad \therefore\ \gamma \doteqdot 1.$$

And for the numerical example ($m=1, Dn=6.7\times10^{-3}$) in the case of the straight pipe (see ZIELKE, 1968), γ is calculated as

$$\frac{p^*_{\text{viscous}}(1,10_-)}{p^*_{\text{nonviscous}}(1,10_-)} \doteqdot 0.78 \qquad \frac{\bar{p}^*_{8.12\text{viscous}}}{\bar{p}^*_{8.12\text{nonviscous}}} \doteqdot 0.4 \qquad \therefore\ \gamma \doteqdot 2.$$

After all, the effect of viscosity on the peak pressure value in the case of the linear tapered pipe is larger than that in the straight pipe. As the peak pressure at the valve is considerably saved by the effect of viscosity, there is little possibility that the maximum and minimum pressures occur after a long time.

5. Conclusions

The analytical investigation has been made with respect to the pressure rise at the valve which is caused by the quick closure of the valve located at the downstream of the tapered pipe line, and the following results have been mainly obtained.

(1) The initial pressure rise

When the valve is closed instantaneously, the maximum and minimum pressures at the valve before the appearance of the reflected pressure wave from the tank are shown in Fig. 5. Especially in the case of $m=1+\delta\,(|\delta|\ll1)$, they are expressed as

$$[p_{\max}, p_{\min}] = [\rho_0 a q_0/A(0), (1-2\delta)\rho_0 a q_0/A(0)].$$

(2) The average pressure value

In order to estimate the intensity of the water hammer, the following parameter has been defined in the present paper;

$$\bar{p}_{02L/a} = \frac{1}{2L/a}\int_0^{2L/a} |p(L,t)|\,dt.$$

The relation between $\bar{p}_{02L/a}/\{\rho_0 a q_0/A(0)\}$ and m is shown in Fig. 7. Especially in the case of $m=1+\delta\,(|\delta|\ll1)$, the parameter as defined above becomes

$$\bar{p}_{02L/a} = (1-\delta)\rho_0 a q_0/A(0).$$

For any finite m, the calculated average pressure value is small compared with $\rho_0 a q_0/\{A(0)/m\}$, which is the estimated average pressure value based upon the initial fluid velocity.

(3) The peak pressure value

According to the numerical example which includes the effect of viscosity, it is highly unlikely that the pressure reaches the peak value after a long time. Under the assumption of nonviscosity ($Dn=0$), an excellent agreement between the numerical solution by the method of characteristics and the present analytical solution has been found for the linear tapered pipe. In the case with the exponential tapered pipe, the agreement is not so good as that in the case with the linear tapered pipe.

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