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On the Numerical Analysis of Transient Flow in a Pipe Filled with Viscous Liquid (by the Method of Characteristics)

(Received January 27, 1970)

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Abstract

With the development of industrial technology, the hydraulic machines and equipments of late have become larger in size, and the non-steady flow of fluid in the pipe conduit of the oil pressure circuit came to attract an increasing amount of attention. The analysis of this problem boils down to the question of how accurately the Navier-Stokes equation can be solved. Many a method has been proposed, which introduces the idea of the impedance and admittance in the electric circuit, in obtaining the analytical solution in a form corresponding to the frequency. However, the processes involved in the proposed solution are too complicated, and an application of these methods to the flow of fluid in the pipe conduit cannot be justified. Furthermore, these methods fail to take into consideration the influence of gravity and the elasticity of the pipe conduit.

This paper, therefore, describes an analysis of this problem by means of the characteristic curve method, taking into account the shortcomings of the above-mentioned methods. The advantage of this method lies in its facility, enabling to obtain numerical solutions taking into consideration such conditions as the characteristics of the ramifications, combining and the varying calibers of the pipe. This paper also reports the manner, in which the pressure wave declines, in case the resistance arrived at through numerical calculation is in proportion to the first power of the velocity, and that in case the resistance is in proportion to the second power of the velocity.

I. Introduction

There are two methods of analyzing the non-steady flow of fluid in the oil pressure circuit. (i) A method based on differential equation: this is further divided into two, (a) one takes into consideration the inertia of the fluid but disregards the compressibility(1), while (b) the other takes into consideration the compressibility but disregards the inertia(2). (ii) Another method based on partial differential equation:

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(a) this treats the fluid as inviscid fluid in one dimension; (b) treats the fluid as viscous fluid in one dimension[3], and (c) treats the fluid as viscous fluid in two dimensions[4],[5],[6]. The last method introduces the idea of Series Impedance and Shunt Admittance of the electric circuit, in obtaining the analytical solution in the from corresponding to the frequency. However, the processes involved in the proposed solution are too complicated, and an application of this method to the analysis of the flow of fluid in the pipe conduit cannot be justified. Furthermore, this analysis disregards the influence of the elasticity of the pipe conduit, its inclination and the gravity.

In an effort to remedy these shortcomings, the authors worked out an analysis by means of a characteristic curve method which reflects the characteristics of the ramification of the pipe conduits, their combination, connection of pipes with varying diameters, the valves and the boundary characteristics.

II. Notations

A : cross.sectional area of conduit
D : inside diameter of conduit
E : modulus of elasticity of pipe wall materia.
F : loss coefficient $=\frac{32}{D^2}\eta$
H : pressure head
H_0 : normal pressure head $=\frac{aU_0}{g}$
H_R : pressure head of reservoir
K : bulk moduls of water
K' : equivalent bulk modulus $=\frac{1}{\frac{1}{K}+\frac{\lambda}{E}}$
L : total length of conduit
M : Mach-number $=\frac{U}{a}$
M_0 : normal Mach-number $=\frac{U_0}{a}$
P : $p+\rho_0 gz$
P_R : reservoir pressure
R : inside radius of conduit
a : velocity of pressure wave
c : coefficient determined by support conditions of conduit
e : thickness of conduit material
f : Darcy-Weisbach friction coefficient

$\boldsymbol{g}$: gravity acceleration vector
g : absolute value of gravity acceleration $=|\boldsymbol{g}|$
h_l : dimensionless hydraulic loss head
$=\frac{1}{H}\left(f\frac{L}{D}\frac{U_0^2}{2g}\right)$ for turbulent flow
$=\frac{1}{P_0}\left(\frac{32}{D^2}\eta U_0 L\right)$ for laminar flow
m : mass within control surface $=\rho A\delta x$
p : pressure
s : dimensionless deviation of density $=\frac{\rho-\rho_0}{\rho_0}$
t : time
u : axial velocity component
v : radial velocity component
$\boldsymbol{w}$: velocity vector
x : positive distance measured from valve to reservoir
z : height of conduit from reference level
ρ : density of water
ρ_0 : normal density of water
η : viscosity
ν : kinematic viscosity
ν_0 : normal kinematic viscosity
τ_i : eigenvalue
λ : elasticity factor $=\frac{E}{dP}(2\varepsilon_1+\varepsilon_2)$
ε_1 : circumferential strain $=\frac{dR}{R}$
ε_2 : longitudinal strain $=\frac{d(\delta x)}{\delta x}$
σ_1 : circumferential stress
σ_2 : longitudinal stress
μ : Poisson's ratio for pipe wall material

III. Fundamental equations

Equation of Navier-Stokes

$$\frac{d\boldsymbol{W}}{dt}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\boldsymbol{W}+\frac{\nu}{3}\nabla(\nabla\cdot\boldsymbol{W})+\boldsymbol{g}. \quad (1)$$

Equation of continuity for cases where the control surface is changeable

$$\frac{\partial m}{\partial t}+\nabla\cdot(m\boldsymbol{W})=0. \quad (2)$$

State equation for oil

$$\frac{dp}{d\rho}=\frac{K}{\rho}. \tag{3}$$

Hook's Law for thin pipe

$$\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix}=\frac{1}{E}\begin{pmatrix}1 & -\mu\\ -\mu & 1\end{pmatrix}\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}. \tag{4}$$

The above four are the fundamental equations. The coordinate axis (Fig. 1-a and b) is designated as (x, r, θ') and the axis x is plotted along the pipe axis, r axis is plotted in the direction of the radius and the θ' axis is along circumferential direction.

Postulations

i. $\nu=\nu_0$ constant

ii. Axious symmetric flow of two dimentions $\boldsymbol{W}=(u, v, o)$ and $u\gg v$. Accordingly, the pressure is uniform on the section, $p=p\ (x, t)$.

iii. Flow pattern changes while maintaining the distribution of velocity in the laminar flow and parabola:

$$U(x, t)=\frac{1}{A}\int_0^k 2\pi ru(a, r, t)dr,$$

$$u(x, r, t)=2U(x, t)\left\{1-\left(\frac{r}{R}\right)^2\right\}.$$

iv. $R\gg L$.

v. The compressibility of the fluid and the elasticity of the wall of the pipe is about the same:

$$\rho=\rho_0(1+s), \qquad s\ll 1.$$

vi. $$\boldsymbol{g}=(-g\sin\theta, 0, 0).$$

From Eq. (1), on the basis of the above-mentioned postulations,

$$\frac{du}{dt}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\nu_0\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-g\sin\theta \tag{5}$$

is obtained. From Eq. (2)

$$\frac{1}{\rho}\frac{d\rho}{dt}+\frac{1}{A}\frac{dA}{dt}+\frac{1}{\delta x}\frac{d(\delta x)}{dt}+\nabla\cdot\boldsymbol{W}=0.$$

By means of Eqs. (3) and (4)

$$\frac{1}{K}\frac{dp}{dt}+\frac{2\varepsilon_1}{dp}\frac{dp}{dt}+\frac{\varepsilon}{dp}\frac{dp}{dt}+\nabla\cdot\boldsymbol{W}=0.$$

Accordingly

$$\frac{1}{K'}\frac{dp}{dt}+\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{\partial u}{\partial x}=0 \tag{6}$$

is obtained.

Where

$$\frac{1}{K'}=\frac{1}{K}+\frac{\lambda}{E},$$

$$\lambda=\frac{E}{dp}(2\varepsilon_1+\varepsilon_2)=\frac{1}{dp}\left\{(2\varepsilon\mu)\sigma_1+(1-2\mu)\sigma_2\right\}=\frac{Dc}{e},$$

$c=1-\mu^2$: for both sides are fixed,

$c=\frac{5}{4}-\mu$: for one side is fixed, the other is free.

We define

$$P=p+\rho_0 gz,$$

$$a=\sqrt{\frac{K'}{\rho_0}}=\sqrt{\frac{K/\rho_0}{1+\frac{KDc}{Ee}}},$$

$$F=\frac{32}{D^2}\eta,$$

and Eqs. (5) and (6) is averaged on the section. By using the postulation (ii) and (iv)

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}=-\frac{1}{\rho_0}\frac{\partial P}{\partial x}-\frac{F}{\rho_0}U, \tag{7}$$

$$\frac{\partial P}{\partial t}+U\left(\frac{\partial p}{\partial \lambda}-\rho_0 g\sin\theta\right)=-\rho_0 a^2\frac{\partial U}{\partial x} \tag{8}$$

are obtained as the first approximation. These are the fundamental equations for analyzing the impact on the oil pressure circuit.

IV. Characteristic curve method

Equations (7) and (8) are the partial differential equations of the non-linear and hyperbolic type.

We can write them in the following vector type:

$$A\boldsymbol{V}+\boldsymbol{V}_t=\boldsymbol{C}, \tag{9}$$

where

$$A=\begin{pmatrix}U, & \frac{1}{\rho_0}\\ \rho_0 a^2, & U\end{pmatrix},\quad \boldsymbol{V}=\begin{pmatrix}U\\ P\end{pmatrix},\quad \boldsymbol{V}_x=\begin{pmatrix}U_x\\ P_x\end{pmatrix},\quad \boldsymbol{V}_t=\begin{pmatrix}U_t\\ P_t\end{pmatrix},\quad C=\begin{pmatrix}-\frac{F}{\rho_0}U\\ \rho_0 gU\sin\theta\end{pmatrix}.$$

Accordingly, the characteristic equation of Eq. (9) is

$$|A-\tau I|=0, \tag{10}$$

on the basis of which the eigenvalues of (i=1, 2) are obtained;

$$\tau_1=U+a,$$

$$\tau_2=U-a.$$

If the eigenvector corresponding to the eigenvalue τ_i is assumed to be $\boldsymbol{h}_i$, we have

$$\boldsymbol{h}_i A=\tau_i \boldsymbol{h}_i \tag{11}$$

$$h_{11}:h_{12}=a:\frac{1}{\rho_0},\ \text{for}\ \tau_1;$$

$$h_{21}:h_{22}=-a:\frac{1}{\rho_0},\ \text{for}\ \tau_2.$$

Multiplying Eq. (9) by eigenvector $\boldsymbol{h}_i$ from the left-hand side, we have

$$\boldsymbol{h}\cdot(\tau_i \boldsymbol{V}_x+\boldsymbol{V}_t)=\boldsymbol{h}_i\cdot\boldsymbol{C},$$

$$\therefore\quad \boldsymbol{h}_i\cdot d\boldsymbol{V}=\boldsymbol{h}_i\cdot\boldsymbol{C}dt. \tag{12}$$

This is the characteristic curve. It is divided into two component parts:

$$dP=-\rho_0 a dU+(\rho_0 g \sin\theta-aF)Udt, \tag{13}$$

$$\text{for}\qquad dx=(U+a)dt,$$

and

$$dP=\rho_0 a dU+(\rho_0 g \sin\theta+aF)Udt, \tag{14}$$

$$\text{for}\qquad dx=(U-a)dt.$$

V. Method[8] of calculation of numerical value

The method is divided into three parts, and each of them is solved as simultaneous equations along the characteristic curve.

(i) The end of the left-hand side

Boundary condition;

$$P=f(U). \tag{15}$$

Characteristic curve;

$$dP=\rho_0 a dU+(\rho_0 g \sin\theta+aF)Udt. \tag{14}$$

(ii) On the pipe line

Characteristic curve of the negative slope;

$$dP = -\rho_0 a dU + (\rho_0 g \sin\theta - aF) U dt. \tag{13}$$

Characteristic curve of the positive slope;

$$dP = \rho_0 a dU + (\rho_0 g \sin\theta + aF) U dt. \tag{14}$$

(iii) The end of the right-hand side
Characteristic curve;

$$dP = -\rho_0 a dU + (\rho_0 g \sin\theta - aF) U dt. \tag{13}$$

Boundary conditions;

$$P = g(U). \tag{16}$$

The results are given at the intersection points. Particularly, in the case of liquid, it being $M = \frac{V}{a} \sim \frac{10}{1000} = 0.01$, the calculation is much simplified by linearizing it as $dx = \pm a dt$. When equation (13) is integrated to ① and ② with reference to Fig. 3-a and equation (14) is integrated to ③ and ②, equations (17) and (18) result:

$$P_2 - P_1 = -\rho_0 a (U_2 - U_1) - \frac{1}{2}(aF - \rho_0 g \sin\theta)(U_2 + U_1)\Delta t, \tag{17}$$

$$P_2 - {}_3 = \rho_0 a (U_2 - U_3) + \frac{1}{2}(aF + \rho_0 g \sin\theta)(U_2 + U_3)\Delta t. \tag{18}$$

Substituting the following $k_1(U)$ and $k_2(U)$ into equations (17) and (18),

$$k_1(U) = \frac{1}{2}(aF - \rho_0 g \sin\theta) V \Delta t,$$

$$k_2(U) = \frac{1}{2}(aF + \rho_0 g \sin\theta) V \Delta t,$$

we have

$$\{P_2 + k_1(U_2)\} - \{P_1 - k_1(U_1)\} = -\rho_0 a (U_2 - U_1), \tag{19}$$

$$\{P_2 - k_2(U_2)\} - \{P_3 + k_2(U_3)\} = +\rho_0 a (U_2 - U_3). \tag{20}$$

If equations (19) and (20) are reproduced in the form of diagrams, they result in Fig. 3-b and c. Generally under this method, k_1 and k_2 become valid in relation to the radom function of V. Particularly, the flow of fluid of low viscosity in the pipe conduit becomes turbulent, accompanying a resistance loss in proportion to the second power of the velocity. Even in this case, only $k(V)$ becomes a quadratic and equations (19) and (20) still remains valid.

VI. Calculation models and their review

All the calculations were made under the assumption that the valves in the normal flow of the fluid is suddenly closed.

i. Fig. 4-a, b, c and d show the changes over time in the pressure at a point right behind the valve, in case the viscosity of the fluid flowing through a horizontal pipe conduit increased. Fig. 4-a shows the case in which the fluid being non-viscous there took place no loss of energy. In this case, this was a perfect rectangular wave and no decline was observed. On the other hand, as the viscosity of the fluid increased the pressure rapidly decreased. In this case, the relations between the maximum pressure and the coefficient of viscosity can be shown by

$$P_{\max} \fallingdotseq 2\rho a U_0\left(1-\frac{16\nu L}{D^2 a}\right). \tag{21}$$

Fig. 5-a, b, c and d shows the changes in the pressure at the center of the pipe conduit, and due to the overlapping pressure of the reflected waves generated from the reservior, the changes in the pressure becomes complicated.

ii. In an effort to determine the effects of the various angle, calculations were conducted on the basis of $\theta=0°$, $30°$, $45°$, $60°$ and $90°$. However, the results showed little influence on the pressure, other than those on the potential energy, as shown in Fig. 6-a, b and 10. Accordingly, even if the influence on the potential energy was ignored, the resulting error would be in the order of 1%. This can be seen also from equations (13) and (14) to be

$$\frac{\rho_0 g \sin\theta U \Delta t}{\rho_0}=\frac{\rho_0 g}{\gamma}\cdot\frac{L\sin\theta}{H_0}\cdot\frac{U_0}{a}\cdot\frac{U}{U_0}\cdot\frac{\Delta t}{\mu}<M_0 .$$

iii. Generally, the fundamental equations applicable to the cases, in which the flow in the pipe is turbulent, are

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}=-g\left(\frac{\partial H}{\partial x}+\frac{f}{D}\cdot\frac{U|U|}{2g}\right)^{(9),(10),(11)}, \tag{22}$$

$$\frac{\partial H}{\partial t}+U\left(\frac{\partial H}{\partial x}-\sin\theta\right)=-\frac{a^2}{g}\frac{\partial U}{\partial x}. \tag{23}$$

The solution of the case, in which 50% of the head was lost under similar boundary conditions, obtained on the basis of the above-mentioned equations, is shown in Fig. 7-a and b. When this was compared with Fig. 4-c and 5-c, it becomes evident that the decline of pressure of such viscous fluid as oil is more precipitous than other fluids, in spite of the fact that the energy loss of these fluids is the same. Fig. 8-b shows a comparison of decline of pressure between the case in which the loss of energy is in proportion to the second power of the velocity and that in which the same is in proportion to the first power of the velocity. According to the results of the theoretical calculation, they are

$$\frac{P-P_R}{\rho_0 a U_0} = \exp\left\{-0.73(1\pm\varepsilon)\frac{FL}{\rho_0 a}\cdot\frac{t}{2\mu}\right\}, \tag{24}$$

$$\frac{H-H_R}{\frac{aU_0}{g}} = \exp\left\{-0.26(1\pm\varepsilon)\sqrt{\frac{fLM_0}{2D}}\cdot\frac{t}{2\mu}\right\}, \tag{25}$$

where $\varepsilon \lesssim 0.2$.

However, in these calculations, the reflection was assumed to have caused no loss of energy, and losses relating to the boundary layer were not taken into consideration. In practice, therefore, the decline of pressure should have been somewhat more rapid than those indicated by the above results.

iv. Fig. 8-a shows the maximum rise of pressure and the minimum decline of pressure occuring when the lost head in the pipe is increased, with the head fixed (by such means as roughening the interior surface of the pipe, raising the viscosity of the fluid and narrowing and lengthening the pipe conduits, etc.). As the loss of head is increased, the rate of rise of the maximum pressure slows down and at the same time the minimum pressure also decreases. Particularly disturbing in this case are the separation of the water column in the case of water hammer, and the generation of air bubbles in the case of oil hammer. It would be of interest to calculate taking these conditions into account.

v. Fig. 9-a and b show the decline of pressure when H_0 is increased as the lost head increases, under conditions in which H_R is fixed. From these figures, it is evident that as the loss of head increases, the tempo of pressure changes becomes accelerated.

VII. Conclusion

Fundamental equations have been formulated taking into consideration the effects of gravity, the elastic deformation of the pipe conduit, and the compressibility of oil, and by means of a diagram analytical method based on the characteristic curve method, which facilitates analysis of the pipe conduits and incorporation of the boundary conditions, a numerical calculation method is shown. This method enables field engineers to obtain solutions by means of repetitive simple calculations, and the authors believe that this is eminently suitable to field engineers who have the benefit of a digital electronic computer, but not the highly sophisticated knowledge of mathematics.

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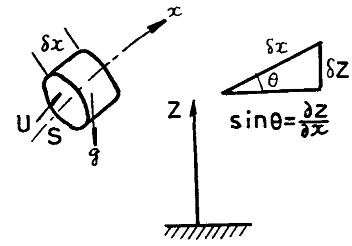


Fig. 1-a. Coordinates.

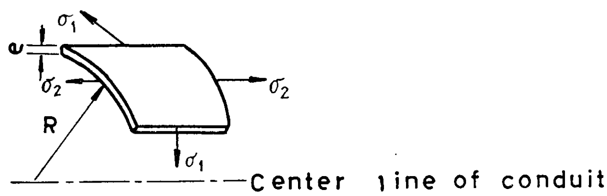


Fig. 1-b. Relations of stress.

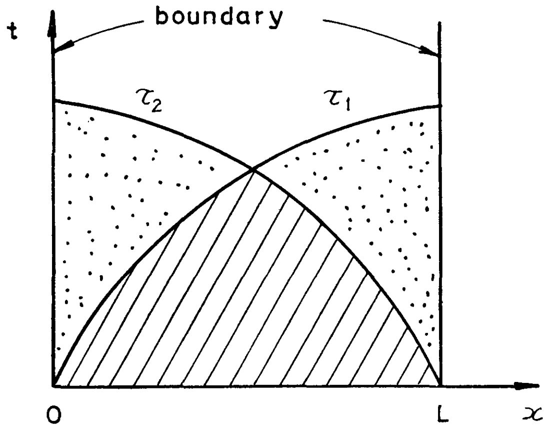


Fig. 2. Physical plane.

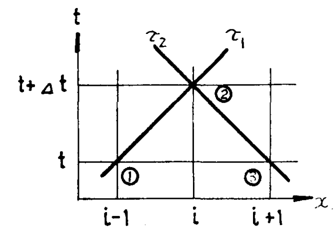


Fig. 3-a. Linearized physical plane.

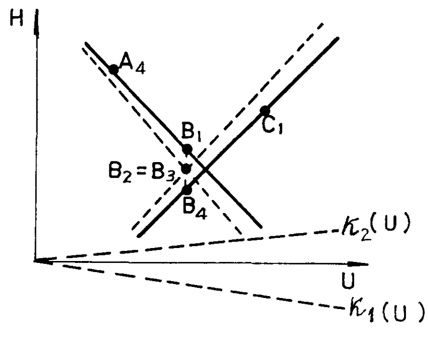


Fig. 3-b. Hodograph plane.

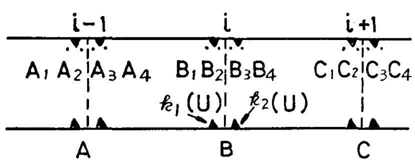


Fig. 3-c. Model of conduit.

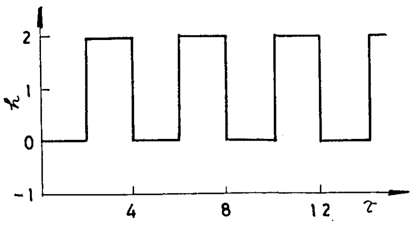


Fig. 4-a. Pressure change at the inlet, $h_l=0$, $\theta=0°$.

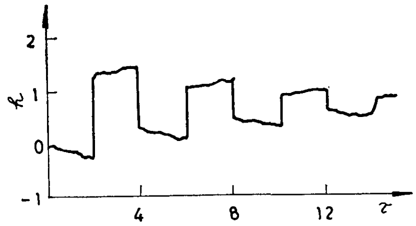


Fig. 4-b. Pressure change at the inlet, $h_l=0.25\,v$, $\theta=0°$.

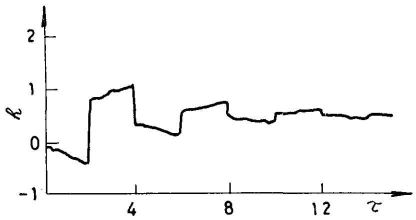


Fig. 4-c. Pressure change at the inlet, $h_l=0.5\,v$. $\theta=0°$.

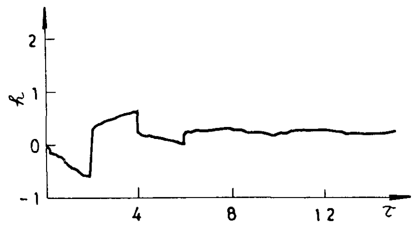


Fig. 4-d. Pressure change at the inlet, $h_l=0.75\,v$, $\theta=0°$.

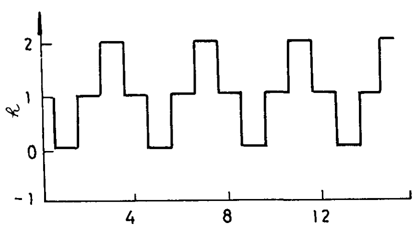


Fig. 5-a. Pressure change at the middle of the conduit, $h_l=0$, $\theta=0°$.

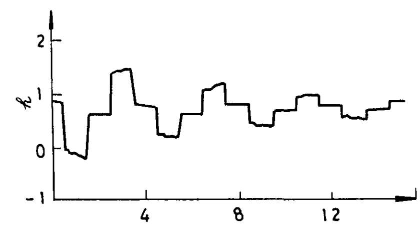


Fig. 5-b. Pressure change at the middle of the conduit, $h_l=0.25\,v$, $\theta=0°$.

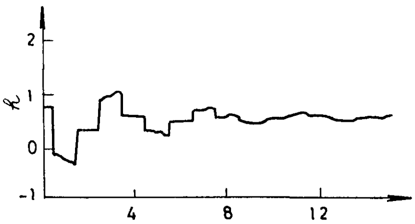


Fig. 5-c. Pressure change at the middle of the conduit, $h_l=0.5\,v$, $\theta=0°$.

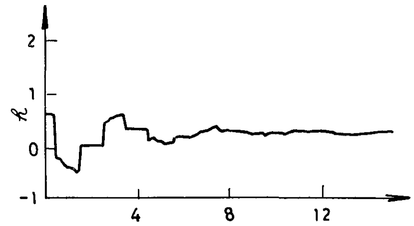


Fig. 5-d. Pressure change at the middle of the conduit, $h_l=0.75\,v$, $\theta=0°$.

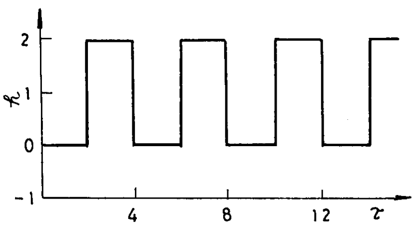


Fig. 6-a. Pressure change at the inlet, $h_l=0$, $\theta=90°$.

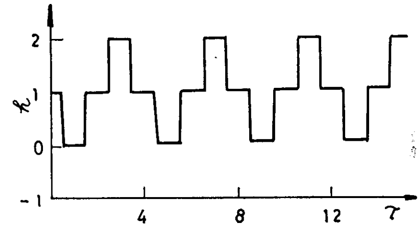


Fig. 6-b. Pressure change at the middle of the conduit, $h_l=0$, $\theta=90°$.

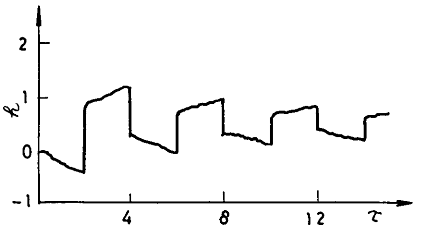


Fig. 7-a. Pressure change at the inlet, $h_l=0.5\,v^2$, $\theta=0°$.

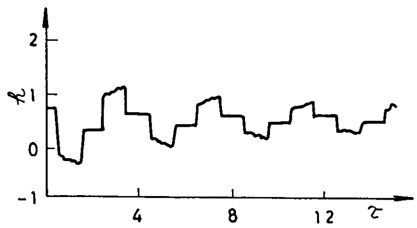


Fig. 7-b. Pressure change at the middle of the conduit, $h_l=0.5\,v^2$, $\theta=0°$.

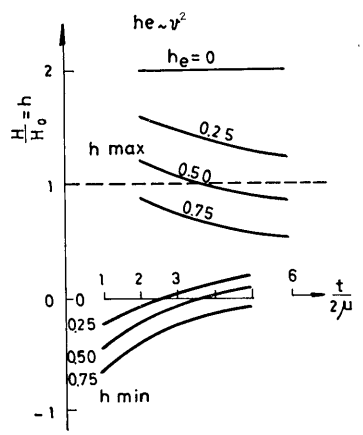


Fig. 8-a₁. Pressure attenuation in proportion to v^2 of h_l, H_0 is constant.

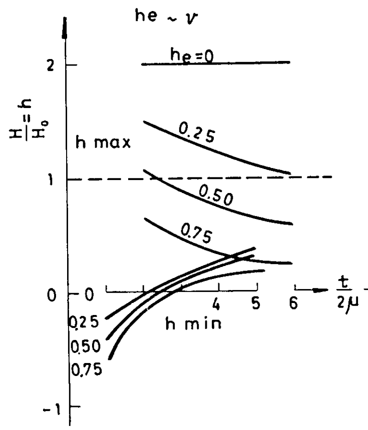


Fig. 8-a₂. Pressure attenuation in proportion to v of h_l, H_0 is constant.

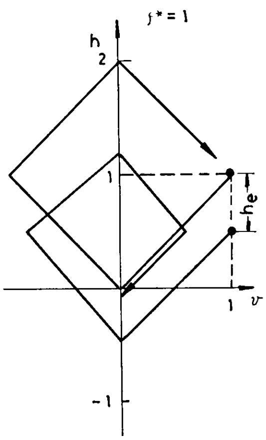
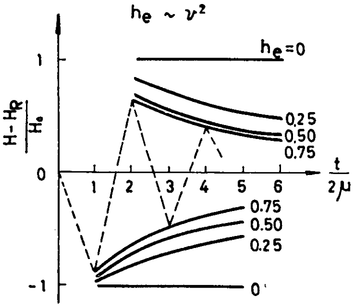


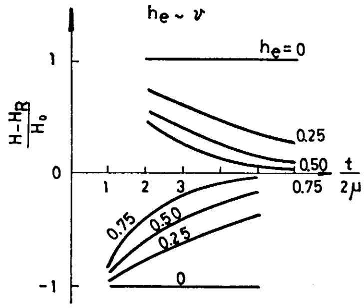
Fig. 8-a₃. Simple illustration.



$$\frac{H-H_R}{H_0}=e^{-0.25(1\pm\varepsilon)\sqrt{\frac{H_0-H_R}{H_0}}\,\frac{t}{2\mu}}$$

$\varepsilon \leq 0.2$

Fig. 8-b$_1$. Pressure attenuation in proportion to v^2 of h_l, H_0 is constant.



$$\frac{H-H_R}{H_0}=e^{-0.72(1\pm\varepsilon)\frac{H-H_R}{H_0}\,\frac{t}{2\mu}}$$

$\varepsilon \leq 0.2$

Fig. 8-b$_2$. Pressure attenuation in proportion to v of h_l, H_0 is constant.

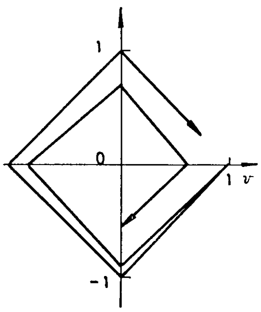


Fig. 8-b$_3$. Simple illustration

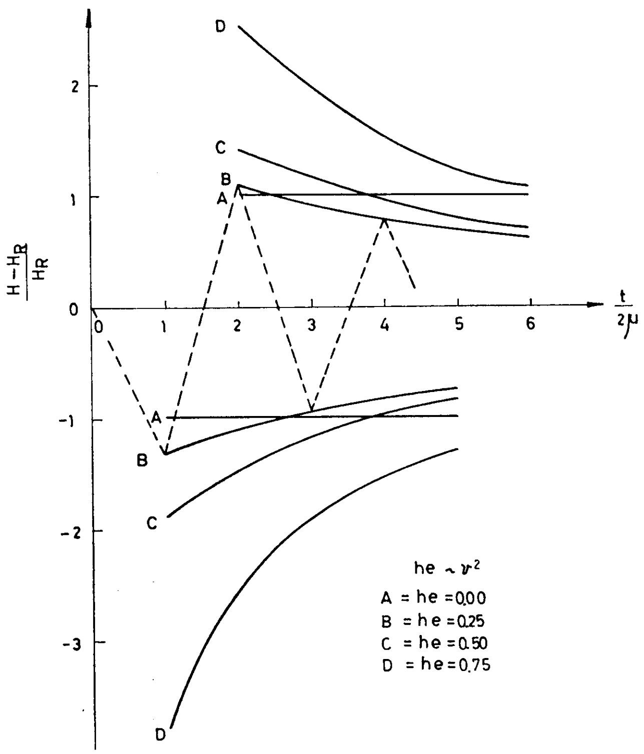


Fig. 9-a. Pressure attenuation in proportion to v of h_l, H_R is constant.

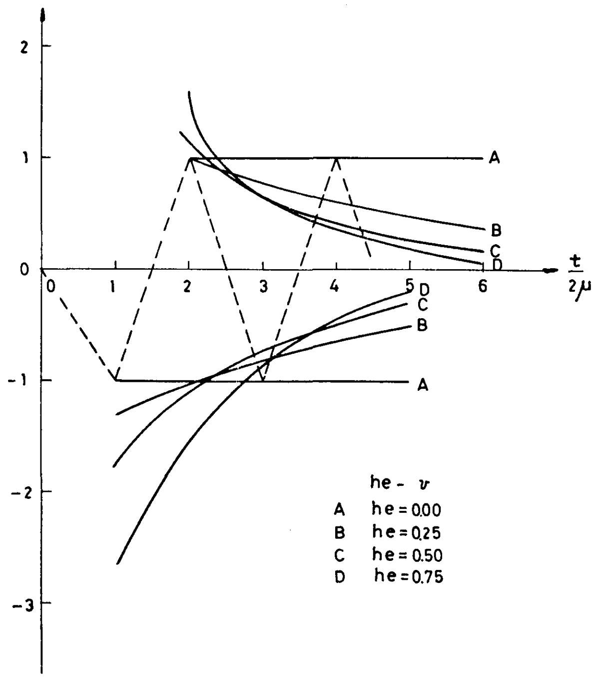


Fig. 9-b. Pressure attenuation in proportion to v^2 of h_l, H_R is constant.

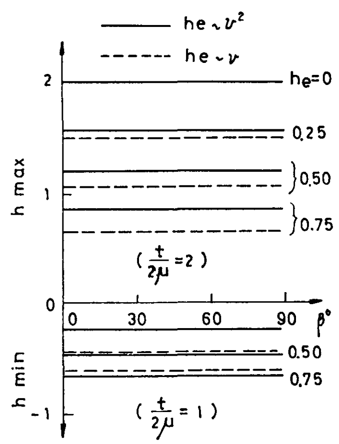


Fig. 10. Relations between maximum pressure rise and slope of conduit.