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On the Theoretical Formulas for Calculating the Time Lag between the Stoppage of Pump and the Closure of the Valve and the Quantity of Reverse Flow

(Received January 27, 1970)

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Abstract

As a simple device for preventing the reverse flow of liquid in the pipe conduit system of the pump, check valves are often used. For analyzing the water-hammer action in the pipe conduit, it is necessary to understand the action characteristics of the check valve. At present, however, little is known about the characteristics of the valve. In order, therefore, to cope with the reduced effects of the water-hammer, a weight or a spring is attached to the valve. The best way to minimize the rise in the pressure is to completely shut the valve at the moment the liquid starts to flow in reverse. A delay in shutting the valve causes a rapid rise in the pressure. Then, how long is the time required to shut the valve completely after the liquid has started to flow in reverse? What is the quantity of reverse flow? These are the questions, which this paper attempts to answer by means of theoretical analysis. The results obtained are shown in a diagram to facilitate the reading of them by field engineers.

I. Introduction

As a simple device for preventing the reverse flow of liquid in the pipe conduit system of the pump, check valves[1],[2] are often used. For analyzing the water-hammer action in the conduit, it is necessary to understand the overall characterisitics of the pump, in general, and the action characteristics of the check valve in particular. At present, however, little is known about the characteristics of the valve. In order, therefore, to cope with the reduced effects of the water-hammer, a weight or a spring is attached to the valve. The best way to minimize the rise in the pressure is to completely shut the valve at the moment the liquid in the conduit starts to flow in reverse. A delay in shutting the valve causes a rapid rise of the pressure.

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Then, what is the time lag between the point of time at which the pump stops and that at which the liquid starts to flow in reverse? What is the quantity of reverse flow? To calculate the time lag and the quantity of reverse flow is the subjects of this paper. In order to calculate the quantity of reverse flow, one must first calculate the conduit constant ρ^* with the given pump, the pump constant K^*, and the time lag T_a^*, intervening between the stoppage of the pump and the start of the reverse flow, which is the function of the friction loss h_l. However, the diagram available for the calculation of the time lag T_a^* ignores the head loss through friction, as in the case of the diagram of J. Parmakian.(3) The diagram failed to show the characteristic curve of the pump which was assumed for the calculation, neither is it clear in what scope this diagram is applicable. Other diagrams include Professor Kito's,(4) who treated the matter as one of the simple dynamics, and Professor Kinno's,(5) who calculated the quantity assuming the loss head as the orifice at the discharge end. These, however, appear to allow considerable error of the time intervals in the graphical calculation.

Mindful of these shortcomings of the existing diagrams, the authors distributed the head loss through friction throughout the conduit, the number of division of the conduit having been so arranged as to conform to $K^*\Delta t^* \leqq 0.1$, and T_a^* was obtained through a theoretical calculation of the water-hammer action, using a complete characteristic curve $(n_s=1800)$(6) of the pump. This paper describes the processes involved in the calculation of the case mentioned above.

II. Notations

D: inside diameter of conduit

E: modulus of elasticity of pipe wall material

F: elliptic integral of the first kind

GD^2: flywheel effect of rotating parts of pump

H: pressure head

H_l: hydraulic loss head

H_0: normal pressure head

I_1: moment of area of the first order $=\int_{S_1} r dm$

I_2: moment of area of the second order $=\int_{S_1} r^2 dm$

K: bulk modulus of water

K^*: pump constant $=\dfrac{120 g M_0 L}{\pi G D^2 N_0 a}$

L: total length of conduit

M: pump input torque

M_0: normal pump input torque

Q: discharge of conduit
Q_0: normal discharge of conduit
Q_b: discharge at starting time of value
S: sectional area of conduit $=\frac{\pi}{4}D^2$
S_1: area of valve
S_2: area of difference between sectional area of conduit and projected area of valve (Eq. 20)
T_a^*: time from pump stoppage to starting time in reverse flow (Fig. 5)
T_b^*: time from pump stoppage to valve starting time (Fig. 5)
T_c^*: time from valve starting time to complete closure of valve (Fig. 5)
V: velocity
V_0: normal velocity
V_1: velocity at the valve
V_2: velocity in the area S_2
W: weight of valve
X: =Eq. 28
Y: =Eq. 32
a: velocity of pressure wave $=\sqrt{\frac{K/\rho_0}{1+\frac{KD}{Ee}}}$
e: thickness of conduit material
f: Darcy-Weisbach friction coefficient
g: gravity acceleration
h^*: dimensionless pressure head
h_l^*: dimensionless hydraulic loss head
k: =Eq. 16
m: mass
m^*: dimensionless torque $=\frac{M}{M_0}$
n^*: dimensionless pump speed $=\frac{N}{N_0}$
r: radius
$\tilde{r}$: equivalent radius
t: time
q^*: dimensionless discharge $=\frac{Q}{Q_0}$
x^*: dimensionless distance measured positive from valve to reservoir
α: given acceleration working on the valve
β: angle of inclined conduit
γ: specific weight of liquid
δ: $=\frac{H_{l_0}}{L}$

η: hydraulic loss coefficient $=\dfrac{fLV_0^2}{2gDH_0}$

ζ: influence coefficient (Eq. 4)

θ: angle of valve opening

θ_0: normal angle of valve opening

λ: $=\dfrac{\alpha}{g}$

μ: traveling time of wave from valve to reservoir $=\dfrac{L}{a}$

ξ: $=\dfrac{S_1}{S}$

ρ_0: density of fluid

ρ^*: pipe constant $=\dfrac{aQ_0}{gSH_0}$

φ: =Eq. (5)

ϕ: =Eq. (17)

Subscripts

a: starting time in reverse flow

b: valve starting time

c: complete closure of valve

0: initial state

1: position of valve

2: between sectional area of conduit and area of valve

*: dimensionless

$\cdot$: time derivative

III. Fundamental equations

3. 1. *Kinematic Equation of the Valve and its Solution*

The flow of the fluid, particularly those on both sides of the valve, set in motion by the stoppage of the pump appears to be very complicated. This makes it difficult to calculate analytically the impact registered on the valve by the movement of the fluid. So, it is assumed that a given acceleration α works on the valve. The valve revolves with its center at point A and a stopper is installed at point C, at which the valve rests when the flow of the fluid reverses.

When the composition of the force working in the direction of θ on the micro-mass dm is assumed to be

$$dF=\{g \sin(\theta+\beta)+\alpha \cos\theta\}dm\,, \tag{1}$$

and if the relations of the torque working on the valve is assumed to be

$$I_1=\int rdm\,, \qquad I_2=\int r^2dm\,, \qquad \tilde{r}=\frac{I_2}{I_1}\,, \qquad \lambda=\frac{\alpha}{g}\,,$$

$$I_2\ddot{\theta}=-\int rdF=-\{g\sin(\theta+\beta)+\alpha\cos\theta\}I_1\,. \tag{2}$$

Therefrom

$$\ddot{\theta}+\frac{g}{\tilde{r}}\{\lambda\cos\theta+\sin(\theta+\beta)\}=0 \tag{3}$$

is obtained.

If this is simplified by means of

$$\zeta=\sqrt{\lambda^2+2\lambda\sin\beta+1} \tag{4}$$

and

$$\varphi=\tan^{-1}\frac{\lambda+\sin\beta}{\cos\beta} \tag{5}$$

the kinematic equation

$$\ddot{\theta}+\frac{g\zeta}{\tilde{r}}\sin(\theta+\varphi)=0 \tag{6}$$

ensue therefore.

When t is equated to 0, $\theta=\theta_0$ and the solution of the kinematic equation (6) is obtained on the basis of the initial conditions of $\dot{\theta}=\dot{\theta}_0=0$. When this is multiplied by $\dot{\theta}$ and integrated to $t=0\sim t$, we have

$$\dot{\theta}=-\sqrt{\frac{2g\zeta}{\tilde{r}}\{\cos(\theta+\varphi)-\cos(\theta_0+\varphi)\}}\,. \tag{7}$$

Accordingly, when the time needed for a complete closure of the valve is designated as T_c,

$$T_c=\int_0^{T_c}dt=\sqrt{\frac{\tilde{r}}{2g\zeta}}\int_0^{\theta_0}\frac{d\theta}{\sqrt{\cos(\theta+\varphi)-\cos(\theta_0+\varphi)}} \tag{8}$$

$$=\sqrt{\frac{\tilde{r}}{2g\zeta}}\cdot I \tag{9}$$

results from them, where

$$I=\int_0^{\theta_0}\frac{d\theta}{\sqrt{\cos(\theta+\varphi)-\cos(\theta_0+\varphi)}}\,. \tag{10}$$

Now let us consider the integral I. If the variables are transformed continuously:

$$x=\theta+\varphi\,,$$

$$y^2=\cos x-\cos x_0,$$

then we have

$$I=\int_0^{\theta_0} f(\theta)\,d\theta=\int_{\varphi}^{x_0} g(x)\,dx=\int_{y_1}^{0} h(y)dy\,, \tag{11}$$

where

$$x_0=\theta_0+\varphi\,,$$

$$y_1=\sqrt{\cos\varphi-\cos(\theta_0+\varphi)}=\sqrt{2\sin\frac{\theta_0}{2}\sin\left(\frac{\theta_0}{2}+\varphi\right)}\,,$$

$$h(y)=\frac{-2}{\sqrt{\{(1+\cos x_0)+y^2\}\{(1-\cos x_0)-y^2\}}}\,.$$

When, therefore, the integral equations (12) and (13) are applied,

$$\int_0^x \frac{dx}{\sqrt{(a^2-x^2)(b^2+x^2)}}=\frac{1}{\sqrt{a^2+b^2}}\,[F(k,\phi(x))]_0^x \tag{12}$$

$$k=\frac{a}{\sqrt{a^2+b^2}}$$

$$\phi(x)=\sin^{-1}\sqrt{\frac{1+(b/a)^2}{1+(b/x)^2}}$$

$$F(k,\phi)=\int_0^{\phi}\frac{d\phi}{\sqrt{1-k^2\sin^2\phi}}=\int_0^{\sin\phi}\frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}}\,, \tag{13}$$

the integral I becomes by equating $a=\sqrt{1-\cos x_0}$ and $b=\sqrt{V+\cos x_0}$,

$$I=2\int_0^{y_1}\frac{dy}{\sqrt{(a^2-y^2)(b^2+y^2)}}=\sqrt{2}\,[F(k,\phi(y))]_0^{y_1}$$

$$=\sqrt{2}\,F(k,\phi(y_1))\,. \tag{14}$$

When (9) is substituted by (12), the solutions (15), (16) and (17) are obtained:

$$T_c=\sqrt{\frac{\tilde{r}}{g\zeta}}\,F(k,\phi)\,, \tag{15}$$

$$k=\sin\left(\frac{\varphi+\theta_0}{2}\right)\,, \tag{16}$$

$$\phi=\sin^{-1}\frac{\sqrt{2\{\cos\varphi-\cos(\varphi+\theta_0)\}}}{\sqrt{(1+\cos\varphi)\{1-\cos(\varphi+\theta_0)\}}}\,. \tag{17}$$

3. 2. *Equation of Equilibrium of Valve*

It is assumed that the valve maintains an equilibrium at an opening angle of θ against the flow Q of the fluid in the pipe. The relations between Q and θ, as existing at the time of the equilibrium of the valve, can be determined. Now, it is assumed that the direction of flow remains unchanged. In this case, the force applying in the direction of the flow onto a unit area of the valve by the changing velocity of the flow is given as

$$f=\frac{\gamma Q}{gS_1}(V_2-V_1), \tag{18}$$

and according to the equation of continuity, Q becomes

$$Q=VS=V_1S_1=V_2S_2. \tag{19}$$

The projected area of the valve to the section of the pipe conduit is

$$S_2=S-S_1\cos\theta=S(1-\xi\cos\theta). \tag{20}$$

Accordingly, the equilibrium of the moment is

$$\int rg\sin(\theta+\beta)dm=\int rf\cos\theta dS_1. \tag{21}$$

When the area of the valve is equal to the sectional area of the pipe conduit ($S=S_1$) and Eqs. (18), (19) and (20) are substituted into Eq. (21),

$$Q=gS_1\sqrt{\frac{I_1}{\gamma\int rdS_1}}\Theta(\theta) \tag{22}$$

$$\Theta(\theta)=\frac{\sqrt{\sin(\theta+\beta)(1-\cos\theta)}}{\cos\theta} \tag{23}$$

are obtained. Also, in case $S\neq S_1$, it becomes

$$\Theta(\theta)=\frac{\sqrt{\sin(\theta+\beta)(\xi-\cos\theta)}}{\sqrt{\cos\theta(1-\xi+\cos\theta)}}.$$

Particularly, when the valve is of uniform thickness, $dm=\sigma dS$ ($\sigma=M/S$=face density, $Mg=W$)

$$Q=\sqrt{\frac{WS_1g}{\gamma}}\Theta(\theta). \tag{24}$$

3. 3. *Equation of Inertia of Fluid and its Solution*

The dropping movement of an object receiving a drag in proportion to the square of the velocity is expressed by

$$m\ddot{y}=-c\dot{y}^2-mg. \tag{25}$$

When the movement of the fluid in the pipe conduit from the valve to the reservior (length$=L$) is assumed to be turbulent flow, the loss head H_l is

$$H_l=f\frac{L}{D}\cdot\frac{V^2}{2g}, \tag{26}$$

and the mass of the fluid in the pipe conduit is $\frac{\gamma LS}{g}$. And since $(\alpha+g\sin\beta)$ corresponds to the acceleration g of formula (25), the equation of inertia of the fluid becomes

$$\frac{\gamma LS}{g}\cdot\frac{dV}{dt}=-\gamma Sf\cdot\frac{L}{D}\cdot\frac{V|V|}{2g}-\frac{\gamma LS}{g}(\alpha+g\sin\beta) \tag{27}$$

in accordance with Newton's second law. Accordingly, if formula (27) is de-dimensionalized by

$$t^*=\frac{t}{\mu}=\frac{at}{L}, \qquad v^*=\frac{V}{V_0}=\frac{Q}{Q_0}=q^*,$$

$$h_l^*=\frac{H_{l_0}}{H_0}=f\frac{L}{DH_0}\cdot\frac{V_0^2}{2g}, \qquad \delta=\frac{H_{l_0}}{L}, \qquad \rho^*=\frac{AQ_0}{gSH_0}$$

$$X=\sqrt{\frac{\delta}{\lambda+\sin\beta}}, \tag{28}$$

a differential equation

$$dt^*=-\frac{\rho^*X}{h_l^*}\cdot\frac{Xdv^*}{X^2v^*|v^*|+1} \tag{29}$$

is obtained. If this is integrated with respect to $t^*=0\sim t^*$ and $v^*=1\sim v^*$,

$$t^*=\frac{\rho^*X}{h_l^*}\{\tan^{-1}X-\tan^{-1}v^*X\} \tag{30}$$

is obtained. Where $v^*\geqq 0$. Accordingly, the time T_a^* at which the reverse flow starts is determined by substituting $\nu^*=0$ into Eq. 30:

$$T_a^*=\frac{\rho^*}{h_l^*}X\tan^{-1}X=\frac{\rho^*}{h_l^*}Y, \tag{31}$$

$$Y=X\tan^{-1}X. \tag{32}$$

T_a^* is theoretically calculated against the given pump system. Since ρ^*, h_l^*, δ and $\sin\beta$ are already established, x or y become the function of only λ, or only α $(=\lambda g)$ and thus the presumed acceleration α is determined.

IV. Method of determining the quantity of reverse flow

On the basis of formula (30), the time T_b^* at which the valve starts operating is calculated by equating $v^*=v_b^*$,

$$T_b^*=\frac{\rho^*}{h_l^*}\{Y-X\tan^{-1}v_b^*X\}. \tag{33}$$

v_b^* is calculated by equation $\theta=\theta_0$ of formula (22), that is to say,

$$v_b^* = \frac{V_b}{V_0} = \frac{Q_b}{V_0 S} = \frac{g S_1}{V_0 S}\sqrt{\frac{I_1}{\gamma\int r dS_1}}\Theta(\theta_0)\,. \tag{34}$$

The time T_c^* intervening between the start of operation of the valve and the complete closure of the valve is obtained by de-dimensionalizing formula (15);

$$T_c^* = \frac{a}{L}\sqrt{\frac{\tilde{r}}{g\zeta}}F(k,\psi)\,. \tag{35}$$

Accordingly, the time lag is determined by

$$\varDelta t^* = T_b^* + T_c^* - T_a^*\,. \tag{36}$$

In case of the velocity of flow $v^* \leqq 0$ under formula (29), it becomes

$$dt^* = \frac{\rho^* X}{h_l^*}\cdot\frac{X dv^*}{(Xv^*)^2 - 1}\,, \tag{37}$$

and when it is integrated to $t^* = 0 \sim \varDelta t^*$ and $v^* = 0 \sim -\varDelta v^*$,

$$\varDelta t^* = \frac{\rho^* X}{2h_l^*}\log\left|\frac{1 + X\varDelta v^*}{1 - X\varDelta v^*}\right| \tag{38}$$

is obtained. Generally speaking, it is often the case $0 < X\varDelta v^* < 1$, and then $\varDelta v^*$ is solved as

$$\varDelta v^* = \varDelta q^* = \frac{1}{X}\tanh\frac{h_l^*\varDelta t^*}{\rho^* X} \tag{39}$$

$$= \sqrt{\frac{\lambda + \sin\beta}{H_{l_0}/L}}\tanh\left\{\frac{h_l^*}{\rho^*}\sqrt{\frac{\lambda + \sin\beta}{H_{l_0}/L}}(T_b^* + T_c^* - T_a^*)\right\}. \tag{40}$$

Particularly, when $X\varDelta v^* > 1$, it becomes as follows:

$$\varDelta q^* = \frac{1}{X}\coth\left(\frac{h_l^*}{\rho^* X}\varDelta t^*\right). \tag{41}$$

V. Diagram for calculating T_a^*

T_a was calculated through theoretical computation of the water hammer action on the basis of the complete characteristic curve of the pump for $h_l^* = 0.3$, 0.5 and 0.7 under various combinations of the pump constant K^* and the pipe conduit constant ρ^*, by means of digital electronic computer. The equations of the characteristic curve used in the calculation are

$$dh^* = +\rho^* dq^* + \eta q^*|q^*|dt^*\,, \qquad dx^* = -dt^*\,; \tag{42}$$

$$dh^* = -\rho^* dq^* - \eta q^*|q^*|dt^*\,, \qquad dx^* = +dt^* \tag{43}$$

and the equation of inertia likewise used is

$$dn^* = -K^* m^* dt^* . \tag{44}$$

In selecting the number of division of the pipe conduit, $K^*dt^* \leqq 0.1$, such integer numbers as those, which will maintain the maximum rate of decrease in the number of revolution of the pump at less than ten per cent, are adopted. The results are shown in Figs. 6-b, 6-c and 6-d. For reference purpose, the diagram of J. Parmakian is given in Fig. 6-a, which ignores the loss head of the fluid in the pipe conduit. The case of lost head represented on the horizontal axis is shown in Fig. 6-e.

VI. Diagram showing the calculation of X and Y

Fig. 7-a, a′, b, b′, c and c′ represent Eqs. (28) and (32) reproduced in diagrams, and δ=constant is shown by lines and from the top, δ=10, 8, 6, 4, 2, 1, 0.8, 0.6, 0.4, 0.2, 0.1, 0.08, 0.06, 0.04, and 0.02.

VII. Order of calculation

(i) ρ^*, K^*, h_l^*, β, δ are calculated against the given pump system.
(ii) $\tilde{r}$, θ_0 of the given check valve are calculated.
(iii) T_a^* is determined on the basis of Fig. 6.
(iv) Y is determined using formula (31).
(v) λ is determined by means of the chart of Y (Fig. 7-a′, b′ and c′).
(vi) X is determined using λ calculated under (v) above. (Fig. 7-a, b, and c)
(vii) ζ and ϕ are calculated on the basis of Eqs. (4), (5) and Fig. 2-b and c.
(viii) $\Theta(\theta_0)$ is calculated. (Eq. (23) or Fig. 4)
(ix) k and ϕ is calculated (Eqs. (16) and (17) or Fig. 3-a and b).
(x) $F(k, \phi)$ is calculated (Eq. (13) or Fig. 3-c).
(xi) v_b^* is determined on the basis of Eq. (34) and then T_b^* is calculated by substituting it into Eq. (33)
(xii) T_c^* is calculated (Eq. (35)).
(xiii) Δt^* is calculated (Eq. (36)).
(xiv) Δq^* is calculated (Eq. (39)).

VIII.

8. 1. *Equation for calculating equivalent*

single pendulum: $$\tilde{r} = r , \tag{45}$$

disk: $$\tilde{r} = \frac{\left(\mu_1^2 + \dfrac{1}{4}\right) R_1}{\mu_1} , \tag{46}$$

double disk:
$$\tilde{r}=\frac{\rho_1 t_1 R_1^4\left(\mu_1^2+\frac{1}{4}\right)+\rho_2 t_2 R_2^4\left(\mu_2^2+\frac{1}{4}\right)}{\rho_1 t_1 R_1^3\mu_1+\rho_2 t_2 R_2^3\mu_2}, \tag{47}$$

where ρ_i denotes the density of the valve body, $\mu_i=\frac{r}{R_i}$, and t_i denotes the thickness of the valve.

8. 2. *Angle of slope* β

8. 3. *Influence coefficient*

When the valve shows the influence of the presumed acceleration α and α is equal to zero, $S=1$. (the presumed acceleration α represents the aggregate force of the fluid which works on the valve.)

IX. Conclusion

By means of theoretical analysis of the action of the check valve, the quantity of reverse flow at the moment of full closure of the valve and the time lag were theoretically calculated. This enabled a closer analysis of the water hammer action of the fluid in the pipe conduit equipped with a check valve. Moreover, by reproducing all pertinent formulas in the form of diagram, they are made easy to read by field engineers.

Finally, the authors wish to acknowledge the valuable assistance rendered by Hideya Makino in the calculation of all numerical values used in this paper.

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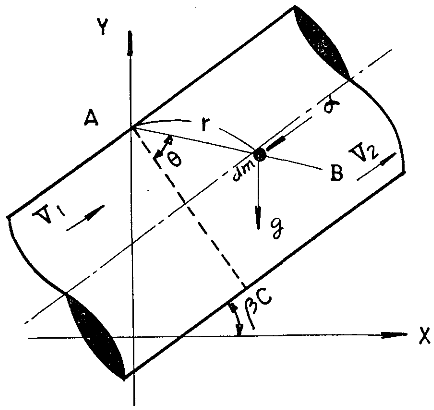

Fig. 1. Force relationship acting on the valve.

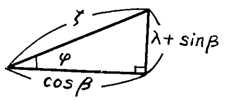

Fig. 2-a. Definition of φ.

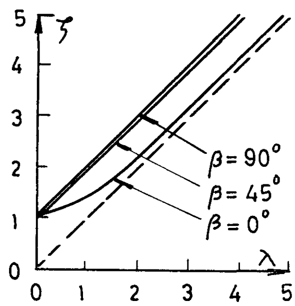

Fig. 2-b. ζ-diagram on the equation (4).

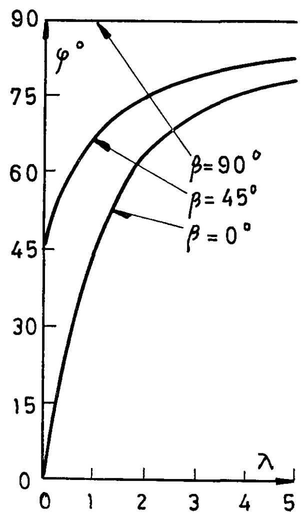

Fig. 2-c. φ-diagram on the equation (5).

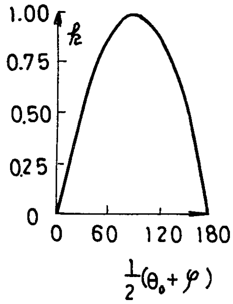

Fig. 3-a. k-diagram on the equation (16).

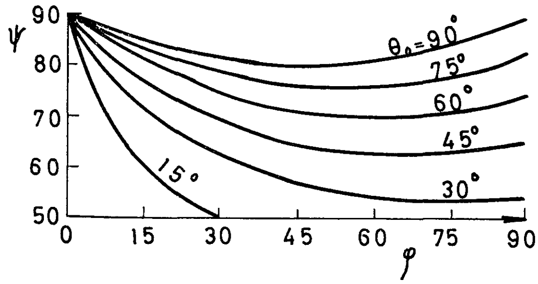

Fig. 3-b. ψ-diagram on the equation (17).

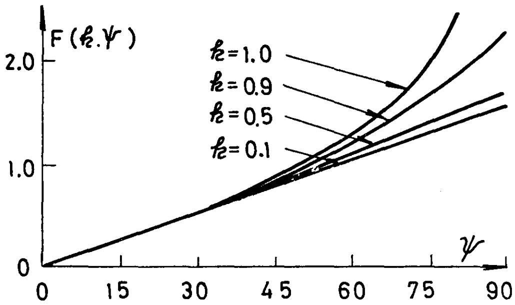

Fig. 3-c. Elliptic integral of the first kind.

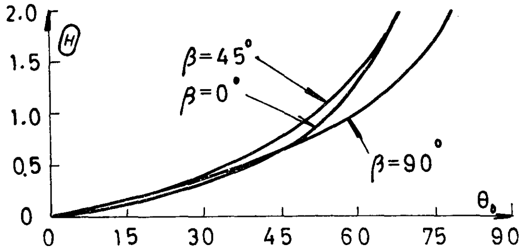

Fig. 4. $\Theta(\theta)$-diagram on the equation (23).

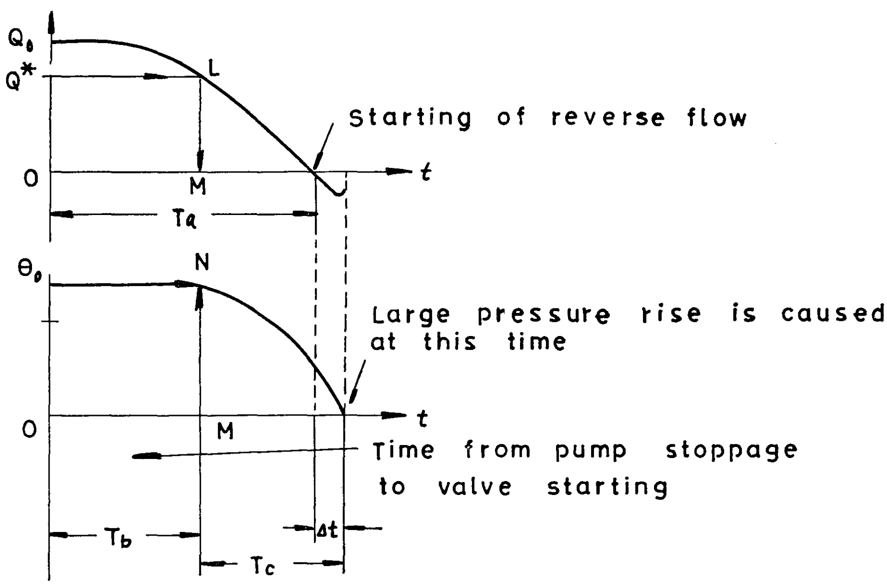

Fig. 5. Valve closing and reverse flow.

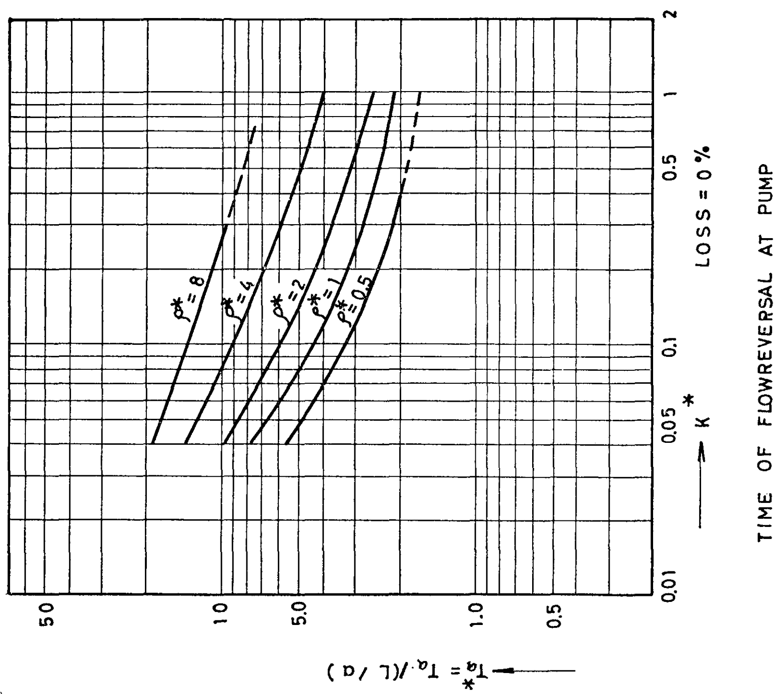

Fig. 6-a. T_a^*-diagram on the 0%-friction loss.

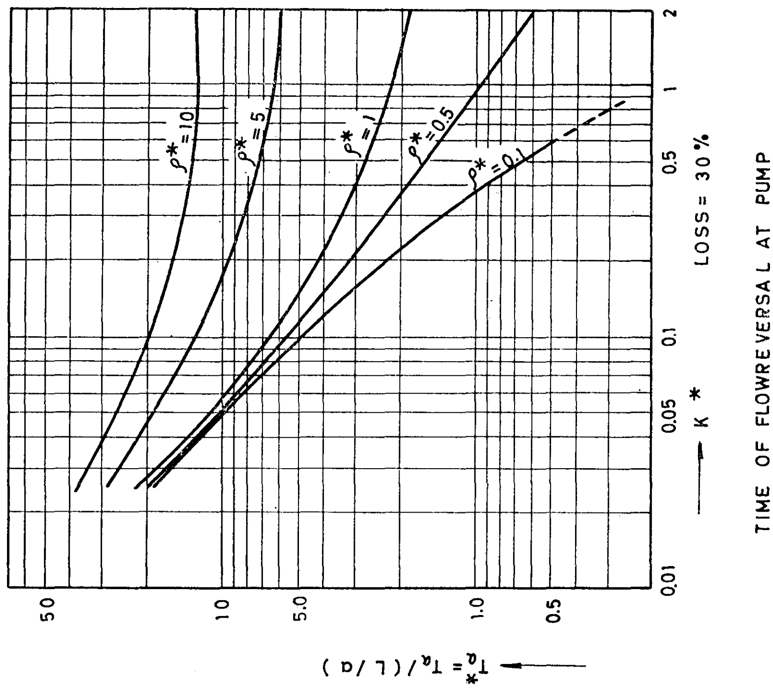

Fig. 6-b. T_a^*-diagram on the 30%-friction loss.

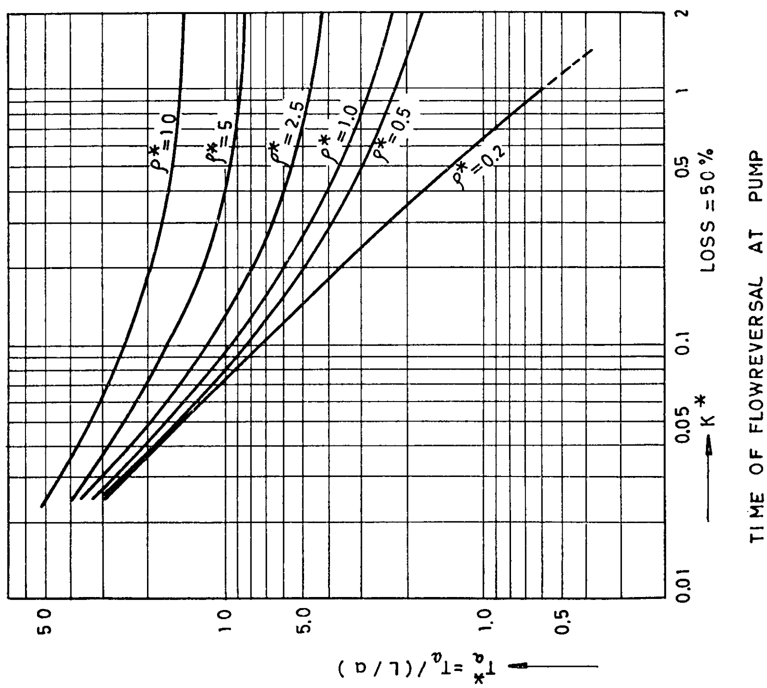

Fig. 6-c. T_a^*-diagram on the 50%-friction loss.

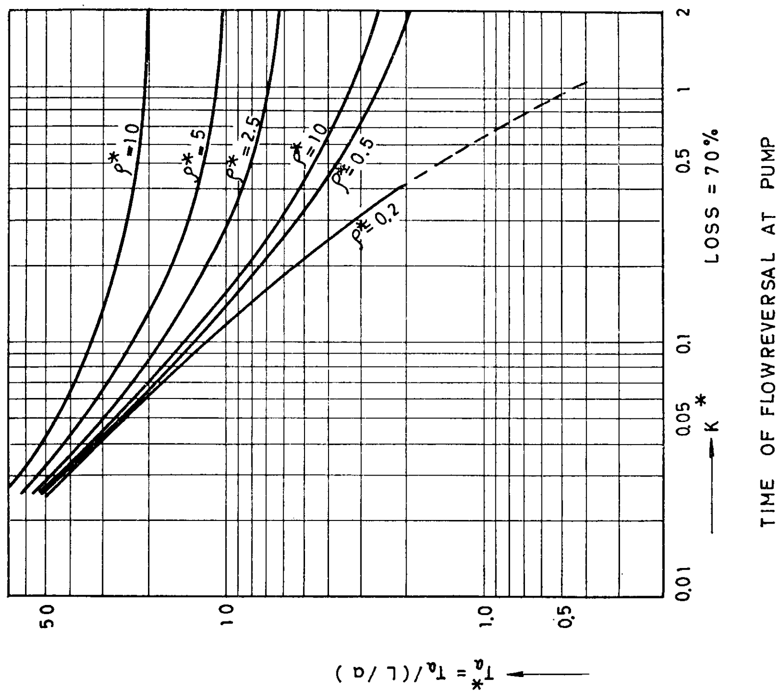

Fig. 6-d. T_a^*-diagram on the 70%-friction loss.

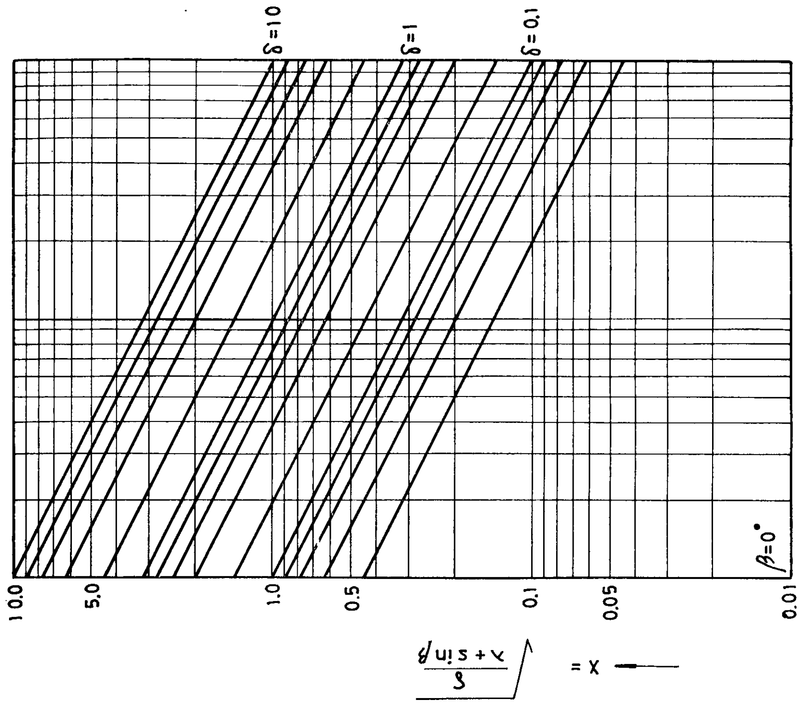

Fig. 7-a. X-diagram on the equation (28), $\beta=0°$.

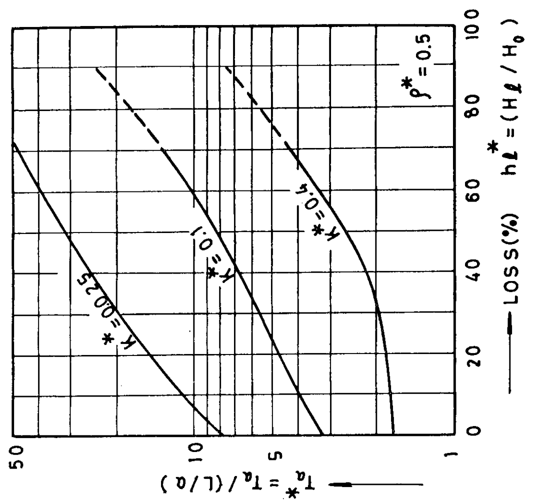

Fig. 6-e. Relations between T_a^* and h_l^*.

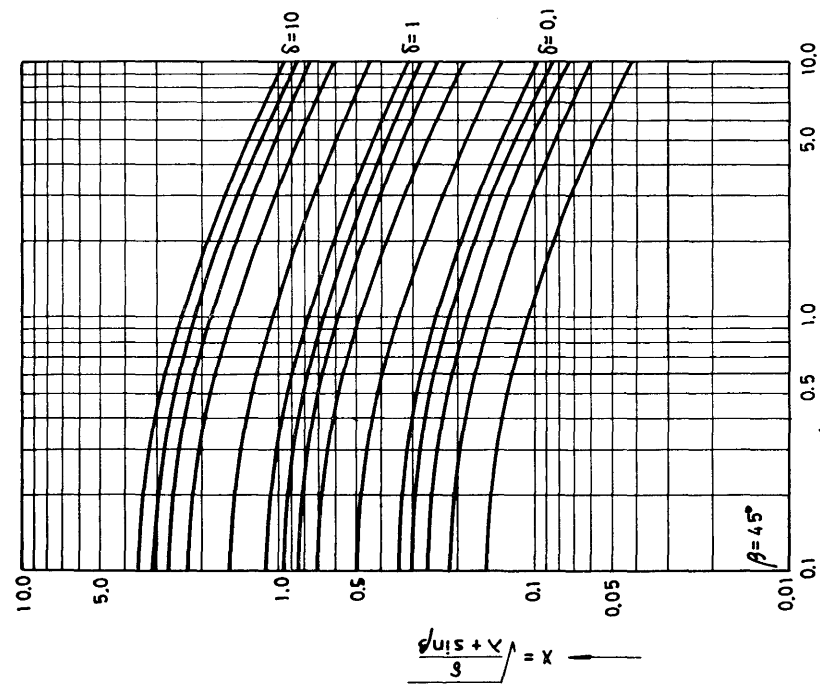

Fig. 7-b. X-diagram on the equation (28), $\beta=45°$.

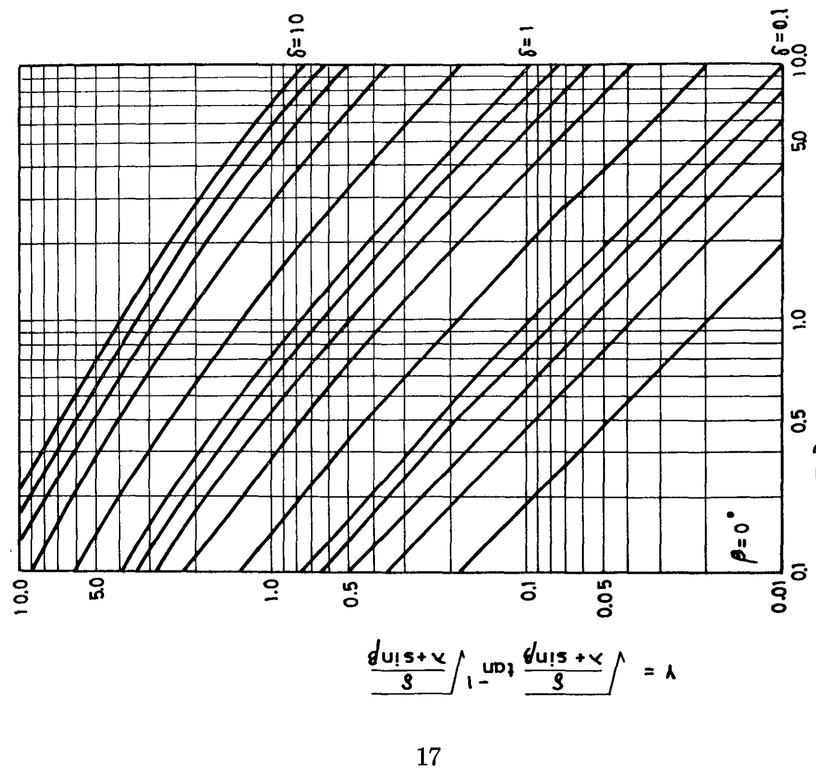

Fig. 7-a′. Y-diagram on the equation (32), $\beta=0°$.

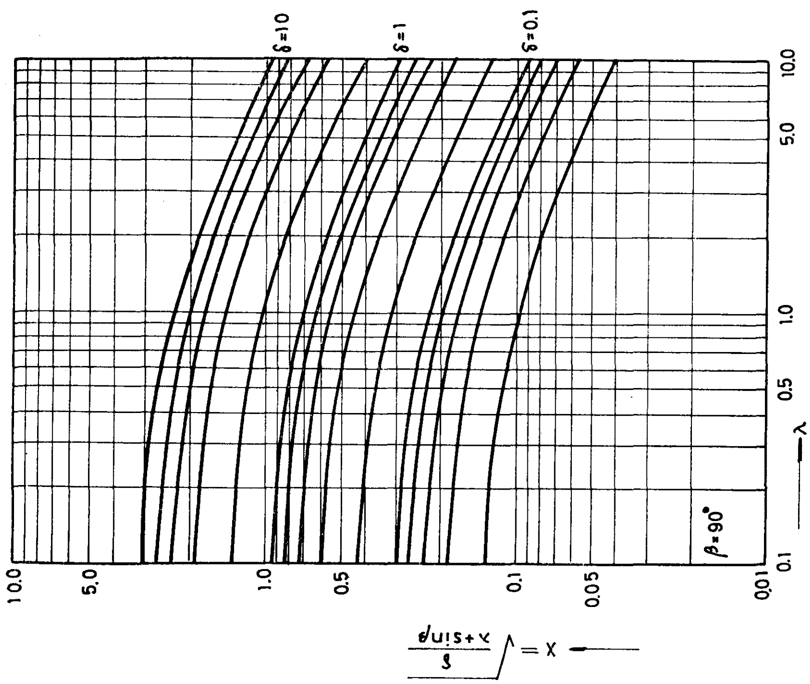

Fig. 7-c. X-diagram on the equation (20), $\beta=90°$.

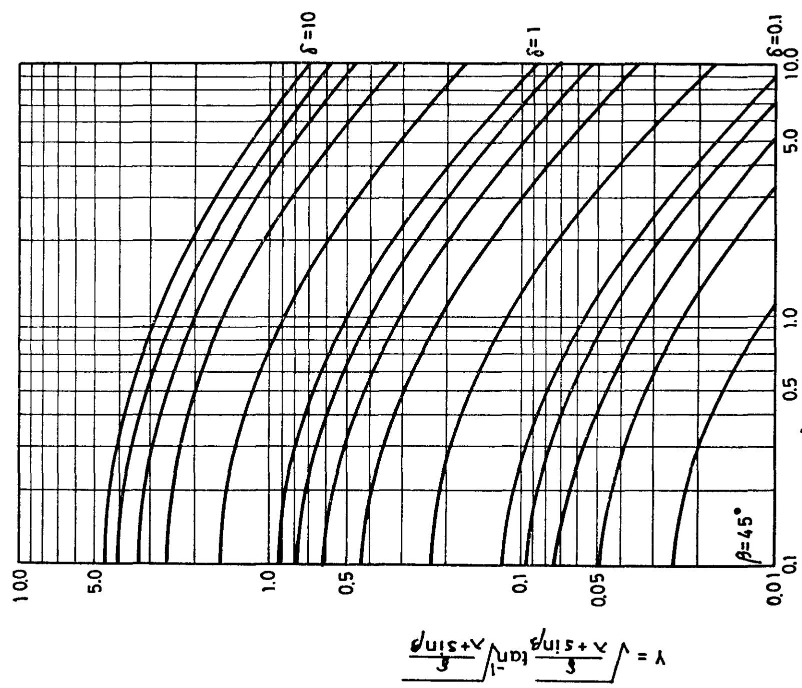

Fig. 7-b'. Y-diagram on the equation (32), $\beta=45°$.

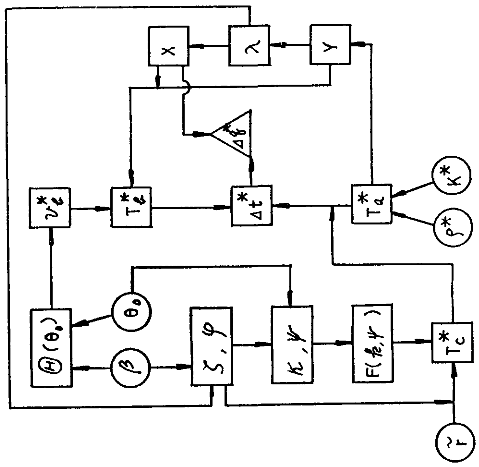

Fig. 8. Calculation process.

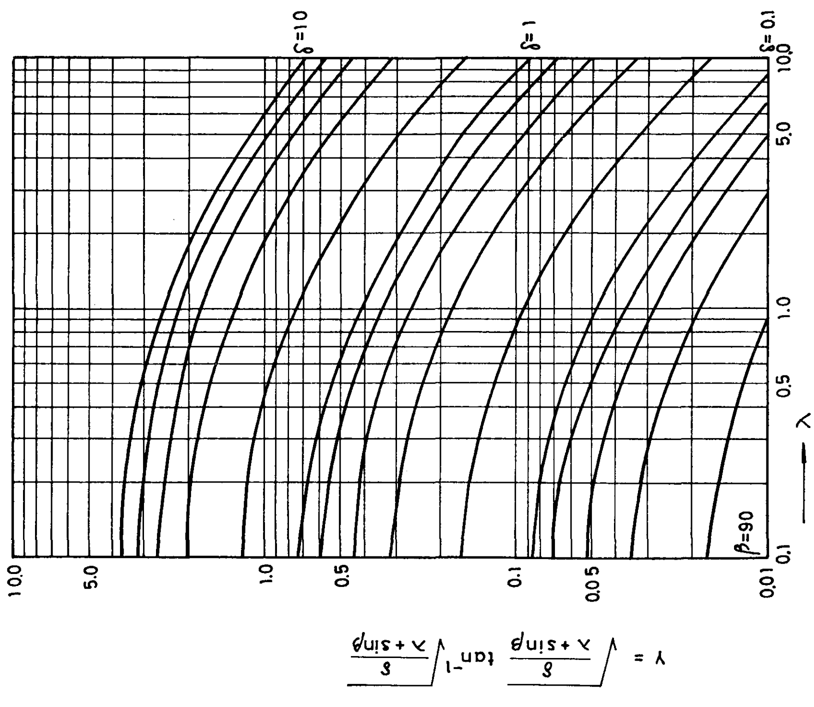

Fig. 7-c′. Y-diagram on the equation (32), $\beta=90°$.

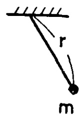

Fig. 9-a. Single pendulum.

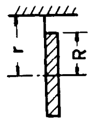

Fig. 9-b. Disk.

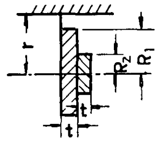

Fig. 9-c. Double disk.

Table 1. Horizontal flow $\beta=0$ and vertical flow $\beta=\frac{\pi}{2}$.

	φ	ψ	K	ζ	$\Theta(\theta)$
$\beta=0$	$\tan^{-1}\lambda$	$\sin^{-1}\sqrt{\frac{2\{\cos\varphi-\cos(\varphi+\theta_0)\}}{(1+\cos\varphi)\{1-\cos(\varphi+\theta_0)\}}}$	$\sin\left(\frac{\theta_0+\varphi}{2}\right)$	$1+\lambda^2$	$\frac{\sqrt{(1-\cos\theta)\sin\theta}}{\cos}$
$\beta=\frac{\pi}{2}$	$\frac{\pi}{2}$	$\sin^{-1}\sqrt{\frac{2\sin\theta_0}{1+\sin\theta_0}}$	$\sin\left(\frac{\theta_0}{2}+\frac{\pi}{4}\right)$	$1+\lambda$	$\frac{1-\cos\theta}{\cos\theta}$