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ON THE FINANCIAL CONSTRAINT OF PRODUCTION

DENZO KAMIYA

Technology is not the only constraint of production. Productive activities of the firm are limited, as well, by financial conditions. A careful treatment of the firm's behaviour cannot neglect the financial constraint of production, particularly, in view of the fact that a short-run production plan is closely related to a long-run investment plan. For, in respect to the latter, the firm cannot expect to secure proceeds enough to finance the project, until after it has actually made a substantial amount of expenditure.

The purpose of the present paper is to propose a systematic way by which we may deal with this rather neglected aspect of production. After a brief discussion on the fundamental point of view, I shall begin the main part by showing that all the important quantities in the theory of production can be derived from two easily observable quantities, namely, the rate of expenditures and the rate of proceeds. Then, I shall discuss how the production plan is related to the finance requirement of the firm, and proceed to a detailed analysis of the effects of alternative finance plans on the various value ratios in the balance sheet of the firm. The paper is concluded by an exposition of a tentative model of the theory of the firm. It should be emphasised here that the way in which financial conditions constrain productive activities depends partly on the objective of the firm.

I. PRODUCTION PLAN AND FINANCE PLAN

The firm makes, in theory, three kinds of decision, namely, (a) decision on production, (b) decision on investment, and (c) decision on finance.

Underlying the distinction between the first and the second kinds of decision, is the recognition that the production of a firm is normally repeated in similar cycles. For the purpose of exposition, let us divide time into number of disjoint periods of equal length, and call them weeks.[1] Now, decision on production is concerned with what commodities are to be produced and in what quantities in the current week. It involves decision on the way in which various goods and service are combined in the production. Decision on

[1] A Week, in Hicks' analysis, is defined as such a period of time during which variations in prices can be neglected. See Hicks (1939), p. 122. It is the shortest unit period of time in his intertemporal general equilibrium theory. He called it a Week to distinguish it from Marshall's Day. In the former, output is variable to a certain extent, while in the latter, it is fixed. I use the word, week, here, merely as a means of drawing a line between production and investment.

investment, on the other hand, is concerned with what goods are to be kept in stock and in what quantities for the production of future weeks. A detailed study has to make distinctions between stocks of raw materials, goods in process, finished goods,[2] and durable means of production. Both acts of production and investment incur various expenditures, and the firm has to finance them in one way or another. Decision on finance is concerned with how theses expenditures are to be financed. In the real world, there is a variety of ways to finance the firm's expenditures. But, for theoretical purposes, it suffices to classify them into three main categories, namely, retention of profits, issue of shares, and use of loans. What decisions the firm would make, of course, depends on the objective of the firm.

Of fundamental importance is the fact that decision on production is related to decision on investment in such a way that, often, it is impossible to separate the one from the other.[3] On starting the production of a week, the firm decides the way in which various goods and services are combined. But, what goods and services the firm would decide to use, for the production of the current week, very much depends on what it plans to produce in future weeks. This dependence is basically due to a technological fact that some of the goods used in the production have the following two properties:

(i) They are related to the products only indirectly, so that their inputs need not be changed as the level of production is changed.[4]
(ii) The same material may be used repeatedly over number of weeks.

Most of durable means of production have these properties.[5] If the firm decided to buy such a good for the current production, it may have done so with an intention to use it for the future production, as well. Then, decision on production is mixed with decision on investment, and the two are inseparable. In other words, the current production plan is part of a connected intertemporal plan which includes decisions on the production of all the future weeks. For, the investment plan is obviously a reflection of the future production plan.

In what follows, we shall treat the firm as an agent making decisions on two separate plans, namely, intertemporal plan of production and intertemporal

[2] A broader view of the production is adopted here, that the production is completed when the products finally part with the firm.

[3] Cf. Hicks (1939), p. 123.

[4] Inputs of this kind of goods have to be specified in two dimensions, quantity and intensity. The level of production can be changed by changing intensity, while quantity is kept fixed.

[5] Hicks has pointed out that if there is no durable goods, there cannot be much intertemporal substitution. See Hicks (1939), p. 208. It is the present author's view that the technological fact as stated above is the fundamental reason why it is so.

plan of finance. We shall refer to the former simply as production plan, and the latter, as finance plan. The interactions between the two plans will become clear as the argument develops.

II. EXPENDITURES AND PROCEEDS, THE BASIC QUANTITIES OF THE ANALYSIS

Let us begin with describing the productive activities of the firm by flow of its total expenditure and that of its total proceed. In view of the fact that the acts of current production and investment are inseparable, this way of starting the argument is perhaps inevitable, to deal with the dynamic behaviour of the firm. Moreover, this approach to the behaviour of the firm would turn out to be particularly convenient when we analyse the interactions between production plan and finance plan.

Let $x(t)$ be the total expenditure at time t on goods and services that the firm purchases for production and investment, and $y(t)$ be the total proceed at time t from products that the firm sells in the market. Both $x(t)$ and $y(t)$ represent the rate of flow at a point of time. Therefore, for instance, the total expenditure in a period beginning at time t_0 and ending at time t_1 is given by the following integral.

$$\int_{t_0}^{t_1} x(u)\, du \, ,$$

and similarly for the total proceed. All the other flow quantities as well are expressed, in this chapter, as the rate of flow at a point of time. Their amounts in a period of time may always be obtained as in the above example.

An important aspect of the behaviour of the firm over time is described by a pair of functions, $x(t)$ and $y(t)$, of time t. The values of $x(t)$ and $y(t)$ at t corresponding to a past time represent expenditures and proceeds realised at that time, while those at t corresponding to a future time represent expenditures and proceeds planned for that time. The values of $x(t)$ and $y(t)$ planned for a future t may not be realised, when t becomes the present. Most of the basic concepts in the theory of the firm are derived from these two quantities, total expenditure and total proceed, which are defined at each point of time.

III. SURPLUS OF PRODUCTION

First of all, the difference between the firm's total proceed from sales of products and its total expenditure on goods and services is its net receipt before deduction of interests on debts. In what follows, let us refer to this difference simply as *net receipt*. Denote by $z(t)$, net receipt at time t. Then

$$z(t) = y(t) - x(t)\,. \tag{1}$$

This is neither surplus nor profits in the ordinary sense of the words, for $z(t)$ does not take into account the gains expected from the future production.

Gains from the future production can be evaluated by the present discounted value of future expected net receipts. Let us call it *value of the firm*. To be precise, let $V(t)$ stand for value of the firm at time t. Then, $V(t)$ is defined as

$$V(t) = \int_t^\infty z(u)e^{-r(u-t)}\,du\,, \tag{2}$$

where r is the rate of discount[6] specific to the firm in question. It is to be noted here that value of the firm, so defined, takes into account not only future net receipts expected from the assets of which the firm is already in possession, but also those from the assets which the firm would intend to buy in the future.

The rate of change in value of the firm with respect to time is called *net investment* of the firm. Denoting net investment at time t by $v(t)$, we have[7]

$$v(t) = \frac{d}{dt}V(t) = rV(t) - z(t)\,. \tag{3}$$

This is the fundamental valuational relationship of the act of investment, but hidden behind the heterogeneity of capital goods. Again, the value of net investment at time t, as defined in (3), reflects not only future net receipts expected from operating capital goods which were purchased at time t, but also those expected from operating capital goods which would be purchased at times later than t.[8] This implies that the value of $v(t)$ at each point of time cannot be determined in isolation, but its values at different points of time are determined all together. This definition of net investment is based on the view that the investment plan for each point of time is part of a long term

[6] The rate of discount is a datum of the present analysis.

[7] The first equality is definitional. The second equality appears in Samuelson (1937), p. 471. It is derived from a mathematical relationship between integral and derivative, as shown below.

$$V(t) = e^{rt}\int_t^\infty z(u)e^{-ru}\,du\,.$$

Therefore,

$$\begin{aligned}\frac{d}{dt}V(t) &= re^{rt}\int_t^\infty z(u)e^{-ru}\,du + e^{rt}\frac{d}{dt}\int_t^\infty z(u)^{-ru}\,du \\ &= rV(t) - e^{rt}z(t)e^{-rt} \\ &= rV(t) - z(t)\,.\end{aligned}$$

[8] Of course, it reflects, as well, any change due to passing of time in expected future stream of net receipt from capital goods older than those purchased at time t.

production plan. The planned course of $v(t)$ is, in fact, implicit in the planned course of $x(t)$ and $y(t)$. The former can always be derived from the latter by formulae (1)–(3).

The natural definition, then, of surplus of the firm at one point of time would be the sum of net receipt and net investment of the firm at that point of time.[9] If $m(t)$ is surplus at time t,

$$m(t) = z(t) + v(t) . \tag{4}$$

This is equal to value added less wages and salaries. Obviously, this concept is equivalent to *produit net* of the Physiocrats and the net income of Ricardo. It is also equivalent to the surplus value of Marx, but defined in terms of market values.[10] Determination of the social aggregate of this quantity was the central problem of the classical theory of income distribution. The surplus in this sense is not apparently a price paid for some services, as wages, interests and rentals may be so regarded, and therefore deserves a particular attention, in the theory of income distribution. Using (2) and (3), (4) may be rewritten as

$$m(t) = rV(t) = r\int_t^\infty z(u)e^{-r(u-t)}\,du . \tag{5}$$

Thus, it may be observed that the theory of the surplus must involve discussions about (a) how expected net receipts are determined, and (b) how the rate of discount is determined. In this paper, we shall deal with the former question.

From the point of view of an individual firm, surplus, $m(t)$, does not represent the net income, of which it can dispose[11] at will. If the firm has any debt outstanding, interests must be deducted from surplus to determine its net income or *profit*. Let $n(t)$ be profit at time t. Then,

$$n(t) = m(t) - iD(t) , \tag{6}$$

where i is the rate of interest on debts, and $D(t)$ is the amount of debts outstanding at time t. This is the theoretical counterpart of the net corporate income in the income account of a corporation. This concept is important in analysing the behaviour of individual firms, which, in turn, determines the surplus of the economy as a whole. Interest payments are transfers of the surplus from one individual to another, due to debts outstanding. But the determination of the total amount of the surplus is influenced by the way in which these transfers are made.

[9] The firm may have a positive surplus when net receipt is negative, if net investment is large enough. Cf. Hicks (1939), p. 195, n. 1.

[10] The surplus value of Marx is defined in terms of labour value.

[11] Apart from taxes imposed on the firm.

IV. COST OF PRODUCTION

In defining surplus and profit of the firm, we made no use of any cost concept, but of course it is implicit in these definitions. Rewrite (4), using (1), to obtain

$$y(t) = m(t) + x(t) - v(t) .$$

This equality show explicitly that $x(t) - v(t)$ must be deducted from the current proceed, $y(t)$, to obtain the current surplus, $m(t)$, at time t. Therefore, the definition of the current cost of production, or simply *cost of production*, consistent with the definition of surplus given by (4), is total expenditure less net investment. Let $c(t)$ be cost of production at time t. Then, we have

$$c(t) = x(t) - v(t) . \tag{7}$$

The total expenditure of the firm at any point of time is composed of two parts, namely, expenditures on goods and services which are used for the current production, and expenditures on goods and services[12] which are used for the future production. Only the former should enter into the current cost of production. But it is not possible to assign each item of the firm's expenditures to one category or the other, because some of the goods and services are used for both current and future production. This is what is described by Keynes as the "interlocking character of the production of what is currently sold with total production."[13] Thus, the only way in which we are able to ascertain the current cost of production is by first evaluating the net investment and then deducting it from the total current expenditure.[14]

The current cost of production is influenced partly by the future production plan of the firm. For, the value of the net investment is determined by the future production plan of the firm. The static cost curves, which relate total, average and marginal costs to the current level of production, are not suitable for dealing with this influence of the future on the current cost production.

V. FINANCE REQUIREMENT OF THE FIRM

The problem of finance arises obviously when the total proceed from the sales of products falls short of the total expenditure on goods and services plus interests on debts outstanding. It also arises when net investment is positive, because profit of the firm exceeds, by definition, its net receipt after deduction of interest costs, by an amount equal to net investment. Therefore, the firm is obliged to retain part of its profit, unless it finds some external

[12] Services which are used to produce goods to be sold in the future, or to be used in the future production.

[13] Keynes (1936), p. 67.

[14] This is the idea underlying Keynes' concept of user cost. See Keynes (1936), p. 67.

sources to finance its net investment. Put it in another way, net investment represents, by definition, that part of the firm's expenditures which is not covered by the cost of production. After all, the total amount of finance that the firm requires is the sum of (a) excess of cost of production plus interest costs over total proceed, and (b) net investment, if both are positive. Ob course, the firm needs no finance for either one of (a) and (b) which is negative, and if both are negatived the firm would have no finance requirement at all.[15]

In mathematical notations, the firm's *finance requirement* at time t, denoted by $e(t)$, may be defined as follows.

$$e(t) = \max\,[c(t) + iD(t) - y(t), 0] + \max\,[v(t), 0]\,, \tag{8}$$

where $\max\,[a, b]$ indicates whichever is larger of a and b. When $v(t)$ is positive, (8) may be written as

$$e(t) = \max\,[x(t) + iD(t) - y(t), v(t)]\,. \tag{8'}$$

Translating back to the ordinary language, we may say that, when the net investment is positive, the total amount of finance required is equal to whichever, is larger of (a) excess of expenditures on goods and services plus interests on debts outstanding over the total proceed from the sales of products, and (b) net investment. When $v(t)$ is negative, (8) is reduced to

$$e(t) = \max\,[c(t) + iD(t) - y(t), 0]\,, \tag{8''}$$

and the total amount of finance required is equal to whichever is larger of (a) excess of cost of production plus interest costs over the total proceed from the sales of products, and (b) zero.

From what has been said above, it is clear that if net profit at time t, $n(t)$, is positive,

$$e(t) = \begin{cases} v(t)\,, & \text{if } v(t) > 0\,, \\ 0\,, & \text{otherwise.} \end{cases} \tag{9}$$

VI. EFFECTS OF THE FIRM'S DECISION ON THE STREAM OF NET RECEIPTS

So far we have been mainly concerned with various definitions, and relationships which are directly derived from these definitions. Specifically, we have given precise and mutually consistent definitions to such concepts as value of the firm, net investment, surplus, profit, cost of production and

[15] When the net investment is negative, *i.e.*, when a disinvestment is made, the firm has an excess of finance equal in amount to the value of disinvestment, which the firm may use for a financial investment, or for cancelling some of its debts outsanding or for financing the loss on its production account, if any.

finance requirement. The basic quantity underlying those definitions is net receipt, which is simply the difference between the firm's total proceed from the sales of products and its total expenditure on goods and services to be used in the production. Let us now turn to the analysis of the firm's decision making as to the production and finance plans which determine the stream of the firm's expenditures and proceeds over time.

Theoretical models of the firm's decision making may be classified by the motive of the firm, on the one hand, and by the structure of the market, on the other. Following discussions are confined to the case where the number of competitors is very large both in the market in which the firm sells its products and in those in which it buys goods and services required in the production. We shall suppose that the number is so large that an individual firm may ignore the reactions of its rivals to a particular policy that it adopts. Thus, the conditions of demand for products and the conditions of supply of goods and services required in the production are *data* to an individual firm, independent of the acts of other firms. However, we shall not suppose that the prices of various goods and services are fixed from the point of view of an individual firm. In other words, the case that we are going to deal with is of market structure which was characterised by Bain[16] as the atomistic competition with product differentiation. The other market structures in Bain's classification are atomistic competition without product differentiation, various oligopolies and monopolies. There are two reasons why we may confine our discussions to the case of atomistic competition with product differentiation, one negative, and the other, positive. Firstly, oligopolies involve complications which deserve a separate treatment, and apart from these complications, there are many important questions to be answered about the firm's decision making as to its production and finance plans. Secondly, the case of atomistic competition with product differentiation includes, as a special case, the atomistic competition without product differentiation, and, moreover, as far as the behaviour of an individual firm is concerned, the formal aspects of the arguments are essentially the same in both cases of atomistic competition with product differentiation and of monopoly. As for the objective of the firm, we shall consider several alternatives.

The decision of the firm is influenced both by the past and by the future. The past may be represented by the stock of various goods in existence which is the embodiment of the past decisions of the firm. The basic factors related to the future are (a) the conditions of technology, (b) the conditions of market of various goods and services, and (c) the conditions of financial market. The conditions of technology determine the range of feasible combinations in the production of various goods and services. The conditions of market of the

[16] Bain (1968), pp. 31-32.

firm's products determine the relationship between the quantity sold of each product and its price, as well as the way in which that relationship is influenced by expenditures for the sales promotion. The conditions of market of various goods and services required for the production determine the relationship between the stream of purchase over time of those goods and services, and the stream of expenditures over the time on them. Finally, the conditions of financial market determine the relationship between the amount of loans that the firm obtains and interests to be paid, as well as the relationship between the amount of shares newly issued and the sacrifices made of the old share holders. What those conditions would be in the future is, of course, not known for certain. The firm, therefore, acts on the basis of expectations concerning these conditions. As expectations formed in the past prove to be disappointed, the firm revises the past plans.[17]

Now let us consider, given the firm's expectations about the future, how the firm can control the expected stream[18] of its future net receipts over time, and how the latter is constrained by the external forces. It is convenient to argue in two stages. In the first stage, suppose that the firm has determined the kinds and quantities of products to be sold at each point of time in the future. Let us call a plan which specifies the kinds and quantities of products to be sold at each point of time in the future, as *output plan*. Now, given an output plan, there are two ways by which the firms can control the expected stream of its future net receipts. Firstly, the firm can change its sales strategy. By changing the sales strategy, the firm can change the prices at which the products are sold as planned. The firm's expenditures would as well be affected. Thus, the expected stream of net receipts is changed. Secondly, alternative ways are open to the firm, both at a point of time and over time, of combining various goods and services to realise the given output plan,[19] though the variety is limited by the conditions of technology. Thus, by changing combination of inputs, the firm can control the stream of its expenditures, and therefore, that of net receipts. Let us call a plan which specifies the directions and amounts of expenditures at each point of time in the future, as *expenditure plan*. It is determined, given an output plan, as a result of the firm's decision on the sales promotion and the combination of inputs.[20] What has been said above is then that, given an output plan,

(i) there are more than one expenditure plans that the firm may adopt,

[17] In the last section of this paper, we shall discuss the revision of past plans in more detail.

[18] It is the expected course of net receipts in the future that is considered here. It is not the actual course that the firm will follow in the future. The expected course would affect the actual course by influencing what the firm would decide currently to do at each point of time.

[19] See the discussion on p. 52 for the substitution over time.

[20] An output plan, together with an expenditure plan, constitutes a production plan.

because the firm has a variety of ways of promoting sales and combining various inputs, and

(ii) the expected stream of future net receipts is determined depending on which expenditure plan the firm would adopt.

Among the expenditure plans compatible with an output plan, there is such an expenditure plan that maximises value of the firm. The existence of the maximum for any output plan is obvious because the firm cannot raise indefinitely the prices at which the products are sold as planned, without incurring increasingly larger expenditures for the sales promotion under given conditions of market for the products, and because the firm has to pay positive prices for the inputs that it purchases.[21]

In the second stage, let us consider the effect of changes in the output plan on the expected stream of future net receipts, supposing that the objective of planning is to maximise value of the firm. As argued above, to any output plan, there corresponds an expenditure plan that maximises value of the firm. The maximum value, of course, depends on the given output plan. By planning to expand the sales of products faster, the firm can expect, to a certain extent, to increase value of the firm. But the latter cannot be increased indefinitely. There are two reasons for believing that there is a limit to value of the firm.[22] Firstly, the markets which an individual firm faces of various goods and services are limited. Thus, the faster the firm plans to expand quantities of the products that it intends to sell, the lower would be the prices at which it can expect to sell these products as it has planned. Or, if the firm tries to keep their prices high, the larger amount of expenditure would be required for the sales promotion.[23] Furthermore, it is probable that the firm is obliged to pay a higher price per unit of input, if it accelerates the expansion beyond a certain extent. Secondly, there is a limit to the speed at which the capacities of the managerial services may be increased.[24] Thus, inefficiencies are caused in the management, by expanding the quantities of the

[21] For the sake of simplicitly, we shall ignore the multiplicity of the maximising expenditure plan, because it does not alter the following arguments in any essential way.

[22] Penrose has pointed out three factors limiting the growth of the firm, namely, managerial ability, product or factor markets, and uncertainty and risk. Penrose (1959), p. 43. Of these, only the first and the second are relevant to the consideration of the effet of the firm's production plan on value of the firm. The third factor, uncertainty and risk, is one of the determinants of the rate of discount, which, in the first approximation, may be regarded as independent of the firm's decision on the production plan. It is through their constraining effects on the expected stream of future net receipts and on the rate of discount, that these factors have influences on the firm's decision, and thus, on its growth.

[23] See E. A. G. Robinson (1931), pp. 120–121, and Penrose (1959), p. 44.

[24] As Penrose wrote, this is ultimately due to the fact that the management works as a team, *i.e.*, as a unit. See Penrose (1959), p. 46. This idea of the management team is further developed in Marris (1964), pp. 114–118.

products too fast, or by diversifying the products too fast. The increasing inefficiencies would, in turn, cause increases in the expenditures, which would eventually dominate increases in the proceeds. That is to say, value of the firm would eventually begin to fall, as the rate of expansion of the quantities of the products and the rate of diversification of products are made higher and higher. To use Marris' phrase, this is a dynamisation of the old concept of diseconomies of scale.[25] It is true that the limit set to value of the firm by the conditions of market may be evaded by the diversification of the products,[26] but the firm would eventually be confronted with the managerial limit.

From the above argument about the ways by which the firm can control the expected stream of future net receipts, and various forces constraining the latter, we may draw following conclusions. Value of the firm has a certain maximum, and the firm can attain that maximum value by choosing an appropriate production plan, *i.e.*, an output plan and an expenditure plan. The argument is qualified on two accounts. Firstly, value of the firm is supposed to be determined on the basis of the expectations of the firm itself. Secondly, the maximum possible efficiency of the management is supposed to be dependent only on the long run trends of expansion and diversification, and not on the short run fluctuations in the quantities of products. The effect of the latter can rather easily be incorporated into the analysis. But, since it diverts our attention too far away from our central problem, we shall ignore this effect throughout the present work.

Whether or not the firm would desire to maximize value of the firm is another question.[27] There are other aspects to which the firm might pay attention of the production plan. Above all a production plan has the financial aspect, to the problem of which we shall now turn.

VII. NET WORTH AND VALUE OF SHARES OF THE FIRM

We have seen that to each production plan, a stream of net receipts is associated. Finance requirement at each point of time is derived from the latter by formula (8). Of course, it is partly influenced by the extent to which the firm resorted to debt financing in the past. For, the debts raised in the past oblige the firm now to pay interests. The firm has three alternative sources of finance, namely, retained profits, fund raised by the new issue of shares, and loans. Now, let us examine different consequences of the various finance plans of the firm.

[25] Marris (1964), p. 117.

[26] See Penrose (1959), p. 44.

[27] The traditional theory, as one can see in *Value and Capital* by Hicks, takes it for granted that the objective of the firm is to maximise the value of the firm. See Hicks (1939), p. 202.

As we recognise the fact that there are alternative sources of finance to the firm, the distinction emerges among the value of the firm, the net worth of the firm, and the total market value of shares of the firm. To compare consequence of different finance plans, we have to give them precise definitions. The first has already been defined by (2), above. The *net worth of the firm* is defined as the value of the firm less the total value of debts outstanding. Let us denote the net worth of the firm at time t, by $A(t)$. Then, using the notations already defined,

$$A(t) = V(t) - D(t) \,. \tag{10}$$

Of course, if the firm has no debt outstanding, then, the net worth of the firm is equal to the value of the firm.

The total market value of shares is the sum of value of shares evaluated at the price at which they are traded in the stock market. A realistic theory of share valuation would have to take into account whatever factors which are regarded by the speculators as influencing the share prices. That is beyond the scope of the present work. What we are going to deal with is the theoretical value of shares. The definition presupposes that the stock market is rational[28] to a certain extent. Thus, the *total market value of shares of the firm* is defined as the present discounted value of the total dividends expected to be paid in the future. A complication arises from the fact that the number of shares may be increasing, for the firm may issue new shares. Part of the total dividends expected to be paid in the future will go to shares which are to be issued in the future. Only that part of the future expected dividends that accrues to the shares already in existence has to be taken into account to determine the total market value of shares at each point time. To be precise, let $p(t)$ be the total amount of dividends expected to be paind at time t, and $N(t)$ be the number of shares at time t. Then, the total market value of shares of the firm at time t, denoted by $B(t)$, is defined as[29]

$$B(t) = \int_t^\infty \frac{N(t)}{N(u)} p(u) e^{-r(u-t)} \, du \,. \tag{11}$$

This definition of the value of shares presupposes too much knowledge on the part of the participants in the stock market. But, we shall, first of all, examine what we can say about the consequences of different finance plans on the basis of this definition, and then afterwards, we shall see how these consequences work themselves out in the real world of limited information.

The sum of the total market value of shares, $B(t)$, and the total value of

[28] The nature of the rationality is self-explanatory in the following definition.

[29] If the retention ratio is ρ, then $p(t)$ is given by

$$p(t) = (1 - \rho) n(t) \,.$$

debts outstanding, $D(t)$, is the market value of the firm in the sense of Modigliani and Miller.[30] Their theory suggests that, given the stream of surplus, $m(t)$, and the rate of discount, r, this sum is determined independent of the finance plan.[31] But their arguments are not concerned with the intertemporal finance plan, and therefore, the fact is ignored that part of the surplus, $m(t)$, in the future may go the shareholders or to the creditors of the debts who do not exist at the present. The market value of the firm, $B(t) + D(t)$, may very well be influenced by the intertemporal financial plan. We shall discuss this influence in detail, below.

The concept of valuation ratio became important in the recent growth theories.[32] Let us define *valuation ratio* as the ratio of the total market value of shares of the firm, $B(t)$, to the value of the firm, $V(t)$. The latter is of course the present discounted value of the future expected net receipts. In the definition of Marris and Kaldor, the denominator is called book value of net assets or capital employed, but treated virtually as a quantity of malleable capital. The definition suggested above escapes this ambiguity, and fits well into Marris' theory of takeover. Denote the valuation ratio at time t, by $\sigma(t)$. Then,

$$\sigma(t) = \frac{B(t)}{V(t)}. \tag{12}$$

VIII. COMPARISON OF ALTERNATIVE FINANCE PLANS

The value of the firm, $V(t)$, as defined in (2), is determined by the production plan of the firm, independent of its finance plan, But the total market value of shares, $B(t)$, as defined in (11), is influenced by the finance plan of the firm. Therefore, the valuation ratio is determined by the financial policy of the firm. For analysing the behaviour of the firm, it is useful to see what the valuation ratio turns out to be, depending on the financial plan, supposing that the production plan has already been determined. We shall examine below four critical cases. In all of those cases examined, it is supposed that both the profit and the net investment are always non-negative. Therefore, the firm's demand for finance is always equal in amount to the net investment.[33]

[30] See Modigliani and Miller (1958), p. 268. Their market value of the firm is defined as the expectation of a stochastic variable.

[31] Their Proposition I. See Modigliani and Miller (1958), pp. 268–271. This proposition is crucial to their theory of share valuation and investment.

[32] See Marris (1964), p. 22 and Kaldor (1966), p. 317. Incidentally, the market value of the firm, which is the numerator in Marris' definition of the valuation ratio, is the total market value of shares of the firm, and therefore, is not the market value in the sense of Modigliani and Miller.

[33] See p. 57 above.

Case (i): *The firm finances all the net investment by retained profits, and it has no debt outstanding at any time.* Obviously, such a financial plan is possible only if the profit is always larger than the net investment.

In this case, the profit, $n(t)$, is equal to the surplus, $m(t)$, at any time, for there is no interest payment. Then, since the profit retained for finance purpose is equal to the net investment, the total dividends paid at time t, $p(t)$, is given by[34]

$$p(t) = m(t) - v(t) = z(t) .$$

Furthermore, since the number of shares is unchanged throughout the time, ratio $N(t)/N(u)$ is equal to unity for any pair of u and t. That is to say, all of the total dividens to be paid in the future accrue to the shares which are in existence now. Thus, from formula (11), the total market value of shares of the firm at time t, $B(t)$, is given by

$$B(t) = \int_t^\infty z(u)e^{-r(u-t)}\, du .$$

The right hand side is nothing but the definition of $V(t)$. Hence, at any time, the total market value of shares of the firm is equal to the value of the firm, and the valuation ratio is unity. As the value of debts is always zero, the net worth of the firm, as well, is equal to the value of the firm, at any time. Thus, we have

$$A(t) = B(t) = V(t) .$$

It should be obvious that these values are also equal to the market value of the firm in the sense of Modigliani and Miller.

Case (ii): *The firm finances all the net investment by issues of new shares, and it has no debt outstanding at any time.* At each point of time, the total amount of dividends is larger than that of the previous case, by the amount of net investment. But, the effect of the increase in the total dividends on the total market value of shares of the firm is offset by the converse effect of the increase in the number of shares. It can be shown that the firm can make the total market value of shares, at most, equal to the value of the firm.[35] Unlike the previous case, the net investment may exceed the profit.

Te see that, in this case, the total market value of shares of the firm cannot exceed the value of the firm, we have to recall the fundamental relationship of the share valuation, (11). Let $o(t)$ be the amount of dividends per share at time t, and $q(t)$ be the price of a unit of share at time t. Then, from (11) we have

$$q(t) = \int_t^\infty o(u)e^{-r(u-t)}\, du .$$

[34] The second equality follows from the definition of $m(t)$ given in (4), p. 55.

[35] This is a result of Modgliani and Miller. See Modigliani and Miller (1961), pp. 415–416.

Differentiating this equality[36] with respect to t, we have[37]

$$\frac{dq}{dt} = rq - o \,. \tag{13}$$

By definition,

$$B = qN \,.$$

Differentiate this equality with respect to t, to obtain

$$\frac{dB}{dt} = q\frac{dN}{dt} + N\frac{dq}{dt} \,,$$

and then using (13),

$$\frac{dB}{dt} = q\frac{dN}{dt} + rqN - oN \,.$$

Again, by definition,

$$p = oN \,.$$

Therefore,

$$\frac{dB}{dt} = rB - p + q\frac{dN}{dt} \,. \tag{14}$$

This is a relationship of general applicability.[38] It holds regardless of what finance plan the firm chooses.

Now remember that the firm has no debt outstanding at any time, in the case being examined. Therefore, the total dividends at time t, $p(t)$, is equal to the surplus at time t, $m(t)$. On the other hand, the net investment is entirely financed by new issue of shares. Since the firm cannot sell new shares at a price higher than that of old shares, the rate of change in the number of shares must be at least as large as v/q. If the firm does sell new shares at the price of old shares prevailing at the time of issue,

$$q\frac{dN}{dt} = v \,.$$

Therefore, (14) turns out to be[39]

$$\frac{dB}{dt} = rB - z \,.$$

Solving this differential equation, we have again

$$B(t) = \int_t^\infty z(u)e^{-(u-t)}\,du \,,$$

[36] Alternatively, (13) may be regarded as the basic relationship, from which it follows that the price of a share is the present discounted value of the future dividends. See, for instance, Solow (1971), p. 339.

[37] Notation (t) may be dropped without causing any confusion.

[38] It can be drived also directly from (11) by differentiation.

[39] Definitional relationship (4) is used.

which is equal to $V(t)$, the value of the firm.[40] Of course, the firm could sell new shares at a price lower than that of old shares prevailing at the time of issue. If the firm plans to sell new shares at lower prices than the prevailing prices of old shares at some points of time in the future, the present total market value of shares of the firm becomes smaller than the value of the firm. For, then, the rate of growth of the number of shares must be larger than v/q, and a larger proportion of future total dividends accrues to shares to be issued in the future.[41]

Thus, we have the same consequences as in the previous case, if new shares are issued always at the price of old shares prevailing at the time of issue. If new shares are issued at lower prices, the total market value of shares of the firm is smaller than the value of the firm, and the valuation ratio is less than unity. But the net worth of the firm is still equal to the value of the firm.

Case (iii): *The firm finances all the net investment by debts, after it has raised the initial capital.* The net investment may exceed the profit. In this case, the consequences depend on the level of the rate of interest. We shall examine the consequences supposing that the rate of interest is fixed.

Let V_0 be the amount of the initial capital. Then as a consequence of the finance plan in question,

$$D(t) = V(t) - V_0 .$$

In this case, as no profit is retained, the total amount of dividends is equal to the profit at each point of time. Thus, the total amount of dividends at time t, $p(t)$, is given by[42]

$$p(t) = \left(1 - \frac{i}{r}\right) m(t) + iV_0 .$$

Then, since the number of shares is unchanged throughout the time,

$$\begin{aligned} B(t) &= \int_t^\infty p(u)e^{-r(u-t)}\, du \\ &= \left(1 - \frac{i}{r}\right) \int_t^\infty m(u)e^{-r(u-t)}\, du + \frac{i}{r} V_0 . \end{aligned}$$

That is to say, the total market value of shares of the firm is given as the weighted average of the initial capital and the present discounted value of the future expected surpluses, the weights being determined by the ratio of the rate of interest to the rate of discount.

[40] Cf. Modigliani and Miller (1961), pp. 415–416. Their argument is based on a discrete period model.

[41] Modigliani and Miller (1961) do not refer to this possibility.

[42] Insert the above expression of $D(t)$ into the definition of profit, (6), and then use relationship (5).

As the surplus has been defined as the sum of net receipt and net investment, its present discounted value is equal to the sum of the value of the firm, which is the present discounted value of future expected net receipts, and the present discounted value of future expected net investment. Denote the value of the latter at time t by $H(t)$, *i.e.*,

$$H(t) = \int_t^\infty v(u)e^{-r(u-t)}\, du \,. \tag{15}$$

Then,

$$\int_t^\infty m(u)e^{-r(u-t)}\, du = V(t) + H(t) \,. \tag{16}$$

Now, $B(t)$ is expressed as

$$B(t) = \left(1 - \frac{i}{r}\right)(V(t) + H(t)) + \frac{i}{r} V_0 \,.$$

This is witten in yet another form as

$$B(t) = V(t) + H(t) - \frac{i}{r}(V(t) - V_0 + H(t)) \,.$$

Differentiate both sides with respect to t, to obtain[43]

$$\frac{dB}{dt} = (r - i)H \,.$$

Hence, when the rate of interest is equal to the rate of discount, the total market value of shares of the firm, $B(t)$, stays at the value of initial capital, V_0, and the valuation ratio keeps decreasing as the value of the firm keeps growing.

However, when the rate of interest is smaller than the rate of discount, the total market value of shares is expected to be growing at a rate equal to $(r - i)H$. Moreover, if the ratio of H to $V - V_0 + H$ is greater than the ratio of i to r, the total market value of shares is greater than the value of the firm, and the valuation ratio is greater than unity. That is to say, roughly speaking, the firm can raise the valuation ratio to a level above unity by financing the net investment by debts, if the rate of interest is lower than the rate of discount, and if the value of net investment planned to be made in the future is sufficiently large relative to the value of net investment which has already been made.

The net worth of the firm is equal to the value of the initial capital, regardless of the relative level of i and r. If i is equal to r, the total market value

[43] Applying the same mathematical relationship as explained in footnote 7 on p. 54,

$$\frac{dB}{dt} = \left(1 - \frac{i}{r}\right) r(V + H) - \left(1 - \frac{i}{r}\right) m \,.$$

Then, insert relationship (6) into the right hand side.

of shares is equal to the net worth of the firm, *i.e.*,

$$A(t) = B(t) = V_0 .$$

Again, if i is equal to r,

$$B(t) + D(t) = V(t) ,$$

that is, the market value of the firm in the sense of Modigliani and Miller[44] is equal to the value of the firm defined as the present discounted value of the future expected net receipts.[45]

Case (iv): *The firm finances the net investment by retained profits and debts, in such a way that the ratio of the value of the firm is fixed at a certain prescribed level.* The ratio of the value of debts to the firm is called the gearing ratio.[46] Let δ be the level at which the gearing ratio is to be fixed. Then, a fraction δ of the net investment has to be financed by debts, and the remainder, by retained profits, at each point of time. Therefore, the profit must be larger than a fraction $1 - \delta$ of the net investment, at each point of time.

In this case, the profit at time t, $n(t)$, turns out to be[47]

$$n(t) = \left(1 - \frac{i}{r}\delta\right) m(t) .$$

Then, because the amount of profits required to be retained at time t is $(1 - \delta)v(t)$, and because the surplus at time t is the sum of the net receipt and the net investment at time t, the total amount of dividends expected to be paid at time t, $p(t)$, is given by[48]

$$p(t) = \left(1 - \frac{i}{r}\delta\right) z(t) + \left(1 - \frac{i}{r}\right) \delta v(t) .$$

44 For the definition, see Modigliani and Miller (1958), p. 268.

45 If i is smaller than r, then B exceeds V_0 by an amount equal to $(1 - i/r)(V - V_0 + H)$, and therefore, $B + D$ also exceeds V by the same amount. See p. 67, above.

46 See Marris (1964), pp. 131-132. We shall ignore here the distinction between liquid and illiquid assets.

47 By definition, $n(t) = m(t) - iD(t)$, and by assumption, $D(t) = \delta V(t)$. Finally, using (6),

$$m(t) - i\delta V(t) = \left(1 - \frac{i}{r}\delta\right) m(t) .$$

48
$$\begin{aligned} p(t) &= n(t) - (1 - \delta)v(t) \\ &= \left(1 - \frac{i}{r}\delta\right) m(t) - (1 - \delta)v(t) \\ &= \left(1 - \frac{i}{r}\delta\right) (z(t) + v(t)) - (1 - \delta)v(t) \\ &= \left(1 - \frac{i}{r}\delta\right) z(t) + \left(1 - \frac{i}{r}\right) \delta v(t) . \end{aligned}$$

The number of shares remains unchanged throughout the time. Thus, the total market value of shares of the firm at time t, $B(t)$, is

$$B(t) = \int_t^\infty p(u)e^{-r(u-t)}\,du$$
$$= \left(1 - \frac{i}{r}\delta\right)\int_t^\infty z(u)e^{-r(u-t)}\,du + \left(1 - \frac{i}{r}\right)\delta\int_t^\infty v(u)e^{-r(u-t)}\,du\,.$$

Using definitions (2) and (15), $B(t)$ is written as

$$B(t) = \left(1 - \frac{i}{r}\delta\right)V(t) + \left(1 - \frac{i}{r}\right)\delta H(t)\,,$$

or alternatively, as

$$B(t) = V(t) + \delta\left(H(t) - \frac{i}{r}(V(t) + H(t))\right).$$

Differentiating with respect to t, we have[49]

$$\frac{dB}{dt} = (r - i)\delta H + (1 - \delta)V\,.$$

Hence, if the rate of interest is equal to the rate of discount, the total market value of shares of the firm is expected always to be a fraction $1 - \delta$ of the value of the firm, and therefore, the valuation ratio remains constant at $1 - \delta$. Furthermore, the total market value of shares of the firm is equal to the net worth of the firm. Thus,

$$A(t) = B(t) = (1 - \delta)V(t)\,,$$

and

$$B(t) + D(t) = V(t)\,.$$

The second equality shows that, again, the market value of the firm in the sense of Modigliani and Miller is equal to value of the firm defined as the present discounted value of the future expected net receipts.

If the rate of interest is smaller than the rate of discount, the total market value of shares of the firm is larger or smaller than the value of the firm according as $H/(V + H)$ is larger or smaller than i/r. That is to say, the valuation ratio is larger or smaller than unity, according as $H/(V + H)$ is larger or smaller than i/r. Further examination of the above equalities reveals the fact that, if $H/(V + H)$ is larger than i/r, then the valuation ratio is the larger, the larger is the gearing ratio, and that, if $H/(V + H)$ is smaller than i/r, then the valuation ratio is the larger, the smaller is the gearing ratio. In the former case, the valuation ratio is maximised when the gearing ratio is unity. In the latter case, the valuation ratio is maximised when the gearing ratio is zero,

[49] Apply the same mathematical relationship as in footnote 7 on p. 54, to the derivative of H.

if the profit is large enough. However, the smaller is the gearing ratio, the larger is the amount required of profits to finance the net investment. Therefore, given a production plan, there is a limit below which the gearing ratio cannot be reduced, because of the constraint imposed by the profit. The maximum of the valuation ratio is attained at such a lower limit to the gearing ratio.

Again, if the rate of interest is smaller than the rate of discount, the total market value of shares of the firm exceeds the net worth of the firm by an amount equal to $(1 - i/r)\delta(V + H)$. Therefore, the market value of the firm in the sense of Modigliani and Miller also exceeds the value of the firm defined as the present discounted value of the future expected net receipts, by the same amount.

The percentage rate of growth of the total market value of shares of the firm can easily be calculated from the above equalities. It is given by

$$\frac{1}{B}\frac{dB}{dt} = \frac{r(1-\delta)V + (r-i)\delta H}{(r-i\delta)V + (r-i)\delta H}.$$

Hence, by denoting the percentage rate of growth of B and that of V respectively be g_B and g_V, we have

$$g_B - g_V = \frac{(r-i)\delta(V+H)}{(r-i\delta)V + (r-i)\delta H}\left(\frac{rH}{V+H} - g_V\right).$$

Therefore, the valuation ratio is expected to be increasing or decreasing according as $H/(V + H)$ is larger or smaller than g_V/r. Of course, if the former is equal to the latter, the valuation ratio is expected to be unchanged.

IX. CONCLUSIONS ABOUT ALTERNATIVE FINANCE PLANS

From the examination of the four critical cases as made above, we may draw following conclusions.

i. The total market value of shares of the firm is the same, *ceteris paribus*, whether the firm finances the net investment by retained profits or by new issues of shares, provided that both old and new shares are traded at the theoretical value.[50] In other words, from the point of view of the present shareholders, the two ways of financing the net investment are indifferent with each other, under the proviso stated above.[51]

ii. If the firm uses only retained profits or new issues of shares to finance the net investment, the total market value of shares of the firm is equal to the

[50] See p. 62, for the concept of the theoretical value of shares.

[51] If we take into account an institutional fact that new issues of shares incur the costs of issue, financing by retained profits is the better alternative to the present shareholders.

value of the firm, provided that both old and new shares are traded in the market at the theoretical value.

iii. It is only by debt financing that the firm is able to raise the valuation ratio to a level above unity. Two conditions are required for having the valuation ratio larger than unity. These are (a) the rate of interest is lower than the rate of discount, and (b) the value of the net investment to be made in the future is sufficiently large[52] relative to the value of net investment which has already been made. If either one of (a) and (b) is not satisfied[53] debt financing reduces the total market value of shares of the firm below the value of the firm, and the valuation ratio, below unity.

iv. So long as the two conditions, (a) and (b) above, are satisfied, the valuation ratio can be increased by increasing the gearing ratio. On the other hand, when either one of (a) and (b) is not satisfied, the valuation ratio is decreased by increasing the gearing ratio.

v. If the firm imposes a certain upper limit on the gearing ratio, *and* if it does ont issue new shares, some production plan which are technologically feasible cannot be adopted. For, the profit may be insufficient to finance the net investment. If the firm does impose an upper limit on the gearing ratio for one reason or another,[54] the only was to make all the technologically feasible production plans financially feasible as well, is to issue new shares at times of high net investment.

vi. Debt financing gives rise to discrepancy between the value of the firm and the total market value of shares of the firm. Production plan that maximises the former is not necessarily the one that maximises the latter.

X. OPINION OF THE MANAGEMENT AND OPINION OF THE MARKET

So far, we have been arguing under the assumption that the rate of discount to calculate the value of the firm and the rate of discount to calculate the theoretical value of shares are the same. However, there is a reason for believing that they are not the same. The former is related to the calculation, made by the management, of the depreciation allowance and the current cost of production. Therefore, it reflects the opinion of the management as to risk and uncertainty about the future stream of net receipts. On the other hand, the latter reflects the opinion of the stock market as to the risk and uncertainty about the future stream of net receipts. The opinion of the market may differ from that of the management, even apart from the speculative factors.

[52] For the precise meaning, see p. 67 and p. 69.

[53] If (a) is not satisfied, (b) cannot be satisfied either in normal circumstances, in which the net investment is not persistently negative.

[54] For example, increasing probability of insolvency for higher gearing ratio.

Taking into account the fact that the two rates of discount are different from each other, let r_B stand for the rate of discount of the stock market. r is the rate of discount of the management, as before. Now, we may conceive two present discounted values of the future expected net receipts, V and V_B, discounted at r and r_B, respectively. Then, the valuation ratio may be decomposed into two factors, as shown below,

$$\frac{B}{V} = \frac{V_B}{V}\frac{B}{V_B}.$$

Denoting V_B/V and B/V_B by σ_1 and σ_2, respectively, we have

$$\sigma = \sigma_1\sigma_2 . \qquad (17)$$

All the conclusions about the relationship between V and B, as enumerated above in the previous section, hold, *mutatis mutandis*, for V_B and B. Above all, the value of σ_2, which is the ratio of B to V_B, may differ from unity, only if there is a spread between r_B and i. Given the production plan, the value of σ_2, when r_B differs from i, is determined depending on what financial plan the firm adopts. It is worth noting that if the firm plans to have no debt, σ_2 turns out to be unity, and the valuation ratio is reduced to σ_1, regardless of the proportion of retained profits and new issues of shares to finance the net investment.[55] Thus, we have the following formulae for the valuation ratio[56] for the cases where the firm plans to have no debt,

$$\sigma = \frac{V_B}{V}.$$

As for σ_1, it follows immediately from the definition, that its value may differ from unity only if there is a spread between r and r_B. The value of σ_1, when r differs from r_B, is determined depending on what production plan the firm adopts, and is independent of the financial plan of the firm.[57] Moreover, it is obvious that σ_1 is larger or smaller than unity, according as r is larger or smaller than r_B, irrespective of the production plan of the firm.

XI. STEADY GROWTH OF THE FIRM

At this stage, let us see the implications of the foregoing analysis for the case of steady growth of the firm.

[55] This is the case treated by Kahn. See Kahn (1972), Essay 10. Notice that the retention ratio, which in his case determines the proportion of retained profits and new issues, does not appear in his formula for the valuation ratio.

[56] This is a generalised form of Kahn's formula (5) in Kahn (1972), p. 214. This point is elucidated further by an example given in the next section of the present chapter.

[57] Discussion of Kahn (1972) is devoted entirely to the analysis of the effect of the production plan on the valuation ratio. The retention ratio is defined at the outset, but plays no role in the subsequent arguments about the valuation ratio.

Let us suppose that the basic quantity of our analysis, the net receipt, is growing at a steady rate g, so that $z(t)$ is written as

$$z(t) = z_0 e^{gt} . \tag{18}$$

Then, it can easily been shown that the value of the firm, $V(t)$, the net investment, $v(t)$, and the present discounted value of the future net investment, $H(t)$, are all growing at the same steady rate g, and their initial values[58] are $z_0/(r - g)$, $gz_0/(r - g)$, and $gz_0/(r - g)^2$, respectively. Of course, the surplus, $m(t)$, which is the sum of $z(t)$ and $v(t)$, as well as its present discounted value which is the sum of $V(t)$ and $H(t)$ is growing at the same steady rate g, and their initial values are $rz_0/(r - g)$, and $rz_0/(r - g)^2$, respectively. The present values of the streams of $z(t)$, $v(t)$, and $m(t)$ discounted at r_B, when r_B is different from r, are obtained from those discounted at r, by substituting r for r_B. It is to be noted here that[59] for all t,

$$\frac{H}{V + H} = \frac{g}{r} . \tag{19}$$

Suppose further that the gearing ratio is fixed at δ, and see what can be said about the valuation ratio.[60] Firstly, the value of σ_1 can be obtined immediately from the values of V and V_B in a steady state. It is given by[61]

$$\sigma_1 = \frac{r - g}{r_B - g} , \tag{20}$$

and is constant over time. Secondly, equality (19) suggests that σ_2 is also constant over time.[62] In fact, it is calculated to be

$$\sigma_2 = \left(1 - \delta \frac{i - g}{r_B - g}\right) . \tag{21}$$

Thus, both σ_1 and σ_2 are constant over time, and therefore the valuation ratio σ is constant over time.

XII

Let us finally turn to the analysis of the firm's decision making concerning the production and finance plan.

So far, we have been arguing as if the entire stream of the future net receipts is known once the production plan is determined. But it is not the case in the

[58] Their values at $t = 0$.

[59] Calculation of all the present values presupposes that the rate of discount is larger than the rate of growth.

[60] The proportion of retained profits and new issues to finance the net investment does not matter in the determination of the valuation ratio. See conclusion i on p. 70.

[61] This is Kahn's formula. See Kahn (1972), p. 214.

[62] See p. 70.

world of uncertainty. Expectations formed at the present may be disappointed in the future, when the future becomes the present. Uncertainty is essentially due to the lack of information. But information cannot be obtained without costs. Under these circumstances, it may not be worthwhile, from the point of view of an individual firm, to try to forecast minute details of fluctuations expected to take place in the future. The firm may well be content if it can form a rough idea about trends. If we may suppose so, the analysis of steady growth of the previous section can be applied. In the rest of this paper, we shall examine the implications of the hypothesis that to each production plan, the firm associates a trend of net receipts as expressed by (18). Now, a production plan can be represented by two parameters, z_0 and g.

Some additional considerations must be made, before we discuss the choice of the firm as to the production and finance plans. First of all, the range of values that z_0 and g can take is not unbounded. Given the condition of technology, and the conditions of production inherited from the past, the range of values of z_0 and g is constrained by the limits of the market and of the managerial efficiency. The discussion of Section VI above suggests that its boundary is represented by a decreasing function relating z_0 to g. Thus, we have

$$(22) \qquad z_0 = z_0(g),$$

with

$$\frac{dz_0}{dg} < 0.$$

Secondly, if g can be made larger than r or r_B, we have an absurd consequence that the value of the firm or the total market value of shares can be made infinitely large. A plausible assumption is that as g becomes larger, the risk and uncertainty as to the future net receipts become also larger, and the increasing risk and uncertainty prevent the value of the firm or the total market value of shares from getting infinitely large. This assumption may be represented as

$$(23) \qquad r = r(g), \qquad \text{and} \qquad r_B = r_B(g),$$

with

$$r(g) > g, \qquad \text{and} \qquad r_B(g) > g.$$

$r(g)$ and $r_B(g)$ may, in fact, be constant, if there is an upper limit to g.

Finally, it is reasonable to suppose that the lender's risk[63] increases as the gearing ratio of the firm rises, and on the whole the firm has to pay interests

[63] See Keynes (1936), p. 144, for the definition.

at a higher rate.[64] Then, we have the following relationship,

$$i = i(\delta)\,, \tag{24}$$

with

$$\frac{di}{d\delta} > 0\,.$$

Now we have considered all the fundamental quantitative relationships governing the behaviour of the firm. They are exhibited in equalities (20)–(24). Let us illustrate their implications by showing the consequences of alternative objectives of the firm.[65]

Of the objective of the firm, three alternatives occur immediately to our mind, namely, maximisation of the value of the firm, maximisation of the total market value of shares of the firm, and maximisation of the rate of growth.

If two functions, $r(g)$ and $r_B(g)$, of g coincide with each other, the value of the firm is equal to the total market value of shares of the firm, irrespective of what is the choice of the firm as to the production and finance plans. For, (20) and (21) show that both σ_1 and σ_2 are identically equal to unity. Therefore, the distinction between the first and the second alternatives disappears. Obviously, the finance plan does not matter if the objective is to maximise the value of the firm. In fact, the maximum condition is given by

$$\frac{z'}{z} = \frac{r' - 1}{r - g}\,. \tag{25}$$

The traditional treatment, as by Hicks,[66] may be regarded as being concerned with this special case.

In more general cases, the first and the second objectives lead to different choices as to the production and finance plans on the part of the firm. This is because the maximum conditions for the second objective are given by[67]

$$\frac{z'}{z} = \frac{2(r_B' - 1)}{r_B - 1} - \frac{r_B' - (1 - \delta)}{(r_B - i\delta) - (1 - \delta)g}\,, \tag{26 a}$$

$$(i - g) + i'\delta = 0\,. \tag{26 b}$$

It is clear that (26 a) and (26 b) do not necessarily imply (25). Notice also

[64] The interest rate on a particular loan contracted on a fixed condition is, of course, not influenced by a change in the gearing ratio. However, since rates of interest on new loans become higher as the gearing ratio goes up, the overall rate of interest becomes higher.

[65] Solow (1971), treats a similar problem. He ignores the financial problems, on the ground which the present author is not able to justify.

[66] Hicks (1939).

[67] Take partial derivatives of $B_0 = \sigma V_0$ with respect to g and δ, and equate them to zero. The result is obtained by using the definition of σ, and (20) and (21).

that in maximisation of the total market value of shares, the finance plan does matter, even if we ignore the dependence of i on δ.

Which of the first and the second objectives leads to a higher rate of growth depends on the second order derivatives, and we cannot draw a simple conclusion.

If the firm's objective is a higher rate of growth, it would eventually have to sacrifice either the value of the firm, or the total market value of shares of the firm, or the valuation ratio. Equality (26 b) above is the condition for having maximum total market value of shares of the firm, as well as for maximum valuation ratio.[68] Therefore, if the firm desires to minimise the sacrifices in the total market value of shares of the firm, or in the valuation ratio, as it aims at a higher rate of growth, it ought to adopt a finance plan such that (26 b) is satisfied for any given rate of growth. In other words, for any rate of growth, the gearing ratio satisfying (26 b) minimises the sacrifices in the total market value of shares of the firm and in the valuation ratio. Put it in yet another way, the firm which tries to maximise the rate of growth, keeping a certain minimum valuation ratio[69] has to choose the gearing ratio that satisfies (26 b).

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[68] Maximum with respect to δ. This is because the only factor in $B_0 = \sigma V_0$ involving δ is σ.

[69] This is the type of the firm treated in Marris (1967).