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A NON-QUASI-COMPETITIVE COURNOT DUOPOLY WITH STABILITY

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Abstract: In the classical Cournot duopoly model, it is well-known that for a wide class of demand functions and for concave cost functions, the quasi-competitiveness of the duopoly may be lost, resulting in an increase on the commodity price. In all the known examples, this phenomenon is accompanied by the loss of the stability of the model. This paper presents a classical Cournot duopoly model with a unique feature: the loss of quasi-competitiveness with tability of the equilibrium.

JEL Classification Number: Primary D43; Secondary C62
Key words: Cournot equilibrium, non-cooperative duopoly, quasi-competitiveness, stability

1. INTRODUCTION

THE CLASSIC MODEL of Cournot oligopoly equilibrium was designed bearing in mind the mathematical modeling of the "effects of competition". Changing from a monopoly to a duopoly situation, one of these effects should be, as any reasonable person would agree, the reduction in the price of the commodity. This effect is called *quasi-competitiveness* in the specialized literature. There exist models, though, that do not present it. Frank Jr. and Quandt (1963) offer a model in which duopoly price is greater than monopoly price; their inverse demand function is somewhat "kinked" though and the feeling that these kinks are responsible for the rise in price is unavoidable. McManus (1962) and later McManus (1964) offer more general models in which a loss of quasi-competitiveness may occur. In fact, McManus (1964) relates quasi-competitiveness with the uniqueness of the equilibrium. In a more thorough analysis Ruffin (1971) presents a classic Cournot equilibrium in which a new entry breaks the quasi-competitiveness, violating at the same time the stability of the model. Ruffin, in fact, directly relates a condition for stability established by Hahn (1962) to quasi-competitiveness. Hahn's condition requires the uniqueness of the equilibrium; Okuguchi and Suzumura (1971) prove that Hahn's stability condition ensures uniqueness of the equilibrium. Lastly, Okuguchi (1974) proves that the uniqueness of

Okuguchi and Suzumura proves quasi-competitiveness despite losing stability.[1] A very good summary of these results can be found in Okuguchi (1976) and, from a more general point of view, Daughety (1988). A good reference for the generalization to multi-product firms can be found in Okuguchi and Szidarovsky (1999). More recently, under the assumption that each firm faces a production adjustment cost at each time period, Szidarovzky and Yen (1995) study dynamic oligopolies and find the necessary and sufficient condition for a global asymptotical stability. On the other hand, using lattice-theoretic methods, Amir and Lambson (2000) throw new light into the existence of static Cournot equilibrium. They obtain two minimal set of assumptions on the (derivatives of) demand and cost functions that guarantee that industry (equilibrium) price decreases [increases] with the number of competing firms whenever inverse demand or price decreases faster [slower] at any given output level than does marginal cost at all lower output levels. The dynamic stability of the equilibria, though, is not considered. The present state of the art can be found in Okuguchi and Szidaeovsky (1999) or Vives (2000).

In this paper, we build a model in which, starting from any linear decreasing inverse demand function we find an increasing piece-wise linear cost function in two pieces such that the model has the unique following features:

1. A unique Cournot equilibrium point is reached.
2. Monopoly price, p_1, is lower than the equilibrium price for duopoly, p_2.
3. The equilibrium point is stable in a sense that will be seen presently.

We hope our model may add some information to the clarification of the interdependence of the three aspects of Cournot oligopoly: uniqueness of equilibrium, stability and quasi-competitiveness. Our cost function is concave for the levels of production of interest; we would not be able to break quasi-competitiveness otherwise (Szidarovszky and Yakowitz (1982) prove uniqueness and quasi-competitiveness assuming strictly convex cost functions and decreasing differentiable demand functions). Our contribution shows that the phenomenon of the loss of quasi-competitiveness is not incompatible with a unique Cournot equilibrium point which is globally stable.

In section 2, after building the model, we discuss monopoly and duopoly maximizing outputs, the reaction curves and the necessary assumptions required to achieve our results. We prove the existence and uniqueness of a Cournot solution and, lastly, we study its stability under an adjustment mechanism proportional to the difference between actual firm output and profit maximizing output. In section 3 we exhibit a numerical example of our model.

[1] The models dealt with by these authors vary slightly in their assumptions concerning demand and cost functions: some require differentiability, others only continuity or even semi-continuity. Others consider increasing marginal costs, others not. Some consider all the firms identical and others consider different costs for each firm, etc.

2. THE MODEL

We assume a linear market demand function for the industry of our homogeneous commodity, $p = a - bq$, $a, b > 0$ and a continuous, piece-wise linear cost function:

$$C(q) = \begin{cases} c_1 + d_1 q & \text{if } 0 \leq q < q_m \\ c_2 + d_2 q & \text{if } q_m \leq q \leq a/b, \end{cases} \tag{1}$$

where $c_1, c_2, d_1, d_2 > 0$, $d_1 > d_2$, and q_m is a given point in the interval $(0, a/b)$. The continuity of $C(q)$ at $q = q_m$ requires that

$$c_2 = c_1 + (d_1 - d_2)q_m \,. \tag{2}$$

The requirement $d_1 > d_2$ is necessary in our model as for linear demands and convex cost functions it is well-known that the model is quasi-competitive, (see Quandt (1967) or Ruffin (1971)).

2.1. *Monopoly*

In a monopoly situation, the profit function of our sole firm is:

$$\Pi(q) = \begin{cases} -bq^2 + (a - d_1)q - c_1 & \text{if } 0 \leq q < q_m \\ -bq^2 + (a - d_2)q - c_2 & \text{if } q_m \leq q \leq a/b, \end{cases}$$

that can be written as

$$\Pi(q) = \begin{cases} \Pi_1(q) = -b\left(q - \dfrac{a - d_1}{2b}\right)^2 + \left(\dfrac{(a - d_1)^2}{2^2 b} - c_1\right) & \text{if } 0 \leq q < q_m \\ \Pi_2(q) = -b\left(q - \dfrac{a - d_2}{2b}\right)^2 + \left(\dfrac{(a - d_2)^2}{2^2 b} - c_2\right) & \text{if } q_m \leq q \leq a/b. \end{cases}$$

The profits are thus denoted separately: $\Pi_1(q)$ in the output interval $[0, q_m)$ and $\Pi_2(q)$ in the interval $[q_m, a/b]$. The global profit function, $\Pi(q)$, has a derivative for each q in $(0, a/b)$ except for $q = q_m$.

Equation (3) represents two parabolas: $\Pi_1(q)$ to the left of q_m and $\Pi_2(q)$ to the right of q_m. They both connect at q_m (see Figure 4 at the end of the paper). $\Pi_1(q)$ has vertex (q_1^c, Π_1^c) with

$$q_1^c = \frac{a - d_1}{2b} \quad \text{and} \quad \Pi_1^c = \frac{(a - d_1)^2}{2^2 b} - c_1 = b(q_1^c)^2 - c_1 \,, \tag{4}$$

and $\Pi_2(q)$ has vertex (q_2^c, Π_2^c) with

$$q_2^c = \frac{a - d_2}{2b} \quad \text{and} \quad \Pi_2^c = \frac{(a - d_2)^2}{2^2 b} - c_2 = b(q_2^c)^2 - c_2 \,. \tag{5}$$

As $d_1 > d_2$, we have $q_1^c < q_2^c$ and we assume that the point q_m satisfies

$$q_1^c < q_m < q_2^c \,.$$

In order to have $q_1^c > 0$, we also require that $0 < d_i < a$, $(i = 1, 2)$. Notice that from $0 < d_2$ we have $q_2^c < a/(2b)$. Summing up, we assume:

$$0 < q_1^c < q_m < q_2^c < \frac{a}{2b}. \tag{6}$$

It will be convenient to introduce the following parameters:

$$\lambda = q_1^c/q_2^c \quad \text{and} \quad \mu = q_m/q_2^c. \tag{7}$$

Now, expression (6) becomes:

$$0 < \lambda < \mu < 1. \tag{8}$$

2.2. Duopoly

Let us now suppose that a new firm with the same cost function enters the industry. We are now in a situation of duopoly. Let us denote by Q_m the total output in the case of monopoly and Q_d the total industry output in case of duopoly. Let p_1 and p_2 be the respective prices. Using the notations just introduced in (4), (5) and (7) we state the following result:

THEOREM 1. *If*

$$\frac{1}{2} < \lambda < \frac{3}{4} \tag{9}$$

and

$$\max\left\{\lambda, \frac{1}{6}(5-\lambda)\right\} < \mu < \frac{1}{2}(1+\lambda), \tag{10}$$

then $Q_d < Q_m$, and, consequently, $p_2 > p_1$. Moreover, the duopoly outputs are reached for the Cournot equilibrium point

$$\left(\frac{2}{3}q_1^c, \frac{2}{3}q_1^c\right). \tag{11}$$

This equilibrium point is unique.

Proof. Let Π_j be the profit of firm j $(j = 1, 2)$ and let $\mathbf{q} = (q_1, q_2)$ be the output vector:

$$\Pi_1(\mathbf{q}) = \begin{cases} \Pi_{1,1} = -bq_1^2 + ((a-d_1) - bq_2)q_1 - c_1 & \text{if } 0 \le q_1 < q_m \\ \Pi_{1,2} = -bq_1^2 + ((a-d_2) - bq_2)q_1 - c_2 & \text{if } q_m \le q_1 \le a/b, \end{cases}$$

$$\Pi_2(\mathbf{q}) = \begin{cases} \Pi_{2,1} = -bq_2^2 + ((a-d_1) - bq_1)q_2 - c_1 & \text{if } 0 \le q_2 < q_m \\ \Pi_{2,2} = -bq_2^2 + ((a-d_2) - bq_1)q_2 - c_2 & \text{if } q_m \le q_2 \le a/b. \end{cases}$$

As above, using (4) and (5), these profit functions can be written as

$$(12)\quad \Pi_1(\mathbf{q}) = \begin{cases} \Pi_{1,1} = -b\left(q_1 - \left(q_1^c - \frac{1}{2}q_2\right)\right)^2 + b\left(q_1^c - \frac{1}{2}q_2\right)^2 - c_1 \\ \qquad \text{if } 0 \le q_1 < q_m \\ \Pi_{1,2} = -b\left(q_1 - \left(q_2^c - \frac{1}{2}q_2\right)\right)^2 + b\left(q_2^c - \frac{1}{2}q_2\right)^2 - c_2 \\ \qquad \text{if } q_m \le q_1 \le a/b \end{cases}$$

and

$$(13)\quad \Pi_2(\mathbf{q}) = \begin{cases} \Pi_{2,1} = -b\left(q_2 - \left(q_1^c - \frac{1}{2}q_1\right)\right)^2 + b\left(q_1^c - \frac{1}{2}q_1\right)^2 - c_1 \\ \qquad \text{if } 0 \le q_2 < q_m \\ \Pi_{2,2} = -b\left(q_2 - \left(q_2^c - \frac{1}{2}q_1\right)\right)^2 + b\left(q_2^c - \frac{1}{2}q_1\right)^2 - c_2 \\ \qquad \text{if } q_m \le q_2 \le a/b\,. \end{cases}$$

From equation (13), the local maxima of $\Pi_2(\mathbf{q})$ in each of the intervals separated by q_m are

$$(14)\qquad \Pi_2^{\max} = \begin{cases} b\left(q_1^c - \frac{1}{2}q_1\right)^2 - c_1 \\ b\left(q_2^c - \frac{1}{2}q_1\right)^2 - c_2\,, \end{cases}$$

and these are reached for values of q_2 that depend on q_1. This gives the *reaction curve* of firm 2 respect to the output of firm 1:

$$(15)\qquad R_2(q_1) = \begin{cases} q_1^c - \frac{1}{2}q_1 & \text{if } 0 \le q_2 < q_m \\ q_2^c - \frac{1}{2}q_1 & \text{if } q_m \le q_2 \le a/b\,. \end{cases}$$

In the same way we have the reaction curve of firm 1 respect to the output of firm 2:

$$R_1(q_2) = \begin{cases} q_1^c - \frac{1}{2}q_2 & \text{if } 0 \le q_1 < q_m \\ q_2^c - \frac{1}{2}q_2 & \text{if } q_m \le q_1 \le a/b\,. \end{cases}$$

The graph of each reaction curve (see Figure 1) is, in general, the graph of a correspondence and not the graph of a function. $R_2(q_1) = q_m$ for $q_1 = 2(q_2^c - q_m)$ and thus, by (15), if $q_1 \in [0, 2(q_2^c - q_m)]$, $R_2(q_1)$ takes two values. From these two possible values of R_2, firm 2 will choose the one that maximizes its profit. Using this value as the only image, we will change R_2 into a proper function.

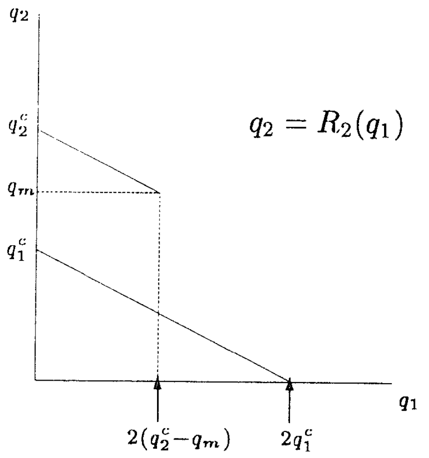


Figure 1

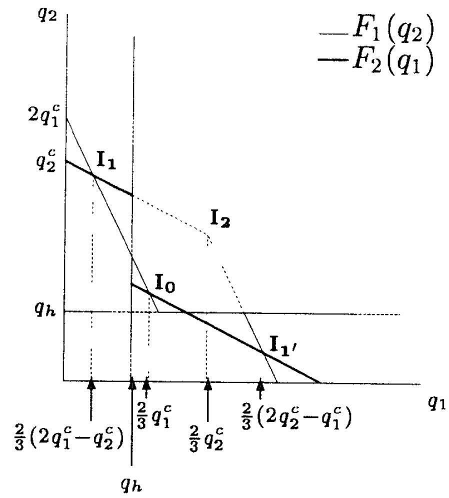


Figure 2

Consequently, we are interested in finding the $q_1 \in [0, 2(q_2^c - q_m)]$ for which we have $\Pi_{2,2}(R_2(q_1)) \geq \Pi_{2,1}(R_2(q_1))$. By (14) we get:

$$c_2 - c_1 \leq b\left(\left(q_2^c - \frac{1}{2}q_1\right)^2 - \left(q_1^c - \frac{1}{2}q_1\right)^2\right). \tag{16}$$

From (2), (4) and (5) we have

$$c_2 - c_1 = 2b(q_2^c - q_1^c)q_m, \tag{17}$$

and replacing this value of $c_2 - c_1$ in (16), after some algebra we reach

$$2b(q_2^c - q_1^c)q_m \leq b(q_2^c + q_1^c - q_1)(q_2^c - q_1^c),$$

which duly simplified takes us to

$$q_1 \leq q_2^c + q_1^c - 2q_m \equiv q_h.$$

Notice that $q_h < 2(q_2^c - q_m)$. After the correct reaction has been chosen, each reaction curve becomes a function (see Figure 2),

$$F_2(q_1) = \begin{cases} q_2^c - \frac{1}{2}q_1 & \text{if } 0 \leq q_1 \leq q_h \\ q_1^c - \frac{1}{2}q_1 & \text{if } q_h < q_1 \leq a/b, \end{cases} \tag{18}$$

and

$$F_1(q_2) = \begin{cases} q_2^c - \frac{1}{2}q_2 & \text{if } 0 \leq q_2 \leq q_h \\ q_1^c - \frac{1}{2}q_2 & \text{if } q_h < q_2 \leq a/b. \end{cases} \tag{19}$$

The intersections of the reaction functions (18) and (19) are the Cournot points of the model.

The *possible* intersections are

$$\mathbf{I_0} = \left(\frac{2}{3}q_1^c, \frac{2}{3}q_1^c\right); \qquad \mathbf{I_1} = \left(\frac{2}{3}(2q_1^c - q_2^c), \frac{2}{3}(2q_2^c - q_1^c)\right);$$

$$\mathbf{I_2} = \left(\frac{2}{3}q_2^c, \frac{2}{3}q_2^c\right); \qquad \mathbf{I_{1'}} = \left(\frac{2}{3}(2q_2^c - q_1^c), \frac{2}{3}(2q_1^c - q_2^c)\right).$$

Not all of these intersections will take place at the same time. Depending on the value of the parameter q_h some of them will not take place. Let us classify the different possibilities in terms of q_h and, consequently, in terms of q_m.

The function $F_2(q_1)$ presents a single discontinuity at $q_h = q_2^c + q_1^c - 2q_m$. From Figure 2 and from the possible positions of q_h, we infer:

- If $\frac{2}{3}(2q_2^c - q_1^c) \le q_h \le q_2^c$ or, equivalently, $\frac{1}{2}q_1^c \le q_m \le \frac{1}{6}(-q_2^c + 5q_1^c)$, the only existing intersection is $\mathbf{I_2}$.
- If $\frac{2}{3}q_2^c \le q_h < \frac{2}{3}(2q_2^c - q_1^c)$ or, equivalently, $\frac{1}{6}(-q_2^c + 5q_1^c) < q_m \le \frac{1}{2}(\frac{1}{3}q_2^c + q_1^c)$, then the existing intersections are $\mathbf{I_2}$, $\mathbf{I_1}$ and $\mathbf{I_{1'}}$.
- If $\frac{2}{3}q_1^c \le q_h < \frac{2}{3}q_2^c$ or, equivalently, $\frac{1}{2}(\frac{1}{3}q_2^c + q_1^c) < q_m \le \frac{1}{2}(q_2^c + \frac{1}{3}q_1^c)$, then $\mathbf{I_1}$ and $\mathbf{I_{1'}}$ exist.
- If $\frac{2}{3}(2q_1^c - q_2^c) \le q_h < \frac{2}{3}q_1^c$ or, equivalently, $\frac{1}{2}(q_2^c + \frac{1}{3}q_1^c) < q_m \le \frac{1}{6}(5q_2^c - q_1^c)$, then we have $\mathbf{I_0}$, $\mathbf{I_1}$ and $\mathbf{I_{1'}}$ as the intersection points.
- Lastly, if $0 < q_h < \frac{2}{3}(2q_1^c - q_2^c)$, or, equivalently, $\frac{1}{6}(5q_2^c - q_1^c) < q_m < \frac{1}{2}(q_1^c + q_2^c)$, we will only have $\mathbf{I_0}$.

In our case, from (10), replacing λ and μ by their values we get

$$0 < q_h < \frac{2}{3}(2q_1^c - q_2^c)$$

and, consequently, our model has a unique Cournot equilibrium point, $\mathbf{I_0}$. Notice that the first inequality in (9) is equivalent to $2q_1^c - q_2^c > 0$.

If $\mathbf{I_0}$ is the equilibrium point in duopoly,

$$Q_d = \frac{4}{3}q_1^c.$$

Let us now find Q_m. Maximum profit under monopoly can only be Π_1^c or Π_2^c as in (4) and (5). It is seen at once that, in our case, $\Pi_1^c < \Pi_2^c$. Indeed,

$$b(q_1^c)^2 - c_1 < b(q_2^c)^2 - c_2$$

is equivalent to

$$c_2 - c_1 < b(q_2^c + q_1^c)(q_2^c - q_1^c).$$

Now, replacing $c_2 - c_1$ by the expression obtained in (17), we reach

$$q_m < \frac{1}{2}(q_1^c + q_2^c),$$

which is exactly $\mu < \frac{1}{2}(1 + \lambda)$, the second inequality in (10). Consequently,

$$Q_m = q_2^c.$$

Comparing now Q_m and Q_d, we see immediately that $Q_d < Q_m$ as the second inequality in (9), $\lambda < 3/4$, is precisely $\frac{4}{3}q_1^c < q_2^c$.

It is worth noticing that by the double inequality in (10), the parameter μ has room to exist only if $\lambda > 1/2$, which is precisely the first inequality in (9). It is also clear that the second inequality in (9), which is needed to ensure $Q_m = q_2^c$, makes $q_h > 0$ as can be seen from Figure 2. This last condition, makes it possible for firm 2 to maximize its output at $q_2 = q_2^c$ if $q_1 = 0$. □

2.3. Stability of the model

We will study the stability of our duopoly model assuming that each firm adjusts its output proportionally to the difference between its actual profit and its profit maximizing output, a common adjustment system in the literature, see Hahn (1962); Fisher (1961); Quandt (1967):

$$\begin{cases} \dot{q}_1 = k_1(F_1(q_2) - q_1 \\ \dot{q}_2 = k_2(F_2(q_1) - q_2)\,, \end{cases} \tag{20}$$

where F_1, F_2 are the reaction functions of (18) and (19); k_1 and k_2 are the 'speeds' of adjustment, $k_1, k_2 > 0$, and $t = 0$ is the moment firm 2 enters the market.

We consider that the second firm enters the market when firm 1 is already maximizing its profit producing q_2^c. Under these conditions and the same assumptions of theorem 1 we may state:

THEOREM 2. *The Cournot equilibrium point* $\mathbf{I_0}$ *is stable no matter the entry output of firm* 2 *within the range* $[0, q_2^c]$.

Proof. If we consider as possible outputs all the points $(q_1, q_2) \in [0, a/b]\times[0, a/b]$, the reaction of each firm depends on the position of the actual outputs $(q_1(t), q_2(t))$ in each of the four regions which divide the square $[0, a/b] \times [0, a/b]$. Figure 3 shows these four regions which we will call I, I′, II and III according to the following description:

- Region I: $0 \le q_1 \le q_h$ and $q_h < q_2 \le a/b$.
- Region I′: $q_h < q_1 \le a/b$ and $0 \le q_2 \le q_h$.
- Region II: $0 \le q_1 \le q_h$ and $0 \le q_2 \le q_h$.
- Region III: $q_h < q_1 \le a/b$ and $q_h < q_2 \le a/b$.

In this way, the system of differential equations (20) can be split in four systems, one for each region above, in which the corresponding reaction functions are continuous and linear. To cover these four possibilities, we introduce A_1 and A_2 to denote either q_1^c or q_2^c depending on the region we are in. Thus, in Region I, $A_1 = q_1^c$, $A_2 = q_2^c$; in Region I′, $A_1 = q_2^c$, $A_2 = q_1^c$; in Region II, $A_1 = q_2^c$, $A_2 = q_2^c$ and, lastly, in Region III, $A_1 = q_1^c$, $A_2 = q_1^c$. Using this convention, each of the four linear systems in (20) can be written as

$$\begin{cases} \dot{q}_1 = k_1\left(A_1 - \frac{1}{2}q_2 - q_1\right) \\ \dot{q}_2 = k_2\left(A_2 - \frac{1}{2}q_1 - q_2\right)\,. \end{cases}$$

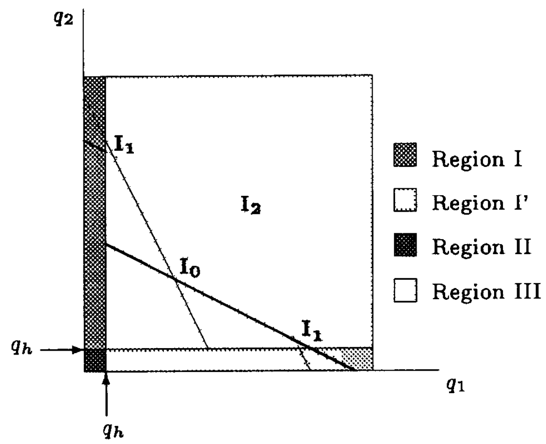


Figure 3. Stability study: The four regions

The general solution is:

$$(21)\qquad \begin{cases} q_1(t) = D_1 e^{-\alpha_1 t} + E_1 e^{-\alpha_2 t} + \dfrac{2}{3}(2A_1 - A_2) \\ q_2(t) = D_2 e^{-\alpha_1 t} + E_2 e^{-\alpha_2 t} + \dfrac{2}{3}(2A_2 - A_1)\,, \end{cases}$$

where α_1 and α_2 are real and positive

$$\alpha_1 = \frac{(k_1 + k_2) - \sqrt{(k_1 + k_2)^2 - 3k_1k_2}}{2}\,;$$

$$\alpha_2 = \frac{(k_1 + k_2) + \sqrt{(k_1 + k_2)^2 - 3k_1k_2}}{2}\,;$$

and D_1, E_1, D_2, E_2 are real constants that depend on k_1, k_2, and the initial conditions $q_1(0)$ and $q_2(0)$. The stationary solution is obviously

$$\left(\frac{2}{3}(2A_1 - A_2), \frac{2}{3}(2A_2 - A_1)\right)\,,$$

that is to say, each of the four points $\mathbf{I_0}$, $\mathbf{I_1}$, $\mathbf{I_{1'}}$, $\mathbf{I_2}$, depending on the region in which the motion starts.[2]

The fact that α_1 and α_2 in (21) are strictly positive ensures the global stability of each of the attractors if the motion of $(q_1(t), q_2(t))$ **stays within the region** of validity of the system of differential equations. If the motion of our output vector takes it from one region to another, the values of A_1 and A_2 change and the stationary solution with them. A more detailed study of the motion of our output vector $(q_1(t), q_2(t))$ is then required.

If $q_2(0) > q_h$, the initial point $(q_1(0), q_2(0))$ is located in Region III where $A_1 = q_1^c$, $A_2 = q_1^c$ and the attractor is $\mathbf{I_0}$. If $q_2(0) \leq q_h$, then $(q_1(0), q_2(0))$ is located in Region I′ but the corresponding attractor is also placed in Region III and the orbit

[2] Let us recall that, under the assumptions of our model as set in (9) and (10) we have $q_2^c < 2q_1^c$, and $\mathbf{I_0}$ is the unique Cournot point that exists.

of $(q_1(t), q_2(t))$ eventually will enter Region III. When this happens, the system of prevailing equations will be the one whose attractor is $\mathbf{I_0}$. The actual orbits may enter or leave different regions depending on the sign of $\dot{q}_i$. To be more specific, if $(2q_1^c - q_2^c)/2 < q_h < 2(2q_1^c - q_2^c)/3$ there would be orbits going from Region III into Region I′ through a point on the segment with endpoints $(2q_1^c - 2q_h, q_h)$ and (q_2^c, q_h).

It is easy to see that, in this case, the second component of the orbit, $q_2(t)$, must attain a minimum value exactly **on** the line $q_2 = q_2^c - (1/2)q_1$, and then will re-enter Region III through a point on the segment with endpoints $(q_2^c - \frac{1}{2}q_h, q_h)$ and $(2q_1^c - 2q_h, q_h)$.

After that, the orbit does not enter Region I′ again. A similar behavior may occur when we start from an initial point $q_2(0)$ very near to q_2^c and a value for $k_1 \gg k_2$. In this case, the orbit could move from Region III to Region I behaving in a similar way as before. □

Let us remark that the assumption $\lambda > 1/2$ is essential for the stability of the process as allowing $q_2^c > 2q_1^c$ breaks the stability for some production entries $q_2(0) \in [0, q_2^c]$. To see that, if we proceed as in the proof of theorem 1, we would have as one of the intersections

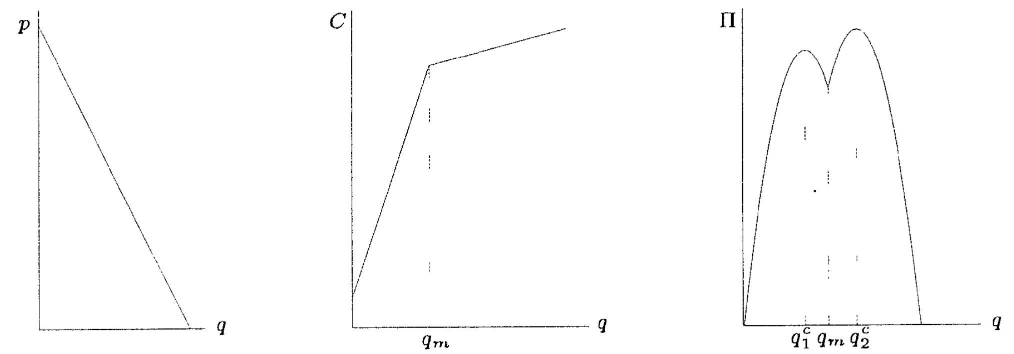
$$\mathbf{I_{1'}} = \left(\frac{2}{3}|2q_2^c - q_1^c|, -\frac{2}{3}|q_2^c - 2q_1^c|\right).$$

Consequently, in this case, the attractor of an initial state in Region I′ would be placed in the non-positive zone for q_2. That would mean that the output of firm 2, $q_2(t)$, would decrease till become zero. Firm 2 would leave the market and $\mathbf{I_0}$ would not be stable.

3. A NUMERICAL EXAMPLE

We finish by exhibiting a numerical example of our model. Let our demand function be $p = 100 - 2q$, and let the cost function of our 2 firms be:

$$C(q) = \begin{cases} 10 + 47.2q & \text{if } 0 \le q < 18 \\ 787.6 + 4q & \text{if } 18 \le q < 5. \end{cases} \tag{22}$$



Demand Cost Monopoly profit

Figure 4

The main parameters in our model are

1. $q_2^c = 24$.
2. $\lambda = 0.55$ and thus it satisfies (9):

$$1/2 < \lambda < (5 + 1)/(2 \times 5)\,.$$

3. Hence $q_1^c = \lambda q_2^c = 13.2$.
4. $\mu = 0.75$ and consequently satisfies (10):

$$0.7416666\ldots = \max\{\lambda, (1/6)(5 - \lambda)\} < \mu < (1/2)(1 + \lambda) = 0.775\,.$$

5. Thus $q_m = \mu q_2^c = 18$.

Monopoly total output is 24 at a price $p_1 = 52$. Duopoly total output is 17.60 at the price $p_2 = 64.80$.

4. CONCLUSIONS

We have exhibited a classical Cournot duopoly model in which monopoly price is lower than the price for the free entry of a new firm. The equilibrium solution reached is unique and stable under habitual adjustment mechanisms. This is new in the literature as the loss of quasi-competitiveness has been always linked to instability.

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