

7-21-2005

A non-singular black hole model as a possible end-product of gravitational collapse

Manasse Mbonye

Follow this and additional works at: <http://scholarworks.rit.edu/article>

Recommended Citation

Mbonye, Manasse, "A non-singular black hole model as a possible end-product of gravitational collapse" (2005). *Physical Review D*, 72 (), 8 pages. Accessed from <http://scholarworks.rit.edu/article/183>

This Article is brought to you for free and open access by RIT Scholar Works. It has been accepted for inclusion in Articles by an authorized administrator of RIT Scholar Works. For more information, please contact ritscholarworks@rit.edu.

A non-singular black hole model as a possible end-product of gravitational collapse.

*Manasse R. Mbonye^{1,2} and Demosthenes Kazanas¹

¹*NASA/Goddard Space Flight Center
Mail Code 663,
Greenbelt, MD 20785*

²*Department of Physics,
Rochester Institute of Technology,
84 Lomb Drive, Rochester, NY 14623.*

May 29, 2006

Abstract

In this paper we present a non-singular black hole model as a possible end-product of gravitational collapse. The depicted spacetime which is type $[II, (II)]$, by Petrov classification, is an exact solution of the Einstein equations with and contains two horizons. The equation of state $p_r(\rho)$ in the radial direction, is a well-behaved function of the density $\rho(r)$ and smoothly reproduces vacuum-like behavior near $r = 0$ while tending to a polytrope at larger r , low ρ values. The final equilibrium configuration comprises a de Sitter-like inner core surrounded by a family of 2-surfaces Σ of matter fields with variable equation of state. The fields are all concentrated in the vicinity of the radial center $r = 0$. The solution depicts a spacetime that is asymptotically Schwarzschild at large r , while it becomes de Sitter-like as $r \rightarrow 0$. Possible physical interpretations of the macro-state of the black hole interior in the model are offered. We find that the possible state admits two equally viable interpretations, namely either a quintessential intermediary region or a phase transition in which a two-fluid system is in both dynamic and thermodynamic equilibrium. We estimate the ratio of pure matter present to the total energy and in both cases find it to be virtually the same, being ~ 0.83 . Finally, the well-behaved dependence of the density and pressure on the radial coordinate provides some insight on dealing with the information loss paradox.

Keywords: non singular black holes

* NAS/NRC Senior Research Associate; e-mail: mrmmps@rit.edu

Pacs: 04.70.Bw, 04.20.Jb

1 Introduction

It is generally accepted that the formation of a black hole proceeds with the formation of a marginally trapped surface (the apparent horizon) into which matter is collapsing and one that encloses a region of space interior to which the light cones are tipped [1] so much so that even outgoing null geodesics proceed toward ever decreasing values of the radial coordinate. The famous singularity theorems [2][3][4], then posit that under certain assumptions on energy conditions of matter, the eventual state of the collapse is an infinite density singularity surrounded by an event horizon (defined by the outermost trapped surface). It is, however, a fact that the existence of such a singularity is neither easy to verify nor refute observationally because of the limits set by the black hole event horizon. In the absence of such observational evidence, the compelling logic in the arguments leading to the singularity theorems has generally shaped the widespread conviction that indeed a black hole must contain a physical singularity.

As is known, however, the picture of a singular black hole opens up several puzzles. For example, the existence of spacetime singularities results in irretrievably total information loss [5]. Moreover, the infinite tidal forces that result from such a singularity formation lead to a breakdown in the descriptive power of General Relativity [6]. As a result it is generally believed that the perceived final collapse state implied by the singularity theorems could be but a manifestation of the incompleteness of classical gravity and that a correct (quantum) theory of gravity will dispense with this singular final state and admit a state of finite (albeit very high) curvature. An altogether different view is that the singularities implied by the aforementioned theorems simply reflect our lack of knowledge of the properties of matter under extreme conditions. Thus it has been suggested that the singularity theorems could be circumvented if (some of) the conditions imposed on the properties of collapsing matter were relaxed. Indeed modern frameworks (like string theory) attempting to formulate a quantum theory of gravity seek the existence of a fundamental length scale and hence a singularity free spacetime. There are also philosophical questions as to whether spacetime can indeed accommodate singularities of any kind, be they from gravitational collapse or vacuum-induced [7].

As a result of these and other considerations a renewed interest has grown lately with regard to spacetime singularities and their existence [8]. Indeed the concept of non-singular collapse dates back to Sakharov's consideration of the equation of state $p = -\rho$ for a superdense fluid and Gliner's view [9] that such a fluid could be the final state of gravitational collapse. Such ideas are now attracting a growing amount of attention. For example, Ellis [10] has argued that in the case of a closed universe a trapped surface does not necessarily lead to a singularity. Recently the present authors have investigated an analogous

argument with regard to black holes in [11]. It was pointed out in [11] that under physically acceptable energy conditions the expansion of the outgoing mode which assumes increasingly negative values in a trapped region can eventually turn around to (even) assume a positive signature inside a black hole. Such behavior suggests the existence some interior region, surrounding the core, which is not trapped. In turn, this feature implies the absence of a singularity in such a spacetime. In recent years, several non-singular black hole solutions have been found. Dymnikova, for example, [12] has constructed an exact solution of the Einstein field equations for a non-singular black hole containing at the core a fluid de-Sitter-like fluid with anisotropic pressure. This solution is Petrov Type [(II),(II)] by classification¹. Another line of investigation in this area, initiated by Markov [13], suggests a limiting curvature approach. This idea has been explored further by several authors, see for example [14][15][16][17][18][19].

A common feature in all these treatments is that the geometry of the spacetime in question is Schwarzschild at large r as expected by Birkoff's theorem, and de Sitter-like at small r values as implied by the $p = -\rho$ equation of state assumed to prevail near $r = 0$. These approaches can be put in two broad classes. (a) Those for which the transition to this "exotic" state of matter can be placed well inside the Schwarzschild horizon [12] and (b) those which replace the entire volume of the Schwarzschild metric interior to $r = 2M$ by a substance with the $p = -\rho$ equation of state [6], dispensing in this way with the presence of the horizon altogether and the "information paradox" problems it engenders.

The problem of transition from the usual matter equation of state to that appropriate for these non-singular solutions is usually not addressed at all. This problem is particularly acute for models of the second class discussed above, since the density at horizon formation scales like $\rho \simeq 10^{16} (M/M_\odot)^{-2} \text{ g cm}^{-3}$ and for objects of mass $M \geq 10^8 M_\odot$, is not greater than that of water, which is known to have an equation of state very different from $p = -\rho$. Moreover, issues to do with direct matching of an external Schwarzschild vacuum to an interior de Sitter have previously been discussed [20] based on junction conditions [21]. In fact, in terms of gravitational collapse the associated difficulties have been used to suggest modifications in some treatments [22]. The present work is motivated by these questions and is undertaken as a first effort to provide some answers by constructing models using an explicit equation of state with the desired properties. It would appear that the junction constraints [21] when applied to the end-product of non-singular gravitational collapse imply the matter fields across the Schwarzschild/de Sitter boundary should have a radially dependent equation of state of the form $1 \lesssim w(r) \lesssim -1$ that smoothly changes the matter-energy from a stiff fluid to a cosmological constant.

The model sketched in this paper describes the geometry of a body that passed during its collapse through a stiff fluid state to settle into the final state of a non-singular black hole. The solution allows for a matter fluid region (with a family of equations of state $0 \leq w(r) \leq 1$ which envelopes an intermediate

¹We shall present a solution with the same asymptotic behavior as in [12] which however is different in both classification (it is Petrov Type [II,(II)] and physical interpretation.

region with a family of equations of state $-1 < w(r) < 0$ which in turn envelopes a de Sitter like region with $w = -1$ at the center. Our model differs both from the traditional singular black hole solutions which introduce matter at the singularity and from the relatively new non-singular solutions which usually introduced only a de sitter-like spacetime inside the black hole. The main feature of our model is that it

- (1) introduces gravitating matter inside a non-singular black hole and
- (2) offers a reasonable explanation of how part of this matter can evolve towards a de Sitter-like vacuum to provide the radial tension or negative pressure that supports the remaining matter fields against the formation of a singularity.

The rest of the paper is organized as follows. In section 2 we summarize the equations to be solved. We also construct the working equation of state and discuss some of its desirable features. In section 3 we solve the Einstein Field Equations for the desired geometry and obtain an exact solution. In section 4 we highlight the physical implications of the model. Section 5 concludes the discussion.

2 Problem formulation

2.1 The field equations

It is assumed, for simplicity, that the collapsed object can be reasonably described by a spherical, static geometry. The desired line element then takes the general form

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (2.1)$$

where $\lambda(r)$ and $\nu(r)$ are to be determined from the Einstein Field Equations

$$G_{\mu\nu} = -8\pi GT_{\mu\nu}. \quad (2.2)$$

Under the assumed spacetime symmetry, Eqs.2 reduce to

$$e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi G\rho(r), \quad (2.3)$$

$$e^{-\lambda} \left(\frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = 8\pi Gp_r(r), \quad (2.4)$$

and

$$e^{-\lambda} \left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right) = 8\pi p_{\perp}(r). \quad (2.5)$$

Here $\rho(r) = T_0^0$ is the energy density, $p_r = T_1^1$ is the radial pressure and $(p_{\theta} = T_2^2) = (p_{\varphi} = T_3^3) = p_{\perp}$ is the tangential pressure. In this treatment the fluid is anisotropic with $T_2^2 = T_3^3 \neq T_1^1$ and $T_1^1 \neq T_0^0 \neq T_2^2$. The spacetime is thus Petrov Type [II, (II)], where () implies a degeneracy in the eigenvalues of the Weyl tensor.

2.2 Equation of state

In the remaining part of this section we construct the working equation of state $p_r(\rho)$ in the radial direction which reproduces the characteristics of the collapsed body as highlighted in the previous section. We justify the choice by pointing out its desired features. On the other hand, the tangential pressure equation of state $p_\perp(\rho)$ will be derived later from the Einstein Field Equations (Eq. 2.5) using the radial pressure function $p_r(\rho)$ to be constructed.

The main desired features are that (1) the equation of $p_r(\rho)$ change from the one associated with the usual matter, at low densities, to that associated with the de Sitter geometry at sufficiently high densities near the center of the configuration and; (2) that this change be well-behaved. To this end we consider an equation of state that takes the general form² where m and $1/n$ are, yet to be determined, positive real numbers and α is a free parameter to be constrained. This equation implies presence of a

$$p_r(\rho) = \left[\alpha - (\alpha + 1) \left(\frac{\rho}{\rho_{\max}} \right)^m \right] \left(\frac{\rho}{\rho_{\max}} \right)^{1/n} \rho, \quad (2.6)$$

maximum limiting density ρ_{\max} concentrated in a region of size $r_0 = \sqrt{\frac{1}{G\rho_{\max}}}$, the core region of the configuration. One notes that the main desirable features are readily apparent in the functional form of Eq. 2.6. At low densities, $\left(\frac{\rho}{\rho_{\max}} \right)^m < \frac{\alpha}{\alpha+1}$, the equation of state reduces to that of a polytrope of index n ($p_r \propto \rho^{1+1/n}$) while for $\left(\frac{\rho}{\rho_{\max}} \right)^m > \frac{\alpha}{\alpha+1}$ the pressure decreases to eventually approach that of the vacuum $p_r = -\rho_{\max}$ as $\rho \rightarrow \rho_{\max}$. We look for the simplest form of Eq. 2.6 by judiciously constraining both the indices m and n and the parameter α .

Since Eq. 6 already satisfies the general features desired by the model, its specific form, based on an appropriate choice of the indices m and n and the parameter α is made through the demand that it must satisfy the following basic conditions.

- (1) It must have no pathologies, i.e.
 - (i) the sound speed $\frac{dp}{d\rho}$ can not be a maximum at $\rho = 0$
 - (ii) in order to rule out superluminal behavior, $\frac{dp}{d\rho} \not\geq 1$, the maximum sound speed, i.e. the value of $\frac{dp}{d\rho}$ at the point where $\frac{d^2p}{d\rho^2} = 0$ must be given by $\frac{dp}{d\rho} \Big|_{\rho_{stiff}} = c_s^2 = c = 1$.
- (2) It must satisfy minimum acceptable energy conditions, namely the Weak Energy Condition, $\rho \geq 0$, $\rho + p_r \geq 0$ and the Dominant energy Condition, $\rho \geq 0$, $p_r \in [-\rho, +\rho]$.

The $[m = 1, 1/n = 0]$ and the $[m = 2, 1/n = 0]$ cases are both pathological (they do not satisfy (i) above) and will therefore be discarded. The next simplest potential choices are $[m = 1, n = 1]$ and $[m = 2, n = 1]$. Both these forms, apparently, manifest no pathologies of the previous cases. In both cases the sound

²This choice may not be unique.

speed vanishes $c_s \rightarrow 0$ at vanishing density $\rho \rightarrow 0$. Furthermore for $\rho > 0$ the sound speed c_s is initially an increasing function of ρ , as expected in reality??. One can therefore demand in each case (i.e. for fixed m and n values) that the maximum sound speed coincide with that of light i.e. the extremum sound speed at $\frac{d^2 p}{d\rho^2} = 0$ be given by $\frac{dp}{d\rho} = 1$ in order to rule out superluminal behavior. These conditions also suffice to determine the parameter α and allow the fulfillment of the energy conditions in (2). Solving $\frac{d^2 p}{d\rho^2} = 0$ and $\frac{dp}{d\rho} = 1$ simultaneously for the $[m = 1, n = 1]$ case, i.e. fixes α at $\frac{3}{4} - \frac{1}{4}\sqrt{33}$ or $\frac{1}{4}\sqrt{33} + \frac{3}{4}$ (we keep only the positive solution). Similarly for the $[m = 2, n = 1]$ one fixes α to $\alpha = 2.2135$. We will select the quartic function corresponding to $m = 2, n = 1$ as the representative form for our model equation of state, in part because, unlike the cubic form, it unambiguously provides only one possible value of α and makes the interpretation simpler. Thus Eq. 2.6 now takes the form

$$p_r(\rho) = \left[\alpha - (\alpha + 1) \left(\frac{\rho}{\rho_{\max}} \right)^2 \right] \left(\frac{\rho}{\rho_{\max}} \right) \rho, \quad (2.7)$$

with the constraint that $\alpha = 2.2135$. It is the equation of state we adapt throughout the remaining part of the paper.

A few words on the behavior of our assumed equation of state. While no explicit microscopic prescription leading to its form is presently given, we consider that its softening past the “stiff” ($c_s^2 = dp/d\rho = 1$) state and its eventual conversion to an equation of state appropriate to that of the vacuum ($p = -\rho$), is effected by the coupling of matter to a scalar field akin to that of Higgs that gives rise to particle masses. The dominance of the pressure by terms which render it negative is considered to imply that eventually the energy associated with the self interaction of this field provides the dominant contribution to the energy momentum tensor, which is assumed to be that of a perfect fluid.

In this paper no attempt is made to follow the dynamical evolution of the collapse. Instead it is assumed that the collapsed body has already reached static equilibrium from its collapse. Thus, we only investigate the characteristics of the various static 2-surfaces as the radial coordinate decreases from the matter surface $r = R$ to the body center $r = 0$. Since our anticipated solution will allow matter fields we can, with no loss of generality, take for initial conditions on the collapsed body surface R to be $\rho = 0$, $p_r = 0$. We therefore expect to have four regions in this spacetime which must be satisfied by the expected solution.

Region I: Schwarzschild vacuum: $R < r < \infty$, $p = \rho = 0$

Region II: Regular matter fields: $r_\varepsilon < r < R$, $\rho > 0, 0 < p \leq \rho$,

Region III: Quintessential fields: $r_0 < r < r_\varepsilon$, $0 < \rho < \rho_{\max}, -\rho_{\max} < p < 0$,

Region IV: Λ -vacuum: $0 \leq r \leq r_0$, $p = -\rho = -\rho_{\max}$.

The solution to Eqs. 2.1-2.5, must satisfy the following asymptotic conditions at large r and small r , respectively.

(i) The spacetime must be asymptotically Schwarzschild for large r , i.e. for

$R < r < \infty$

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{\left(1 - \frac{2M}{r} \right)} dr^2 - r^2 (d\theta + \sin^2 \theta d\varphi^2). \quad (2.8)$$

where for the black hole R is some hypersurface such that $R < 2M$, $M = 4\pi \int_0^\infty \rho(r) r^2 dr$ being the total mass.

(ii) The spacetime must be asymptotically de Sitter

$$ds^2 = - \left(1 - \frac{r^2}{r_0^2} \right) dt^2 + \frac{1}{\left(1 - \frac{r^2}{r_0^2} \right)} dr^2 + r^2 (d\theta + \sin^2 \theta d\varphi^2). \quad (2.9)$$

for $0 \leq r \leq (r_0 < R)$. Here, $r_0 = \sqrt{\frac{3}{8\pi G \rho_{\max}}} = \sqrt{\frac{3}{\Lambda}}$ signals the onset of de Sitter behavior, where $\rho_{\max} = \rho|_{r \rightarrow 0}$ is the upper-bound on the density of the fields of order of Planck density ρ_{Pl} .

The asymptotic conditions in Eqs. 2.8 and 2.9 imply there is an interior region which includes the family of surfaces $\Sigma = \{\Sigma_{II} \cup \Sigma_{III}\}$ and which interfaces with region I on the outer side and region IV on the inner side. The entire spacetime must therefore satisfy regularity conditions at the two interfaces $r = R$ and $r = r_0$. Put simply, such conditions guarantee (i) continuity of the mass function and (ii) continuity of the pressure across the interfacing hypersurfaces. Thus at each interface, i.e. (I, II) and (III, IV) we must have

$$[\rho^+ - \rho^-]_{|r=r_i} = 0, \quad (2.10)$$

$$[p^+ - p^-]_{|r=r_i} = 0,$$

where $r_i = \{R, r_0\}$ and $+$, $-$ refer to the exterior and interior values respectively and $p = \{p_r, p_\perp\}$. A desirable feature of the model (as we find later) is the smooth continuity of the density and pressure between region II and region III. This feature removes the problem having to match the two regions through junction conditions between the regular matter fields and the vacuum-like field, since here such conditions will be satisfied naturally.

3 The solution

We now solve the Einstein equations 2.3-2.5 for the spacetime of the model. Integration of Eq. 2.3 gives

$$e^{-\lambda} = 1 - 8\pi \frac{1}{r} \int_0^r \rho(r') r'^2 dr' = 1 - \frac{2m(r)}{r}, \quad (3.1)$$

where, as stated before, $m(r)$ is the mass enclosed by the a 2-sphere of radius r . Further, using Eqs. 2.7 and 11, integration of the T_1^1 Eq. 2.4, gives

$$e^\nu = \left(1 - \frac{2m(r)}{r} \right) e^{\int 8\pi \left[\alpha - (\alpha+1) \left(\frac{\rho}{\rho_{\max}} \right)^2 \right] \left(\frac{\rho}{\rho_{\max}} \right) \rho \frac{dr}{1 - \frac{2m(r)}{r}}}, \quad (3.2)$$

where $\alpha = 2.2135$.

Eqs. 3.1 and 3.2 into Eq. 2.1 give

$$ds^2 = - \left(1 - \frac{2m(r)}{r}\right) e^{\Gamma(r)} dt^2 + \frac{1}{1 - \frac{2m(r)}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (3.3)$$

where

$$\Gamma(r) = \int 8\pi \left[\alpha - (\alpha + 1) \left(\frac{\rho}{\rho_{\max}}\right)^2 \right] \left(\frac{\rho}{\rho_{\max}}\right) \left(\frac{r}{r - 2m(r)}\right) \rho dr \quad (3.4)$$

Eq. 3.3 is the general solution for the geometry of our model. It describes the spacetime of a non-singular black hole with both matter and a de Sitter core. We shall describe below a particular solution resulting from the choice of a prescribed density function $\rho(r)$.

To formally complete the solution one must also solve equation 5 for the tangential pressure $p_{\perp} = \{p_{\theta}, p_{\varphi}\}$. On doing this one finds (see also [23] [24]) that

$$p_{\perp} = p_r + \frac{r}{2} p_r' + \frac{1}{2} (p_r + \rho) \left[\frac{Gm(r) + 4\pi G r^3 p_r}{r - 2Gm(r)} \right], \quad (3.5)$$

where p_r is given by Eq. 2.7. Note that in this model the last term in Eq. 3.6 is not vanishing, in general, as is the case considered in some previous treatments where it was assumed that $p_r = -\rho$ for all ρ (see for example [25]). Eq.3.6 is a generalization of the Tolman-Oppenheimer-Volkoff equation [26].

One can now discuss the results of Eqs.3.1-3.4. The mass $m(r)$ enclosed a 2-surface at any radial coordinate $0 < r < R$ is given by $m(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$. The entire mass of the body M is given by $M = \int_0^{\infty} m(r) dr = \int_0^{\infty} \rho(r) r^2 dr$. It is then clear that outside the mass of the body $T_0^0 = 0$. Further, as ρ vanishes outside the body, so does the pressure (see Eqs. 2.7 and 3.3). In particular, from Eq. 3.3 one observes that in the limit $\rho \rightarrow 0$ and $m(r) \rightarrow M$ the lapse function and the shift vector give $e^{\nu} = e^{-\lambda} = 1 - \frac{2GM}{r}$. Thus outside the mass (i.e. for large r) the spacetime becomes asymptotically Schwarzschild, with the metric given by Eq. 2.8. Further, in the limit $r \rightarrow r_0$, $\rho \rightarrow \rho_{\max}$ (and as both Eqs. 2.7 and 3.4 show) $p_r \rightarrow -\rho_{\max}$ and $p_{\perp} \rightarrow -\rho_{\max}$. It follows, therefore that for $0 \leq r < r_0$ the fluid has constant positive density ρ_{\max} and constant negative pressure $-\rho_{\max}$ and takes on the character of a de Sitter spacetime, through the equation of state $p = -\rho_{\max}$.

In order to establish the actual asymptotic behavior of the spacetime at small r one has to choose a density profile for the matter. We choose to use the one employed by Dymnikova [12] because of its simplicity and convenient form for integration over the source volume $\rho = \rho_{\max} \exp\left[-\frac{r^3}{r_g(r_0)^2}\right]$. The mass $m(r)$ enclosed by a 2-sphere at the radial coordinate r is then given by $m(r) = \int_0^r \rho_{\max} \left\{ \exp\left[-\frac{r'^3}{r_g(r_0)^2}\right] \right\} r'^2 dr' = M \left[1 - \exp\left(-\frac{r^3}{r_g(r_0)^2}\right) \right]$, where M

is the total mass $M = \int_0^\infty \rho_{\max} \left\{ \exp \left[-\frac{r^3}{r_g(r_0)^2} \right] \right\} r^2 dr$. The entire mass is essentially concentrated in a region of size $R \simeq (r_0^2 r_g)^{1/3}$, which in general is close to $r = 0$, with its precise value depending on the value of the maximum density assumed ρ_{\max} .

The introduction of the density function $\rho = \rho_{\max} \exp \left[-\frac{r^3}{r_g(r_0)^2} \right]$ in Eq. 3.3 provides a particular solution for the model. In the limit $r \rightarrow 0$, it is seen that the solution Eq. 3.3 asymptotically leads to Eq. 2.9 as the spacetime becomes asymptotically de Sitter.

4 Physical Interpretation

Eqs. 3.3, 3.4, 3.5 along with the prescribed density profile, form the solution in this model describing the spacetime of a non-singular black hole containing both matter and a de Sitter core. A heuristic interpretation of the form of the equation of state used has been given in the previous section. The field whose presence leads to the asymptotic relation $p = -\rho$ has not been specified but it must be similar to the quintessence field used in cosmology. In this specific case, it could in fact be the Higgs field that gives rise to the particle masses. As such the maximum density could be roughly $\rho_{\max} \simeq m_H^4$, the latter being the Higgs field expectation value. This could be either of order 1 TeV or even of order of the Plank mass if, as argued, that should be the typical mass of all scalars coupled to gravity.

The depicted scenario allows two viable, albeit, different interpretations in describing the internal structure of such a black hole. The two interpretations have the common feature that in both cases a fluid with a negative pressure, in the interior, supports the matter fields in the outer region against any further collapse. The effect in both cases is to create a non-singular black hole.

4.1 The quintessential picture

The first interpretation is directly based on the analytical functional form of the equation of state depicted in Eq. 2.7. Here, as one moves from the surface R of the collapsed body towards the center, one first encounters region II where from the functional form of the equation of state one crosses a family Σ_{II} of 2-surfaces with matter-like $w > 0$. At the critical point ρ_c in the $\rho - p_r$ space $\frac{dp_r}{d\rho} = 0$ thereafter the pressure p_r begins to drop with increasing density ρ . At some point $\rho_c = \left(\frac{\alpha}{\alpha+1} \right)^{\frac{1}{2}} \rho_{\max}$, the fluid temporarily becomes pressureless $w \rightarrow 0$, implying that $T_1^1 = G_1^1 \rightarrow 0$. Beyond this, for smaller values of r one enters region III in which one is now crossing a family of 2-surfaces Σ_{III} for which the equation of state is given by $-1 < w < 0$, smoothly decreasing from $w = 0$ towards $w = -1$. In this picture we will refer to this as the quintessential region. Evidently Σ_{III} exhausts the entire family of quintessential fluids $-1 < w < 0$. This fluid family provides some of the negative pressure against the implosion

of the matter fields in the outer region. The rest of the negative pressure is provided by the constant density ρ_{\max} , constant pressure $p = -\rho_{\max}$, inner core $0 < r < r_0$ that mimics the cosmological constant. This interpretation is valid provided the equation of state $p_r(\rho)$ is, indeed in reality, a well-behaved function of r for $\rho_c < \rho < \rho_{\max}$.

In this quintessential picture, one can estimate the amount remaining pure matter (the total field for which $w \geq 0$) as a fraction of the total energy of the system. This matter lies in the outer region $r_\varepsilon \leq r \leq R$ (i.e. between points B and C in Figure 1). Since B is given by, $\frac{dp_r}{d\rho}|_{\rho_c} = 0$ and C is given by $p_r|_{\rho_\varepsilon} = 0$, one can infer from Eq. 7 that in this region the density function ρ is bounded by $\left[\rho_c = \left(\frac{\alpha}{3(\alpha+1)}\right)^{\frac{1}{2}} \rho_{\max}\right] \leq \rho \leq \left[\rho_\varepsilon = \left(\frac{\alpha}{(\alpha+1)}\right)^{\frac{1}{2}} \rho_{\max}\right]$. To determine the matter mass \mathcal{M}_{matter} enclosed we use the density profile $\rho(r) = \rho_{\max} \exp\left[-\frac{r^3}{r_0^2 r_g}\right]$ and perform the integral $\int \rho r^2 dr$ with the appropriate limits. It is useful to change the independent variable from r to ρ . Then $r^2 dr = \frac{1}{3} r_0^2 r_g \left(\frac{\rho_m}{\rho}\right) d\left(\frac{\rho}{\rho_m}\right)$ and we have that $\mathcal{M}_{matter} = 4\pi \int \rho r^2 dr = \frac{4}{3} \pi r_0^2 r_g \int_0^{\rho_\varepsilon} d\rho$. Thus

$$\mathcal{M}_{matter} = \frac{4}{3} \pi r_0^2 r_g \left(\frac{\alpha}{\alpha+1}\right)^{\frac{1}{2}} \rho_{\max}. \quad (4.1)$$

Using $\alpha = 2.2135$ as adopted earlier for our model, gives $M_{matter} = 0.82995 \left(\frac{4}{3} \pi r_0^2 r_g\right) \rho_{\max}$. On the other hand the total mass $M = 4\pi \int \rho \exp\left[\frac{r^3}{r_0^2 r_g}\right] r^2 dr = \frac{4}{3} \pi r_0^2 r_g \rho_{\max}$. Thus the fractional content of matter in the quintessential picture for the black hole is $\frac{\mathcal{M}_{matter}}{M} = 0.82995 \simeq 0.83$.

4.2 A two-fluid system

One can attempt yet a seemingly equally viable interpretation of the black hole's internal structure. If one holds as in [6] the view of a phase transition of the fluid from the matter-like form to the de Sitter state given by $p = -\rho_{\max}$, then the successive 2-surfaces in the family $r_0 < r < r_c$ will represent a two fluid system of matter fields and the Λ -vacuum which in both dynamic and thermodynamic equilibrium. The fluid begins as pure matter on the 2-surface r_c and gets richer and richer in the Λ -vacuum state as one moves deeper and deeper into the black hole. Then the density $\rho_r(r)$ at any such point on any such 2-surface shell $r_0 < r < R$ is really a sum of two partial densities

$$\rho = \beta(r) \rho_c - [1 - \beta(r)] \rho_{\max}, \quad (4.2)$$

where ρ_c is the critical density just before collapse when $p = \rho_c$ and $0 \leq \beta \leq 1$. The function $\beta(r)$ is the fractional content of pure matter fields to the total surface energy contained in an elementary 2-surface shell $4\pi r^2 dr$ at $r_0 < r < R$. For decreasing r , $\beta(r)$ is a decreasing function, evolving from $\beta(R) = 1$ to $\beta(r_0) = 0$. In terms of the partial pressures of the matter fields and the de

Sitter vacuum, the pressure at any position $r_0 \leq r < R$ is then given by

$$p_r(r) = \sum_{n=0}^3 a_n \left(\frac{\rho_c}{\rho_{\max}} \right)^n [\beta(r) \rho_c - (1 - \beta(r)) \rho_{\max}] \quad (4.2)$$

where the a 's are positive constants given respectively by

$$\begin{aligned} a_0 &= (1 - \beta) \left[\alpha - (\alpha + 1) (1 - \beta)^2 \right], \\ a_1 &= \beta [\alpha + (\alpha + 1) (1 - \beta)], \\ a_2 &= \beta^2 (\alpha + 1) (1 - \beta), \\ a_3 &= \beta^3 (\alpha + 1). \end{aligned} \quad (4.4)$$

This expression based on partial pressures will then replace Eq. 2.7 as the equation of state $p_r(\rho)$ in the event the evolution of matter fields to a de Sitter state involves a spontaneous phase transition. One notes that the pressure p_r still has the desired asymptotic behavior. Thus in the limit $\beta(r) \rightarrow 1$, $p \rightarrow p_c = \left[\alpha - (\alpha + 1) \left(\frac{\rho_c}{\rho_{\max}} \right)^2 \right] \left(\frac{\rho_c}{\rho_{\max}} \right) \rho_c$, while in the limit $\beta(r) \rightarrow 0$, $p \rightarrow -\rho_{\max}$ as expected.

One can now estimate the energy M_{matter} in the pure matter fields as a fraction of the total energy of the black hole in this two-fluid picture. We assume that up to the critical density ρ_c the matter is pure and that the energy content contribution of the de Sitter fluid with an energy density ρ_{\max} starts to grow at $\rho = \rho_c$. Then the matter content can be written as a two piece integral $M_{matter} = 4\pi \int_{r_c}^{\infty} \rho r^2 dr + 4\pi \int_{r_0}^{r_c} \beta(r) \rho_c r^2 dr$. We will assume $\beta(r)$ to be simply linear in r with a form $\beta(r) = \frac{r-r_0}{r_c-r_0}$, which satisfies the requirements imposed on it above. Then substituting for $\beta(r)$ and again noting that $r^2 dr = \frac{1}{3} r_0^2 r_g \left(\frac{\rho_m}{\rho} \right) d \left(\frac{\rho}{\rho_m} \right)$ we get

$$\mathcal{M}_{matter} = \frac{4}{3} \pi \left\{ [r_0^2 r_g] + \left[\frac{1}{4} (r_c + r_0) (r_c^2 + r_0^2) - \frac{1}{3} r_0 \frac{r_c^3 - r_0^3}{r_c - r_0} \right] \right\} \rho_c. \quad (4.5)$$

From the definition of the density function we have that $r = \left[-r_0^2 r_g \ln \left(\frac{\rho}{\rho_{\max}} \right) \right]^{\frac{1}{3}}$. Since $r \rightarrow r_0$ as $\rho \rightarrow \rho_{\max}$ we have that for $0 < r < r_0$, $r(\rho)$ is not well defined since r is now not single-valued in ρ . Further, we don't know the actual size of r_0 . As a result we shall assume, for the purpose of evaluating Eq. 4.5, that both $r = \left[-r_0^2 r_g \ln \left(\frac{\rho}{\rho_{\max}} \right) \right]^{\frac{1}{3}}$ and $r \rightarrow r_0$ as $\rho \rightarrow \rho_{\max}$ hold so that $r_0(\rho_{\max}) \rightarrow 0$. This is equivalent to having the de Sitter vacuum approached only as $r \rightarrow 0$. Thus the value of M_{matter} calculated from here will be an upper bound. Applying this on the integral $\int_{r_0}^{r_c} \beta(r) \rho_c r^2 dr$ part Eq. 4.5 implies $\left[\frac{1}{4} (r_c + r_0) (r_c^2 + r_0^2) - \frac{1}{3} r_0 \left(\frac{r_c^3 - r_0^3}{r_c - r_0} \right) \right] \rightarrow \frac{1}{4} r_c^3 \rho_c$ so that Eq. 4.5 reduces to

$$\mathcal{M}_{matter} = \frac{4}{3} \pi \left[r_0^2 r_g + \frac{1}{4} r_c^3 \right] \left(\frac{\alpha}{3(\alpha + 1)} \right)^{\frac{1}{2}} \rho_{\max} \quad (4.6)$$

Use of $r_c = \left[-r_0^2 r_g \ln \left(\frac{\rho_\varepsilon}{\rho_{\max}} \right) \right]^{\frac{1}{3}}$ and $\alpha = 2.2135$ then gives $\frac{M_{2f}}{M} = 0.47917 [1 + 0.73570] = 0.8316 \simeq 0.83$.

This result agrees with the one found above in the quintessential picture.

5 Conclusion

In this paper we have suggested a possible model for a non-singular black hole as a product of gravitational collapse. It is an exact solution of the Einstein Equations, being Type [II, (II)] by Petrov classification. This model is based on a judicious choice we have made of the equation of state of the collapsed matter. At high densities this equation of state violates some of the energy conditions [2][3] originally used to justify the existence of black hole singularities. In our model this violation leads to non-singular collapse. On the other hand, for the entire density parameter space the equation of state in our approach still satisfies the Weak Energy Condition $\rho \geq 0$, $\rho + p_r \geq 0$, a basic requirement for physical fields.

The solution depicts a spacetime with matter fluids in the outer layers (region II) which, as one moves deeper inside, give way to either a quintessential or a two-fluid region III. Region III, in turn, evolves into region IV, an inner-most core with de Sitter characteristics. The fluids in regions III and IV both provide the negative pressures needed to sustain the outer matter in static equilibrium. The solution has the required asymptotic forms, reducing to the Schwarzschild vacuum solution outside the matter fields and reducing to the de Sitter solution as one approaches the black hole center. The existence of region III renders the interface between matter fields and the de Sitter vacuum to join smoothly. This is because both the density ρ , the radial pressure p_r and the tangential pressure p_\perp profiles and their derivatives are continuous. This character of the fields makes the matching across associated interfaces natural.

Using our model we have, in Section IV, offered two viable interpretations about the possible macro-state of the fields making up the total energy of the black hole. These interpretations are based on two pictures, namely a Quintessential Picture and a Two Fluid Picture. In both cases we have estimated the fractional contribution of the regular matter-like fields as a fraction of the total black hole energy. We find for an upper bound the same value of ~ 0.83 in both cases³. As a corollary, our model suggests that the lower bound for the amount of matter that is found in the “exotic” state can be as small as 17% of the entire configuration of the collapsed matter. Both the size and density of this de Sitter-like central core region will depend on the details of the microscopic physics that lead to the specific equation of state we have employed, which go beyond the scope of the present work. However, these considerations are consistent with our present notions of dynamical particle masses and symmetry breaking, as the density of this state is much higher than those produced

³It is remarkable that the two different interpretations yield the same numerical values of the mass-fractions.

to-date in the laboratory. The precise value is uncertain but it has to be at least as high as m_H^4 , where m_H is the mass of the Higgs field, and it can be as high as the Planck density. Our entire non-singular configuration is well within the horizon of the black hole and in this respect the configuration is very different from some previous treatments, e.g. [6]. In these treatments, the field configuration fills the entire volume interior to $r = 2M$ with a fluid with $p = -\rho$, whose energy density for sufficiently large black holes can be smaller than that of water. This raises the difficult problem of converting matter from the usual equations of state to the “exotic” $p = -\rho$, under usual laboratory conditions.

Finally, the model leaves some issues unresolved. First, the calculation for these mass fractions is based on assuming that in the region $r_0 < r < R$ both the pressure and the density are well-behaved, functions in the radial coordinate r , so that conversely $r(\rho) = -r_0^2 r_g \ln\left(\frac{\rho}{\rho_{\max}}\right)$ well defined in the entire parameter space of ρ . Since in our model $\rho \rightarrow \rho_{\max}$ when $r \rightarrow r_0$, then $r(\rho)$ vanishes at r_0 forcing (in the mass calculation) the de Sitter state to appear virtually at the origin. It is in this sense that $\frac{\mathcal{M}_{\text{matter}}}{M} = 0.83$ is an upper-bound. One can reduce this fraction by using $r(\rho) = -r_0^2 r_g \ln\left(\frac{\rho}{\rho_{\max}}\right) + r_0$ in the calculations, instead. However since we don't know the radial extent the de Sitter field could fill at the core, or whether indeed it fills any space bigger than the Planck length we can not presently constrain $\frac{\mathcal{M}_{\text{matter}}}{M}$ from below. This is one issue for quantum gravity to settle.

Secondly, our classical model can not distinguish between the two pictures offered to determine which choice of the matter macrostate is the correct one. However, the fact that in the region $r_0 < r < R$ the energy-momentum tensor elements ρ and p are well-behaved functions of the radial coordinate r may provide an interesting insight. It suggests that one can associate a unique value of the energy-momentum tensor on each 2-surface in the family $\Sigma = \{\Sigma_{II} \cup \Sigma_{III}\}$. One can further speculate that, such a configuration may even be quantizable. In such a case the information pertaining to the pre-collapse phase of the object would not be lost but would now reside on the (quantized) onion-like family of hyperfaces Σ inside the black hole. This is another issue for quantum gravity to settle. These results suggest a need for further investigation.

Acknowledgement 1 *This research was performed while one of the authors (MRM) held a National Research Council Senior Research Associateship Award at (NASA-Goddard).*

References

- [1] S. Hawking and R. Penrose, *The Nature of Spacetime*, Princeton University Press (1995).
- [2] R. Penrose, *Phys. Rev. Lett.* 14, 57(1965).

- [3] S. W. Hawking and R. Penrose, Proc. Roy. Soc. Lond. A 314, 529(1970).
- [4] S. W. Hawking and G. F. R. Ellis, ApJ 152, 25(1968).
- [5] J. D. Bekenstein, Phys. Rev. D7 2333(1973).
- [6] P.O.Mazur and E. Mottola, gr-qc/010903.
- [7] M. R. Mbonye, gr-qc/0309135; M. R. Mbonye, Mod. Phys. Lett. A19 (2004) 117-134.
- [8] M. Visser and D. L. Wiltshire, Class.Quant.Grav. 21 (2004) 1135-1152.
- [9] E. B. Gliner, gr-qc/9808042.
- [10] G. F. R. Ellis, gr-qc/0304039.
- [11] M. R. Mbonye and D. Kazanas, Non-singular trapped surfaces in gravitational collapse; to appear soon.
- [12] I. Dymnikova, Int. J. Mod. Phys. D5 529(1996). See also I. Dymnikova, gr-qc/0304110.
- [13] Markov [M. A. JEPP Lett. 36 265(1982); Ann. Phys. 155 333(1984)
- [14] Makhov, Mukhanov, and Frolov, Phys. Lett. B216 272 (1987).
- [15] Makhov, Mukhanov, and Frolov, Phys. rev D 41, 383(1990)].
- [16] V. Mukhanov, R. Brandenberger, Phys. Rev. Lett. 68, 1969(1992).
- [17] Brandenberger, V. Mukhanov and A. Sornorger and M Trodden (inprep)
- [18] M. Trodden, V. F. Mukhanov, Brandenberger, Phys. Lett. B316 483(1993).
- [19] Easson and Brandenberger , JHEP, 0106, 024(2001).
- [20] Poisson and Israel, Class. and Quant. Grav. 5, L201(1988).
- [21] O. Grøn, Lett. Cimento, 44, 177(1985)
- [22] M. Visser and D. Wiltshire, gr-qc/0310107.
- [23] R. L. Flowers, E.P.T. Liang, ApJ 188 657(1974).
- [24] I. Dymnikova, gr-qc/0112052.
- [25] I. Dymnikova, gr-qc/0112052.
- [26] R. M. Wald, General Relativity, Univ. of Chicago Press (1984).

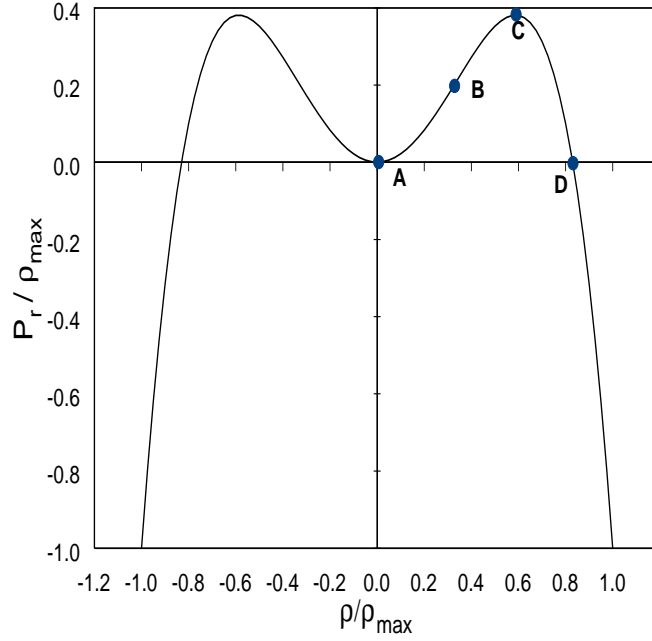


Figure 1: The quartic equation of state functional $p(\rho)$ with $m = 2$, $n = 1$. At A when $\rho = 0$, then $p = 0$ and $\frac{dp}{d\rho} = 0$. The value of the parameter $\alpha = 2.213(5)$ is chosen so that for the entire density parameter space $\rho \geq 0$ the sound speed is not superluminal. The point B giving the maximum slope corresponds to the maximum sound speed, $\frac{d^2p}{dr^2} = 0$, chosen to correspond to the light speed, $\frac{dp}{d\rho} = c^2 = 1$. Point C is the critical point (ρ_c, p_c) , and for $\rho > \rho_c$, then $p(\rho)$ is a decreasing function. At point D or $\rho = \rho_\varepsilon$, the pressure temporarily vanishes..