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LOSS CONE REFILLING RATES IN GALACTIC NUCLEI

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Abstract

A gap in phase space is opened up by a binary supermassive black hole as it ejects stars in a galactic nucleus. This gap must be refilled before the single black hole that subsequently forms can disrupt or accrete stars. We compute loss cone refilling rates for a sample of elliptical galaxies as a function of the mass ratio of the binary that preceded the current, single black hole. Refilling times are of order 10^{10} yr or longer in bright elliptical galaxies. Tidal flaring rates in these galaxies might be much lower than predicted using steady-state models.
Subject headings: stellar dynamics, galaxies: nuclei, black holes

1. INTRODUCTION

The loss cone of a black hole (BH) at the center of a galaxy is defined as the set of orbits that intersect the BH, or that pass within some distance of its center. For instance, the tidal disruption loss cone consists of orbits with pericenters below r_t, the radius at which tidal forces from the BH would disrupt a star. Stars on such orbits are removed in a single orbital period or less, and subsequent feeding of stars to the BH requires a re-population of the loss cone, which is typically assumed to be driven by gravitational encounters between stars.

Classical loss-cone theory (Bahcall & Wolf 1976; Lightman & Shapiro 1977; Cohn & Kulsrud 1978) was directed toward understanding the observable consequences of massive BHs at the centers of globular clusters. Globular clusters are many relaxation times old, and this assumption was built into the theory, by requiring the stellar phase space density near the BH to have reached an approximate steady state under the influence of gravitational encounters. In a collisionally-relaxed cluster, the density of stars near the BH has the Bahcall-Wolf (1976) "zero-flux" form, $\rho \propto r^{-7/4}$, and the dependence of f on J, the stellar angular momentum per unit mass, near the loss-cone boundary is described by the Cohn-Kulsrud (1978) boundary conditions. The feeding rate is determined by the gradients of f with respect to J at the loss cone boundary and by the normalization of f at values of J far from the loss cone, i.e. by the stellar density.

Galactic nuclei differ from globular clusters in that the relaxation time is often very long, typically in excess of 10^{10} yr (Faber et al. 1997). One consequence is that the stellar density profile near the BH need not have the Bahcall-Wolf $r^{-7/4}$ form. But the fact that galactic nuclei are not collisionally relaxed also has implications for the more detailed form of the phase space density near the loss cone boundary, and hence for the feeding rate. In one widely discussed model for the formation of galactic nuclei (Begelman, Blandford, & Rees 1980), a binary BH forms following the merger of two galaxies. The binary ejects stars on orbits such that $J < J_{bin} \approx \sqrt{2GM_{12}a_h}$, with M_{12} the mass of the binary and $a_h \approx G\mu/4\sigma^2$ the binary's semi-major axis at the time of its formation; $\mu = M_1 M_2/M_{12}$ is the reduced mass and σ is the stellar velocity dispersion. Since $a_h \gg r_t$, there will be a gap in angular momentum space around the single BH, of mass $M_\bullet = M_{12}$,

that subsequently forms, corresponding to stars with $J \lesssim J_{bin}$ that were ejected by the binary. Before the single BH can begin to consume stars at the steady-state rate, this gap needs to be re-filled. This argument suggests that the feeding rates of BHs in galaxies that formed via mergers might be much lower than predicted by application of the steady-state theory. Here we solve the time-dependent equations describing the evolution of the stellar phase-space density around a BH in a galactic nucleus and compute the time required for the steady-state feeding rate to be reached.

2. APPROXIMATE TIMESCALES

Diffusion of stars into a single BH is dominated by scattering onto low-angular-momentum orbits (Frank & Rees 1976). The scattering time is

$$T_\theta(r) \approx \theta(r)^2 T_r(r) \tag{1}$$

where $\theta(r)$ is the angle within which a star's velocity vector must lie if it is in the loss cone, and T_r is the relaxation time,

$$T_r(r) = \frac{\sqrt{2}\sigma(r)^3}{\pi G^2 m_\star \rho(r) \log \Lambda}, \tag{2}$$

with $m_\star$ the stellar mass and $\log \Lambda \approx 15$ the Coulomb logarithm. The square-root dependence of θ on T_θ reflects the fact that entry into the loss cone is a diffusive process. If the loss cone was initially emptied of all stars with pericenters below some radius r_0, the time to diffusively refill this angular momentum gap is

$$T_{gap} \approx \frac{r_0}{r} T_r. \tag{3}$$

Equation 3 assumes that $r_t \ll r_0 \lesssim r_h$ with $r_h = GM_{12}/\sigma^2$ the influence radius of the single (coalesced) BH (Frank & Rees 1976). Until the angular momentum gap is filled, the rate of supply of stars to the BH's capture sphere will be much less than its steady-state value.

In the binary coalescence model, most stars with pericenters

$$r_p \lesssim Ka_h = K\frac{G\mu}{4\sigma^2} \tag{4}$$

will have been ejected prior to coalescence by the gravitational slingshot mechanism (Saslaw, Valtonen & Aarseth 1974), with K a constant of order unity. This ejection will

occur even if the binary's main source of angular momentum loss is torques from gas clouds, as long as $|a/\dot{a}|$ exceeds stellar orbital periods near the BHs. Setting $r_0 \approx r_p$ and $r \approx r_h$ (since most of the scattering into the single BH takes place from radii near r_h), we find

$$T_{gap} \approx \frac{K}{4} \frac{q}{(1+q)^2} T_r(r_h) \qquad (5)$$

with $q = M_2/M_1 \leq 1$. For $q = 1$, $T_{gap} \approx T_r/16$, and for $q \ll 1$, $T_{gap} \approx (q/4)T_r$. These times are short compared with T_r but still in excess of 10^{10} yr for the luminous galaxies that are most likely to have experienced mergers.

The principal uncertainties in this estimate of the refilling time are the value of K, and the fact that stars are scattered into the loss cone from a range of distances with different values of T_r. We address these issues by more careful calculations in the following sections.

3. CREATION OF A PHASE-SPACE GAP

The gap is created by the binary BH when its separation reaches $\sim a_h$. The definition given above for a_h is difficult to apply to galaxies in which σ is a function of radius. We therefore adopt an alternative definition in terms of r_h:

$$a_h = \frac{\mu}{M_{12}} \frac{r_h}{4} = \frac{q}{(1+q)^2} \frac{r_h}{4}, \qquad (6)$$

with r_h defined as the radius in the unperturbed galaxy containing a mass in stars equal to twice M_{12}. These definitions are equivalent to the standard ones $r_h = GM_{12}/\sigma^2$, $a_h = G\mu/4\sigma^2$ in a singular-isothermal-sphere nucleus but can also be applied to nuclei in which σ is a function of radius.

Once it forms, the binary shrinks as it ejects stars that pass within a distance$\sim a(t)$ of the binary's center of mass. The coupled evolution of the stellar fluid and the binary is complicated due to processes like re-ejection, the repeated interaction of (non-escaping) stars with the binary, and by the fact that the binary's cross-section is changing with time (Merritt 2004).

To model this process, and to compute the size of the gap in terms of a_h as we have defined it, we carried out a set of Monte-Carlo simulations. Scattering experiments were first carried out for an isolated, circular-orbit binary as a function of the parameters (R_p, V_p), the distance and velocity at closest approach to the binary that a test particle would have if the binary were replaced by a point of mass M_{12} at the binary's center of mass. The changes in energy and angular momentum, ΔE and ΔJ, experienced during the encounter were recorded and the calculation repeated for a large number ($\sim 10^6$) of different (R_p, V_p) values.

In the second step, a set of Monte-Carlo positions and velocities were generated for a set of stars drawn from an isotropic distribution function describing the stars in the unperturbed galaxy. The gravitational potential was fixed as $\Phi(r) = \Phi_\star(r) - GM_{12}/r$ with $\Phi_\star(r)$ the contribution to the potential from the stars. The stellar orbits were integrated forward in time, and stars were given kicks in energy and angular momentum whenever they passed within a distance $r_{crit}(t) = 3a(t)$ of the galaxy center. The kicks were drawn randomly from the stored values of ΔE and ΔJ; the star's orbit in the full potential was related to the Keplerian orbit in the scattering experiments by equating the pericenter distance r_p and velocity at pericenter v_p of the orbit about the galaxy's center with R_p and V_p. Immediately after a kick, a star was
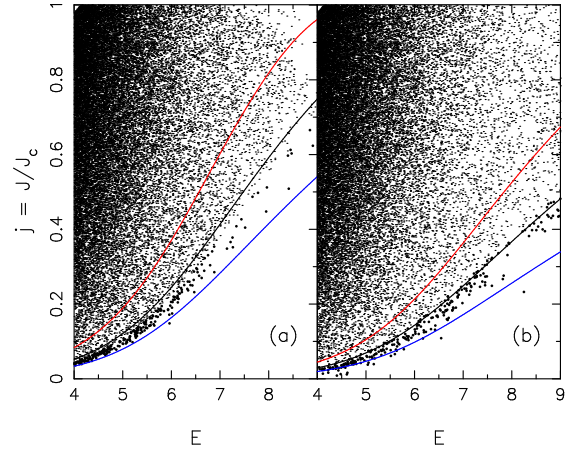

FIG. 1.— Lindblad diagram of stars at the final time step in the Monte-Carlo experiments, with $q = 1$ (a) and $q = 1/8$ (b). Additional parameters are given in the text. Larger dots are stars that are still on orbits that intersect the binary. Lines show j_{gap} for $K = 0.5$ (blue), 1 (black) and 2 (red).

placed on an outgoing orbit on the sphere $r = r_{crit}$ with a velocity computed using its new values of E and J. At the same time, the binary's energy was reduced by $m_\star \Delta E$ and its semi-major axis correspondingly changed. Possible changes in the eccentricity of the binary's orbit were ignored. Stars with initial apocenters within $a(0)$ were flagged and removed; in a real galaxy, such stars would have been ejected during the binary's formation. The stellar potential was taken to be that of a Dehnen (1993) model; initial positions and velocities of the stars were generated from the unique isotropic distribution function $f(E)$ that reproduces the Dehnen mass distribution in the combined potential of the stars and the central point mass (Tremaine et al. 1994). The parameters of the Monte-Carlo integrations were q, M_{12}/M_{gal}, $a(0)$, and γ, the central logarithmic density slope of the Dehnen model.

Figure 1 illustrates the gap created in two integrations with $\gamma = 2$ and $M_{12}/M_{gal} = 0.001$, and with $q = (1, 1/8)$. The plots show the phase-space distribution at a time of ~ 4 in units such that $G = M_{gal} = r_D = 1$ with r_D the Dehnen-model scale length. The larger circles are stars that are still interacting with the binary, i.e. stars with $r_p < r_{crit}$. Most of these stars will eventually be ejected but some will remain in the galaxy as the binary "shrinks away" beneath them. The three curves in Figure 1 are

$$J^2 = 2Ka_h[E - \Phi(Ka_h)] \equiv J_{gap}^2 \qquad (7)$$

with $K = (0.5, 1, 2)$; J_{gap} is the angular momentum of a star with pericenter Ka_h. There is a sharp drop in phase space density at $J \approx J_{gap}$, $K \approx 1$. This relation was found to hold also for other values of γ and q. The location of the gap was found to be only weakly dependent on the choice of $a(0)$. For the integrations of Figure 1, $a(0) \approx 2a_h$, and increasing $a(0)$ by a factor of 10 caused J_{gap} to increase by only $\sim 50\%$. We conclude that equation (7) with $K = 1$ is a good representation of the phase-space gap produced by a binary BH.

4. REFILLING THE GAP

In the Monte-Carlo experiments described above, the binary decay stalls after all the stars on orbits intersecting the binary have been ejected. We assume that some other mechanism, e.g. torques from gas in an accretion disk, then acts to extract angular momentum from the binary, eventually bring-

ing the two components close enough together that emission of gravitational radiation can induce complete coalescence.

The phase-space gap opened up by the binary will then gradually refill on the time scale T_{gap}. We modelled this evolution using the time-dependent Fokker-Planck equation. The following simplifying assumptions were made. (1) Changes in stellar *energy* due to encounters were ignored, i.e., we assumed that $T_{gap} \ll T_r$ (cf. equation 5). (2) Relevant stars are those with $R \equiv J^2/J_c(E)^2 \ll 1$, where $J_c(E)$ is the angular momentum of a circular orbit of energy E. Under this assumption, the low-R form of the velocity diffusion coefficient may be used. (3) The diffusion in J is slow compared with orbital periods so that the orbit-averaged form of the Fokker-Planck equation can be used. We justify this assumption below. Given these assumptions, the evolution of the stellar phase space density $f(J,t;E)$ obeys (Milosavljević & Meritt 2003)

$$\frac{\partial f}{\partial \tau} = \frac{1}{4j}\frac{\partial}{\partial j}\left(j\frac{\partial f}{\partial j}\right) \qquad (8)$$

where $j \equiv J/J_c(E) = R^{1/2}$ is a dimensionless angular momentum variable and $\tau \equiv \alpha(E)t$ is a dimensionless time, with $\alpha(E)$ the orbit-averaged diffusion coefficient at energy E:

$$\alpha(E) = \frac{1}{P(E)}\oint \frac{dr}{v_r}\lim_{R\to 0}\frac{\langle(\Delta R)^2\rangle}{2R}. \qquad (9)$$

and $P(E)$ the period of a radial orbit of energy E. Hence $\tau \approx t/T_r(E)$. Equation (8) can be integrated forward in time given boundary conditions at $j = 0$ and $j = 1$ and initial conditions $f(j,t=0;E)$. Figure 2 shows the result if $f(j,t=0)$ has the form

$$f = 0, \ \ j \le 0.2; \quad f = 1, \ \ 0.2 < j < 1. \qquad (10)$$

These are the initial conditions, at one energy, created by a binary BH with $j_{gap} = 0.2$ in an initially isotropic nucleus. The boundary conditions are

$$f(j \le j_{lc},t) = 0; \quad \frac{\partial f}{\partial j}\big|_{j=1} = 0 \qquad (11)$$

with j_{lc} the scaled angular momentum of a capture orbit; in Figure 2, $j_{lc} = 0.02$. The flux of stars into the BH is given by

$$\mathcal{F}(t;E) = -\frac{d}{dt}\left[\int_{j_{lc}}^1 N(j,t;E)dj^2\right] = \frac{\alpha}{2}\frac{\partial N}{\partial \log j}\big|_{j_{lc}} \qquad (12)$$

with $N(E,R,t)dEdR = 4\pi^2 P(E)J_c^2(E)f(j^2,t;E)dEdR$ the number of stars in the integral-space element $dEdR$.

The orbit-averaged assumption breaks down sufficiently far from the binary, in the "pinhole" regime (Lightman & Shapiro 1977). In the loss cone defined by a *single* BH, the breakdown occurs at binding energies less than E_{crit} where

$$q_{lc}(E) \equiv \frac{\alpha(E)P(E)}{R_{lc}(E)} > 1. \qquad (13)$$

For $E \lesssim E_{crit}$, stars can wander into and out of the loss cone in a single orbital period and the loss cone remains nearly full in spite of capture. Typically $r_{crit} \approx r_h$, the BH's influence radius (Frank & Rees 1976). In the case of the angular momentum gap created by a *binary* BH, the orbit-averaged assumption holds initially even for $r \gg r_{crit}$, since stars need to diffuse over the full angular momentum gap before they can be disrupted. Breakdown of the orbit-averaged approximation only occurs much farther from the center where

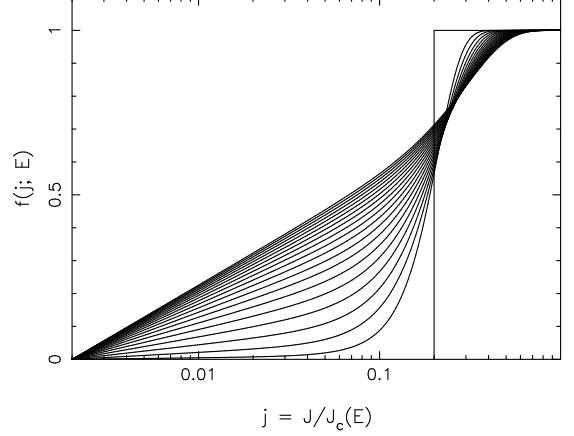$$q_{gap}(E) \equiv \frac{\alpha(E)P(E)}{R_{gap}(E)} > 1. \qquad (14)$$

FIG. 2.— Solution to equation (8) with the initial and boundary conditions given in the text, at dimensionless times $\tau = 0, 1\times 10^{-3}, 2\times 10^{-3}, ..., 4\times 10^{-2}$ where $\tau = \mu(E)t \approx t/T_r$.

Cohn & Kulsrud (1978) showed that the steady-state form of f near the loss cone was $f \sim \ln(j/j_0)$ and derived an expression for $j_0 = j_0(E)$; beyond r_{crit}, $j_0 \approx 0$, i.e. the loss cone is nearly full. We adopted the same expression for j_0 here and set $f = 0, j \le j_0$ at all times. While not strictly correct in the case of a time-dependent loss cone, this boundary condition gives the correct result as $t \to \infty$, and the error in the inferred loss-cone flux at earlier times should be very small since so little of the galaxy is in the full-loss-cone immediately after the binary forms.

We applied this prescription to compute the evolution of f and $\mathcal{F}$ in the "core" galaxies from Paper I. For each galaxy, the function $\alpha(E)$ was computed on a grid of energies and stored. Equation (8) was then integrated forward for the appropriate dimensionless time at each E value and the flux computed from equation (12). We used the same values for the BH mass M and for r_t as in Paper I (only the "$M - \sigma$" values for M from that paper were used.) We tried three different values for the mass ratio of the binary that was assumed to have formed the gap: $q \equiv M_2/M_1 = (0.1, 0.3, 1)$, and j_{gap} was computed from equation (7) for various values of K; following the discussion above, we focus here on the results obtained with $K = 1$. At the end of the integration, the galaxy was assumed to be in its currently-observed state, for which we know $\overline{f}(E)$, the R-averaged distribution function. The correctness of the integrations was tested by verifying that the asymptotic flux was equal to the steady-state values computed in Paper I.

5. RESULTS

Figure 3 shows the energy-dependent evolution of the loss-cone flux in one of the sample galaxies, NGC 4168, for $K = 1$ and $q = 0.1$. Stars with binding energies near $\Phi(r_h)$ are the first to be scattered into the BH, followed by stars with energies slightly above or below E_h. The fact that very little of the flux comes from stars with energies very different from E_h validates our approximate treatment of the loss-cone boundary condition in the full-loss-cone regime. The total flux reaches $(1\%, 10\%, 50\%, 90\%)$ of its steady-state value in a time of $\sim 4.5\times 10^9, 9.8\times 10^9, 1.7\times 10^{10}, 9.7\times 10^{10}$ yr.

We defined two characteristic times associated with loss-cone refilling. t_0 is the elapsed time before a single star would
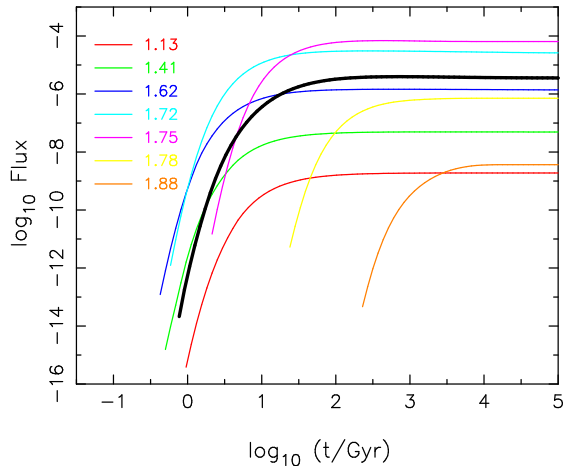
FIG. 3.— Dependence of the loss-cone flux on energy and time in the galaxy NGC 4168 assuming a binary mass ratio of $q = 0.1$. Curves are labelled by their binding energy; the value of the gravitational potential at $r = r_h$ in this galaxy is 1.76. Thick black curve is the total flux, in units of stars per year.
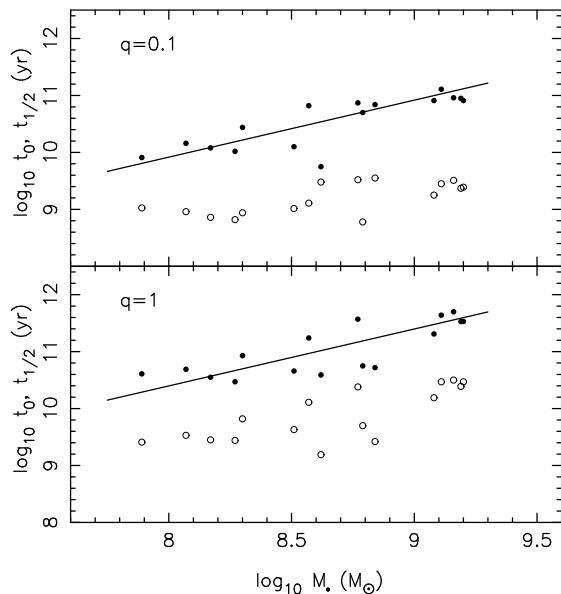

FIG. 4.— Two characteristic times associated with loss-cone refilling in the sample galaxies. t_0 is the elapsed time before the first star is captured and $t_{1/2}$ is the time for the loss-cone flux to reach $1/2$ of its steady-state value. Solid lines are the approximate fitting function, equation (15).

be scattered into the loss cone. Obviously, the value of t_0 might depend rather sensitively on the choice of initial conditions. A more robust measure of the refilling time is $t_{1/2}$, the elapsed time before the loss-cone flux reaches $1/2$ of its

steady-state value. Figure 4 shows values of t_0 and $t_{1/2}$ computed for our sample of galaxies, using $K = 1$ and $q = (0.1, 1)$. t_0 is of order 10^9 yr or greater for all galaxies and for both values of q, and $t_{1/2}$ is roughly an order of magnitude greater. The relation:

$$\frac{t_{1/2}}{10^{11}\text{yr}} \approx C \frac{\mu}{10^8 M_\odot} \approx C \frac{q}{(1+q)^2} \frac{M_\bullet}{10^8 M_\odot} \quad (15)$$

fits the data in Figure 4 tolerably well with $C = 1$.

6. DISCUSSION

The flux of stars into the tidal disruption loss cone of a BH that formed via binary coalescence can be much lower than would be predicted under the steady-state assumption (Syer & Ulmer 1999; Magorrian & Tremaine 1999; Wang & Merritt 2004). Time scales for loss-cone refilling in the nuclei of bright elliptical galaxies are of order 10^{10} yr or longer and scale roughly linearly with BH mass for a fixed (assumed) mass ratio of the binary that preceded the current, single BH. Even if the galaxy merger occurred as long as $\sim 5 \times 10^9$ yr ago, the flux of stars into the BH's tidal disruption sphere could still be less than $\sim 1\%$ of the steady-state value. BH feeding rates in bright elliptical galaxies were already expected to be lower ($\lesssim 10^{-5}$ stars yr^{-1}; Paper I) than in less luminous galaxies, due primarily to their low central densities. Based on the arguments in this paper, stellar disruption rates in these galaxies might be much lower still, making the nuclei of bright ellipticals unlikely sites for observing a tidal flaring event. Event rates in fainter galaxies, on the other hand, should be closer to the steady-state values computed in Paper I since these galaxies are less likely to have experienced a merger since the epoch at which the stellar cusp was formed, and since relaxtion times in their dense nuclei are probably relatively short (Lauer et al. 1998).

We have investigated an extreme, but physically plausible, model in which the region around a newly-coalesced BH was emptied of stars out to radii far greater than the radius of the tidal disruption sphere. But even in less extreme nuclear formation models, the long relaxation times in stellar nuclei imply a slow approach to a steady-state distribution of stars near the loss cone and hence a loss cone flux that can be very different than the value computed via the standard, steady-state theory. Our results highlight the need for a fully time-dependent theory of loss cone evolution that can be applied to systems, like galactic nuclei, that are much less than one relaxation time old.

REFERENCES

Bahcall, J. N. & Wolf, R. A. 1976, ApJ, 209, 214
Begelman, M. C., Blandford, R. D. & Rees, M. J. 1980, Nature, 287, 307
Cohn, H., & Kulsrud, R. M. 1978, ApJ, 226, 1087
Dehnen, W. 1993, MNRAS, 265, 250
Faber, S. M. et al. 1997, AJ, 114, 1771
Ferrarese, L., & Merritt, D. 2000, ApJ, 539, L9
Frank, J., & Rees, M. J. 1976, MNRAS, 176, 633 (FR)
Genzel, R. et al. 2003, ApJ, 594, 812
Hills, J. G. 1975, Nature, 254, 295
Lauer, T. et al. 1998, AJ, 116, 2263
Lightman, A. P. & Shapiro, S. L. 1977, ApJ, 211, 244

Magorrian, J., & Tremaine, S. 1999, MNRAS, 309, 447
Merritt, D. 2004, "Single and Binary Black Holes and their Influence on Nuclear Structure," in Carnegie Observatories Astrophysics Series, Vol. 1: Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Cambridge: Cambridge Univ. Press)
Milosavljevic, M. & Merritt, D. 2003, ApJ, 596, 860
Rest, A. et al. 2001, AJ, 121, 2431
Saslaw, W. C., Valtonen, M. J., & Aarseth, S. J. 1974, ApJ, 190, 253
Syer D., & Ulmer A. 1999, MNRAS, 306, 35
Tremaine, S. et al. 1994, AJ, 107, 634
Wang, J. & Merritt, D. 2004, ApJ, 600, 149 (Paper I)

Young, P. J. 1977, ApJ, 215, 36
Young, P. J., Shields, G. R., & Wheeler, J. C. 1977, ApJ, 212, 367
Zhao, H., Haehnelt, M. G., & Rees, M. J. 2002, Nature, 7, 385