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A Method for Determining Optimal Tenant Mix (Including Location) in Shopping Centers

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Abstract

With assumptions like maximally productive lease structure, equilibrium space allocation, a just-saturated retail market, and zero vacancies, store location would be the remaining variable in obtaining "optimal tenant mix." Here we illustrate an optimization program to maximize tenant location within shopping malls taking into consideration the two complementary effects present in malls: bid rent theory and revised central place theory. Suggested distance and connection matrices for the quadratic assignment program (QAP) solving for an optimal tenant mix are demonstrated.

A Method for Determining Optimal Tenant Mix (Including Location) in Shopping Centers

by Charles C. Carter and Marcus T. Allen

Ith assumptions like maximally productive lease structure, equilibrium space allocation, a just-saturated retail market, and zero vacancies, store location would be the remaining variable in obtaining "optimal tenant mix." Here we illustrate an optimization program to maximize tenant location within shopping malls taking into consideration the two complementary effects present in malls: bid rent theory and revised central place theory. Suggested distance and connection matrices for the quadratic assignment program (QAP) solving for an optimal tenant mix are demonstrated.

Introduction

Research has been done to explore the determinants of store/tenant location within shopping centers, and the retail industry knows a good deal about where types and sizes of stores should best be located. However, there is a prevailing belief among real estate professionals and academics that an "ultimate tenant mix" (including tenants' locations) is discoverable, which should prove very useful to shopping center management in deciding which tenants should be included in a mall and in which locations.[1] However, no exact formula exists, and research continues on the basis of adding to the knowledge of optimal store location and tenant mix.[2] Recent research has made further advances towards solving the problem of ultimate locations for stores, and the "ultimate tenant mix" is another step closer to being discovered.

Research on location was made possible after basic knowledge was found on mechanisms affecting stores in malls. The academic findings generally parallel professional knowledge about the nature of store attributes and location.[3] These findings were learned mostly during the 1990s, all under the rubric of "internalizing externalities." It is the inter-store externalities that determine allocation of space, agency relationships, rent, location, and other characteristics of stores in shopping malls. Benjamin, Boyle, and Sirmans (1990, 1992) led the way with their seminal papers on how retail leases differed from other commercial leases. Rents charged to retail tenants were found to be partly based on tenant characteristics, such as default probability and customer-generating capability.

From there, research followed that showed:
1. Stores ability to generate customer traffic varies, and this ability ranges by store type. Rent subsidies go to those that produce these externalities while rent premiums are paid by those that "free ride." The per-foot subsidies and premiums have been measured (Pashigan & Gould (1998)).
2. Other factors held constant, higher rents are charged by larger, newer shopping centers, and those with more national chain stores (as opposed to stores generated

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[1] See, e.g., Marlow (1992) and Stambaugh (1978).

[2] Some recent findings have been made by Des Rosiers, et al. (2009), that hold promise to better sort out the question of mix, on the matter of the concentration of store types.

[3] For instance, Bean, et al. (1988), at 2 report that Homart Development Company (a shopping center owner and developer) normally negoti-ates low lease rental rates with their anchor stores and that their "profits are made primarily from the rent paid by the non-anchor tenants."

locally). Higher rents are also charged by centers with less vacant space and centers with greater market area purchasing power (Sirmans & Guidry (1993); Gatzlaff, et al. (1994)).[4]

3. Significant vacancy increases, especially among anchor tenants, cause rents in centers to fall substantially after a period of time (Gatzlaff, et al. (1994)).

4. Store space is allocated to stores to the point where net marginal revenue is equal to the marginal cost of space, less an externality term, all adjusted by each store's elasticity of demand (Brueckner (1993)).

5. "Common agency" tells us that stores' leases should be a combination of fixed rent (called "base rent") and percentage rent[5] and that a lease cancellation clause should be included (Miceli and Sirmans (1995)).

6. Fixed rent varies inversely with sales externalities generated by stores, and percentage rent varies positively with fixed rent (and sales externalities)(Wheaton (2000)).

In addition to learning about lease and rent structuring, the following has been learned about store location:

1. The highest customer traffic takes place at the mall's center (usually at or near the food court), and customer traffic tapers off with distance from the center (Carter & Vandell (2005)).

2. Store size increases, rent per square foot decreases, and revenue earned per square foot decreases, in accordance with distance from the mall's center (Carter & Vandell (2005)).

3. Stores of the same type that promote comparative shopping will generally be dispersed as opposed to clustered (Carter & Haloupek (2002)).[6]

4. Concentration of store types, whether promoting comparison shopping or not, tends to lower rents (and lower revenues) after some point (Des Rosiers, et al. (2009); Eppli & Shilling (1995)).

This last finding merits some elucidation in the context of this paper. It's fairly simple to see that too many of one store type (e.g., men's shoes) will have a negative effect on mall economics since it will create a ultra-competitive environment that spreads a limited number of sales across a large number of stores. Fewer numbers of this store type will provide a comparative, not ultra-competitive, environment. The location research mentioned earlier states that some dispersion, rather than clustering of same-type stores, works best. Therefore, number and separation distance are one part of the location problem that needs to be estimated, whose solution would add to knowledge of shopping centers. Finding the distance between stores of the same type would assist in determining the number of each store type, since there is a limited amount of space in a mall and only so many stores of a single type that could fit into that space.

Use of this information, whose parameters can be set out accurately, would lead to a reasonable answer to the question of the "ultimate tenant mix" (including locations) for a standard regional shopping center. The question then becomes how to best use these rules as inputs to form an optimal result (see Section 3). Before forming an optimal result it would be judicious to first discuss in some detail other characteristics of store location in malls.

Author

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[4] Market area purchasing power is a function of the disposable income of the population residing within the center's primary market area (often expressed as a several-mile radius around the center).

[5] Percentage rent is extra rent charged by a shopping center based on a percentage of a store's gross revenue generated over some breakpoint amount. For instance, three percent of gross sales over $500,000 for the year.

[6] Theory on this economic process called "revised central place theory" can be found in the works of McLafferty & Ghosh (1986) and Ghosh (1990)

A Closer Look at Tenant Location in Malls

The problems of store location and mix are not subjective. They should not be discussed in terms of how "more" or "less" of some factor is better or worse for overall revenues or profits of stores or for overall revenues or profits of mall owners. The assumption has always been implied that all of the matters at work in shopping centers create a Pareto optimal equilibrium.[7] As such, any other location than that defined by the equilibrium solution would mean less revenue (profits) for some store in the mall.

Looking at the Problem in Terms of Bulk Store Types

A recent article by Des Rosiers, et. al (2009)[8] uses the Herfindahl index to measure the intra-category retail concentration of 31 categories of stores located in eleven Canadian shopping centers (six in Montreal and five in Quebec). Not surprisingly they found that intra-category retail concentration could affect base rent negatively "as a result of increased bargaining power enjoyed by dominant tenants." However, this effect ranged from low to high among the categories of non-anchor stores. Their finding that base rent follows percentage rents, ceteris paribus, confirms a finding by Pashigan & Gould (1998) as well as Chun (1996). Overall, intra-category retail concentration lowered rents. Another finding, new to the literature, was that higher order stores paid higher base rent than lower order stores,[9] ceteris paribus.

The Heifendahl index measures the concentration of production in an industry in terms of a retail unit's gross leaseable area (GLA), measured from 0 (absence of a store in a retail category) to 1 (all retail activity put out by a single tenant). Each retail category is deemed a market for which store(s) hold a market share, and is more commonly used in economics as a measure of competition among firms in a market. The resulting index is proportional to the average market share, weighted by market share, ranging from 0 to 1.0; that is, ranging from a large number of small firms to a single monopolistic producer. The stores (tenants) here represent all competitors in the "market" and what is understood as defining a "market" is of great importance.

In terms of the overview of shopping center knowledge given above, high concentrations of store categories that result in lower base rents would represent too much GLA for the store type. Separation or dispersion of same-type stores would mean that at some point there would be a limit to the number of stores (or GLA) of that store type in a mall. At some point, the amount of GLA of a store type would be suboptimal.

In working papers by Eppli & Shilling (1993 & 1995) the authors divide the stores into anchor stores and non-anchor (mall) stores. Attempts are then made to separately measure the customer drawing power or revenue produced by each. Additionally, the papers distinguish between revenue generation made by anchor stores, called retail mass agglomeration, and revenue made by non-anchor stores, called retail merchandise type attraction. The authors conclude that more anchor store GLA and more non-anchor store outlets in shopping centers both result in higher revenue for the center. These results are explainable by the simple finding that larger regional and super-regional shopping centers perform better than smaller centers, ceteris paribus (Guidry and Sirmans (1993)). But they do pose an interesting alternative to the Des Rosiers, et al. (2009) model with regard to handling the issue of markets.

Significant parts of these two papers consist of the empirical attempt to estimate the market expansion potential of store categories in a market area extending beyond the

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[7]   Existence of Nash-Courtney equilibrium should follow (Yu (2002)).

[8]   Previous working papers along the same lines are Des Rosiers, et al. (2004) and Shun-Te Yuo, et al. (2003).

[9]   In the literature "high" order stores are those selling usually pricier goods for which customers compare shop; "low" order stores are those selling less expensive goods, those not requiring comparison shopping.

shopping centers. It is implicit that one category of stores in the center does not make up the market. Consequently, much of the analysis using the Herfindahl Index by Des Rosiers, et al. (2009) would not be possible because it assumes market saturation (not over-or under-saturation) for all store categories.

The Way to Look at Store Location and Mix

We will make the assumption that the market areas for the stores of the regional or super regional shopping center are saturated. This means that there is little if any room for growth (or size reduction) for stores of any type, total gross leaseable area (GLA) being in-line with population income and spending. Another assumption is that generally Pareto optimal solutions can be obtained by reviewing characteristics of store types, sizes, and locations. That is, shopping center operations over many years have realized near optimization as they have attempted to maximize value in their centers.

Optimum store mix may be a somewhat ideal concept. Store locations and sizes will go through iterations over time - that have more to do with what management thinks are good results as opposed to what management thinks may be the best results at the time. The perfect outcome may never be realized but best tendencies should be always recognizable. With this in mind we use inputs from research on store location to locate "optimum tenant mix."

Optimal Tenant Mix

A Method for Determining Optimal Store Location

One way to determine the "best" location of stores in shopping centers would be to treat the matter as an optimization problem. In fact, this method has already been tried in management science literature, where the authors seem to be much more interested in the algorithm than with the inputs and results. In "Selecting Tenants in a Shopping Mall" (Bean, et al. (1988)), the authors create a nonlinear integer program that solves for the optimum of tenant mix and locations, solving for the number of tenant types (20 store types) with those types' sizes (three size classes) and three location classes (side aisle leading to a parking lot, main aisle between anchors, and another class). An objective function optimizes present worth of the mall, which is made up of the present values of the stores in the mall. The exercise is premised on the idea that rents of the mall tenants (both base rent and percentage rent) are the primary factor in the value of a regional or super-regional mall.[10]

This exercise seems straightforward, except, of course, for the mathematics. After some reflection the reader should question what the inputs of this model really consist of. The authors make no assumptions as to why or where certain stores should be located nor what sizes these certain tenants should be in any given location. They are given them by Homart's Market Research Group. On pages 4 and 5 the authors say:

" … Homart's Market Research Group can estimate a store's sales over the study horizon given its type, size, location class, and the number of stores of its type in the mall. These revenue figures, together with rental rates depending on the store's characteristics, determine the rental income Homart will receive. Then a present worth coefficient, PW_{ijkl} , is calculated to be the total contribution to present worth of a store of type i, location class j, and size k if it is one of l stores of type i in the mall."

The why's and wherefore's of the inputs are evidently what the Market Research Group found as the best historical rents received for each tenant (store), taking into consideration type, size, location class, and number of stores in the malls. The malls' sizes and shapes are

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[10]   Pashigan and Gould (1998).

not considered, nor are consumers' characteristics or market areas, nor distances from the malls' center or store type dispersion.[11] Consequently, their results are not at all based on theory and their variables may have been important or not important and the ways they were employed may have been well or poorly done.

The authors confess to improving on earlier approaches to solving the tenant mix problem, and gained recognition from Homart Development Company that their results did indeed do a better job than the company had accomplished in the past. This amounts to telling Homart where to put stores of various types and sizes in a mall based on the past sales of different types, sizes, and general locations of stores. Not much is said about what is going on regarding the effects of the stores on each other. The authors do state that their data shows an optimum number of stores of each type exist. They base this on the fact that as the number of stores of each type grows, returns for each store initially grows, but at some point marginal gains decrease and then grow negative as stores begin to compete for the same share of the market.

These authors use twenty different store types, the average number of stores of each type, with three different general mall locations, and three sizes (within the store type's range) of each store type. Their solution includes the number of each store type, with its respective location, and sizes within a hypothetical mall that bring the highest value to the developer. They compare to the mix and locations in actual malls with rents received. Nothing is reported to describe or explain spatial store patterns.

Inputs from Tenant (Store) Location Research

The Data

For our study, tenant, lease, and location data on mall stores from regional and non-regional centers were supplied by two (2) sources that required confidentially. The database consists of 1,012 stores (tenants) doing business during 1991 and 1992 from nine (9) centers located throughout the U. S. All centers are enclosed, of contemporary design, comparable in amenities and occupancy (nearly 100%), making the data set ideal for an optimization exercise.

Shopping centers differed in size from 613,400 square feet to 1,004,400 square feet (819,650 square feet on average). The mall area of the shopping centers (non-anchor stores) differed in sizes from 283,600 to 403,700 square feet (347,033 square feet on average). Six (6) centers were single level, two had two levels, and one had three levels. For our purposes we used data from the six single-level malls.[12] Malls' centers were discerned and stores' distances from the center of each mall were measured and normalized so as to be comparable between malls. Explanations of the data gathering and processing are described in Carter (1999), Carter and Vandell (2005), and Carter and Haloupek (2002).

Descriptive statistics for this sample are set out in Tables 1, 2, 3, and 4.[13] Both sales and rents per square foot for stores generally and for specific store types tend to decrease at a decreasing rate with increased square footage. Average annual non-anchor sales for all nine malls were $361.44 per square foot. Average distances from the center of the mall (normalized) and average sales and rents per square foot are also detailed. For individual malls, average non anchor sales per square foot ranged from $266 to $435. Rents varied similarly. Differences in revenues and rents varied according to income per capita in the

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[11] Nothing beyond the facts that some were new malls and some were relatively new malls was given.

[12] We feel the older, single level mall data give us a less complicated, more basic view of how shopping centers operate. Data from this period would not be in competition with "big box" retailers nor would they reflect more modern nuisances of today's various mall themes.

[13] Distances to malls' centers (in feet) were normalized so as to be comparable across malls, as follows: square feet = (square feet/square feet of mall area) x 1000.

shopping center's market area, e.g., downtown San Francisco (highest) versus suburban Memphis (lowest). Vacancy did not vary a great deal, averaging 3.1%, and none of the vacancy levels were excessively high. Local tenants (businesses originating nearby) made up 37% of the database, while national chains (businesses having stores located in at least a couple of states and a common name) made up 63% of the database.

Our data divide store types into eleven categories, including fast food (food-court). For our hypothetical example, we have chosen a standard "I" shape, small-sized, single level mall with two separated anchor stores, much like the hypothetical regional shopping center set out in *Dollars and Cents of Shopping Centers* (Figure 1) and like the malls' designs from our database (see Carter and Haloupek (2002), Figure 4). Since fast food goes in the middle, that leaves ten store types to distribute in the mall. Since we feel location is a more important variable, the mall is divided into ten (10) sections instead of just three, based primarily on the relative distance to the center of the mall.

Spatially, fast food stores makes up about four percent of store space, but tables, chairs, trash receptacles, and so forth take up sizable space in the food court. Fast food is left in the center of the hypothetical shopping center while the remaining ninety six percent is left as space for the other ten store types. Taking the weighted average space using the descriptive statistics above, the rest of the stores take up the following percent of mall space shown below in Table 4. Numbers of stores of the various store types are also shown in Table 4 for a model mall.

Variables	Mean	Standard Deviation	Minimum	Maximum
Square Feet (SF)	2,394.99	2,233.14	120	27,000
Sales ($/SF)	$361.44	$217.12	$33.00	$1,632.00
Total Rent ($/SF)	$36.64	$25.71	$5.83	$277.00

Table 1

Store Characteristics

Store Type	Mean Rent per SF	Rank	Mean Size in SF (Standard Deviation)	Mean Distance to Center in feet (Normalized)	Rank Distances (Normalized)
All Stores	$37.75	-	2,417.06	295.3 (517.8)	-
(1) Jewelry	$63.58	1	1,239.57 (679.90)	281.6 (485.4)	3 (3)
(2) Cards and Gifts	$30.00	10	2,080.24 (1,219.88)	347.1 (596.7)	9 (9)
(3) Women's Apparel	$30.00	11	3,906.92 (2,485.24)	312.7 (519.8)	5 (5)
(4) Fast Food	$59.05	2	874.69 (859.13)	70.5 (115.4)	1 (1)
(5) Family Apparel	$36.97	4	3,053.07 (1,861.71)	330.2 (547.9)	6 (6)
(6) Men's Apparel	$32.97	6	2,384.16 (1,575.56)	336.0 (556.6)	7 (7)
(7) Leisure & Entertainment	$33.85	5	2,651.53 (3,107.16)	405.1 (688.7)	11 (11)
(8) Home Furnishings	$31.40	7	2,666.49 (3,857.35)	326.7 (571.4)	8 (8)
(9) Men's & Boy's Shoes	$30.43	9	2,397.19 (1,173.98)	396.6 (670.4)	10 (10)
(10) Women's Shoes	$30.95	8	1,733.78 (993.78)	273.0 (453.7)	2 (2)
(11) Specialty Food	$43.00	3	1,272.51 (1,033.13)	294.4 (489.8)	4 (4)

Table 2

Store Characteristics

(11 store types)

Table 2 (continued)

Store Characteristics
(11 store types)

Store Type	Total Rent		Size	
	Mean	St. Deviation	Mean	St. Deviation
All Stores	$37.75/SF	$21.52/SF	2,417.06 SF	
(1) Jewelry	$60.00/SF	$27.00/SF	1,300.72 SF	647.53 SF
(2) Cards and Gifts	$30.95/SF	$12.40/SF	2,088.25 SF	1,172.81 SF
(3) Women's Apparel	$28.25/SF	$16.83/SF	3,906.92 SF	2,475.24 SF
(4) Fast Food				
(5) Family Apparel	$36.97/SF	$20.70/SF	3,053.07 SF	1,861.71 SF
(6) Men's Apparel	$32.97/SF	$15.95/SF	2,384.16 SF	1,575.56 SF
(7) Leisure & Entertainment	$33.85/SF	$17.29/SF	2,651.53 SF	3,107.16 SF
(8) Home Furnishings	$31.40/SF	$15.17/SF	2,666.49 SF	3,857.35 SF
(9) Men's & Boy's Shoes	$30.43/SF	$12.20/SF	2,397.19 SF	1,173.98 SF
(10) Women's Shoes	$31.95/SF	$9.60/SF	1,733.78 SF	993.58 SF
(11) Specialty Food	$43.00/SF	$19.12/SF	1,749.80 SF	1,002.88 SF

Table 3

Characteristics by
Selected Store Types

Variable	Store Type	Observation Frequency	Mean	Standard Deviation	Minimum	Maximum
SF	Gifts	6.5%	2,088.25	1,172.81	665	5,475
SF	Women's Shoes	5.0%	1,733.98	993.78	1,000	6,437
SF	Women's Apparel	18.0%	3,906.92	2,475.24	569	13,915
SF	Jewelry	6.2%	1,300.72	647.53	472	4,278

Variable	Store Type	Metric	Mean	Standard Deviation	Minimum	Maximum
Sales	Gifts	$/SF	$298.82	$123.18	$136	$711
Sales	Women's Shoes	$/SF	$303.53	$116.99	$84	$566
Sales	Women's Apparel	$/SF	$258.18	$112.85	$90	$751
Sales	Jewelry	$/SF	$676.48	$276.02	$213	$1,433

Variable	Store Type	Metric	Mean	Standard Deviation	Minimum	Maximum
TRNT	Gifts	$/SF	$30.95	$12.14	$16	$64
TRNT	Women's Shoes	$/SF	$31.95	$9.60	$12	$51
TRNT	Women's Apparel	$/SF	$28.25	$16.83	$5.83	$77
TRNT	Jewelry	$/SF	$59.82	$27.11	$17.31	$127

Table 4

Percentage Store Types and
Steepness of Bid-Rent Curves

Store Type	Percent of Mall	# of Stores of Same Size	Total Stores SF	Response Curve at Center	Degree Slope (1=greatest)
Fast Food	3.98%	11	9.621.59	$64.37	2
Women's Apparel	29.50%	18.5	71,331.19	$29.81	11
Leisure & Entertainment	24.10%	22	58,333.66	$46.71	5
Family Apparel	8.80%	7	21,371.49	$37.58	6
Home Furnishings	6.80%	6.2	16,532.24	$36.50	9
Men's & Boy's Shoes	6.60%	6.69	16,061.17	$36.50	10
Men's Apparel	6.2%	6.3	15,020.21	$49.07	4
Cards & Gifts	5.6%	6.5	13,521.56	$37.59	8
Jewelry	5.6%	6.2	7,685.33	$75.18	1
Women's Shoes	3.6%	5	8,668.90	$40.99	7
Specialty Food	3.3%	6.2	7,889.56	$51.14	3
Total	**100%**	**100**	**241,706 + anchor store areas**		

Bid-rent Effect on Store Location

Store type locations are figured using results found in Carter (1999) and Carter and Vandell (2005) on how store types line up in terms of the center of the mall, on the basis of a revised bid-rent theory and estimates. Those studies showed that a store type's rents tapered off at different rates as they located further from the malls' centers. In our study we calculated the relative losses of rent per square foot for the different store types, and determined where stores would range along the stretch from the mall's center to the mall's end, where the mall meets an anchor store. Standard deviations of store type rents were

Figure 1

25 stores, rectangular from top to bottom, increasing in size from right to left
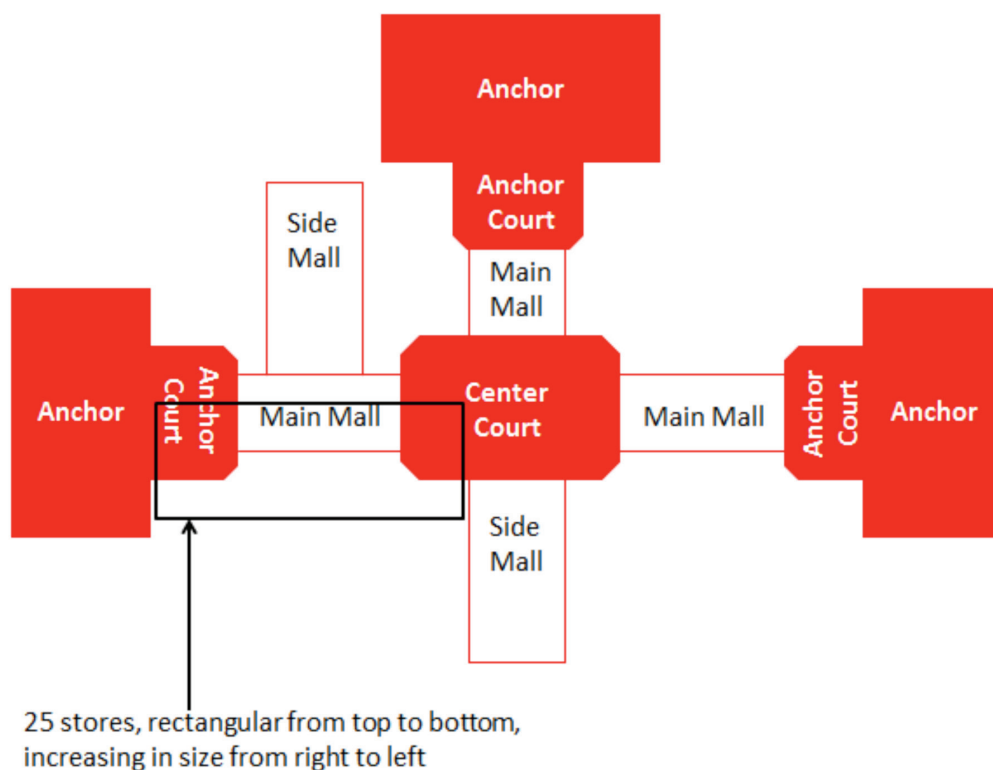
also considered in determining maximum and minimum limits for tenant types and rent per square foot in the optimization program set out below. A model "I" shaped, small regional shopping center houses 100 stores of ten different store types (the food court makes eleven store types), making up 241,706 square feet (excluding anchor store area).[14]

As shown in Table 4, jewelry stores have the steepest bid rent curve and, consequently, locate nearest the center of the mall, followed by fast food (food-court), [15] specialty food, men's apparel, and the rest. The least steep bid-rent curve was for women's apparel, whose average store size was the largest of the store types. Both expansions of the mall area from the mall center would be identical for the model "I" shaped shopping center, since store locations and store characteristics, store types and sizes, are meant to be identical on both sides.

Revised Central Place Theory Effect on Store Location

The second factor affecting store location is the dispersion of same-type, comparison shopping stores as demonstrated in Carter and Haloupek (2002). In addition to the bid rent factor there is the tendency for stores that promote comparison shopping to form a cluster.[16] The cluster is not one where stores of the same type all locate next to each other. Rather, they locate near enough to each other that the average shopper can make the most of the average shopping trip (an average trip consists of three store visits).[17] This means locations that make for convenient trips consist of about two store visits for single purpose shopping and about four store visits for multi-purpose shopping. The result for stores of the same type is that they locate on both sides of the malls' center and not usually next to each other.

[14] A limitation on data may include that only the distance of stores from the center are normalized and can be readily compared from one shopping center to another. Other data, store size, rents per square feet, and sales per square feet, are from their respective shopping centers and will not be readily comparable for several reasons, including size of the shopping center and income differences of the populace making up the trade areas. See, e.g., Guidry and Sirmans. It should be noted, however, that these limitations did not prevent very useful analysis, e.g., Pashigan and Gould (1998).

[15] This was the result of many of the food-courts not being located in exactly the center of the malls from which the data was taken.

[16] The main article on this theory taken from the literature was Ingene and Ghosh (1990), though other articles preceded that one (see, e.g., Eaton and Lipsey (1979) and De Palma, et al. (1985).

[17] Three store visits per trip is the average as found by Stillerman, Jones & Co. (1994), National Benchmarks, consumer research. See Carter and Haloupek (2002).

In Carter and Haloupek (2002) the comparison shopping stores were women's apparel, men's apparel, men's and boy's shoes, and women's shoes. To this list we add family apparel, cards and gifts, and jewelry stores, leaving fast food (food-court), home furnishings, and specialty food. Fast food (food-court), of course, makes up a class of its own, while home furnishing stores arguably sell "high-order" or shopping goods, not comparison goods. Though specialty food (e.g., Cinnabon, Aunt Anne's Pretzels) is hard to categorize, it doesn't seem to fit the category of comparison shopping goods used here.

Quadratic Assignment Location (QAP)

The problem of location optimization most simply can be viewed as a quadratic assignment location problem with a linear objective function.[18] The objective function maximizes the landlord's/developer's rents. Maximizing rents is largely a matter of maximizing mall stores' (tenants') rents, since anchor stores receive the rent subsidies and the landlords' (developers') profits come entirely from the mall area.[19]

Without fast food (food-court), the ten stores' space breaks down into the following square feet per store category and store numbers:

1) 71,331 SF women's apparel (WA) (18.5 stores)
2) 58,334 SF leisure and entertainment (LE) (22 stores)
3) 21,371 SF family apparel (FA) (7 stores)
4) 16,532 SF home furnishings (HF) (6.2 stores)
5) 16,061 SF men's & boy's shoes (MS) (6.7 stores)
6) 15,020.42 SF men's apparel (MA) (6.3 stores)
7) 13,522 SF cards & gifts (CG) (6.5 stores)
8) 7,685 SF jewelry (J) (6.2 stores)
9) 8,669 SF women's shoes (WS) (5 stores)
10) 7,890 SF specialty food (SF) (6.2 stores)

The store categories will line up, from closest to furthest from the food-court (furthest to closest to each of the anchors) in both directions of the mall. Their order in both directions follows the steepness of their store types' bid-rent curves as set out in Table 4. With two anchor stores, one at each end of the mall, the linear mall area is about 720 feet long and the aisles are 30 feet wide. Stores are rectangular, about 130- 150 feet deep and about 30 feet wide. For instance, a large store making up 4,500 square feet might be 150' x 30', while a smaller store of 2,700 square feet might be 135' by 20.'[20] Total square footage of store space is about 250,000 square feet.

The model mall can most simply be viewed as four banks of stores, one on each side of the aisles with two aisles radiating out in opposite directions from the mall center (food-court). In this way the assumption is that the optimized bank of stores' dimensions will be quadrupled, and these four banks of stores will make up the stores in the mall. The single bank of stores is as follows: WA WA WA WA MS MS HF HF CG CG WS WS WS FA FA LE LE LE LE LE MA MA SF SF J J (25 stores total; see abbreviations above). To this sequence is added a dispersion factor so that store types (except for home furnishings and specialty foods) disperse, so the sequence becomes: WA WA WA MS WA MS HF HF CG WS CG WS FA LE FA LE LE LE MA LE MA SF J SF J.[21]

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[18] QAPs can be converted to linear integer programs. See Schrage (1997), p 327.

[19] See, e.g., Gould and Pashigan (2005).

[20] Side aisles in actual malls help to balance out the differences between large, rectangular stores and smaller stores whose lengths are not so deep.

[21] Note there are slight discrepancies in sequences of about half of the eleven store types, between the order of mean rents by store types and the order in which store type rents would be at the center of the mall (see "response curve at center" above). The latter denotes the bid rent curves for store types as found in Carter and Vandell (2005). We are convinced these discrepancies (set out below) can be rectified with more

Views of the scatter plots of the locations of individual stores of store types, show that, although they can be shown to form normal distributions around distance means from malls' centers, the distributions are flat (as relatively large standard deviations show).[22] The mall owner/developer would make more profit if these stores' locations showed a less flat distribution (the normal distributions had less width and more height). It is a safe assumption given the research that higher rents would follow from more optimal positioning of stores of different store types. One can surmise that a given stores' locations would be "better" if circumstances in the malls allowed for this kind of optimization.

The question of what rent is owed has to do with store-type rent spreads around their means. Rents for different store types were found to be normally distributed around their means.[23] Normal distribution of rents was determined after the data was transformed so that rents among the several malls were comparable, much as distance measure data were normalized in Carter (1999) and Carter and Vandell (2005).[24] Using the formula for the normal or Gaussian distribution to represent rents, a modest increase in height (median rents) and narrowing of width (lower standard deviation) of store-type rents would be what a mall owner (developer) would pursue, by locating stores of the same store type more in line with their optimal distances from the mall center.

A decrease of one-tenth of the variance in the rent distribution, by moving stores of a certain type closer together around their mean distance from the mall center, would result in an increase in mean rents of about five and four tenths percent (5.4 %). Inputting one-tenth lower variance, the normal or Gaussian function shows an increase in mean rents of 5.4%.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Retaining comparison-shopping stores' dispersion as set out in Carter and Haloupek (2002) would be part of an overall distribution to maximum rents. Rents could also increase to reflect higher rent on account of dispersion of stores of the same type, which is more reflective of that pattern. The measure of such an increase would need to be determined empirically. The quadratic assignment programs' distance and weight (connection) matrixes, respectively, are as set out in Appendix A.

In the connection matrix, highest mean rent ($63.58/SF for jewelry) x 1.054 equals $67.01/SF. The result is $67.01/SF, representing rent for jewelry stores whose rent performance is raised 5.4% through better location. Each of the ten store types are represented in this fashion, e.g., for Men's Apparel, $32.97/SF x 1.054 = $34.75/SF.

Lastly, inputting different sized stores for each store type to reflect the standard deviation or variance of store type sizes observed is needed. The distance and connection matrices are set out in Appendices A and B. The quadratic assignment location problem

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careful measurement and regression analysis.

	mean rent	rank	rent at store center	rank
1) jewelry	$63.58/SF	1	$75.18/SF	1
2) fast food		2		2
3) specialty food	$43.00/SF	3	$51.14/SF	3
4) men's apparel	$32.97/SF	6	$49.07/SF	4
5) leisure & entertainment	$33.85/SF	5	$46.71/SF	5
6) women's shoes	$36.97/SF	4	$40.99/SF	6
7) family apparel	$30.00/SF	10	$37.58/SF	7
8) cards & gifts	$30.95/SF	8	$37.59/SF	8
9)home furnishings	$31.43/SF	7	$36.50/SF	9
10) men's & boy's shoes	$30.43/SF	9	$36.50/SF	10
11) women's apparel	$30.00/SF	11	$29.81/SF	11

The sequence used here is from the bid rent curve findings, while the mean rents are from the data.

[22]   The multiple regression statistics from Carter and Vandell (2005) and Carter (1999) also show this.

[23]   The Kolmogorov-Smirnoff and Shapiro-Wilks tests were used.

can be represented as set out in Appendix C. The fact that this is a QAP can be judged by the makeup of the objective function. Notice here that rather than solving a problem we demonstrate what the ingredients of a QAP problem looks like (analysis having been done in Carter (1999) and Carter and Vandell (2005)).

Conclusion

The ultimate tenant mix will vary by overall size of the regional mall, given that square footage and location of stores depend on a maximization solution to a quadratic assignment problem. The assumptions are: 1) mall space is homogenous (except for build-out), 2) the retail market for all goods in shopping centers is at equilibrium (not over-or-under-saturated), 3) leases are maximally productive (containing both minimum and percentage rent clauses and a cancellation clause), and 4) space is allocated optimally, to the point where net marginal revenue is equal to the marginal cost of space less an externality term, adjusted by each store's elasticity of demand. Stores' square footage depend on optimal square footage for store types, balancing shoppers' convenience in comparison shopping (positive externalities) with a too competitive environment should a store type become too concentrated.

The remaining factor affecting rents (and ultimately developer's and stores' profits) pertains to location. We have shown that in the simplest case of an "I" shaped suburban mall stores line up in each aisle, from the center food court to the anchor stores, following 1) bid rent theory generally and 2) revised central place theory for comparison shopping store types. These two economic forces can be represented in a weight (connection) matrix, which along with a distance matrix, become the inputs for solving a quadratic assignment problem for ultimate tenant mix.

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Appendix A

Distance Matrix

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
				0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
					0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
						0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
							0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
								0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
									0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
										0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
											0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
												0	1	2	3	4	5	6	7	8	9	10	11	12	13
													0	1	2	3	4	5	6	7	8	9	10	11	12
														0	1	2	3	4	5	6	7	8	9	10	11
															0	1	2	3	4	5	6	7	8	9	10
																0	1	2	3	4	5	6	7	8	9
																	0	1	2	3	4	5	6	7	8
																		0	1	2	3	4	5	6	7
																			0	1	2	3	4	5	6
																				0	1	2	3	4	5
																					0	1	2	3	4
																						0	1	2	3
																							0	1	2
																								0	1
																									0

Appendix B

Weight (connection) matrix

0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3	3	3	4	2	3	1	2	0
	0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3	3	3	4	2	3	1	2
		0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3	3	3	4	2	3	1
			0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3	3	3	4	2	3
				0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3	3	3	4	2
					0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3	3	3	4
						0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3	3	3
							0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3	3
								0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3	3
									0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5	3
										0	10	10	10	11	9	10	8	9	7	6	7	5	6	4	5
											0	10	10	10	11	9	10	8	9	7	6	7	5	6	4
												0	10	10	10	11	9	10	8	9	7	6	7	5	6
													0	10	10	10	11	9	10	8	9	7	6	7	5
														0	10	10	10	11	9	10	8	9	7	6	7
															0	10	10	10	11	9	10	8	9	7	6
																0	10	10	10	11	9	10	8	9	7
																	0	10	10	10	11	9	10	8	9
																		0	10	10	10	11	9	10	8
																			0	10	10	10	11	9	10
																				0	10	10	10	11	9
																					0	10	10	10	11
																						0	10	10	10
																							0	10	10
																								0	10
																									0

Appendix C

$$\text{Maximize} \quad \sum_{i=1}^{n} RI_i \, S_i \, X_i$$

Subject to:

1. $\sum^{m} \sum^{p} S_{i,j} \, X_{i,j} \geq LS_j$,   (min. space allocated to a tenant type)

2. $\sum^{m} \sum^{p} S_{i,j} \, X_{i,j} \leq US_j$,   (max. space allocated to a tenant type)

3. $\sum^{m} \sum^{p} S_{i,j} \, X_{i,j} \geq LL_j$,   (min. space allocated to a tenant of a tenant type)

4. $\sum^{m} \sum^{p} S_{i,j} \, X_{i,j} \leq UL_j$,   (max. space allocated to a tenant of a tenant type)

5. $\sum^{o} S_{i,j} \, X_{i,j} \leq GLA \quad 250{,}000 \text{ SF}$

where

RI	= rental income per square feet for a specific tenant type
S	= space for a specific tenant type
X_i	= 1, 0
LS	= min. amount of space allocated to a specific tenant type
US	= max. amount of space allocated to a specific tenant type
LL	= min. amount of space available for tenant of a tenant type
UL	= max. amount of space available for tenant of a tenant type
GLA	= shopping center gross leaseable area (250,000 SF)

n = number of tenants (100), for i = 1, 2, …, n
m = number of tenant types (10), j = 1, 2, … ,m
p = number of tenant type sizes (3), k = 1, 2, …, p