

**ON PSEUDO COMPATIBLE P-FUZZY SOFT RELATIONS****V. Ramadas* & B. Anitha****

* Professor, Department of Mathematics, PRIST University, Thanjavur, Tamilnadu

** Research Scholar, Department of Mathematics, PRIST University,
Thanjavur, Tamilnadu**Cite This Article:** V. Ramadas & B. Anitha, "On Pseudo Compatible P-Fuzzy Soft Relations", International Journal of Applied and Advanced Scientific Research, Volume 3, Issue 1, Page Number 7-11, 2018.**Abstract:**

We introduce the notion of pseudo compatible P-fuzzy soft relations of a sub group, cossets of a group, strongest fuzzy soft relations and how they are related with fuzzy soft normal subgroups.

Key Words: Soft Set, Null Soft Set, Injection Function, Fuzzy Set, P-Fuzzy Soft Middle Cosset, Pseudo Fuzzy Cosset, Strongest Fuzzy Relation & Compatible Fuzzy Soft Set.

Introduction:

The concept of fuzzy sets was first introduced by Zadeh [23]. Rosenfeld [16] used this concept to formulate the notion of fuzzy groups. Since then, many other fuzzy algebraic concepts based on the Rosenfeld's fuzzy groups were developed. Anthony and Sherwood [1] redefined fuzzy groups in terms of t- norm which replaced the min operations of Rosenfeld's definition. Some properties of these redefined fuzzy groups, which we call t- fuzzy groups, have been developed by Sherwood [18], sessa [17], sidky and misharf (19). However the definition of t- fuzzy groups seems to be too general. Soft set theory was introduced in 1999 by Molodtsov [15] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 21, 23]. Moreover, Atagun and Sezgin [5] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Ali et al. [4] introduced several operations of soft sets and Sezgin and Atagun [21] studied on soft set operations as well. Furthermore, soft set relations and functions [6] and soft mappings [14] with many related concepts were discussed. Here we introduce the notion of pseudo compatible P-fuzzy soft relations of a subgroup, cossets of a group, strongest fuzzy soft relations and how they are related with fuzzy soft normal subgroups.

Section-2 Preliminaries:

In this section, we recall basic definitions of soft set theory that are useful for subsequent sections. For more detail see the papers [[11], [15].] Throughout the paper, U refers to an initial universe, E is a set of parameters and $P(U)$ is the power set of U . \subset and \supset stand for proper subset and super set, respectively.

Definition 2.1 [22]: A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . Note that a soft set (F, A) can be denoted by F_A . In this case, when we define more than one soft set in some subsets A, B, C of parameters E , the soft sets will be denoted by F_A, F_B, F_C , respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E , the soft sets will be denoted by F_A, G_A, H_A , respectively. For more details, we refer to [11, 17, 18, 26, 29, 7]. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.2 [12]: Let $\lambda, \mu \in S(U)$. Then

- (i) If $\lambda(e) = \emptyset$ for all $e \in E$, λ is said to be a null soft set, denoted by \emptyset .
- (ii) If $\lambda(e) = U$ for all $e \in E$, λ is said to be an absolute soft set, denoted by U .
- (iii) λ is a soft subset of μ , denoted $\lambda \subseteq \mu$, if $\lambda(e) \subseteq \mu(e)$ for all $e \in E$.
- (iv) Soft union of λ and μ , denoted by $\lambda \cup \mu$, is a soft set over U and defined by $\lambda \cup \mu: E \rightarrow P(U)$ such that $(\lambda \cup \mu)(e) = \lambda(e) \cup \mu(e)$ for all $e \in E$.
- (v) $\lambda = \mu$, if $\lambda \subseteq \mu$ and $\lambda \supseteq \mu$.
- (vi) Soft intersection of λ and μ , denoted by $\lambda \cap \mu$, is a soft set over U and defined by $\lambda \cap \mu: E \rightarrow P(U)$ such that $(\lambda \cap \mu)(e) = \lambda(e) \cap \mu(e)$ for all $e \in E$.
- (vii) Soft complement of λ is denoted by λ^c and defined by $\lambda^c: E \rightarrow P(U)$ such that $\lambda^c(e) = U/\lambda(e)$ for all $e \in E$.

Definition 2.3 [12]: Let E be a parameter set, $S \subset E$ and $\lambda: S \rightarrow E$ be an injection function. Then $S \cup \lambda(s)$ is called extended parameter set of S and denoted by ξ_S . If $S=E$, then extended parameter set of S will be denoted by ξ .

Definition 2.4 [6]: The relative complement of the soft set F_A over U is denoted by F_A^c , where $F_A^c : A \rightarrow P(U)$ is a mapping given as $F_A^c(a) = U \setminus F_A(a)$, for all $a \in A$.

Definition 2.5 [6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$. The restricted intersection of F_A and G_B is denoted by $F_A \Psi G_B$, and is defined as $F_A \Psi G_B = (H, C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

Definition 2.6 [6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B = (H, C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

Definition 2.7 [12]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B . Then we can define the soft set $\psi(F_A)$ over U , where $\psi(F_A) : B \rightarrow P(U)$ is a set valued function defined by $\psi(F_A)(b) = \bigcup \{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$, $= \emptyset$ otherwise for all $b \in B$. Here, $\psi(F_A)$ is called the soft image of F_A under ψ . Moreover we can define a soft set $\psi^{-1}(G_B)$ over U , where $\psi^{-1}(G_B) : A \rightarrow P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all $a \in A$. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ .

Definition 2.8 [13]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B . Then we can define the soft set $\psi^*(F_A)$ over U , where $\psi^*(F_A) : B \rightarrow P(U)$ is a set-valued function defined by $\psi^*(F_A)(b) = \bigcap \{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$, $= \emptyset$ otherwise for all $b \in B$. Here, $\psi^*(F_A)$ is called the soft anti image of F_A under ψ .

3. Structures of Fuzzy Soft Subgroup:

Definition 3.1: A mapping $\mu : X \rightarrow [0, 1]$, where X is an arbitrary non-empty set is called a fuzzy soft subset in X .

Definition 3.2: Let G be any group. A mapping $\mu : G \rightarrow [0, 1]$ is a fuzzy soft subgroup of G if (FSG1) $\mu(xy) \geq \min \{ \mu(x), \mu(y) \}$ (FSG2) $\mu(x^{-1}) = \mu(x)$ for all $x, y \in G$.

Example:

Let Z be the additive group of all integers. For any integer n , nZ denote the set of all integers multiples of n .

(i.e) $nZ = \{ 0, \pm n, \pm 2n, \pm 3n, \dots \}$. We have $Z > 2Z > 4Z > 8Z > 16Z$. Define $\mu : Z \rightarrow [0, 1]$ by $\mu(x) = 1$, if $x \in 16Z$; $= 0.7$, if $x \in 8Z - 16Z$; $= 0.5$ if $x \in 4Z - 8Z$; $= 0.2$ if $x \in 2Z - 4Z$; $= 0$ if $x \in Z - 2Z$. It can be easily verified that μ is fuzzy soft sub group of Z . If the Supplementary condition (FSG₃) $\mu(e_G) = 1$ are satisfied, then the fuzzy soft group is called a standardized fuzzy soft group where e_G is an identity of the group (G, \cdot)

Proposition 3.3:

A fuzzy soft subset μ of a group ' G ' is a fuzzy soft subgroup of \hat{G} if and only if $\mu(xy^{-1}) \geq \min \{ \mu(x), \mu(y) \}$ for every x, y in G

Proof:

Let ' μ ' be a fuzzy soft subgroup of \hat{G} . Form ' μ ' is a fuzzy group (FSG₁) and (FSG₂) are satisfied. $\mu(xy^{-1}) \geq \min \{ \mu(x), \mu(y^{-1}) \} = \min \{ \mu(x), \mu(y) \}$ conversely let $\mu(xy^{-1}) \geq \min \{ \mu(x), \mu(y) \}$ in equality be satisfied. Choosing $y = x$ we get that $\mu(xx^{-1}) = \mu(e) \geq \min \{ \mu(x), \mu(x^{-1}) \} = \mu(x)$. Hence for $x=e$. $\mu(y^{-1}) = \mu(ey^{-1}) \geq \min \{ \mu(e), \mu(y) \} = \mu(x)$ consequently $\mu(xy^{-1}) \geq \min \{ \mu(x), \mu(y^{-1}) \} = \min \{ \mu(x), \mu(y) \}$

Remarks 3.4: Let ' μ ' be a fuzzy soft sub group of a group ' G ' and $x \in G$. then $\mu(xy) = \mu(y)$ for every $y \in G$ if and only if $\mu(x) = \mu(e)$

Definition 3.5: Let ' μ ' be a fuzzy soft sub group of a group ' G '. For any $a \in G$. are defined by $(a\mu)(x) = \mu(a^{-1}x)$ for every $x \in G$ is called the P-fuzzy soft cosset of the group G determined by ' a ' and ' μ '

Definition 3.6: Let ' μ ' be the fuzzy soft sub group of a group G . then for any $a, b \in G$ a P-fuzzy soft middle cosset $a\mu b$ of the group G is defined by $(a\mu b)(x) = \mu(a^{-1}xb)$ for every $x \in G$.

Definition 3.7: Let ' μ ' be a fuzzy soft sub group of G and $a \in G$. Then the P-pseudo fuzzy cosset $(a\mu)^p$ is defined by $(a\mu)^p(x) = p(a)\mu(x)$ for every $x \in G$ and for some $p \in P$.

Example:

Let $G = \{1, w, w^2\}$ be a group with respect to multiplication where ' w ' denotes the cube root of unity. Define a map $\mu : G \rightarrow [0, 1]$ by

$$\mu(x) = \begin{cases} 0.7 & \text{if } x = 1 \\ 0.3 & \text{if } x = w, w^2 \end{cases}$$

The pseudo fuzzy soft cosset $(a\mu)^p$ for $p(x) = 0.4$ for every $x \in G$ to be equal to 0.28 if $x = 1$ and 0.12 if $x = w, w^2$.

Definition 3.8: Let μ and λ be any two fuzzy soft subsets of a set ' X ' and $p \in P$. the P-pseudo fuzzy soft double cosset to $(\mu x \lambda)^p$ is defined as $((\mu x \lambda)^p) = (x\mu)^p \cap (x\lambda)^p$ for $x \in X$.

Definition 3.9: Let λ and ' μ ' be two fuzzy soft subgroups of a group 'G' then λ and μ are said to be P- fuzzy soft conjugate subgroups of G if for some $g \in G$ $\lambda(x) = \mu(g^{-1} x g)$ for every $x \in G$.

4. Some Properties of Pseudo Fuzzy Soft Cosets:

Proposition 4.1:

Let ' μ ' be a fuzzy soft subgroup of a group 'G'. Then P-pseudo fuzzy soft coset $(a \mu)^p$ is a fuzzy soft sub group of 'G' for every $a \in G$.

Proof: Let ' μ ' be a fuzzy soft sub group of G, for every x, y in G we have $(a \mu)^p(x y^{-1}) = p(a) \mu(x y^{-1}) \geq p(a) \min \{ \mu(x), \mu(y) \} = \min \{ p(a) \mu(x), p(a) \mu(y) \} \geq \min \{ (a \mu)^p(x), (a \mu)^p(y) \}$ for every $x \in G$. This proves that $(a \mu)^p$ is a fuzzy soft subgroup of G.

Remark 4.2: A fuzzy soft subgroup ' μ ' of a group G is said to be positive fuzzy soft subgroup of 'G' if ' μ ' is positive fuzzy soft subset of the group 'G'.

Proposition 4.3:

Every P- pseudo fuzzy soft double coset is a fuzzy soft subgroup of a group 'G'

Proof:

- (i) $(\mu x \lambda)^p(x y) = \{ (x \mu)^p \cap (x \lambda)^p \} (xy) = (x \mu)^p(x y)$ and $(x \lambda)^p(xy)$ }
 $= p(x) \mu(x y)$ and $p(x) \lambda(x y)$
 $\geq p(x) \min \{ \mu(x), \mu(y) \}$ and $p(x) \min \{ \lambda(x), \lambda(y) \}$
 $\geq \min \{ p(x) \mu(x), p(x) \mu(y) \}$ and $\min \{ p(x) \lambda(x), p(x) \lambda(y) \}$
 $\geq \min \{ p(x) \mu(x), p(x) \mu(x) \}, \min \{ p(x) \mu(y) \text{ and } p(x) \lambda(y) \}$
 $= \min \{ (x \mu)^p \cap (x \lambda)^p \} (x), (x \mu)^p \cap (x \lambda)^p (y)$
 $\geq \min \{ (\mu x \lambda)^p(x), (\mu x \lambda)^p(y) \}$
 - (ii) $(\mu x \lambda)^p(x) = \{ (x \mu)^p \cap (x \lambda)^p \} (x) = (x \mu)^p \cap (x \lambda)^p(x)$
 $= p(x) \mu(x)$ and $p(x) \lambda(x) = p(x) \mu(x)^{-1}$ and $p(x) \lambda(x)^{-1}$ (since λ and μ are fuzzy subsets) =
 $(x \mu)^p(x)^{-1}$ and $(x \lambda)^p(x)^{-1} = \{ (x \mu)^p \cap (x \lambda)^p \} (x)^{-1} = (\mu x \lambda)^p(x)^{-1}$
- Theorem is proved.

Proposition 4.4:

Every P-fuzzy soft middle coset of a group 'G' is a fuzzy soft subgroup of G.

Proof:

Let $a \mu b$ be a P-fuzzy soft middle coset of the group 'G' and ' λ ' and ' μ ' be two P-conjugate fuzzy soft subgroups of G.

- (i) $(a \mu b)(x y) = \mu(a^{-1} x y b^{-1}) = \lambda(x y)$ [$\because \lambda$ and μ conjugate fuzzy soft subgroups]
 $\geq \min \{ \lambda(x), \lambda(y) \} \geq \min \{ \mu(a^{-1} x b^{-1}), \mu((a^{-1} y b^{-1})) \}$
 $\geq \min \{ (a \mu b)(x), (a \mu b)(y) \}$
- (ii) $(a \mu b)(x) = \mu(a^{-1} x b^{-1}) = \mu(a^{-1} x^{-1} b^{-1})$ (\because ' μ ' fuzzy sub group) = $(a \mu b)(x^{-1})$ Theorem is proved.

Definition 4.5: Let 'G' be a group. A fuzzy soft subgroup ' μ ' of 'G' is called normal if $\mu(x) = \mu(y^{-1} x y)$ for all x, y in G. (or) A fuzzy soft subgroup μ_H of G is called a fuzzy soft normal subgroup of 'G' if $\mu_H(x y) = \mu_H(y x)$ for all x, y in G.

Proposition 4.6:

Every P-pseudo fuzzy soft coset is a fuzzy soft normal subgroup of a group 'G'

Proof:

Let $(a \mu)^p$ be any P-pseudo fuzzy soft coset. $a \in G$ and for some $p \in P$. Now $(a \mu)^p(x) = p(a) \mu(x) = p(a) \min \{ \mu(e), \mu(x) \} = p(a) \min \{ \mu(y^{-1} y), \mu(x) \}$
 $\geq p(a) \min \{ \min \{ \mu(y)^{-1}, \mu(y) \}, \mu(x) \} \geq p(a) \min \{ \mu(y)^{-1}, \min \{ \mu(y), \mu(x) \} \}$
 $= p(a) \mu(y^{-1} x y)$ for all $y \in G$.

Aliter:

Let $(a \mu)^p$ be any P-pseudo fuzzy soft coset and $a \in G$ for some $p \in P$, Let μ_H is a fuzzy soft normal subgroup of G. Now $(a \mu_H)^p(x y) = p(a) \mu_H(x y) = p(a) \mu_H(y x) = (a \mu_H)^p(y x)$ (μ_H is fuzzy soft normal) = $(a \mu_H)^p(y x)$

Proposition 4.7:

The intersection of two P-pseudo fuzzy soft coset normal subgroup is also fuzzy soft normal subgroup of a group.

Proof:

Let $(a \mu)^p$ and $(b \mu)^p$ be any two P-pseudo fuzzy soft coset normal subgroup of G.

$$(a \mu)^p(x) = (a \mu)^p(y^{-1} x y), y \in G \text{--- (1)}$$

$$(b \mu)^p(x) = (b \mu)^p(y^{-1} x y), y \in G \text{--- (2)}$$

Now, $\{ (a \mu)^p \cap (b \mu)^p \} (x) = ((a \cap b) \mu)^p(x) = p(a \cap b) \mu(x) = p(a) \cdot p(b) \mu(x) = p(a) \cdot \mu(x)$ and $p(b) \mu(x) = (a \mu)^p(x)$ and $(b \mu)^p(x) = (a \mu)^p(y^{-1} x y)$ and $(b \mu)^p(y^{-1} x y)$ by ((i) & (ii)) = $p(a) \cdot p(b) \mu(y^{-1} x y) = ((a \cap b) \mu)^p(y^{-1} x y) = \{ (a \mu)^p \cap (b \mu)^p \} (y^{-1} x y)$.

Theorem is proved

Aliter:

Let $(a \mu_H)^p \cap (b \mu_H)^p (x y) = ((a \cap b) \mu_H)^p (x y) = p(a \cap b) \mu(x y) = p(a \cap b) \mu_H(y x)$ (μ_H is fuzzy soft normal) $= (a \cap b) \mu_H^p (y x) = \{a \cap b\}^p \cap (b \mu_H)^p (y x)$

Proposition 4.8:

P-Pseudo fuzzy soft double cosset is a fuzzy soft normal subgroup of a group 'G'

Proof:

Let $(\mu x \lambda)^p$ be any P- pseudo fuzzy soft double cosset for $x \in X$.

$$\begin{aligned} \text{Now } (\mu x \lambda)^p (x) &= \{(\mu x)^p \cap (x \lambda)^p\}(x) = (\mu x)^p(x) \cap (x \lambda)^p(x) \\ &= p(x) \cap \mu(x) \cap p(x) \lambda(x) = p(x) \min\{\mu(x), \mu(e)\} \cap p(x) \min\{\lambda(x), \lambda(e)\} \\ &= p(x) \min\{\mu(x), \mu(y^{-1}y)\} \cap p(x) \min\{\lambda(x), \lambda(y^{-1}y^{-1})\} \\ &\geq p(x) \min\{\mu(x), \min\mu(y^{-1}), \mu(y)\} \cap \\ &\quad p(x) \min\{\lambda(x), \min\{\lambda(y^{-1}), \lambda(y)\}\} \\ &= p(x) \min\{\mu(y^{-1}), \mu(xy)\} \cap p(x) \min\{\lambda(y^{-1}), \lambda(xy)\} \\ &= p(x) \mu(y^{-1}xy) \cap p(x) \lambda(y^{-1}xy) = \{x \mu\}^p \cap (x \lambda)^p (y^{-1}xy) \\ &= (\mu x \lambda)^p (y^{-1}xy) \end{aligned}$$

Theorem is proved.

Proposition 4.9:

P-Fuzzy soft middle cossets forms a fuzzy soft normal subgroup of G.

Proof:

$$\begin{aligned} (a \mu b)(x) &= \mu(a^{-1}x b^{-1}) = \lambda(x) = \min\{\lambda(x), \lambda(e)\} \\ &= \min\{\lambda(x), \lambda(y^{-1}y)\} \geq \min\{\lambda(x), \min\{\lambda(y^{-1}), \lambda(y)\}\} \\ &= \min\{\lambda(y^{-1}) \min\{\lambda(x), \lambda(y)\}\} = \min\{\lambda(y^{-1}), \lambda(xy)\} = \lambda(y^{-1}x) \\ &= \mu(a^{-1}(y^{-1}xy) b^{-1}) = (a \mu b)(y^{-1}xy) \end{aligned}$$

Definition 4.10: The strong fuzzy soft α -cut is defined as $A^+_{\alpha} = \{x/A(x) > \alpha\}$ where A is any fuzzy soft set .

Definition 4.11: Let 'A' be a fuzzy soft set in a set S. Then the strongest fuzzy soft relation on 'S' (ie) fuzzy soft relation on 'A' is $\mu_A(x,y) = \min\{A(x), A(y)\}$.

Definition 4.12: Cartesian Product: Let λ and μ be any two fuzzy soft sets in X. Then the cartesian Product of λ and μ is $\lambda \times \mu: x \times x \rightarrow [0, 1]$ defined by $(\lambda \times \mu)(x,y) = \min\{\lambda(x), \mu(y)\}$ for all $x,y \in X$.

Proposition 4.13:

Let μ_A be a strongest Fuzzy soft relation on 'S' and ' A^+_{α} ' be the strong α -cut .Then μ_A forms a strong α - cut fuzzy soft group on S.

Proof:

Let $A:S \rightarrow [0,1]$ be any function and μ_A be the strongest fuzzy soft relation on S.

(i) Let $x,y \in S$

$$\mu_A(x,y) = \min\{A(x), A(y)\} \geq \min\{\alpha, \alpha\} \geq \alpha$$

(ii) $\mu_A(x^{-1},y^{-1}) = \min\{A(x^{-1}), A(y^{-1})\} = \min\{A(x), A(y)\} = \mu_A(x,y)$

(ii) $\mu_A(e, e) = \min\{A(e), A(e)\} = \min\{1,1\} = 1$

μ_A forms a strong fuzzy group α - cut on S.

Proposition 4.14:

Let λ and μ be strong fuzzy soft α - cuts on S. Then $\lambda \times \mu$ is a strong fuzzy soft group α - cut.

Proof: Let $x,y \in S$ and $\lambda: x \times x \rightarrow [0,1]$ be any function.

(i) $(\lambda \times \mu)(x,y) = \min\{\lambda(x), \mu(y)\} \geq \min\{\alpha, \alpha\} \geq \alpha$

(ii) $(\lambda \times \mu)(x^{-1},y^{-1}) = \min\{\lambda(x^{-1}), \mu(y^{-1})\} = \min\{A(x), A(y)\} = (\lambda \times \mu)(x, y)$

(ii) $(\lambda \times \mu)(e,e) = \min\{\lambda(e), \mu(e)\} = \min\{1,1\} = 1$

$(\lambda \times \mu)$ forms a strong fuzzy soft group α -cut on S.

Remark 4.15: i) $\min(a,b)^i = \min\{a^i, b^i\}$ for all Positive integer 'i'

ii) $\mu_A^i(x,y) = (\mu_A(x,y))^i = \min\{A(x), A(y)\}^i = \min\{A^i(x), A^i(y)\}$

Proposition 4.16:

Let μ_A^i and μ_A^j be two strong fuzzy soft relations and A_{α}^+ be strong fuzzy soft α - cut .Then $\mu_A^i \cup A^j$ forms a strong fuzzy soft α -cut on S.

Proof:

Since $i < j$

$$\begin{aligned} \mu_A^i \cup A^j(x,y) &= \{(A^i \cup A^j)(x), (A^i \cup A^j)(y)\} = \min\{\max\{A^i(x), A^j(x)\}, \max\{A^i(y), A^j(y)\}\} \\ &= \max\{\min\{A^i(x), A^i(y)\}, \min\{A^j(x), A^j(y)\}\} \\ &= \max\{\min\{A(x), A(y)\}^i, \min\{A(x), A(y)\}^j\} \\ &\geq \max\{\min\{\alpha, \alpha\}^i, \min\{\alpha, \alpha\}^j\} \\ &\geq \max\{\min\{\alpha^i, \alpha^i\}, \min\{\alpha^j, \alpha^j\}\} \geq \max\{\alpha^i, \alpha^j\} \geq \alpha^i \end{aligned}$$

$\mu_{A \cup A^+}^i$ is a strong fuzzy soft α -cut on S.

Remark 4.17: Let μ_A^i and $\mu_{A^+}^j$ be two strong fuzzy soft relations and A_α^+ be strong fuzzy soft α -cut. Then $\mu_{A \cup A^+}^i$ is a strong fuzzy soft α -cut on S.

Proof: It is obvious

Definition 4.18 : A fuzzy soft binary relation μ on a semi group 'S' is called P-fuzzy soft compatible iff $\mu(ac, bd) \geq \min \{ \mu(a,b), \mu(c,d) \}$ for all $a,b,c,d \in S$.

Proposition 4.19:

Let μ_A be the strongest fuzzy soft relation on S. Then A_α^+ is a strong α -cut then μ_A forms P-fuzzy soft compatible.

Proof:

Now $\mu_A(ac, bd) = \min \{ A(ac), A(bd) \} \geq \min \{ \min \{ A(a), A(c) \}, \min \{ A(b), A(d) \} \}$
 $\geq \min \{ \mu_A(a,b), \mu_A(b,d) \}$

3.18 Proposition:

Let μ_A be a P-fuzzy soft compatible. Then μ_A is a strong fuzzy soft α -cut.

Proof:

Now $\mu_A(ac, bd) \geq \{ \mu_A(a, b), \mu_A(b, d) \} = \min \{ \min \{ A(a), A(b) \}, \min \{ A(b), A(d) \} \} > \min \{ \min \{ \alpha, \alpha \}, \min \{ \alpha, \alpha \} \} > \alpha$. Hence μ_A is P-fuzzy soft compatible forms a strong fuzzy soft α -cut.

Conclusion:

Here we introduce the notion of pseudo compatible P-fuzzy soft relations of a subgroup, cosets of a group, strongest fuzzy soft relations and how they are related with fuzzy soft normal subgroups. One can obtain the similar ideal into Soft G-modular and L- fuzzy structures.

References:

1. J. M. Anthony and H. Sherwood, Fuzzy groups redefined J. Math. Anal. Appl.69 (1979), 124-130.
2. Acar U., Koyuncu F., Tanay B., Soft sets and soft rings, Comput. Math. Appl., 59(2010), 3458-3463.
3. Aktas. H., C. agman N., Soft sets and soft groups, Inform. Sci., 177(2007), 2726-2735.
4. Ali M.I.,Feng F., Liu X., Min W.K., Shabir M., On some new operations in soft set theory, Comput. Math. Appl., 57(2009), 1547-1553.
5. Atagun A.O., Sezgin A., Soft substructures of rings, fields and modules, Comput. Math. Appl., 61(3) (2011), 592-601.
6. Babitha K.V., Sunil J.J., Soft set relations and functions, Comput. Math. Appl., 60(7)(2010), 1840-1849.
7. Feng F., Jun Y.B., Zhao X., Soft semirings, Comput. Math. Appl., 56(2008), 2621-2628.
8. Feng F., Liu X.Y., Leoreanu-Fotea V., Jun Y.B., Soft sets and soft rough sets, Inform. Sci., 181(6) (2011), 1125-1137.
9. Jun Y.B., Soft BCK/BCI-algebras, Comput. Math. Appl., 56(2008), 1408 -1413.
10. Jun Y.B., Park C.H., Applications of soft sets in ideal theory of BCK/ BCI-algebras, Inform. Sci., 178(2008), 2466-2475.
11. Jun Y.B., Lee K.J., Zhan J., Soft p-ideals of soft BCI-algebras, Comput. Math. Appl., 58(2009), 2060-2068.
12. Kazancı O., Yılmaz S., Yamak S., Soft sets and soft BCH-algebras, Hacet. J. Math. Stat., 39(2)(2010), 205-217.
13. Majumdar P., Samanta S.K., on soft mappings, Comput. Math. Appl., 60 (9)(2010), 2666-2672.
14. Molodtsov D., Soft set theory-first results, Comput. Math. Appl., 37(1999), 19-31.
15. A. Rosenfeld, Fuzzy groups, J.math.Anal.Appl.35 (1971), 512- 517.
16. S. Sessa, on fuzzy subgroups and fuzzy ideals under triangular norms, Fuzzy sets and fuzzy systems, 13, (1984), 95-100.
17. H. Sherwood, Product of fuzzy subgroups, Fuzzy sets and systems, 11, (1983), 79-89.
18. F. I. Sidky and M. Atif Misherf, Fuzzy cosets and cyclic and abelian fuzzy subgroups, Fuzzy sets and systems, 43, (1991), 243-250.
19. Sezgin A., Atagun A.O., Ayg un E., A note on soft near-rings and idealistic soft near-rings, Filomat, 25(1)(2011), 53-68.
20. Sezgin A., Atagun A.O., on operations of soft sets, Comput. Math. Appl., 61(5) (2011), 1457-1467.
21. Zhan J., Jun Y.B., Soft BL-algebras based on fuzzy sets, Comput. Math. Appl., 59(6) (2010), 2037-2046. Fuzzy sets and systems, 43, (1991), 243-250.
22. L.A.Zadeh, Fuzzy sets, Inform and control, 8, (1965), 338-353.