World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:11, No:10, 2017

Subclasses of Bi-Univalent Functions Associated with Hohlov Operator

Rashidah Omar, Suzeini Abdul Halim, Aini Janteng

International Science Index, Mathematical and Computational Sciences Vol:11, No:10, 2017 waset.org/Publication/10008113

Abstract**—**The coefficients estimate problem for Taylor-Maclaurin series is still an open problem especially for a function in the subclass of bi-univalent functions. A function $f \in A$ is said to be bi-univalent in the open unit disk D if both f and f^{-1} are univalent in D. The symbol A denotes the class of all analytic functions f in D and it is normalized by the conditions $f(0) = f'(0) - 1 = 0$. The class of bi-univalent is denoted by $\boldsymbol{\sigma}$. The subordination concept is used in determining second and third Taylor-Maclaurin coefficients. The upper bound for second and third coefficients is estimated for functions in the subclasses of bi-univalent functions which are subordinated to the function φ. An analytic function f is subordinate to an analytic function g if there is an analytic function w defined on D with $w(0) = 0$ and $|w(z)| < 1$ satisfying $f(z) = g[w(z)]$. In this paper, two subclasses of bi-univalent functions associated with Hohlov operator are introduced. The bound for second and third coefficients of functions in these subclasses is determined using subordination. The findings would generalize the previous related works of several earlier authors.

Keywords**—**Analytic functions, bi-univalent functions, Hohlov operator, subordination.

I. Introduction

Let A denote the class of functions f which are analytic in the open unit disk $D := \{z: z \in \boldsymbol{C}, |z| < 1\}$ and normalized by the following Taylor-Maclaurin series:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \ (z \in D). \tag{1}$$

Recall that the convolution of two analytic functions $f, h \in A$ is the analytic function defined as

$$(f * h)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n ,$$

where $f(z)$ is given by (1) and $h(z) = z + \sum_{n=2}^{\infty} b_n z^n$.

For the complex parameters a, b and c ($c \neq 0, -1, -2, -3, \cdots$, the Gaussian hypergeometric function is defined as

$$\begin{aligned} {}_2F_1(a, b, c; z) &= \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} z^n \\ &= 1 + \sum_{n=2}^{\infty} \frac{(a)_{n-1} (b)_{n-1}}{(c)_{n-1!}(n-1)!} z^{n-1} \ (z \in D) \end{aligned} \tag{2}$$

Rashidah Omar is with the Mathematics Department, Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara, Cawangan Sabah, Malaysia (phone:6088-325151; e-mail: ashidah@hotmail.com).

Suzeini Abdul Halim is with the Institute of Mathematical Sciences, Faculty of Science, Universiti Malaya, Malaysia (e-mail: suzeini@um.edu.my).

Aini Janteng is with the Mathematics Department, Faculty of Science and Natural Resources, Universiti Malaysia Sabah, Malaysia (e-mail: aini_jg@ums.edu.my).

where $(\alpha)_n$ is the Pochhammer symbol defined, in terms of gamma function, by

$$(\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)} = \begin{cases} 1 & (n = 0) \\ \alpha(\alpha+1)(\alpha+2)\cdots(\alpha+n-1) & (n = 1,2,3,\cdots). \end{cases}$$

Using the Gaussian hypergeometric function given by (2), Hohlov [1] introduced the operator $I_{a,b;c}$ as:

$$I_{a,b;c} f(z) = z\,{}_2F_1(a, b, c; z) * f(z) = z + \sum_{n=2}^{\infty} \emptyset_n a_n z^n \ (z \in D)$$

where

$$\emptyset_n = \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}.$$

In particular, if $b = 1$ then $I_{a,b;c}$ reduces to the Carlson-Shaffer operator. Also, the Hohlov operator is a generalization of the Ruscheweyh operator and Bernadi-Libera-Livingston operator.

Let S denote the subclass of functions in A which are univalent in D. According to the Koebe one-quarter theorem [2], it ensures that the images of D under every univalent function f in S contain a disk of radius $\frac{1}{4}$. Thus, every univalent function f on D has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z, z \in D$$

and

$$f(f^{-1}(w)) = w, |w| < r_0(f), r_0(f) \geq \frac{1}{4}.$$

A function $f \in A$ is said to be bi-univalent in D if both f and f^{-1} are univalent on D. Let σ denote the class of bi-univalent functions in D given by the Taylor-Maclaurin series expansion (1). Some examples of functions in the class σ are $\frac{z}{1-z}$, $-\log(1-z)$ and $\frac{1}{2} log\left(\frac{1+z}{1-z}\right)$.

In 1967, Lewin [3] developed the class of bi-univalent function σ and showed that $|a_2| < 1.51$. On the other hand, for the most general families of functions given by (1), the initial bounds for bi-starlike were conjectured in [4] that $|a_2| \leq \sqrt{2}$ and $|a_2| \leq 1$ for bi-convex functions [5]. The coefficient problem for each of the following Taylor-Maclaurin coefficients $|a_n|$ $(n \in \boldsymbol{N} \backslash \{1,2\}; \boldsymbol{N} := \{1,2,3,\cdots\})$ is still an open problem.

An analytic function f is subordinate to an analytic function g, denoted as $f(z) \prec g(z)$ if there is an analytic function w defined on D with $w(0) = 0$ and $|w(z)| < 1$ satisfying $f(z) = g\big(w(z)\big)$. Ma and Minda [6] unified various subclasses of starlike and convex functions. An analytic function φ with

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positive real part is considered in the unit disk $D, \varphi(0) = 1, \varphi'(0) > 0$, and φ maps D onto a region starlike with respect to 1 and symmetric with respect to the real axis. The function φ has a series expansion of the form

$$\varphi(z) = 1 + B_1 z + B_2 z^2 \cdots \quad (3)$$

where all coefficients are real and $B_1 > 0$. The classes of Ma-Minda starlike and convex functions consist of function $f \in A$ satisfying the subordination $\frac{zf'(z)}{f(z)} \prec \varphi(z)$ and $1 + \frac{zf''(z)}{f'(z)} \prec \varphi(z)$, respectively.

Recently, the estimate for second and third coefficients of bi-univalent functions is investigated by [7]-[14]. Besides that, there are several authors who determined the initial bounds for the subclasses of bi-univalent functions associated with operator such as in [15]-[21].

Motivated by [12], [19], we introduce two new subclasses of bi-univalent functions associated with Hohlov operator based on Ma-Minda concept. Furthermore, the bound for second and third coefficients of functions in these subclasses are obtained. The results would generalize the previous related works of several earlier authors.

Using the Hohlov operator $I_{a,b;c}$, we introduce the following two subclasses of bi-univalent functions.

Definition 1. A function $f \in \sigma$ is said to be in the class $\mathcal{H}_\sigma^{a,b;c}(\varphi)$, where φ is given in (3), if the following subordinations hold:

$$\left[I_{a,b;c} f(z)\right]' \prec \varphi(z)$$

and

$$\left[I_{a,b;c} g(w)\right]' \prec \varphi(w)$$

where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - \cdots \quad (4)$$

Note that for $a = c$ and $b = 1$, the class $\mathcal{H}_\sigma^{a,b;c}(\varphi)$ reduces to the class $\mathcal{H}_\sigma(\varphi)$ introduced by Ali et al. [12]. On the other hand, if $\varphi(z) = \left(\frac{1+z}{1-z}\right)^\alpha$ or $\varphi(z) = \frac{1+(1-2\beta)z}{1-z}$ with $a = c$ and $b = 1$, the subclasses introduced by [14] are obtained.

Definition 2. Let φ is given in (3) and $\lambda \geq 1$. A function $f \in \sigma$ is said to be in the class $\mathfrak{B}_\sigma^{a,b;c}(\varphi, \lambda)$ if the following conditions are satisfied:

$$(1-\lambda)\frac{I_{a,b;c} f(z)}{z} + \lambda\left[I_{a,b;c} f(z)\right]' \prec \varphi(z)$$

and

$$(1-\lambda)\frac{I_{a,b;c} g(w)}{w} + \lambda\left[I_{a,b;c} g(w)\right]' \prec \varphi(w)$$

where the function g is given by (4).

For special cases, the class $\mathfrak{B}_\sigma^{a,b;c}(\varphi, \lambda)$ reduces to the previous classes introduced by several authors. For examples:

i) If $a = c$, $b = 1$ and $\lambda = 1$, the class $\mathfrak{B}_\sigma^{a,b;c}(\varphi, \lambda)$ reduces to [12].

ii) If $a = c$, $b = 1$ and $\lambda = 1$ with $\varphi(z) = \left(\frac{1+z}{1-z}\right)^\alpha$ and $(z) = \frac{1+(1-2\beta)z}{1-z}$, we obtain the subclasses defined by [14].

iii) If $= c$, $b = 1$, $\varphi(z) = \left(\frac{1+z}{1-z}\right)^\alpha$ and $\varphi(z) = \frac{1+(1-2\beta)z}{1-z}$, the class $\mathfrak{B}_\sigma^{a,b;c}(\varphi, \lambda)$ reduces to subclasses introduced in [13].

To establish the bounds for coefficients a_2 and a_3, we state the well-known lemma that is used to obtain the bounds.

Lemma 1. If $p \in \wp$ then $|p_k| \leq 2$ for each k, where $\wp$ is the family of all functions p analytic in D, $Re\, p(z) > 0, p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots$ for $z \in D$.

II. Coefficient Estimates for the Function in $\mathcal{H}_\sigma^{a,b;c}(\varphi)$

We begin by finding the bound for second and third coefficients for functions in the class $\mathcal{H}_\sigma^{a,b;c}(\varphi)$

Theorem 1. Let f given by (1) be in the class $\mathcal{H}_\sigma^{a,b;c}(\varphi)$ then

$$|a_2| \leq \frac{B_1\sqrt{B_1}}{\sqrt{\left|3\emptyset_3 B_1^2 + 4\emptyset_2^2(B_1 - B_2)\right|}}$$

and

$$|a_3| \leq B_1\left[\frac{B_1}{4\emptyset_2^2} + \frac{1}{3\emptyset_3}\right]. \quad (5)$$

Proof. For $f \in \mathcal{H}_\sigma^{a,b;c}(\varphi)$ and $g = f^{-1}$, there exist analytic functions $u, v: D \to D$ with $u(0) = v(0) = 0$, satisfying

$$\left[I_{a,b;c} f(z)\right]' = \varphi(u(z))$$

and

$$\left[I_{a,b;c} g(w)\right]' = \varphi(v(w)). \quad (6)$$

Define the functions p and q as

$$p(z) \coloneqq \frac{1+u(z)}{1-u(z)} = 1 + p_1 z + p_2 z^2 + \cdots$$

and

$$q(z) \coloneqq \frac{1+v(z)}{1-v(z)} = 1 + q_1 z + q_2 z^2 + \cdots$$

or, equivalently

$$u(z) = \frac{p(z)-1}{p(z)+1} = \frac{1}{2}\left[p_1 z + \left(p_2 - \frac{p_1^2}{2}\right)z^2 + \cdots\right]$$

and

$$v(z) = \frac{q(z)-1}{q(z)+1} = \frac{1}{2}\left[q_1 z + \left(q_2 - \frac{q_1^2}{2}\right)z^2 + \cdots\right]. \quad (7)$$

Functions p and q are analytic in D with $p(0) = 1 = q(0)$ and have positive real parts in D.

It follows from (6) and (7), together with (3) that

$$\left[I_{a,b;c} f(z)\right]' = \varphi\left[\frac{p(z)-1}{p(z)+1}\right]$$

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and

$$\left[I_{a,b;c}g(w)\right]' = \varphi\left[\frac{q(w)-1}{q(w)+1}\right]$$

where

$$\varphi\left[\frac{p(z)-1}{p(z)+1}\right] = 1 + \frac{1}{2}B_1p_1z + \left[\frac{1}{2}B_1\left(p_2 - \frac{p_1^2}{2}\right) + \frac{1}{4}B_2p_1^2\right]z^2 + \cdots \quad (8)$$

and

$$\varphi\left[\frac{q(w)-1}{q(w)+1}\right] = 1 + \frac{1}{2}B_1q_1w + \left[\frac{1}{2}B_1\left(q_2 - \frac{q_1^2}{2}\right) + \frac{1}{4}B_2q_1^2\right]w^2 + \cdots \quad (9)$$

On the other hand,

$$\left[I_{a,b;c}f(z)\right]' = 1 + 2\emptyset_2a_2z + 3\emptyset_3a_3z^2 + \cdots \quad (10)$$

and

$$\left[I_{a,b;c}g(w)\right]' = 1 - 2\emptyset_2a_2w + 3\emptyset_3(2a_2^2 - a_3)w^2 - \cdots \quad (11)$$

Now, equating the coefficients from (8)-(11), we have

$$2\emptyset_2a_2 = \frac{1}{2}B_1p_1 \quad (12)$$

$$3\emptyset_3a_3 = \frac{1}{2}B_1\left(p_2 - \frac{p_1^2}{2}\right) + \frac{1}{4}B_2p_1^2 \quad (13)$$

$$-2\emptyset_2a_2 = \frac{1}{2}B_1q_1 \quad (14)$$

and

$$3\emptyset_3(2a_2^2 - a_3) = \frac{1}{2}B_1\left(q_2 - \frac{q_1^2}{2}\right) + \frac{1}{4}B_2q_1^2. \quad (15)$$

From (12) and (14), we obtain

$$p_1 = -q_1 \quad (16)$$

and

$$\frac{32\emptyset_2^2a_2^2}{B_1^2} = p_1^2 + q_1^2. \quad (17)$$

Now, from (13), (15) and (17), we get

$$6\emptyset_3a_2^2 = \frac{1}{2}B_1(p_2+q_2) - \frac{8\emptyset_2^2a_2^2}{B_1} + \frac{8\emptyset_2^2B_2a_2^2}{B_1^2}$$

$$a_2^2 = \frac{B_1^3(p_2+q_2)}{4\left[3\emptyset_3B_1^2 + 4\emptyset_2^2(B_1 - B_2)\right]}.$$

Applying Lemma 1 for coefficients p_2 and q_2, we have

$$|a_2| \le \frac{B_1\sqrt{B_1}}{\sqrt{\left|3\emptyset_3B_1^2 + 4\emptyset_2^2(B_1 - B_2)\right|}}.$$

Next, by subtracting (15) from (13) and further computations, it leads to

$$6\emptyset_3a_3 - 6\emptyset_3a_2^2 = \frac{1}{2}B_1(p_2 - q_2).$$

Then, it follows from (16) and (17) that

$$a_3 = \frac{B_1^2p_1^2}{16\emptyset_2^2} + \frac{B_1(p_2 - q_2)}{12\emptyset_3}.$$

Applying Lemma 1 for coefficients p_1, p_2 and q_2, yields

$$|a_3| \le B_1\left[\frac{B_1}{4\emptyset_2^2} + \frac{1}{3\emptyset_3}\right]$$

which completes the proof of Theorem 1.

Remark 1. For $a = c$ and $b = 1$, the results reduce to Theorem 1 in [12].

Remark 2. For $a = c$ and $b = 1$, the class of strongly starlike functions, the function φ is given by

$$\varphi(z) = \left(\frac{1+z}{1-z}\right)^\alpha = 1 + 2\alpha z + 2\alpha^2z^2 + \cdots, \quad (0 < \alpha \le 1)$$

which gives $B_1 = 2\alpha$ and $B_2 = 2\alpha^2$. Then, the inequalities (5) reduce to the result in [14, Theorem 1]. Furthermore, in the case

$$\varphi(z) = \frac{1+(1-2\beta)z}{1-z}$$
$$= 1 + 2(1-\beta)z + 2(1-\beta)z^2 + \cdots \quad (0 \le \beta < 1).$$

By letting $B_1 = B_2 = 2(1-\beta)$, the inequalities in (5) reduce to Theorem 2 in [14].

III. Coefficient Estimates for the Function in $\mathfrak{B}_\sigma^{a,b;c}(\varphi,\lambda)$

In this section, we determine the bound $|a_2|$ and $|a_3|$ for functions in the class $\mathfrak{B}_\sigma^{a,b;c}(\varphi,\lambda)$.

Theorem 2. Let f given by (1) be in the class $\mathfrak{B}_\sigma^{a,b;c}(\varphi,\lambda)$, $\lambda \ge 1$. Then

$$|a_2| \le \frac{B_1\sqrt{B_1}}{\sqrt{\left|B_1^2(1+2\lambda)\emptyset_3 + (B_1 - B_2)(1+\lambda)^2\emptyset_2^2\right|}}$$

and

$$|a_3| \le \frac{B_1}{(1+2\lambda)\emptyset_3} + \frac{B_1^2}{(1+\lambda)^2\emptyset_2^2}. \quad (18)$$

Proof. For $f \in \mathfrak{B}_\sigma^{a,b;c}(\varphi,\lambda)$, there are analytic functions $u, v: D \to D$ with $u(0) = v(0) = 0$, such that

$$(1-\lambda)\frac{I_{a,b;c}f(z)}{z} + \lambda\left[I_{a,b;c}f(z)\right]' = \varphi(u(z)) \quad (19)$$

and

$$(1-\lambda)\frac{I_{a,b;c}g(w)}{w} + \lambda\left[I_{a,b;c}g(w)\right]' = \varphi(v(w)). \quad (20)$$

Since

$$(1-\lambda)\frac{I_{a,b;c}f(z)}{z} + \lambda\left[I_{a,b;c}f(z)\right]'$$
$$= 1 + (1+\lambda)\emptyset_2a_2z + (1+2\lambda)\emptyset_3a_3z^2 + \cdots$$

and

$$(1-\lambda)\frac{I_{a,b;c}g(w)}{w} + \lambda\left[I_{a,b;c}g(w)\right]'$$

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$$= 1 - (1+\lambda)\emptyset_2 a_2 w + (1+2\lambda)\emptyset_3(2a_2^2 - a_3)w^2 - \cdots$$

Then, from (8), (9), (19) and (20), it follows that

$$(1+\lambda)\emptyset_2 a_2 = \frac{1}{2}B_1 p_1 \quad (21)$$

$$(1+2\lambda)\emptyset_3 a_3 = \frac{1}{2}B_1\left(p_2 - \frac{p_1^2}{2}\right) + \frac{1}{4}B_2 p_1^2 \quad (22)$$

$$-(1+\lambda)\emptyset_2 a_2 = \frac{1}{2}B_1 q_1 \quad (23)$$

and

$$(1+2\lambda)\emptyset_3(2a_2^2 - a_3) = \frac{1}{2}B_1\left(q_2 - \frac{q_1^2}{2}\right) + \frac{1}{4}B_2 q_1^2. \quad (24)$$

From (21) and (23) yield

$$p_1 = -q_1 \quad (25)$$

Now, from (22), (24) and (25) lead to

$$a_2^2 = \frac{B_1^3(p_2+q_2)}{4[B_1^2(1+2\lambda)\emptyset_3 + (B_1 - B_2)(1+\lambda)^2\emptyset_2^2]}$$

which yields the estimate on $|a_2|$ as described in (18).

Proceeding similarly as in the earlier proof, making use (22)-(25) shows that

$$a_3 = \frac{B_1^2 p_1^2}{4(1+\lambda)^2\emptyset_2^2} + \frac{B_1(p_2-q_2)}{4(1+2\lambda)\emptyset_3}.$$

Then, applying Lemma 1 for coefficients p_1, p_2 and q_2, we readily get

$$|a_3| \le \frac{B_1}{(1+2\lambda)\emptyset_3} + \frac{B_1^2}{(1+\lambda)^2\emptyset_2^2}$$

which completes the proof of Theorem 2.

Remark 3.

i) For $\lambda = 1$, the result reduces to Theorem 1.

ii) For $a = c$, $b = 1$ and $\lambda = 1$, the inequalities (18) reduces to Theorem 1 in [12].

iii) If $a = c$, $b = 1$ and $\lambda = 1$ with $\varphi(z) = \left(\frac{1+z}{1-z}\right)^{\alpha}$ and $(z) = \frac{1+(1-2\beta)z}{1-z}$, the inequalities (18) reduces to Theorem 1 and Theorem 2 in [14] respectively.

iv) If $= c$, $b = 1$, $\varphi(z) = \left(\frac{1+z}{1-z}\right)^{\alpha}$ and $\varphi(z) = \frac{1+(1-2\beta)z}{1-z}$, the result reduces to Theorem 2.1 and Theorem 2.2 given by [13].

ACKNOWLEDGMENT

This research is supported by Malaysian Ministry of Higher Education grant FRGS/1/2016/STG06/UiTM/02/1 and Universiti Teknologi Mara Cawangan Sabah.

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Rashidah Omar was born on 2 November 1974 in Kelantan, Malaysia. The author obtained the first degree in Pure Mathematics (Universiti Kebangsaan Malaysia, 1993-1998), MSc in Mathematics (Universiti Kebangsaan Malaysia, 1998-1999) and Phd in Complex Analysis (Universiti Malaya, Malysia, 2009-2013).

Currently, she is working as a senior lecturer at Universiti Teknologi Mara, Cawangan Sabah, Kota Kinabalu, Malaysia. The area of research is Complex Analysis and Geometric Function Theory.

Dr Rashidah is a member of PERSAMA (Malaysian Mathematical Sciences Society).