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Fuzzy Subalgebras and Fuzzy Ideals of BCI-Algebras with Operators

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Abstract—The aim of this paper is to introduce the concepts of fuzzy subalgebras, fuzzy ideals and fuzzy quotient algebras of BCI-algebras with operators, and to investigate their basic properties.

Keywords—BCI-algebras, BCI-algebras with operators, fuzzy subalgebras, fuzzy ideals, fuzzy quotient algebras.

I. Introduction

THE fuzzy set is a generalization of the classical set and it has been applied to many mathematical branches such as groups, rings, ideals and obtained many theories about fuzzy set since Zadeh [13] first raised the concept of fuzzy set in 1965.

BCK/BCI-algebras are two classes of logical algebras, which were introduced by Imai and Iseki [1], [2]. In 1991, Xi [3] applied the concept of fuzzy sets to BCK-algebras, since then fuzzy BCK/BCI-algebras have been extensively investigated by several researchers. Jun et al. [4], [5] introduced the concepts of fuzzy positive implicative ideals and fuzzy commutative ideals of BCK-algebras. Meng et al. [6] introduced the concept of fuzzy implicative ideals of BCK-algebras. Jun et al. [7] introduced the concept of commutative ideals of BCI-algebras, Liu and Meng [9], [10] introduced the concepts of fuzzy positive implicative ideals and fuzzy implicative ideals of BCI-algebras. In 1993, Zheng [8] defined operators in BCK-algebras and introduced the concept of BCI-algebras with operators and gave some isomorphism theorems of it. Next, Liu [12] introduced the university property of direct products of BCI-algebras. In 2002, Liu [11] introduced the concept of the fuzzy quotient algebras of BCI-algebras.

In this paper, we introduce the definitions of fuzzy subalgebras, fuzzy ideals and fuzzy quotient algebras of BCI-algebras with operators, Moreover, the basic properties were discussed and many results have been obtained, which enriches the theory of BCK/BCI-algebras.

II. Preliminaries

We recall some definitions and propositions which will be needed.

An algebra $\langle X;*,0\rangle$ of type (2,0) is called a BCI-algebra, if it satisfies the following conditions:

$$BCI-(1)\left(\left(x*y\right)*\left(x*z\right)\right)*\left(z*y\right)=0,$$
$$BCI-(2)\left(x*\left(x*y\right)\right)*y=0,\ BCI-(3)\,x*x=0,$$
$$BCI-(4)\,x*y=0 \text{ and } y*x=0 \text{ imply } x=y,$$

for all $x,y,z\in X$. We can define $x*y=0$ if and only if $x\le y$, then the above conditions can be written as:

1. $(x*y)*(x*z)\le z*y$,
2. $x*(x*y)\le y$,
3. $x\le x$,
4. $x\le y$ and $y\le x$ imply $x=y$,

for all $x,y,z\in X$. If a BCI-algebra satisfies the identity $0*x=0$, then it is called a BCK-algebra.

Definition 1. If $\langle X;*,0\rangle$ is a BCI-algebra, A is a non-empty subset of X, and $x*y\in A$ for all $x,y\in A$, then $\langle A;*,0\rangle$ is called a subalgebra of $\langle X;*,0\rangle$.

Definition 2. [10] A fuzzy set in a set S is a function A from S into $[0,1]$.

Definition 3. [4] If $\langle X;*,0\rangle$ is a BCI-algebra, a fuzzy set A of X is called a fuzzy subalgebra of X if for all $x,y\in X$, it satisfies:

$$A(x*y)\ge A(x)\wedge A(y).$$

Definition 4. [5] $\langle X;*,0\rangle$ is a BCI-algebra, a fuzzy subset A of X is called a fuzzy ideal of X if it satisfies:

1. $A(0)\ge A(x),\forall x\in X$,
2. $A(x)\ge A(x*y)\wedge A(y),\forall x,y\in X$.

Definition 5. [6] $\langle X;*,0\rangle$ is a BCI-algebra, M is a non-empty set, if there exists a mapping $(m,x)\to mx$ from $M\times X$ to X which satisfies

$$m(x*y)=(mx)*(my),\forall x,y\in X,m\in M.$$

then M is called a left operator of X, X is called a BCI-algebra with left operator M, or $M-$BCI-algebra for short.

Proposition 1. Let $\langle X;*,0\rangle$ be a $M-$BCI-algebra, if A is a

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International Science Index, Mathematical and Computational Sciences Vol:11, No:6, 2017 waset.org/Publication/10007869

fuzzy ideal of it, and $x*y \le z$, then $A(x) \ge A(y) \wedge A(z)$ for all $x, y, z \in X$.

Definition 6. Let A and B be fuzzy sets of set X, then the direct product $A \times B$ of A and B is a fuzzy subset of $X \times X$, define $A \times B$ by

$$A \times B(x, y) = A(x) \wedge B(y), \forall x, y \in X.$$

Definition 7. [6] Let $\langle X;*,0\rangle$ and $\langle \bar{X};*,0\rangle$ be two $M-$BCI-algebras, if f is a homomorphism from $\langle X;*,0\rangle$ to $\langle \bar{X};*,0\rangle$, and $f(mx) = mf(x)$ for all $x \in X$, $m \in M$, then f is called a homomorphism with operators.

Definition 8. $\langle X;*,0\rangle$ is a $M-$BCI-algebra, let B be a fuzzy set of X, and A be a fuzzy relation of B, if

$$A_B(x, y) = B(x) \wedge B(y) \text{ for all } x, y \in X,$$

then A is called a strong fuzzy relation of B. In the following parts, X always means an $M-$BCI-algebra unless otherwise specified.

III. Fuzzy Subalgebras of BCI-Algebras with Operators

Definition 9. If $\langle X;*,0\rangle$ is an $M-$BCI-algebra, A is a non-empty subset of X, and $mx \in A$ for all $x \in A, m \in M$, then $\langle A;*,0\rangle$ is called an $M-$subalgebra of $\langle X;*,0\rangle$.

Definition 10. $\langle X;*,0\rangle$ is a $M-$BCI-algebra, A is a fuzzy subalgebra of X , if $A(mx) \ge A(x)$ for all $x \in X, m \in M$, then A is called an $M-$fuzzy subalgebra of X.

Example 1. If A is an $M-$fuzzy subalgebra of X , then X_A is an $M-$fuzzy subalgebra of X, define X_A by

$$X_A : X \to [0,1], X_A(x) = \begin{cases} 1, x \in A \\ 0, x \notin A. \end{cases}$$

Proof. (1) For all $x, y \in X$, if $x, y \in A$, then $x*y \in A$, therefore

$$X_A(x*y) = 1 \ge X_A(x) \wedge X_A(y),$$

if there exists at least one which does not belong to A between x and y, for example $x \notin A$, thus

$$X_A(x*y) \ge 0 = X_A(x) \wedge X_A(y),$$

therefore X_A is a fuzzy subalgebra of X.

(2) For all $x \in X$, $m \in M$, if $x \in A$, then $mx \in A$, therefore

$$X_A(mx) = 1 \ge X_A(x),$$

if $x \notin A$, then

$$X_A(mx) \ge 0 = X_A(x),$$

therefore X_A is an $M-$fuzzy subalgebra of X.

Proposition 3. A is an $M-$fuzzy subalgebra of X if only if A_t is an $M-$subalgebra of X, where A_t is a non-empty set, define X_A by

$$A_t = \{x \mid x \in X, A(x) \ge t\}, \forall t \in [0,1].$$

Proof. Suppose A is an $M-$fuzzy subalgebra of X, A_t is a non-empty set, $t \in [0,1]$, then we have

$$A(x*y) \ge A(x) \wedge A(y).$$

If $x \in A_t, y \in A_t$, then

$$A(x) \ge t, A(y) \ge t,$$

thus

$$A(x*y) \ge A(x) \wedge A(y) \ge t,$$

thus we have

$$x*y \in A_t.$$

For all $x \in X, m \in M$, if A is an $M-$fuzzy subalgebra of X, hence

$$A(mx) \ge A(x) \ge t,$$

thus

$$mx \in A_t,$$

therefore A_t is an $M-$subalgebra of X . Conversely, suppose A_t is an $M-$subalgebra of X, then we have $x*y \in A_t$. Let $A(x) = t$, then

$$A(x*y) \ge t = A(x) \ge A(x) \wedge A(y).$$

For all $x \in X$, $m \in M$, if A_t is an $M-$subalgebra of X, then we have

$$A(mx) \ge t = A(x),$$

therefore A is an $M-$fuzzy subalgebra of X.

Proposition 4. Suppose X, Y are $M-$BCI-algebra, f is a mapping from X to Y, if A is an $M-$fuzzy subalgebra of

International Science Index, Mathematical and Computational Sciences Vol:11, No:6, 2017 waset.org/Publication/10007869

the Y, then $f^{-1}(A)$ is an $M-$fuzzy subalgebra of X.

Proof. Let $y \in Y$, suppose f is a epimorphism, then there exists x in X, we have $y = f(x)$. If A is an $M-$fuzzy subalgebra of Y, then we have

$$A(x*y) \geq A(x) \wedge A(y), A(mx) \geq A(x).$$

For all $x, y \in X, m \in M$,

(1) $f^{-1}(A)(x*y) = A(f(x)*f(y)) \geq A(f(x)) \wedge A(f(Y))$
$= f^{-1}(A)(x) \wedge f^{-1}(A)(y)$;

(2) $f^{-1}(A)(mx) = A(f(mx)) = A(mf(x)) \geq A(f(x))$
$= f^{-1}(A)(x)$.

Therefore $f^{-1}(A)$ is an $M-$fuzzy subalgebra of X.

IV. Fuzzy Ideals of BCI-Algebras with Operators

Definition 11. $\langle X;*,0\rangle$ is an $M-$BCI-algebra, A is a fuzzy ideal of X, if $A(mx) \geq A(x)$ for all $x \in X, m \in M$, then A is called an $M-$fuzzy ideal of X.

Example 2. If A is an $M-$ fuzzy ideal of X, then X_A is an $M-$ fuzzy ideal of X, define X_A by

$$X_A : X \to [0,1], X_A(x) = \begin{cases} 1, x \in A \\ 0, x \notin A. \end{cases}$$

Proof. (1) For all $x, y \in X$, if $x, y \in A$, then $x*y \in A$, therefore

$$X_A(0) = 1 \geq X_A(x),\ X_A(x) = 1 \geq X_A(x*y) \wedge X_A(y),$$

if there exists at least one which does not belong to A between x and y, for example $x \notin A$, thus

$$X_A(0) = 1 \geq X_A(x),\ X_A(x) \geq X_A(x*y) \wedge X_A(y) = 0,$$

therefore X_A is a fuzzy ideal of X.

(2) For all $x \in X$, $m \in M$, if $x \in A$, then $mx \in A$, therefore

$$X_A(mx) = 1 \geq X_A(x).$$

If $x \notin A$, then

$$X_A(mx) \geq 0 = X_A(x),$$

therefore X_A is an $M-$fuzzy ideal of X.

Proposition 5. A is an $M-$fuzzy ideal of X if only if A_t is an $M-$ideal of X, where A_t is non-empty set, define A_t by

$$A_t = \{x \mid x \in X, A(x) \geq t\}, \forall t \in [0,1].$$

Proof. Suppose A is an $M-$fuzzy ideal of X, A_t is non-empty set, $t \in [0,1]$, then we have

$$A(0) \geq A(x) \geq t,$$

thus $0 \in A_t$. If $x*y \in A_t, y \in A_t$, then

$$A(x*y) \geq t, A(y) \geq t,$$

thus

$$A(x) \geq A(x*y) \wedge A(y) \geq t,$$

thus we have

$$x \in A_t.$$

For all $x \in X$, $m \in M$, if A is an $M-$fuzzy ideal of X, hence

$$A(mx) \geq A(x) \geq t,$$

thus

$$mx \in A_t,$$

therefore A_t is an $M-$ideal of X. Conversely, suppose A_t is an $M-$ideal of X, then we have $0 \in A_t, A(0) \geq t$. Let $A(x) = t$, thus $x \in A_t$, we have

$$A(0) \geq t = A(x),$$

suppose there is no

$$A(x) \geq A(x*y) \wedge A(y),$$

then there exist $x_0, y_0 \in X$, we have

$$A(x_0) < A(x_0 * y_0) \wedge A(y_0),$$

let $t_0 = A(x_0 * y_0) \wedge A(y_0)$, then

$$A(x_0) < t_0 = A(x_0 * y_0) \wedge A(y_0),$$

if $x_0 * y_0 \in A_{t_0}, y_0 \in A_{t_0}$, then we have

$$x_0 \in A_{t_0},$$

then

$$A(x_0) \geq t_0,$$

International Science Index, Mathematical and Computational Sciences Vol:11, No:6, 2017 waset.org/Publication/10007869

which is inconsistent with $A(x_0) < t_0 = A(x_0 * y_0) \wedge A(y_0)$, then we have

$$A(x) \geq A(x * y) \wedge A(y).$$

For all $x \in X, m \in M$, if A_t is an $M-$ideal of X, then we have

$$A(mx) \geq t = A(x),$$

therefore A is an $M-$fuzzy ideal of X.

Proposition 6. Suppose X, Y are $M-$BCI-algebras, f is a mapping from X to Y, A is an $M-$fuzzy ideal of Y, then $f^{-1}(A)$ is an $M-$fuzzy ideal of X.

Proof. Let $y \in Y$, suppose f is an epimorphism, then there exists $x \in X$, we have $y = f(x)$. If A is an $M-$fuzzy ideal of Y, then we have

$$A(0) \geq A(y) \text{ or } A(f(0)) \geq A(y).$$

For all $x, y \in X, m \in M$,

(1) $f^{-1}(A)(0) = A(f(0)) = A(0) \geq A(f(x)) = f^{-1}(A)(x)$;

(2) $f^{-1}(A)(x) = A(f(x))$
$\geq A(f(x) * f(y)) \wedge A(f(y)) = A(f(x * y)) \wedge A(f(y))$
$= f^{-1}(A)(x * y) \wedge f^{-1}(A)(y)$;

(3) $f^{-1}(A)(mx) = A(f(mx)) = A(mf(x)) \geq A(f(x)) = f^{-1}(A)(x)$.

Therefore $f^{-1}(A)$ is an $M-$fuzzy ideal of X.

V. Fuzzy Quotient BCI-Algebras with Operators

Definition 12. Let A be an $M-$fuzzy ideal of X, for all $a \in X$, fuzzy set A_a on X defined as:

$$A_a : X \to [0,1]$$
$$A_a(x) = A(a * x) \wedge A(x * a), \forall x \in X.$$

Denote $X/A = \{A_a : a \in X\}$.

Proposition 7. Let $A_a, A_b \in X/A$, then $A_a = A_b$ if only if $A(a * b) = A(b * a) = A(0)$.

Proof. Let $A_a = A_b$, then we have $A_a(b) = A_b(b)$, thus

$$A(a * b) \wedge A(b * a) = A(b * b) \wedge A(b * b) = A(0).$$

That is $A(a * b) = A(b * a) = A(0)$. Conversely, suppose that $A(a * b) = A(b * a) = A(0)$. For all $x \in X$, since

$$(a * x) * (b * x) \leq a * b, (x * a) * (x * b) \leq b * a.$$

It follows from Proposition 1 that

$$A(a * x) \geq A(b * x) \wedge A(a * b), A(x * a) \geq A(x * b) \wedge A(b * a).$$

Hence

$$A_a(x) = A(a * x) \wedge A(x * a) \geq A(b * x) \wedge A(x * b) = A_b(x).$$

That is $A_a \geq A_b$. Similarly, for all $x \in X$, since

$$(b * x) * A(a * x) \leq b * a, (x * b) * A(x * a) \leq a * b.$$

It follows from Proposition 1 that

$$A(b * x) \geq A(a * x) \wedge A(b * a), A(x * b) \geq A(x * a) \wedge A(a * b).$$

Hence

$$A_b(x) = A(b * x) \wedge A(x * b) \geq A(a * x) \wedge A(x * a) = A_a(x).$$

That is $A_b \geq A_a$. Therefore, $A_a = A_b$. we complete the proof.

Proposition 8. Let $A_a = A_{a'}, A_b = A_{b'}$, then $A_{a*b} = A_{a'*b'}$.

Proof. Since

$$\begin{aligned}((a * b) * (a' * b')) * (a * a') &= ((a * b) * (a * a')) * (a' * b') \\ &\leq (a' * b) * (a' * b') \leq b' * b, \\ ((a' * b') * (a * b)) * (b * b') &= ((a' * b') * (b * b')) * (a * b) \\ &\leq (a' * b) * (a * b) \leq a' * a.\end{aligned}$$

Hence

$$A((a * b) * (a' * b')) \geq A(a * a') \wedge A(b' * b) = A(0),$$

$$A((a' * b') * (a * b)) \geq A(b * b') \wedge A(a' * a) = A(0).$$

Therefore

$$A((a * b) * (a' * b')) = A((a' * b') * (a * b)) = A(0),$$

it follows from Proposition 7 that $A_{a*b} = A_{a'*b'}$. we completed the proof.

Let A be an $M-$fuzzy ideal of X. The operation "$*$" of R/A is defined as:

$$\forall A_a, A_b \in R/A, A_a * A_b = A_{a*b}.$$

By Proposition 7, the above operation is reasonable.

Proposition 9. Let A be an $M-$fuzzy ideal of X, then

International Science Index, Mathematical and Computational Sciences Vol:11, No:6, 2017 waset.org/Publication/10007869

$R/A=\{R/A;*,A_0\}$ is an $M-$BCI-algebra.

Proof. For all $A_x, A_y, A_z \in R/A$,

$$\left(\left(A_x * A_y\right)*\left(A_x * A_z\right)\right)*\left(A_z * A_y\right) = A_{((x*y)*(x*z))*(z*y)} = A_0;$$
$$\left(A_x *\left(A_x * A_y\right)\right)* A_y = A_{(x*(x*y))*y} = A_0;\ A_x * A_x = A_{x*x} = A_0;$$

if $A_x * A_y = A_0, A_y * A_x = A_0$, then

$$A_{x*y} = A_0, A_{y*x} = A_0,$$

it follows from Proposition 7 that

$$A(x*y)=A(0), A(y*x)=A(0),$$

hence

$$A_x = A_y.$$

Therefore $R/A=\{R/A;*,A_0\}$ is a BCI-algebra. For all $A_x \in R/A, m\in M$, we define $mA_x = A_{mx}$. Firstly, we verify that $mA_x = A_{mx}$ is reasonable. If $A_x = A_y$, then we verify

$$mA_x = mA_y,$$

that is to verify

$$A_{mx} = A_{my}.$$

We have

$$A(mx*my)=A(m(x*y))\geq A(x*y)=A(0)$$

and

$$A(my*mx)=A(m(y*x))\geq A(y*x)=A(0),$$

so we have

$$A(mx*my)=A(my*mx)=A(0),$$

that is, $A_{mx}=A_{my}$. In addition, for all $m\in M, A_x, A_y \in R/A$,

$$m\left(A_x * A_y\right) = mA_{x*y} = A_{m(x*y)} = A_{(mx)*(my)} = A_{mx} * A_{my} = mA_x * mA_y.$$

Therefore $R/A=\{R/A;*,A_0\}$ is an $M-$BCI-algebra.

Definition 13. Let μ be an $M-$fuzzy subalgebra of X, and A be an $M-$fuzzy ideal of X, we define a fuzzy set of X/A as:

$$\mu/A: X/A \to [0,1],\ \ \mu/A(A_t)=\sup_{A_x=A_t}\mu(x), \forall A_t \in X/A.$$

Proposition 10. μ/A is an $M-$fuzzy subalgebea of X/A.

Proof. For all $A_x, A_y \in X/A$,

$$\begin{aligned}\mu/A\left(A_x * A_y\right) &= \mu/A\left(A_{x*y}\right)\\ &= \sup_{A_z=A_{x*y}} \mu(z) \geq \sup_{A_s=A_x, A_t=A_y} \mu(s*t) \geq \sup_{A_s=A_x, A_t=A_y} \mu(s)\wedge\mu(t)\\ &= \sup_{A_s=A_x,} \mu(s) \wedge \sup_{A_t=A_y,} \mu(t) = \mu/A\left(A_x\right)\wedge \mu/A\left(A_y\right).\end{aligned}$$

For all $m\in M, A_x \in R/A$,

$$\mu/A\left(A_{mx}\right)=\sup_{A_{mz}=A_{mx}}\mu(mz)\geq \sup_{A_z=A_x}\mu(z)=\mu/A\left(A_x\right).$$

Therefore, μ/A is an $M-$fuzzy subalgebra of X/A.

VI. Direct Products of Fuzzy Ideals in BCI-Algebras with Operators

Proposition 11. Suppose A and B are $M-$fuzzy ideals of X, then $A\times B$ is an $M-$fuzzy ideal of $X\times X$.

Proof. (1)Let $(x,y)\in X\times X$, then

$$A\times B(0,0)=A(0)\wedge B(0)\geq A(x)\wedge B(y)=A\times B(x,y),$$

thus for all $(x,y)\in X\times X, A\times B(0,0)\geq A\times B(x,y)$;

(2) For all $(x_1,x_2),(y_1,y_2)\in X\times X$, we have

$$\begin{aligned}&A\times B\left(\left(x_1,x_2\right)*\left(y_1,y_2\right)\right)\wedge A\times B\left(y_1,y_2\right)\\ &= A\times B\left(x_1*y_1, x_2*y_2\right)\wedge A\times B\left(y_1,y_2\right)\\ &=\left(A\left(x_1*y_1\right)\wedge B\left(x_2*y_2\right)\right)\wedge A\left(y_1\right)\wedge B\left(y_2\right)\\ &=\left(A\left(x_1*y_1\right)\wedge A\left(y_1\right)\right)\wedge\left(B\left(x_2*y_2\right)\wedge B\left(y_2\right)\right)\\ &\leq A\left(x_1\right)\wedge B\left(x_2\right)\\ &= A\times B\left(x_1,x_2\right),\end{aligned}$$

thus for all $(x_1,x_2),(y_1,y_2)\in X\times X$, we have

$$A\times B\left(x_1,x_2\right)\geq A\times B\left(\left(x_1,x_2\right)*\left(y_1,y_2\right)\right)\wedge A\times B\left(y_1,y_2\right);$$

(3) For all $(x,y)\in X\times X$, we have

$$\begin{aligned}A\times B\left(m(x,y)\right)&=A\times B(mx,my) = A(mx)\wedge B(my)\\ &\geq A(x)\wedge B(y)=A\times B(x,y),\end{aligned}$$

thus for all $\forall(x,y)\in X\times X$, we have

$$A\times B\left(m(x,y)\right)\geq A\times B(x,y).$$

Therefore $A\times B$ is an $M-$fuzzy ideal of $X\times X$.

Proposition 12. Suppose A and B are fuzzy sets of X, if

International Science Index, Mathematical and Computational Sciences Vol:11, No:6, 2017 waset.org/Publication/10007869

$A\times B$ is an $M-$fuzzy ideal of $X\times X$, then A or B is an $M-$fuzzy ideal of X.

Proof. Suppose A and B are $M-$fuzzy ideals of X, then for all $(x_1,x_2),(y_1,y_2)\in X\times X$, we have

$$\begin{aligned}A\times B(x_1,x_2)&\geq A\times B((x_1,x_2)*(y_1,y_2))\wedge A\times B(y_1,y_2)\\&=A\times B((x_1*y_1),(x_2*y_2))\wedge A\times B(y_1,y_2),\end{aligned}$$

if $x_1=y_1=0,$ then

$$A\times B(0,x_2)\geq A\times B(0,x_2*y_2)\wedge A\times B(0,y_2),$$

we have $A\times B(0,x)=A(0)\wedge B(x)=B(x)$, so $B(x_2)\geq B(x_2*y_2)\wedge B(y_2)$. If $A\times B$ is an $M-$fuzzy ideal of X, then

$$A\times B(m(x,y))\geq A\times B(x,y),\forall(x,y)\in X\times X,$$

let $x=0$, then

$$\begin{aligned}&A\times B(m(x,y))=A\times B(mx,my)=A(mx)\wedge B(my)=B(my)\\&\geq A(x)\wedge B(y)=A(0)\wedge B(y)=B(y),\end{aligned}$$

thus we have $B(my)\geq B(y)$ for all $y\in X,m\in M$. Therefore B is an $M-$fuzzy ideal of X.

Proposition 13. If B is a fuzzy set, A is a strong fuzzy relation A_B of B, then B is a $M-$ fuzzy ideal of X if only if A_B is an $M-$fuzzy ideal of $X\times X$.

Proof. If B is an $M-$fuzzy ideals of X, then for all $(x,y)\in X\times X$, we have

$$A_B(0,0)=B(0)\wedge B(0)\geq B(x)\wedge B(y)=A_B(x,y);$$

for all $(x_1,x_2),(y_1,y_2)\in X\times X$, we have

$$\begin{aligned}A_B(x_1,x_2)&=B(x_1)\wedge B(x_2)\\&\geq(B(x_1*y_1)\wedge B(y_1))\wedge(B(x_2*y_2)\wedge B(y_2))\\&=(B(x_1*y_1)\wedge B(x_2*y_2))\wedge(B(y_1)\wedge B(y_2))\\&=A_B(x_1*y_1,x_2*y_2)\wedge A_B(y_1,y_2)\\&=A_B((x_1,x_2)*(y_1,y_2))\wedge A_B(y_1,y_2);\end{aligned}$$

for all $(x,y)\in X\times X,m\in M,$

$$\begin{aligned}A_B(m(x,y))&=A_B(mx,my)=B(mx)\wedge B(my)\\&\geq B(x)\wedge B(y)=A_B(x,y).\end{aligned}$$

Therefore, if B is an $M-$fuzzy ideal of X, then A_B is an $M-$fuzzy ideal of $X\times X$. Conversely, suppose A_B is an $M-$fuzzy ideal of $X\times X$, then $\forall(x_1,x_2)\in X\times X$, we have

$$B(0)\wedge B(0)=A_B(0,0)\geq A_B(x,x)=B(x)\wedge B(x);$$

for all $(x_1,x_2),(y_1,y_2)\in X\times X$, we have

$$\begin{aligned}B(x_1)\wedge B(x_2)&=A_B(x_1,x_2)\\&\geq A_B((x_1,x_2)*(y_1,y_2))\wedge A_B(y_1,y_2)\\&=A_B(x_1*y_1,x_2*y_2)\wedge A_B(y_1,y_2)\\&=(B(x_1*y_1)\wedge B(x_2*y_2))\wedge(B(y_1)\wedge B(y_2))\\&=(B(x_1*y_1)\wedge B(y_1))\wedge(B(x_2*y_2)\wedge B(y_2));\end{aligned}$$

let $x_2=y_2=0,$ then

$$B(x_1)\wedge B(0)\geq(B(x_1*y_1)\wedge B(y_1))\wedge B(0),$$

if A_B is an $M-$fuzzy ideal of $X\times X$, then

$$\begin{aligned}&A_B(m(x,y))\geq A_B(x,y),\forall x,y\in X\times X,m\in M,\\&B(mx)\wedge B(my)=A_B(mx,my)\geq A_B(x,y)=B(x)\wedge B(y),\end{aligned}$$

if $x=0,$ then

$$B(0)\wedge B(my)=A_B(0,my)\geq A_B(0,y)=B(0)\wedge B(y),$$

namely, $B(my)\geq B(y)$. Therefore B is an $M-$fuzzy ideal of X.

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