International Journal of Applied and Advanced Scientific Research (IJAASR)
Impact Factor: 5.655, ISSN (Online): 2456 - 3080
(www.dvpublication.com) Volume 2, Issue 2, 2017

REPRESENTATIONS OF EDGE REGULAR BIPOLAR FUZZY GRAPHS

V. Ramadass* & D. Kalpana**
* Professor, Department of Mathematics, Prist University, Tanjore, Tamilnadu
** Research Scholar, Department of Mathematics, Prist University, Tanjore, Tamilnadu

Cite This Article: V. Ramadass & D. Kalpana, "Representations of Edge Regular Bipolar Fuzzy Graphs", International Journal of Applied and Advanced Scientific Research, Volume 2, Issue 2, Page Number 267-272, 2017.

Abstract:

In this paper the concept of an edge regular bipolar fuzzy graph is introduced and some relationship between degree of a vertex and degree of an edge in bipolar fuzzy graphs are studied. Also some properties of an edge regular bipolar fuzzy graphs are provided.

Key Words: bipolar fuzzy graph, Degree of a Vertex, Order of a Bipolar Fuzzy Graph, Size of a Bipolar Fuzzy Graph & Regular Bipolar Fuzzy Graph

1. Introduction:

In 1965,L.A.Zadeh[1] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. In many domains, it is important to be able to deal with bipolar fuzzy information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several direction. Akram[2] introduced the concept of bipolar fuzzy graphs and defined different operations on it. A.Nagoorgani and K.Radha[3,4] introduced the concept of regular fuzzy graphs in 2008 and discussed about the degree of a vertex in some fuzzy graphs. K.Radha and N.Kumaravel[5] introduced the concept of edge degree, total edge degree and discussed about the degree of an edge in some fuzzy graphs. S.Arumugam and S.Velammal[6] discussed edge domination in fuzzy graphs.A.Nagoorgani and M.Baskar Ahamed[7] discussed order and size in fuzzy graph. A.Nagoorgani and J.Malarvizhi discussed properties of μ complement of a fuzzy graph. In this paper we introduce edge regular bipolar fuzzy graph. We provide some relationship between degree of a vertex and degree of an edge. Also we study some properties of edge regular bipolar fuzzy graph.

2. Preliminaries:

2.1 Bipolar Fuzzy Graph [1]: By a bipolar fuzzy graph, we mean a pair G = (A,B) where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on V such that $\mu_B^P\,(x,y) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N\,(x,y) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all$(x,y) \in E$. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E respectively. Note that B is symmetric bipolar fuzzy relation on A. we use the notation xy for an element of E. Thus,
G = (A,B) is a bipolar fuzzy graph of $G^* = (V,E)$ if $\mu_B^P\,(xy) \le \min(\mu_A^P(x), \mu_A^P(y))$ and
$\mu_B^N\,(xy) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all $xy \in E$.

2.2 Degree of a Vertex [8]: Let G:(A,B) be a bipolar fuzzy graph on $G^*:(V,E)$. The degree of a vertex $(\mu_A^P(x), \mu_A^N(x))$ is $d_G(\mu_A^P(x), \mu_A^N(x)) = \sum_{x \ne y}(\mu_B^P(xy), \mu_B^N(xy)) = \sum_{xy \in E}(\mu_B^P(xy), \mu_B^N(xy))$

The minimum degree of G is $\delta(G) = \cap\, \{d_G(\mu_A^P(x), \mu_A^N(x)),\ \forall\, x \,\epsilon\, V\}$

The maximum degree of G is $\delta(G) = \cup\, \{d_G(\mu_A^P(x), \mu_A^N(x)),\ \forall\, x \,\epsilon\, V\}$

2.3 Total Degree of a Vertex [4]: Let G:(A,B) be a bipolar fuzzy graph on $G^*:(V,E)$. The total degree of a vertex $(\mu_A^P(x), \mu_A^N(x))$ is $t\,d_G(\mu_A^P(x), \mu_A^N(x)) = \sum_{\substack{x \ne y \\ xy \in E}} (\mu_B^P(xy), \mu_B^N(xy)) + (\mu_A^P(x), \mu_A^N(x))$

2.4 Degree of an Edge in a Graph [6]: Let $G^*:(V,E)$ be a graph and let $e = uv\,\epsilon E$ be an edge in G^*. Then the degree of the edge $uv\,\epsilon E$ is defined by $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$

2.5 Degree of an Edge in a Bipolar Fuzzy Graph [5]: Let G:(A,B) be a bipolar fuzzy graph on $G^*:(V,E)$. The degree of an edge $(\mu_B^P(xy), \mu_B^N(xy))$ is

$$d_G(\mu_B^P(xy), \mu_B^N(xy)) = d_G(\mu_A^P(x), \mu_A^N(x)) + d_G\big(\mu_A^P(y), \mu_A^N(y)\big) - 2(\mu_B^P(xy), \mu_B^N(xy))$$
$$= \sum_{\substack{xz \in E \\ x \ne z}}(\mu_B^P(xz), \mu_B^N(xz)) + \sum_{\substack{yz \in E \\ y \ne z}}(\mu_B^P(yz), \mu_B^N(yz))$$

The minimum edge degree of G is $\delta_E(G) = \cap\, \{d_G(\mu_B^P(xy), \mu_B^N(xy)),\ \forall\, xy \,\epsilon\, E\}$
The maximum edge degree of G is $\delta(G) = \cup\, \{d_G(\mu_B^P(xy), \mu_B^N(xy)),\ \forall\, xy \,\epsilon\, E\}$

2.6 Example:

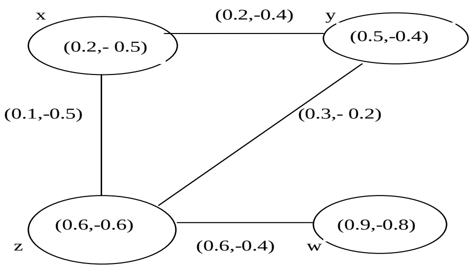

Figure 2.1

$d_G(\mu_A^P(x), \mu_A^N(x)) = (0.3, -0.9)$
$t\, d_G(\mu_A^P(x), \mu_A^N(x)) = (0.5, -1.4)$
$\delta(G) = \cap \{d_G(\mu_A^P(x), \mu_A^N(x)),\ \forall\, x \epsilon\, V\} = \cap \{(0.3, -0.9), (0.5, -0.6), (1.0, -1.1), (0.6, -0.4)\}$
$\delta(G) = (0.3, -1.1)$
$\Delta(G) = \cup \{d_G(\mu_A^P(x), \mu_A^N(x)),\ \forall\, x \epsilon\, V\} = \cup \{(0.3, -0.9), (0.5, -0.6), (1.0, -1.1), (0.6, -0.4)\}$
$\Delta(G) = (1.0, -0.4)$
$d_G(\mu_B^P(xy), \mu_B^N(xy)) = (0.4, -0.7)$
$t\, d_G(\mu_B^P(xy), \mu_B^N(xy)) = (0.6, -1.1)$
$\delta_E(G) = \cap \{d_G(\mu_B^P(xy), \mu_B^N(xy)),\ \forall\, xy \epsilon\, E\} = \cap \{(0.4, -0.7), (0.9, -1.3), (1.1, -1.0), (0.4, -0.7)\}$
$\delta_E(G) = (0.4, -1.3)$
$\Delta_E(G) = \cup \{d_G(\mu_B^P(xy), \mu_B^N(xy)),\ \forall\, xy \epsilon\, E\} = \cup \{(0.4, -0.7), (0.9, -1.3), (1.1, -1.0), (0.4, -0.7)\}$
$\Delta_E(G) = (1.1, -0.7)$

2.7 Order of a Bipolar Fuzzy Graph [7]: The order of a bipolar fuzzy graph G is defined by

$$O(G) = \sum_{x \in V} \left(\mu_A^P(x), \mu_A^N(x)\right).$$

2.8 Size of a Bipolar Fuzzy Graph [7]: The size of a bipolar fuzzy graph G is defined by

$$S(G) = \sum_{xy \in E} \left(\mu_B^P(xy), \mu_B^N(xy)\right).$$

2.9 Regular Bipolar Fuzzy Graph [3]: Let G: (A,B) be a bipolar fuzzy graph on $G^*: (V, E)$. If each vertex in G has same degree k then G is said to be regular bipolar fuzzy graph (or) k-regular bipolar fuzzy graph.

2.10 Theorem [7]: Let G: (A,B) be a bipolar fuzzy graph on $G^*: (V, E)$. Then

$$\sum_{x \in V} d_G\left(\mu_A^P(x), \mu_A^N(x)\right) = 2S(G).$$

2.11 Theorem [3]: Let G: (A,B) be a bipolar fuzzy graph on $G^*: (V, E)$. Then

$$\sum_{x \in V} t\, d_G\left(\mu_A^P(x), \mu_A^N(x)\right) = 2S(G) + O(G).$$

3. Some Results Based on Vertex Degree and Edge Degree:

3.1 Theorem: Let G: (A,B) be a bipolar fuzzy graph on a cycle $G^*: (V, E)$. Then

$$\sum_{x \in V} d_G\left(\mu_A^P(x), \mu_A^N(x)\right) = \sum_{xy \in E} d_G\left(\mu_B^P(xy), \mu_B^N(xy)\right).$$

ie) $\sum_{v \in V} d_G(v) = \sum_{uv \in E} d_G(uv)$.

Proof:

Let G^* be a cycle $v_1 v_2 v_3 \ldots\ldots v_n v_1$.

Then $\sum_{i=1}^{n} d_G(v_i\, v_{i+1}) = d_G(v_1 v_2) + d_G(v_2 v_3) + \cdots .. d_G(v_n v_1)$, where $v_{n+1} = v_1$

$= d_G(v_1) + d_G(v_2) - 2\mu(v_1 v_2) + d_G(v_2) + d_G(v_3) - 2\mu(v_2 v_3) + \cdots d_G(v_n) + d_G(v_1) - 2\mu(v_n v_1)$

$= 2(d_G(v_1) + d_G(v_2) + \cdots d_G(v_n)) - 2(\mu(v_1 v_2) + 2\mu(v_2 v_3) + \cdots + 2\mu(v_n v_1))$

$= 2\sum_{i=1}^{n} d_G(v_i) - 2\sum_{i=1}^{n} \mu(v_i\, v_{i+1})$

$= \sum_{v_i \in V} d_G(v_i) + \sum_{v_i \in V} d_G(v_i) - 2S(G)$, since $S(G) = \sum_{uv \in E} \mu(uv)$.

$= \sum_{v_i \in V} d_G(v_i) + 2S(G) - 2S(G)$, since $\sum_{v_i \in V} d_G(v_i) = 2S(G) = \sum_{v_i \in V} d_G(v_i)$

Hence $\sum_{v \in V} d_G(v) = \sum_{uv \in E} d_G(uv)$.

3.2 Theorem: Let G: (A,B) be a bipolar fuzzy graph on $G^*: (V, E)$. Then

$$\sum_{xy \in E} d_G\left(\mu_B^P(xy), \mu_B^N(xy)\right) = \sum_{xy \in E} d_{G^*}\left(\mu_B^P(xy), \mu_B^N(xy)\right)\left(\mu_B^P(xy), \mu_B^N(xy)\right).$$

Where $d_{G^*}\left(\mu_B^P(xy), \mu_B^N(xy)\right) = d_{G^*}\left(\mu_A^P(x), \mu_A^N(x)\right) + d_{G^*}\left(\mu_A^P(y), \mu_A^N(y)\right) - 2$ for all $x, y \in V$

Proof:

By the definition of edge degree in a bipolar fuzzy graph, edge degree is sum of the membership values of its adjacent edges.

Therefore in $\sum_{xy \in E} d_G\left(\mu_B^P(xy), \mu_B^N(xy)\right)$,every edge contributes its membership value exactly number of edges adjacent to that edge times.

Thus, in $\sum_{xy \in E} d_G\left(\mu_B^P(xy), \mu_B^N(xy)\right)$,each $\left(\mu_B^P(xy), \mu_B^N(xy)\right)$ appears $d_{G^*}\left(\mu_B^P(xy), \mu_B^N(xy)\right)$ times and these are the only values in that sum.

Hence $\sum_{xy\in E} d_G(\mu_B^P(xy), \mu_B^N(xy)) = \sum_{xy\in E} d_{G^*}(\mu_B^P(xy), \mu_B^N(xy))(\mu_B^P(xy), \mu_B^N(xy))$.

3.3 Theorem: Let G: (A,B) be a bipolar fuzzy graph on a k-regular bipolar fuzzy graph $G^*: (V, E)$. Then $\sum_{xy\in E} d_G(\mu_B^P(xy), \mu_B^N(xy)) = 2(k-1)S(G)$.

Proof:

By theorem 3.2,

$\sum_{xy\in E} d_G(\mu_B^P(xy), \mu_B^N(xy)) = \sum_{xy\in E} d_{G^*}(\mu_B^P(xy), \mu_B^N(xy))(\mu_B^P(xy), \mu_B^N(xy))$.

$= \sum_{xy\in E}(d_{G^*}(\mu_A^P(x), \mu_A^N(x)) + d_{G^*}(\mu_A^P(y), \mu_A^N(y)) - 2)(\mu_B^P(xy), \mu_B^N(xy))$.

Since G^* is k-regular bipolar fuzzy graph, $d_{G^*}(\mu_A^P(x), \mu_A^N(x)) = k$ for all $x \in V$

$\sum_{xy\in E} d_G(\mu_B^P(xy), \mu_B^N(xy)) = (k + k - 2) \sum_{xy\in E}(\mu_B^P(xy), \mu_B^N(xy))$

$= 2(k-1) \sum_{xy\in E}(\mu_B^P(xy), \mu_B^N(xy))$

Hence $\sum_{xy\in E} d_G(\mu_B^P(xy), \mu_B^N(xy)) = 2(k-1)S(G)$,since$S(G) = \sum_{xy\in E}(\mu_B^P(xy), \mu_B^N(xy))$.

4. Edge Regular Bipolar Fuzzy Graphs:

4.1 Edge Regular Bipolar Fuzzy Graph: Let G: (A,B) be a bipolar fuzzy graph on $G^* : (V, E)$. If each edge in G has same degree k, then G is said to be an edge regular bipolar fuzzy graph (or) k-edge regular bipolar fuzzy graph.

4.2 Remark: G is k-edge regular bipolar fuzzy graph if and only if $\delta_E(G) = \Delta_E(G) = k$.

4.3 Remark: A complete bipolar fuzzy graph need not be edge regular. For example, in figure 4.3, G is not bipolar edge regular, but it is complete.

4.4 Example: Consider the following bipolar fuzzy graph G: (A,B)

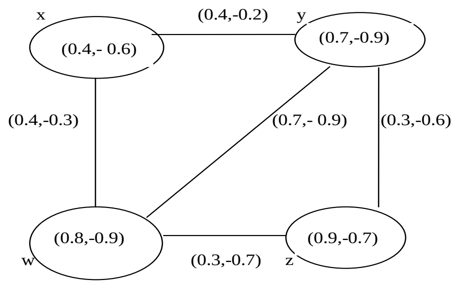

Figure 4.1

Here, G is (1.4,-1.8) edge regular bipolar fuzzy graph. But it is not a regular bipolar fuzzy graph.

4.5 Example: Consider the following bipolar fuzzy graph G: (A,B)

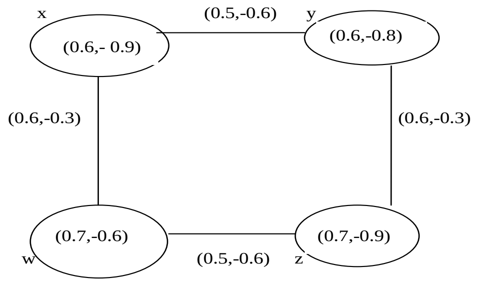

Figure 4.2

Here G is (1.1,-0.9) regular bipolar fuzzy graph. But it is not an edge regular bipolar fuzzy graph.

4.6 Example: In the following figure 4.3, G is neither edge regular nor regular bipolar fuzzy graph.

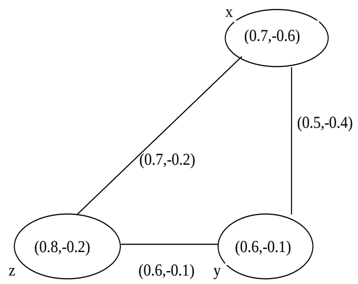

Figure 4.3

4.7 Example: Consider the following bipolar fuzzy graph G: (A,B).

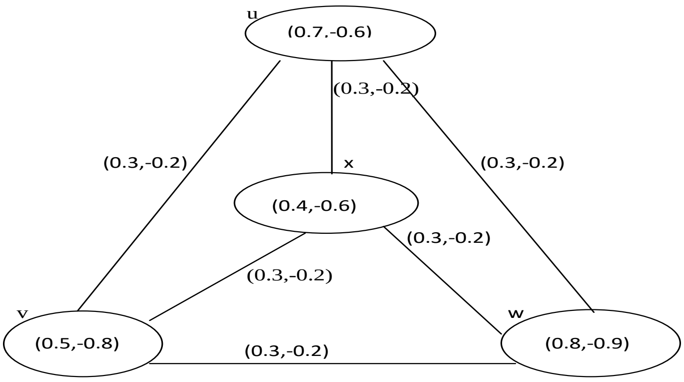

Figure 4.4

Here G is both (1.2,-0.8) edge regular bipolar fuzzy graph and (0.9,-0.6) regular bipolar fuzzy graph.

4.8 Remark: From the above examples, it is clear that in general there does not exist any relationship between edge regular and regular bipolar fuzzy graph.

4.9. Theorem: Let B be a constant function in G: (A,B) on $G^*: (V, E)$. If G is regular, then G is an edge regular bipolar fuzzy graph.

Proof:

Let $\left(\mu_B^P(xy), \mu_B^N(xy)\right) = c$ for all $xy \in E$, where c is a constant.

Assume that G is a k-regular bipolar fuzzy graph.

Then $d_G\left(\mu_A^P(x), \mu_A^N(x)\right) = k$ for all $x \in V$

By definition of edge degree,

$d_G(\mu_B^P(xy), \mu_B^N(xy)) = d_G(\mu_A^P(x), \mu_A^N(x)) + d_G\left(\mu_A^P(y), \mu_A^N(y)\right) - 2\left(\mu_B^P(xy), \mu_B^N(xy)\right)$, for all $xy \in E$

=k+k-2c for all $xy \in E$

$d_G(\mu_B^P(xy), \mu_B^N(xy)) = 2(k - c)$ for all $xy \in E$

Hence G is an edge regular bipolar fuzzy graph.

4.10 Remark: The converse of above theorem need not br true. It can be seen from the following figure. Here B is a constant function and G is (0.9,-1.2) edge regular bipolar fuzzy graph but not a regular bipolar fuzzy graph.

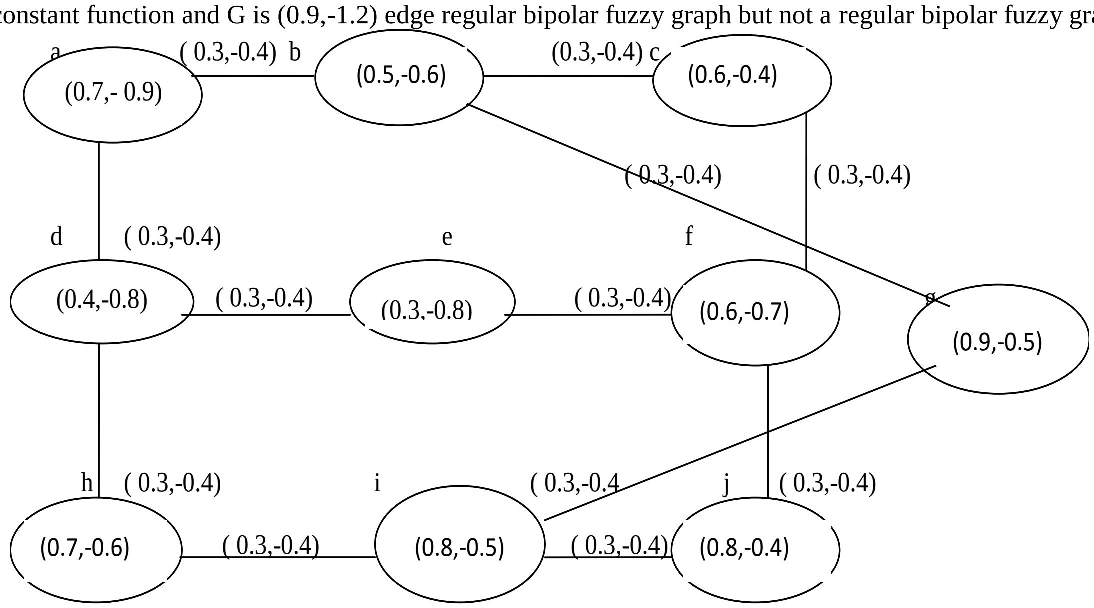

Figure 4.5

4.11Theorem: Let G: (A,B) be a bipolar fuzzy graph on a k-regular bipolar fuzzy graph $G^*: (V, E)$. Then E is a constant function if and only if G is both regular and edge regular bipolar fuzzy graph.

Proof:

Let G: (A,B) be a bipolar fuzzy graph on a k-regular bipolar fuzzy graph $G^*: (V, E)$.

Assume that E is a constant function.

Let $\left(\mu_B^P(xy), \mu_B^N(xy)\right) = c$ for all $xy \in E$, where c is a constant.

To prove that G is both regular and edge regular bipolar fuzzy graph.

Since G^* is k-regular, $d_{G^*}\left(\mu_A^P(x),\mu_A^N(x)\right) = k$ for all $x \in V$

$\therefore\ d_G\left(\mu_A^P(x),\mu_A^N(x)\right) = \sum_{xy\in E}\left(\mu_B^P(xy),\mu_B^N(xy)\right)$ for all $x \in V$

$= \sum_{xy\in E} c$ for all $x \in V$ $= c\ d_{G^*}\left(\mu_A^P(x),\mu_A^N(x)\right)$ for all $x \in V$

$d_G\left(\mu_A^P(x),\mu_A^N(x)\right) =$ ck for all $x \in V$

Hence G is a regular bipolar fuzzy graph.

Now,

$$d_G\left(\mu_B^P(xy),\mu_B^N(xy)\right) = \sum_{\substack{xz\in E\\ x\neq z}}\left(\mu_B^P(xz),\mu_B^N(xz)\right) + \sum_{\substack{zy\in E\\ z\neq y}}\left(\mu_B^P(zy),\mu_B^N(zy)\right), \text{ for all } xy \in E$$

$$= \sum_{\substack{xz\in E\\ x\neq z}} c + \sum_{\substack{zy\in E\\ z\neq y}} c \text{ for all } xy \in E$$

$$= c\ (\ d_{G^*}\left(\mu_A^P(x),\mu_A^N(x)\right) - 1) +\ c\ (\ d_{G^*}\left(\mu_A^P(y),\mu_A^N(y)\right) - 1) \text{ for all } xy \in E$$

= c(k-1) + c(k-1) for all $xy \in E$

$\therefore d_G\left(\mu_B^P(xy),\mu_B^N(xy)\right) = 2c(k-1)$ for all $xy \in E$

Hence G is also edge regular bipolar fuzzy graph.

Conversely,assume that G is both regular and edge regular bipolar fuzzy graph.

Let $d_G\left(\mu_A^P(x),\mu_A^N(x)\right) = k_1$ for all $x \in V$

And $d_G\left(\mu_B^P(xy),\mu_B^N(xy)\right) = k_2$ for all $xy \in E$

To prove that E is a constant function.

By definition of edge degree,

$$d_G(\mu_B^P(xy),\mu_B^N(xy)) = d_G(\mu_A^P(x),\mu_A^N(x)) + d_G\left(\mu_A^P(y),\mu_A^N(y)\right) - 2(\mu_B^P(xy),\mu_B^N(xy)), \text{ for all } xy \in E$$

$$k_2 = k_1 + k_1 - 2\left((\mu_B^P(xy),\mu_B^N(xy)\right), \text{ for all } xy \in E$$

$\left((\mu_B^P(xy),\mu_B^N(xy)\right) = (2k_1 - k_2)/2$ for all $xy \in E$

Hence E is a constant function.

4.12 Theorem: Let G: (A,B) be a bipolar fuzzy graph on $G^*:(V,E)$ such that E is a constant function. Then G is edge regular if and only if G^* is edge regular bipolar fuzzy graph.

Proof:

Let $\left(\mu_B^P(xy),\mu_B^N(xy)\right) = c$ for all $xy \in E$, where c is a constant.

Assume that G is edge regular bipolar fuzzy graph.

Then $d_G\left(\mu_B^P(xy),\mu_B^N(xy)\right) = k$ for all $xy \in E$

Now,

$$d_G\left(\mu_B^P(xy),\mu_B^N(xy)\right) = \sum_{\substack{xz\in E\\ x\neq z}}\left(\mu_B^P(xz),\mu_B^N(xz)\right) + \sum_{\substack{zy\in E\\ z\neq y}}\left(\mu_B^P(zy),\mu_B^N(zy)\right), \text{ for all } xy \in E$$

$$k = \sum_{\substack{xz\in E\\ x\neq z}} c + \sum_{\substack{zy\in E\\ z\neq y}} c \text{ for all } xy \in E$$

$$= c\ (\ d_{G^*}\left(\mu_A^P(x),\mu_A^N(x)\right) - 1) +\ c\ (\ d_{G^*}\left(\mu_A^P(y),\mu_A^N(y)\right) - 1) \text{ for all } xy \in E$$

k/c = $d_{G^*}\left(\mu_A^P(x),\mu_A^N(x)\right) +\ d_{G^*}\left(\mu_A^P(y),\mu_A^N(y)\right) - 2$for all $xy \in E$

$d_{G^*}\left(\mu_B^P(xy),\mu_B^N(xy)\right) = k/c$ for all $xy \in E$

Hence G^* is edge regular bipolar fuzzy graph.

Conversely,assume that G^* is m edge regular bipolar fuzzy graph.

Then

$d_{G^*}\left(\mu_B^P(xy),\mu_B^N(xy)\right) = m$ for all $xy \in E$

Now,

$$d_G\left(\mu_B^P(xy),\mu_B^N(xy)\right) = \sum_{\substack{xz\in E\\ x\neq z}}\left(\mu_B^P(xz),\mu_B^N(xz)\right) + \sum_{\substack{zy\in E\\ z\neq y}}\left(\mu_B^P(zy),\mu_B^N(zy)\right), \text{ for all } xy \in E$$

$$= \sum_{\substack{xz\in E\\ x\neq z}} c + \sum_{\substack{zy\in E\\ z\neq y}} c \text{ for all } xy \in E$$

$$= c\ (\ d_{G^*}\left(\mu_A^P(x),\mu_A^N(x)\right) - 1) +\ c\ (\ d_{G^*}\left(\mu_A^P(y),\mu_A^N(y)\right) - 1) \text{ for all } xy \in E$$

$$= c\ d_{G^*}\left(\mu_B^P(xy),\mu_B^N(xy)\right)$$

= cm for all $xy \in E$

Hence G is edge regular bipolar fuzzy graph.

4.13 Theorem: Let G: (A,B) be a bipolar fuzzy graph on $G^*:(V,E)$ such that E is a constant function. Then G is regular if and only if G^* is regular bipolar fuzzy graph.

Proof:

Let $\left(\mu_B^P(xy),\mu_B^N(xy)\right) = c$ for all $xy \in E$, where c is a constant.

Assume that G is regular bipolar fuzzy graph.

Let $d_G\left(\mu_A^P(x),\mu_A^N(x)\right) = k$ for all $x \in V$

Then $d_G\left(\mu_A^P(x),\mu_A^N(x)\right) = \sum_{\substack{xy\in E\\ x\neq y}}\left(\mu_B^P(xy),\mu_B^N(xy)\right)$, for all $x \in V$

$$k = \sum_{\substack{xy\in E\\ x\neq y}} c$$

$$k = c\, d_{G^*}\left(\mu_A^P(x), \mu_A^N(x)\right) = m \text{ for all } x \in V$$

$$d_{G^*}\left(\mu_A^P(x), \mu_A^N(x)\right) = k/c \text{ for all } x \in V$$

Hence G^* is regular bipolar fuzzy graph.

Conversely, assume that G^* is m-regular bipolar fuzzy graph.

Then $d_{G^*}\left(\mu_A^P(x), \mu_A^N(x)\right) = m$ for all $x \in V$

$$d_G\left(\mu_A^P(x), \mu_A^N(x)\right) = \sum_{\substack{xy\in E\\ x\neq y}} \left(\mu_B^P(xy), \mu_B^N(xy)\right), \text{ for all } x \in V$$

$$d_G\left(\mu_A^P(x), \mu_A^N(x)\right) = \sum_{\substack{xy\in E\\ x\neq y}} c, \text{ for all } x \in V$$

$$d_G\left(\mu_A^P(x), \mu_A^N(x)\right) = c\, d_{G^*}\left(\mu_A^P(x), \mu_A^N(x)\right), \text{ for all } x \in V$$

$$d_G\left(\mu_A^P(x), \mu_A^N(x)\right) = cm, \text{ for all } x \in V$$

Hence G is regular bipolar fuzzy graph.

4.14 Theorem: The size of a k-edge regular bipolar fuzzy graph G : (A,B) on a k_1- edge regular bipolar fuzzy graph $G^* : (V, E)$ is $\frac{qk}{k_1}$ where $q = |E|$

Proof:

The size of G is S(G) = $\sum_{\substack{xy\in E\\ x\neq y}} \left(\mu_B^P(xy), \mu_B^N(xy)\right)$

Since G is k-edge regular bipolar fuzzy graph and G^* is k_1 edge regular bipolar fuzzy graph,

$$d_G\left(\mu_B^P(xy), \mu_B^N(xy)\right) = k,\ d_{G^*}\left(\mu_B^P(xy), \mu_B^N(xy)\right) = k_1 \text{ for all } xy \in E$$

By theorem 3.2

$$\sum_{xy\in E} d_G\left(\mu_B^P(xy), \mu_B^N(xy)\right) = \sum_{xy\in E} d_{G^*}\left(\mu_B^P(xy), \mu_B^N(xy)\right)\left(\mu_B^P(xy), \mu_B^N(xy)\right).$$

Thus $\sum_{xy\in E} d_G\left(\mu_B^P(xy), \mu_B^N(xy)\right) = \sum_{xy\in E} k_1 \left(\mu_B^P(xy), \mu_B^N(xy)\right).$

$$qk = k_1\, S(G)$$

Hence S(G) = $\frac{qk}{k_1}$

5. Conclusion:

In this paper, we have found some relationship between degree of a vertex and degree of an edge in bipolar fuzzy graphs and studied some results between regular and edge regular bipolar fuzzy graphs. The results discussed may be useful for further study on totally edge regular bipolar fuzzy graphs.

6. References:

1. L.A.Zadeh,Fuzzy sets, Information and control 8(1965) 338-353.
2. M.Akram, Bipolar fuzzy graphs, Information sciences,DOI 10.1016/j.ins 2011.07.037,2011.
3. A.Nagoorgani,K.Radha, On regular fuzzy graphs, journal of physical sciences,Vol.12,33-44,2008.
4. A.Nagoorgani, and K.Radha The degree of a vertex in some fuzzy graphs, International journal of algorithms, computing and Mathematics, Vol 2,No.3,Aug 2009.
5. K.Radha and N.Kumaravel, The degree of an edge in Cartesian product and composition of two fuzzy graphs, International journal of applied mathematics and statistical sciences (IJAMSS) IASET, Vol.2,Issue2 65-78 may 2013.
6. S.Arumugam and S.Velammal, edge domination in graphs,Taiwanese journal of Mathematics, Vol.2,PP.173-179 June 1998.
7. A. Nagoorgani,M.Basheer Ahamed order and size in fuzzy graph, Bulletin of pure and applied sciences,Vol.22E(No.1),145-148,2003.
8. A.Nagoorgani and j.Malarvizhi properties of μ-complement of a fuzzy graph,international journal of algorithms, Computing and Mathematics, Vol.2,No.3,73-83, 2009.