

ANALYSIS OF LATERAL VIBRATIONS OF THE AXIALLY
LOADED HELICAL SPRING

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It is shown in this paper that the proposed concept by Kruzelecki and Życzkowski (1990) of the equivalent rod can be applied in calculations of natural lateral vibrations of springs and that the obtained results will be nearer FEM results than the standard model based on the Timoshenko equivalent beam. The model, created on this base, allows one to calculate natural frequencies of the clamped-clamped spring. It is also shown that models based on the equivalent beam concept, which are easier to apply than the models treating the spring as the spatially curved rod, have only a slightly smaller accuracy. It is also indicated that in the most common practice making of manufacturing end coils of springs, the natural frequencies differ significantly from the frequencies calculated by means of all tested methods. The performed simulations show that differences between the first and the second as well as the third and the fourth natural frequency of the spring are small and, therefore, the axially symmetrical equivalent beam model can be used without a large error. The diagram allowing one to determine whether the desired frequencies are lower or higher than the cut-off frequency is developed for the presented model.

Keywords: helical spring vibrations, axially loaded spring, coil spring, Timoshenko equivalent beam

1. Introduction

Lateral vibrations of helical springs constitute essential hazards for their operation safety, since they can cause high stress amplitudes and contribute to the loss of stability. On account of that, looking for ways of limiting natural vibrations of helical springs and the development of calculation methods allowing one to determine these limitations efficiency is essential. Equally essential is the accurate determination of natural frequencies.

Problems related to determination the above value can be considered when the spring is treated as a spatially curved rod or using the concept of the equivalent beam. Analyses of spring vibrations treated as the spatially curved rod were presented, among others, by Love (1899), Wittrick (1966), Mottershead (1980), Pearson and Wittrick (1986) Jiang *et al.* (1991), Yildirim (1999) Becker *et al.* (2002), Lee and Thompson (2001), Lee (2007), Stander and Du Preez (1992), Nagaya *et al.* (1986), Taktak *et al.* (2008), Yu and Yang (2010). The analyses enabled a relatively accurate description of the effect, but due to problem complexity it was very difficult to apply and – depending on assumptions – to obtain analytical results in a closed form. For practical reasons much more suitable is the equivalent rod or beam model (Haringx, 1949; Della Pietra and Della Valle, 1982; Kobelev, 2014; Michalczyk, 2014). However, the beam model does not take into account several factors related to dynamic effects in springs. It assumes, among others, that the equivalent beam modelling the spring in the case of lateral vibrations is prismatic and has axially symmetrical parameters characterising stiffness and mass distribution. It can be expected that this assumption in the case of clamped-clamped springs (with fixed ends) – the most often met case in practice – can be the reason of significant errors, since the spring lateral

stiffness is not the same in all directions and its distribution round the axis depends on the partial coil number. The problem of lateral vibrations of helical springs treated as the equivalent beam was considered in the paper by Kobelev (2014). However, only the simplest case, such as the simply-supported beam, was solved in that study.

A new concept of an equivalent rod, which could be applied to analysis of the helical spring static stability was proposed by Kruzelecki and Życzkowski (1990). That idea, in contrast to the model of Timoshenko and Gere (1961), took into account the influence of the lead angle of the helix line on its flexural, shearing and compression rigidities. In addition, that concept allowed the analysis of spring buckling in two planes.

The aim of the hereby paper is the analysis of the possibility of application of the Kruzelecki and Życzkowski (1990) concept to the determination of lateral vibrations of axially loaded helical springs. The results obtained on the basis of such formulated model will be compared with the classical model of the Timoshenko equivalent beam, with the numerical simulation results and with the results achieved by other authors. The presented model additionally takes into account the non-rotational way of mounting of the end coils.

2. Analysis

For an element of the equivalent beam modelling the axially helical spring loaded by a force P , as shown in Fig. 1, the following equations of motion can be written (Haringx, 1949)

$$\frac{\partial N}{\partial \psi} + Q = 0 \quad m_e \frac{\partial^2 y}{\partial t^2} = N \frac{\partial \psi}{\partial x} - \frac{\partial Q}{\partial x} \quad m_e r_g^2 \frac{\partial^2 \psi}{\partial t^2} = N \varphi + Q + \frac{\partial M}{\partial x} \quad (2.1)$$

The angle ψ originates only from bending, while the angle φ originates only from the equivalent beam shearing

$$\psi + \varphi = -\frac{\partial y}{\partial x} \quad \frac{\partial \psi}{\partial x} = \frac{M}{\alpha} \quad \varphi = \frac{Q}{\beta} \quad (2.2)$$

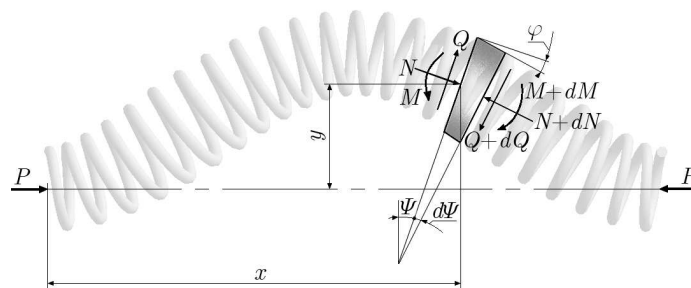


Fig. 1. Model of the equivalent rod for analysis of lateral vibrations of the helical spring

The coefficients α and β are up-to-date flexural and shearing rigidities of the equivalent beam, respectively. The spring mass for its length unit is marked by m_e , while the radius of gyration of the equivalent beam cross-section is marked by r_g . Limiting the vibration analysis to small deflections only, it can be assumed that $N \approx P$. Making use of (2.2) in (2.1) on this assumption, after transformations, we obtain

$$\beta \left(1 + \frac{P}{\beta}\right) \varphi = m_e r_g^2 \frac{\partial^2 \psi}{\partial t^2} - \alpha \frac{\partial^2 \psi}{\partial x^2} \quad - m_e \frac{\partial^2 \varphi}{\partial t^2} + \beta \frac{\partial^2 \varphi}{\partial x^2} = m_e \frac{\partial^2 \psi}{\partial t^2} + P \frac{\partial^2 \psi}{\partial x^2} \quad (2.3)$$

Eliminating φ from equations (2.3) and transforming, we obtain the wave equation of lateral vibrations of the spring under the axial static load (Haringx, 1949)

$$\frac{\partial^4 \psi}{\partial x^4} - \left(\frac{m_e}{\beta} + \frac{m_e r_g^2}{\alpha} \right) \frac{\partial^4 \psi}{\partial x^2 \partial t^2} + \frac{m_e^2 r_g^2}{\alpha \beta} \frac{\partial^4 \psi}{\partial t^4} + \frac{P}{\alpha} \left(1 + \frac{P}{\beta} \right) \frac{\partial^2 \psi}{\partial x^2} + \frac{m_e}{\alpha} \left(1 + \frac{P}{\beta} \right) \frac{\partial^2 \psi}{\partial t^2} = 0 \quad (2.4)$$

Since we are considering the model with undamped vibrations, separating variables in equation (2.4), the function $\psi(x, t)$ can be written as the product of the amplitude $\Psi(x)$ and the time function $T(t) = \sin \omega t$. Analogous expressions can be also written for the functions φ and y

$$\psi(x, t) = \Psi(x) \sin \omega t \quad \varphi(x, t) = \Phi(x) \sin \omega t \quad y(x, t) = Y(x) \sin \omega t \quad (2.5)$$

Substituting proper derivatives (2.5)₁ into equation (2.4) and performing transformations, we obtain the equation for amplitudes in the form

$$\Psi^{IV} + \left[\left(\frac{m_e}{\beta} + \frac{m_e r_g^2}{\alpha} \right) \omega^2 + \frac{P}{\alpha} \left(1 + \frac{P}{\beta} \right) \right] \Psi^{II} - \left[\frac{m_e}{\alpha} \left(1 + \frac{P}{\beta} \right) \omega^2 - \frac{m_e^2 r_g^2}{\alpha \beta} \omega^4 \right] \Psi = 0 \quad (2.6)$$

Denoting for simplification

$$b = \left(\frac{m_e}{\beta} + \frac{m_e r_g^2}{\alpha} \right) \omega^2 + \frac{P}{\alpha} \left(1 + \frac{P}{\beta} \right) \quad c = \frac{m_e}{\alpha} \left(1 + \frac{P}{\beta} \right) \omega^2 - \frac{m_e^2 r_g^2}{\alpha \beta} \omega^4 \quad (2.7)$$

The characteristic equation for (2.6), after substituting $\Psi(x) = Ce^{rx}$, takes the form

$$r^4 + br^2 - c = 0 \quad (2.8)$$

Substituting $q = r^2$, we obtain a quadratic equation $q^2 + bq - c = 0$, whose roots are

$$q_1 = \frac{1}{2} \left(-b + \sqrt{b^2 + 4c} \right) \quad q_2 = \frac{1}{2} \left(-b - \sqrt{b^2 + 4c} \right) \quad (2.9)$$

The expression, which occurs in square roots in (2.9) can be transformed, using (2.7), into

$$b^2 + 4c = \frac{m_e^2}{\alpha^2 \beta^2} (\alpha - r_g^2 \beta)^2 \omega^4 + 2 \frac{m_e}{\alpha} \left(1 + \frac{P}{\beta} \right) \left(\frac{P}{\beta} + \frac{P r_g^2}{\alpha} + 2 \right) \omega^2 + \frac{P^2}{\alpha^2} \left(1 + \frac{P}{\beta} \right)^2 \quad (2.10)$$

This expression, similarly as b , is higher than null for an arbitrary frequency ω , which means

$$\bigwedge_{\omega \geq 0} q_2 < 0$$

Thus, the solution form of equation (2.6) depends on sign of the root q_1 . The cut-off frequency value ω_b at which the solution form of equation (2.8) changes, is calculated by equating the right-hand side of equation (2.9)₁ to null

$$\omega_b = \sqrt{\frac{\beta}{m_e r_g^2} \left(1 + \frac{P}{\beta} \right)} \quad (2.11)$$

Thus

$$\bigwedge_{\omega < \omega_b} \Psi(x) = C_1 \cosh(k_1 x) + C_2 \sinh(k_1 x) + C_3 \cos(k_2 x) + C_4 \sin(k_2 x) \quad (2.12)$$

where

$$k_1 = \sqrt{\frac{1}{2} \left(-b + \sqrt{b^2 + 4c} \right)} \quad k_2 = \sqrt{\frac{1}{2} \left(b + \sqrt{b^2 + 4c} \right)} \quad (2.13)$$

and

$$\bigwedge_{\omega > \omega_b} \Psi(x) = C_5 \cos(k_3 x) + C_6 \sin(k_3 x) + C_7 \cos(k_4 x) + C_8 \sin(k_4 x) \quad (2.14)$$

where

$$k_3 = \sqrt{\frac{1}{2}(b - \sqrt{b^2 + 4c})} \quad k_4 = k_2 \quad (2.15)$$

The way of determining the cut-off frequency ω_b for short, not loaded beams, was studied by Stephen and Puchegger (2006) and Majkut (2009). From the investigations of the author (which are presented in the further part of this paper) it results that even for relatively short springs, the first natural frequencies of lateral vibrations are lower than the cut-off frequency ω_b . Therefore, in further considerations, expression (2.12) is used.

The spring deflection angle resulting only from shearing, on the basis of (2.3)₁, equals

$$\varphi = \frac{1}{\beta(1 + \frac{P}{\beta})} \left(m_e r_g^2 \frac{\partial^2 \psi}{\partial t^2} - \alpha \frac{\partial^2 \psi}{\partial x^2} \right) \quad (2.16)$$

Making use of (2.2)₃, (2.5)_{2,3} and (2.16) in equation (2.1)₂, after transformations, we obtain the equation of amplitudes in the form

$$Y = - \left[\frac{P(1 + \frac{P}{\beta}) + m_e \omega^2 r_g^2}{m_e \omega^2 (1 + \frac{P}{\beta})} \Psi^I + \frac{\alpha}{m_e \omega^2 (1 + \frac{P}{\beta})} \Psi^{III} \right] \quad (2.17)$$

Using derivatives (2.12) in equation (2.17) and transforming, we finally obtain the equation of displacements in the form

$$Y(x) = (-Ak_1 - Bk_1^3)C_1 \sinh(k_1 x) + (-Ak_1 - Bk_1^3)C_2 \cosh(k_1 x) \\ + (Ak_2 - Bk_2^3)C_3 \sin(k_2 x) + (-Ak_2 + Bk_2^3)C_4 \cos(k_2 x) \quad (2.18)$$

where, for notation simplification, the following marks are introduced

$$A = \frac{P(1 + \frac{P}{\beta}) + m_e \omega^2 r_g^2}{m_e \omega^2 (1 + \frac{P}{\beta})} \quad B = \frac{\alpha}{m_e \omega^2 (1 + \frac{P}{\beta})} \quad (2.19)$$

The boundary conditions, being the clamped-clamped spring, correspond properly with the real operation conditions of a majority of springs applied in industry and, therefore, such a model will be considered. These conditions are

$$\mathbf{1)} \quad y(0, t) = 0 \quad \mathbf{2)} \quad y(L, t) = 0 \quad \mathbf{3)} \quad \psi(0, t) = 0 \quad \mathbf{4)} \quad \psi(L, t) = 0 \quad (2.20)$$

where L is the real height of the spring.

From conditions **1**, **3** and **4**, we obtain the following relationships

$$C_2 = \frac{-\cosh(k_1 L) + \cos(k_2 L)}{\sinh(k_1 L) + \frac{Ak_1 + Bk_1^3}{-Ak_2 + Bk_2^3} \sin(k_2 L)} C_1 \quad C_3 = -C_1 \\ C_4 = \frac{Ak_1 + Bk_1^3}{-Ak_2 + Bk_2^3} C_2 \quad (2.21)$$

On the basis of the second boundary condition (2.20), we obtain the frequency equation

$$0 = (-Ak_1 - Bk_1^3)C_1 \sinh(k_1 L) + (-Ak_1 - Bk_1^3)C_2 \cosh(k_1 L) + (Ak_2 - Bk_2^3)C_3 \sin(k_2 L) + (-Ak_2 + Bk_2^3)C_4 \cos(k_2 L) \quad (2.22)$$

The models proposed by Kruźelecki and Życzkowski (1990) have been used for the determination of the equivalent stiffness of the beam modelling the spring. The equivalent flexural rigidity and shear stiffness are expressed by

$$\alpha = \frac{2EJ \sin \delta}{2 + \nu \cos^2 \delta} \quad \beta = \frac{EJ \sin \delta}{R^2(1 + \nu \sin^2 \delta)} \frac{2\pi n}{\int_{\phi_0}^{\phi_0 + 2\pi n_0} \sin^2 \phi \, d\phi} \quad (2.23)$$

where EJ is the product of Young's modulus and area moment of inertia of the wire section, ν is Poisson's ratio. The values δ and R are the lead angle of the helix and half of the diameter of the nominal spring for an arbitrary force P , respectively. The angle ϕ , occurring in (2.23)₂, is the angle marked by the lead radius between the transverse force vector and an arbitrary cross-section of the spring wire. Since the transverse force direction may not coincide with the direction determined by the spring axis and the beginning of the helix line, the angle coordinate of the spring beginning ϕ_0 can generally differ from zero.

As compared to the widely applied in the references equivalent stiffness, see e.g. Timoshenko and Gere (1961), Guido *et al.* (1978), in which the lead angle of the helix δ is assumed as constant and equal to zero, expressions (2.23) allow one to take into account a non-zero value of the angle δ dependent on the axial force P . The relationships given by Kruźelecki and Życzkowski (1990) take also into account the nominal spring radius R change as well as the change of the active coils number n , under the influence of the force P . The authors assumed that the way of supporting the spring ends enables their free rotation. In such a case, during compression of the spring the number of its coils usually decreases (Michalczyk, 2009). Since the wire length is constant, it results in an additional increase in its nominal radius R . Its value as well as the value of the up-to-date helix line pitch angle δ can be determined by rearrangement of the relationships between the torsion and curvature of the helix wire and the force P given by Haringx (1948)

$$\frac{PR^2(1 + \nu) \cos \delta}{EJ} = \sin \delta \cos \delta - \frac{R}{R_0} \sin \delta_0 \cos \delta_0 \quad \frac{PR^2 \sin \delta}{EJ} = \cos^2 \delta - \frac{R}{R_0} \cos^2 \delta_0 \quad (2.24)$$

In order to realise the condition of free rotation of the spring ends, one of them has to be supported on a thrust bearing. Only in such a case equations (2.24) can be used for calculating R and δ .

However, most often, springs are supported in a way making free rotations of the end coils impossible and, therefore, this case will be considered further. The number of active coils n of the spring loaded by the force P is equal – due to the clamped-clamped condition – to the number of active coils n_0 of the not loaded spring. On this assumption and making use of the geometrical relations, it is possible to determine the helix line pitch angle δ and the spring nominal radius R under load

$$\delta = \arcsin \left(\left(1 - \frac{P}{\gamma_0} \right) \sin \delta_0 \right) \quad R = R_0 \frac{\cos \delta}{\cos \delta_0} \quad (2.25)$$

The equivalent shear stiffness γ_0 occurring in equation (2.25)₁ can be expressed by the equation (Kruźelecki and Życzkowski, 1990)

$$\gamma_0 = \frac{EJ \sin \delta_0}{R_0^2(1 + \nu \cos^2 \delta_0)} \quad (2.26)$$

Knowing the nominal radius R of the axially loaded clamped-clamped spring, it is possible to calculate its equivalent radius of gyration r_g . In the case of modelling the radius of gyration of cylindrical helical springs, the equivalent rod constitutes the thin-walled cylinder of the average radius equal to the nominal spring radius R and of the mass equal to the spring mass. On this assumption, making use of (2.25)₂, the equivalent radius of gyration r_g can be written in the form

$$r_g = \frac{\sqrt{2}}{2} R_0 \frac{\cos \delta}{\cos \delta_0} \quad (2.27)$$

Introducing (2.25)₁, (2.26), (2.27) into (2.11), (2.22), (2.23), we can calculate cut-off frequencies and natural frequencies of lateral vibrations.

3. Simulations, results and discussion

3.1. Analysis of natural frequencies of lateral vibrations of not axially loaded springs

Comparisons of the results obtained on the basis of numerical FEM simulations of models (given in references) in which the spring is considered as a spatially curved rod of the classic model of the Timoshenko equivalent beam (applied up-to-date in all investigations concerning lateral vibrations of the spring modelled as the equivalent rod) with the proposed in this paper model using the concept of the equivalent beam given by Kruzelecki and Życzkowski (1990) is presented below. As the first example, the spring of the same parameters as those used in work by Lee and Thompson (2001) is analysed. These parameters are as follows: total spring length $L = 332$ mm, spring nominal radius $R_0 = 65$ mm, wire radius $r = 6$ mm, density $\rho = 7800$ kg/m³, number of active coils $n = 6$, Young's modulus $E = 209000$ MPa, Poisson's ratio $\nu = 0.28$. The first four forms of lateral vibrations of the clamped-clamped spring not loaded by a longitudinal force obtained by the FEM in the ANSYS software are presented in Fig. 2.

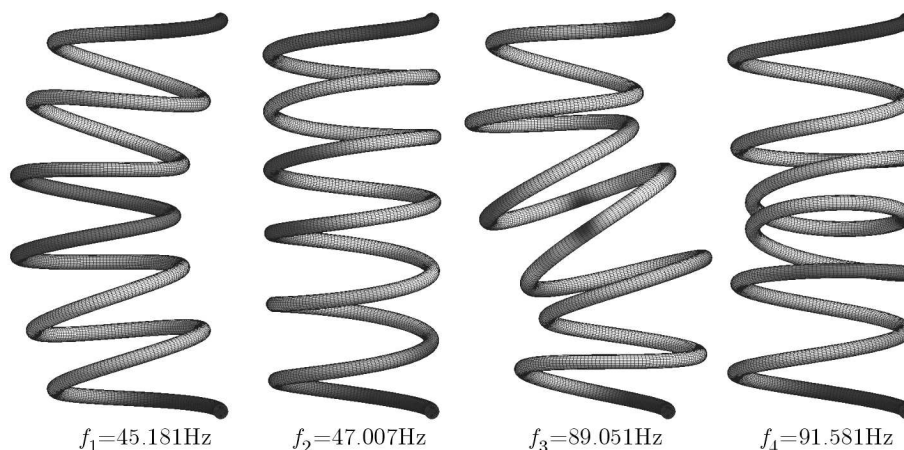


Fig. 2. Forms and frequencies of lateral natural vibrations of the clamped-clamped spring

As can be seen in Fig. 2, the frequency f_2 is only insignificantly higher than f_1 . In the case of frequencies f_4 and f_3 , the difference is even smaller.

The comparison of natural frequencies of the spring not loaded by a longitudinal force obtained by means of FEM, the dynamic stiffness method (Lee and Thompson, 2001), transfer matrix method (Pearson, 1982), standard model of the Timoshenko equivalent beam (STB) and the model presented in this paper (MTB) is presented in Table 1. In the methods presented by Lee and Thompson (2001) and Pearson (1982), the spring is treated as a spatially curved rod. The solution, in both methods, is obtained by means of a numerical method.

It should be noticed, that in methods using the concept of the equivalent rod modelled as acylinder, all frequencies of lateral vibrations are doubled due to the axial symmetry.

Table 1

	f_1	f_2	f_3	f_4
FEM model (Fig. 2)	45.181	47.007	89.051	91.581
Dynamic stiffness method	45.135	46.951	88.976	91.586
Transfer matrix method	45.13	46.95	88.97	91.59
Equivalent Timoshenko beam (STB)	45.975	45.975	94.088	94.088
Model presented in the work (MTB)	45.764	45.764	93.62	93.62

Comparing the results given in Table 1 it is seen that the application of the modified Timoshenko beam model (MTB) with taking into account non-rotational ends support and using stiffness given in the paper by Kruzelecki and Życzkowski (1990) provides, in the case of the analysed not axially loaded spring, results very similar to those ones obtained on the basis of the standard Timoshenko beam model (STB). The largest difference between the frequencies obtained from the FEM simulation and the frequencies obtained from the two last models did not exceed 6%.

The most often occurring case in the engineering practice is the spring of end coils bent and ground off, since only this way of support (except cases of using special spring holders or spring guided on a pin or in a cylindrical sleeve) ensures its stable operation. It can be expected that the way of making the end coils has an influence on the natural frequencies. In order to illustrate this problem, FEM analyses have been performed for a spring of the same parameters as before but with different shapes of neutral coils. The model of such spring and its first four lateral vibration modes are presented in Fig. 3.

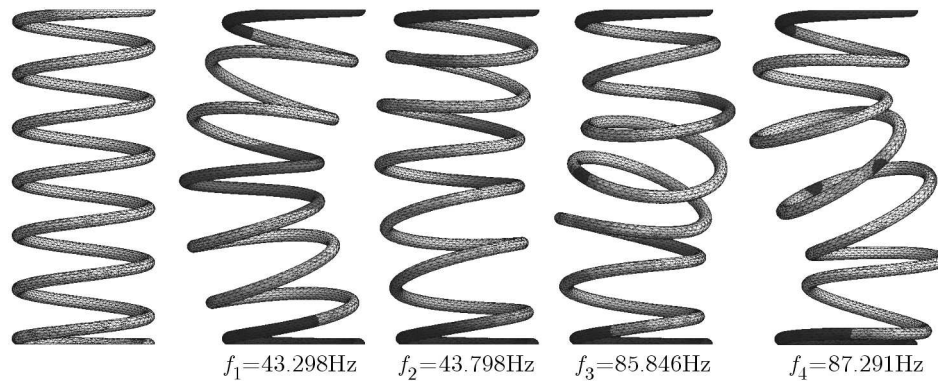


Fig. 3. Forms and frequencies of lateral natural vibrations of the clamped-clamped spring with typical industrial end coils

Comparing data contained in Table 1 and 2, it can be noticed that in the case of the spring with end coils bent and ground off, a better similarity of the FEM results for the models proposed by Lee and Thompson (2001) and Pearson (1982) is not obtained (at least for the first two frequencies) than the results obtained on the basis of the models based on the equivalent beam concept.

Table 2

	f_1	f_2	f_3	f_4
FEM model with bent coils (Fig. 3)	43.298	43.798	85.846	87.291

3.2. Analysis of natural frequencies of lateral vibrations of axially loaded springs

Helical springs applied in the industry most often operate under a certain preliminary axial load, usually caused by the supported machine, weight or preliminary tension, e.g. in overload clutches, valves or in variable-speed transmission systems. Thus, the influence of the axial force (approximately static) on natural frequency vibrations of helical springs is essential. Diagrams of the first four natural frequencies of lateral vibrations obtained (as before) from FEM analyses and from the solution presented in this study are presented in Fig. 4. The FEM simulations were carried out in two stages: in the first one, the statistical analysis was performed at the determined spring load, while in the second, the modal analysis was performed for such a loaded model. Analyses and calculations were performed for the following spring relative deflections: 0; 0.125; 0.25; 0.375; 0.5. Simulations were carried out in the ANSYS packet, Mechanical APDL module.

As can be seen in Fig. 4, along with an increase in the axial force P and thus an increase in the spring relative deflection, the model presented in this study improves convergence with the FEM in relation to the standard model of the equivalent beam.

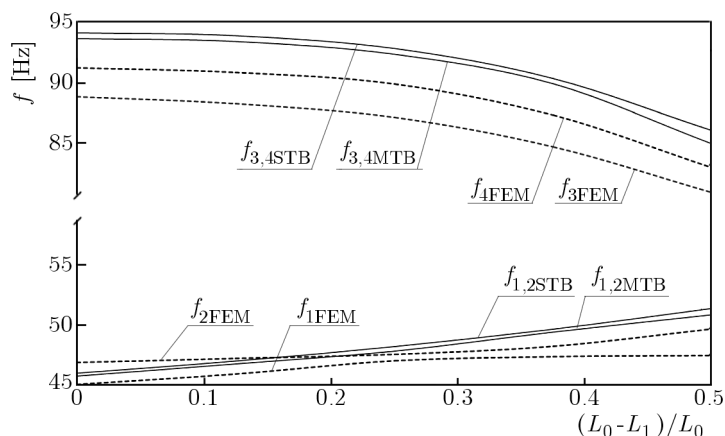


Fig. 4. Comparison of the first four natural frequencies of lateral vibrations obtained from FEM simulations with the first two double frequencies calculated on the basis of the standard model of the equivalent beam (STB) and the model presented in this study (MTB)

3.3. Cut-off frequencies of lateral vibrations of helical springs according to the presented model

As has been mentioned above, equations (2.12), (2.18) and (2.22) are applicable only in the case when the cut-off frequency ω_b determined by equation (2.11) is larger than the looked for natural frequencies. Thus, from the point of view of analysis of spring lateral vibrations, it is important to find out whether the looked for vibrations form is above or below the cut-off frequency ω_b . This problem was analysed by Guido *et al.* (1978), where the authors indicated that for slenderness $L_0/D_0 = 1$ (or less) natural frequencies always correspond to solution (2.13). However, they did not provide neither slenderness values nor relative deflections at which the successive natural frequencies can be calculated from (2.12). On the basis of the studies performed at various material and geometrical parameters, it can be stated that the spring slenderness and its deflection decide whether the given natural frequency is higher or lower than the cut-off frequency.

The diagram allowing one to determine whether equation (2.12) or (2.14) should be used in calculations of natural frequencies of lateral vibrations is presented in Fig. 5. The up-to-date spring length L_1 related to the length of a not loaded spring L_0 is marked in the vertical axis,

while the not loaded spring slenderness $L_0/2R_0$ in the horizontal axis. If, e.g. we are interested in natural frequencies of lateral vibrations of the clamped-clamped spring of a slenderness being 2.5 and compressed to 0.6 of its initial height (point *A* in Fig. 5), then the first two double frequencies $\omega_{1,2}$ and $\omega_{3,4}$ can be calculated using the solution in form (2.12) and (2.22), while in order to find the parameters of higher frequencies, the solution in form (2.14) should be used.

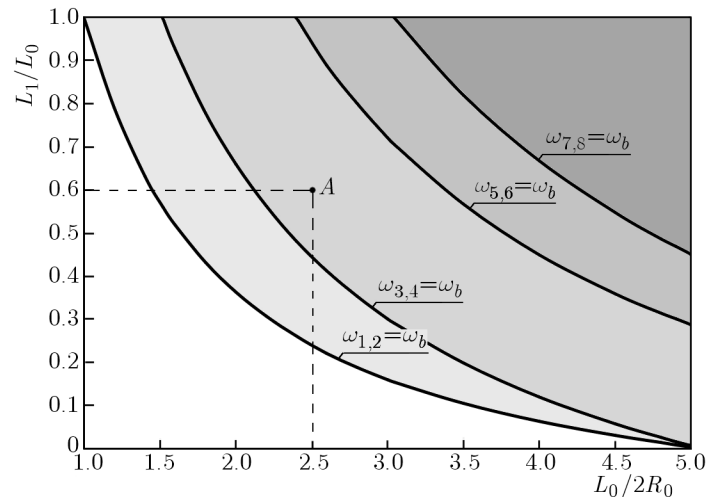


Fig. 5. Curves separating the zones in which the cut-off frequency ω_b is higher than the successive frequencies of lateral natural vibrations

4. Conclusions

The modified model of the equivalent beam, presented in this study, allows one to determine form (2.18) and frequencies (2.22) of lateral natural vibrations of the axially loaded spring. The results obtained on the basis of the presented model, using the concept given by Kruzelecki and Życzkowski (1990) are slightly closer to the results of the FEM analysis than the standard model based on the Timoshenko equivalent beam. The developed model allows calculation of natural frequencies of the axially loaded spring and supported in a way making free rotations of its end coils impossible. It is also shown that the models based on the equivalent beam concept, easier in applications than models treating the spring as a spatially curved rod, have only insignificantly smaller accuracy. The largest difference between the frequencies obtained from the FEM simulation for the spring without bent end coils (Fig. 2) and the frequencies obtained from the presented model is a little bit above 5%. Comparing the frequencies obtained from the FEM simulation for the spring without bent end coils (Fig. 2) with the frequencies obtained from the FEM simulation for the spring of typical industrial end coils (Fig. 3) can be noticed that the differences are even larger – the maximum difference for the second frequency exceeded 6%. Thus, the way of making the end coils significantly influences the natural frequencies. In the most often met in practice case of spring end coils being bent and ground off, the natural frequencies differ significantly from the frequencies calculated by all tested methods. The performed simulations indicated also that the differences between the first and the second as well as the third and the fourth natural frequency are small for springs and, therefore, the axially symmetric equivalent beam model can be applied for calculating (without large error) these frequencies. The diagram (Fig. 5) allowing one to determine which form of the solution to equation (2.6) should be applied in calculating frequencies and forms of natural vibrations of the spring of the given slenderness and static deflection has also been developed for the presented model.

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