NLOS MITIGATION IN TOA-BASED INDOOR LOCALIZATION BY NONLINEAR FILTERING UNDER SKEW t-DISTRIBUTED MEASUREMENT NOISE

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ABSTRACT

Wireless localization by time-of-arrival (TOA) measurements is typically corrupted by non-line-of-sight (NLOS) conditions, causing biased range measurements that can degrade the overall positioning performance of the system. In this article, we propose a localization algorithm that is able to mitigate the impact of NLOS observations by employing a heavy-tailed noise statistical model. Modeling the observation noise by a skew t-distribution allows us to, on the one hand, employ a computationally light sigma-point Kalman filtering method while, on the other hand, be able to effectively characterize the positive skewed non-Gaussian nature of TOA observations under LOS/NLOS conditions. Numerical results show the enhanced performance of such approach.

***Index Terms*—** Robust filtering, NLOS mitigation, skew t-distribution, sigma-point Kalman filter

1. INTRODUCTION

The problem under study concerns the derivation of efficient filtering methods, that are robust in challenging applications/scenarios such as the LOS/NLOS propagation conditions in indoor localization systems [1]. The state-space models (SSM) of interest are expressed as

$$\mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{u}_k, \quad \mathbf{u}_k \sim \mathcal{N}(0, \mathbf{Q}_k), \tag{1}$$
$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{n}_k, \qquad \mathbf{n}_k \sim \text{non-Gaussian} \tag{2}$$

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$ and $\mathbf{y}_k \in \mathbb{R}^{n_y}$ are the states and observations of the system at time k. $\mathbf{f}$ and $\mathbf{h}$ are the process and measurement functions, known and possibly nonlinear, and both process and observation noises, $\mathbf{u}_k$ and $\mathbf{n}_k$, assumed mutually independent. In real–life systems we do not have complete knowledge of the system dynamics, thus the measurement noise statistics are assumed to be unknown to a certain extent. In contrast, the process noise covariance $\mathbf{Q}_k$ is considered known in this contribution. The components of the measurement noise are assumed to be independent, each one distributed according to a parametric heavy-tailed non-Gaussian distribution, $n_{k,i} \sim \mathcal{D}(\boldsymbol{\theta}_i)|_{i=1,\dots,n_y}$, with $\boldsymbol{\theta}_i$ representing the unknown parameters of the non-Gaussian distribution.

A key point is to assume that the univariate measurement noise component distributions, $\mathcal{D}(\boldsymbol{\theta}_i)$, can be written in a conditionally (hierarchical) Gaussian form. These distributions are typically written using symmetric Gaussian scale mixtures (a.k.a. scale mixture of normals) [2, 3], which include among others the Gaussian, Student-t and symmetric α-stable ($S\alpha S$) distributions. But skewness can also be accounted within this context, for instance using the normal variance-mean representation introduced in [4] or the skew t distribution hierarchical formulation given in [5].

In the literature, some contributions dealing with conditionally Gaussian SSMs, heavy-tailed and skewed distributions were proposed. A particle filter (PF) solution for linear SSMs in $S\alpha S$ noise was presented in [6]. This idea was further explored in [7] for nonlinear systems, and generalized to other symmetric distributions in [8]. The key idea was to take advantage of the conditionally Gaussian form and use a sigma-point filter [9, 10] for the nonlinear state estimation. A robust filtering variational Bayesian (VB) approach was considered for linear systems in [11], and further extended to nonlinear SSMs in [12] considering a symmetric Student-t measurement noise. But symmetric distributions may not always be appropriate to characterize the system noise. Recently, two interesting approaches to deal with linear SSMs under skewed noise was proposed, the first one uses a marginalized PF solution [13] and the other considers a VB solution [14]. Notice that these contributions deal with either nonlinear systems corrupted by symmetric distributed noises or linear SSMs under skewed noise. Regarding the problem at hand, a skewed t-distributed noise was recently proposed to address the NLOS problem in TOA-based positioning [15].

In this contribution, we are interested in a TOA-based tracking application, where the LOS/NLOS propagation is modeled using a skew t-distributed measurement noise [15]. Whereas a sigma-point filter deals with the nonlinear state estimation problem, a learning method is used to estimate the time-varying skew t-distribution parameters.

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2. SYSTEM MODEL

2.1. TOA-based Localization

We consider a localization problem in which a target is moving in a plane and a set of N sensors are placed at known locations to measure range information to the target. Ranges are then processed to track the target. The state vector is composed of position and velocity components, $\mathbf{p}_k \triangleq (x_k, y_k)^\top$ and $\mathbf{v}_k \triangleq (\dot{x}_k, \dot{y}_k)^\top$, respectively. A linear constant acceleration model is adopted for state evolution, and thus

$$\underbrace{\begin{pmatrix} x_k \\ y_k \\ \dot{x}_k \\ \dot{y}_k \end{pmatrix}}_{\mathbf{x}_k} = \underbrace{\begin{pmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\mathbf{F}} \mathbf{x}_{k-1} + \underbrace{\begin{pmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{pmatrix}}_{\mathbf{G}} \mathbf{u}_k \,,$$

where the zero-mean Gaussian process noise $\mathbf{u}_k$ models an acceleration of 0.1 m/s^2, i.e., $\mathbf{u}_k \sim \mathcal{N}(\mathbf{0}, 10^{-4} \cdot \mathbf{I}_2)$.

Let us use $\mathbf{x}_{k,i} = [x_{k,i}, y_{k,i}]^\top$ to denote the known 2-dimensional position of the i-th anchor node. Then, the observed range from each node i to the target, denoted as $\hat{\rho}_{k,i}$ are modeled as $\hat{\rho}_{k,i} = \rho_i(\mathbf{x}_k) + n_{k,i}$, $i \in \{1, \ldots, N\}$, with $n_{k,i}$ denoting the ranging error and $\rho_i(\mathbf{x}_k) \triangleq \rho_{k,i} = \|\mathbf{x}_k - \mathbf{x}_{k,i}\|$ the true distance from the i-th node to the target node at k. The resulting measurement equation is

$$\boldsymbol{\rho}_k = [\rho_{k,1}, \cdots, \rho_{k,N}]^\top \tag{3}$$

$$= \underbrace{\begin{pmatrix} \|\mathbf{x}_k - \mathbf{x}_{k,1}\| \\ \vdots \\ \|\mathbf{x}_k - \mathbf{x}_{k,N}\| \end{pmatrix}}_{\mathbf{h}_k(\mathbf{x}_k)} + \underbrace{\begin{pmatrix} n_{k,1} \\ \vdots \\ n_{k,N} \end{pmatrix}}_{\mathbf{n}_k} , \tag{4}$$

with measurement noise $\mathbf{n}_t$ being nominally distributed according to $\mathbf{n}_t \sim \mathcal{N}(\mathbf{0}, \sigma^2 \cdot \mathbf{I}_N)$. The standard deviation of the range estimates, σ, depends on the technology under consideration. For instance, for Global Navigation Satellite Systems (GNSS) receivers without additional aiding this could be in the order of 5 to 10 meters. In the case of ultra-wideband (UWB) devices this could be reduced to 0.1 to 1 meter. We use the latter technology in the simulations section.

2.2. Measurement noise for NLOS characterization

Under NLOS conditions, the Gaussian noise modeling is known to be too simplistic, and the receiver is likely to estimate distances to the anchors larger than the true ones [1]. Such biased range estimates may be seen as outliers to the nominal model. To solve this problem and get a more accurate observation model, one must resort to heavy-tailed noise distributions to effectively model outliers and positive skewed distributions to cope with the true NLOS behavior. As recently proposed in [15], the observation noise components are assumed to be independently univariate skewed t-distributed [5] to properly model the NLOS scenario,

$$n_{k,i} \sim \mathcal{ST}\left(\mu, \sigma^2, \lambda, \nu\right), \tag{5}$$

which is parametrized by its location, scale and skewness parameters, $\mu \in \mathbb{R}$, $\sigma^2 \in \mathbb{R}^+$, and $\lambda \in \mathbb{R}$, respectively, and the degrees of freedom $\nu \in \mathbb{R}^+$. For $\lambda = 0$ the skew t-distribution becomes the standard Student's t-distribution $\mathcal{T}\left(\mu, \sigma^2, \nu\right)$; if $\nu \to \infty$ it becomes the skew normal distribution $\mathcal{SN}\left(\mu, \sigma^2, \lambda\right)$; and if in addition $\lambda \to 0$ it becomes the normal distribution $\mathcal{N}\left(\mu, \sigma^2\right)$.

It was pointed out in the introduction that we are interested in hierarchical representations of the non-Gaussian noise distribution. A skew t-distribution hierarchical representation allowing a conditionally Gaussian form is

$$n|\gamma, \tau \sim \mathcal{N}\left(\mu + \lambda\gamma, \tau^{-1}\sigma^2\right) \tag{6}$$

$$\gamma|\tau \sim \mathcal{N}_+\left(0, \tau^{-1}\right) \; ; \; \tau \sim \mathcal{G}\left(\frac{\nu}{2}, \frac{\nu}{2}\right) \tag{7}$$

where $\mathcal{N}_+(\cdot,\cdot)$ and $\mathcal{G}(\cdot,\cdot)$ are the positive truncated normal and gamma distributions. While τ produces the heavy-tails, γ controls the skewness of the distribution.

2.3. SSM for the TOA-based localization problem

Considering the localization problem at hand, together with the skewed t-distribution hierarchical Gaussian form presented in the previous section, the SSM is defined as

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{G}\mathbf{u}_k, \quad \mathbf{u}_k \sim \mathcal{N}(0, \mathbf{Q}), \tag{8}$$

$$\boldsymbol{\rho}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{n}_k, \qquad n_{k,i} \sim \mathcal{ST}(\mu, \sigma^2, \lambda, \nu) \tag{9}$$

where the measurement noise components are distributed as

$$n_{k,i}|\gamma_{k,i}, \tau_{k,i} \sim \mathcal{N}\left(m_{k,i}, s_{k,i}^2\right) \tag{10}$$

$$m_{k,i} = \mu + \lambda\gamma_{k,i} \; ; \; s_{k,i}^2 = \tau_{k,i}^{-1}\sigma^2 \tag{11}$$

$$\gamma_{k,i}|\tau_{k,i} \sim \mathcal{N}_+\left(0, \tau_{k,i}^{-1}\right) \; ; \; \tau_{k,i} \sim \mathcal{G}\left(\frac{\nu}{2}, \frac{\nu}{2}\right) \tag{12}$$

If we define $\boldsymbol{\theta} = [\mu, \sigma^2, \lambda, \nu]$, $\boldsymbol{\psi}_{k,i}(\boldsymbol{\theta}) = [\gamma_{k,i}(\boldsymbol{\theta}), \tau_{k,i}(\boldsymbol{\theta})]$ and $\boldsymbol{\phi}_k = [\boldsymbol{\psi}_{k,1}(\boldsymbol{\theta}), \ldots, \boldsymbol{\psi}_{k,N}(\boldsymbol{\theta})]$, the complete measurement noise can be rewritten as

$$\mathbf{n}_k|\boldsymbol{\phi}_k \sim \mathcal{N}\left(\mathbf{m}_k(\boldsymbol{\phi}_k), \mathbf{R}_k(\boldsymbol{\phi}_k)\right) \tag{13}$$

$$\mathbf{m}_k(\boldsymbol{\phi}_k) = [m_{k,1}(\boldsymbol{\psi}_{k,1}(\boldsymbol{\theta})), \ldots, m_{k,N}(\boldsymbol{\psi}_{k,N}(\boldsymbol{\theta}))]^\top$$

$$\mathbf{R}_k(\boldsymbol{\phi}_k) = \mathrm{diag}\left(s_{k,1}^2(\boldsymbol{\psi}_{k,1}(\boldsymbol{\theta})), \ldots, s_{k,N}^2(\boldsymbol{\psi}_{k,N}(\boldsymbol{\theta}))\right)$$

Using these definitions we can define the pseudo-observation $\mathbf{y}_k \triangleq \boldsymbol{\rho}_k - \mathbf{m}_k(\boldsymbol{\phi}_k)$, which is instrumental to deal with the noise mean with the filtering method.

3. JOINT TRACKING AND MODEL ESTIMATION

We are interested in tracking the states of the system and, simultaneously, inferring the unknown model parameters. The joint *a posteriori* distribution casts all the information about the states and the system model provided by the observations. Using the SSM formulation (8)-(9), and the conditionally Gaussian form in (13), the conditional posterior $p(\mathbf{x}_k|\boldsymbol{\phi}_k, \mathbf{y}_{1:k})$ turns to be Gaussian and thus can be computed using a Gaussian filter [16]. We propose to use the quadrature Kalman filter (QKF) [9, 10], which resorts to Gauss-Hermite quadrature rules to approximate the integrals in the optimal solution (sketched in Algorithm 1).

3.1. Gaussian filtering

The optimal Bayesian filtering solution is given by the posterior distribution $p(\mathbf{x}_k|\boldsymbol{\phi}_k, \mathbf{y}_{1:k})$, which is typically computed using two steps, *prediction* and *update*. If the system is nonlinear and Gaussian, both the transition density $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ and the likelihood $p(\mathbf{y}_k|\mathbf{x}_k, \boldsymbol{\phi}_k)$ are Gaussian distributed. Under this assumption, the predictive and posterior densities can be approximated as $p(\mathbf{x}_k|\boldsymbol{\phi}_k, \mathbf{y}_{1:k-1}) = \mathcal{N}\left(\hat{\mathbf{x}}_{k|k-1}, \boldsymbol{\Sigma}_{k|k-1}\right)$ and $p(\mathbf{x}_k|\boldsymbol{\phi}_k, \mathbf{y}_{1:k}) = \mathcal{N}\left(\hat{\mathbf{x}}_{k|k}, \boldsymbol{\Sigma}_{k|k}\right)$, respectively [9] with[1]

$$\hat{\mathbf{x}}_{k|k-1} = \int \mathbf{f}(\mathbf{x}_{k-1}) p(\mathbf{x}_{k-1}|\boldsymbol{\phi}_{k-1}, \mathbf{y}_{1:k-1}) d\mathbf{x}_{k-1}$$

$$\boldsymbol{\Sigma}_{k|k-1} = \int \mathbf{f}^2(\mathbf{x}_{k-1}) p(\mathbf{x}_{k-1}|\boldsymbol{\phi}_{k-1}, \mathbf{y}_{1:k-1}) d\mathbf{x}_{k-1} - \hat{\mathbf{x}}^2_{k|k-1} + \mathbf{Q}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \left(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}\right) \quad \text{s.t.} \quad \mathbf{y}_k \triangleq \boldsymbol{\rho}_k - \mathbf{m}_k(\boldsymbol{\phi}_k)$$

$$\boldsymbol{\Sigma}_{k|k} = \boldsymbol{\Sigma}_{k|k-1} - \mathbf{K}_k \boldsymbol{\Sigma}_{y,k|k-1} \mathbf{K}_k^\top$$

where the Kalman gain, measurement prediction, innovation covariance and cross-covariance are obtained as

$$\mathbf{K}_k = \boldsymbol{\Sigma}_{xy,k|k-1} \boldsymbol{\Sigma}^{-1}_{y,k|k-1}$$

$$\hat{\mathbf{y}}_{k|k-1} = \int \mathbf{h}(\mathbf{x}_k) p(\mathbf{x}_k|\boldsymbol{\phi}_{k-1}, \mathbf{y}_{1:k-1}) d\mathbf{x}_k$$

$$\boldsymbol{\Sigma}_{y,k|k-1} = \int \mathbf{h}^2(\mathbf{x}_k) p(\mathbf{x}_k|\boldsymbol{\phi}_{k-1}, \mathbf{y}_{1:k-1}) d\mathbf{x}_k - \hat{\mathbf{y}}^2_{k|k-1} + \mathbf{R}_k(\boldsymbol{\phi}_k)$$

$$\boldsymbol{\Sigma}_{xy,k|k-1} = \int \mathbf{x}_k \mathbf{h}^\top(\mathbf{x}_k) p(\mathbf{x}_k|\boldsymbol{\phi}_{k-1}, \mathbf{y}_{1:k-1}) d\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1} \hat{\mathbf{y}}^\top_{k|k-1}$$

3.2. Estimation of noise latent variables $\boldsymbol{\phi}_k$

In practice, at time k, the filter requires a particular value of $\mathbf{m}_k(\boldsymbol{\phi}_k)$ and $\mathbf{R}_k(\boldsymbol{\phi}_k)$ to be executed. This means a particular realization of the random variables $\gamma_{k,i}$ and $\tau_{k,i}$ for each node i. The approach we followed operates in the update step of the filter, where the observations can be considered independent among anchor nodes and thus to follow a $\mathcal{N}(y_{k,i}; h(\mathbf{x}_k) + \mu + \lambda\gamma_{k,i}, s^2_{k,i})$. Therefore, we would like to infer from a single observation the values that $\gamma_{k,i}$ and $\tau_{k,i}$ took. The ML solution is the trivial solution where $\hat{\gamma}_{k,i} = y_{k,i} - h(\mathbf{x}_k) - \mu$ coincides with the innovation error of the filter and $\tau_{k,i}$ is related to the innovation's covariance.

However, following a Bayesian approach, we have defined a priori distributions for the variables of interest which can be used in the inference problem. Actually, they could be of great help in this case where only a single observation is available and the ML is likely to provide unreliable estimates. All information regarding these unknowns is contained in

$$p(\gamma_{k,i}, \tau_{k,i}|y_{k,i}) \propto p(y_{k,i}|\lambda\gamma_{k,i}, \tau_{k,i}) p(\gamma_{k,i}|\tau_{k,i}) p(\tau_{k,i}) \tag{14}$$

where the likelihood is described by the Gaussian previously mentioned. Particularly, we use a normalized likelihood

$$p(y_{k,i}|\gamma_{k,i}, \tau_{k,i}) = \mathcal{N}\left(\frac{y_{k,i} - h(\mathbf{x}_k) - \mu}{\sigma}; \tilde{\gamma}_{k,i}, \tau^{-1}_{k,i}\right) \tag{15}$$

where $\tilde{y}_{k,i} \triangleq \frac{y_{k,i} - h(\mathbf{x}_k) - \mu}{\sigma}$ and $\tilde{\gamma}_{k,i} \triangleq \lambda\gamma_{k,i}/\sigma$. $\hat{\mathbf{x}}_{k|k-1}$ is used instead of $\mathbf{x}_k$, which is unknown. In this situation, it is possible to obtain analytically a solution for the posterior distributions of $\gamma_{k,i}$ and $\tau_{k,i}$ thanks to the conjugate nature of the prior distributions [17]. The prior distributions were defined

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Algorithm 1 Quadrature Kalman Filter (QKF)

Require: $\mathbf{y}_{1:K}$, $\hat{\mathbf{x}}_{0|0}$, $\boldsymbol{\Sigma}_{0|0} = \mathbf{S}_{0|0}\mathbf{S}_{0|0}^\top$, $\mathbf{Q}$ and $\boldsymbol{\theta}$

Define M sigma–points and weights $\{\boldsymbol{\xi}_i, \omega_i\}_{i=1,\ldots,M}$ by using Gauss-Hermite quadrature rules [10].

for $k = 1$ to K **do**

  Prediction (time update)

  Propagate the sigma-points:

$$\mathbf{x}_{i,k-1|k-1} = \mathbf{S}_{k-1|k-1}\boldsymbol{\xi}_i + \hat{\mathbf{x}}_{k-1|k-1},\ i = 1, \ldots, M.$$
$$\tilde{\mathbf{x}}_{i,k|k-1} = \mathbf{f}(\mathbf{x}_{i,k-1|k-1}),\ i = 1, \ldots, M.$$

  Estimate the predicted state and corresponding covariance:

$$\hat{\mathbf{x}}_{k|k-1} = \sum_{i=1}^M \omega_i \tilde{\mathbf{x}}_{i,k|k-1}.$$
$$\boldsymbol{\Sigma}_{k|k-1} = \sum_{i=1}^M \omega_i \tilde{\mathbf{x}}^2_{i,k|k-1} - \hat{\mathbf{x}}^2_{k|k-1} + \mathbf{Q}$$

  Update (filtering estimate)

  Propagate the sigma-points:

$$\mathbf{x}_{i,k|k-1} = \mathbf{S}_{k|k-1}\boldsymbol{\xi}_i + \hat{\mathbf{x}}_{k|k-1},\ i = 1, \ldots, M.$$
$$\tilde{\mathbf{y}}_{i,k|k-1} = \mathbf{h}(\mathbf{x}_{i,k|k-1}),\ i = 1, \ldots, M.$$

  Estimate the predicted measurement:

$$\hat{\mathbf{y}}_{k|k-1} = \sum_{i=1}^M \omega_i \tilde{\mathbf{y}}_{i,k|k-1}.$$

  Estimate noise variables $\boldsymbol{\phi}_k$ using (20) $\Rightarrow \hat{\boldsymbol{\phi}}_k$

  Estimate the innovation covariance matrix:

$$\boldsymbol{\Sigma}_{y,k|k-1} = \sum_{i=1}^M \omega_i \tilde{\mathbf{y}}^2_{i,k|k-1} - \hat{\mathbf{y}}^2_{k|k-1} + \mathbf{R}_k(\hat{\boldsymbol{\phi}}_k)$$

  Estimate the cross-covariance matrix:

$$\boldsymbol{\Sigma}_{xy,k|k-1} = \sum_{i=1}^M \omega_i \tilde{\mathbf{x}}_{i,k|k-1} \tilde{\mathbf{y}}^\top_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1} \hat{\mathbf{y}}^\top_{k|k-1}$$

  Estimate the state and corresponding error covariance:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \left(\boldsymbol{\rho}_k - \mathbf{m}_k(\hat{\boldsymbol{\phi}}_k) - \hat{\mathbf{y}}_{k|k-1}\right)$$
$$\boldsymbol{\Sigma}_{k|k} = \boldsymbol{\Sigma}_{k|k-1} - \mathbf{K}_k \boldsymbol{\Sigma}_{y,k|k-1} \mathbf{K}_k^\top$$

  where the Kalman gain is $\mathbf{K}_k = \boldsymbol{\Sigma}_{xy,k|k-1}\boldsymbol{\Sigma}^{-1}_{y,k|k-1}$.

end for

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[1] We write $(\mathbf{x})^2$, $(\mathbf{y})^2$, $\mathbf{f}^2(\cdot)$ and $\mathbf{h}^2(\cdot)$ as the shorthand for $\mathbf{x}\mathbf{x}^\top$, $\mathbf{y}\mathbf{y}^\top$, $\mathbf{f}(\cdot)\mathbf{f}^\top(\cdot)$ and $\mathbf{h}(\cdot)\mathbf{h}^\top(\cdot)$, respectively. We omitted the dependence with time of $\mathbf{f}_{k-1}(\cdot)$ and $\mathbf{h}_k(\cdot)$ for the sake of clarity.

in (12). For the sake of simplicity, we do not consider the truncated Gaussian distribution in which case the derivations are simplified [5]. Then,

$$p(\tilde{\gamma}_{k,i}|\tau_{k,i}) = \mathcal{N}\left(\tilde{\gamma}_{k,i}\,;\,0,\lambda^2\tau_{k,i}^{-1}\right) \quad (16)$$

$$p(\tau_{k,i}) = \mathcal{G}\left(\tau_{k,i}\,;\,\frac{\nu}{2},\frac{\nu}{2}\right) \quad (17)$$

and it turns out that the posterior marginals of interest are

$$p(\tilde{\gamma}_{k,i}|y_{k,i}) = \mathcal{T}_{2\alpha}\left(\frac{\tilde{y}_{k,i}}{2},\frac{\beta}{2\alpha}\right) \quad (18)$$

$$p(\tau_{k,i}|y_{k,i}) = \mathcal{G}\left(\alpha,\beta\right) \quad (19)$$

with $\alpha = \frac{\nu}{2}+\frac{1}{2}$ and $\beta = \frac{\nu}{2}+\frac{\tilde{y}_{k,i}^2}{4}$. Recall that we are interested in point estimates, required to run the filter at time k. From (18) and (19) we can compute the corresponding modes and use them as our estimates

$$\hat{\gamma}_{k,i} = \frac{|\tilde{y}_{k,i}|}{2}\frac{\sigma}{\lambda} \quad , \quad \hat{\tau}_{k,i} = \frac{\alpha-1}{\beta} \quad (20)$$

where we took into account that $\hat{\gamma}_{k,i} \in \mathbb{R}^+$ by construction.

4. COMPUTER SIMULATIONS

The proposed filtering method was validated in a realistic scenario composed of $N = 6$ anchor nodes, circularly deployed in a 40×40 m^2 area. The considered ranging technology was UWB, and the parameters $\boldsymbol{\theta}$ of the model adjusted as in [15], where the authors adjusted the parameters to match real data. Particularly, we considered $\mu = -0.1$ m, $\sigma = 0.3$ m, $\lambda = 0.6$ m, and $\nu = 4$. For stability reasons, a square-root formulation of the QKF was used in the simulations [18].

Simulation results compare the root mean square error (RMSE) performance of three kinds of QKF. Namely, $i)$ a QKF operating under the Gaussian assumption without accounting for the heavy tails of the measurement noise. In this case, the noise is assumed zero-mean, additive, white, and normally distributed such that $n_{k,i} \sim \mathcal{N}(0,\sigma^2)$ with $\sigma = 0.3$ m; $ii)$ a clairvoyant QKF that knows exactly the realization of the latent variables $\boldsymbol{\phi}_k$ at each instant k and thus can use $\mathbf{m}_k(\boldsymbol{\phi}_k)$ and $\mathbf{R}_k(\boldsymbol{\phi}_k)$; and $iii)$ the proposed QKF that estimates $\boldsymbol{\phi}_k$ and thus uses $\mathbf{m}_k(\hat{\boldsymbol{\phi}}_k)$ and $\mathbf{R}_k(\hat{\boldsymbol{\phi}}_k)$ in the filtering process. All these methods consider $\boldsymbol{\theta}$ known.

From Fig. 1 we can see that although the clairvoyant filter outperforms the rest, our solution tends to its performance. On the other hand, the QKF operating under the fully Gaussian assumption (alien to the tails) shows peaks due to NLOS-induced outliers. Nevertheless, the differences are not severe. Recall that the parameter regulating the heavy tails of the noise distribution is ν. The larger this value, the more it resembles a Gaussian distribution. With $\nu = 4$ we are in such situation and thus the similar performance. A second experiment was conducted with $\nu = 2$, in which case the effect of NLOS is clear for the Gaussian filter, whereas our solution hardly degrades with respect to the clairvoyant method.

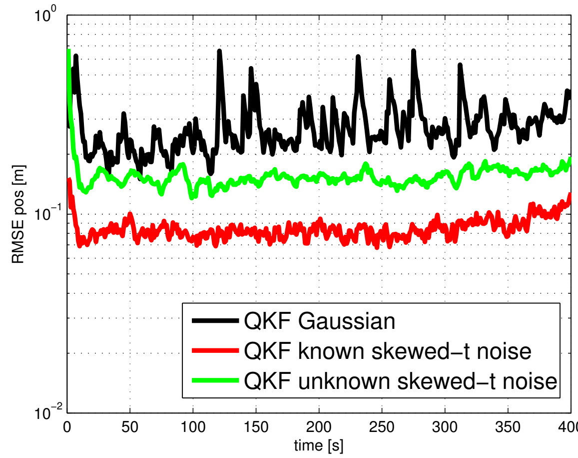

Fig. 1. RMSE of position for $\nu = 4$.

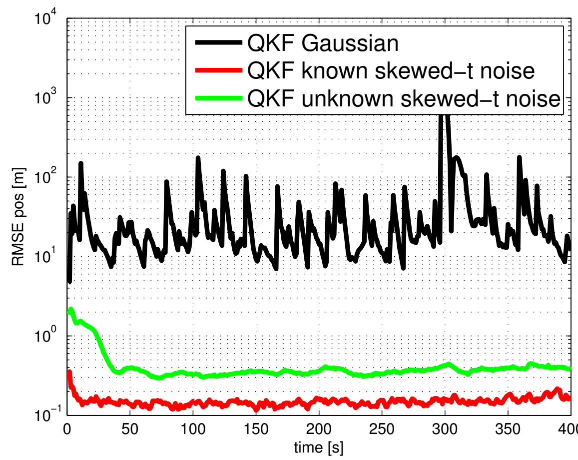

Fig. 2. RMSE of position for $\nu = 2$.

5. CONCLUSIONS

This paper presented a filter design framework/methodology to robustly deal with NLOS situations in TOA-based localization. In NLOS situations, range estimates are typically biased and have larger variances than LOS measurements. This can be incorporated in the model as a heavy-tailed distribution, which can be formulated as a conditional Gaussian distribution and thus tracking can be efficiently implemented via Gaussian filters. The article proposed as well an algorithm to estimate in execution time the noise statistics. Simulation results show improved performance and the promising capabilities of such approach.

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