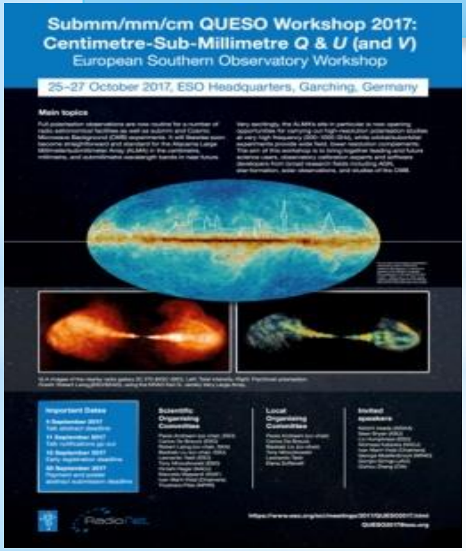
# Evidence of Cosmic Strings by Observation of the Alignments of Quasar Polarization Axes at Mpc Scales

Centimetre-Sub-Millimetre European Southern Observatory Workshop
Garching, October 25 - 27, 2017

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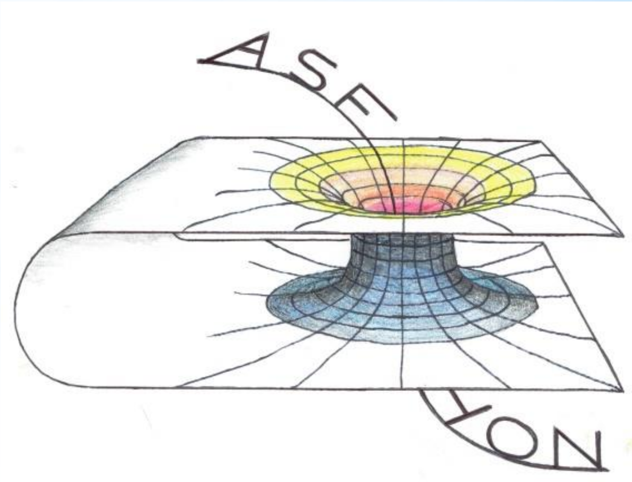

Presentation based on:

R.Slagter, S.Pan: Found. of Phys. 2016 ( communicated by t'Hooft)
R.Slagter: Journ. of Mod. Phys. 2016 & 2017
S.Pan, J.de Haro, A. Paliathanasir, R.Slagter: Mon.Not. Roy. Ast. Soc.2016
R.Slagter: Astrophys. Journ. 1983, 1986 [multiple scale analysis on HF pert.]

# What has a theoretical physicists to do with workshop on polarimetry at ESO?

We shall see:

Observations on **quasars** at different redshifs could provide us evidence of cosmic strings

Especially z $\approx 3$

Ideas are welcome . [ERC grant proposal submitted]

# Possible evidence of cosmic strings via alignment of quasar polarization axis?

There appeared two investigations on polarization vectors on BH and quasars:

D.Hutsemekers, et al, Alignment of quasar polarizations with large-scale structures

A.Taylor, et al, Alignment of Radio Galaxies in deep radio imaging of ELAIS N1

## Alignment of quasar polarizations with large-scale structures*

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Received ; accepted:



**ABSTRACT**

We have measured the optical linear polarization of quasars belonging to Gpc-scale quasar groups at redshift $z \sim 1.3$. Out of 93 quasars observed, 19 are significantly polarized. We found that quasar polarization vectors are either parallel or perpendicular to the directions of the large-scale structures to which they belong. Statistical tests indicate that the probability that this effect can be attributed to

## Alignments of Radio Galaxies in Deep Radio Imaging of ELAIS N1

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[3] *National Radio Astronomy Observatory, Socorro, New Mexico, USA*



Accepted XXX. Received YYY; in original form ZZZ

**ABSTRACT**

We present a study of the distribution of radio jet position angles of radio galaxies over an area of 1 square degree in the ELAIS N1 field. ELAIS N1 was observed with the Giant Metrewave Radio Telescope at 612 MHz to an rms noise level of 10 $\mu$Jy and angular resolution of $6'' \times 5''$. The image contains 65 resolved radio galaxy jets. The spatial distribution reveals a prominent alignment of jet position angles along a "filament" of about 1°. We examine the possibility that the apparent alignment arises from an underlying random distribution and find that the probability of chance

# Overview

I. *What are Topological Defects in Cosmology?*

a. Origin: *superconductivity* *[Ginsburg-Landau theory]*

b. *The only survivor: Cosmic String [no monopoles,..]*

c. *CS can cause primordial structure: scale-invariant.*

II. *Application to: warped brane world models with U(1)*

*+ scalar-gauge field (in the brane)*

*Spin-off:*

***a.*** ***Self-acceleration of FLRW possible without $\Lambda$?***

***[Slagter, Pan: Found of Phys, 2016]***

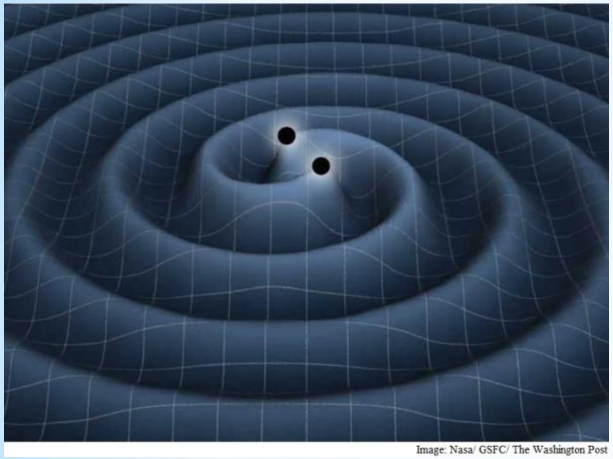
►► ***b.*** ***Evidence Cosmic Strings via alignment of quasar polarization?***

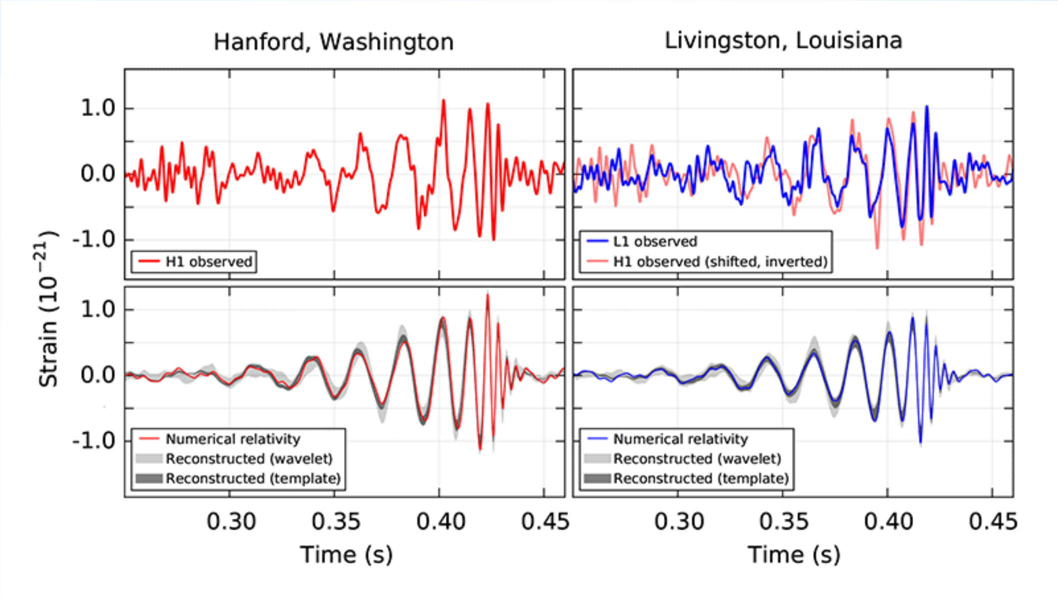
***[Slagter: Journ Mod Phys,2016, 2017]***

***Ann of Physics, 2017 [subm]***

# General Relativity

**GR** is by far the ***best tested theory*** : recently: **gravitational waves** detected:

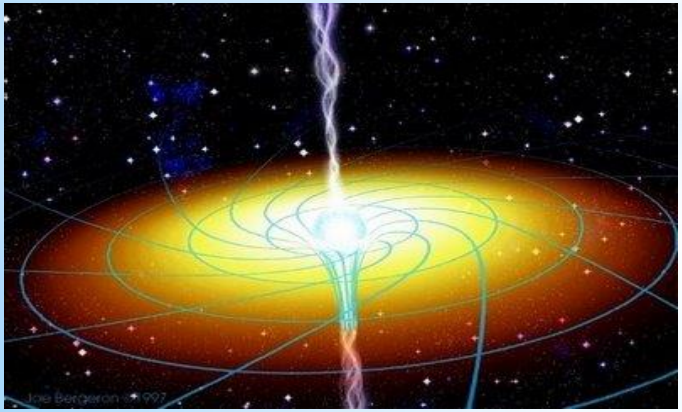


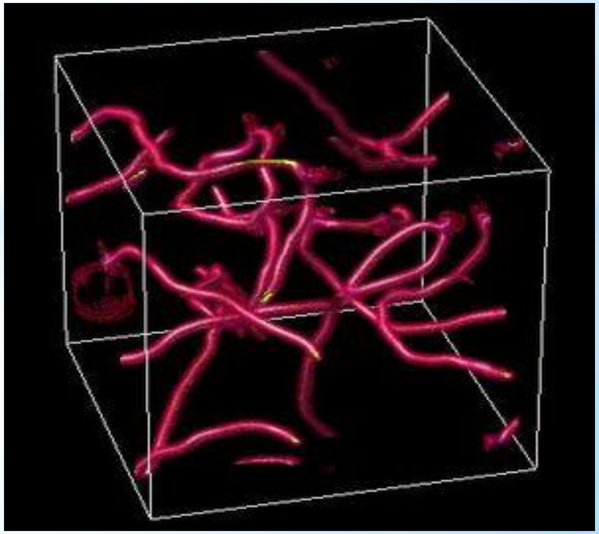

Total amount of energy $\sim \mathbf{10^{40} J}$

The two **most interesting** compact objects in GR:

**Kerr black hole:**



**Cosmic strings:**



# Severe problems of GR + QFT

1. Hiarchy-problem ( why is gravity so weak?)

2. What is dark-energy (needed for accelerated universe)  $\Lambda$ needed??

3. Then: huge discrepancy between $\rho_\Lambda \sim 10^{-120}$ and $\rho_{vac.} \sim 10^{-3}$
   + incredibly fine-tuned: $\Omega_\Lambda \sim \Omega_{Mat}$

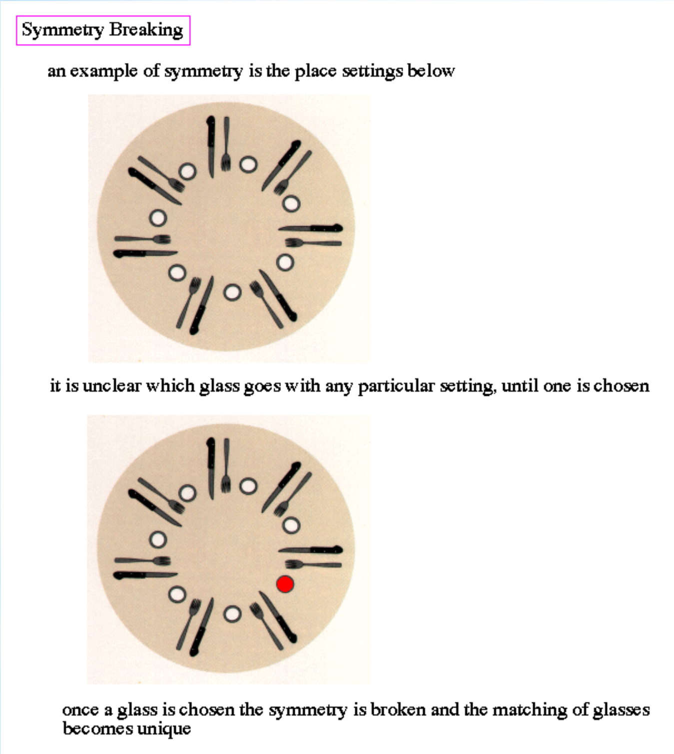
4. What happens at the Planck length?  TOE possible?

5. The black hole war: Hawking--‘t Hooft
   Desperately needed: quantum-gravity model

6. Do we need higher-dimensional worlds?  [are we a “hologram” ]

**7. How do we make gravity conformal (scale-) invariant
   Klein-Gordon ( massless) and Maxwell: are CI
   Vacuum Einstein-dilaton: is CI
   our world is non-vacuum: Is the conformal factor linked to dilaton,
   in order to explain mass spectrum by symmetry breaking

# Symmetry breaking: the ultimate route to understand particle physics and general relativity at the planck scale $L_{pl} = \sqrt{\hbar G/c^3} = 1.6\ 10^{-33} cm$




Conformal (scale-) invariance:

** At high energies: restmass particles negligible effects. So in TOE no explicit mass scales

** renormalizable ( dimensionless coupling c.)
** quantum theory of gravity possible ('t Hooft 2014,2017) without singulatities

** Symmetry methods very successful: standard model: Higgs mechanism.

** will be an experimental constraint!!

** AdS/CFT correspondence in stringtheory?:
holographic principle: conformal field theory =boundary of higher dim spacetimes.

# Present State of our Universe

► The expansion of our universe is **accelerating:**

$H_0 = 71.9 \pm 2.7$ [$H0LiCOW$, 2017]  $H_0 = 67.9 \pm 1.5$ [$\Lambda CDM$]  **New physics?**

► One needs **dark energy** with an effectively negative pressure, $p < -\frac{1}{3}\rho$

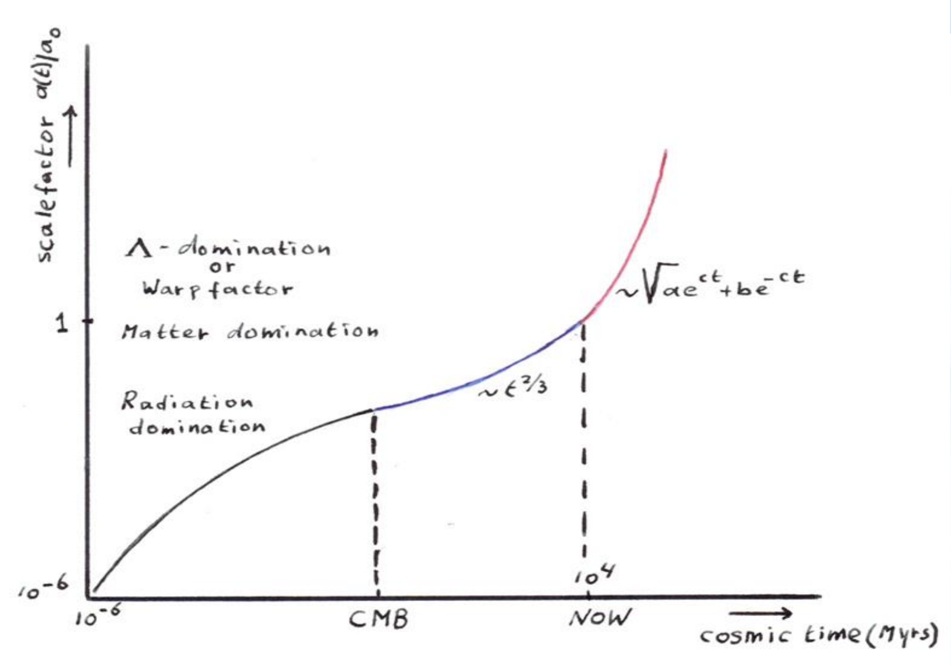
$\Lambda$CDM: **w = -1**  **[ Planck 2015: $w > -1$ ? ]**

► We should live now in the cosmological constant dominated era (and approx. )

$\Omega_\Lambda = 0.73$  $\Omega_M = \Omega_{DM} (= 0.23) + \Omega_B (= 0.046)$

► **Dark Energy Survey** [DES 2017]: **wCDM**: $\Omega_{DM} = 0.301$ **w = -0.8±0.2**

**Eucid(2020):** will give dicisive answers: modify gravity, $\Lambda$ , or: conformal field theory



# The scalar-gauge field in GR

The abelian scalar (Higgs) field with gaugegroup U(1) has **lived up its reputation**!!

1. As **order parameter** in super conductivity: Ginzburg-Landau model

2. The U(1)-scalar-gauge field in **standard model** of particle physics (**Higgs mech**.)

3. The special $\phi^4$ self interacting **Nielsen-Olesen vortex** solution

4. Needed in **inflationairy** model [ horizon-flatness problems solved?]

5. **General Relativistic-cosmic** string solution

6. **Super-massive cosmic strings**: can build-up huge mass in the extra-dimension of the bulk spacetime ( warped spacetimes)

7. **NEW:** Connection with secular instability of an initial axially sym. Configuration

** **a kind of a second-order "phase-transition"**

** **the breaking of the non-axially sym $\sim e^{im\varphi}$**

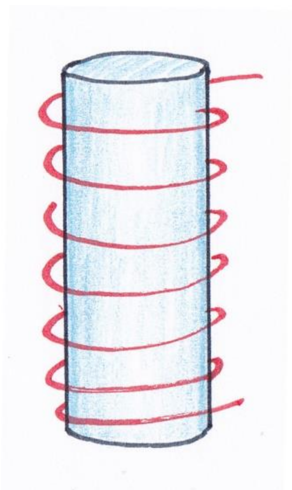
**►►► quasar alignment? Quasar-confinement for large red-shift must be of primordial origin.**

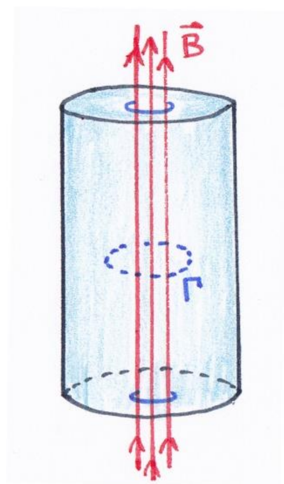
# A. Super-conductivity

**Gisnzberg-Landau model:** **Type II Super-conductivity**

► Formation of the supercond. state: Cooper current by the **Meissner effect**:

If one places a super cond. cylinder in a solenoid ► magn. field is expelled from cyl.






► Increasing magn.field:

**vortices** are formed [Abrikosov-vortex]

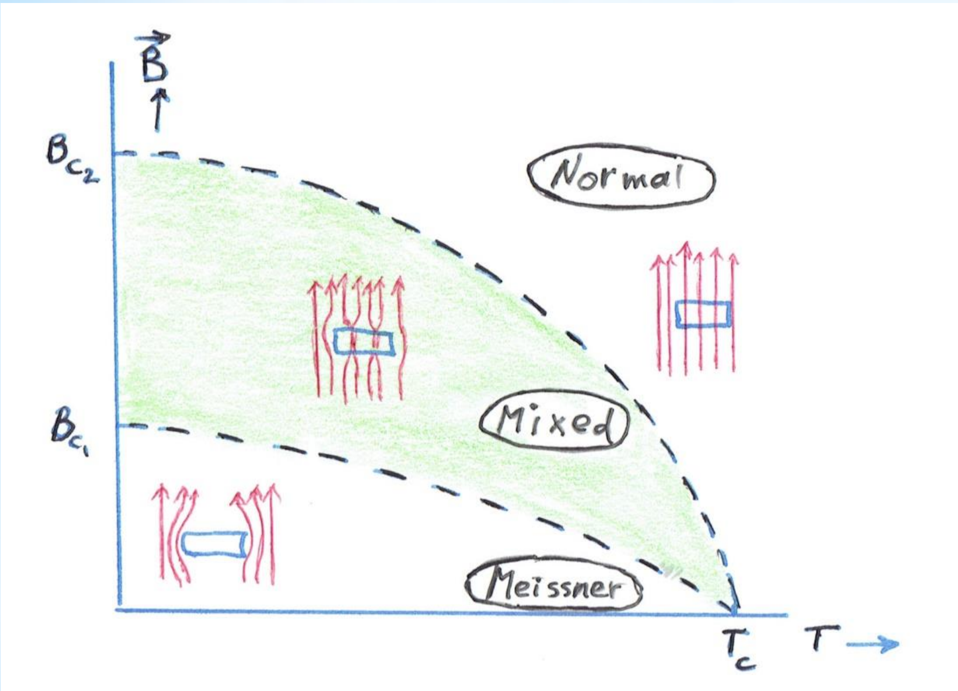
► The magn.flux is quantized:

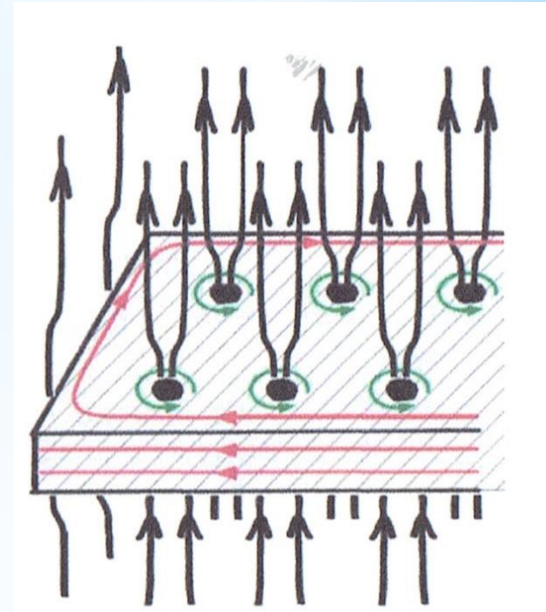
$$\Phi = \oint_\Gamma A.dr = \frac{h}{q}\oint \nabla\phi.dr = n\frac{2\pi h}{q}$$

n = winding number

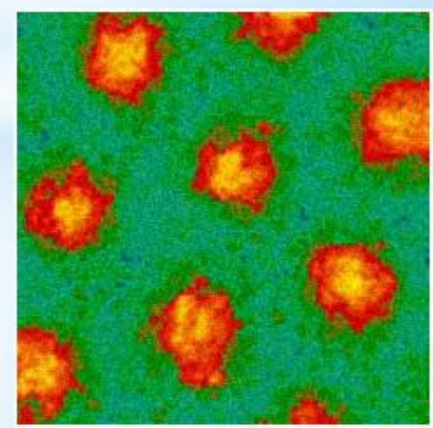
# B. Abrikosov-vortices

► Energetically favorable to form LATTICE of quantum vortices often forming a triangular lattice



Vortices in NbSe2 superconductor.mp4



There are 2 critical values

- ► $B < B_{c1}$ : Meissner effect
- ► $B_{c1} < B < B_{c2}$ : small "tubes" where B penetrates: vortices
- ► $B > B_{c2}$ : normal state

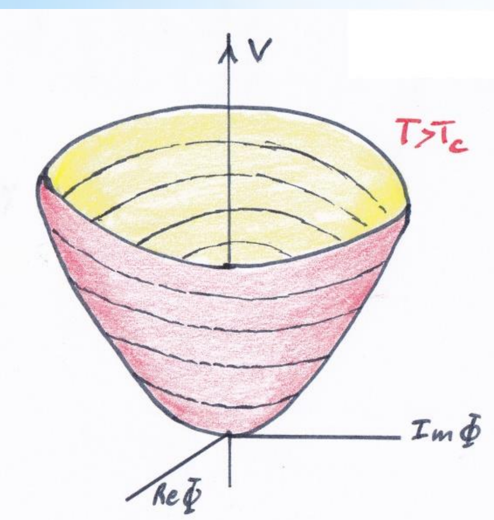
# C.The topological formulation: The Nielsen-Olesen vortex

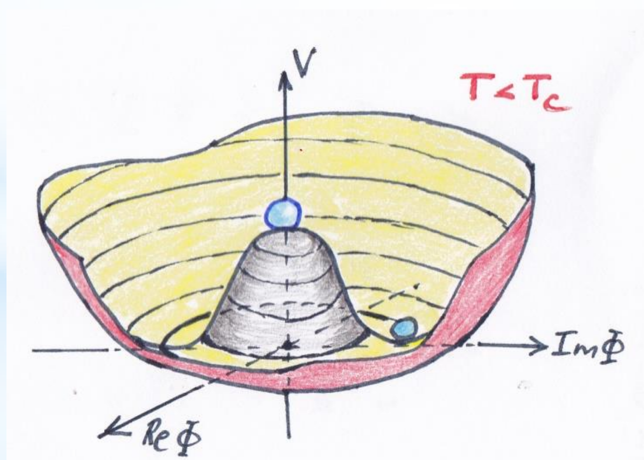
Now: QFT: Let us consider the U(1) scalar-gauge field:
the complex scalar field $\Psi$ will be coupled minimally to the gauge field $A_\mu$
($\beta$ coupling const; $\eta$ VEV)

$$\mathcal{L} = -\frac{1}{4} F_{ab} F^{ab} - \frac{1}{2} D_a \Phi (D^a \Phi)^* - \frac{1}{8} \beta (|\Phi|^2 - \eta^2)^2$$

With
$$F_{ab} = \partial_a A_b - \partial_b A_a, \quad D_a = \partial_a + ieA_a$$

So we replaced in GL model $\frac{q}{h} = e$.



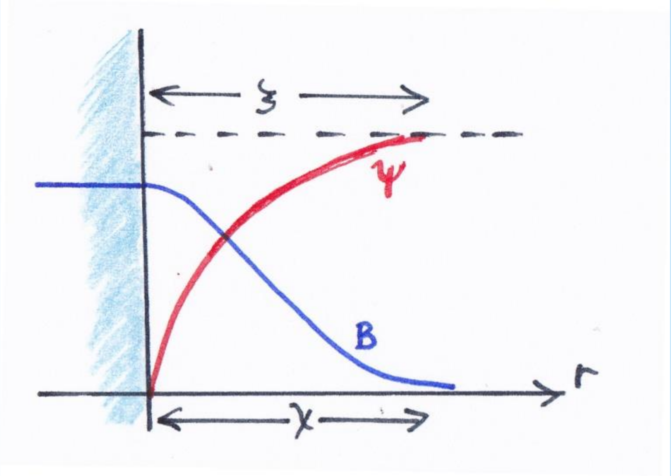

When temp. drops, scalar develops a degenerated vacuum [= SC state in GL]
In polar coord.:

$$\Phi = \Psi(\rho, t) e^{in\varphi}$$
$$A_a = \frac{1}{e} [B(\rho, t) - n] \nabla_a \varphi$$

n = number of flux quanta

**Note:** These vortices can be used to describe the dual strings [Nambu-Goto]

# C. The Nielsen-Olesen vortex




**Typical solution**: two characteristic lengths:
coherence length $\xi$
penetration length $\chi$

The action is invariant under the gauge-transf.:

$$\Phi \to e^{i\chi}\Phi, \quad A_a \to A_a + \partial_a \chi$$

However, the vacuum state NOT, hence the EM-gauge symmetry is broken

**SO: The** vortex is a spatial localized structure around which the order parameter has a none trival winding: it is a topological defect, where the normal state intrudes and magnetic flux penetrates.

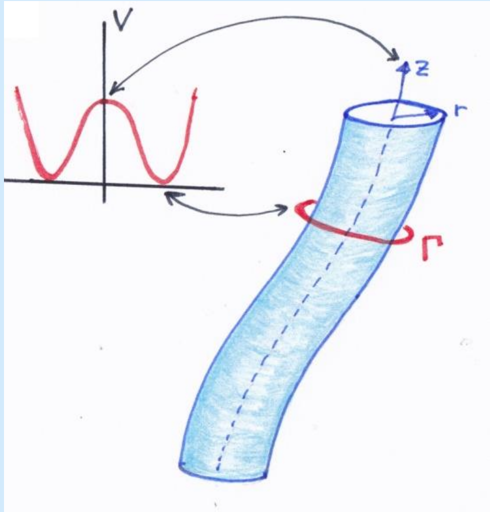
**Ginzburg-Landau parameter**: $\kappa = \chi/\xi$ [exeptional $\Phi^4 model$ $\kappa = 1/\sqrt{2}$ ]

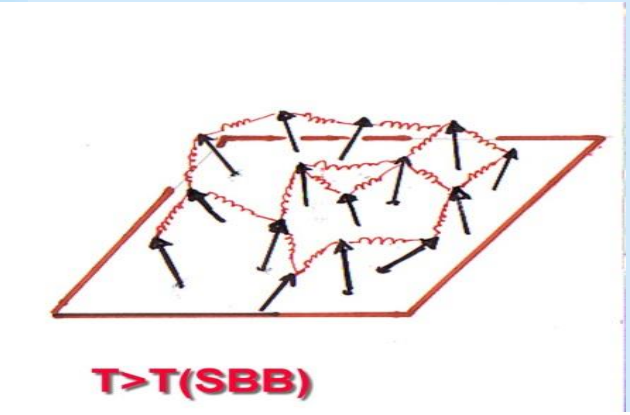
The vortex number n [$= \frac{1}{2\pi} \int F$ ] equals the winding number of $\Phi$

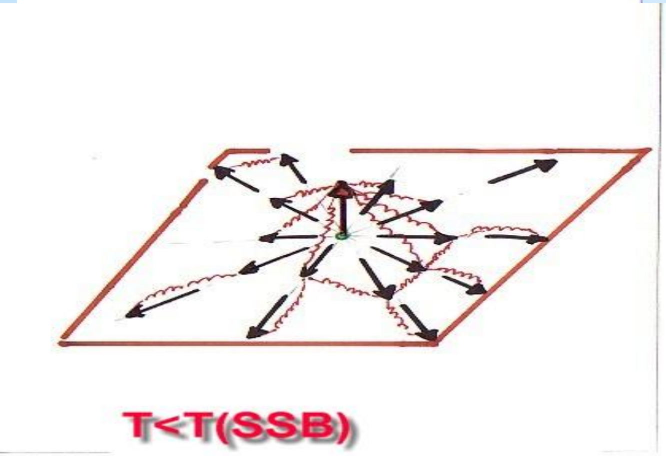
# E. Trapped energy of false vacuum

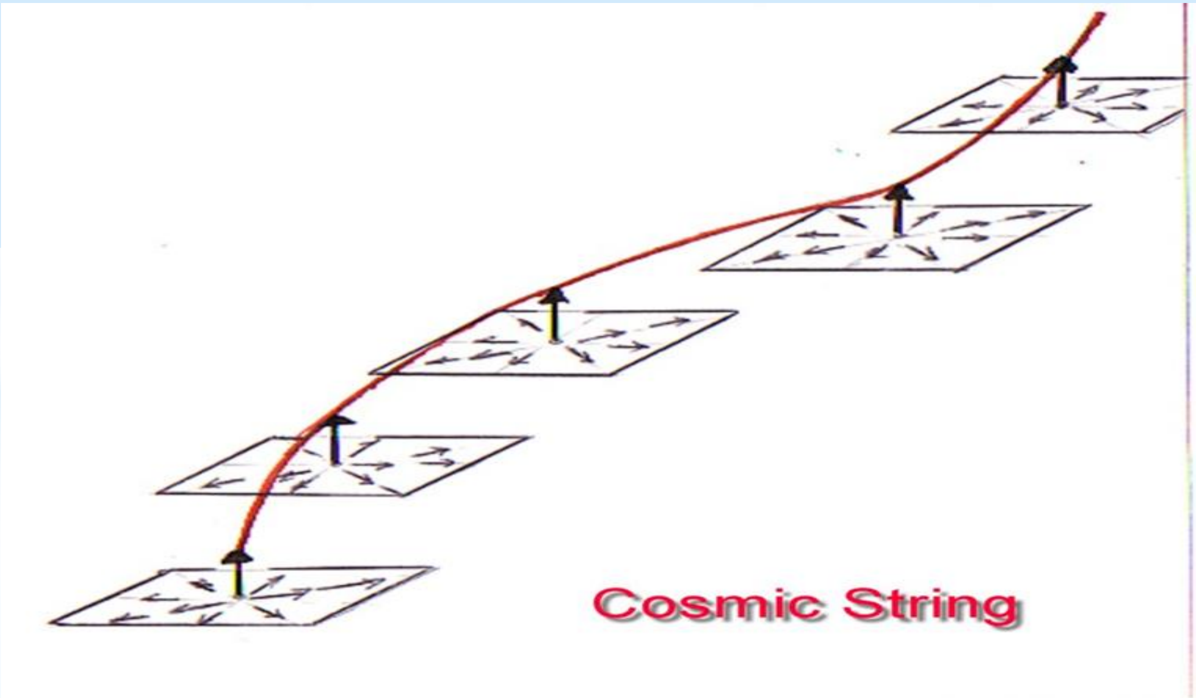
► Trapped energy of the false vacuum

► One " Higgs-pencil" cannot follow the symmetry in the plane: if it lays down, symmetry will be broken. At this point there is a lot of potential energy stored in the scalar field configuration



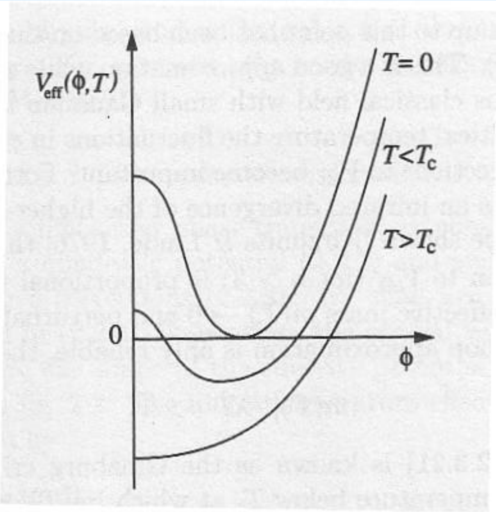



# First and second order phase transition

► In reality, Φ is a quantum field, so **V(Φ)** must be modified due to radiative corrections. For the Goldstone model, the second order phase transition is described by the high-temperature effective potential

$$U_{eff}(\Phi,T) = m(T)^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4, m^2 = \frac{\lambda}{12}(T^2 - 6\eta^2)$$




In Hot Big Bang model the universe starts at very high temperature. When universe cools down below $T_c$, Φ develops an expectation value:

$|\Phi| = (T_c^2 - T^2)^{1/2}$

► **The phase φ takes again different values at different regions of space.**

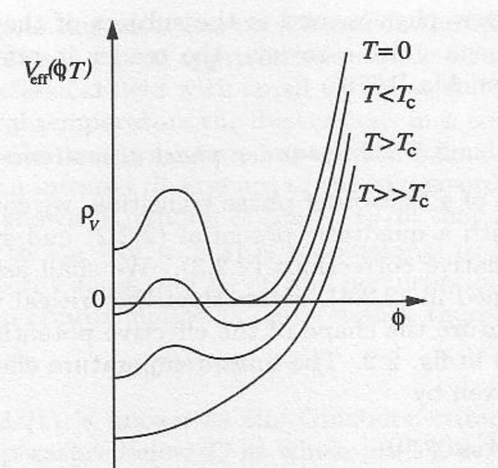
**Consider now the first order effective potential**

$$U_{eff} = m(T)^2|\Phi|^2 + \frac{3e^2}{16\pi^2}|\Phi|^4 ln(\frac{|\Phi|^2}{\sigma^2}),$$

$m^2 = \mu_0^2 + \frac{1}{4}e^2T^2$

**difference: symmetric phase below $T_c$ remains meta-stable if $\mu_0^2 < 0$**

**application: Inflation**



# GR: The self-gravitating NO-string

- It came as a big surprise that there exists vortex-like solutions in GR.
- Field equations: $$ds^2 = -e^A dt^2 + e^B dz^2 + dr^2 + e^C d\varphi^2$$

$$G_{\mu\nu} = \kappa^2{}_4 T_{\mu\nu} \qquad D_\mu D^\mu \Phi - 2\frac{\partial U}{\partial \Phi^*} = 0 \qquad \nabla^\mu F_{\mu\nu} - \frac{1}{2} ie[\Phi(D_\nu\Phi)^* - \Phi^*(D_\nu\Phi)] = 0$$

$$A_\nu = \frac{(P-n)}{e}\nabla_\nu\varphi \qquad \Phi = X\, e^{in\varphi}$$

To restore boost inv: A=B

[ K= $e^{A+\frac{C}{2}}$ ]

$$\partial_{rr}K = \frac{1}{2}\kappa_4{}^2\,\eta^2[-\frac{3}{4}K(X^2-1)^2 - 2e^{2A}\frac{X^2P^2}{K} + \frac{e^{2A}}{\alpha K}(\partial_r P)^2]$$

$$\partial_{rr}A = -\frac{\partial_r A\partial_r K}{K} + \kappa_4{}^2\,\eta^2[-\frac{1}{4}(X^2-1)^2 + \frac{e^{2A}}{\alpha K^2}(\partial_r P)^2]$$

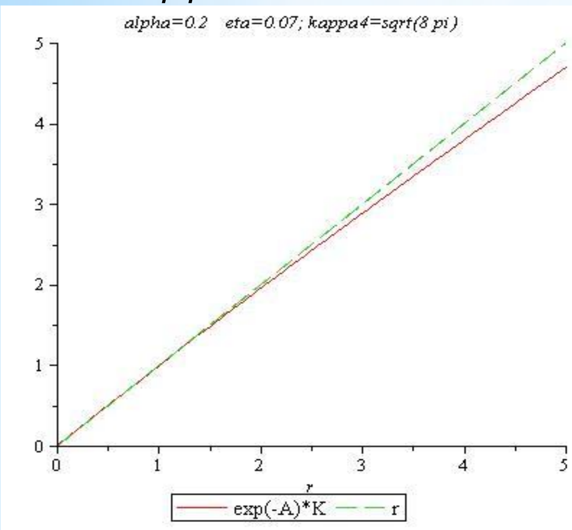
$$\partial_{rr}X = -\frac{\partial_r X\partial_r K}{K} + \frac{1}{2}X(X^2-1)^2 + \frac{e^{2A}}{K^2}XP^2$$

$$\partial_{rr}P = -2\partial_r P\partial_r A + \frac{\partial_r P\partial_r K}{K} + \alpha X^2 P$$

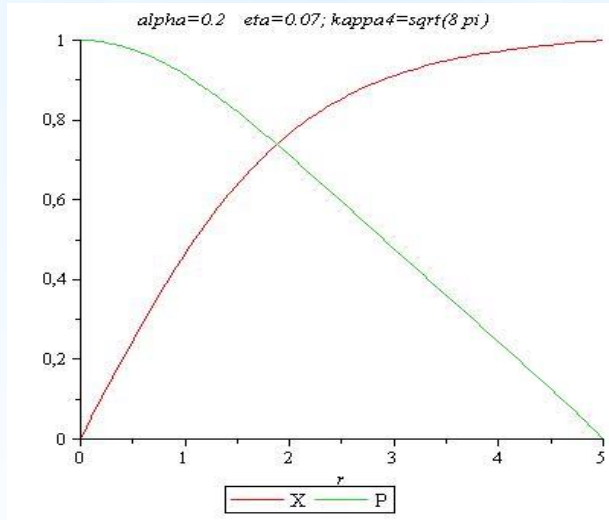
$$\alpha = \frac{e^2}{\beta} = \frac{m_A{}^2}{m_\Phi{}^2}$$

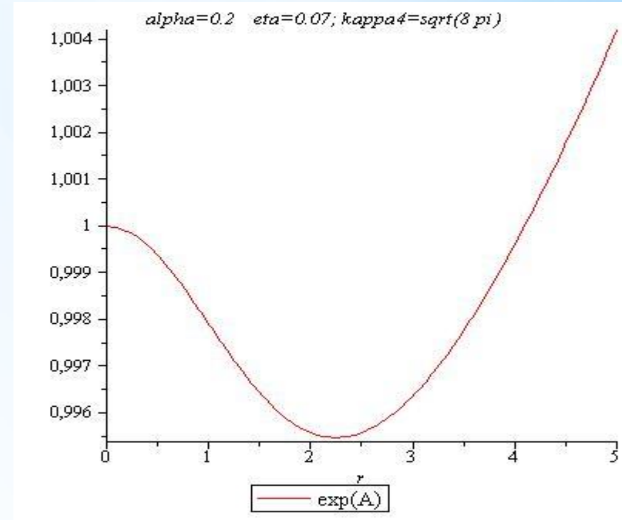
# Typical numerical solution

$g_{\varphi\varphi} - comp$:




Where did we see this before?




The metric becomes asymptotically [Garfinkle, 1987] :

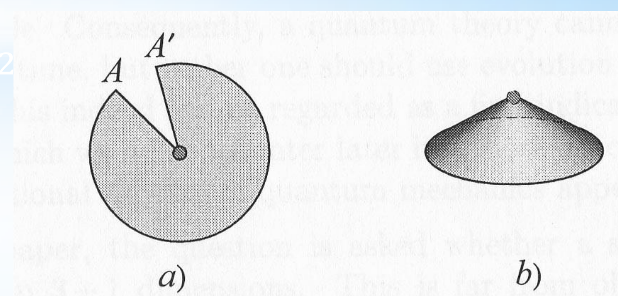
$$ds^2 = -e^{a_0}(dt^2 - dz^2) + dr^2 + e^{-2a_0}[k_2 r + a_2]^2 d\varphi^2$$

This metric can be brought to **Minkowski** by the change of variables

$$ds^2 = -(dt^2 - dz^2) + dr^2 + d\varphi'^2$$

►However:

$$\mathbf{0 \le \varphi' \le 2\pi e^{-a_0} k_2 < 2\pi}$$




Look at $g_{\varphi\varphi}$ component: angle deficit

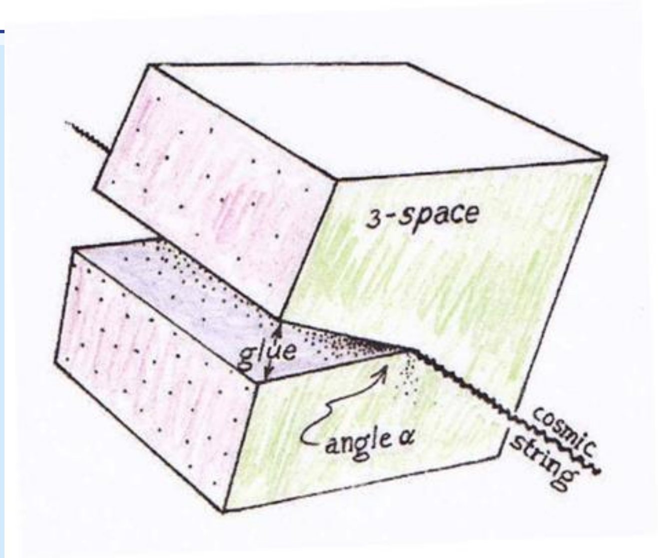
# The conical spacetime

►angle deficit :
$\Delta\theta = 2\pi(1 - e^{-a_0}\mathbf{k}_2)$ [$k_2$ determined by η, $m_A$ / $m_\Phi$ ]

► On proves: $\Delta\theta = \kappa_4{}^2\,\mu + \frac{\pi}{2}\int_0^\infty e^{-A}K\,(\frac{dA}{dr})^2\,dr$

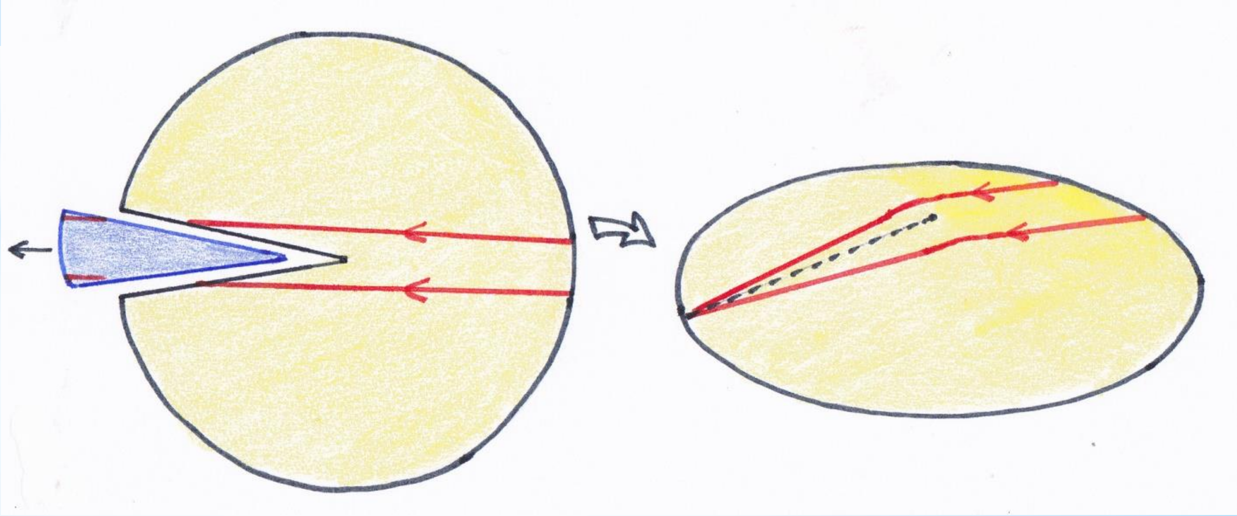
With $\mu \sim \eta^2$ the linear energy density

$$\mu = 2\pi \int_0^\infty e^{-A} K \sigma dr$$



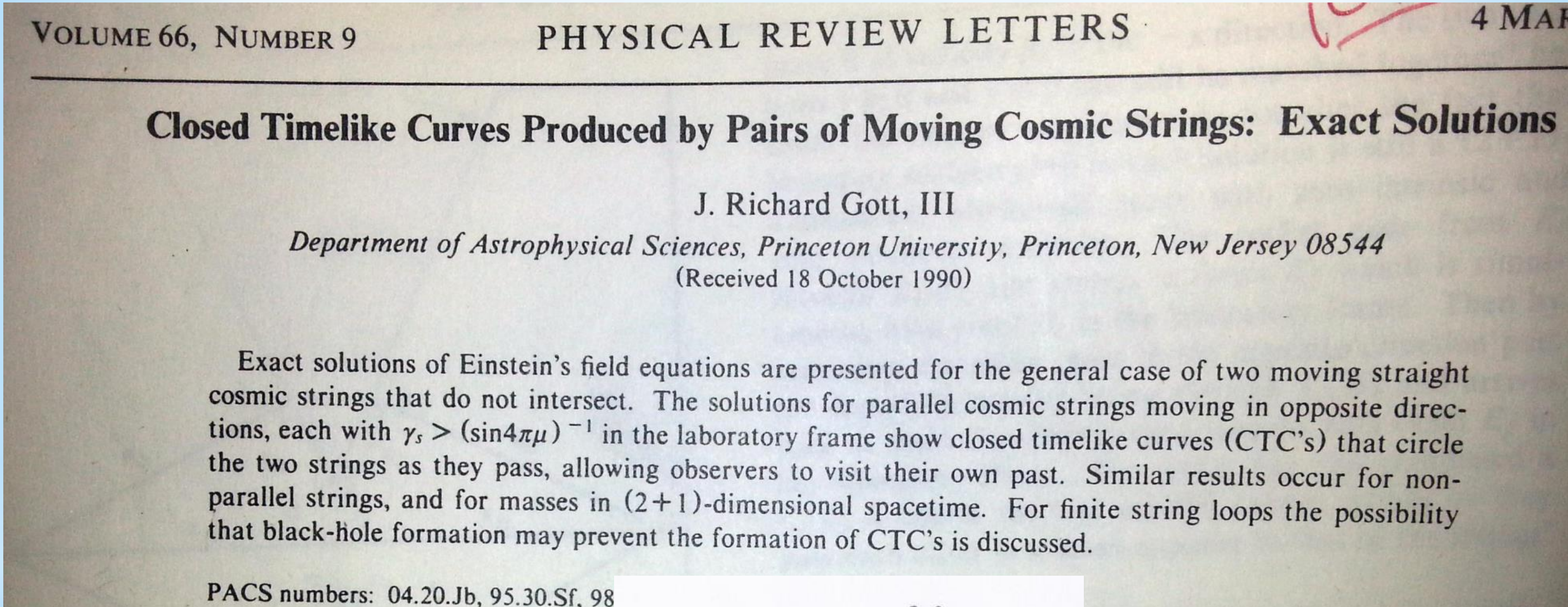

►The angle deficit will increase with the energy scale of symmetry breaking. Further, for GUT scale, $\eta \sim 10^{16}$ GeV, so the mass per unit length is $G\mu \sim 10^{-6}$
Numerical analysis of super massive cosmic strings, shows that the solution becomes singular at finite distance of the string or the angle deficit becomes greater than 2π [angle surplus]

Double-images:



# Time machines?

**In 1990 there appeared a shocking article:**


VOLUME 66, NUMBER 9 PHYSICAL REVIEW LETTERS 4 MAR

**Closed Timelike Curves Produced by Pairs of Moving Cosmic Strings: Exact Solutions**

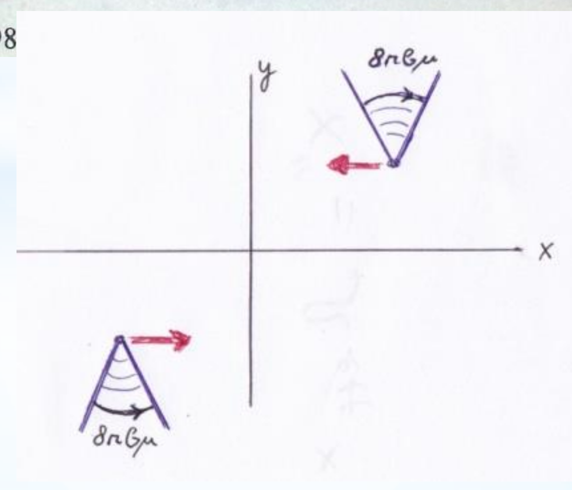
J. Richard Gott, III

*Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey 08544*

(Received 18 October 1990)

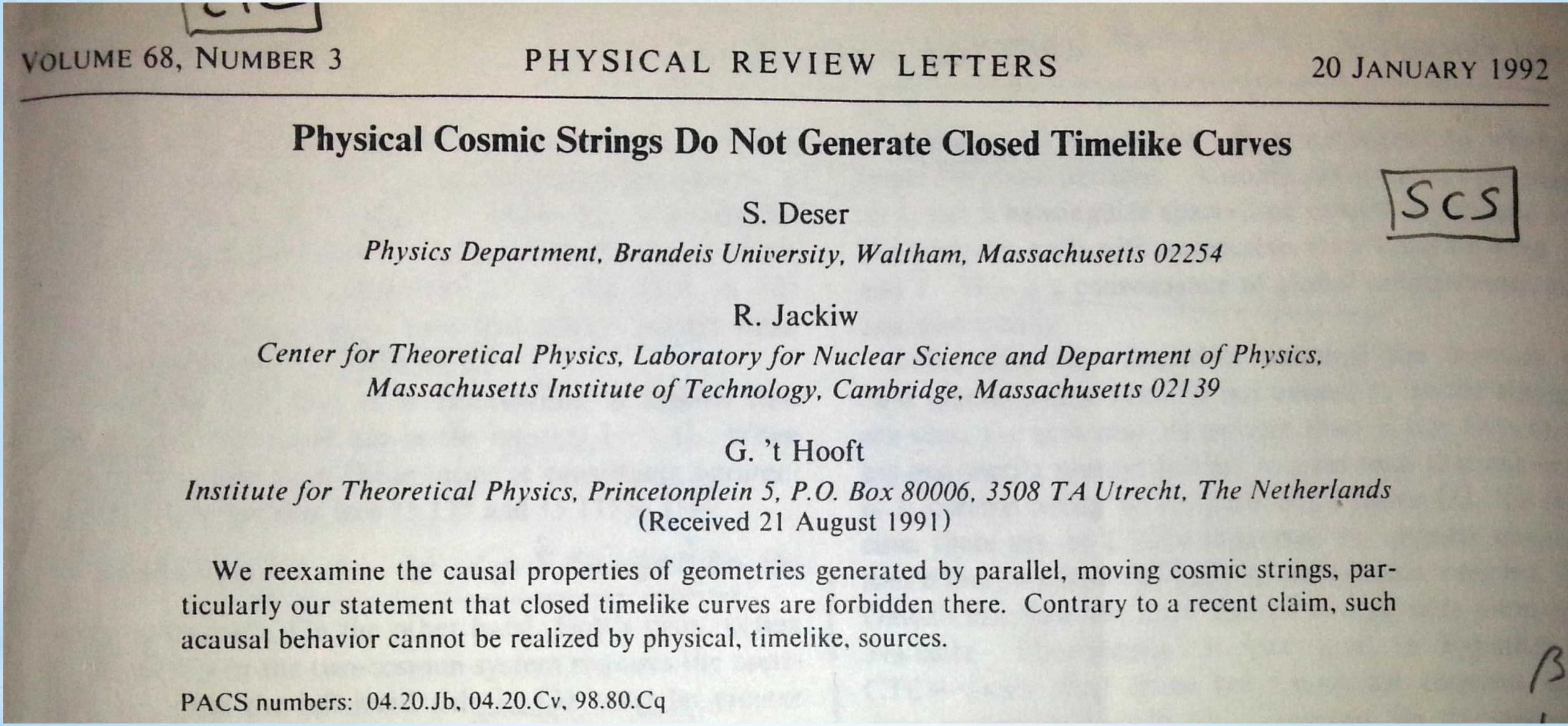
Exact solutions of Einstein's field equations are presented for the general case of two moving straight cosmic strings that do not intersect. The solutions for parallel cosmic strings moving in opposite directions, each with $\gamma_s > (\sin 4\pi\mu)^{-1}$ in the laboratory frame show closed timelike curves (CTC's) that circle the two strings as they pass, allowing observers to visit their own past. Similar results occur for nonparallel strings, and for masses in (2+1)-dimensional spacetime. For finite string loops the possibility that black-hole formation may prevent the formation of CTC's is discussed.

PACS numbers: 04.20.Jb, 95.30.Sf, 98



# Chronology protection is saved!

**In 1992: proof of the impossibility**



VOLUME 68, NUMBER 3 PHYSICAL REVIEW LETTERS 20 JANUARY 1992

**Physical Cosmic Strings Do Not Generate Closed Timelike Curves**

S. Deser

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R. Jackiw

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(Received 21 August 1991)

We reexamine the causal properties of geometries generated by parallel, moving cosmic strings, particularly our statement that closed timelike curves are forbidden there. Contrary to a recent claim, such acausal behavior cannot be realized by physical, timelike, sources.

PACS numbers: 04.20.Jb, 04.20.Cv, 98.80.Cq

**Two kinds of people:** believers and non – believers

Several hundred of articles on this subjec!!

# Time machines?

- Some physicists believe in timemachines around CS:

**Suppose two CS moving in opposite direction:**

gott ctc 2.wmv

**'t Hooft [1990-1994]: NO**

**<u>However</u>: In 2+1 dimensions: "cosmons "**
**example of self-gravitating particles**
**quantizable? ['t Hooft 1990]**

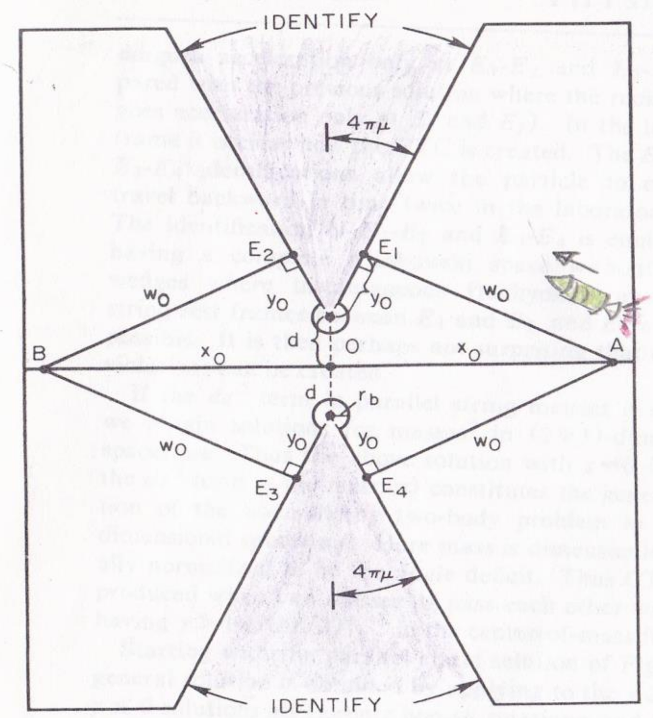


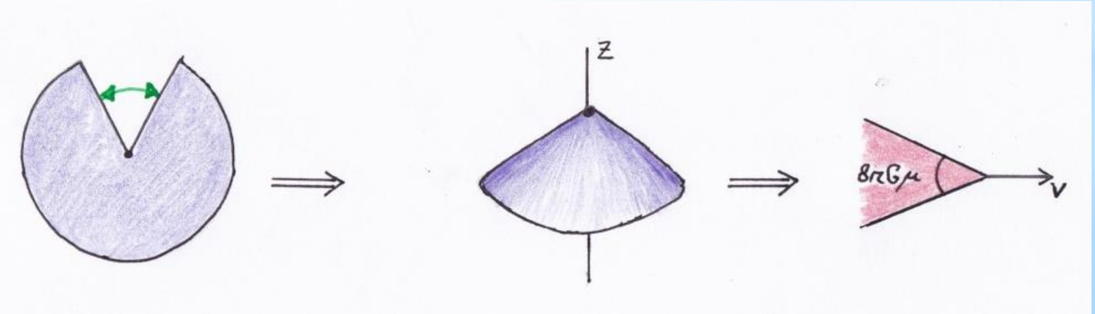
FIG. 1. Two-parallel-string static solution: $(x,y)$ plane.

Delete $dz^2$ :

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 (1\text{-}4G\mu)^2 \, d\varphi^2$$

In 3-dim: localy flat spacetime!

Still there is mass!=angle deficit



# Cosmological Cosmic Strings [Gregory,1989]

Question: **What about cylindrical** GW from CS in expanding universe?
[Importance of **cyl symm grav waves** was already noticed
by Einstein-Rosen[**1936**]]

- U(1) CS can be embedded into a flat 4D FRW along the polar axis
- **However:** The approx spacetime becomes conical:[ not pleasant]

$$ds^2 = a(t)^2[-dt^2 + dr^2 + K(r)^2 dz^2 + (1-4\pi G\mu)^2 S(r)^2 d\varphi^2]$$

and can be matched on the well known FLRW spacetime by suitable transformation

$$ds^2 = a(t)^2\left[-dt^2 + \frac{dR^2}{1-kR^2} + R^2 d\theta^2 + (1-4\pi G\mu)^2 R^2 sin^2\theta d\varphi^2\right]$$

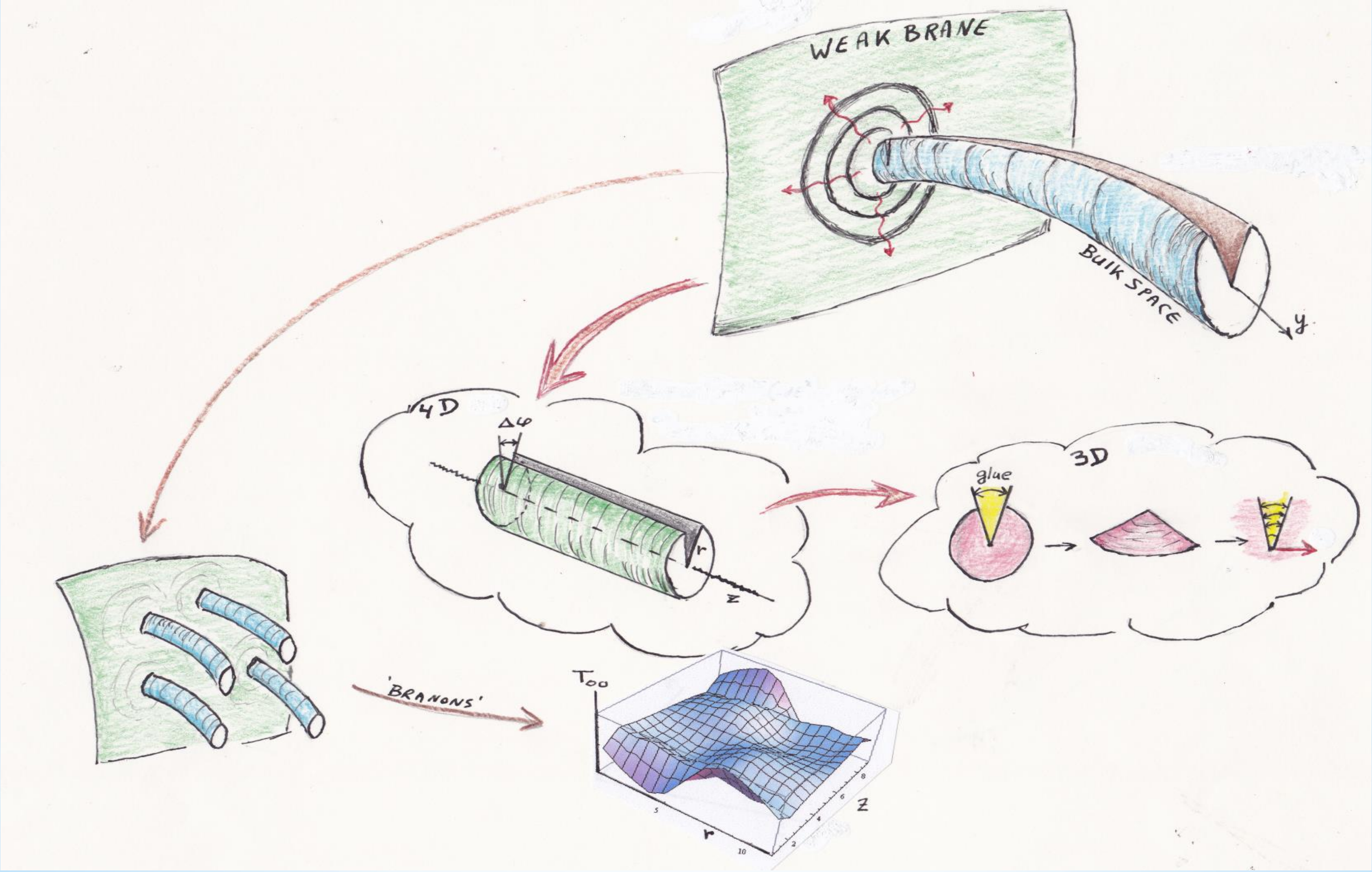
- Result: No contribution from the gravitation waves from the CS because

C-energy $\frac{r_{CS}}{R_H} \sim \frac{\dot{a}}{a} \sim \mathbf{10^{-20}}$ **extremely small**

- Disturbances are damped rapidly by $(\frac{r_{CS}}{R_H})^2$
- Asymptotic conical ST ( angle deficit) is **problematic**. Also found in radiative cyl. Einstein-Rosen ST: C-energy related to angle deficit [just as mass is related to angle deficit for CS].

So: Surviving disturbances must be very small ( otherwise conflict with observ)

# Artist impression of a cosmic string in 5D, 4D and 3D




Randall-Sundrum : large extra dimension [CERN?]

# Lessons from the abelian U(1) n-vortices solution

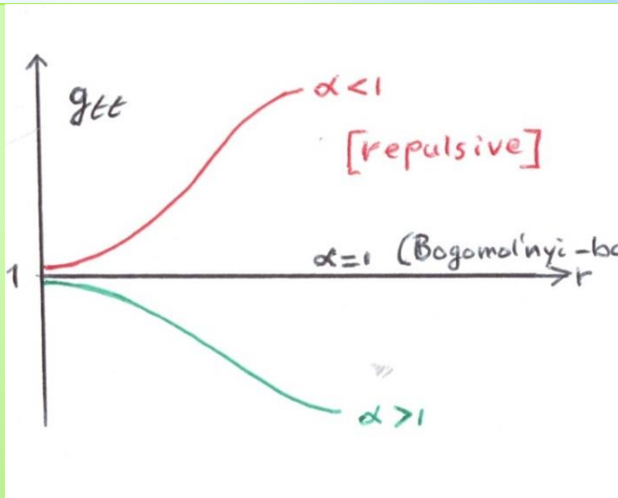
n-vortex solution parameter:

$$\alpha = \frac{e^2}{\beta} = \frac{{m_A}^2}{{m_\Phi}^2}$$

A. For type II finite superconductors:

** Flux tubes arrage in a regular lattice for $\alpha > 1$ ( vortex- vortex repulsive)

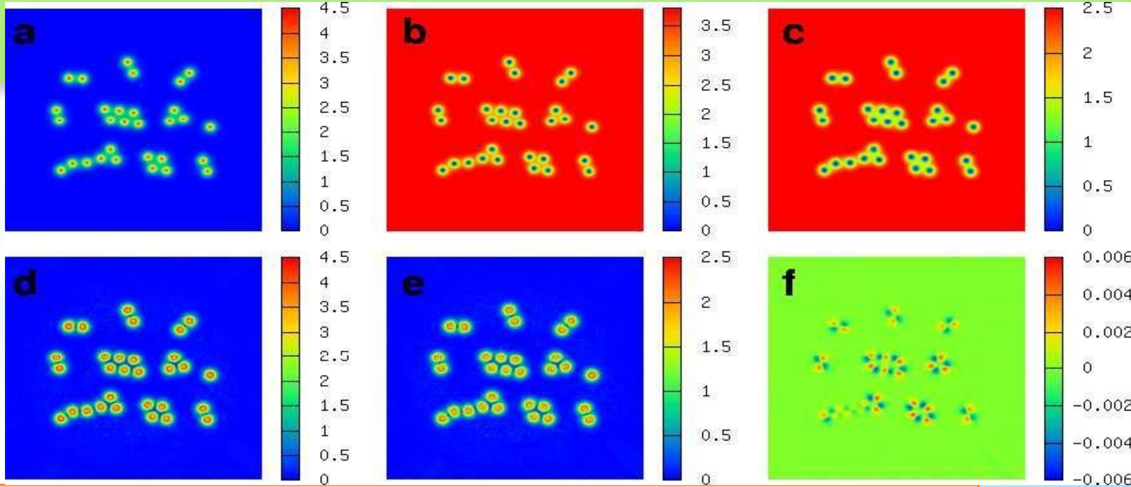
** For fixed n, $\alpha > 1$: maximizes the vortex-vortex separation [in fact: **unstable!]**




** Formation of vortex-clusters observed from n-vortex! (**"semi-Meissner"-** effect)

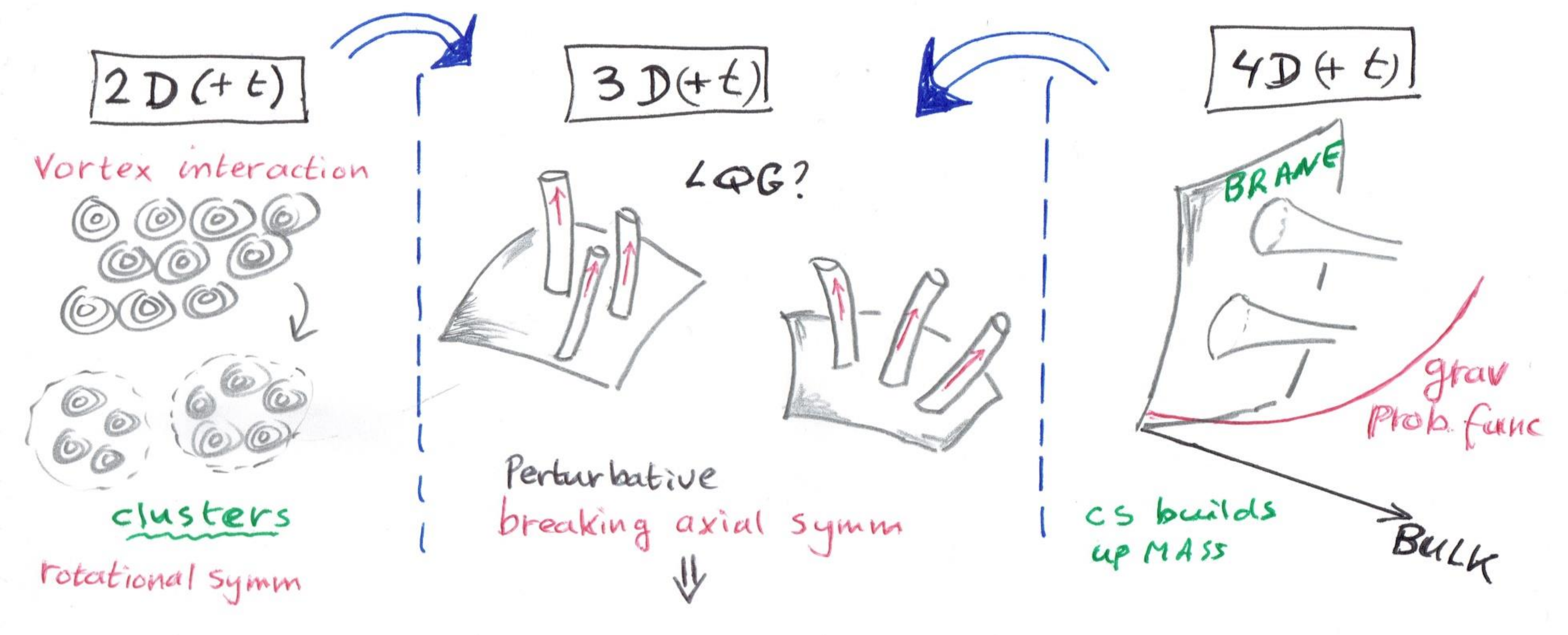
(Carlstrom,..,2011)

(Solve time-dep GL-eq.)




This is just what we need in polarization alignment in LQG's!!
[different in separated LQG's?]

# Entanglement Cosmic Strings from early stages



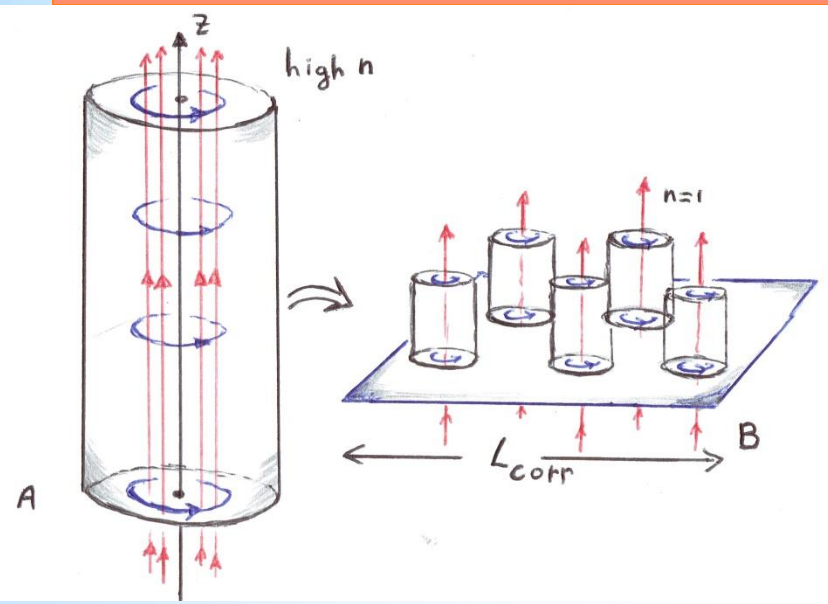

We shall see in our perturbative MS- approach: Emergent $\varphi$ -dependency!

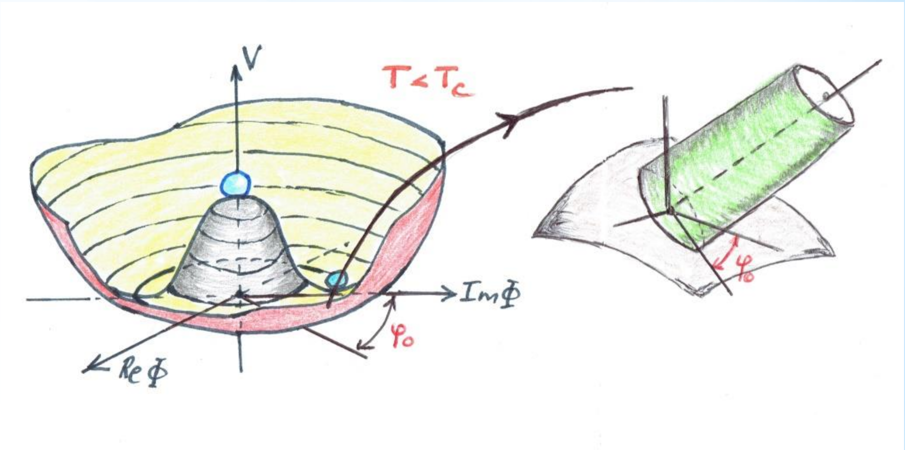
- ►Polarization axes-entanglement
- ►Different in the different LQG's
- ►$T_{zz}^{(i)} \sim \varphi$ -dependent
- ►$T_{t\varphi}^{(i)} \neq 0$ **[temporary broken axial symm]**
- ►$T_{\varphi\varphi}^{(i)}$ changes sign
- ►Amplification by warpfactor from 5D[necessary!] otherwise to light

Symmetry breaking ►►► gravity come into play ◄◄◄ amplification

# Entanglement Cosmic Strings from early stages

First and second order perturbations of the scalar and gauge fields in higher winding number-mode will decay into vortices of lower winding number till the groundstate (n=1) is reached.




$$^4\mathbf{T}^{(0)}_{\mathbf{zz}} = e^{4\bar{\psi}-2\bar{\gamma}}\dot{Y}h_{14}(\partial_t\bar{X}-\partial_r\bar{X})\mathbf{cos}[(n_2-n_1)\varphi] + \frac{e^{6\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^4 r^2\epsilon^2}\dot{B}(\partial_t\bar{P}-\partial_r\bar{P})$$

$$^4\mathbf{T}^{(0)}_{\mathbf{t}\varphi} = \bar{X}\bar{P}\dot{Y}\,\mathbf{sin}[(n_2-n_1)\varphi]$$

Recovery of axial symmetry by emission ofgravitational waves

$^4\mathbf{T}^{(1)}_{\mathbf{t}\varphi}$, $^4\mathbf{T}^{(1)}_{\mathbf{zz}}$ Will contribute to second order effect. Terms: $\sin(n_3-n_1)$ .....

# Related: Spontaneous symmetry breaking and Equatorial eccentricity

Secular and dynamical second-harmonic instabilities: related to

** second-order phase-transitions with **equatorial eccentricity** ($\varepsilon$) as order-parameter in self-gravitating compact objects: breaking axisymmetric symmetry: azimuthal angle comes into play

** phase transition of meridional eccentricity takes place on a time-scale comparable with the emission of grav waves in order to restore $\varepsilon$=1 [vorticity loss]

** restore of stationary axially symmetric configuration [ i.e. SO(2) symm] from **discrete** subgroup: symm only under rotations by ± 180$^o$ [ in our case: higher order eq.: ± 90$^o$]

** Chandrasehkar(1973!): quasi-stationary non-axisymmetric deformation with φ-dependence of the form $e^{im\varphi}$ (m integer)

** In GR terms: $T_{t\varphi}{}^{(i)} \neq 0 \rightarrow 0$

** points of **bifurcation** from the Maclaurin and Jacobi ellipsoids:

$\varepsilon$=0.813 : Jacobi bifurcation

$\varepsilon$=0.953: onset of non-axisymm dyn instability

$\varepsilon$=0.999: onset of axisymm dyn instability

Calculations done in perturbation approach: also a higher-order effect!

# Status of Cosmic Strings [by numerical simulation]

► Cosmic strings → **nonlinearities** already at high redshifts.
► Cosmic strings lead to perturbations which are non-Gaussian.
► Cosmic strings predict specific geometrical patterns in position space.
► CS are predicted in many models **beyond the "Standard Model".**
and **inevitably form** in the early universe and **persist** to the present time;
► By searching for **cosmological signatures** of strings we can constrain particle physics models beyond the Standard Model [more profound at **high redshifts**!]
► width $r_{cs} \sim \frac{1}{\sqrt{\beta}\eta}$ mass $G\mu \sim \eta^2$
► network forms at $t = t_{sb}$ ( symm break phase transition); separation increases
► **correlation** length $\xi(t)$ : value of $\Phi$ in two regions independent, if these regions are seperated $> \xi$
► $\xi(t)$ cannot exceed causal horizon $d_H(t) \sim t$. So $\xi(t) < t$
► $\xi(t)$ at $t = t_{sb}$ for U(1) model: $T_{sb} \approx T_{GL}$ and $\xi(T_{GL}) \approx \frac{1}{\lambda\eta}$

hor. size at $T_{GL}$ : $d_H \sim \frac{m_{pl}}{T_{GL}^2} \sim \frac{m_{pl}}{\eta^2}$ So $\frac{\xi}{d_H} \sim \frac{\eta}{\lambda m_{pl}}$

► evolution **not sensitive** to details of initial state.
► cosmological signatures of strings are **proportional to $G\mu$**
► CS are constrained from cosmology: CMB: $G\mu \leq 3.3\ 10^{-7}$ (otherwise conflict with the observed acoustic oscillations in the CMB angular power spectrum

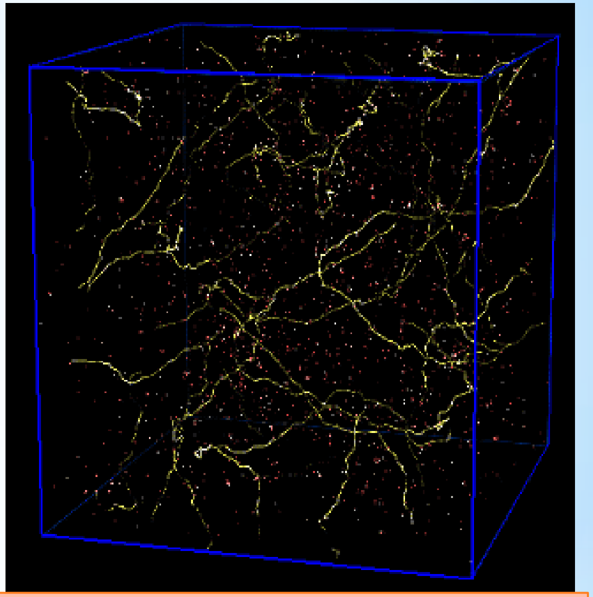
GW and PULSAR timing: $G\mu \leq 10^{-7}$

# Cosmic Strings evolution [Kibble mechanism: "Toy"-model]

- ► Let phase $\varphi$ vary on the correlation scale $\xi$ just after symmetry breaking scale.
- ► **simulate** different azimuthal $\varphi$ values on a lattice [monte-carlo method]

**Result:**

network of long strings: **snapshot:**



►Divide the time interval into Hubble expansion times.

►In each Hubble expansion time the network of long strings is described by a set of straight string segments with length $\xi(t) \sim c_1 t$

►Fixed number N of segments per Hubble volume.

So **if** the **azimuthal angle** ( the phase of the Higgs field) varies at the time of symm. breaking on the correlation length $\xi \rightarrow$ can translate to later time ( **quasar axes** align.)

**[NOTE:** So it would be of interest to obtain data for **different z-values** $\approx 3$
Not yet available (VLT: z< 1.5) ]

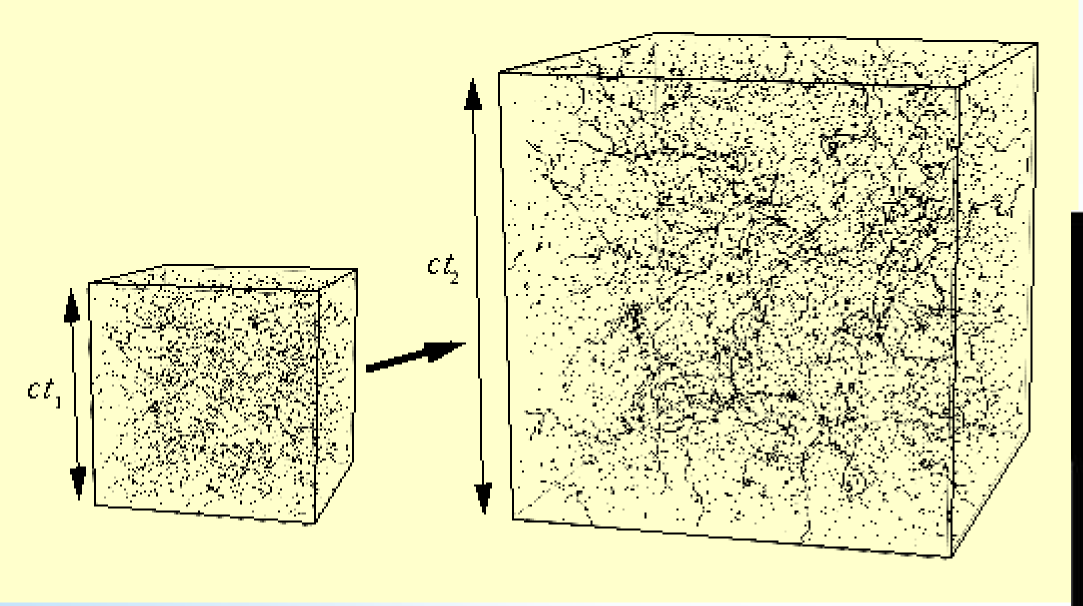
Spin-off: quasar-alignment can deliver evidence for cs!

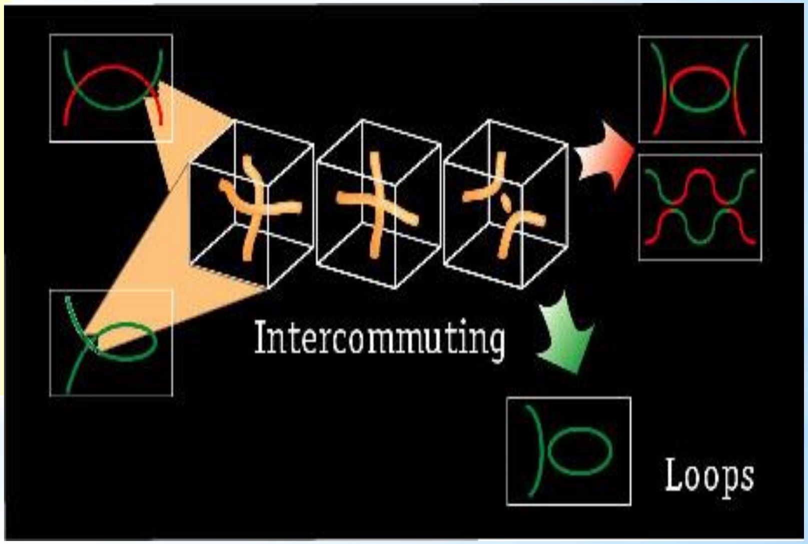
[however: we need **massive** cosmic strings: coming from the bulk]]

# Cosmic Strings evolution: One-scale model

**one-scale model**: scaling solution: $\xi(t) \sim t$    length: $l \sim G\mu t$
long string density: $\rho_{st} = \frac{\mu}{\xi^2}$

►string evolution is described as `**scaling**' or **scale-invariant**, that is, the properties of the network look the same at any particular time $t$ if they are scaled (or multiplied) by the change in the time  ["self-similar" evolution]




►they "shake off" loops
[so they do not overclose universe]

►Interaction properties of long cs:
probably **non-intercommuting** ( no signals) and separation increases

# Cosmic Strings evolution: One-scale model

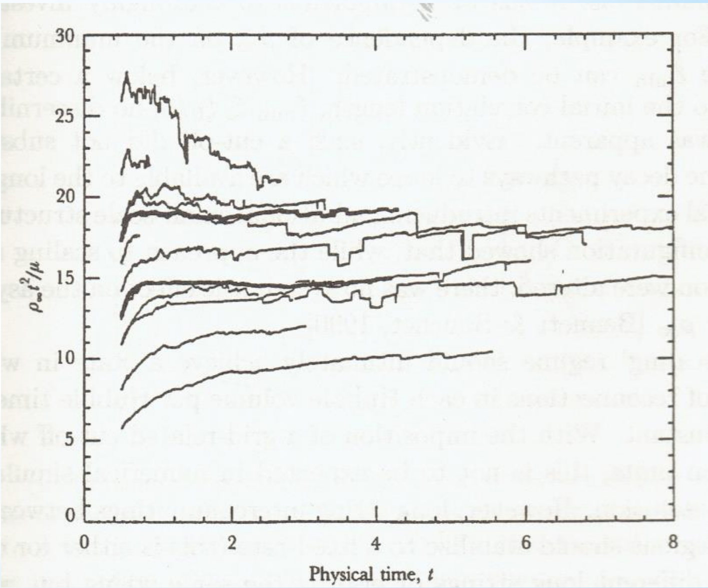
Numerical models:

►string network evolves toward a "scaling" regime

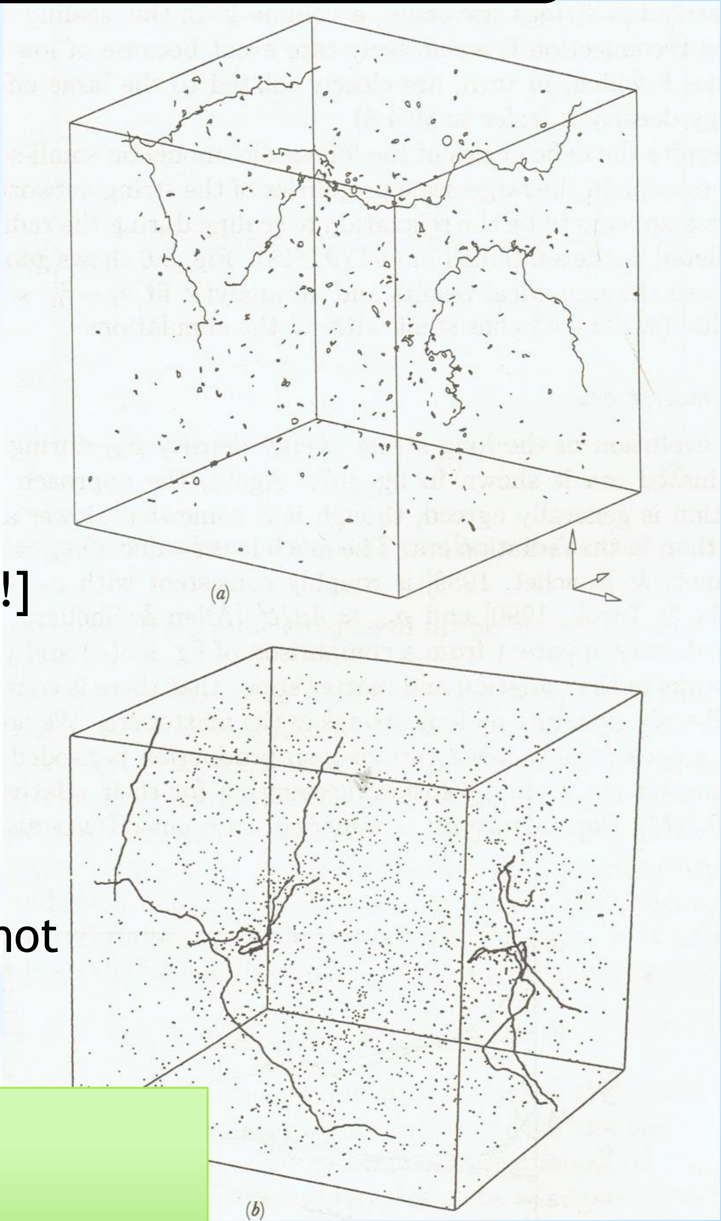
The characteristic scale ξ of the "infinite" long string network remains constant relative to $d_H$.

The energy does not grow with scale factor, because energy losses by small loops.

**All simulations:** driven towards a stable fixed point $\rho_\infty t^2 = const$



# Cosmic Strings evolution: One-scale model




**a.** Evolution of string network during radiation dominated era. Box side-length $L \approx \frac{d_H}{4}$
After exp by factor 4

**b.** Matter dominated era $L \approx \frac{d_H}{2}$ after exp. by factor 16

**However**: long-string substructure possible!
[needed for observed quasar-alignment!]
Heavily dependent on intercommuting or non-intercommuting strings.

Non-intercomm: domination of cosmic strings by increase of energy density.
If NOT in conflict with standard cosm model[may not dominate too early!]: then:

- ►very light and may not dominate too early:
- ► $G\mu \leq 10^{-30}$, $\eta \leq 10^4 GeV$
- ►so unable to provide energy density perturbations

# Problems for Cosmic Strings from Observations

► density perturbations : $\frac{\delta\rho}{\rho} \sim G\mu = \eta^2 / M_p{}^2 \sim 10^{-6}$ for GUT scale

► **They could**: 1. produce large-scale structure 3. lensing effect
2. anisotropy in MBR 4. GW by chopping off loops

► Now: inconsistencies with new CBM power spectrum COBE, WMAP

► They cannot provide a satisfactory explanation for the magnitude of the initial density perturbations [**too light**]

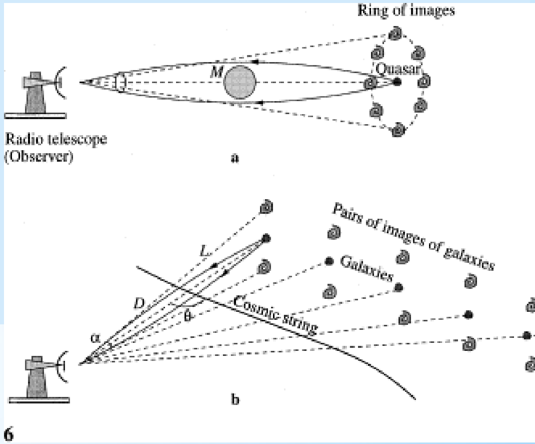
► How to handle **super-massive CS** with $G\mu >> 1$ [ phase transition at energy much larger than GUT ].

This is **interesting** for perturbation analysis and entanglement of quasars

[The angle deficit will increase with the **energy scale** of symmetry breaking]

► where is the axially symmetric gravitational lensing-effect?

► Cosmological CS: late-time conical residu [unwanted]
[Gregory, 1989]




So **Exit CS** study??

# Rescue of CS

**reborn CS** → Go to warped 5D Randall-Sundrum model

- ▶ in the brane: unobservable **angle deficit** [no double images]
- ▶ asymptotically: no conical space time [**Slagter**, 2012, IJMPD]
- ▶ **No conflict with: CMB-spectrum**
- ▶ **The effective** 4D spacetime of the CS in agreement with GUT;

▶ **CS can be produced in superstring theory [ F- and D-strings]**

▶ **Super massive CS with Gμ >> 1 will be warped down to GUT scale on the brane**
[no singularities at finite distance of core as in the standard model]

▶ **Disturbances** in the spatial components of the stress-energy tensor cause cylindrical symmetric waves, **amplified** due to the presence of the **bulk space** with **warp factor**

[don't fade away as in standard model]

▶▶ **Mass:** $\mu = 2\pi F \int_0^\infty e^{-A} K\sigma dr$ **with F the WARPFACTOR**

so: building up a huge mass in the bulk : KK-modes on brane

▶▶ **Test of RS type models against observational constraint possible !**

Cern: KK-particles detectable?

# The Quasars link

Peculiar results from observations:

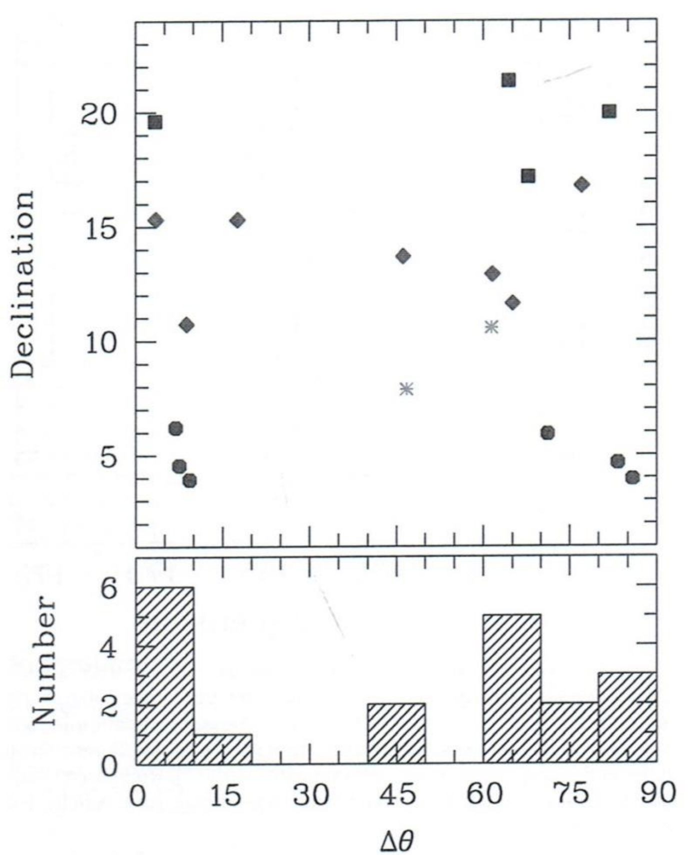



**Fig. 5.** Bottom: The distribution of the acute angle Δθ (in degree) between quasar polarizations and the orientation of their host large-scale structure. Top: Δθ is plotted against the object declination (in degree) to illustrate the behavior of the different quasar groups (1: squares, 2: lozenges, 3: asterisks, 4: hexagons; colors as in Fig. 4).

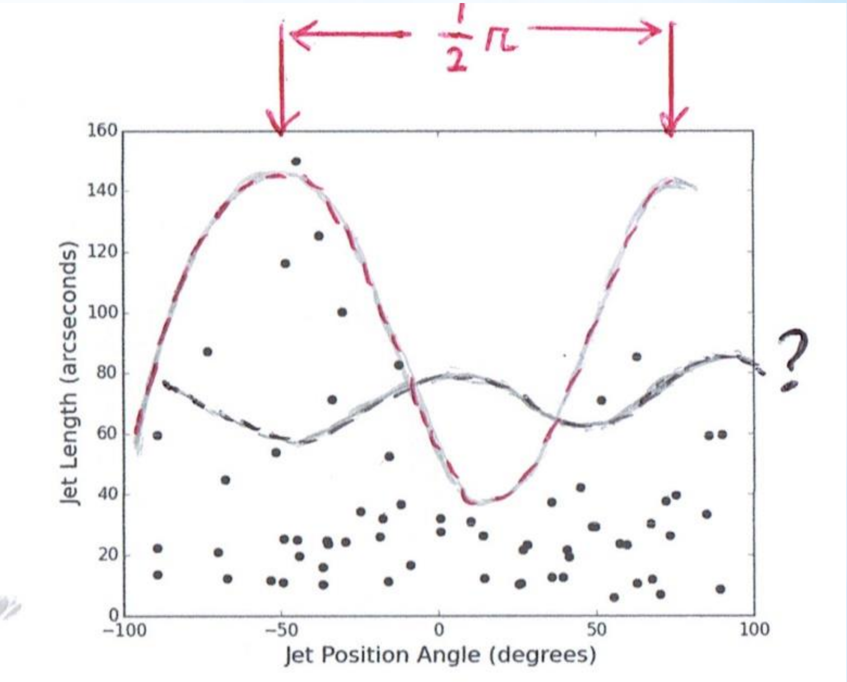



**Figure 4.** The length of the 64 radio jets plotted against jet position angle. The longest jets are preferentially present in the excess of object with polarisation angle $\sim -40°$.

# The Quasars link

Resuls from observation **Sloan Digital Sky Survey DR7** [ 355 quasars]

I. Optical [ and possible radio]- polarization alignment observed in LQG's on Gpc-scale
--- probably morphological
--- note: matter density fluctuations cannot explainthis effect; it is beyond the homogeneity scale

II. In different LQG's different position angles.

III. At large red shift: polarization vectors either parallel or perpendicular
[ this cannot be explaned by considering two pol in one quasar as suggested]
statistical evidence: probability of randomness: <0.1%!

IV. Slightly z-dependency.

VI. Peculiar: The significance depends on the number of quasars in the LQG's!
low density: preferential pol
high density: perpendicular pol possible

We shall see: **all in agreement with our model**

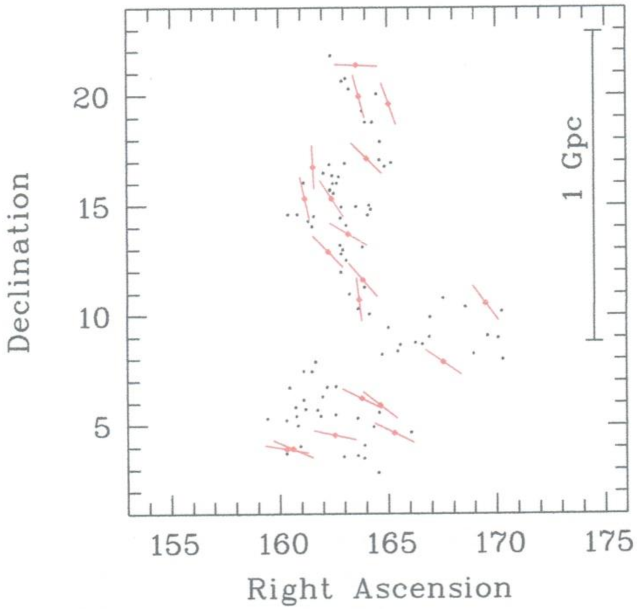



Figure 4.6: The polarization vectors of the 19 quasars with $p_{\rm lin} \geq 0.6\%$ are superimposed on the large-scale structure after rotation of the polarization angles according to $\tilde{\psi} = \mathrm{mod}(\psi, 90°) + 90°$. A clear correlation is seen but we nevertheless caution against exaggerated visual impression since polarization angles are now in the range $90° - 180°$. Right ascensions and declinations are in degree. The comoving distance scale is indicated as in Fig. 4.4.

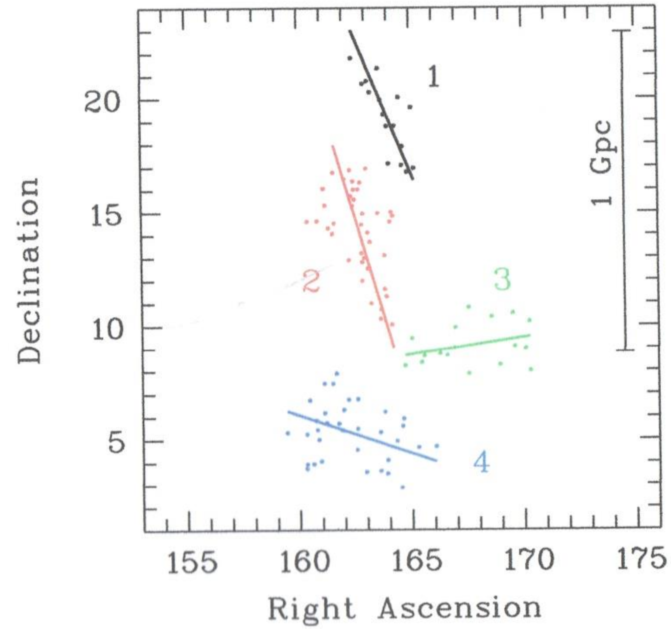



**Fig. 4.** The quasar groups and their orientations on the sky. Right ascensions and declinations are in degree. The superimposed lines illustrate the orientations of the four groups labelled 1, 2, 3, 4. The comoving distance scale at redshift $z = 1.3$ is indicated assuming a flat Universe with $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ and $\Omega_m = 0.27$.

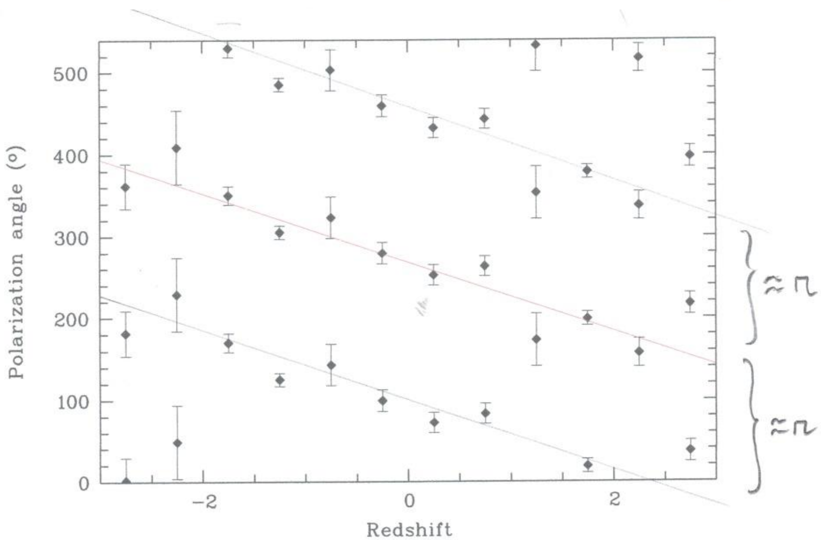



Figure 3: The quasars polarization angles, averaged over redshift bins $\Delta z = 0.5$, as a function of the redshift. Redshifts are counted positively for object located in the North Galactic Cap and negatively for those on in the South Galactic Cap. Only the 183 quasars belonging to the A1–A3 axis are considered. Error bars represent 68 % angular confidence intervals for the circular mean (Fisher

# Why Warped 5D Space times?

Solves:

- ► Coincidence-problem: $\Omega_\Lambda \sim \Omega_M$
- ► Finetuning-problem: $\rho_{\Lambda,obs} \sim 10^{-57} GeV^4$ $\rho_{\Lambda,theor} \sim 1\ TeV^4$
- ► Ad hoc modifications: of the Friedmann equation risky, specially when considering density perturbations: do it covariantly

► Disturbances don't survive in 4D models : at least some of them are needed for the observed large-scale structures [here: quasar alignment]

In warped 5D model: they do survive and

- ► No Λ needed
- ► solves hierarchy problem [ why is gravity so weak]

So modify GR : D-branes. 1. Dvali-Gabadadze- Porrati (DGP)

⇒2. Randall-Sundrum (RS)

In general:

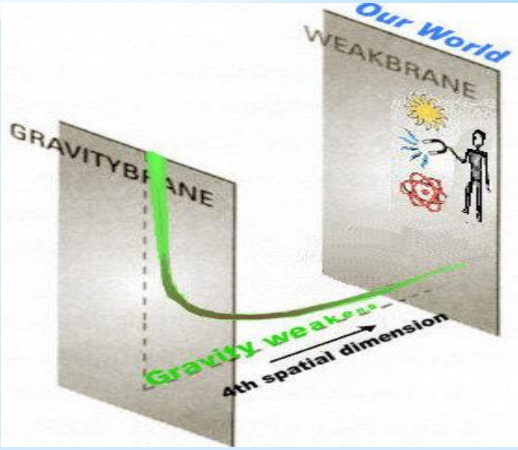
Gravity leakage at late-times initiates acceleration, due to weakening of gravity on the brane . not due to any negative pressure field.

**4D gravity is recovered at high energy via the lightest KK modes of the graviton**

# Brane world models of Randall-Sundrum

► Large extra dimension [ no curled-up tiny-dim. ]

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$




► At low energy: gravity localized at the brane: GR recovered. Modification to the weak field eq. Negative bulk $\Lambda$ prevents gravity to leak into extra dimensions (squeezes gravity closer to the weak brane)

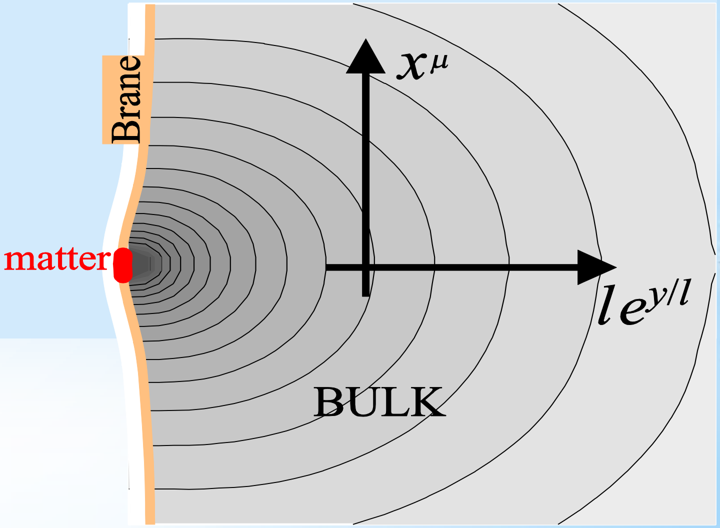
► At high energy: gravity "leaks" into the bulk

► Solves hierarchy problem

► The 5D graviton effects ( KK modes) **detectable?**

► Because of the exponential warping is the effective scale on visible brane at y=L:

$$M_p^2 = M_5^3\,(1-e^{-2kL})/k$$



# The warped 5D model with the U(1) scalar-gauge field

We consider the warped spacetime: $[{}^4g_{\mu\nu} = {}^5g_{\mu\nu} - n_\mu n_\nu]$ (n normal to brane)

$$ds^2 = \mathcal{W}(t,r,y)^2\left[e^{2(\gamma(t,r)-\psi(t,r))}\left(-dt^2 + dr^2\right) + e^{2\psi(t,r)}dz^2 + r^2e^{-2\psi(t,r)}d\varphi^2\right] + dy^2$$

With W the warpfactor. We reside on the BRANE y=0. Gravity can prop. in BULK
We consider: scalar-gauge field in brane: [empty BULK; only $\Lambda_5$ ]

$$\Phi = \eta X(t,r)e^{i\varphi}, \qquad A_\mu = \frac{1}{\varepsilon}[P(t,r) - 1]\nabla_\mu\varphi, \qquad V(\Phi) = \frac{1}{8}\beta(\Phi^2 - \eta^2)^{\,2}$$

From the 5D-eq:
**[Slagter-Pan;2016]**
**Found of Phys**

$$\mathcal{W} = \frac{e^{\sqrt{-\frac{1}{6}\Lambda_5}(y-y_0)}}{\alpha\sqrt{r}}\sqrt{(d_1e^{\alpha t} - d_2e^{-\alpha t})(d_3e^{\alpha r} - d_4e^{-\alpha r})}$$

The modified 4D effective Einstein equations:

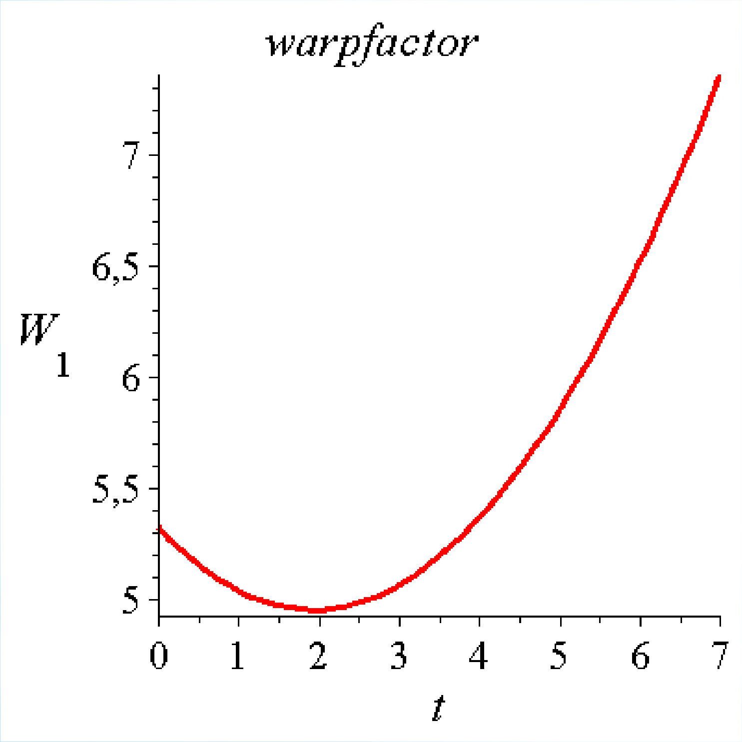
$${}^4G_{\mu\nu} = -\Lambda_{eff}\,{}^4g_{\mu\nu} + \kappa_4^2\,{}^4T_{\mu\nu} + \kappa_5^4\,S_{\mu\nu} - \mathcal{E}_{\mu\nu}$$

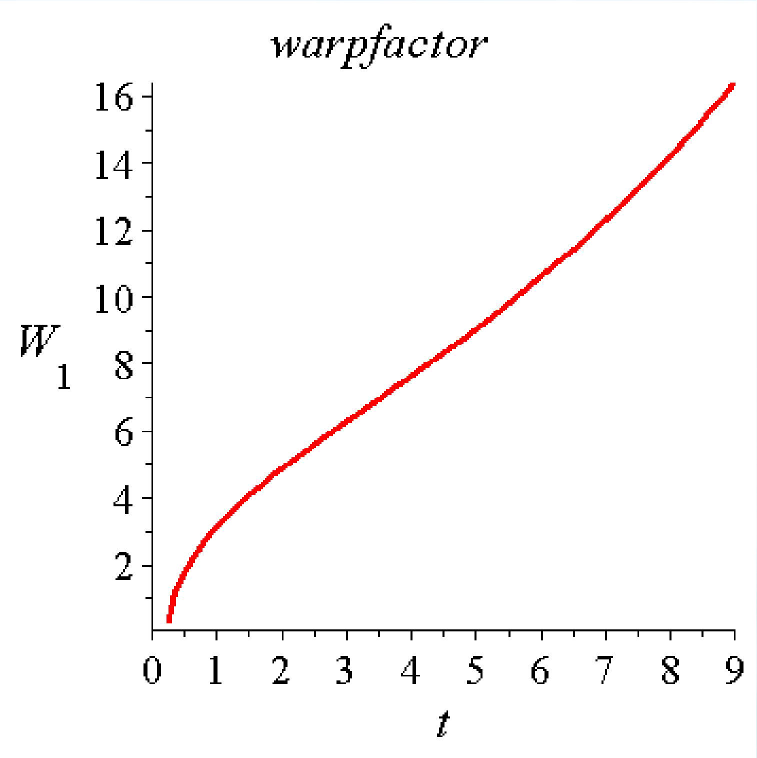
**S** is the quadratic term in the energy-momentum tensor [from extrinsic curv. terms in proj. Einstein tensor]
$\mathcal{E}$ is part of the 5D Weyl tensor C and carries inf.of grav.field outside the brane

$$\mathcal{E}_{\mu\nu} = {}^5C_{\alpha\gamma\beta\delta}n^\gamma n^\delta\,{}^4g_\mu^\alpha\,{}^4g_\nu^\beta$$

$\Lambda_{eff} = 0$ **(RS-finetuning)**

# Exact solutions




Slagter-Pan;2016--Found of Phys

# The warped 5D model with the U(1) scalar-gauge field

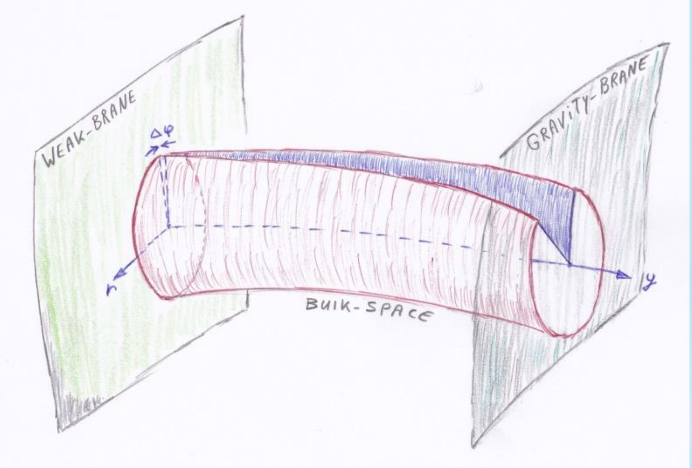
The scalar-gauge field equations:

$$D^\mu D_\mu \Phi = 2\frac{dV}{d\Phi^*} \qquad {}^4\nabla^\mu F_{\nu\mu} = \frac{1}{2} i\varepsilon(\Phi(D_\nu\Phi)^* - \Phi^* D_\nu \Phi)$$

With $D_\mu \Phi = {}^4\nabla_\mu \Phi + i\epsilon A_{\mu\Phi}$.

►The scalar gauge field can build-up a **huge mass** per unit length (or **angle-deficit**) by the warpfactor W: **$G\mu \sim 1$**

**►Can induce massive KK-modes felt on the brane.**
[while manifestation on **brane** will be warped down to GUT scale consistent with observation]




►**Disturbances** can cause cyl. symm waves amplified by the warpfactor and could survive natural damping due to the expansion of the universe.

►Could possible explane "**self-acceleration**" **[ dark energy]** with $\Lambda_{eff}$=0 !

# The nonlinear wave approximation in 5D GenRel

We expand:

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(x) + \frac{1}{\omega} h_{\mu\nu}(x,\xi,\chi,..) + \frac{1}{\omega^2} k_{\mu\nu}(x,\xi,\chi,..) + \cdots$$

$$A_\mu = \bar{A}_\mu(x) + \frac{1}{\omega} B_\mu(x,\xi,\chi,..) + \frac{1}{\omega^2} C_\mu(x,\xi,\chi,..) + \cdots$$

$$\Phi = \bar{\Phi}(x) + \frac{1}{\omega} \Psi(x,\xi,\chi,..) + \frac{1}{\omega^2} \Xi(x,\xi,\chi,..) + \ \ldots$$

We define

$$\frac{dg_{\mu\nu}}{dx^\sigma} = g_{\mu\nu,\sigma} + \omega l_\sigma \dot{g}_{\mu\nu} + \breve{\omega} k_\nu \breve{g}_{\mu\nu} + .. \quad g_{\mu\nu,\sigma} = \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \qquad \dot{g}_{\mu\nu} = \frac{\partial g_{\mu\nu}}{\partial \xi}$$

The rapid variations occur in the directions of $l_\mu$ , $k_\mu$ transversal to the sub-minifolds of constant phase .

For the time being: only $l_\mu = \frac{\partial \Theta}{\partial x^\mu}$ [ now $\Theta = t - r$]

The perturbations can be **φ-dependent! We write:**

$$\bar{\Phi} = \bar{X}\, e^{in_1\phi} \qquad \Psi = Y\ e^{in_2\phi} \qquad \Xi = Z\ e^{in_3\phi}$$

So we break-up the original vortex in 3 different **windingnumbers.**

**Still stable?: We shall see: YES.**

**$\varphi$ −dependency enters in perturbation equations**

We write:

$$\Gamma^{\alpha}_{\mu\nu} = \bar{\Gamma}^{\alpha}_{\mu\nu} + \Gamma^{\alpha(0)}_{\mu\nu} + \frac{1}{\omega}\Gamma^{\alpha(1)}_{\mu\nu} + \ \ldots$$

$$R^{\sigma}_{\mu\tau\nu} = \omega\, R^{\sigma(-1)}_{\mu\tau\nu} + \bar{R}^{\sigma}_{\mu\tau\nu} + R^{\sigma(0)}_{\mu\tau\nu} + \frac{1}{\omega}\, R^{\sigma(1)}_{\mu\tau\nu} + \cdots$$

with $$\Gamma^{\sigma(0)}_{\mu\nu} = \tfrac{1}{2}\bar{g}^{\beta\sigma}\left(l_\mu \dot{h}_{\beta\nu} + l_\nu \dot{h}_{\beta\mu} - l_\beta \dot{h}_{\mu\nu}\right)$$

$$\Gamma^{\alpha(1)}_{\mu\nu} = \tfrac{1}{2}\left(h^{\sigma}_{\mu:\nu} + h^{\sigma}_{\nu:\mu} - h^{:\sigma}_{\mu\nu} - \ l_\nu \dot{k}^{\sigma}_{\mu} + l_\mu \dot{k}^{\sigma}_{\nu} - l^{\sigma}\dot{k}_{\mu\nu}\right) - h^{\sigma}_{\rho}\Gamma^{\rho(0)}_{\mu\nu}$$

We substitute the expansions into the fieldequations and subsequently put zero the various powers of ω

From the $\omega^1$ **Einstein**: $${}^4G^{(-1)}_{\mu\nu} = -\mathcal{E}^{(-1)}_{\mu\nu}$$ ("gauge" cond)

**Scalar**: $$l^\mu l_\mu \ddot{\Psi} = 0$$ **[note: this is the Eikonal eq., or $\ddot{\Psi}$ ]**

**gaugefield**: $$l^\mu \ddot{B}_\mu = 0$$

Normally one imposes a priori gauge-conditions: $$l^\mu\left(\ddot{h}_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}\ddot{h}\right) = 0$$

The contribution of $\mathcal{E}^{(-1)}_{\mu\nu}$ changes the conditions on $h_{\mu\nu}$

**Further**: we take $l^\mu l_\mu = 0$ (Eikonal cond)

$l^\mu l_\mu \neq 0$ means that $h_{\mu\nu}$ arises from a coord transformation.

# The effective brane $\omega^0$ Einstein equations

The $\boldsymbol{\omega^{(0)}}$- Einstein equations:

$$\boldsymbol{{}^4\overline{G}_{\mu\nu} + {}^4G^{(0)}_{\mu\nu} = -\,\Lambda_{eff}\,{}^4\overline{g}_{\mu\nu} + \kappa_4^2({}^4\overline{T}_{\mu\nu} + \,{}^4T^{(0)}_{\mu\nu}) + \kappa_5^4\left(\overline{S}_{\mu\nu} + S^{(0)}_{\mu\nu}\right) - (\overline{\mathcal{E}}_{\mu\nu} + \overline{\mathcal{E}}^{(0)}_{\mu\nu})}$$

where the part of the Weyl tensor is:

$$\mathcal{E}_{\mu\nu} = \; n^\gamma n^\delta\,{}^4g_\mu^\alpha\,{}^4g_\nu^\beta[{}^5R_{\alpha\gamma\beta\delta} - \frac{1}{3}({}^5g_{\alpha\gamma}\,{}^5R_{\delta\beta} - {}^5g_{\alpha\delta}\,{}^5R_{\gamma\beta} - {}^5g_{\beta\delta}\,{}^5R_{\gamma\alpha} + {}^5g_{\beta\delta}\,{}^5R_{\gamma\alpha}) + \frac{1}{12}({}^5g_{\alpha\gamma}\,{}^5g_{\delta\beta} - {}^5g_{\alpha\delta}\,{}^5g_{\gamma\beta})\,{}^5R]$$

Now we take only $\boldsymbol{h_{11}, h_{44} \quad h_{13} \quad h_{14} \quad h_{55} \;\neq\; 0}$ **[consistent with gauge c.]**

One can also **integrate** the equations wrt **to ξ :** <u>propagation equations</u>

<u>**Then:**</u> substitute back these equations: $(\Lambda_{eff} = 0\ (RS\ finetuning)$

$$\boldsymbol{{}^4\overline{G}_{\mu\nu} = \kappa_4^2\,{}^4\overline{T}_{\mu\nu} + \kappa_5^4\overline{S}_{\mu\nu} - \,\overline{\mathcal{E}}_{\mu\nu} + \frac{1}{\tau}\int\left(\kappa_4^2T^{(0)}_{\mu\nu} + \,\kappa_5^4\,S^{(0)}_{\mu\nu} \,-\, {}^4G^{(0)}_{\mu\nu} \,-\, \mathcal{E}^{(0)}_{\mu\nu}\right)d\xi}$$

<u>one says</u>: ► $\boldsymbol{-\int \mathcal{E}^{(0)}_{\mu\nu}\, d\xi}$ is the KK-mode contribution of the perturbative 5D graviton

► can play the role of effective CC ( same sign)

► is an extra “back-reaction” term which contain $\boldsymbol{\dot{h}_{55}}$

# The background Einstein equations to order $\omega^{(0)}$

In our special model, we have decoupled background equations:

$$\partial_{tt}^2 \bar{\mathcal{W}} = -\partial_{rr}^2 \bar{\mathcal{W}} + \frac{2}{\bar{\mathcal{W}}}(\partial_t \bar{\mathcal{W}}^2 + \partial_r \bar{\mathcal{W}}^2) - \bar{\mathcal{W}}(\partial_t \bar{\psi}^2 + \partial_r \bar{\psi}^2) + \frac{\bar{\mathcal{W}}}{r}(\partial_r \bar{\gamma} - \partial_t \bar{\gamma})$$
$$+ 2(\partial_r \bar{\mathcal{W}} - \partial_t \bar{\mathcal{W}})(\partial_t \bar{\psi} - \partial_r \bar{\psi} + \partial_r \bar{\gamma} - \partial_t \bar{\gamma}) + 2\bar{\mathcal{W}} \partial_t \bar{\psi} \partial_r \bar{\psi} - 4\frac{\partial_t \bar{\mathcal{W}} \partial_r \bar{\mathcal{W}}}{\bar{\mathcal{W}}}$$
$$- 2\partial_{tr} \bar{\mathcal{W}} - \frac{3}{4}\kappa_4^2 \left( e^{2\bar{\psi}} \frac{(\partial_t \bar{P} - \partial_r \bar{P})^2}{\bar{\mathcal{W}} r^2 \epsilon^2} + \bar{\mathcal{W}}(\partial_t \bar{X} - \partial_r X)^2 \right)$$

$$\partial_{tt}^2 \bar{\psi} = \partial_{rr}^2 \bar{\psi} + \frac{\partial_t \bar{\psi}}{r} + \frac{2}{\bar{\mathcal{W}}}(\partial_r \bar{\mathcal{W}} \partial_r \bar{\psi} - \partial_t \bar{\mathcal{W}} \partial_r \bar{\psi}) - \frac{\partial_r \bar{\mathcal{W}}}{r \bar{\mathcal{W}}} + \frac{3e^{2\bar{\psi}}}{4\bar{\mathcal{W}}^2 r^2 \epsilon^2}\kappa_4^2 (\partial_t \bar{P}^2 - \partial_r \bar{P}^2 - \bar{\mathcal{W}}^2 \varepsilon^2 \bar{X}^2 \bar{P}^2 e^{2\bar{\gamma} - 2\bar{\psi}})$$

$$\partial_t \bar{\gamma} = \partial_r \bar{\gamma}$$
$$+ \frac{1}{\partial_t \bar{\mathcal{W}} - \partial_r \bar{\mathcal{W}} - \frac{\bar{\mathcal{W}}}{2r}} \Bigg[ \frac{1}{2} \bar{\mathcal{W}}(\partial_t \bar{\psi} - \partial_r \bar{\psi})^2 + \frac{\partial_r \bar{\mathcal{W}}}{r} - \partial_{tr} \bar{\mathcal{W}} + \partial_{rr} \bar{\mathcal{W}} + \frac{2\partial_t \bar{\mathcal{W}} \partial_r \bar{\mathcal{W}}}{\bar{\mathcal{W}}}$$
$$+ (\partial_r \bar{\mathcal{W}} - \partial_t \bar{\mathcal{W}})(\partial_r \bar{\psi} - \partial_t \bar{\psi}) - \frac{\partial_r \bar{\mathcal{W}}^2 + 3\partial_t \bar{\mathcal{W}}^2}{2\bar{\mathcal{W}}}$$
$$+ \kappa_4^2 \frac{\bar{\mathcal{W}}}{16} \left( 7\partial_t \bar{X}^2 + 5\partial_r \bar{X}^2 - 12\partial_t \bar{X} \partial_r \bar{X} + 5e^{2\bar{\gamma}} \frac{\bar{X}^2 \bar{P}^2}{r^2} + 6e^{2\bar{\psi}} \frac{(\partial_r \bar{P} - \partial_t \bar{P})^2}{\bar{\mathcal{W}}^2 r^2 \epsilon^2} \right.$$
$$\left. + \bar{\mathcal{W}}^2 \beta e^{2\bar{\gamma} - 2\bar{\psi}} (\bar{X}^2 - \eta^2)^2 \right) \Bigg]$$

# The Einstein propagation equations to order $\omega^{(0)}$

The equations are:

$$\partial_t \dot{h}_{14} = \partial_r \dot{h}_{14} + \ddot{k}_{14} - \ddot{k}_{24} + 2\Big(\partial_r\bar{\psi} - \partial_t\bar{\psi} + \frac{\partial_t \bar{W}_1 - \partial_r \bar{W}_1}{\bar{W}_1} - \frac{1}{r}\Big)\dot{h}_{14}$$
$$+2\kappa_4^2 e^{2\bar{\gamma}-2\bar{\psi}}\bar{W}_1^2 \bar{X}\bar{P}\dot{Y}\,\mathbf{sin}[(n_2-n_1)\varphi] + \partial_\varphi\Big[\bar{W}_1^2 e^{2\bar{\gamma}-2\bar{\psi}}\dot{h}_{55} - \dot{h}_{11} - \frac{e^{2\bar{\gamma}}}{r^2}\dot{h}_{44}\Big]$$

$$\partial_t \dot{h}_{11} = \partial_r \dot{h}_{11} + \frac{e^{2\bar{\gamma}}}{r^2}(\partial_r\bar{\psi} - \partial_t\bar{\psi} - \frac{1}{2r})\dot{h}_{44}$$
$$+\frac{2}{\bar{W}_1}\Big(\partial_t\bar{W}_1 - \partial_r\bar{W}_1 + \bar{W}_1(\partial_r\bar{\psi} - \partial_t\bar{\psi} + \partial_t\bar{\gamma} - \partial_r\bar{\gamma})\Big)\dot{h}_{11}$$
$$+\frac{1}{2}e^{2\bar{\gamma}-2\bar{\psi}}\bar{W}_1^2\Big(\frac{\partial_r\bar{W}_1 - \partial_t\bar{W}_1}{\bar{W}_1} + \frac{1}{2r}\Big)\dot{h}_{55} + \kappa_4^2 e^{2\bar{\gamma}-2\bar{\psi}}\bar{W}_1^2(\partial_t\bar{X} - \partial_r\bar{X})\dot{Y}\,\mathbf{cos}[(n_2-n_1)\varphi]$$

$$\partial_t \dot{h}_{44} = \partial_r \dot{h}_{44}\Big(2\partial_r\bar{\psi} - 2\partial_t\bar{\psi} - \frac{3}{2r} + \frac{\partial_t\bar{W}_1 - \partial_r\bar{W}_1}{\bar{W}_1}\Big)\dot{h}_{44} + \frac{\kappa_4^2}{\epsilon}(\partial_r\bar{P} - \partial_t\bar{P})\dot{B}$$
$$+\frac{1}{2}\bar{W}_1^2 r^2 e^{-2\bar{\psi}}(\partial_t\bar{\psi} - \partial_r\bar{\psi} + \frac{1}{2r})\dot{h}_{55}$$

*Note*: term $\cos[(n_2 - n_1)\varphi]$. Choose $(n_2 - n_1) = 2$ and we have $\cos(2\varphi)$ , so two extremal values on $2\pi$ mod $\frac{1}{2}\pi$.

h44 interact with EM pert B even when scalarfield is absent!

►These propagation equations are linear in the first order derivative.
Appearance of combinations of $\ddot{h}_{\mu\nu}$ and $\ddot{k}_{\mu\nu}$ terms:
distortion of the shape of the waves

►The equation for $\dot{h}_{55}$ is as expected: $\dot{\boldsymbol{h}}_{\mathbf{55}} = \boldsymbol{\mathcal{M}}_{\mathbf{1}}(\boldsymbol{t}, \boldsymbol{r}, \boldsymbol{\varphi}, \boldsymbol{\xi}).\ \boldsymbol{\mathcal{M}}_{\mathbf{2}}(\boldsymbol{y})$ :
the brane part must be separable from the bulk part.

►There is an interaction between the HF perturbations from the bulk, the matterfields on the brane and the evolution of $\dot{h}_{ij}$

►The bulk contribution $\dot{h}_{55}$ is amplified by the warpfactor!

►It is a reflection of the massive KK-modes felt on the brane.

►Effectively a **dark-energy** term in Einstein equations

**<u>However:</u>** a more general solution must be investigated with $\boldsymbol{\kappa}_{\mathbf{5}}^{\mathbf{4}}\left(\bar{\boldsymbol{S}}_{\boldsymbol{\mu\nu}} + \boldsymbol{S}_{\boldsymbol{\mu\nu}}^{(\mathbf{0})}\right)$

For example in $\bar{\psi}_{tt}$: terms at rhs:

$$\kappa_5^4 \int \left(\Psi \dot{B}(\bar{X}_t - \bar{X}_r)(\bar{P}_t - \bar{P}_r)\cos[(n_i - n_j)\varphi\right) d\xi$$

# $\omega^1$ Einstein equations

The $\boldsymbol{\omega}^{(\mathbf{1})}$- Einstein equations:

$$^4G_{\mu\nu}^{(1)} = \kappa_4^2\,{}^4T_{\mu\nu}^{(1)} + \kappa_5^4\, S_{\mu\nu}^{(1)} - \mathcal{E}_{\mu\nu}^{(1)}$$

For example:

$$\begin{aligned}
\partial_{\mathbf{t}}\dot{\mathbf{k}}_{\mathbf{55}} = \partial_r\dot{k}_{55} + \frac{1}{2}(\partial_{rr}h_{55} - \partial_{tt}h_{55}) + \frac{1}{2}\partial_t h_{55} - \partial_r h_{55})\dot{h}_{55} + \frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2}(\partial_t\dot{h}_{55} - \partial_r\dot{h}_{55})h_{11} \\
\kappa_4^2\frac{e^{2\bar{\gamma}}}{r^2}\bar{X}Y\Big(2\bar{P}(n_1 - n_2 - \bar{P}) + e^{-2\bar{\psi}}r^2\bar{W}_1^2\beta(\eta^2 - \bar{X}^2)\mathbf{cos}[(n_2 - n_1)\varphi] \\
+2\frac{e^{\bar{\psi}}}{\bar{W}_1^2 r^2}\Big[\kappa_4^2\frac{e^{2\bar{\gamma}}}{r^2}\bar{X}^2\bar{P}^2 + \partial_{rr}\bar{\psi} - \partial_{tt}\bar{\psi} + 2\frac{e^{2\bar{\psi}}}{\bar{W}_1^2 r^2}\Big(\partial_r\bar{\psi} - \partial_t\bar{\psi} + \frac{\partial_t\bar{W}_1 - \partial_r\bar{W}_1}{\bar{W}_1} - \frac{3}{2r}\Big)\dot{h}_{44} \\
+\frac{\partial_r\bar{\psi}}{r} - \frac{\partial_r\bar{W}_1}{r}\Big]h_{44} + 2\frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2 r^2}\Big[\kappa_4^2 e^{2\bar{\gamma}}\Big(\bar{X}^2\bar{P}^2 + \frac{1}{8}e^{-2\bar{\psi}}\beta(\bar{X}^2 - \eta^2)^2 r^2\bar{W}_1^2\Big) \\
+\frac{\partial_t\bar{W}_1^2 - \partial_r\bar{W}_1^2}{\bar{W}_1^2} - \frac{\partial_{tt}\bar{W}_1 + \partial_{rr}\bar{W}_1}{\bar{W}_1}r^2\Big]h_{11} + \frac{e^{2\bar{\psi}}}{\bar{W}_1^2 r^2}\Big[2(\partial_r\bar{\psi} - \partial_t\bar{\psi} + \frac{\partial_t\bar{W}_1 - \partial_r\bar{W}_1}{\bar{W}_1})\partial_\varphi h_{14} \\
+2(\partial_r h_{44} - \partial_t h_{44})\dot{h}_{44} + \partial_{r\varphi}h_{14} - \partial_{t\varphi}h_{14} + \ddot{k}_{11} + \frac{e^{2\bar{\gamma}}}{r^2}\ddot{h}_{44}\Big] - 2\kappa_4^2\frac{e^{2\bar{\gamma}}}{r^2}\bar{X}^2\bar{P}B.
\end{aligned}$$

Integration wrt $\xi$ : 2-th order wave equation for $h_{55}$

Substituting back: equation for $\dot{k}_{55}$ [constraint eq.]

**Cauchy problem** solved ! [true dynamical system]

$$
\begin{aligned}
\partial_t \dot{k}_{14} = \partial_r \dot{k}_{14} + 2\Big(\partial_r\bar{\phi} - \partial_t\bar{\phi} + \frac{\partial_t \bar{W}_1 - \partial_r \bar{W}_1}{\bar{W}_1} - \frac{1}{r}\Big)\dot{k}_{14} + \frac{1}{2}\dot{h}_{55}(\partial_r h_{14} - \partial_t h_{14} \\
+h_{14}(\partial_r \dot{h}_{55} - \partial_t \dot{h}_{55})) + \kappa_4^2 e^{2\bar{\gamma}-2\bar{\psi}}\Big[\bar{W}_1^2\Big(\partial_t \bar{X} Y(n_1 - n_2 - \bar{P}) + \bar{X}(\bar{P}\partial_t Y + \epsilon \dot{Y} B)\Big)\mathbf{sin}[(n_2 - n_1)\varphi] \\
+\bar{W}_1^2 \bar{X}\bar{P}\dot{Z}\mathbf{sin}[(n_3 - n_1)\varphi] + (\partial_t \bar{X} - \partial_r \bar{X})\dot{Y} h_{14}\mathbf{cos}[(n_2 - n_1)\varphi]\Big] + \frac{e2\bar{\gamma}}{r^2}(h_{44}\dot{h}_{55} - \frac{1}{2}\dot{h}_{44}\partial_\varphi h_{55}) \\
+\frac{1}{2}\dot{h}_{55}(\partial_\varphi h_{11} - \bar{W}_1^2 \partial_\varphi h_{55}) + \frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2 r^4}(h_{44}\partial_\varphi \dot{h}_{44} - e^{-4\bar{\gamma}} r^4 h_{11}\partial_\varphi \dot{h}_{11}) + \partial_\varphi\Big[e^{2\bar{\gamma}-2\bar{\psi}}(\bar{W}_1^2 \partial_t \bar{\psi} - \bar{W}_1 \partial_t \bar{W}_1)h_{55} \\
+2h_{11}(\partial_r \bar{\gamma} + 2\frac{\partial_t \bar{W}_1}{\bar{W}_1} - \partial_t \bar{\psi}) - \partial_t h_{11} + \frac{1}{2}(\dot{k}_{22} - \dot{k}_{11} - \frac{e^{2\bar{\gamma}}}{r^2}\dot{k}_{44} + e^{2\bar{\gamma}-2\bar{\psi}}\bar{W}_1^2 \dot{k}_{55}) + 2e^{2\bar{\gamma}}(\frac{\partial_t \bar{W}_1}{\bar{W}_1} - 2\frac{\partial_t \bar{\psi}}{r^2})h_{44} \\
-\frac{e^2\bar{\gamma}}{r^2}(\partial_t h_{44} + e^{-2\bar{\psi}} r^2 \bar{W}_1^2 \partial_t h_{55})\Big] + \ddot{l}_{14} - \ddot{l}_{24} + 2\kappa_4^2 \bar{W}_1^2(\bar{X}^2 \epsilon \bar{P} B_0 - \frac{1}{8}\beta h_{14}(\bar{X}^2 - \eta^2)^2) + \mathcal{H}.
\end{aligned}
$$

Now we observe terms in $k_{14}$ with respect to $h_{14}$ :$\mathbf{sin}[(n_3 - n_1)\varphi]$

So to next order, the maxima can be out-of phase w.r.t first-order: $\mathbf{sin}[(n_2 - n_1)\varphi]$

for example: $(n_2 - n_1) = 2$ $(n_3 - n_1) = 4$

Integration wrt $\xi$: second-order PDE for $h_{11}$ !! [ just as for $h_{55}$ ]

back-reaction terms appear from bulk.

# The $\omega^0$ scalar-gauge field equations

Simplified case: $l_\mu = [1,-1,0,0,0]$

Then: first order gauge field: $\boldsymbol{B_\mu = [B_0, B_0, 0, B, 0]}$

From the **gauge field** eq: : The $\bar{A}_\mu$ is as the unperturbed case( after int.wrt $\xi$)

The first order perturbations:

$$\partial_t \dot{\Psi} = \partial_r \dot{\Psi} + \left[\frac{\partial_r \bar{\mathcal{W}} - \partial_t \bar{\mathcal{W}}}{\bar{\mathcal{W}}} + \frac{1}{2r}\right] \dot{\Psi}$$

$$\partial_t \dot{B} = \partial_r \dot{B} + \left[\partial_r \bar{\psi} - \partial_t \bar{\psi} - \frac{1}{2r}\right] \dot{B} + e^{2\bar{\psi}} \frac{(\partial_r \bar{P} - \partial_t \bar{P})}{2r^2 \bar{\mathcal{W}}^2 \varepsilon} \dot{h}_{44}$$

$$\partial_t \dot{B}_0 = \partial_t \dot{B}_0 - e^{2\bar{\gamma}} \frac{\partial_\varphi \dot{B}}{r^2} - \varepsilon e^{2\bar{\gamma} - 2\bar{\psi}} \bar{\mathcal{W}}^2 \bar{X} \dot{\Psi} \, \mathbf{sin}(\boldsymbol{n_2 - n_1})\boldsymbol{\varphi} + e^{2\bar{\psi}} \frac{(\partial_t \bar{P} - \partial_r \bar{P})}{2r^2 \bar{\mathcal{W}}^2 \varepsilon} \dot{h}_{14}$$

- ► We observe: **φ-dependent** parts arise, amplified by warpfactor!
- ► One needs: $\boldsymbol{l^\mu \bar{A}_\mu = 0}$ , otherwise real and imaginary parts interacts as propagation progresses.
- ►We omitted for time being $C_\mu$ and the $\boldsymbol{\kappa_5^4 \left(\bar{S}_{\mu\nu} + S_{\mu\nu}^{(0)}\right)}$ term
- ►Approximate wave solution no longer axially symmetric! [also found by Choquet B]
- ► The $\bar{W}^2$ -term in eq. for $B_0$: peculiar behavior
- ► The linear dv system $(\dot{h}_{ij}, \dot{B}, \dot{B}0, \dot{Y})$ can be solved by integration( **Choquet-B**,1977)

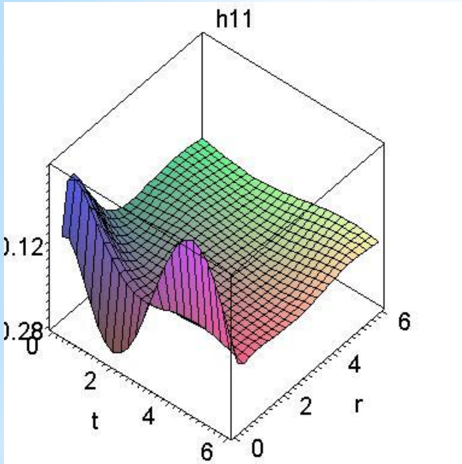
$$n^i \partial_i \vec{U} = A.\vec{U}$$

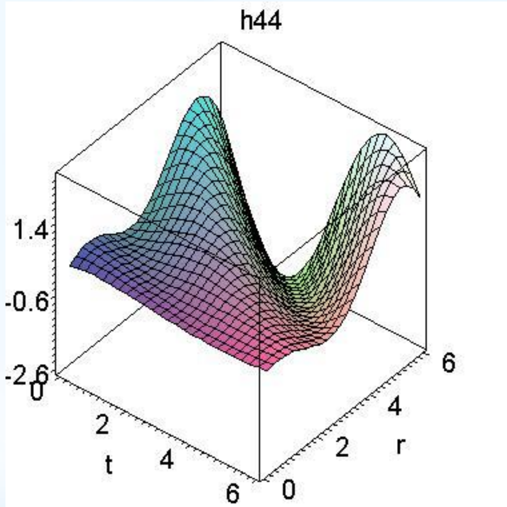
With $\vec{U} = \left(\dot{h}_{11},\ \dot{h}_{44},\ \dot{h}_{14},\ \dot{h}_{55},\ \dot{B},\ \dot{B}o,\ \dot{Y}\right)$

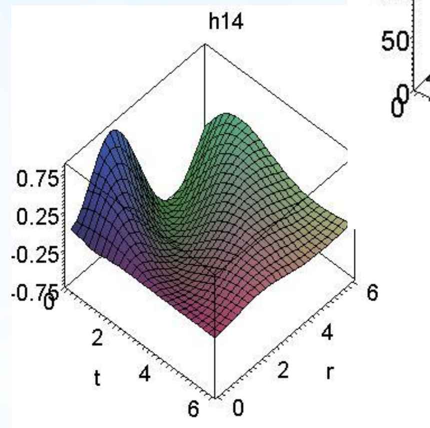
and A:

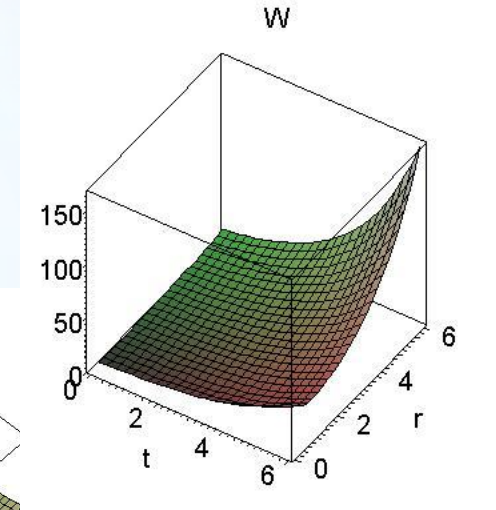
$$A = \begin{pmatrix}
2\left(\frac{W_t - W_r}{W} + \psi_r - \psi_t + \bar{\gamma}_t - \bar{\gamma}_r\right) & \frac{e^{2\bar{\gamma}}}{r^2}\left(\bar{\psi}_r - \bar{\psi}_t - \frac{1}{2r}\right) & 0 & e^{2\bar{\gamma}-2\bar{\psi}} W^2 \left(\frac{W_r - W_t}{2W} + \frac{1}{4r}\right) & 0 & 0 & K_4^2 (\bar{X}_t - \bar{X}_r) \cos(u_2 - u_1)\varphi \\
0 & \frac{W_t - W_r}{W} + 2(\bar{\psi}_r - \bar{\psi}_t) - \frac{3}{2r} & 0 & \frac{1}{2} W^2 e^{-2\bar{\psi}} r^2 \left(\bar{\psi}_t - \bar{\psi}_r + \frac{1}{2r}\right) & K_4^2 \frac{\bar{P}_t - \bar{P}_r}{\varepsilon} & 0 & 0 \\
-\partial\varphi & -\frac{e^{2\bar{\gamma}}}{r^2}\partial\varphi & \frac{2W_t - 2W_r}{W} + \bar{\psi}_r - \bar{\psi}_t - \frac{2}{r} & W^2 e^{2\bar{\gamma}-2\bar{\psi}} \partial\varphi & 0 & 0 & 2K_4^2 e^{2\bar{\gamma}-2\bar{\psi}} W^2 \bar{X} \bar{P} \sin(u_2 - u_1)\varphi \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{e^{2\bar{\psi}}(\bar{P}_t - \bar{P}_r)}{2r^2 W^2 \varepsilon} & 0 & 0 & \bar{\psi}_r - \bar{\psi}_t - \frac{1}{2r} & 0 & 0 \\
0 & 0 & \frac{e^{2\bar{\psi}}(\bar{P}_t - \bar{P}_r)}{r^2 W^2 \varepsilon} & 0 & -\frac{e^{2\bar{\gamma}}}{r^2}\partial\varphi & 0 & \varepsilon e^{2\bar{\gamma}-2\bar{\psi}} W^2 \bar{X} \sin(u_2 - u_1)\varphi \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{W_r - W_t}{W} + \frac{1}{2r}
\end{pmatrix}$$

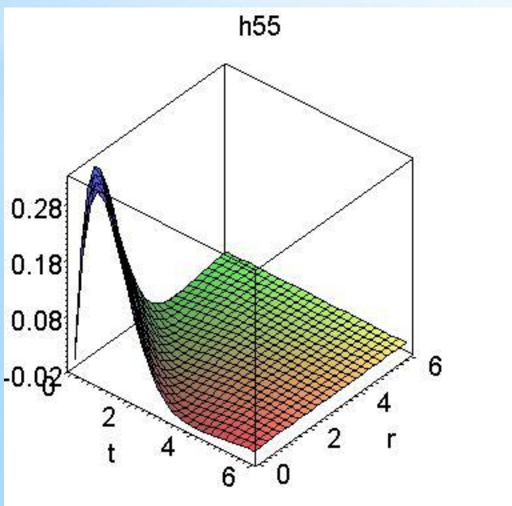
# Typical simplified solution of the first order equations

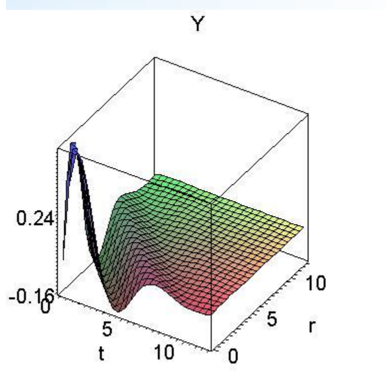


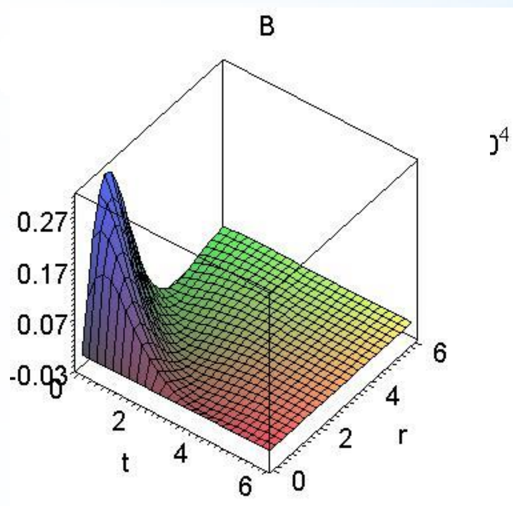



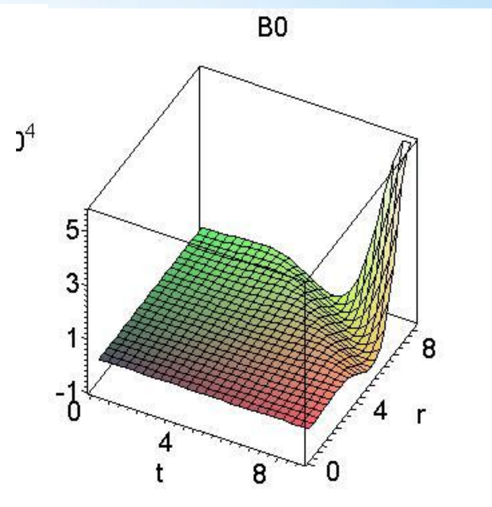



# The scalar background field equation

After integration we obtain for the background scalar field

$$\bar{D}^{\alpha}\bar{D}_{\alpha}\bar{\Phi} - \frac{1}{2}\beta\bar{\Phi}(\bar{\Phi}\bar{\Phi}^{*} - \eta^{2}) = \frac{1}{\tau}\int \left(h^{\mu\nu}l_{\mu}l_{\nu}\ddot{\Psi} + \bar{g}^{\mu\nu}\Gamma^{\alpha(0)}_{\mu\nu}\dot{\Psi}\right) d\xi$$

► There is a "backreaction" from the HF perturbations

►

# $\omega^1$ matter field equations ( 2-order)

For the scalar field:

$$\partial_t \dot{\mathbf{Z}} = \partial_r \dot{Z} + +\Big(\frac{\partial_r \bar{W}_1 - \partial_t \bar{W}_1}{\bar{W}_1} + \frac{1}{2r}\Big)\dot{Z}$$

$$-\frac{1}{2}\beta \bar{W}_1^2 \bar{X}^2 Y e^{2\bar{\gamma}-2\bar{\psi}} \mathbf{cos}(n_3 + n_2 - 2n_1)\varphi] + \Big[\partial_{tt} Y - \partial_{rr} Y + \frac{\partial_r Y}{r}$$

$$+2\frac{\partial_r Y \partial_r \bar{W}_1 - \partial_t Y \partial_t \bar{W}_1}{\bar{W}_1} + \frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2}\Big((\partial_r Y - \partial_t Y)\dot{h}_{11} + \dot{Y}(\partial_t h_{11} - \partial_r h_{11} - \frac{1}{2r})$$

$$-\frac{e^{2\bar{\gamma}}}{r^2} Y(n_2 - n_1 + \bar{P})^2 + 2\frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2} h_{11}\dot{Y}\Big(\partial_r \bar{\gamma} - \partial_t \bar{\gamma} + \partial_r \bar{\psi} - \partial_t \bar{\psi} + \frac{\partial_r \bar{W}_1 - \partial_t \bar{W}_1}{\bar{W}_1}$$

$$+\frac{\partial_r \dot{Y} - \partial_t \dot{Y}}{\dot{Y}}\Big) - \beta e^{2\bar{\gamma}-2\bar{\psi}} Y \bar{X}^2 \bar{W}_1^2 + 2\frac{e^{2\bar{\psi}}}{\bar{W}_1^2 r^2} h_{44}\dot{Y}\partial_r \bar{\psi} - \partial_t \bar{\psi} - \frac{1}{2r}\Big]\mathbf{cos}[(n_3 - n_2)\varphi]$$

$$+\Big[\frac{e^{2\bar{\psi}-2\bar{\gamma}}}{2\bar{W}_1^2}\Big((\partial_t \bar{X} - \partial_r \bar{X})(\dot{k}_{11} - \dot{k}_{22} + \partial_t \bar{X}\partial_t h_{11} - \partial_r \bar{X}\partial_r h_{11} + h_{11}\Big((\partial_{rr}\bar{X} - \partial_{tt}\bar{X})\bar{W}_1^2$$

$$+2\frac{\partial_r \bar{X}\partial_r \bar{W}_1 - \partial_t \bar{X}\partial_t \bar{W}_1}{\bar{W}_1} + 2(\partial_t \bar{X}\partial_t \bar{\psi} - \partial_r \bar{X}\partial_r \bar{\psi} + \partial_r \bar{X}\partial_r \bar{\gamma} - \partial_t \bar{X}\partial_t \bar{\gamma})\Big)$$

$$+2\frac{e^{2\bar{\psi}}}{\bar{W}_1^2 r^2} h_{44}(\partial_r \bar{X}\partial_r \bar{\psi} - \partial_t \bar{X}\partial_t \bar{\psi} - \frac{\partial_r \bar{X}}{2r})\Big]\mathbf{cos}[(n_3 - n_1)]$$

of the form: $(..)\mathbf{cos}(\boldsymbol{n_3} + \boldsymbol{n_2} - 2\boldsymbol{n_1})\boldsymbol{\varphi} + (..)\,\mathbf{cos}(\boldsymbol{n_3} - \boldsymbol{n_2})\boldsymbol{\varphi} + (..)\mathbf{cos}(\boldsymbol{n_3} - \boldsymbol{n_1})$

Numerical solution needed, because there is a coupling with 1-st order terms

**Again:** equation can be seen as second order wave-eq for Y

# Energy-momentum tensor components

**Energy-current components:**

$$ {}^{4}\mathbf{T}^{(\mathbf{0})}_{\mathbf{t}\varphi} = \bar{X}\bar{P}\dot{Y}\,\mathbf{sin}[(n_2 - n_1)\varphi] $$

$$ {}^{4}\mathbf{T}^{(\mathbf{1})}_{\mathbf{t}\varphi} = \left[\partial_t \bar{X} Y (n_1 - n_2 - \bar{P}) + \bar{X}(\bar{P}\partial_t Y + \epsilon B \dot{Y})\right]\mathbf{sin}[(n_2 - n_1)\varphi] $$

$$ + \frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2}\dot{Y} h_{14}(\partial_t \bar{X} - \partial_r \bar{X})\mathbf{cos}[(n_2 - n_1)\varphi] + h_{14}\left[\frac{e^{4\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^4 r^2 \epsilon^2}(\partial_t \bar{P} - \partial_r \bar{P})^2\right. $$

$$ + \frac{1}{8}\beta(\bar{X}^2 - \eta^2)^2 + \frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2}(\partial_r \bar{W}_1^2 - \partial_t \bar{W}_1^2) + \frac{1}{2}e^{2\bar{\psi}}\frac{\bar{X}^2\bar{P}^2}{\bar{W}_1^2 r^2} + \bar{X}^2\bar{P}\epsilon B_0^2 $$

# Energy-momentum tensor components

$$ {}^{4}\mathbf{T}^{(\mathbf{0})}_{\mathbf{tt}} = \dot{Y}^2 + \dot{Y}(\partial_t \bar{X} + \partial_r \bar{X})\mathbf{cos}[(n_2 - n_1)\varphi] + \frac{e^{2\bar{\psi}}}{\bar{W}_1^2 r^2 \epsilon}(\epsilon \dot{B}^2 + \dot{B}(\partial_t \bar{P} + \partial_r \bar{P} $$

$$ {}^{4}\mathbf{T}^{(\mathbf{1})}_{\mathbf{tt}} = \Big[\frac{1}{2}e^{2\bar{\gamma}-2\bar{\psi}}\bar{W}_1^2\bar{X}Y(\bar{X}^2 - \eta^2) + \partial_r\bar{X}\partial_r Y + \partial_t\bar{X}\partial_t Y $$

$$ + \frac{e^{2\bar{\gamma}}}{r^2}\bar{X}\bar{P}Y(n_2 - n_1 + \bar{P})\Big]\mathbf{cos}[(n_2 - n_1)\varphi] - 2\epsilon\bar{X}\dot{Y}B_0\mathbf{sin}[(n_2 - n_1)\varphi] $$

$$ + \dot{Z}(\partial_t\bar{X} + \partial_r\bar{X})\mathbf{cos}[(n_3 - n_1)\varphi] + 2\dot{Y}\dot{Z}\mathbf{cos}[(n_3 - n_2)\varphi] $$

$$ - \frac{e^{4\bar{\psi}}}{\bar{W}_1^4 r^4 \epsilon^2}\Big((\frac{1}{2}(\partial_t\bar{P}^2 + \partial_r\bar{P}^2) + \epsilon\dot{B}(\partial_t\bar{P} + \partial_r\bar{P}) + \epsilon^2\dot{B}^2\Big)h_{44} $$

$$ -\Big(\frac{1}{8}(\bar{X}^2 - \eta^2)^2 - \frac{e^{2\bar{\psi}}}{2\bar{W}_1^2 r^2}\bar{X}^2\bar{P}^2\Big)h_{11} + \frac{e^{2\bar{\psi}-2\bar{\gamma}}}{2\bar{W}_1^2 r^4}\bar{X}^2\bar{P}^2 h_{44} + \dot{Y}(\partial_t Y + \partial_r Y) $$

$$ + e^{2\bar{\psi}}\bar{W}_1^2 r^2 \epsilon\dot{C}(\partial_t\bar{P} + \partial_r\bar{P} + 2\epsilon\dot{B}) + \frac{\epsilon e^{2\bar{\gamma}}}{r^2}\bar{X}^2\bar{P}^2 B $$

So 4 periodic functions ! Numerical solution needed.

# Conclusions

**How to detect Cosmis Strings:** **I.** Perturbation can lead to signatures in temperature anisotropy , polarization and non-Gaussian spectra of the CMB?

**II.** Gravitational waves [loop decay]?

**III.** Lensing?

**NOT FOUND!**

**Alternative:** **Via quasar alignment of polarization axes.**

Fractional azimuthal-angle dependent wave-like structure found in first- and second-orde perturbation equations using MS-method. Dependent of winding number

Abrikosov n-vortices are unlikely [energy is reduced if they split up into singlevortex]
[n is winding number or topol. charge]

**However:** contrib. of the 5D Weyl tensor: warpfactor enters the GR equations
[kind of dark-energy]

The symm breaking of the Higgs field $\leftrightarrow$ SO(2) breaking of the axially symm. In discrete subgroup of rot. about $180^o$
Return to a axially symm. by emission of GW [restore of SO(2)]

**General:** conformal (scale-) invariance is the missing symmetry in physics!!
spontaneously broken just as in standard model the SU(3)

# Conclusions

**Azimuthal-angle φ dep. in energy momentum tensor:**

${}^4T_{tt}^{(0)}$ : $cos[(n_2 - n_1)\varphi]$ ${}^4T_{tt}^{(1)}$ : $sin[(n_2 - n_1)\varphi]$

${}^4T_{t\varphi}^{(0)}$ : $\sin[(n_2 - n_1)\varphi]$

For $n_2 - n_1 = 2$ ► **2 extremal values on [0,π] mod (½π)**

► **out of phase of next order term**

$n_3 - n_1 = 4$ ► $n_3 + n_2 - 2n_1 = 6$ ► $n_3 - n_2 = 2$

Terms in scalar perturbations and ${}^4T_{mn}^{(i)} \sim (n_{i+1} - n_i - \bar{P})^i$

So: instable by the breakup of vortices? [ as in exceptional $\phi^4$ model]

**NO:** suppression by **warpfactor**

Careful comparison of this **spectrum** with preferred orientations of **quasars**:

All features of alignment of pol. axes in LQG explainable !

## evidence of cosmic strings?

Prospect: new data for high-redshift needed [ on his way…]

Then: next order results can be tested.