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On some pseudo Smarandache function related triangles

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Abstract Two triangles $T(a,b,c)$ and $T(a',b',c')$ are said to be pseudo Smarandache related if $Z(a) = Z(a'), Z(b) = Z(b'), Z(c) = Z(c')$, where $Z(.)$ is the pseudo Smarandache function, and $T(a,b,c)$ denotes the triangle with sides a, b and c. This paper proves the existence of an infinite family of pairs of dissimilar Pythagorean triangles that are pseudo Smarandache related.

Keywords Smarandache function, pseudo Smarandache function, Smarandache related triangles, pseudo Smarandache related triangles, Pythagorean triangles.

§1. Introduction and result

The Smarandache function, denoted by S(n), is defined as follows.

Definition 1.1. For any integer $n \geq 1$, ($\mathbb{Z}^+$ being the set of all positive integers),

$$S(n) = \min\{m : m \in \mathbb{Z}^+, \quad n|m!\}.$$

The following definition is due to Sastry [6].

Definition 1.2. Two triangles $T(a,b,c)$ (with sides of length a, b and c) and $T(a',b',c')$ (with sides of length a', b' and c'), are said to be Smarandache related if

$$S(a) = S(a'), S(b) = S(b'), S(c) = S(c').$$

Definition 1.3. A triangle $T(a,b,c)$ is said to be Pythagorean if and only if one of its angles is $90°$.

Thus, a triangle $T(a,b,c)$ (with sides of length a, b and c) is Pythagorean if and only if $a^2 + b^2 = c^2$.

Sastry[6] raised the following question : Are there two distinct dissimilar Pythagorean triangles that are Smarandache related? Recall that two triangles $T(a,b,c)$ and $T(a',b',c')$ are similar if and only if the corresponding three sides are proportional, that is, if and only if

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

Otherwise, the two triangles are dissimilar.

The following result, due to Ashbacher [1], answers the question in the affirmative.

Theorem 1.1. There are an infinite family of pairs of dissimilar Pythagorean triangles that are Smarandache related.

The proof of Theorem 1.1 is rather simple : For any prime $p \geq 17$, the two families of dissimilar Pythagorean triangles $T(3p, 4p, 5p)$ and $T(5p, 12p, 13p)$ are Smarandache related, since $S(3p) = S(4p) = S(5p) = S(12p) = S(13p) = p$.

Ashbacher [1] introduced the concept of pseudo Smarandache related triangles, defined as follows.

Definition 1.4. Two triangles $T(a, b, c)$ and $T(a^{'}, b^{'}, c^{'})$ are said to be pseudo Smarandache related if $Z(a) = Z(a^{'})$, $Z(b) = Z(b^{'}), Z(c) = Z(c^{'})$.

In Definition 1.4 above, $Z(.)$ denotes the pseudo Smarandache function. Recall that the pseudo Smarandache function is defined as follows.

Definition 1.5. For any integer $n \geq 1$, $Z(n)$ is the smallest positive integer m such that $1 + 2 + \cdots + m \equiv \frac{m(m+1)}{2}$ is divisible by n. Thus,

$$Z(n) = \min\{m : m \in Z^{+}, n | \frac{m(m+1)}{2}\}; n \geq 1.$$

Ashbacher [1] used a computer program to search for dissimilar pairs of Pythagorean triangles that are pseudo Smarandache related. He reports some of them, and conjectures that there are an infinite number of pairs of dissimilar Pythagorean triangles that are pseudo Smarandache related.

In this paper, we prove the conjecture of Ashbacher [1] in the affirmative. This is done in Theorem 3.1 in Section 3. We proceed on the same line as that followed by Ashbacher. However, in the case of the pseudo Smarandache related triangles, the proof is a little bit more complicated. The intermediate results, needed for the proof of Theorem 3.1, are given in the next Section 2.

§2. Some preliminary results

The following result, giving the explicit expressions of $Z(3p)$, $Z(4p)$ and $Z(5p)$, are given in Majumdar [5].

Lemma 2.1. If $p \geq 5$ is a prime, then

$$Z(3p) = \begin{cases} p-1, & \text{if } 3|(\mathrm{p}-1), \\ p\,, & \text{if } 3|(\mathrm{p}+1). \end{cases}$$

Lemma 2.2. If $p \geq 5$ is a prime, then

$$Z(4p) = \begin{cases} p-1, & \text{if } 8|(p-1), \\ p, & \text{if } 8|(p+1), \\ 3p-1, & \text{if } 8|(3p-1), \\ 3p, & \text{if } 8|(3p+1). \end{cases}$$

Lemma 2.3. If $p \geq 7$ is a prime, then

$$Z(5p) = \begin{cases} p-1, & \text{if } 10|(p-1), \\ p, & \text{if } 10|(p+1), \\ 2p-1, & \text{if } 5|(2p-1), \\ 2p, & \text{if } 5|(2p+1). \end{cases}$$

The explicit expressions of $Z(12p)$ and $Z(13p)$ are given in the following two lemmas.

Lemma 2.4. If $p \geq 13$ is a prime, then

$$Z(12p) = \begin{cases} p-1, & \text{if } 24|(p-1) \\ p, & \text{if } 24|(p+1), \\ 3p-1, & \text{if } 8|(3p-1), \\ 3p, & \text{if } 8|(3p+1), \\ 7p-1, & \text{if } 24|(7p-1), \\ 7p, & \text{if } 24|(7p+1). \end{cases}$$

Proof. By definition,

$$Z(12p) = \min\{m : 12p|\frac{m(m+1)}{2}\} = \min\{m : p|\frac{m(m+1)}{24}\}. \quad (1)$$

If $p|m(m+1)$, then p must divide either m or $m+1$, but not both, and then 24 must divide either $m+1$ or m respectively. In the particular case when 24 divides $p-1$ or $p+1$, the minimum m in (1) may be taken as $p-1$ or p respectively. We now consider the following eight cases that may arise :

Case 1 : p is of the form $p = 24a + 1$ for some integer $a \geq 1$.
In this case, $24|(p-1)$. Therefore, $Z(12p) = p - 1$.

Case 2 : p is of the form $p = 24a + 23$ for some integer $a \geq 1$.
Here, $24|(p+1)$, and so, $Z(12p) = p$.

Case 3 : p is of the form $p = 24a + 5$ for some integer $a \geq 1$, so that $8|(3p+1)$.
In this case, the minimum m in (1) may be taken as $3p$. That is, $Z(12p) = 3p$.

Case 4 : p is of the form $p = 24a + 19$ for some integer $a \geq 1$.
Here, $8|(3p-1)$, and hence, $Z(12p) = 3p - 1$.

Case 5 : p is of the form $p = 24a + 7$ for some integer $a \geq 1$.
In this case, $24|(7p-1)$, and hence, $Z(12p) = 7p$.

Case 6 : p is of the form $p = 24a + 17$ for some integer $a \geq 1$.
Here, $24|(7p+1)$, and hence, $Z(12p) = 7p + 1$.

Case 7 : p is of the form $p = 24a + 11$ for some integer $a \geq 1$.
In this case, $8|(3p-1)$, and hence, Z(12p) = 3p - 1.

Case 8 : p is of the form $p = 24a + 13$ for some integer $a \geq 1$.
Here, $8|(3p+1)$, and hence, $Z(12p) = 3p$.

Lemma 2.5. For any prime $p \geq 17$,

$$Z(13p) = \begin{cases} p-1, & \text{if } 13|(p-1), \\ p, & \text{if } 13|(p+1), \\ 2p-1, & \text{if } 13|(2p-1), \\ 2p, & \text{if } 13|(2p+1), \\ 3\text{p}-1, & \text{if } 13|(3p-1), \\ 3p, & \text{if } 13|(3p+1), \\ 4\text{p}-1, & \text{if } 13|(4p-1), \\ 4p, & \text{if } 13|(4p+1), \\ 5\text{p}-1, & \text{if } 13|(5p-1), \\ 5p, & \text{if } 13|(5p+1), \\ 6\text{p}-1, & \text{if } 13|(6p-1), \\ 6p, & \text{if } 13|(6p+1). \end{cases}$$

Proof. By definition,

$$Z(13p) = \min\{m : 13p|\frac{m(m+1)}{2}\} = \min\{m : p|\frac{m(m+1)}{26}\}. \quad (2)$$

We have to consider the twelve possible cases that may arise :

Case 1 : p is of the form $p = 13a + 1$ for some integer $a \geq 1$.
In this case, $13|(p-1)$, and so, $Z(13p) = p - 1$.

Case 2 : p is of the form $p = 13a + 12$ for some integer $a \geq 1$.
Here, $13|(p+1)$, and hence, $Z(13p) = p$.

Case 3 : p is of the form $p = 13a + 2$ for some integer $a \geq 1$.
In this case, $13|(6p+1)$, and hence, $Z(13p) = 6p$.

Case 4 : p is of the form $p = 13a + 11$ for some integer $a \geq 1$.
Here, $13|(6p-1)$, and hence, $Z(13p) = 6p - 1$.

Case 5 : p is of the form $p = 13a + 3$ for some integer $a \geq 1$.
In this case, $13|(4p+1)$, and hence, $Z(13p) = 4p$.

Case 6 : p is of the form $p = 13a + 10$ for some integer $a \geq 1$.
Here, $13|(4p-1)$, and hence, $Z(13p) = 4p - 1$.

Case 7 : p is of the form $p = 13a + 4$ for some integer $a \geq 1$.
In this case, $13|(3p+1)$, and hence, $Z(13p) = 3p$.

Case 8 : p is of the form $p = 13a + 9$ for some integer $a \geq 1$.
Here, $13|(3p-1)$, and hence, $Z(13p) = 3p - 1$.

Case 9 : p is of the form $p = 13a + 5$ for some integer $a \geq 1$.
In this case, $13|(5p+1)$, and hence, $Z(13p) = 5p$.

Case 10 : p is of the form $p = 13a + 8$ for some integer $a \geq 1$.
Here, $13|(5p-1)$, and hence, $Z(13p) = 5p - 1$.

Case 11 : p is of the form $p = 13a + 6$ for some integer $a \geq 1$.
In this case, $13|(2p+1)$, and hence, $Z(13p) = 2p$.

Case 12 : p is of the form $p = 13a + 7$ for some integer $a \geq 1$.
Here, $13|(2p-1)$, and hence, $Z(13p) = 2p - 1$.

§3. Main result

We are now state and prove the main result of this paper in the following theorem.

Theorem 3.1. There are an infinite number of pairs of dissimilar Pythagorean triangles that are pseudo Smarandache related.

Proof. We consider the pair of dissimilar Pythagorean triangles

$$T(3p, 4p, 5p) \quad \text{and} \quad T(5p, 12p, 13p), \tag{3}$$

where p is a prime of the form

$$p = (2^3 \cdot 3 \cdot 5 \cdot 13)n + 1 = 1560n + 1, n \in Z^+. \tag{4}$$

By Lemma 2.1 - Lemma 2.5,

$$Z(3p) = Z(4p) = Z(5p) = Z(12p) = Z(13p) = p - 1,$$

so that the triangles $T(3p, 4p, 5p)$ and $T(5p, 12p, 13p)$ are pseudo Smarandache related. Now, since there are an infinite number of primes of the form (4) (by Dirichlet's Theorem, see, for example, Hardy and Wright [3], Theorem 15, pp. 13), we get the desired infinite number of pairs of dissimilar Pythagorean triangles that are pseudo Smarandache related.

It may be mentioned here that, the family of pairs of triangles (3), where p is a prime of the form

$$p = 1560n - 1, n \in Z^+, \tag{5}$$

also forms (dissimilar) pseudo Smarandache related Pythagorean triangles.

§4. Some remarks

The pseudo Smarandache function $Z(n)$ is clearly not bijective. However, we can define the inverse $Z^{-1}(m)$ as follows :

$$Z^{-1}(m) = \{n \in Z^+ : Z(n) = m\} \quad \text{for any integer} \quad m \geq 3, \tag{6}$$

with

$$Z^{-1}(1) = 1, Z^{-1}(2) = 3. \tag{7}$$

As has been pointed out by Majumdar [5], for any $m \in Z^+$, the set $Z^{-1}(m)$ is non-empty and bounded with $\frac{m(m+1)}{2}$ as its largest element. Clearly, $n \in Z^{-1}(m)$ if and only if the following two conditions are satisfied :

(1) n divides $\frac{m(m+1)}{2}$,

(2) n does not divide $\frac{\ell(\ell+1)}{2}$ for any ℓ with $1 \leq \ell \leq m-1$.

We can look at (6) from a different point of view : On the set Z^+ , we define the relation $\Re$ as follows :

$$\text{For any } n_1, n_2 \in Z^+, n_1 \Re n_2 \quad \text{if and only if} \quad Z(n_1) = Z(n_2). \tag{8}$$

It is then straightforward to verify that $\Re$ is an equivalence relation on Z^+ . It is well-known that an equivalence relation induces a partition (on the set Z^+) (see, for example, Gioia [2], Theorem 11.2, pp. 32). The sets $Z^{-1}(m), m \in Z^+$, are, in fact, the equivalence classes induced by the equivalence relation $\Re$ on Z^+ , and possesses the following two properties :

(1) $\sum\limits_{m=1}^{\infty} Z^{-1}(m) = \mathbb{Z}^+$

(2) $Z^{-1}(m_1) \cap Z^{-1}(m_2) = \emptyset$, if $\mathrm{m}_1 \neq m_2$.

Thus, for any $n \in \mathbb{Z}^+$, there is one and only one $m \in \mathbb{Z}^+$ such that $n \in Z^{-1}(m)$.

A different way of relating two triangles has been considered by Ashbacher [1], which is given in the following definition.

Definition 4.1. Given two triangles, $T(a, b, c)$ and $T(a', b', c')$, where

$$a + b + c = 180 = a' + b' + c', \tag{9}$$

they are said to be pseudo Smarandache related if

$$Z(a) = Z(a'), Z(b) = Z(b'), Z(c) = Z(c').$$

The difference in Definition 1.4 and Definition 4.1 is that, in the former, the sides of the pair of triangles are pseudo Smarandache related, while their angles, measured in degrees, are pseudo Smarandache related in the latter case. Note that the condition (9) merely states the fact the sum of the three angles of a triangle is 180 degrees.

Using a computer program, Ashbacher searched for pseudo Smarandache related pairs of triangles (in the sense of Definition 4.1). He reports three such pairs.

However, in this case, the equivalence classes $Z^{-1}(m), m \in Z^+$, might be of some help. The condition (9) can be dealt with by considering the restricted sets $Z^{-1}(m|\pi)$:

$$Z^{-1}(m|\pi) = \{n \in Z^+ : Z(n) = m, 1 \leq n \leq 178\}. \tag{10}$$

Table 4.1 gives such restricted sets related to our problem.

Clearly, two pseudo Smarandache related triangles $T(a, b, c)$ and $T(a', b', c')$ must satisfy the following condition :

$$a, a' \in Z^{-1}(m_1|\pi); b, b' \in Z^{-1}(m_2|\pi), c, c' \in Z^{-1}(m_3|\pi) \quad \text{for some} \quad m_1, m_2, m_3 \in Z^+.$$

Thus, for example, choosing

$$a, a^{'} \in \{2, 6\} = Z^{-1}(3|\pi); b, b^{'} \in \{44, 48, 88, 132, 176\} = Z^{-1}(32|\pi),$$

we can construct the pseudo Smarandache related triangles $T(2, 48, 130)$ and $T(6, 44, 130)$. Again, choosing $a, a', b, b' \in \{25, 50, 75, 100, 150\} = Z^{-1}(24|\pi)$, we can form the pair of pseudo Smarandache related triangles $T(25, 150, 5)$ and $T(75, 100, 5)$ with the characteristic that

$$Z(a) = Z(a^{'}) = 24 = Z(b) = Z(b^{'}) \quad (\text{and } \ Z(c) = Z(c^{'}) = 4).$$

Choosing $a, a^{'}, b, b^{'} \in \{8, 20, 24, 30, 40, 60, 120\} = Z^{-1}(15|\pi)$, we see that the equilateral triangle $T(60, 60, 60)$ is pseudo Smarandache related to the triangle $T(20, 40, 120)$!

Ashbacher [1] cites the triangles $T(4, 16, 160)$ and $T(14, 62, 104)$ as an example of a pseudo Smarandache related pair where all the six angles are different. We have found three more, given below :

(1) $T(8, 12, 160)$ and $T(40, 36, 104)$,

(with $Z(8) = Z(40) = 15, Z(12) = Z(36) = 8, Z(160) = Z(104) = 64$),

(2) $T(16, 20, 144)$ and $T(124, 24, 32)$,

(with $Z(16) = Z(124) = 31, Z(20) = Z(24) = 15, Z(144) = Z(32) = 63$),

(3) $T(37, 50, 93)$ and $T(74, 75, 31)$,

(with $Z(37) = Z(74) = 36, Z(50) = Z(75) = 24, Z(93) = Z(31) = 30$).

Table 4.1. Values of $Z^{-1}(m|\pi) = \{n \in Z^{+} : Z(n) = m, 1 \leq n \leq 180\}$

m	$Z^{-1}(m\|\pi)$	m	$Z^{-1}(m\|\pi)$	m	$Z^{-1}(m\|\pi)$
1	$\{1\}$	35	$\{90\}$	80	$\{81, 108, 162, 180\}$
2	$\{3\}$	36	$\{37, 74, 111\}$	82	$\{83\}$
3	$\{2, 6\}$	39	$\{52, 130, 156\}$	83	$\{166\}$
4	$\{5, 10\}$	40	$\{41, 82, 164\}$	84	$\{170\}$
5	$\{15\}$	41	$\{123\}$	87	$\{116, 174\}$
6	$\{7, 21\}$	42	$\{43, 129\}$	88	$\{89, 178\}$
7	$\{4, 14, 28\}$	43	$\{86\}$	95	$\{152\}$
8	$\{9, 12, 18, 36\}$	44	$\{99, 110, 165\}$	96	$\{97\}$
9	$\{45\}$	45	$\{115\}$	100	$\{101\}$
10	$\{11, 55\}$	46	$\{47\}$	102	$\{103\}$
11	$\{22, 33, 66\}$	47	$\{94, 141\}$	106	$\{107\}$
12	$\{13, 26, 39, 78\}$	48	$\{49, 56, 84, 98, 147, 168\}$	108	$\{109\}$
13	$\{91\}$	49	$\{175\}$	111	$\{148\}$
14	$\{35, 105\}$	51	$\{102\}$	112	$\{113\}$
15	$\{8, 20, 24, 30, 40, 60, 120\}$	52	$\{53, 106\}$	120	$\{121\}$
16	$\{17, 34, 68, 136\}$	53	$\{159\}$	124	$\{125\}$
17	$\{51, 153\}$	54	$\{135\}$	126	$\{127\}$
18	$\{19, 57, 171\}$	55	$\{140, 154\}$	127	$\{64\}$
19	$\{38, 95\}$	56	$\{76, 114, 133\}$	128	$\{172\}$
20	$\{42, 70\}$	58	$\{59\}$	130	$\{131\}$
21	$\{77\}$	59	$\{118, 177\}$	136	$\{137\}$
22	$\{23\}$	60	$\{61, 122\}$	138	$\{139\}$
23	$\{46, 69, 92, 138\}$	63	$\{32, 72, 96, 112, 144\}$	148	$\{149\}$
24	$\{25, 50, 75, 100, 150\}$	64	$\{80, 104, 160\}$	150	$\{151\}$
25	$\{65\}$	65	$\{143\}$	156	$\{157\}$
26	$\{27, 117\}$	66	$\{67\}$	162	$\{163\}$
27	$\{54, 63, 126\}$	67	$\{134\}$	166	$\{167\}$
28	$\{29, 58\}$	69	$\{161\}$	168	$\{169\}$
29	$\{87, 145\}$	70	$\{71\}$	172	$\{173\}$
30	$\{31, 93, 155\}$	71	$\{142\}$	178	$\{179\}$
31	$\{16, 62, 124\}$	72	$\{73, 146\}$	255	$\{128\}$
32	$\{44, 48, 88, 132, 176\}$	78	$\{79\}$		
34	$\{85, 119\}$	79	$\{158\}$		

Ashbacher [1] reports that, an exhaustive computer search for pairs of dissimilar search for all pseudo Smarandache related triangles $T(a,b,c)$ and $T(a^{'},b^{'},c^{'})$ $(a+b+c=180=a^{'}+b^{'}+c^{'})$ with values of a in the range $1 \le a \le 178$, revealed that a cannot take the following values :

$$\begin{aligned} 1,15,23,35,41,45,51,59,65,67,71,73,77,79,82,83,86,87,89, \\ 90,91,97,101,102,105,107,109,113,115,116,118,121,123,125,126,127, \\ 131,134,135,137,139,141,142,143,148,149,151,152,153,157,158,159, \\ 161,163,164,166,167,169,170,171,172,173,174,175,176,177,178. \end{aligned} \tag{11}$$

Can the table of the sets $Z^{-1}(m|\pi)$, Table 4.1, be utilized in explaining this observation? Of course, if a is too large, then there is a possibility that no two dissimilar triangles exist: Very large value of a very often forces the two triangles to be similar. For example, if $a = 174 \in Z^{-1}(87|\pi)$, then the three possible pairs of values of (b,c) are $(1,5)$, $(2,4)$ and $(3,3)$. Note that the two sets $Z^{-1}(1|\pi) = \{1\}$ and $Z^{-1}(2|\pi) = \{3\}$ are singleton. Thus, if $a^{'} = 174$, then the two triangles $T(a,b,c)$ and $T(a^{'},b^{'},c^{'})$ must be similar. On the other hand, if $a^{'} = 116 \in Z^{-1}(87|\pi)$, then we cannot find $b^{'},c^{'}$ with $b^{'}+c^{'} = 64$. Now, we consider the case when $a = 164$. This case is summarized in the tabular form below :

a=164	$a^{'}$=164	$a^{'}$=82	$a^{'}$=41	Remark	
(1,15)	Similar Triangle	Not Possible	Not Possible	Both belong to singleton sets	(b,c)
(2,14)	Similar Triangle	Not Possible	Not Possible	$b^{'}+c^{'} \le 34$	
(3,13)	Similar Triangle	Not Possible	Not Possible	3 belongs to a singleton set	
(4,12)	Similar Triangle	Not Possible	Not Possible	$b^{'}+c^{'} \le 64$	
(5,11)	Similar Triangle	Not Possible	Not Possible	$b^{'}+c^{'} \le 65$	
(6,10)	Similar Triangle	Not Possible	Not Possible	$b^{'}+c^{'} \le 16$	
(7,9)	Similar Triangle	Not Possible	Not Possible	$b^{'}+c^{'} \le 57$	
(8,8)	Similar Triangle	Not Possible	Not Possible		

However, if $a = 165$, we can get two dissimilar pseudo Smarandache related triangles, namely, $T(165,7,8)$ and $T(81,21,60)$.

A closer look at the values listed in (11) and those in Table 4.1 reveals the following facts:
(1) The numbers not appearing in any 180 degrees triplets are all belong to singleton sets, with the exception of 3 (though we conjecture that 3 cannot appear in any triplet), 47, 64, and 103. For the last three cases, we can form the following examples :

(a) $T(47,76,57)$ and $T(47,114,19)$ (with $Z(76) = 56 = Z(114), Z(57) = 18 = Z(19)$),

(b) $T(64,12,104)$ and $T(64,36,80)$ (with $Z(12) = 8 = Z(36), Z(104) = 64 = Z(80)$),

(c) $T(103,11,66)$ and $T(103,55,22)$ (with $Z(11) = 10 = Z(55), Z(66) = 11 = Z(22)$).

(2) In most of the cases, if a value does not appear in the triplet, all other values of the corresponding $Z^{-1}(m|\pi)$ also do not appear in other triplet, with the exception of $145 \in Z^{-1}(29|\pi)$ ($87 \in Z^{-1}(29|\pi)$ does not appear in any triplet), and $146 \in Z^{-1}(72|\pi)$($73 \in Z^{-1}(72|\pi)$ does not appear in any triplet). In this connection, we may mention the following pairs of triplets :

(a) $T(145,14,21)$ and $T(145,28,7)$ (with $Z(14)=7=Z(28), Z(21)=6=Z(7)$),

(b) $T(146,4,30)$ and $T(146,14,20)$ (with $Z(4)=7=Z(14), Z(30)=15=Z(20)$).

(c) Both $54, 63 \in Z^{-1}(72|\pi)$ can appear in one or the other triplet, but $126 \in Z^{-1}(72|\pi)$ cannot appear in any triplet.

It is also an interesting problem to look for 60 degrees and 120 degrees pseudo Smarandache related pairs of triangles, in the sense of Definition 4.1, where a 60 (120) degrees triangle is one whose one angle is 60 (120) degrees. We got the following two pairs of pseudo Smarandache related 60 degrees triangles :

(1) $T(8,60,112)$ and $T(24,60,96)$, (with $Z(8)=Z(24)=15, Z(112)=Z(96)=63$),

(2) $T(32,60,88)$ and $T(72,60,48)$, (with $Z(32)=Z(72)=63, Z(88)=Z(48)=32$), while our search for a pair of pseudo Smarandache related 120 degrees triangles went in vain.

We conclude the paper with the following open problems and conjectures. The first two have already been posed by Ashbacher [1].

Problem 1. Are there an infinite number of pairs of dissimilar 60 degrees triangles that are pseudo Smarandache related, in the sense of Definition 1.4?

Problem 2. Is there an infinite family of pairs of dissimilar 120 degrees triangles that are pseudo Smarandache related, in the sense of Definition 1.4?

Ashbacher [1] reports that a limited search on a computer showed only four pairs of dissimilar 60 degrees psedo Smarandache related triangles, while the number is only one in the case of 120 degrees triangles.

In the formulation of Definition 4.1, the number of pairs of dissimilar Smarandache related or pseudo Smarandache related triangles is obviously finite. In fact, since the number of integer solutions to

$$\begin{cases} a+b+c=180 \\ a \geqslant 1, b \geqslant 1, c \geqslant 1 \end{cases}$$

is $C(179,2)$ (see, for example, Johnsonbaugh [4], Theorem 4.5, pp. 238), the number of such triangles cannot exceed $C(179,2)$. But this is a very crude estimate. The next problems of interest are

Problem 3. Is it possible to find a tight or better upper limit to the number of pairs of dissimilar triangles that are pseudo Smarandache related, in the sense of Definition 4.1?

Problem 4. Is it possible to find a good upper limit to the number of pairs of dissimilar 60 degrees triangles that are pseudo Smarandache related, in the sense of Definition 4.1?

Problem 5. Is it possible to find another pair of 60 degrees pseudo Smarandache related triangles, in the sense of Definition 4.1, where all the six angles are acute?

By a limited search, we found only two pairs of dissimilar 60 degrees pseudo Smarandache related triangles, the ones already mentioned above.

Conjecture 1. There is no pair of dissimilar 120 degrees triangles that are Pseudo Smarandache related, in the sense of Definition 4.1.

Conjecture 2. There is no pair of dissimilar 90 degrees triangles that are Pseudo Smarandache related, in the sense of Definition 4.1.

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