Variational Principles and Perceptual Color Correction of Digital Images

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ABSTRACT

Variational principles appear in a vast number of scientific disciplines. They often provide a 'view from above' which permits to better comprehend and analyse problems. In this paper we show how variational techniques can be used to modelise perceptual color correction of digital images. We will show that the basic human visual phenomenology defines a unique class of functionals whose minimization gives rise to color enhancement. This framework provides a unified home for noticeable models of perceptual color correction, as e.g. Retinex.

Categories and Subject Descriptors

I.4.3 [Image Processing and computer vision]: Enhancement, Filtering; I.4.8 [Scene Analysis]: Color

Keywords

Perceptual color correction, local contrast enhancement

1. INTRODUCTION

Human vision is a process of great complexity: it begins with the photochemical transitions that occurs when light is captured by the three different types of cone pigments inside the retina, then an electric impulse passes through the nervous system and reaches the brain, where it is analyzed and interpreted. However, retina photochemistry, nervous impulse propagation and brain interpretation are not fully understood, hence a *deterministic* characterization of the visual process is unavailable. For this reason, the majority of color perception models follow a *descriptive* approach, trying to simulate macroscopic features of color vision, rather than reproduce neurophysiological activity.

Here we will show that a set of basic *phenomenological characteristics* can be translated into mathematical axioms to be fulfilled by a variational energy functional defined on the space of image functions. Remarkably, there is only one class of energy functionals able to comply with all the axioms at once. Once such a perceptual energy is fixed, the Euler-Lagrange equations corresponding to its minimization give rise to a computational algorithm that can be used to perform perceptual color correction of digital images. The advantage of this point of view relies in the intertwining between the algorithm equation and the corresponding vari- ational energy, which permits to better understand the al- gorithm behavior in terms of important image features as tone dispersion or contrast.

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Furthermore, by comparing different energy functionals, one can discover interrelations between models that can be very difficult to be found without a variational formulation. Before providing the basic details, let us fix the notation. Given a discrete RGB image, we denote by $\mathfrak{I}=\{1,\dots,W\}\times\{1,\dots,H\}\subset\mathbb{Z}^2$ its spatial domain, $W, H \geq 1$ being integers; $x=(x1, x2)$ and $y=(y1, y2)$ denote the coordinates of two arbitrary pixels in I. Let us consider a normalized dynamic range in [0, 1], and denote a color image function by $\vec{I}:\mathfrak{I}\to[0,1]^3$, $\vec{I}(x)=(I_R(x),I_G(x),I_B(x))$, where $I_c(x)$ is the intensity level of the pixel $x \in$ I in the chromatic channel $c \in \{R, G, B\}$. All computations will be performed on the scalar components of the image, thus treating independently each channel, written, for simplicity, as $I(x)$.

2. GENERAL FUNCTIONAL ENERGY FOR PERCEPTUAL COLOR CORRECTION

We detail here the general functional energy that corresponds to a perceptually-inspired color correction algorithm. In order to understand its meaning, let us remember that human color perception is characterized by *both local and global features*: contrast enhancement has a local nature, i.e. *spatially variant*, while visual adaptation is global.

These basic considerations imply that a perceptually inspired energy functional should be composed by two terms: one spatially-dependent contrast term $C_w(I)$, whose minimization must leads to a local contrast enhancement coherent with Weber-Fechner's law of contrast perception, and one global dispersion term $D(I)$, whose minimization must lead to a control of the departure from both original point- wise values and the middle gray, which, in our normalized dynamic range, is 1/2.

It can be proven that the most general energy functional $E^{\varphi}_{w,\alpha,\beta}(I)=C^{\varphi}_w(I)+D_{\alpha,\beta}(I)$ is composed by these terms

$$C^{\varphi}_w(I) := \frac{1}{4}\iint_{\mathfrak{I}} w(x,y)\,\varphi\left(\frac{\min(I(x),I(y))}{\max(I(x),I(y))}\right)dxdy, \quad (1)$$

the functional parameter φ being a strictly increasing positive function, and

$$\begin{aligned}D_{\alpha,\beta}(I) &= \alpha\int_{\mathfrak{I}}\left(\frac{1}{2}\log\frac{1}{2I(x)}-\left(\frac{1}{2}-I(x)\right)\right)dx\\ &+\beta\int_{\mathfrak{I}}\left(I_0(x)\log\frac{I_0(x)}{I(x)}-(I_0(x)-I(x))\right)dx.\end{aligned} \quad (2)$$

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Let us discuss the meaning of $C_{w\varphi}(I)$ and $D_{\alpha,\beta}(I)$, starting with the contrast term. Notice that the ratio defined by *min(I(x), I(y))/ max(I(x), I(y))* is minimized when the minimum decreases and the maximum increases, which of course corresponds to a contrast stretching. Thus, minimizing an increasing function of that ratio will produce a contrast enhancement. Moreover, recalling that a global illuminant change can be represented as the transformation $I(x) \mapsto \lambda I(x),\ \lambda > 0$, the contrast enhancement term will be unaffected by such a change, coherently with the color constancy property. Finally, it can be proven that this definition of contrast is coherent with Weber-Fechner's law.

The dispersion term is based on the relative entropy distance [1] between *I* and *1/2* (first term) and between I0 and I (second term). Given the statistical interpretation of entropy, we can say *that minimizing $D_{\alpha,\beta}(I)$ amounts to minimizing the disorder of intensity levels around 1/2 and around the original data $I_0(x)$.* Thus, $D_{\alpha,\beta}(I)$ accomplishes the required tasks of a dispersion term.

By minimizing the energy $E_{w,\alpha,\beta}$, e.g. through a gradient *descent technique*, we have the explicit algorithm implementation of this model:

$$\partial_t \log I = -\delta E^{\varphi}_{w,\alpha,\beta}(I), \quad (3)$$

t being the evolution parameter and δ the first variation of *E*. For practical implementations this scheme must be discretized: choosing a finite evolution step $\Delta t > 0$ and setting $I^k(x) = I_{k\Delta t}(x),\ k \in \mathbb{N}$, with $I^0(x)$ being the original image, we have

$$I^{k+1}(x) = \frac{I^k(x) + \Delta t\left(\frac{\alpha}{2} + \beta I^0(x) + \frac{1}{2}R^{\varphi}_{w,I_k}(x)\right)}{1 + \Delta t(\alpha + \beta)}. \quad (4)$$

As can be seen, α and β represent the strength of the attachment to ½ and to I^0, respectively, while $R^{\varphi}_{w,Ik}$ represents the contrast enhancement.

3. INTERRELATIONS BETWEEN PERCEPTUAL COLOR CORRECTION MODELS

By selecting particular functional forms for the function φ, we can embed in the previous framework two well known perceptually-inspired color correction algorithms, more precisely if we set: φ ≡ id, i.e. the identity function, then the previous model coincides with a symmetrized version of the Retinex algorithm [2, 4, 5].
If φ is chosen to be the natural logarithm: φ ≡ log, then the previous model coincides with that presented in [3].
This shows that the difference between the model presented in [3] and the symmetric version of the original Retinex algorithm relies only in the shape of the function φ applied to the contrast variable. Such a result was not at all obvious without a variational formulation.

Finally, it was proven in [6] that histogram equalization can be seen as the minimization of a functional composed by a global contrast term and a quadratic dispersion term, so our model can be seen as an improvement of histogram equalization, where contrast is modified locally and with a more sound perceptual basis and dispersion is controlled with an entropic function instead of a quadratic one. This last property is important when dealing with dark image areas, where the entropic dispersion is much stronger than the quadratic one, thus providing a better noise control in the enhancement process. To show this effect let us consider in Figure 1 a result of our model (with φ ≡ id) compared with that obtained by histogram equalization.

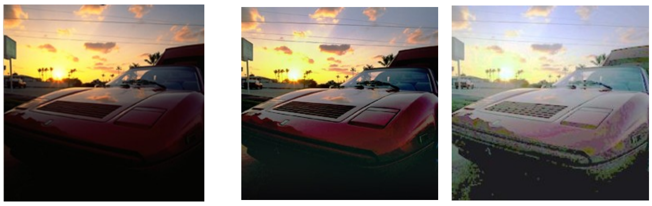

Figure 1: *Left*: Original image. *Middle*: Image filtered with the proposed model. *Right*: Image after histogram equalization.

4. CONCLUSION

We have discussed a variational framework in which color correction is realized through the minimization of energy functionals suitably designed in order to comply with a set of phenomenological properties of the human visual system. This framework embeds several known color perception models and permits to devise new ones by changing a functional parameter.

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