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τ*- GENERALIZED SEMICLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT
In this paper, we introduce a new class of sets called τ^*-generalized semiclosed sets and τ^*-generalized semiopen sets in topological spaces and study some of their properties.

KEYWORDS: $\tau^* - gs\ closed$ set, $\tau^* - gs\ open$ set

INTRODUCTION
In 1970, Levine[6] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham[5] introduced the concept of the closure operator c1* and a new topology τ^* and studied some of their properties. S.P.Arya[2], P.Bhattacharyya and B.K.Lahiri[3], J.Dontchev[4], H.Maki, R.Devi and K.Balachandran[9], [10], P.Sundaram and A.Pushpalatha[12], A.S.Mashhour, M.E.Abd E1-Monsof and S.N.E1-Deeb[11], D.Andrijevic[1] and S.M.Maheshwari and P.C.Jain[9], Ivan Reilly [13], A.Pushpalatha, S.Eswaran and P.RajaRubi [14] introduced and investigated generalized semi closed sets, semi generalized closed sets, generalized semi preclosed sets, α −generalized closed sets, generalized-α closed sets, strongly generalized closed sets, preclosed sets, semi-preclosed sets and α −closed sets, generalized preclosed sets and τ^*-generalized closed sets respectively. In this paper, we obtain a new generalization of preclosed sets in the weaker topological space(X, τ^*).

Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, int(A), $cl(A)$, cl^*(A), $scl(A), spcl(A), cl_\alpha(A)$, $cl_p(A)$ and A^c denote the interior, closure, $closure^*$, semi-closure, semi-preclosure, α-closure, preclosure and complement of A respectively.

PRELIMINARIES
We recall the following definitions

Definition: 2.1

A subset A of a topological space (X , τ) is called

(i) Generalized closed (briefly g-closed)[6] if cl(A)⊆G whenever A⊆G and G is open in X

(ii) Semi-generalized closed(briefly sg-closed)[3] if scl(A)⊆G whenever A⊆G and G is semi open in X.

(iii) Generalized semi-closed (briefly gs-closed)[2] if scl(A)⊆G whenever A⊆G and G is open in X.

(iv) α- closed[8] if cl(int(cl(A))) ⊆ A

(v) α-generalized closed (briefly αg-closed)[9] if cl_α(A) ⊆ G whenever A⊆G and G is open in X.

(vi) Generalized α-closed (briefly gα-closed)[10] if spcl(A) ⊆ G whenever A⊆G and G is open in X.

(vii) Generalized semi-preclosed (briefly gsp-closed)[2] if scl(A) ⊆ G whenever A ⊆ G and G is open in X.

(viii) Strongly generalized closed (briefly strongly g-closed)[12] if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is g-open in X.

(ix) Preclosed[11] if $cl(int(A)) \subseteq A$

(x) Semi-closed[7] if $int(cl(A)) \subseteq A$

(xi) Semi-preclosed (briefly sp-closed)[1] if $int(cl(int(A))) \subseteq A$.

(xii) Generalized preclosed (briefly gp-closed)[13] if $cl_p A \subseteq G$ whenever $A \subseteq G$ and G is open.

The complements of the above mentioned sets are called their respective open sets.

Definition: 2.2

For the subset A of a topological X, the generalized closure operator cl^*[5] is defined by the intersection of all g-closed sets containing A.

Definition: 2.3

For the subset A of a topological X, the topology τ^* is defined by $\tau^* = \{G: cl^*(G^c) = G^c\}$.

Definition: 2.4

For the subset A of a topological X,

(i) the semi-closure of A$\left(briefly\ scl(A)\right)$[7] is defined as the intersection of all semi-closed sets containing A.

(ii) the semi-Pre closure of A$\left(briefly\ spcl(A)\right)$[1] is defined as the intersection of all semi-preclosed sets containing A.

(iii) the $\alpha - closure$ of A $\left(briefly\ cl_\alpha(A)\right)$[8] is defined as the intersection of all $\alpha - closed$ sets containing A.

(iv) the preclosure of A, denoted by $cl_p(A)$[13], is the smallest preclosed set containing A.

Definition: 2.5

A subset A of a topological space X is called τ^* generalized closed set $(briefly\ \tau^* - gclosed)$[14] if $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* - open$. The complement of $\tau^* - generalized$ closed set is called the $\tau^* - generalized$ open set $(briefly\ \tau^* - gopen)$.

Definition: 2.6

A subset A of a topological space X is called $\tau^* - generalized$ preclosed $(briefly\ \tau^* - gp - closed)$[15] if $cl^*(cl_p(A)) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* - generalized$ open. The complement of $\tau^* - generalized$ preclosed set is called the $\tau^* - generalized$ preopen set (briefly $\tau^* - gp - open$).

$\tau^* - Generalized$ SEMICLOSED SETS IN TOPOLOGICAL SPACES

In this section, we introduce the concept of τ^*- generalized semiclosed sets in topological spaces.

Definition: 3.1

A subset A of a topological space X is called $\tau^* - generalized$ semiclosed $(briefly\ \tau^* - gsclosed)$ if $cl^*(scl(A)) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* -$ open. The complement of $\tau^* - generalized$ semiclosed set is called the $\tau^* - generalized$ semiopen set (briefly $\tau^* - gsopen$).

Example: 3.2

Let $X = \{a, b, c\}$ and let $\tau = \{\phi, X, \{a\}, \{a, b\}, \{c\}, \{a, c\}\}$. Here (X, τ^*) is τ^*-generalized semiclosed

Theorem: 3.3

Every closed set in X is $\tau^* - gsclosed$.

Proof:

Let A be a closed set. Let $A \subseteq G$. Since A is closed, $cl(A) = A \subseteq G$. But $cl^*(scl(A)) \subseteq cl(A)$. Thus, we have $cl^*(scl(A)) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* - open$. Therefore A is $\tau^* - gsclosed$.

Theorem: 3.4

Every $\tau^* - closed$ set in X is $\tau^* - gsclosed$.

Proof:

Let A be a $\tau^* - closed$ set. Let $A \subseteq G$ where G is $\tau^* - open$ Since A is $\tau^* - closed$, $cl^*(scl(A)) = A \subseteq G$. Thus, we have $cl^*(scl(A)) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* - open$. Therefore A is $\tau^* - gs$ closed.

Theorem: 3.5

Every g-closed set in X is a τ^*–gsclosed set but not conversely.

Proof:

Let A be a g-closed set. Assume that $A \subseteq G$, G is $\tau^* - open$ in X. Then $cl(A) \subseteq G$, Since A is g-closed. But $cl^*(scl(A)) \subseteq cl(A)$. Therefore $cl^*(scl(A)) \subseteq G$. Hence A is $\tau^* - gsclosed$.

The converse of the above theorem need not be true as seen from the following example.

Example: 3.6

Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{a, b\}, \{c\}, \{a, c\}\}$. Then, the set {a,c} is $\tau^* - gp$-closed but not g-closed.

Remark: 3.7

The following example shows that $\tau^* - gp - closed$ sets are independent from sp-closed, sg-closed set, $\alpha - closed\ set$, preclosed set, gs-closed set, gsp-closed set, $\alpha g - closed$ set and g$\alpha - closed$ set.

Example: 3.8

Let $X = \{a, b, c\}$ and $Y = \{a, b, c, d\}$ be the topological spaces.

(i) Consider the topology $\tau = \{X, \phi, \{a\}\}$. Then the sets $\{a\}, \{a, b\}\ and\ \{a, c\}$ are $\tau^* - gs - closed$ but not sp-closed.

(ii) Consider the topology $\tau = \{X, \phi, \{a, b\}\}$. Then the sets $\{a\}$ and $\{b\}$ are sp-closed but not $\tau^* - gs - closed$.

(iii) Consider the topology $\tau = \{X, \phi\}$. Then the sets $\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\} and\ \{a, c\}$ are $\tau^* - gs - closed$ but not sg-closed.

(iv) Consider the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then the sets $\{a\} and\ \{b\}$ are sg-closed but not $\tau^* - gs - closed$.

(v) Consider the topology $\tau = \{X, \phi, \{a\}\}$. Then the sets $\{a\}, \{b\}, \{c\}, \{a, b\} and\ \{a, c\}$ are $\tau^* - gs - closed$ but not $\alpha - closed$.

(vi) Consider the topology $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. Then the set $\{b\}$ is $\alpha - closed$ but not $\tau^* - gs - closed$.

(vii) Consider the topology $\tau = \{X, \phi, \{a\}\}$. Then the sets $\{a\}\ \{a, b\}\ and\ \{a, c\}$ are $\tau^* - gs - closed$ but not preclosed.

(viii) Consider the topology $\tau = \{X, \phi, \{b\}, \{a, b\}\}$. Then the set $\{a\}$ is pre-closed but not $\tau^* - gs - closed$.

(ix) Consider the topology $\tau = \{X, \phi\}$. Then the sets $\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\ and\ \{a, c\}$ are $\tau^* - gs - closed$ but not gs-closed.

(x) Consider the topology $\tau = \{Y, \phi, \{a\}, \{a, b, c\}, \{a, b, d\}\}$. Then the sets $\{b\}\ \{b, c\}\ and\ \{b, d\}$ are $gs - closed$ but not $\tau^* - gs$ –closed.

(xi) Consider the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then the sets $\{b\} and\ \{a, b\}$ are $gsp - closed$ but not $\tau^* - gs$ –closed.

(xii) Consider the topology $\tau = \{Y, \phi, \{a\}\}$. Then the set $\{a\}$ is $\tau^* - gs - closed$ but not gsp-closed.

(xiii) Consider the topology $\tau = \{X, \phi, \{a\}\}$. Then the set $\{a\}$ is $\tau^* - gs - closed$ but not αg-closed.

(xiv)Consider the topology $\tau = \{Y, \phi, \{a\}, \{a, b, c\}, \{a, b, d\}\}$. Then the set $\{b\}, \{b, c\}, \{b, d\}\ are\ \alpha g - closed$ but not $\tau^* - gs - closed$.

(xv) Consider the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Then the set $\{b\}$ is $\tau^* - gs - closed$ but not g$\alpha - closed$.

(xvi)Consider the topology $\tau = \{Y, \phi, \{a\}, \{a, b, c\}, \{a, b, d\}\}$. Then the set $\{b\}, \{b, c\} and\ \{b, d\}$ are $g\alpha - closed$ but not $\tau^* - gs - closed$.

Theorem: 3.9

For any two sets A and B $cl^*(scl(A \cup B)) = cl^*(scl(A)) \cup cl^*(scl(B))$

Proof:

Since $A \subseteq A \cup B$, we have $cl^*(scl(A)) \subseteq cl^*(scl(A \cup B))$ and Since $B \subseteq A \cup B$, we have $cl^*(scl(B)) \subseteq cl^*(scl(A \cup B))$. Therefore $cl^*(scl(A)) \cup cl^*(scl(B)) \subseteq cl^*(scl(A \cup B))$. Also, $cl^*(scl(A))$ and $cl^*(scl(B))$ are the closed sets. Therefore $cl^*(scl(A)) \cup cl^*(scl(B))$ is also a closed set. Again, $A \subseteq cl^*(scl(A))$ and $B \subseteq cl^*(scl(B))$, Implies $\cup B \subseteq cl^*(scl(A)) \cup cl^*(scl(B))$. Thus, $cl^*(scl(A)) \cup cl^*(scl(B))$ is a closed set containing $A \cup B$. Since $cl^*(scl(A \cup B))$ is the smallest closed set containing $A \cup B$. We have $cl^*(scl(A \cup B)) \subseteq cl^*(scl(A)) \cup cl^*(scl(B))$. Thus, $cl^*(scl(A \cup B)) = cl^*(scl(A)) \cup cl^*(scl(B))$

Theorem: 3.10

Union of two $\tau^* - gsclosed$ sets in X is a $\tau^* - gsclosed\ set\ in\ X$.

Proof:

Let A and B be two $\tau^* - gsclosed\ sets$. Let $A \cup B \subseteq G$, where G is $\tau^* - open$. Since A and B are $\tau^* - gs - closed\ sets$, $cl^*(scl(A)) \cup cl^*(scl(B)) \subseteq G$. But by theorem 3.9 $cl^*(scl(A)) \cup cl^*(scl(B)) = cl^*(scl(A \cup B))$. Therefore $cl^*(scl(A \cup B)) \subseteq G$. Hence $A \cup B$ is a $\tau^* - gsclosed$ set.

Theorem: 3.11

A subset A of X is $\tau^* - gsclosed$ if and only if $cl^*(scl(A)) - A$ contains no non-empty $\tau^* - closed$ set in X.

Proof:

Let A be a $\tau^* - gsclosed$ set. Suppose that F is a non-empty $\tau^* - closed$ subset of $cl^*(scl(A)) - A$. Now, $F \subseteq cl^*(scl(A)) - A$. Then $F \subseteq cl^*(scl(A)) \cap A^c$, Since $cl^*(scl(A)) - A = cl^*(scl(A)) \cap A^c$. Therefore $F \subseteq cl^*(scl(A))$ and $F \subseteq A^c$. Since F^c is a τ^*-open set and A is a $\tau^* - gsclosed$, $cl^*(scl(A)) \subseteq F^c$. That is $F \subseteq cl^*(scl(A)) \cap [cl^*(scl(A))]^c = \phi$. That is $F = \phi$, a contradiction. Thus, $cl^*(scl(A)) - A$ contain no non-empty $\tau^* - closed$ set in X. Conversely, assume that $cl^*(scl(A)) - A$ contains no non-empty $\tau^* - closed$ set. Let $A \subseteq G$, G is $\tau^* - open$. Suppose that $cl^*(scl(A))$ is not contained in G. then $cl^*(scl(A)) \cap G^c$ is a non-empty $\tau^* - closed$ set of $cl^*(scl(A)) - A$ which is a contradiction. Therefore, $cl^*(scl(A)) - A \subseteq G$ and hence A is $\tau^* - gsclosed$.

Corollary: 3.12

A subset A of X is $\tau^* - gsclosed$ if and only $cl^*(scl(A)) - A$ contains no non-empty closed set in X.

Proof:

The proof follows from the theorem 3.11 and the fact that every closed set is $\tau^* - closed$ set in X.

Corollary: 3.13

A subset A of X is $\tau^* - gs - closed$ if and only if $cl^*(scl(A)) - A$ contains no non-empty open set in X.

Proof:

The proof follows from the theorem 3.11 and the fact that every open set is $\tau^* - open$ set in X.

Theorem: 3.14

If a subset A of X is $\tau^* - gsclosed$ and $A \subseteq B \subseteq cl^*(scl(A))$, then B is $\tau^* - gsclosed$ set in X.

Proof:

Let A be a $\tau^* - gsclosed$ set such that $A \subseteq B \subseteq cl^*(scl(A))$. Let U be a $\tau^* - open$ set of X such that $B \subseteq U$. Since A is $\tau^* - gsclosed$, we have $cl^*(scl(A)) \subseteq U$.

Now, $cl^*(scl(A)) \subseteq cl^*(scl(B)) \subseteq cl^*(cl^*(scl(A))) = cl^*(scl(A)) \subseteq U$.

That is $cl^*(scl(B)) \subseteq U$, U is $\tau^* - open$.

Therefore B is $\tau^* - gsclosed$ set in X.

The converse of the above theorem need not be true as seen from the following example.

Example: 3.15

Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. Then A and B are $\tau^* - gsclosed$ sets in (X, τ). But $A \subseteq B$ is not a subset of $cl^*(scl(A))$.

Theorem: 3.16

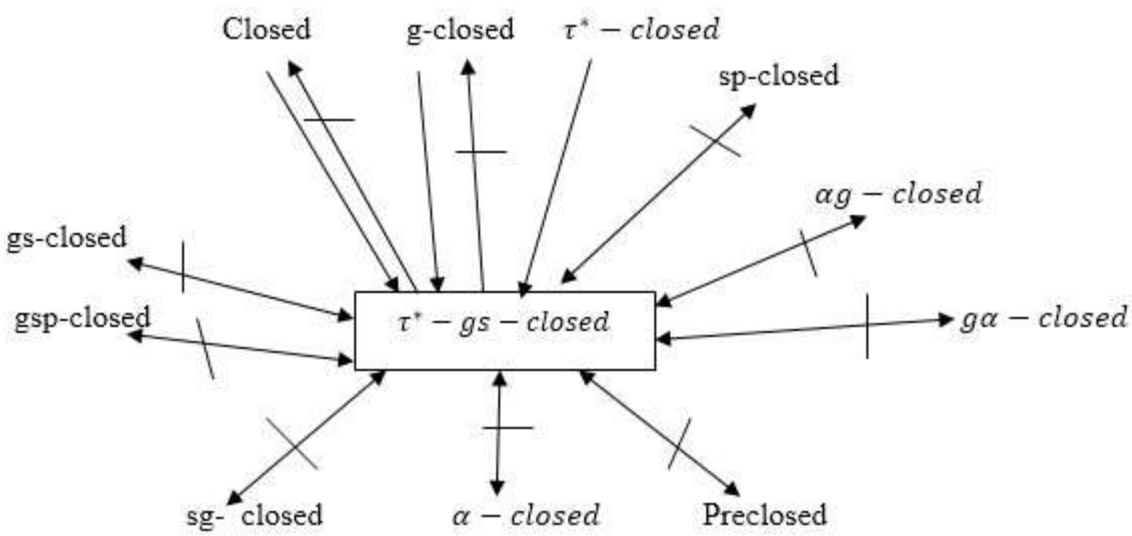
Let A be a $\tau^* - gsclosed$ in (X, τ). Then A is $g - closed$ if and only if $cl^*(scl(A)) - A$ is $\tau^* - open$.

Proof:

Suppose A is g-closed in X. Then, $cl^*(scl(A)) = A$ and so $cl^*(scl(A)) - A = \phi$ which is $\tau^* - open$ in X. Conversely, suppose $cl^*(scl(A)) - A$ is $\tau^* - open$ in X. Since A is $\tau^* - gsclosed$, by the theorem 3.11, $cl^*(scl(A)) - A$ contains no non-empty $\tau^* - closed$ set in X. Then, $cl^*(scl(A)) - A = \phi$. Hence, A is g-closed.

Remark 3.17

From the above discussion, we obtain the following implications.



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