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Applications of Multifrequency EIT to Respiratory Control

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Introduction

What is the problem we are trying to solve?

Our focus is on the use of EIT in respiratory intensive care.

In this setting we often focus on how EIT can be interpreted to help clinicians.

It should also be a priority to adapt EIT to give clinicians familiar and requested information such as:

- ventilation distributions,
- parameters for ventilation models (compliance, resistance),
- procedures to integrate EIT with ventilator control.

Approach

Single Frequency/Conductivity

Coupling reconstructed conductivity maps from single frequency EIT to linear compartmental models of lung function gives possible methods for generating this information.

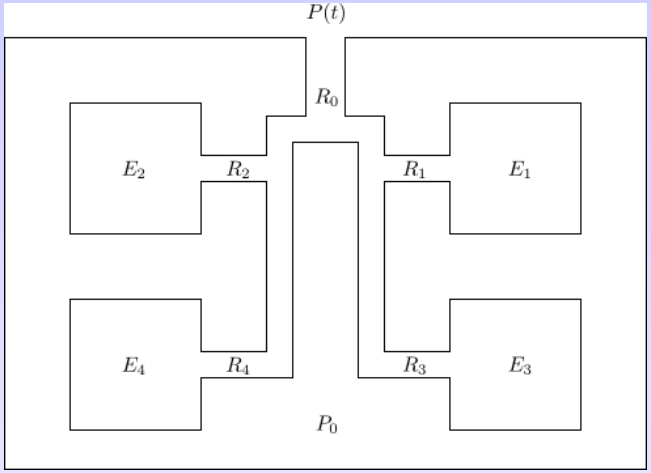
Multiple Frequency/Admittance

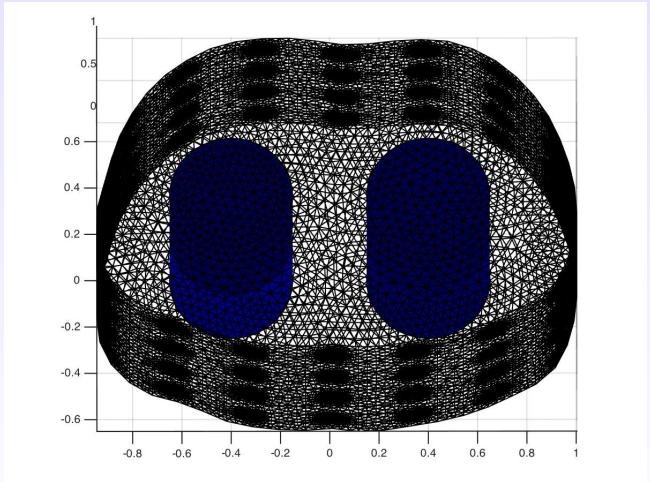
Reconstructing the full complex admittance map at multiple frequencies offers:

- improved accuracy of lung **homogenisation** models,
- further contrasts of different tissues,
- possible identification of blood and perfusion levels.

Model

Our current approach is to couple EIT reconstructions with a linear compartmental ordinary differential equation (ODE) model of lung function.





The parameters in the model are

- Compartmental Elastance (E_i)
- Compartmental Resistance (R_i)
- Tracheal Resistance (R_0)

These are combined into a matrix ODE of the form

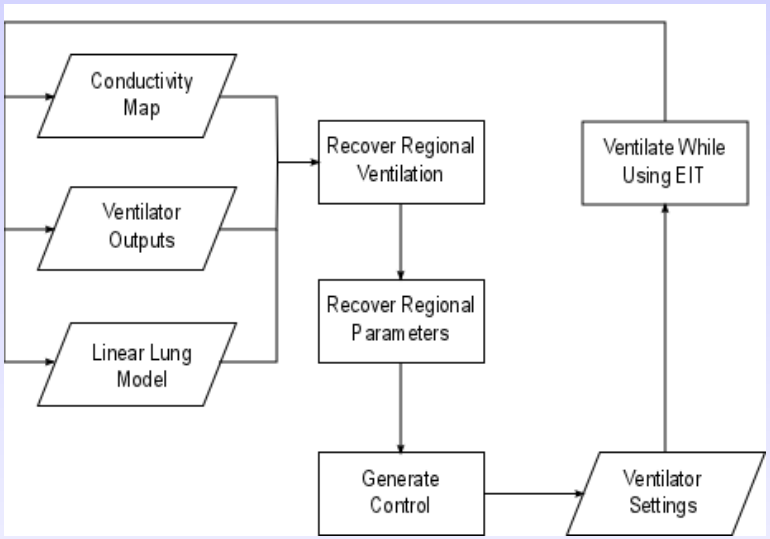
$$E\mathbf{x}(t) + R\dot{\mathbf{x}}(t) = \mathbf{p}(t)$$

where:

- $\mathbf{x}(t)$ - compartmental volumes at time t
- $\dot{\mathbf{x}}(t)$ - compartmental flows at time t
- $\mathbf{b}$ - vector of ones
- $p(t)$ - applied pressure at time t

Procedures

We have used this model to design a possible workflow incorporating time series values of conductivity from EIT into mechanical ventilation, giving the workflow shown below.



1. Recover regional ventilation - numerically differentiate time series values and fit to flow at ventilator.
2. Recover parameters - formulate ODE as single matrix multiplication and invert for parameters.
3. Generate control - use analytic formula to optimise a given pressure profile in terms of H^1 semi-norm.

Knowing further regional information about the patient can improve the accuracy of the first two steps and help improve the **control function** we generate.

Admittance

EIT reconstructions **homogenise** the electrical properties of all the tissues present in a reconstructed element.

Current practice is to relate changes in conductivity to air content, but finding other properties requires more measurements.

Other components of **admittance** could give the required data to fit more quantities of interest.

Relative permittivity values of tissues [1]

Tissues	1 kHz	10 kHz
Liver	1.2×10^5	5.5×10^4
Lungs	8.5×10^4	2.5×10^4
Fat	5.0×10^4	2.0×10^4
Blood*	3.4×10^3	3.4×10^3

Different tissue types show:

- contrast in permittivity values
- different dispersion relations under changing frequency

*Blood values estimated numerically [2]

Multiple Frequencies

Of particular interest is the contrast and dispersion behaviour of blood under different frequencies.

The dispersion relations for blood have been studied under many different situations

$$\epsilon_r = \frac{3 \times 10^{-8}}{\epsilon_0(1 + (\omega \cdot 1.3 \times 10^{-7})^2)}$$

Relative permittivity approximation used by Nopp [2]

- dispersion relations change in vitro over time
- dispersion relations change for levels of coagulation
- largest changes in dispersion shown above 300kHz

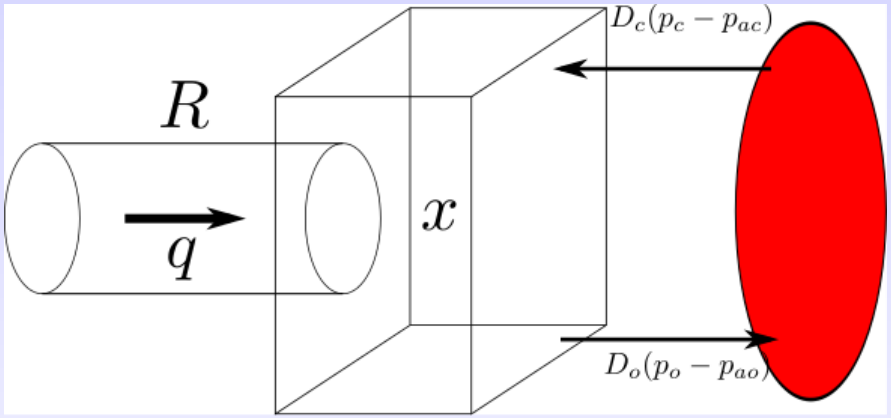
The ability to find regional blood flow or perfusion has applications for improving accuracy of lung models and directing efforts to optimise ventilation control.

Air Flow Parameters

Blood content could also be useful to refine models for recovery and use of parameters.

Current model applies airway resistance to the time derivative of compartmental volume.

This model can be extended to include gas transfer and transport [3].



- q - flux through airways
- D - diffusion coefficients
- p_o, p_c - partial pressures of gases
- p_{ao}, p_{ac} - partial pressures of blood gases

$$\dot{\mathbf{x}} = q - D_o(p_o - p_{ao}) - D_c(p_c - p_{ac})$$

The airway flux q is the only part of this which is effected by airway resistance. More accurate parameter recovery and control requires regional blood flow rates and gas saturation.

Ventilation and Perfusion

The rate at which blood is supplied to a region of lung, or perfusion, can also be combined with ventilation rates to give a weighting for lung control.

Defining a ventilation perfusion V/Q ratio can help quantify regional respiratory dead space and over distension.

- Low V/Q - low rates of oxygen transfer to blood.
- Average V/Q - balanced
- High V/Q - increased risks of overdistension with diminishing returns for gas transfer.

This could be a good way to weight control optimisation.

Methods for control

A system of ODEs $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$ is **controllable** if it can be steered from any state $\mathbf{x_0}$ to any other state $\mathbf{x_T}$ in time T by a control $\mathbf{u}$ [4].

Our system of ODEs is controllable when lung compartments have sufficiently different parameters.

The control

$$u(s) = -B^T \exp\{(T-s)A\} Q_T^{-1} (\exp\{TA\}\mathbf{x_0} - \mathbf{x_T})$$

is the unique control taking $\mathbf{x_0}$ to $\mathbf{x_T}$ in time T which has minimal 2-norm [4].

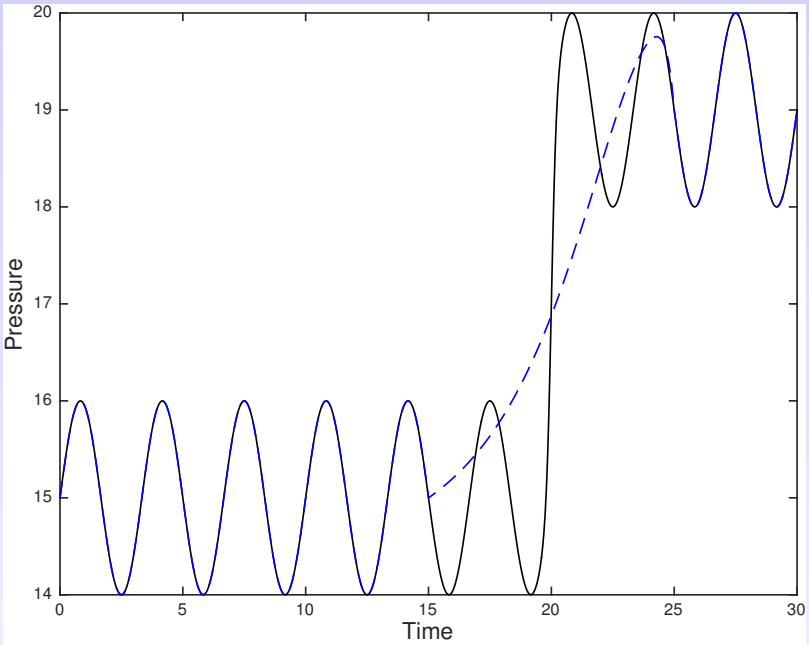
Reformulating our ODE system as

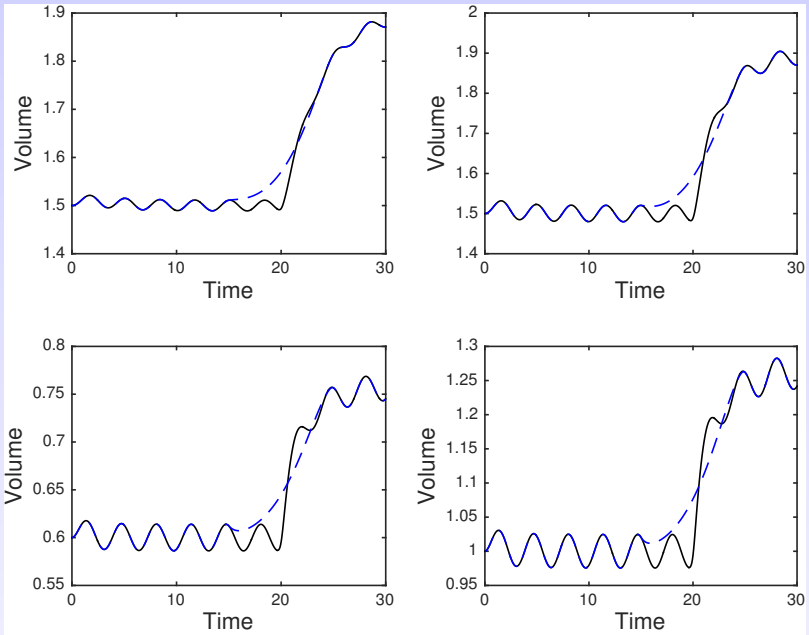
$$\begin{pmatrix} \dot{\mathbf{x}} \\ \dot{p} \end{pmatrix} = \begin{bmatrix} -R^{-1}E & R^{-1}\mathbf{b} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \mathbf{x} \\ p \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

we can find the control pressure taking $\mathbf{x_0}$ to $\mathbf{x_T}$ with minimized H^1 seminorm

Control Demonstration

Using this technique to refine existing pressure controls gives the following results





[VIDEO]

Optimising Target State

The formula for an optimal control pressure derivative can be rewritten as the matrix equation

$$\mathbf{u} = M\left[\begin{pmatrix} \mathbf{x}_T \\ p_T \end{pmatrix} - \exp\left\{T\begin{bmatrix} -R^{-1}E & R^{-1}\mathbf{b} \\ 0 & 0 \end{bmatrix}\right\}\begin{pmatrix} \mathbf{x}_0 \\ p_0 \end{pmatrix}\right].$$

By defining a diagonal matrix of perfusion states, Q, and a vector of ideal ventilation perfusion ratios, $\mathbf{r}$, we can formulate an optimisation

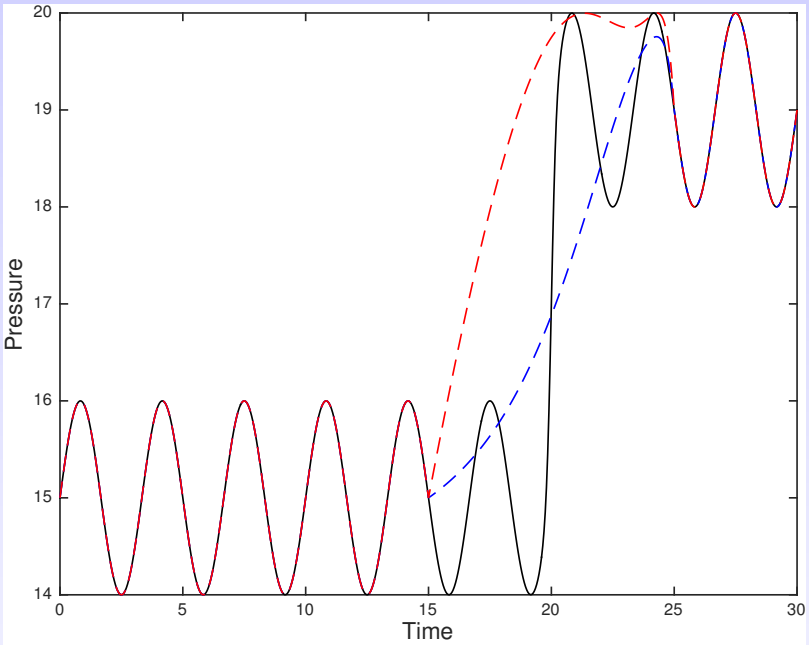
$$\mathbf{x}_T = \arg\min_{\mathbf{x}} \left[\mathbf{x}^* Q^{-2}\mathbf{x} - 2\mathbf{r}^* Q^{-1}\mathbf{x}\right],$$

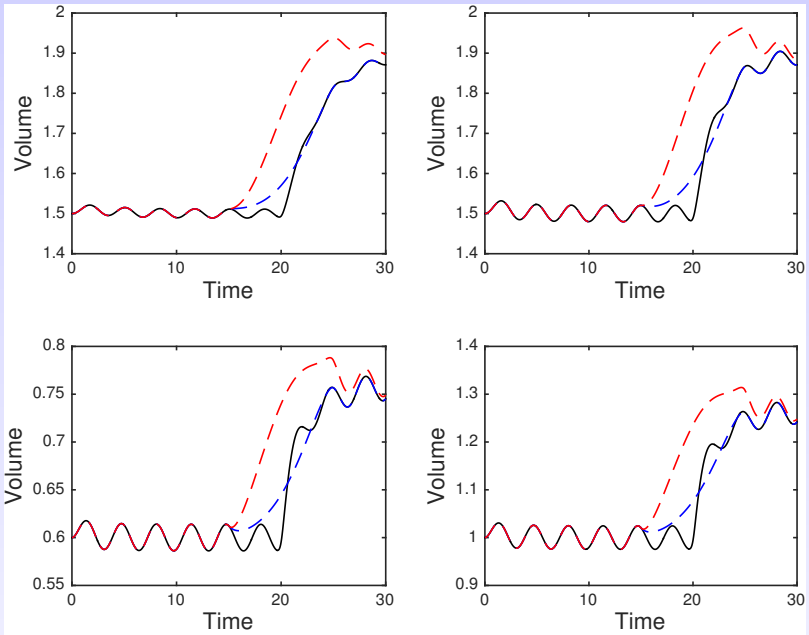
$$\begin{bmatrix} \tilde{M} \\ -\tilde{M} \end{bmatrix}\mathbf{x} \leq \begin{pmatrix} \mathbf{p}_{\max} \\ -\mathbf{p}_{\min} \end{pmatrix} + \mathbf{f}(\mathbf{x}_0, p_0, p_T, T),$$

- $\tilde{M}$ - matrix M adjusted for time integration
- $\mathbf{p}_{\max}$ - maximum allowable pressure
- $\mathbf{p}_{\min}$ - minimum allowable pressure
- $\mathbf{f}$ - adjustment for initial conditions

Optimisation Demonstration

Using this technique to optimise the target inflation state gives the following results





[VIDEO]

Thank you!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Prioritised Optimisation

We may prefer to optimise around recruitment levels instead. However, when the lungs are uniformly under recruited it would be preferable for highly perfused areas to be prioritised. Hence the cost functional should probably be of the form

$$\alpha \| Q^{-1}\mathbf{x} - \mathbf{r}_1 \|^2 + \beta \| V^{-1}\mathbf{x} - \mathbf{r}_2 \|^2$$

- α - priority of perfusion ratio
- β - priority of recruitment
- V - diagonal matrix of compartment volumes

Breath to Breath Optimisation

Using these techniques to optimise the target inflation states after each breath gives the following results