Textbook of Physics – Simplified

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for 12th Standard

by
Dr. P. S. Aithal
M.Sc. (Physics), Ph.D. (Physics),
Post Doc. in Quantum Optics, Post Doc. in Photonics
&
Prof. Shubhrajyotsna Aithal
M.Sc. (Material Science), M.Phil. (Chemistry), (Ph.D.)

Srinivas Group of Institutions
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Chapter 1
ELECTRIC CHARGES AND FIELDS
8M [1M - 1Q; 2M - 1Q; 5M - 1Q (NP)/(LA)]

1.1 Electric charges and their properties: Additivity of charges, quantisation of charges and conservation of charges -

There are two types of charges, namely positive and negative and their effects tend to cancel each other. Smallest electric charge is charge of electron e = 1.6 x 10^{-19} coulombs. If the sizes of charged bodies are very small as compared to the distances between them, we treat them as *point charges*. All the charge content of the body is assumed to be concentrated at one point in space.

Properties of Electric charges :

(1) Type of charges :

Only two types of charges exist namely positive and negative.

(2) Nature of charges :

(i) *like charges repel* and (ii) *unlike charges attract* each other.

(3) Additivity of charges :

If a system contains two point charges q_1 and q_2, the total charge of the system is obtained simply by adding algebraically q_1 and q_2 , i.e., charges add up like real numbers or they are scalars like the mass of a body. If a system contains n charges q_1, q_2, q_3, ..., q_n, then the total charge of the system is $q_1 + q_2 + q_3 + ... + q_n$. Charge has magnitude but no direction, similar to the mass. However, there is one difference between mass and charge. Mass of a body is always positive whereas a charge can be either positive or negative. Ex : +2C + 5C - 3C = +4C

(4) Conservation of charges :

Within an isolated system consisting of many charged bodies, due to interactions among the bodies, charges may get redistributed but it is found that *the total charge of the isolated system is always conserved*.

It is not possible to create or destroy net charge carried by any isolated system although the charge carrying particles may be created or destroyed in a process. Sometimes nature creates charged particles: a neutron turns into a proton and an electron. The proton and electron thus created have equal and opposite charges and the total charge is zero before and after the creation.

(5) Quantisation of charge :

The electric charge exists in discrete packets and is always an integral multiple of *e. i.e.,* $q = \pm ne$; where n = 1, 2,3, 4, ---- and e =1.6 x 10^{-19} coulombs is termed as *quantisation of charge*.

(6) Scalar quantity :

Electric charge is scalar quantity.

1.2 Coulomb's law: Statement, explanation (only in free space) and expression in vector form -

Coulomb measured the force between two point charges and found that *it varied inversely as the square of the distance between the charges and was directly proportional to the product of the magnitude of the two charges and acted along the line joining the two charges*. Thus, if two point charges q_1, q_2 are separated by a distance r in vacuum, the magnitude of the force (F) between them is given by

$$F = k\frac{|q_1 \ q_2|}{r^2} \qquad \text{----------} \quad (1.1)$$

Putting this value of k in Eq. (1.1), we see that for $q_1 = q_2 = 1$ C, $r = 1$ m, $F = 9 \times 10^9$ N. That is, 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude *in vacuum* experiences an electrical force of repulsion of magnitude 9×10^9 N. One coulomb is evidently too big unit to be used. In practice, in electrostatics, one uses smaller units like 1 mC or 1 μC.

The constant k in Eq. (1.1) is usually put as $k = 1/4\pi\varepsilon_0$ for later convenience, so that Coulomb's law is written as

$$F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2} \quad \text{--------} \quad (1.2)$$

ε_0 is called the *permittivity of free space* . The value of ε_0 in SI units is $\varepsilon_0 = 8.854 \times 10^{-12}$ $C^2N^{-1}m^{-2}$.

Coulomb's Law in Vector form :

Coulomb's force law between two point charges q_1 and q_2 located at r_1 and r_2 is then expressed as

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_o} \frac{q_1 \; q_2}{r_{21}^2} \hat{\mathbf{r}}_{21}$$

The force F_{12} on charge q_1 due to charge q_2, is

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 \; q_2}{r_{12}^2} \hat{\mathbf{r}}_{12} = -\mathbf{F}_{21}$$

1.3 Definition of SI unit of charge -

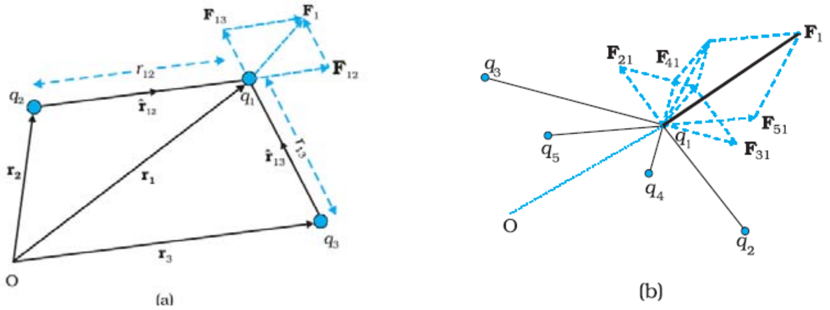
The SI unit of charge is Coulomb (C).

One coulomb of charge is that charge which when placed at rest in vacuum at a distance of one metre from an equal and similar stationary charge repels with a force of 9×10^9 N. One coulomb is evidently too big a unit to be used. In practice, in electrostatics, one uses smaller units like 1 mC or 1 μC. In CGS system, the unit of charge is *statcoulomb* or *esu of charge*.

Electric force between multiple charges : Principle of Superposition

Experimentally it is verified that *force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges*. This is termed as the ***principle of superposition***.

Consider a system of three charges q_1, q_2 and q_3, as shown in Fig (a).



The force on one charge, say q_1, due to two other charges q_2, q_3 can therefore be obtained by performing a vector addition of the forces due to each one of these charges. Thus, if the force on q_1 due to q_2 is denoted by F_{12}, F_{12} is given by following Eqn. even though other charges are present.

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

In the same way, the force on q_1 due to q_3, denoted by F_{13}, is given by

$$\mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13}$$

which again is the Coulomb force on q_1 due to q_3, even though other charge q_2 is present.
Thus the total force F_1 on q_1 due to the two charges q_2 and q_3 is given as

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13}$$

The above calculation of force can be generalised to a system of charges more than three as shown in Fig (b). Then,

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{\mathbf{r}}_{1i}$$

The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of electrostatics is basically a consequence of Coulomb's law and the **superposition principle**.

1.4 Electric field: Definition of electric field - Mention of expression for electric field due to a point charge -

The *electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point*. The charge *Q*, which is producing the electric field, is called a *source charge* and the charge *q*, which tests the effect of a source charge, is called a *test charge*.

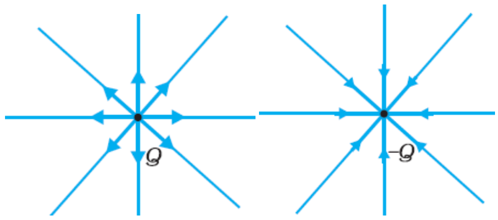
The electric field produced by the charge Q at a point r is given as

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$, is a unit vector from the origin to the point $\mathbf{r}$.
We obtain the force F exerted by a charge Q on a charge q, as

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{\mathbf{r}}$$

the SI unit of electric field as N/C.



If a charge q is brought at any point around Q, Q itself is bound to experience an electrical force due to q and will tend to move. A way out of this difficulty is to make q negligibly small. The force F is then negligibly small but the ratio F/q is finite and defines the electric field:

$$\mathbf{E} = \lim_{q\to 0}\left(\frac{\mathbf{F}}{q}\right)$$

Properties of Electric Field :

(i) The field exists at every point in three-dimensional space.

(ii) For a positive charge, the electric field will be directed **radially outwards** from the charge. On the other hand, if the source charge is negative, the electric field vector, at each point, points **radially inwards**.

(iii) Since the magnitude of the force F on charge q due to charge Q depends only on the distance r of the charge q from charge Q, the magnitude of the electric field E will also depend only on the distance r. Thus at equal distances from the charge Q, the magnitude of its electric field E is same.

(iv) Electric field is a vector quantity whose magnitude and direction are uniquely determined at every point in the field.

Physical significance of electric field :

The accelerated motion of charge q_1 produces electromagnetic waves, which then propagate with the speed c, reach q_2 and cause a force on q_2. The notion of field elegantly accounts for the time delay. Thus, even though electric and magnetic fields can be detected only by their effects (forces) on charges, they are regarded as physical entities, not merely mathematical constructs.

1.5 Superposition principle: Statement, application to find the force between multiple charges.

***Superposition Principle*:** The principle is based on the property that the forces with which two charges attract or repel each other are not affected by the presence of a third (or more) additional charge(s). For an assembly of charges q_1, q_2, q_3, ..., the force on any charge, say q_1, is the vector sum of the force on q_1 due to q_2, the force on q_1 due to q_3, and so on. For each pair, the force is given by the Coulomb's law for two charges.

The electric field E at a point due to a charge configuration is the force on a small positive test charge q placed at the point divided by the magnitude of the charge. Electric field due to a point charge q has a magnitude $|q|/4\pi\,\varepsilon_0 r^2$; it is radially outwards from q, if q is positive, and radially inwards if q is negative. Like Coulomb force, electric field also satisfies superposition principle.

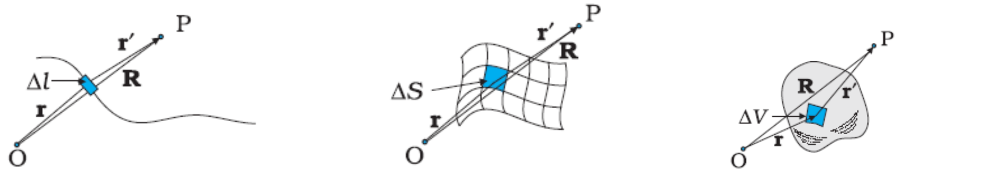
1.6 Application of superposition principle to find electric field for a system of charges.

If there are a number of stationary charges, the net electric field (intensity) at a point is the vector sum of the individual electric fields due to each charge.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \ldots\ldots \vec{E}_n$$

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots\ldots\ldots\right]$$

1.7 Continuous charge distribution: Definitions of surface, linear and volume charge densities -



Line charge $\Delta Q = \lambda \Delta l$ Surface charge $\Delta Q = \sigma \Delta S$ Volume charge $\Delta Q = \rho \Delta V$

Consider an area element ΔS (Fig.) on the surface of the conductor and specify the charge ΔQ on that element.

(i) We then define a ***surface charge density*** σ at the area element by $\sigma = \Delta Q/\Delta S$.

The units for s are C/m^2.

(ii) Similar considerations apply for a line charge distribution and a volume charge distribution. The ***linear charge density*** λ of a wire is defined by $\lambda = \Delta Q/\Delta l$

where Δl is a small line element of wire on the macroscopic scale that, however, includes a large number of microscopic charged constituents, and ΔQ is the charge contained in that line element. The units for λ are C/m.

(iii) The ***volume charge density*** (sometimes simply called charge density) is defined in a similar manner: $\rho = \Delta Q/\Delta V$

where ΔQ is the charge included in the macroscopically small volume element ΔV that includes a large number of microscopic charged constituents. The units for ρ are C/m^3.

1.8 Mention of expression for electric field due to a continuous charge distribution.

The field due to a continuous charge distribution can be obtained in much the same way as for a system of discrete charges. Suppose a continuous charge distribution in space has a charge density ρ. Choose any convenient origin O and let the position vector of any point in the charge distribution be ρ. The charge density ρ may vary from point to point, i.e., it is a function of ρ. Divide the charge distribution into small volume elements of size ΔV. The charge in a volume element ΔV is $\rho\Delta V$.

Now, consider any general point P (inside or outside the distribution) with position vector R (Fig.). Electric field due to the charge $\rho\Delta V$ is given by Coulomb's law:

$$\Delta \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{\rho\, \Delta V}{r'^2} \hat{\mathbf{r}}'$$

where r' is the distance between the charge element and P, and ˆr' is a unit vector in the direction from the charge element to P. By the superposition principle, the total electric field due to the charge distribution is obtained by summing over electric fields due to different volume elements:

$$\mathbf{E} \cong \frac{1}{4\pi\varepsilon_0} \sum_{all\ \Delta V} \frac{\rho\, \Delta V}{r'^2} \hat{\mathbf{r}}'$$

Note that r, r', ˆr' all can vary from point to point.

Hence, using Coulomb's law and the superposition principle, electric field can be determined for any charge distribution, discrete or continuous or part discrete and part continuous.

1.9 Electric dipole: Definition of electric dipole and dipole moment -

An electric dipole is a pair of equal and opposite point charges q and $-q$, separated by a distance $2a$. The line connecting the two charges defines a direction in space. By convention, the direction from $-q$ to q is said to be the direction of the dipole. The mid-point of locations of $-q$ and q is called the centre of the dipole. Its dipole moment vector **p** has magnitude $2qa$ and is in the direction of the dipole axis from $-q$ to q.

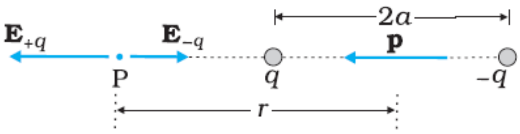
$\therefore$ Electric dipole moment, $\boldsymbol{p = q2a = 2qa}$.

The total charge of the electric dipole is obviously zero. This does not mean that the field of the electric dipole is zero. Since the charge q and $-q$ are separated by some distance, the electric fields due to them, when added, do not exactly cancel out. However, at distances much larger than the separation of the two charges forming a dipole ($r >> 2a$), the fields due to q and $-q$ nearly cancel out. The electric field due to a dipole therefore falls off, at large distance, faster than like $1/r^2$ (the dependence on r of the field due to a single charge q).

1.10 Derivation of electric field due to a dipole (a) at any point on its axis (b) at any point on its equatorial plane -

(i) For points on the axis

Let the point P be at distance r from the centre of the dipole on the side of the charge q, as shown in Fig. (a).



(a)

Then

$$\mathbf{E}_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{\mathbf{p}}$$

where ˆp is the unit vector along the dipole axis (from $-q$ to q). Also

$$\mathbf{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{\mathbf{p}}$$

The total field at P is

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{\mathbf{p}}$$

$$= \frac{q}{4\pi\varepsilon_o}\frac{4ar}{(r^2-a^2)^2}\hat{\mathbf{p}}$$

For $r >> a$

$$\mathbf{E} = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{\mathbf{p}} \qquad (r >> a)$$

The *dipole moment vector* p of an electric dipole is defined by

p = $q \times 2a$ ˆp

that is, it is a vector whose magnitude is charge q times the separation $2a$ (between the pair of charges q, $-q$) and the direction is along the line from $-q$ to q.

At a point on the dipole axis

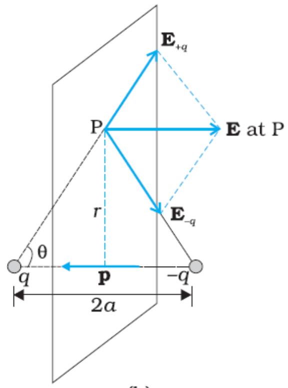
$$\mathbf{E} = \frac{2\,\mathbf{p}}{4\pi\varepsilon_o r^3} \qquad (r >> a)$$

(ii) For points on the equatorial plane

The magnitudes of the electric fields due to the two charges $+q$ and $-q$ are given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2 + a^2}$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2 + a^2}$$



(b)

and are equal.

The directions of E_{+q} and E_{-q} are as shown in Fig. (b).

The components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to ˆp . We have

$$\mathbf{E} = -\,(E_{+q} + E_{-q})\cos\theta\;\hat{\mathbf{p}}$$

$$= -\frac{2\,q\,a}{4\,\pi\,\varepsilon_o\,(r^2 + a^2)^{3/2}}\,\hat{\mathbf{p}}$$

At large distances ($r >> a$), this reduces to

$$\mathbf{E} = -\frac{2\,q\,a}{4\,\pi\,\varepsilon_o\,r^3}\,\hat{\mathbf{p}} \qquad (r >> a)$$

From Eqs. (1.15) and (1.18), it is clear that the dipole field at large distances does not involve q and a separately; it depends on the product qa. This suggests the definition of dipole moment.

The *dipole moment vector* p of an electric dipole is defined by

p = *q* × 2*a* ˆp

that is, it is a vector whose magnitude is charge q times the separation $2a$ (between the pair of charges q, $-q$) and the direction is along the line from $-q$ to q.

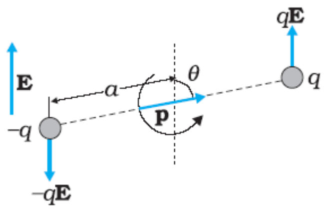
At a point on the equatorial plane

$$\mathbf{E} = -\frac{\mathbf{p}}{4\pi\varepsilon_o r^3} \qquad (r >> a)$$

1.11 Derivation of the torque on an electric dipole in an uniform electric field and expression in vector form.

In a uniform electric field E, a dipole experiences a torque τ given by τ = p × E, but experiences no net force.

Proof :



Consider a permanent dipole of dipole moment **p** in a uniform external field E, as shown in Fig. There is a force qE on q and a force $-q$E on $-q$. The net force on the dipole is zero, since E is uniform. However, the charges are separated, so the forces act at different points, resulting in a torque on the dipole. When the net force is zero, the torque (couple) is independent of the origin. Its magnitude equals the magnitude of each force multiplied by the arm of the couple (perpendicular distance between the two antiparallel forces).

Magnitude of torque $\tau = q\,E \times 2\,a \sin\theta = 2\,q\,a\,E \sin\theta = p\,E\sin\theta$
Its direction is normal to the plane of the paper, coming out of it. The magnitude of $p \times E$ is also $p\,E \sin\theta$ and its direction is normal to the paper, coming out of it.
In vector notation,

$$\vec{\tau} = \vec{p} \times \vec{E}$$

This torque will tend to align the dipole with the field E. When p is aligned with E, the torque is zero. When p is parallel to E or antiparallel to E, then the net torque is zero, but there is a net force on the dipole if E is not uniform.
i.e. if $\theta = 0^\circ$, $\tau = 0$; if $\theta = 90^\circ$, $\tau = pE$ (maximum); if $\theta = 180^\circ$, $\tau = 0$;
S.I. Unit of Torque is Newton.meter (Nm).
Note : If the dipole is placed in a non−uniform electric field at an angle θ, in addition to a torque, it also experiences a force.

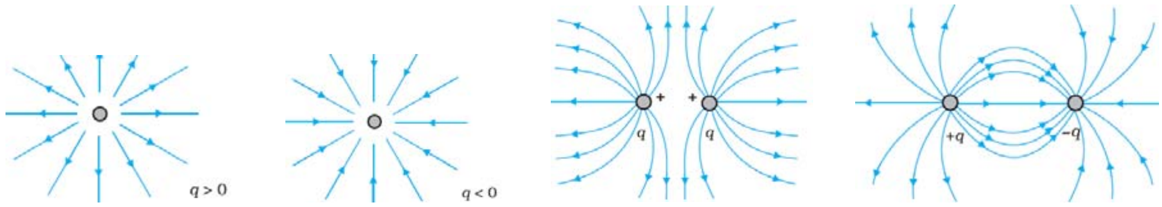
Physical significance of dipoles :
In most molecules, the centres of positive charges and of negative charges lie at the same place. Therefore, their dipole moment is zero. CO_2 and CH_4 are of this type of molecules. However, they develop a dipole moment when an electric field is applied. But in some molecules, the centres of negative charges and of positive charges do not coincide. Therefore they have a permanent electric dipole moment, even in the absence of an electric field. Such molecules are called polar molecules. Water molecules, H_2O, is an example of this type. Various materials give rise to interesting properties and important applications in the presence or absence of electric field.

1.12 Electric field lines: Properties and representation -

The picture of field lines was invented by Faraday to develop an intuitive non- mathematical way of visualizing electric fields around charged configurations. Faraday called them *lines of force.*

The field lines follow some important **general properties**:
(i) Field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity.
(ii) In a charge-free region, electric field lines can be taken to be continuous curves without any breaks.
(iii) Two field lines can never cross each other. (If they did, the field at the point of intersection will not have a unique direction, which is absurd.)
(iv) Electro*static* field lines do not form any closed loops. This follows from the conservative nature of electric field.
(v) The tangent to a line of force at any point on charged body gives the direction of the electric field (E) at that point.
(vi) The number of lines per unit area, through a plane at right angles to the lines, is proportional to the magnitude of E. This means that, where the lines of force are close together, E is large and where they are far apart, E is small.

(vii) Each unit positive charge gives rise to $1/\varepsilon_0$ lines of force in free space. Hence number of lines of force originating from a point charge q is N = q/ε_0 in free space.

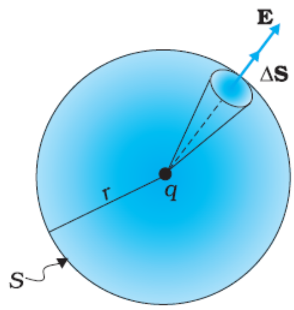


1.13 Electric flux: Concept of electric flux - Area element vector, electric flux through an area element -

Electric flux $\Delta\phi$ through an area element ΔS is defined by $\Delta\phi = E.\Delta S = E\,\Delta S\cos\theta$ which, is proportional to the number of field lines cutting the area element. The angle θ here is the angle between E and ΔS.
The vector area element ΔS is
$\Delta\mathbf{S} = \Delta S\,\hat{n}$
where ΔS is the magnitude of the area element and $\hat{n}$ is normal to the area element, which can be considered planar for sufficiently small ΔS.
For an area element of a closed surface, $\hat{n}$ is taken to be the direction of *outward* normal, by convention.

The unit of electric flux is N C^{-1} m^2.

1.14 Gauss's Law: Statement and its applications to find electric field due to (a) infinitely long straight charged wire, (b) uniformly charged infinite plane sheet and (c) uniformly charged thin spherical shell (field inside and outside) :



Gauss's law states that the *Electric flux (ø) through a closed surface S is* $1/\varepsilon_0$ times the total charge enclosed by *S*.
$\phi = q/\varepsilon_0$
Here, q = total charge enclosed by S.
The law implies that the total electric flux through a closed surface is zero if no charge is enclosed by the surface.
Proof :
Consider the total flux through a sphere of radius *r*, which encloses a point charge *q* at its centre. Divide the sphere into small area elements, as shown in Fig.

The flux through an area element ΔS is

$$\Delta\phi = \mathbf{E}\cdot\Delta\,\mathbf{S} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}\cdot\Delta\mathbf{S}$$

where we have used Coulomb's law for the electric field due to a single charge q. The unit vector $\hat{r}$ is along the radius vector from the centre to the area element. Now, since the normal to a sphere at every point is along the radius vector at that point, the area element ΔS and $\hat{r}$ have the same direction. Therefore,

$$\Delta\phi = \frac{q}{4\pi\varepsilon_0\, r^2}\Delta S$$

since the magnitude of a unit vector is 1.

The total flux through the sphere is obtained by adding up flux through all the different area elements:

$$\phi = \sum_{all\ \Delta S} \frac{q}{4\pi\varepsilon_0\, r^2}\Delta S$$

Since each area element of the sphere is at the same distance r from the charge,

$$\phi = \frac{q}{4\pi\varepsilon_o\, r^2}\sum_{all\ \Delta S}\Delta S = \frac{q}{4\pi\varepsilon_0\, r^2}S$$

Now S, the total area of the sphere, equals $4\pi r^2$. Thus,

$$\phi = \frac{q}{4\pi\varepsilon_0\, r^2}\times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

which is a simple illustration of a general result of electrostatics called ***Gauss's law***.

Significance of Gauss's law :

(i) Gauss's law is true for any closed surface, no matter what its shape or size.

(ii) The term q on the right side of Gauss's law, includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface.

(iii) In the situation when the surface is so chosen that there are some charges inside and some outside, the electric field [whose flux appears on the left side of Eq. (1.31)] is due to all the charges, both inside and outside S. The term q on the right side of Gauss's law, however, represents only the total charge inside S.

(iv) The surface that we choose for the application of Gauss's law is called the Gaussian surface. You may choose any Gaussian surface and apply Gauss's law. However, take care not to let the Gaussian surface pass through any discrete charge. This is because electric field due to a system of discrete charges is not well defined at the location of any charge. (As you go close to the charge, the field grows without any bound.) However, the Gaussian surface can pass through a continuous charge distribution.

(v) Gauss's law is often useful towards a much easier calculation of the electrostatic field *when the system has some symmetry.* This is facilitated by the choice of a suitable Gaussian surface.

(vi) Finally, Gauss's law is based on the inverse square dependence on distance contained in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law.

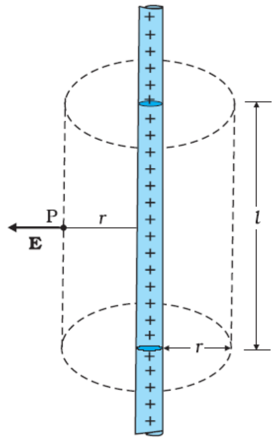
APPLICATIONS OF GAUSS'S LAW :

(a) Field due to an infinitely long straight uniformly charged wire :

Consider an infinitely long thin straight wire with uniform linear charge density λ. The wire is obviously an axis of symmetry. Suppose we take the radial vector from O to P and rotate it around the wire. The points P, P', P'' so obtained are completely equivalent with respect to the charged wire. This implies that the electric field must have the same magnitude at these points. The direction of electric field at every point must be radial (outward if $\lambda > 0$, inward if $\lambda < 0$).

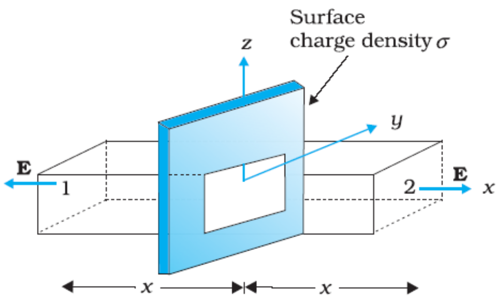
The total field at any point P is radial. Finally, since the wire is infinite, electric field does not depend on the position of P along the length of the wire. In short, the electric field is everywhere

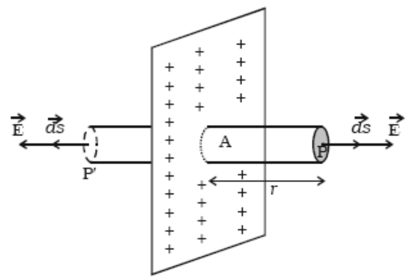
radial in the plane cutting the wire normally, and its magnitude depends only on the radial distance *r*.
To calculate the field, imagine a cylindrical Gaussian surface, as shown in the Fig. Since the field is everywhere radial, flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface, E is normal to the surface at every point, and its magnitude is constant, since it depends only on *r*. The surface area of the curved part is $2\pi rl$, where l is the length of the cylinder.



Flux through the Gaussian surface
= flux through the curved cylindrical part of the surface
$= E \times 2\pi rl$
The surface includes charge equal to λl. Gauss's law then gives
$$E \times 2\pi rl = \lambda l/\varepsilon_0$$
i.e., $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$
Vectorially, E at any point is given by
$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{\mathbf{n}}$$
where $\hat{n}$ is the radial unit vector in the plane normal to the wire passing through the point. E is directed outward if λ is positive and inward if λ is negative.

(b) Uniformly charged infinite plane sheet :





Let σ be the uniform surface charge density of an infinite plane sheet (Fig.). We take the *x*-axis normal to the given plane. By symmetry, the electric field will not depend on *y* and *z* coordinates and its direction at every point must be parallel to the *x*-direction.

We can take the Gaussian surface to be a rectangular parallelepiped of cross sectional area *A*, as shown. (A cylindrical surface will also do.) As seen from the figure, only the two faces 1 and 2 will contribute to the flux; electric field lines are parallel to the other faces and they, therefore, do not contribute to the total flux.

The unit vector normal to surface 1 is in $-x$ direction while the unit vector normal to surface 2 is in the $+x$ direction. Therefore, flux $E.\Delta S$ through both the surfaces are equal and add up. Therefore the net flux through the Gaussian surface is $2\,EA$. The charge enclosed by the closed surface is σA. Therefore by Gauss's law,

$$2\,EA = \sigma A/\varepsilon_0$$
$$\text{or,}\quad E = \sigma/2\varepsilon_0$$

Vectorically,

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{n}} \quad \text{--------} \quad (1)$$

where ˆn is a unit vector normal to the plane and going away from it.

E is directed away from the plate if σ is positive and toward the plate if σ is negative. Note that the above application of the Gauss' law has brought out an additional fact: *E* is independent of *x* also.

For a finite large planar sheet, Eq. (1) is approximately true in the middle regions of the planar sheet, away from the ends.

(c) Field due to uniformly charged thin spherical shell :

Let σ be the uniform surface charge density of a thin spherical shell of radius *R* (Fig.). The situation has obvious spherical symmetry. The field at any point P, outside or inside, can depend only on *r* (the radial distance from the centre of the shell to the point) and must be radial (i.e., along the radius vector).

(i) Field outside the shell: Consider a point P outside the shell with radius vector r. To calculate E at P, we take the Gaussian surface to be a sphere of radius *r* and with centre O, passing through P. All points on this sphere are equivalent relative to the given charged configuration (spherical symmetry). The electric field at each point of the Gaussian surface, therefore, has the same magnitude *E* and is along the radius vector at each point. Thus, E and ΔS at every point are parallel and the flux through each element is *E* Δ*S*. Summing over all Δ*S*, the flux through the Gaussian surface is $E \times 4\pi r^2$. The charge enclosed is $\sigma \times 4\pi R^2$. By Gauss's law

$$E \times 4\pi r^2 = \frac{\sigma}{\varepsilon_0} 4\pi R^2$$

$$\text{Or,}\quad E = \frac{\sigma R^2}{\varepsilon_0 r^2} = \frac{q}{4\pi\varepsilon_0 r^2}$$

where $q = 4\pi R^2\sigma$ is the total charge on the spherical shell.

Vectorially,

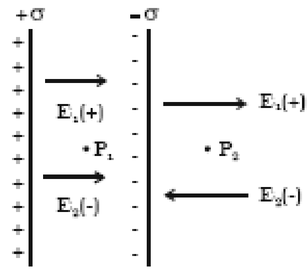
$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

The electric field is directed outward if $q > 0$ and inward if $q < 0$. This, however, is exactly the field produced by a charge *q* placed at the centre O. Thus for points outside the shell, the field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at its centre.

(ii) *Field inside the shell*: In Fig. (b), the point P is inside the shell. The Gaussian surface is again a sphere through P centred at O.

The flux through the Gaussian surface, calculated as before, is $E \times 4\pi r^2$. However, in this case, the Gaussian surface encloses no charge. Gauss's law then gives
$E \times 4\pi r^2 = 0$
i.e., $E = 0$ $(r < R)$
that is, the field due to a uniformly charged thin shell is zero at all points inside the shell. This important result is a direct consequence of Gauss's law which follows from Coulomb's law. The experimental verification of this result confirms the $1/r^2$ dependence in Coulomb's law.

(d) Electric field due to two parallel charged sheets



Consider two plane parallel infinite sheets with equal and opposite charge densities $+\sigma$ and $-\sigma$ as shown in Fig 1. The magnitude of electric field on either side of a plane sheet of charge is $E = \sigma/2\varepsilon_0$ and acts perpendicular to the sheet, directed outward (if the charge is positive) or inward (if the charge is negative).
(i) When the point P_1 is in between the sheets, the field due to two sheets will be equal in magnitude and in the same direction. The resultant field at P_1 is,

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o} \text{ (towards the right)}$$

(ii) At a point P_2 outside the sheets, the electric field will be equal in magnitude and opposite in direction. The resultant field at P_2 is,

$$E = E_1 - E_2 = \frac{\sigma}{2\varepsilon_o} - \frac{\sigma}{2\varepsilon_o} = 0.$$

IMPORTANT FORMULAS :

1. Coulomb's Law
$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$
2. Coulomb's Law in Vector form
$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 \; q_2}{r_{21}^2}\hat{\mathbf{r}}_{21} \quad \text{and} \quad \mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 \; q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = -\mathbf{F}_{21}$$
3. Electric field
$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \quad \text{N/C}$$
4. E = F/q

5. Surface charge density = $\sigma = \Delta Q/\Delta S$ unit C/m^2

6. Linear charge density = $\lambda = \Delta Q/\Delta l$ unit C/m

7. Volume charge density = $\rho = \Delta Q/\Delta V$ unit C/m^3

8. Expression for electric field due to a continuous charge distribution

$$\mathbf{E} \cong \frac{1}{4\pi\varepsilon_0} \sum_{all\ \Delta V} \frac{\rho\ \Delta V}{r'^2} \hat{\mathbf{r}}'$$

9. Electric dipole moment, ***p = q2a = 2qa***
10. The *dipole moment vector* p of an electric dipole is defined by **p = *q* × 2*a* ˆp**

11. Expression for Electric field due to a dipole at any point on its axis is

$$= \frac{q}{4\pi\varepsilon_o} \frac{4ar}{(r^2 - a^2)^2} \hat{\mathbf{p}}$$

For $r >> a$

$$\mathbf{E} = \frac{4qa}{4\pi\varepsilon_0 r^3} \hat{\mathbf{p}} \qquad (r >> a)$$

At a point on the dipole axis

$$\mathbf{E} = \frac{2\mathbf{p}}{4\pi\varepsilon_o r^3} \qquad (r >> a)$$

12. Expression for Electric field due to a dipole at any point on its equatorial plane is

$$\mathbf{E} = -(E_{+q} + E_{-q}) \cos\theta\ \hat{\mathbf{p}}$$

$$= -\frac{2qa}{4\pi\varepsilon_o (r^2 + a^2)^{3/2}} \hat{\mathbf{p}}$$

At large distances ($r >> a$), this reduces to

$$\mathbf{E} = -\frac{2qa}{4\pi\varepsilon_o r^3} \hat{\mathbf{p}} \qquad (r >> a)$$

At a point on the equatorial plane

$$\mathbf{E} = -\frac{\mathbf{p}}{4\pi\varepsilon_o r^3} \qquad (r >> a)$$

13. Expression for torque on an electric dipole in an uniform electric field :
τ = p × E
Magnitude of torque τ = *q E* × 2 *a* sin θ = 2 *q a E* sin θ = p E sin θ
Its direction is normal to the plane of the paper,
In vector form

$$\vec{\tau} = \vec{p} \times \vec{E}$$

14. S.I. Unit of Torque is Newton.meter (Nm).
15. Electric flux Δø through an area element ΔS is defined by Δø = E.ΔS = *E* Δ*S* cos θ which, is proportional to the number of field lines cutting the area element. The angle θ here is the angle between E and ΔS.
The vector area element ΔS is **ΔS** = Δ*S* ˆn
16. The unit of electric flux is N C^{-1} m^2.
17. ***Gauss's law*** states that the *Electric flux (ø) through a closed surface S is* 1/ ε_0 times the total charge enclosed by *S*. ***ø = q/ ε_0***
18. The flux through an area element ΔS is

$$\Delta\phi = \mathbf{E}\cdot\Delta\,\mathbf{S} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}\cdot\Delta\mathbf{S}$$

19. If the magnitude of a unit vector is 1.

$$\Delta\phi = \frac{q}{4\pi\varepsilon_0\, r^2}\Delta S$$

20. Field due to an infinitely long straight uniformly charged wire due to Gauss is

$$E\times 2\pi r l = \lambda l/\varepsilon_0$$

i.e., $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

21. Electric field due to uniformly charged infinite plane sheet using Gauss Law is

$$2\,EA = \sigma A/\varepsilon_0$$

or, $E = \sigma/2\varepsilon_0$

22. Electric Field due to uniformly charged thin spherical shell using Gauss Law is

(i) Field outside the shell :

$$E\times 4\,\pi\,r^2 = \frac{\sigma}{\varepsilon_0}\,4\,\pi\,R^2$$

Or, $E = \dfrac{\sigma R^2}{\varepsilon_0\, r^2} = \dfrac{q}{4\pi\varepsilon_0 r^2}$

where $q = 4\pi R^2\sigma$ is the total charge on the spherical shell.

(ii) *Field inside the shell:*

$E \times 4\pi\, r^2 = 0$

i.e., $E = 0$ $(r < R)$

23. Electric field due to two parallel charged sheets using Gauss Law is

(i) At a point P_1 is in between the sheets

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o}\ \text{(towards the right)}$$

(ii) At a point P_2 outside the sheets

$$E = E_1 - E_2 = \frac{\sigma}{2\varepsilon_o} - \frac{\sigma}{2\varepsilon_o} = 0.$$

ONE MARK QUESTIONS : (1Q)

1. What are point charges?
A: Charges whose sizes are very small compared to the distance between them are called point charges

2. The net charge of a system of point charges -4, +3, -1, +4 (S.I.units)
A: +2

3. What is meant by conservation of charge?
A: The total charge of an isolated system remains always constant

4. What is quantisation of charge?
A: The electric charge is always an integral multiple of 'e' (charge on an electron).

5. Mention the S.I. unit of charge.
A: coulomb (C)

6. Define one coulomb of charge.
A: 1C is the charge that when placed at a distance of 1m from another charge of the same magnitude, in vacuum, experiences an electrical force of repulsion of magnitude 9×10^9N

7. Which principle is employed in finding the force between multiple charges?
A: Principle of superposition

8. Define electric field.
A: Electric field due to a charge at a point in space is defined as the force experienced by a unit positive charge placed at that point.

9. Is electric field a scalar/vector?
A: vector

10. Mention the S.I. unit of electric field.
A: newton per coulomb (NC^{-1})

11. What is the direction of electric field due to a point positive charge?
A: Radially outward

12. What is the direction of electric field due to a point negative charge?
A: Radially inward

13. What is a source charge?
A: The charge which produces the electric field

14. What is a test charge?
A: The charge which detects the effect of the source charge

15. How do you pictorially map the electric field around a configuration of charges?
A: Using electric field lines

16. What is an electric field line?
A: An electric field line is a curve drawn in such a way that the tangent to it at each point represents the direction of the net field at that point

17. What is electric flux?
A: Electric flux over a given surface is the total number of electric field lines passing through that surface.

18. Mention the S.I.unit of electric flux.
A: $Nc^{-1}m^2$

19. What is an electric dipole?
A: An electric dipole is a set of two equal and opposite point charges separated by a small distance

20. What is the net charge of an electric dipole?
A: zero

21. Define dipole moment.
A: Dipole moment of an electric dipole is defined as the product of one of the charges and the distance between the two charges.

22. Is dipole moment a vector / scalar?
A: Vector

23. What is the direction of dipole moment?
A: The dipole moment vector is directed from negative to positive charge along the dipole axis

24. What is the net force on an electric dipole placed in a uniform electric field?
A: Zero

25. When is the torque acting on an electric dipole placed in a uniform electric field maximum?
A: When the dipole is placed perpendicular to the direction of the field.

26. When is the torque acting on an electric dipole placed in a uniform electric field minimum?
A: When the dipole is placed parallel to the direction of the field

27. State Gauss's law.

A: Gauss's law states that 'the electric flux through a closed surface is equal to times the charge enclosed by that surface'

28. What is a Gaussian surface?
A: The closed surface we choose to calculate the electric flux and hence to apply Gauss's law

29. What happens to the force between two point charges if the distance between them is doubled?
A: Decreases 4 times.

30. If two charges kept in 'air' at a certain separation, are now kept at the same separation in 'water' of dielectric constant 80, then what happens to the force between them?
A: Decreases by 80 times.

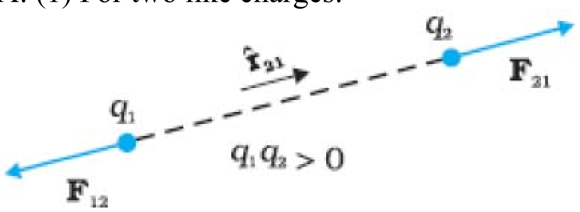
31. On a macroscopic scale is charge discrete or continuous?
A: Continuous.

TWO MARKS QUESTIONS : (1Q)
1. Write the expression for quantisation of charge and explain the terms in it.
A: $q = ne$; n is an integer (+ or -) and e is the charge on an electron

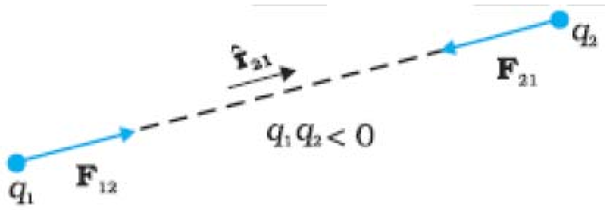
2. State and explain Coulomb's law of electrostatics.
A: Coulomb's law states that 'the electrical force between two point charges is directly proportional to the product of their strengths and is inversely proportional to the square of the distance between them'.
If 'F' represents the electrical force between two point charges q1 and q2 separated by a distance 'r' apart, then according to this law $F= k\,[(q_1.q_2)/r^2]$; $k = (1/4\pi\varepsilon_0)$ is a constant when the two charges are in vacuum.

3. Write Coulomb's law in vector notation and explain it.
A: $\boxed{F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q1q2}{r_{21}^2}\,\widehat{r_{21}}}$; F_{21} → Force on q_2 due to q_1, $\boldsymbol{r_{21}} \rightarrow \boldsymbol{r_2} - \boldsymbol{r_1}$; $\boldsymbol{r_1}$ & $\boldsymbol{r_2}$ are the position vectors of q_1 and q_2 and $\widehat{\boldsymbol{r_{21}}}$ → Unit vector in the direction of $\boldsymbol{r_{21}}$.

4. Write the pictorial representations of the force of repulsion and attraction, between two point charges.
A: (1) For two like charges:



(2) For two unlike charges:



5. Explain the principle of superposition to calculate the force between multiple charges.

A: The principle of superposition:

It is a principle which gives a method to find the force on a given charge due to a group of charges interacting with it. According to this principle "force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges".

To understand this concept, consider a system of charges q_1, q_2……..q_n. The force on q_1 due to q_2 is being unaffected by the presence of the other charges q_3, q_4……..q_n. The total force F_1 on the charge q_1 due to all other charges is then given by superposition principle

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i-2}^{n}\frac{q_i}{r_{1i}^2}\hat{\mathbf{r}}_{1i}$$

6. Mention the expression for the electric field due to a point charge placed in vacuum.

A: Electric field , $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$

7. Write the expression for the electric field due to a system of charges and explain it.

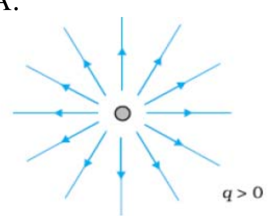
A: Electric field due to a system of charges q_1, q_2, q_3… , q_n described by the position vectors r_1, r_2, r_3, ……………., r_n respectively relative to some origin. Using Coulomb's law and the principle of superposition, it can be shown that the electric field **E** at a point P represented by the position vector **r**, is given by

$$E(r) = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q1}{r1p^2}\widehat{r_{1P}} + \frac{q2}{r2p^2}\widehat{r_{2P}} + \frac{q3}{r3p^2}\widehat{r_{3P}} + \ldots\ldots\ldots\ldots + \frac{qn}{rnp^2}\widehat{r_{nP}}\right\}$$

$$\text{Or } \boxed{E(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i=n}\frac{qi}{rip^2}\widehat{r_{iP}}}$$

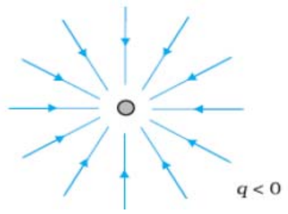
8. Draw electric field lines in case of a positive point charge.

A:



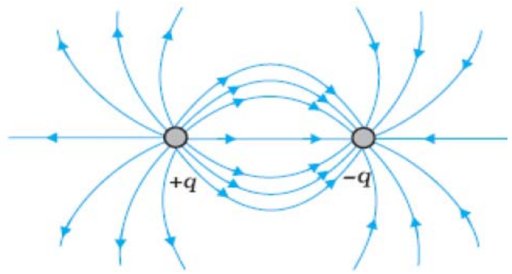
9. Sketch electric field lines in case of a negative point charge.

A:



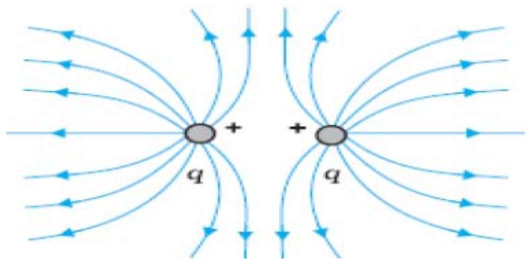
10. Sketch the electric field lines in case of an electric dipole.

A:



11. Sketch the electric field lines in case of two equal positive point charges.

A:



12. Mention any two properties of electric field lines.

A: (1) Electric field lines start from a positive charge and end on a negative charge.

(2) Electric field lines do not intersect each other.

13. Write the expression for the torque acting on an electric dipole placed in a uniform electric field and explain the terms in it.

A: $\tau = PE \sin\theta$; $\tau \rightarrow$ torque, P $\rightarrow$ dipole moment of the electric dipole, E $\rightarrow$ Strength of the uniform electric field and $\theta \rightarrow$ angle between the directions of P and E.

14. Define linear density of charge and mention its SI unit.

A: Linear density of charge is charge per unit length. Its SI unit is Cm^{-1}

15. Define surface density of charge and mention its SI unit.

A: Surface density of charge is charge per unit area. Its SI unit is Cm^{-2}

16. Define volume density of charge and mention its SI unit.

A: Volume density of charge is charge per unit volume. Its SI unit is Cm^{-3}

17. What is the effect of a non-uniform electric field on an electric dipole?

A: In a non-uniform electric field an electric dipole experiences both the torque and a net force.

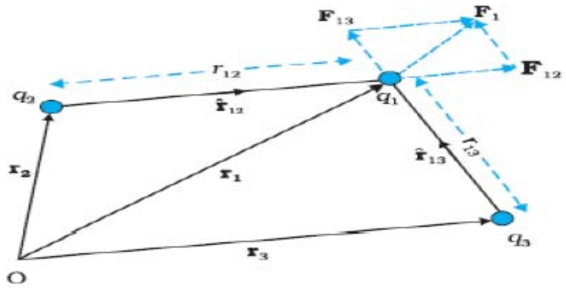
Three mark questions with answers

1. Mention three properties of electric charge.

A: 1.Electrc charge is conserved
2. Electric charge is quantised
3. Electric charge is additive

2. Draw a diagram to show the resultant force on a charge in a system of three charges.

A:



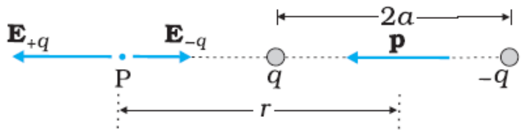
3. Why is the electric field inside a uniformly charged spherical shell, zero? Explain.

A: When a spherical shell is charged, the charges get distributed uniformly over its outer surface and the charge inside the shell is zero. According to Gauss's law, as the charge inside is zero, the electric flux at any point inside the shell will be zero. Obviously the electric field (electric flux per unit area) is also zero.

Five mark questions with answers

1. Obtain an expression for the electric field at a point along the axis of an electric dipole.

Ans : Let the point P be at distance *r* from the centre of the dipole on the side of the charge *q*, as shown in Fig. (a).



(a)

Then

$$\mathbf{E}_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{\mathbf{p}}$$

where ˆp is the unit vector along the dipole axis (from –*q* to *q*). Also

$$\mathbf{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{\mathbf{p}}$$

The total field at P is

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{\mathbf{p}}$$

$$= \frac{q}{4\pi\varepsilon_o}\,\frac{4ar}{(r^2-a^2)^2}\,\hat{\mathbf{p}}$$

For $r >> a$

$$\mathbf{E} = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{\mathbf{p}} \qquad (r >> a)$$

The *dipole moment vector* p of an electric dipole is defined by

p = *q* × 2*a* ˆp

that is, it is a vector whose magnitude is charge q times the separation $2a$ (between the pair of charges q, $-q$) and the direction is along the line from $-q$ to q.

At a point on the dipole axis

$$\mathbf{E} = \frac{2\mathbf{p}}{4\pi\varepsilon_o r^3} \qquad (r >> a)$$

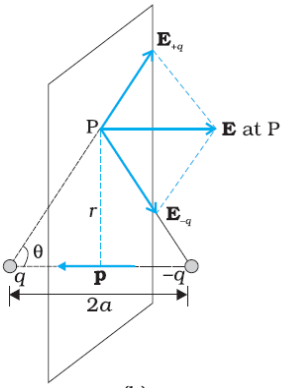
2. Obtain an expression for the electric field at a point on the equatorial plane of an electric dipole.

Ans : The magnitudes of the electric fields due to the two charges $+q$ and $-q$ are given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2 + a^2}$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2 + a^2}$$

and are equal.



(b)

The directions of E_{+q} and E_{-q} are as shown in Fig. (b).

The components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to ˆp . We have

$$\mathbf{E} = -(E_{+q} + E_{-q})\cos\theta\ \hat{\mathbf{p}}$$

$$= -\frac{2qa}{4\pi\varepsilon_o(r^2 + a^2)^{3/2}}\hat{\mathbf{p}}$$

At large distances ($r >> a$), this reduces to

$$\mathbf{E} = -\frac{2qa}{4\pi\varepsilon_o r^3}\hat{\mathbf{p}} \qquad (r >> a)$$

From Eqs. (1.15) and (1.18), it is clear that the dipole field at large distances does not involve q and a separately; it depends on the product qa. This suggests the definition of dipole moment.

The *dipole moment vector* p of an electric dipole is defined by

p = *q* × 2*a* ˆp

that is, it is a vector whose magnitude is charge q times the separation $2a$ (between the pair of charges q, $-q$) and the direction is along the line from $-q$ to q.

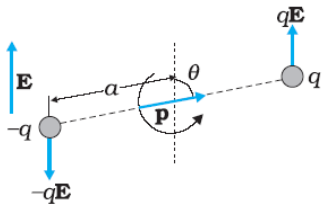
At a point on the equatorial plane

$$\mathbf{E} = -\frac{\mathbf{p}}{4\pi\varepsilon_o r^3} \qquad (r >> a)$$

3. Obtain the expression for the torque acting on an electric dipole placed in a uniform electric field.

In a uniform electric field E, a dipole experiences a torque τ given by τ = p × E, but experiences no net force.

Proof :



Consider a permanent dipole of dipole moment **p** in a uniform external field E, as shown in Fig. There is a force qE on q and a force $-q$E on $-q$. The net force on the dipole is zero, since E is uniform. However, the charges are separated, so the forces act at different points, resulting in a torque on the dipole. When the net force is zero, the torque (couple) is independent of the origin. Its magnitude equals the magnitude of each force multiplied by the arm of the couple (perpendicular distance between the two antiparallel forces).

Magnitude of torque $\tau = q\,E \times 2\,a \sin\theta \quad = 2\,q\,a\,E \sin\theta \; = p\,E \sin\theta$

Its direction is normal to the plane of the paper, coming out of it. The magnitude of p × E is also $p\,E \sin\theta$ and its direction is normal to the paper, coming out of it.

In vector notation,

$$\vec{\tau} = \vec{p} \times \vec{E}$$

This torque will tend to align the dipole with the field E. When p is aligned with E, the torque is zero. When p is parallel to E or antiparallel to E, then the net torque is zero, but there is a net force on the dipole if E is not uniform.

i.e. if $\theta = 0^o$, $\tau = 0$; if $\theta = 90^o$, $\tau = pE$ (maximum); if $\theta = 180^o$, $\tau = 0$;

S.I. Unit of Torque is Newton.meter (Nm).

Note : If the dipole is placed in a non−uniform electric field at an angle θ, in addition to a torque, it also experiences a force.

Physical significance of dipoles :

In most molecules, the centres of positive charges and of negative charges lie at the same place. Therefore, their dipole moment is zero. CO_2 and CH_4 are of this type of molecules. However, they develop a dipole moment when an electric field is applied. But in some molecules, the centres of negative charges and of positive charges do not coincide. Therefore they have a permanent electric dipole moment, even in the absence of an electric field. Such molecules are called polar molecules. Water molecules, H_2O, is an example of this type. Various materials give rise to interesting properties and important applications in the presence or absence of electric field.

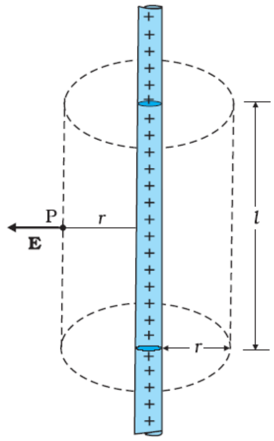
4. Using Gauss's law, obtain an expression for the electric field due to an infinitely long straight uniformly charged conductor.

(a) Field due to an infinitely long straight uniformly charged wire :

Consider an infinitely long thin straight wire with uniform linear charge density λ. The wire is obviously an axis of symmetry. Suppose we take the radial vector from O to P and rotate it around the wire. The points P, P', P'' so obtained are completely equivalent with respect to the charged wire. This implies that the electric field must have the same magnitude at these points. The direction of electric field at every point must be radial (outward if $\lambda > 0$, inward if $\lambda < 0$).

The total field at any point P is radial. Finally, since the wire is infinite, electric field does not depend on the position of P along the length of the wire. In short, the electric field is everywhere radial in the plane cutting the wire normally, and its magnitude depends only on the radial distance r.

To calculate the field, imagine a cylindrical Gaussian surface, as shown in the Fig. Since the field is everywhere radial, flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface, E is normal to the surface at every point, and its magnitude is constant, since it depends only on r. The surface area of the curved part is $2\pi rl$, where l is the length of the cylinder.



Flux through the Gaussian surface
= flux through the curved cylindrical part of the surface
$= E \times 2\pi rl$

The surface includes charge equal to λl. Gauss's law then gives

$$E \times 2\pi rl = \lambda l/\varepsilon_0$$

$$\text{i.e.,} \qquad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

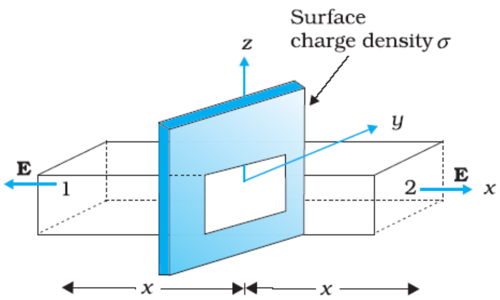
Vectorially, E at any point is given by

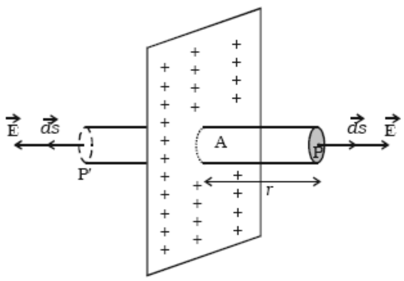
$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\ \hat{\mathbf{n}}$$

where ˆn is the radial unit vector in the plane normal to the wire passing through the point. E is directed outward if λ is positive and inward if λ is negative.

5. Using Gauss's law, obtain an expression for the electric field due to a uniformly charged infinite plane sheet.

<u>Uniformly charged infinite plane sheet :</u>





Let σ be the uniform surface charge density of an infinite plane sheet (Fig.). We take the x-axis normal to the given plane. By symmetry, the electric field will not depend on y and z coordinates and its direction at every point must be parallel to the x-direction.

We can take the Gaussian surface to be a rectangular parallelepiped of cross sectional area A, as shown. (A cylindrical surface will also do.) As seen from the figure, only the two faces 1 and 2 will contribute to the flux; electric field lines are parallel to the other faces and they, therefore, do not contribute to the total flux.

The unit vector normal to surface 1 is in $-x$ direction while the unit vector normal to surface 2 is in the $+x$ direction. Therefore, flux E.ΔS through both the surfaces are equal and add up. Therefore the net flux through the Gaussian surface is 2 EA. The charge enclosed by the closed surface is σ A. Therefore by Gauss's law,

$$2\ EA = \sigma A/\varepsilon_0$$

$$\text{or,}\ \ E = \sigma/2\varepsilon_0$$

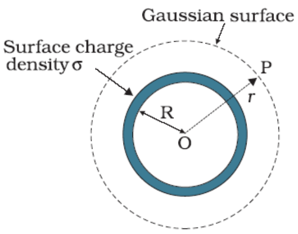
Vectorically,

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{n}} \quad \text{--------} \quad (1)$$

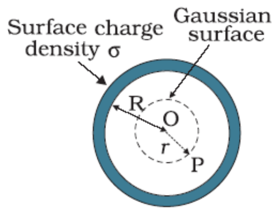
where ˆn is a unit vector normal to the plane and going away from it.
E is directed away from the plate if σ is positive and toward the plate if σ is negative. Note that the above application of the Gauss' law has brought out an additional fact: E is independent of x also.
For a finite large planar sheet, Eq. (1) is approximately true in the middle regions of the planar sheet, away from the ends.

6. Using Gauss's law, obtain an expression for the electric field at an outside point due to a uniformly charged thin spherical shell.

Let σ be the uniform surface charge density of a thin spherical shell of radius R (Fig.). The situation has obvious spherical symmetry. The field at any point P, outside or inside, can depend only on r (the radial distance from the centre of the shell to the point) and must be radial (i.e., along the radius vector).



(a)



(b)

(i) Field outside the shell: Consider a point P outside the shell with radius vector r. To calculate E at P, we take the Gaussian surface to be a sphere of radius r and with centre O, passing through P. All points on this sphere are equivalent relative to the given charged configuration (spherical symmetry). The electric field at each point of the Gaussian surface, therefore, has the same magnitude E and is along the radius vector at each point. Thus, E and ΔS at every point are parallel and the flux through each element is $E\ \Delta S$. Summing over all ΔS, the flux through the Gaussian surface is $E \times 4\pi r^2$. The charge enclosed is $\sigma \times 4\pi R^2$. By Gauss's law

$$E \times 4\pi r^2 = \frac{\sigma}{\varepsilon_0} 4\pi R^2$$

$$\text{Or,}\ \ E = \frac{\sigma R^2}{\varepsilon_0 r^2} = \frac{q}{4\pi\varepsilon_0 r^2}$$

where $q = 4\pi R^2\sigma$ is the total charge on the spherical shell.
Vectorially,

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

The electric field is directed outward if $q > 0$ and inward if $q < 0$. This, however, is exactly the field produced by a charge q placed at the centre O. Thus for points outside the shell, the field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at its centre.

(ii) ***Field inside the shell:*** In Fig. (b), the point P is inside the shell. The Gaussian surface is again a sphere through P centred at O.
The flux through the Gaussian surface, calculated as before, is $E \times 4\pi r^2$. However, in this case, the Gaussian surface encloses no charge. Gauss's law then gives
$E \times 4\pi r^2 = 0$
i.e., $E = 0$ $(r < R)$
that is, the field due to a uniformly charged thin shell is zero at all points inside the shell. This important result is a direct consequence of Gauss's law which follows from Coulomb's law. The experimental verification of this result confirms the $1/r^2$ dependence in Coulomb's law.

Numerical Problems.

1. The electrostatic force on a metal sphere of charge 0.4 μC due to another identical metal sphere of charge -0.8 μC in air is 0.2N. Find the distance between the two spheres and also the force between the same two spheres when they are brought into contact and then replaced in their initial positions.

2. Three small identical balls have charges -3×10^{-12}C, 8×10^{-12}C and 4×10^{-12}C respectively. They are brought in contact and then separated. Calculate (i) charge on each ball (ii) number of electrons in excess or deficit on each ball after contact.

Data : $q_1 = -3 \times 10^{-12}$C, $q_2 = 8 \times 10^{-12}$ C, $q_3 = 4 \times 10^{-12}$ C

Solution : (i) The charge on each ball

$$q = \frac{q_1 + q_2 + q_3}{3} = \left(\frac{-3+8+4}{3}\right) \times 10^{-12} = \quad = 3 \times 10^{-12}\ C$$

(ii) Since the charge is positive, there is a shortage of electrons on each ball.

$$n = \frac{q}{e} = \frac{3 \times 10^{-12}}{1.6 \times 10^{-19}} = 1.875 \times 10^{7}$$

∴ Number of electrons = 1.875×10^{7}.

3. Two insulated charged spheres of charges 6.5×10^{-7}C each are separated by a distance of 0.5m. Calculate the electrostatic force between them. Also calculate the force (i) when the charges are doubled and the distance of separation is halved. (ii) when the charges are placed in a dielectric medium water ($\varepsilon_r = 80$)

Data : $q_1 = q_2 = 6.5 \times 10^{-7}$C, r = 0.5 m

Solution : $F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times (6.5 \times 10^{-7})^2}{(0.5)^2} = 1.52 \times 10^{-2}$ N.

(i) If the charge is doubled and separation between them is halved then,

$$F_1 = \frac{1}{4\pi\varepsilon_o} \frac{2q_1 2q_2}{\left(r/2\right)^2}$$

F_1 = 16 times of F.

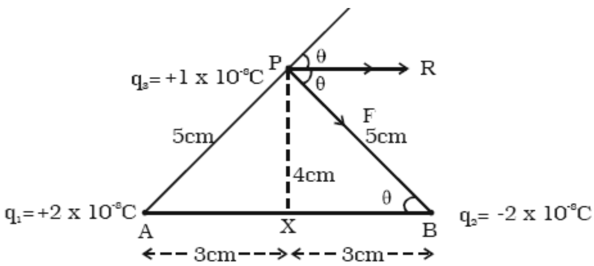
= $16 \times 1.52 \times 10^{-2}$ $\quad F_1 = 0.24$ N

(ii) When placed in water of εr = 80

$$F_2 = \frac{F}{\varepsilon_r} = \frac{1.52 \times 10^{-2}}{80}$$

$F_2 = 1.9 \times 10^{-4}$ N

4. Two small equal and unlike charges 2×10^{-8}C are placed at A and B at a distance of 6 cm. Calculate the force on the charge 1×10^{-8}C placed at P, where P is 4cm on the perpendicular bisector of AB.



Data : $q_1 = +2 \times 10^{-8}$C, $q_2 = -2 \times 10^{-8}$ C $q_3 = 1 \times 10^{-8}$ C at P XP = 4 cm or 0.04 m, AB = 6 cm or 0.06 m

Solution :

From Δ APX, AP = $\sqrt{4^2 + 3^2}$ = 5 cm or 5×10^{-2} m.

A repels the charge at P with a force F (along AP)

$$F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_3}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-8} \times 1 \times 10^{-8}}{(5 \times 10^{-2})^2}$$

= 7.2×10^{-4} N along AP.

B attracts the charge at P with same F (along PB), because BP = AP = 5 cm. To find R, we resolve the force into two components

$$R = F\cos\theta + F\cos\theta = 2F\cos\theta$$

$$= 2 \times 7.2 \times 10^{-4} \times \frac{3}{5} \qquad \left[\because \cos\theta = \frac{BX}{PB} = \frac{3}{5}\right]$$

$$\therefore R = 8.64 \times 10^{-4} \text{ N}$$

5. Compare the magnitude of the electrostatic and gravitational force between an electron and a proton at a distance *r* apart in hydrogen atom. (Given : me = 9.11×10^{-31} kg ; mP = 1.67×10^{-27} kg ; G = 6.67×10^{-11} Nm2 kg–2; e = 1.6×10^{-19} C)

Solution :

The gravitational attraction between electron and proton is

$$F_g = G\frac{m_e m_p}{r^2}$$

Let *r* be the average distance between electron and proton in hydrogen atom. The electrostatic force between the two charges

$$F_e = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2}$$

$$\therefore \quad \frac{F_e}{F_g} = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{G m_e m_p} = \frac{1}{4\pi\varepsilon_o} \frac{e^2}{G m_e m_p}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.11 \times 10^{-31} \times 1.67 \times 10^{-27}}$$

$$\frac{F_e}{F_g} = 2.27 \times 10^{39}$$

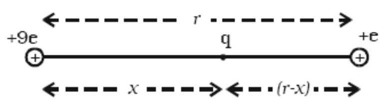
This shows that the electrostatic force is 2.27×10^{39} times stronger than gravitational force.

6. Two point charges +9e and +1e are kept at a distance of 16 cm from each other. At what point between these charges, should a third charge q to be placed so that it remains in equilibrium?

Data : r = 16 cm or 0.16 m; q_1 = 9e and q_2 = e

Solution : Let a third charge q be kept at a distance *x* from +9e and (*r* – *x*) from + e

$$F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon o} \frac{9e \times q}{x^2} = \frac{1}{4\pi\varepsilon_o} \frac{q\ e}{(r-x)^2}$$



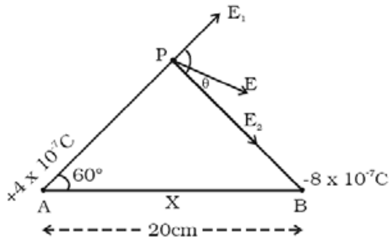
$$\therefore \frac{x^2}{(r-x)^2} = 9 \quad \text{or} \quad \frac{x}{r-x} = 3$$

or $x = 3r - 3x$

$$\therefore \quad 4x = 3r = 3 \times 16 = 48 \text{ cm} \qquad \therefore \quad x = \frac{48}{4} = 12 \text{ cm or } 0.12 \text{ m}$$

∴ The third charge should be placed at a distance of 0.12 m from charge 9e.

7. Two charges 4×10^{-7} C and -8×10^{-7}C are placed at the two corners A and B of an equilateral triangle ABP of side 20 cm. Find the resultant intensity at P.



Data : $q_1 = 4 \times 10^{-7}$ C; $q_2 = -8 \times 10^{-7}$ C; r = 20 cm = 0.2 m
Solution :
Electric field E_1 along AP

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2} = \frac{9\times10^9\times4\times10^{-7}}{(0.2)^2} = 9 \times 10^4 \text{ N C}^{-1}$$

Electric field E_2 along PB.

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r^2} = \frac{9\times10^9\times8\times10^{-7}}{0.04} = 18 \times 10^4 \text{N C}^{-1}$$

$$\therefore \quad E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 120^o}$$

$$= 9\times10^4\sqrt{2^2+1^2+2\times2\times1\left(-\tfrac{1}{2}\right)} \qquad = 9\sqrt{3}\times10^4 = 15.6\times10^4 N\ C^{-1}$$

8. A sample of HCl gas is placed in an electric field of 2.5×10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} C m. Find the maximum torque that can act on a molecule.
Data : E = 2.5×10^4 N C^{-1}, p = 3.4×10^{-30} C m.
Solution : Torque acting on the molecule $\tau = pE \sin\theta$ for maximum torque, $\theta = 90^o$
= $3.4 \times 10^{-30} \times 2.5 \times 10^4$
Maximum Torque acting on the molecule is = 8.5×10^{-26} N m.

9. A sample of HCl gas is placed in an electric field of 2.5×10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} C m. Find the maximum torque that can act on a molecule.
Data : E = 2.5×10^4 N C−1, p = 3.4×10^{-30} C m.
Solution : Torque acting on the molecule $\tau = pE \sin\theta$ for maximum torque, $\theta = 90^o$
= $3.4 \times 10^{-30} \times 2.5 \times 10^4$
Maximum Torque acting on the molecule is = 8.5×10^{-26} N m.

10. A point charge causes an electric flux of -6×10^3 Nm^2 C^{-1} to pass through a spherical Gaussian surface of 10 cm radius centred on the charge. (i) If the radius of the Gaussian surface is doubled, how much flux will pass through the surface? (ii) What is the value of charge?
Data : $\varphi = -6 \times 10^3$ N m^2 C^{-1}; r = 10 cm = 10×10^{-2} m
Solution :
(i) If the radius of the Gaussian surface is doubled, the electric flux through the new surface will be the same, as it depends only on the net charge enclosed within and it is independent of the radius.
∴ $\varphi = -6 \times 10^3$ N m^2 C^{-1}
(ii) ∴ $\varphi = q/\varepsilon_0$ or $q = -(8.85 \times 10^{-12} \times 6 \times 10^3)$ or $q = -5.31 \times 10^{-8}$ C

11. What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Ans :
Charge on the first sphere, $q_1 = 2 \times 10^{-7}$ C
Charge on the second sphere, $q_2 = 3 \times 10^{-7}$ C
Distance between the spheres, r = 30 cm = 0.3 m
Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \quad \text{and} \quad \frac{1}{4\pi \epsilon_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2} \qquad F = \frac{9\times10^9 \times 2\times10^{-7}\times3\times10^{-7}}{(0.3)^2} = 6\times10^{-3} \text{ N}$$

Hence, force between the two small charged spheres is 6×10^{-3} N. The charges are of same nature. Hence, force between them will be repulsive.

12. The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge − 0.8 μC in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

Ans :
(a) Electrostatic force on the first sphere, F = 0.2 N
Charge on this sphere, q_1 = 0.4 μC = 0.4×10^{-6} C
Charge on the second sphere, q_2 = − 0.8 μC = -0.8×10^{-6} C
Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \qquad \text{And, } \frac{1}{4\pi \epsilon_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2} \qquad \text{then} \qquad r^2 = \frac{q_1 q_2}{4\pi \epsilon_0 F}$$

$$= \frac{0.4\times10^{-6}\times8\times10^{-6}\times9\times10^9}{0.2} = \quad = 144\times10^{-4}$$

$$r = \sqrt{144\times10^{-4}} = 0.12 \text{ m}$$

The distance between the two spheres is 0.12 m.

(b) Both the spheres attract each other with the same force. Therefore, the force on the second sphere due to the first is 0.2 N.

13. Check that the ratio ke^2/Gm_em_p is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Ans :
The given ratio is $ke^2/G\, m_em_p$
Where, G = Gravitational constant. Its unit is N m^2 kg^{-2}.
m_e and m_p = Masses of electron and proton. Their unit is kg. e = Electric charge. Its unit is C.
k = constant = $1/4\pi\varepsilon_0$. Its unit is N m^2 C^{-2}.

$$\text{Therefore, unit of the given ratio } \frac{ke^2}{Gm_em_p} = \frac{[\text{Nm}^2 \text{ C}^{-2}][\text{C}^{-2}]}{[\text{Nm}^2 \text{ kg}^{-2}][\text{kg}][\text{kg}]} = \text{M}^0\text{L}^0\text{T}^0$$

Hence, the given ratio is dimensionless.
e = 1.6×10^{-19} C
G = 6.67×10^{-11} N m^2 kg^{-2}
m_e = 9.1×10^{-31} kg
m_p = 1.66×10^{-27} kg
Hence, the numerical value of the given ratio is

$$\frac{ke^2}{Gm_e m_p} = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{6.67\times10^{-11}\times9.1\times10^{-3}\times1.67\times10^{-22}} \approx 2.3\times10^{39}$$

This is the ratio of electric force to the gravitational force between a proton and an electron, keeping distance between them constant.

14. (a) Explain the meaning of the statement 'electric charge of a body is quantised'. (b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Ans:

(a) Electric charge of a body is quantized. This means that only integral (1, 2, …., n) number of electrons can be transferred from one body to the other. Charges are not transferred in fraction. Hence, a body possesses total charge only in integral multiples of electric charge.

(b) In macroscopic or large scale charges, the charges used are huge as compared to the magnitude of electric charge. Hence, quantization of electric charge is of no use on macroscopic scale. Therefore, it is ignored and it is considered that electric charge is continuous.

15. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

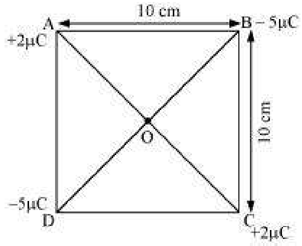
Ans:

Rubbing produces charges of equal magnitude but of opposite nature on the two bodies because charges are created in pairs. This phenomenon of charging is called charging by friction. The net charge on the system of two rubbed bodies is zero. This is because equal amount of opposite charges annihilate each other. When a glass rod is rubbed with a silk cloth, opposite natured charges appear on both the bodies. This phenomenon is in consistence with the law of conservation of energy. A similar phenomenon is observed with many other pairs of bodies.

16. Four point charges $q_A = 2\ \mu C$, $q_B = -5\ \mu C$, $q_C = 2\ \mu C$, and $q_D = -5\ \mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?

Ans:

The given figure shows a square of side 10 cm with four charges placed at its corners. O is the centre of the square.



where,

(Sides) AB = BC = CD = AD = 10 cm

(Diagonals) AC = BD = $10\sqrt{2}$ cm

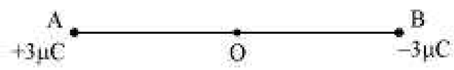
AO = OC = DO = OB = $5\sqrt{2}$ cm

A charge of amount 1μC is placed at point O.

Force of repulsion between charges placed at corner A and centre O is equal in magnitude but opposite in direction relative to the force of repulsion between the charges placed at corner C and centre O. Hence, they will cancel each other. Similarly, force of attraction between charges placed at corner B and centre O is equal in magnitude but opposite in direction relative to the force of attraction between the charges placed at corner D and centre O. Hence, they will also cancel each other. Therefore, net force caused by the four charges placed at the corner of the square on 1 μC charge at centre O is zero.

17. (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?
(b) Explain why two field lines never cross each other at any point?
Ans :
(a) An electrostatic field line is a continuous curve because a charge experiences a continuous force when traced in an electrostatic field. The field line cannot have sudden breaks because the charge moves continuously and does not jump from one point to the other.
(b) If two field lines cross each other at a point, then electric field intensity will show two directions at that point. This is not possible. Hence, two field lines never cross each other.

18. Two point charges $q_A = 3$ μC and $q_B = -3$ μC are located 20 cm apart in vacuum.
(a) What is the electric field at the midpoint O of the line AB joining the two charges?
(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?
Ans :
(a) The situation is represented in the given figure. O is the mid-point of line AB.
Distance between the two charges, AB = 20 cm
∴AO = OB = 10 cm
Net electric field at point O = E



Electric field at point O caused by +3μC charge,

$$E_1 = \frac{3\times10^{-6}}{4\pi\epsilon_0 (AO)^2} = \frac{3\times10^{-6}}{4\pi\epsilon_0 (10\times10^{-2})^2}\ \text{N/C} \quad \text{along OB}$$

Where,

$$\frac{1}{4\pi\epsilon_0} = 9\times10^9\ \text{N m}^2\,\text{C}^{-2}$$

ε_0 = Permittivity of free space
Magnitude of electric field at point O caused by −3μC charge,

$$E_2 = \left|\frac{-3\times10^{-6}}{4\pi\epsilon_0 (OB)^2}\right| = \frac{3\times10^{-6}}{4\pi\epsilon_0 (10\times10^{-2})^2}\ \text{N/C} \quad \text{along OB}$$

$\therefore$ E = $E_1 + E_2$

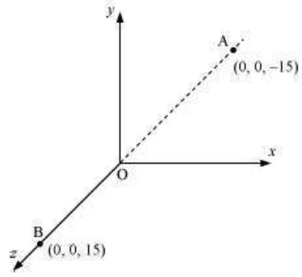
$$= 2\times[(9\times10^9)\times\frac{3\times10^{-6}}{(10\times10^{-2})^2}]$$

[Since the values of E1 and E2 are same, the value is multiplied with 2]

= 5.4×10^6 N/C along OB
Therefore, the electric field at mid-point O is 5.4×10^6 N C^{-1} along OB.
(b) A test charge of amount 1.5×10^{-9} C is placed at mid-point O.
$q = 1.5 \times 10^{-9}$ C
Force experienced by the test charge = F
∴F = qE
= $1.5 \times 10^{-9} \times 5.4 \times 10^6$
= 8.1×10^{-3} N

The force is directed along line OA. This is because the negative test charge is repelled by the charge placed at point B but attracted towards point A.
Therefore, the force experienced by the test charge is 8.1×10^{-3} N along OA.

19. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, −15 cm) and B: (0, 0, +15 cm), respectively. What are the total charge and electric dipole moment of the system?
Ans :
Both the charges can be located in a coordinate frame of reference as shown in the given figure.



At A, amount of charge, $q_A = 2.5 \times 10^{-7}$C
At B, amount of charge, $q_B = -2.5 \times 10^{-7}$ C
Total charge of the system,
$q = q_A + q_B \quad = 2.5 \times 10^{7}\ \mathrm{C} - 2.5 \times 10^{-7}\ \mathrm{C} \quad = 0$
Distance between two charges at points A and B,
d = 15 + 15 = 30 cm = 0.3 m
Electric dipole moment of the system is given by,
$p = q_A \times d = q_B \times d$
$= 2.5 \times 10^{-7} \times 0.3$
$= 7.5 \times 10^{-8}$ C m along positive z-axis
Therefore, the electric dipole moment of the system is 7.5×10^{-8} C m along positive z–axis.

20. An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5 × 104 N C^{-1}. Calculate the magnitude of the torque acting on the dipole.
Ans :
Electric dipole moment, $p = 4 \times 10^{-9}$ C m
Angle made by p with a uniform electric field, θ = 30°
Electric field, $E = 5 \times 10^{4}\ \mathrm{N\ C^{-1}}$
Torque acting on the dipole is given by the relation,
$\tau = pE \sin\theta$

$$= 4\times10^{-9}\times5\times10^{4}\times\sin 30 \qquad = 20\times10^{-5}\times\frac{1}{2} \qquad = \quad =10^{-4}\ \mathrm{N\,m}$$

Therefore, the magnitude of the torque acting on the dipole is 10^{-4} N m.

21. A polythene piece rubbed with wool is found to have a negative charge of 3 × 10−7 C.
(a) Estimate the number of electrons transferred (from which to which?)
(b) Is there a transfer of mass from wool to polythene?
Ans :
(a) When polythene is rubbed against wool, a number of electrons get transferred from wool to polythene. Hence, wool becomes positively charged and polythene becomes negatively charged.
Amount of charge on the polythene piece, $q = -3 \times 10^{-7}$ C
Amount of charge on an electron, $e = -1.6 \times 10^{-19}$ C
Number of electrons transferred from wool to polythene = n, n can be calculated using the relation, q = ne, $\therefore n = q/e = \{-3 \times 10^{-7}) / (-1.6 \times 10^{-19}) \quad = = 1.87 \times 10^{12}$
Therefore, the number of electrons transferred from wool to polythene is 1.87×10^{12}.
(b) Yes.

There is a transfer of mass taking place. This is because an electron has mass,
$m_e = 9.1 \times 10^{-3}$ kg
Total mass transferred to polythene from wool, $m = m_e \times n = 9.1 \times 10^{-31} \times 1.85 \times 10^{12}$
$= 1.706 \times 10^{-18}$ kg
Hence, a negligible amount of mass is transferred from wool to polythene.

22. (a) Two insulated charged copper spheres A and B have their centers separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.
(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?
Ans :
(a) Charge on sphere A, qA = Charge on sphere B, qB = 6.5 × 10−7 C
Distance between the spheres, r = 50 cm = 0.5 m
Force of repulsion between the two spheres,

$$F = \frac{q_A q_B}{4\pi \epsilon_0 r^2} \qquad F = \frac{9\times10^9 \times \left(6.5\times10^{-7}\right)^2}{(0.5)^2}$$

$= 1.52 \times 10^{-2}$ N
Therefore, the force between the two spheres is 1.52×10^{-2} N.
(b) After doubling the charge, charge on sphere A, q_A = Charge on sphere B, $q_B = 2 \times 6.5 \times 10^{-7}$ C $= 1.3 \times 10^{-6}$ C
The distance between the spheres is halved.
$\therefore$ r = 0.5/2 = 0.25 m
Force of repulsion between the two spheres,

$$F = \frac{q_A q_B}{4\pi \epsilon_0 r^2} \qquad = \frac{9\times10^9 \times 1.3\times10^{-6} \times 1.3\times10^{-6}}{(0.25)^2} \qquad = = 16 \times 1.52 \times 10^{-2}$$

= 0.243 N
Therefore, the force between the two spheres is 0.243 N.

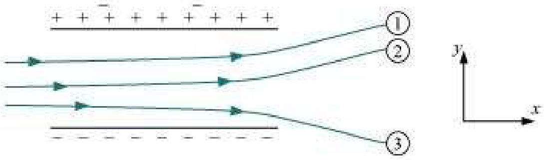
23. Suppose the spheres A and B in Exercise 22 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?
Ans :
Distance between the spheres, A and B, r = 0.5 m
Initially, the charge on each sphere, $q = 6.5 \times 10^{-7}$ C
When sphere A is touched with an uncharged sphere C, q/2 amount of charge from A will transfer to sphere C. Hence, charge on each of the spheres, A and C, is q/2.
When sphere C with charge q/2 is brought in contact with sphere B with charge q, total charges on the system will divide into two equal halves given as, (q/2 + q)/2 = 3q/4.

Each sphere will share each half. Hence, charge on each of the spheres, C and B, is 3q/4. Force of repulsion between sphere A having charge q/2 and sphere B having charge

$$\frac{3q}{4} = \frac{\frac{q}{2}\times\frac{3q}{4}}{4\pi\in_0 r^2} = \frac{3q^2}{8\times 4\pi\in_0 r^2} = 9\times10^9\times\frac{3\times(6.5\times10^{-7})^2}{8\times(0.5)^2} = 5.703\times10^{-3}\text{ N}$$

Therefore, the force of attraction between the two spheres is 5.703 × 10^{-3} N.

24. Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Ans :

Opposite charges attract each other and same charges repel each other. It can be observed that particles 1 and 2 both move towards the positively charged plate and repel away from the negatively charged plate. Hence, these two particles are negatively charged. It can also be observed that particle 3 moves towards the negatively charged plate and repels away from the positively charged plate. Hence, particle 3 is positively charged.

The charge to mass ratio (emf) is directly proportional to the displacement or amount of deflection for a given velocity. Since the deflection of particle 3 is the maximum, it has the highest charge to mass ratio.

25. Consider a uniform electric field E = 3 × 103 îN/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?

Ans :

(a) Electric field intensity, $\vec{E}$ = 3 × 10^3 î N/C

Magnitude of electric field intensity, $|\vec{E}|$ = 3 × 10^3 N/C

Side of the square, s = 10 cm = 0.1 m

Area of the square, A = s^2 = 0.01 m^2

The plane of the square is parallel to the y-z plane. Hence, angle between the unit vector normal to the plane and electric field, θ = 0°

Flux (Φ) through the plane is given by the relation,

$\Phi = |\vec{E}| A \cos\theta$

$= 3\times10^3\times0.01\times\cos 0°$

$= 30$ N m^2/C

(b) Plane makes an angle of 60° with the x-axis. Hence, θ = 60°

Flux, $\Phi = |\vec{E}| A \cos\theta$

$= 3\times10^3\times0.01\times\cos 60°$

$= 30\times1/2 = 15$ Nm^2/C

26. What is the net flux of the uniform electric field of Exercise 25 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Ans :

All the faces of a cube are parallel to the coordinate axes. Therefore, the number of field lines entering the cube is equal to the number of field lines piercing out of the cube. As a result, net flux through the cube is zero.

27. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^{3} N m^2/C. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Ans :

(a) Net outward flux through the surface of the box, $\Phi = 8.0 \times 10^{3}$ N m^2/C

For a body containing net charge q, flux is given by the relation,

$\Phi = q/\varepsilon_0$

$= 8.854 \times 10^{-12} \times 8.0 \times 10^{3}$

$= 7.08 \times 10^{-8}$

$= 0.07$ μC

Therefore, the net charge inside the box is 0.07 μC.

(b) No

Net flux piercing out through a body depends on the net charge contained in the body. If net flux is zero, then it can be inferred that net charge inside the body is zero. The body may have equal amount of positive and negative charges.

28. A point charge +10 μC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge 10 cm.)

Ans :

The square can be considered as one face of a cube of edge 10 cm with a centre where charge q is placed. According to Gauss's theorem for a cube, total electric flux is through all its six faces.

$\Phi_{Total} = q/\varepsilon_0$

Hence, electric flux through one face of the cube i.e., through the square, $\Phi = \Phi_{Total}/6$

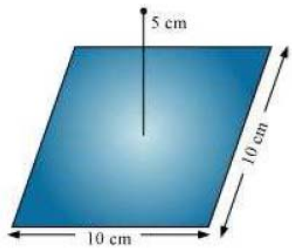
$= 1/6\ (q/\varepsilon_0)$

Where,

ϵ_0 = Permittivity of free space

$= 8.854 \times 10^{-12}$ N−1C^2 m^{-2}

$q = 10$ μC $= 10 \times 10^{-6}$ C



$$\therefore \phi = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.854 \times 10^{-12}}$$

$= 1.88 \times 10^{5}$ N m^2 C^{-1}

Therefore, electric flux through the square is 1.88×10^{5} N m^2 C^{-1}.

29. A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Ans :

Net electric flux (ΦNet) through the cubic surface is given by, $\Phi_{Net} = q/\varepsilon_0$

Where,

ϵ_0 = Permittivity of free space

$= 8.854 \times 10^{-12}$ N−1C^2 m^{-2}

q = Net charge contained inside the cube = 2.0 μC = 2×10^{-6} C

$$\therefore \phi_{Net} = \frac{2\times10^{-6}}{8.854\times10^{-12}}$$

= 2.26×10^{5} N m^2 C^{-1}

The net electric flux through the surface is 2.26×10^{5} N m^2C^{-1}.

30. A point charge causes an electric flux of -1.0×10^3 Nm^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

Ans :

(a) Electric flux, $\Phi = -1.0 \times 10^3$ N m^2/C

Radius of the Gaussian surface,

r = 10.0 cm

Electric flux piercing out through a surface depends on the net charge enclosed inside a body. It does not depend on the size of the body. If the radius of the Gaussian surface is doubled, then the flux passing through the surface remains the same i.e., -10^3 N m^2/C.

(b) Electric flux is given by the relation,

$\Phi = q/\varepsilon_0$

Where,

q = Net charge enclosed by the spherical surface

ϵ_0 = Permittivity of free space = 8.854×10^{-12} $N^{-1}C^2$ m^{-2}

$\therefore q = \Phi\, \varepsilon_0$

$= -1.0 \times 10^3 \times 8.854 \times 10^{-12}$

$= -8.854 \times 10^{-9}$ C

= −8.854 nC

Therefore, the value of the point charge is −8.854 nC.

31. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?

Ans :

Electric field intensity (E) at a distance (d) from the centre of a sphere containing net charge q is given by the relation,

$$E = \frac{q}{4\pi \epsilon_0 d^2}$$

Where,

q = Net charge = 1.5×10^3 N/C

d = Distance from the centre = 20 cm = 0.2 m

ϵ_0 = Permittivity of free space

And, $1/4\pi\varepsilon_0 = 9 \times 10^9$ N m^2 C^{-2}

$$q = E(4\pi \epsilon_0)d^2$$

$$= \frac{1.5\times10^3\times(0.2)^2}{9\times10^9}$$

= 6.67×10^9 C

= 6.67 nC
Therefore, the net charge on the sphere is 6.67 nC.

32. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m^2. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?
Ans :
(a) Diameter of the sphere, d = 2.4 m
Radius of the sphere, r = 1.2 m
Surface charge density, $\sigma = 80.0\ \mu C/m^2 = 80 \times 10^{-6}\ C/m^2$
Total charge on the surface of the sphere,
Q = Charge density × Surface area
$= \sigma\, 4\pi r^2$
$= 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2$
$= 1.447 \times 10^{-3}$ C
Therefore, the charge on the sphere is 1.447×10^{-3} C.
(b) Total electric flux (Φ_{Total}) leaving out the surface of a sphere containing net charge Q is given by the relation,
$\Phi_{Total} = Q/\varepsilon_0$
Where, ε_0
= Permittivity of free space = $8.854 \times 10^{-12}\ N^{-1}C^2\ m^{-2}$, $Q = 1.447 \times 10^{-3}$ C

$$\phi_{Total} = \frac{1.44 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$= 1.63 \times 10^{8}\ N\ C^{-1}\ m^2$
Therefore, the total electric flux leaving the surface of the sphere is $1.63 \times 10^{8}\ N\ C^{-1} m^2$.

33. An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density.
Ans :
Electric field produced by the infinite line charges at a distance d having linear charge density λ is given by the relation,

$$E = \frac{\lambda}{2\pi \epsilon_0 d} \qquad \lambda = 2\pi \epsilon_0 dE$$

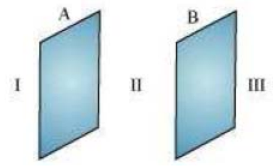
Where,
d = 2 cm = 0.02 m
$E = 9 \times 10^4$ N/C
ε_0 = Permittivity of free space

$$\lambda = \frac{0.02 \times 9 \times 10^4}{2 \times 9 \times 10^9}$$

= 10 μC/m
Therefore, the linear charge density is 10 μC/m.

34. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} C/m^2. What is E: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

Ans :
The situation is represented in the following figure.



A and B are two parallel plates close to each other. Outer region of plate A is labelled as I, outer region of plate B is labelled as III, and the region between the plates, A and B, is labelled as II.
Charge density of plate A, $\sigma = 17.0 \times 10^{-22}$ C/m^2
Charge density of plate B, $\sigma = -17.0 \times 10^{-22}$ C/m^2
In the regions, I and III, electric field E is zero. This is because charge is not enclosed by the respective plates.
Electric field E in region II is given by the relation, $E = \sigma/\varepsilon_0$
Where,
$\therefore \varepsilon_0$ = Permittivity of free space = 8.854×10^{-12} N−1C^2 m^{-2}

$$E = \frac{17.0\times10^{-22}}{8.854\times10^{-12}}$$

= 1.92×10^{-10} N/C
Therefore, electric field between the plates is 1.92×10^{-10} N/C.

35. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 N C^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. ($g = 9.81$ m s^{-2}; $e = 1.60 \times 10^{-19}$ C).
Ans :
Excess electrons on an oil drop, n = 12
Electric field intensity, $E = 2.55 \times 10^4$ N C^{-1}
Density of oil, $\rho = 1.26$ gm/cm^3 = 1.26×10^3 kg/m^3
Acceleration due to gravity, $g = 9.81$ m s^{-2}
Charge on an electron, $e = 1.6 \times 10^{-19}$ C
Radius of the oil drop = r
Force (F) due to electric field E is equal to the weight of the oil drop (W)
F = W
$Eq = mg$ or $Ene = (4/3)\pi r^3 \times \rho \times g$

Where,
q = Net charge on the oil drop = ne
m = Mass of the oil drop

= Volume of the oil drop × Density of oil = $(4/3)\pi r^3 \times \rho$

$$\therefore r = \sqrt[3]{\frac{3Ene}{4\pi\rho g}} = \sqrt[3]{\frac{3\times2.55\times10^{4}\times12\times1.6\times10^{-19}}{4\times3.14\times1.26\times10^{3}\times9.81}}$$

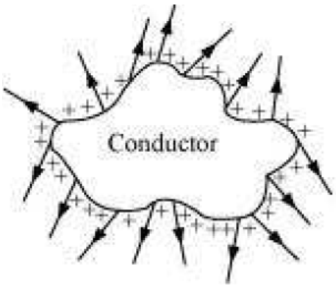
$$= \sqrt[3]{946.09\times10^{-21}} \quad = 9.82\times10^{-7}\ \text{m}$$

= 9.82×10^{-4} mm
Therefore, the radius of the oil drop is 9.82×10^{-4} mm.

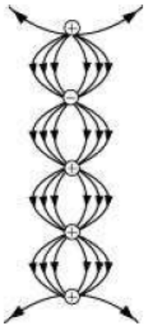
36. Which among the curves shown in Fig. 1.35 cannot possibly represent electrostatic

field lines?

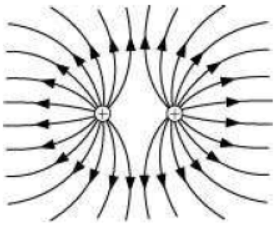
(a)



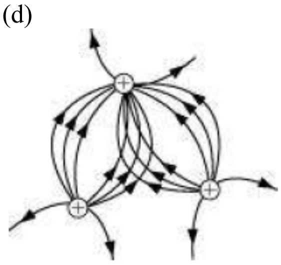
(b)



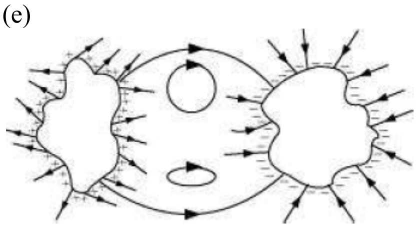
(c)



(d)



(e)



Ans :

(a) The field lines showed in (a) do not represent electrostatic field lines because field lines must be normal to the surface of the conductor.

(b) The field lines showed in (b) do not represent electrostatic field lines because the field lines cannot emerge from a negative charge and cannot terminate at a positive charge.

(c) The field lines showed in (c) represent electrostatic field lines. This is because the field lines emerge from the positive charges and repel each other.

(d) The field lines showed in (d) do not represent electrostatic field lines because the field lines should not intersect each other.
(e) The field lines showed in (e) do not represent electrostatic field lines because closed loops are not formed in the area between the field lines.

37. In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative z-direction?
Ans :
Dipole moment of the system, $p = q \times dl = -10^{-7}$ C m
Rate of increase of electric field per unit length,

$$\frac{dE}{dl} = 10^{+5}\ N\ C^{-1}$$

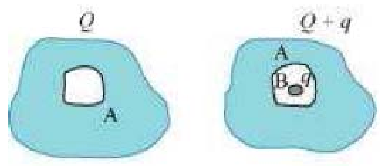
Force (F) experienced by the system is given by the relation,
$F = qE$

$$F = q\frac{dE}{dl} \times dl \qquad = p \times \frac{dE}{dl}$$

$= -10^{-7} \times 10^{-5}$
$= -10^{-2}$ N
The force is -10^{-2} N in the negative z-direction i.e., opposite to the direction of electric field. Hence, the angle between electric field and dipole moment is 180°.
Torque (τ) is given by the relation,
$\tau = pE \sin 180°$
$= 0$
Therefore, the torque experienced by the system is zero.

38. (a) A conductor A with a cavity as shown in Fig. (a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is Q + q [Fig. (b)]. (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.
Ans :
(a) Let us consider a Gaussian surface that is lying wholly within a conductor and enclosing the cavity. The electric field intensity E inside the charged conductor is zero.
Let q is the charge inside the conductor and ε_0 is the permittivity of free space. According to Gauss's law,
Flux,

$$\phi = \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$$



(a) (b)

Here, E = 0, means $q/\varepsilon_0 = 0$; since $\varepsilon_0 \neq 0$, $q = 0$
Therefore, charge inside the conductor is zero.
The entire charge Q appears on the outer surface of the conductor.
(b) The outer surface of conductor A has a charge of amount Q. Another conductor B having charge +q is kept inside conductor A and it is insulated from A. Hence, a charge of amount −q

will be induced in the inner surface of conductor A and +q is induced on the outer surface of conductor A. Therefore, total charge on the outer surface of conductor A is Q + q.
(c) A sensitive instrument can be shielded from the strong electrostatic field in its environment by enclosing it fully inside a metallic surface. A closed metallic body acts as an electrostatic shield.

39. (a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where E = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.
(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.
Ans :
(a) Let the equilibrium of the test charge be stable. If a test charge is in equilibrium and displaced from its position in any direction, then it experiences a restoring force towards a null point, where the electric field is zero. All the field lines near the null point are directed inwards towards the null point. There is a net inward flux of electric field through a closed surface around the null point. According to Gauss's law, the flux of electric field through a surface, which is not enclosing any charge, is zero. Hence, the equilibrium of the test charge can be stable.
(b) Two charges of same magnitude and same sign are placed at a certain distance. The mid-point of the joining line of the charges is the null point. When a test charged is displaced along the line, it experiences a restoring force. If it is displaced normal to the joining line, then the net force takes it away from the null point. Hence, the charge is unstable because stability of equilibrium requires restoring force in all directions.

Multiple Choice Type Questions :

1.1 A glass rod rubbed with silk acquires a charge of $+8 \times 10^{-12}$C. The number of electrons it has gained or lost
(a) 5 × 10 −7 (gained) (b) 5 × 107 (lost) (c) 2 × 10 −8 (lost) (d) –8 × 10 −12 (lost)

1.2 The electrostatic force between two point charges kept at a distance d apart, in a medium ε_r = 6, is 0.3 N. The force between them at the same separation in vacuum is
(a) 20 N (b) 0.5 N (c) 1.8 N (d) 2 N

1.3 Electic field intensity is 400 V m^{-1} at a distance of 2 m from a point charge. It will be 100 V m^{-1} at a distance?
(a) 50 cm (b) 4 cm (c) 4 m (d) 1.5 m

1.4 Two point charges +4q and +q are placed 30 cm apart. At what point on the line joining them the electric field is zero?
(a) 15 cm from the charge q (b) 7.5 cm from the charge q
(c) 20 cm from the charge 4q (d) 5 cm from the charge q

1.5 A dipole is placed in a uniform electric field with its axis parallel to the field. It experiences

(a) only a net force (b) only a torque
(c) both a net force and torque (d) neither a net force nor a torque

1.6 If a point lies at a distance x from the midpoint of the dipole, the electric potential at this point is proportional to

(a) $\frac{1}{x^2}$ (b) $\frac{1}{x^3}$

(c) $\frac{1}{x^4}$ (d) $\frac{1}{x^{3/2}}$

1.7 Which of the following quantities is scalar?
(a) dipole moment (b) electric force (c) electric field (d) electric potential

1.8 The unit of permittivity is

(a) $C^2 N^{-1} m^{-2}$ (b) $N m^2 C^{-2}$

(c) $H m^{-1}$ (d) $N C^{-2} m^{-2}$

1.9 The number of electric lines of force originating from a charge of 1 C is
(a) 1.129×10^{11} (b) 1.6×10^{-19} (c) 6.25×10^{18} (d) 8.85×10^{12}

1.10 The electric field outside the plates of two oppositely charged plane sheets of charge density σ is

(a) $\frac{+\sigma}{2\varepsilon o}$ (b) $\frac{-\sigma}{2\varepsilon o}$

(c) $\frac{\sigma}{\varepsilon o}$ (d) zero

1.11 A hollow metal ball carrying an electric charge produces no electric field at points
(a) outside the sphere (b) on its surface
(c) inside the sphere (d) at a distance more than twice

Most Likely Questions : 8 M (1 M - 1Q; 2M-1Q; 5M-1Problem)

1 Mark Question
(1) Draw the electric field lines for a system of two positive point charges.
(2) A cube encloses a charge of 1 C. What is the electric flux through the surface of the cube?

2 Mark Question
(1) Write two properties of an electric charge.
(2) State Coulomb's law in electrostatics and explain it in the case of free space.

5 Mark Question
(1) Obtain the expression for the electric field at a point on the equatorial plane of an electric dipole.

5 Mark Problem
(1) The electrostatic force on a metal sphere of charge 0.4 μC due to another identical metal sphere of charge -0.8 μC in air is 0.2N. Find the distance between the two spheres and also the force between the same two spheres when they are brought into contact and then replaced in their initial positions.

(2) Two point charges q_1 and q_2, of magnitude $+10^{-8}$ C and -10^{-8} C, respectively placed at A and B edges of an equatorial triangle, are placed 0.1 m apart. Calculate the electric fields at points A, B and C of the edges of a triangle.

$f = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$	1 mark
$0.2 = \frac{9\times10^9\ 0.4\times10^{-6}\ 0.8\times10^{-6}}{r^2}$	1 mark
$r = 0.12m$	1 mark
after contact $f = \frac{9\times10^9\ 0.2\times10^{-6}\ 0.2\times10^{-6}}{0.12^2} = 0.025N(repulsive)$	1 mark
Final answer with unit f = 0.025N	1mark

Chapter 2:
ELECTROSTATIC POTENTIAL AND CAPACITANCE
8M
2M - 1Q ; 3M - 2 Q or 3M -1Q; 5M-1Q (NP)

2.1 Electric potential: Definition of electric potential at a point -
Potential energy of charge q at a point (in the presence of field due to any charge configuration) *is the work done by the external force* (equal and opposite to the electric force) *in bringing the charge q from infinity to that point.*

Work done by an external force in bringing a unit positive charge from infinity to a point = electrostatic potential (V) at that point.

In other words, the electrostatic potential (V) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point.

Electric potential is a physical quantity which determines the flow of charges from one body to another. It is a physical quantity that determines the degree of electrification of a body.
SI unit of electric potential is volt (V) or J C^{-1} or Nm C^{-1}. Electric potential at a point is one volt if one joule of work is done in moving one coulomb charge from infinity to that point in the electric field.

2.2 Definition of potential difference -
Electric Potential Difference between any two points in the electric field is defined as the work done in moving (without any acceleration) a unit positive charge from one point to the other against the electrostatic force irrespective of the path followed.

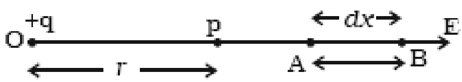
$$W_{AB} = -\int_A^B \vec{E}.\vec{dl} = \frac{q\,q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \text{ or } \frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$\frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\,\frac{1}{r_B} - \frac{q}{4\pi\varepsilon_0}\,\frac{1}{r_A} = V_B - V_A$$

$$\therefore\ \boldsymbol{V_B - V_A = \Delta V = \frac{W_{AB}}{q_0}}$$

1. Electric potential and potential difference are scalar quantities.
2. Electric potential at infinity is zero.
3. Electric potential near an isolated positive charge (q > 0) is positive and that near an isolated negative charge (q < 0) is negative.
4. cgs unit of electric potential is stat volt. 1 stat volt = 1 erg / stat coulomb

2.3 Derivation of electric potential due to a point charge -
Let +q be an isolated point charge situated in air at O. P is a point at a distance r from +q. Consider two points
A and B at distances x and $x + dx$ from the point O (Fig.).



The potential difference between A and B is, dV = −E dx
The force experienced by a unit positive charge placed at A is

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{x^2}$$

$$\therefore \quad dV = -\frac{1}{4\pi\varepsilon_o}\frac{q}{x^2} \cdot dx$$

The negative sign indicates that the work is done against the electric force.
The electric potential at the point P due to the charge $+q$ is the total work done in moving a unit positive charge from infinity to that point.

$$V = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_o x^2} \cdot dx = \frac{q}{4\pi\varepsilon_o r}$$

2.4 Expression for electric potential due a short electric dipole at any point -

Consider an electric dipole consists of two charges q and $-q$ separated by a (small) distance $2a$. Its total charge is zero. It is characterised by a dipole moment vector $\vec{p}$ whose magnitude is $q \times 2a$ and which points in the direction from $-q$ to q (Fig. 2.5).

We know that the electric field of a dipole at a point with position vector $\vec{r}$ depends not just on the magnitude r, but also on the angle between $\vec{r}$ and $\vec{p}$. Further, the field falls off, at large distance, not as $1/r^2$ (typical of field due to a single charge) but as $1/r^3$. To determine the electric potential due to a dipole, we take the origin at the centre of the dipole. Now we know that the electric field obeys the superposition principle. The potential due to the dipole is the sum of potentials due to the charges q and $-q$ *is given by*

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$$

where r_1 and r_2 are the distances of the point P from q and $-q$, respectively.

Now, by geometry,

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta \quad \text{and} \quad r_2^2 = r^2 + a^2 + 2ar\cos\theta$$

We take r much greater than a (r >>1) and retain terms only upto the first order in a/r

$$\cong r^2\left(1 - \frac{2a\cos\theta}{r}\right)$$

Similarly,

$$r_2^2 \cong r^2\left(1 + \frac{2a\cos\theta}{r}\right)$$

Using the Binomial theorem and retaining terms upto the first order in a/r ; we obtain,

$$\frac{1}{r_1} \cong \frac{1}{r}\left(1 - \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)$$

$$\frac{1}{r_2} \cong \frac{1}{r}\left(1 + \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)$$

Using Eqs. (2.9) and (2.13) and $p = 2qa$, we get

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$\mathbf{V = \frac{p \cos\theta}{4\pi\varepsilon_0 r^2}}$ -------------- ( )

Now, $p \cos\theta = \vec{p}\,\hat{r}$

where $\hat{\mathbf{r}}$ is the unit vector along the position vector $\overrightarrow{OP}$. The electric potential of a dipole is then given by

$\mathbf{V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\,\hat{r}}{r^2}}$ when (r >> a) -------- ( )

Equation (2.15) is, as indicated, approximately true only for distances large compared to the size of the dipole, so that higher order terms in a/r are negligible. For a point dipole $\vec{p}$ at the origin, Eq. (2.15) is, however, exact.

From Eq. (2.15), potential on the dipole axis ($\theta = 0, \pi$) is given by

$$V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$ ----------- ( )

(Positive sign for $\theta = 0$, negative sign for $\theta = \pi$.) The potential in the equatorial plane ($\theta = \pi/2$) is zero. Thus

(i) The potential due to a dipole depends not just on r but also on the angle between the position vector $\vec{r}$ and the dipole moment vector $\vec{p}$.

(ii) The electric dipole potential falls off, at large distance, as $1/r^2$, not as $1/r$, characteristic of the potential due to a single charge.

2.5 Comparison of the variation of electric potential with distance between a point charge and an electric dipole -

(i) The potential due to a dipole depends not just on r but also on the angle between the position vector $\vec{r}$ and the dipole moment vector $\vec{p}$.

(ii) The electric dipole potential falls off, at large distance, as $1/r^2$, not as $1/r$, characteristic of the potential due to a single charge.

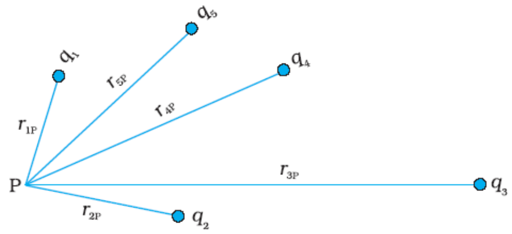
2.6 Application of superposition principle to find electric potential due to a system of charges.

Electrical potential due to system of point charges :

Consider a system of charges $q_1, q_2,\ldots, q_n$ with position vectors $\mathbf{r}_1, \mathbf{r}_2,\ldots\mathbf{r}_n$ relative to some origin (Fig.). The potential V_1 at P due to the charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$ -------- (1)

where r_{1P} is the distance between q_1 and P.



Similarly, the potential V_2 at P due to q_2 and V_3 due to q_3 are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}},\quad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}}$$ -------- (2)

where r_{2P} and r_{3P} are the distances of P from charges q_2 and q_3, respectively; and so on for the potential due to other charges. By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges

$V = V_1 + V_2 + \ldots.. + V_n$ ---------- (3)

$$\boxed{= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \ldots\ldots + \frac{q_n}{r_{nP}}\right)} \quad \text{------------} \quad (4)$$

2.7 Equipotential surfaces: Properties -

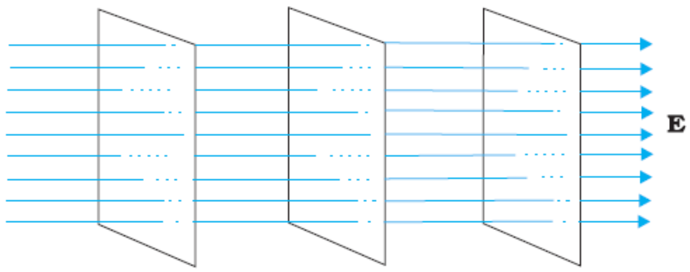
An equipotential surface is a surface with a constant value of potential at all points on the surface. For a single charge q, the potential is given by Eq.

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

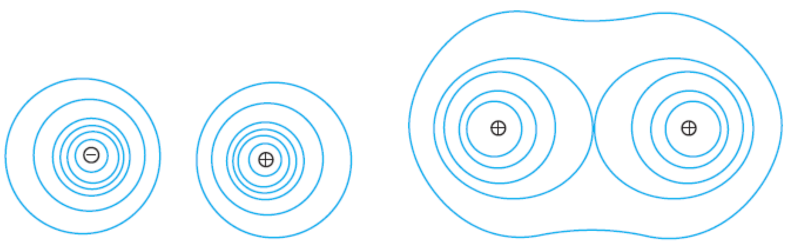
This shows that V is a constant if r is constant. Thus, equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge.

For any charge configuration, equipotential surface through a point is normal to the electric field at that point.

(1) Equipotential surfaces for a uniform electric field :



(2) Some equipotential surfaces for (a) a dipole, (b) two identical positive charges :



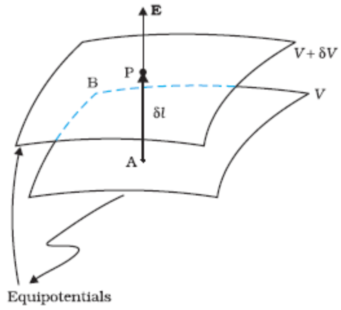
(a) (b)

Properties :

(1) No work is done in moving a test charge from one point to another on an equipotential surface.
(2) The electric field is always perpendicular to the element dl of the equipotential surface.
(3) Equipotential surfaces indicates regions of strong or weak electric fields.
(4) Equipotential surfaces cannot intersect.

2.8 Derivation of the relation between electric field and potential,

Consider two closely spaced equipotential surfaces A and B (Fig.) with potential values V and $V + \delta V$, where δV is the change in V in the direction of the electric field **E**. Let P be a point on the surface B. δl is the perpendicular distance of the surface A from P. Imagine that a unit positive charge is moved along this perpendicular from the surface B to surface A against the electric field. The work done in this process is $|\mathbf{E}|\delta l$.



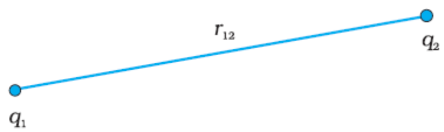
This work equals the potential difference $V_A - V_B$.
Thus,
$|\mathbf{E}|\delta l = V(V + \delta V) = -\delta V$
i.e., $|\mathbf{E}| = -\dfrac{\delta V}{\delta l}$ -------- (1)
Since dV is negative, $\delta V = -|\delta V|$. we can rewrite Eq (1) as
$|\mathbf{E}| = -\dfrac{\delta V}{\delta l} = +\dfrac{|\delta V|}{\delta l}$ -------- (2)

Relation between electric field and potential:
(i) Electric field is defined as negative potential gradient. **E = - dV/dl**
(ii) *Electric field is in the direction in which the potential decreases steepest.*
(iii) *The magnitude of electric field is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.*

2.9 Electric potential energy: Definition of electric potential energy of a system of charges -

Case 1 - Potential Energy due to two charges :
Consider two charges $q1$ and $q2$ with position vector $\vec{r}_1$ and $\vec{r}_2$ relative to some origin. Consider the charges $q1$ and $q2$ initially at infinity and determine the work done by an external agency to bring the charges to the given locations. Suppose, first the charge $q1$ is brought from infinity to the point $\vec{r}_1$. There is no external field against which work needs to be done, so work done in bringing $q1$ from infinity to $\vec{r}_1$ is zero. This charge produces a potential in space given by

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}} \quad \text{-------- (1)}$$



where r_1P is the distance of a point P in space from the location of q_1. From the definition of potential, work done in bringing charge q_2 from infinity to the point $\vec{r}_2$ is q_2 times the potential at $\vec{r}_2$ due to q_1:

$$\text{Work done on } q_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} \quad \text{------- (2)}$$

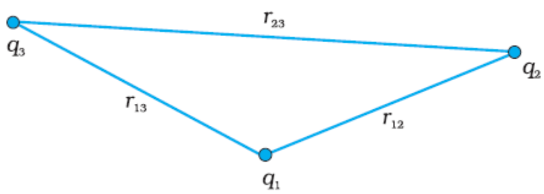
where r_{12} is the distance between points 1 and 2.
Since electrostatic force is conservative, this work gets stored in the form of potential energy of the system. Thus, the potential energy of a system of two charges q_1 and q_2 is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} \quad \text{----------- (3)}$$

Obviously, if q_2 was brought first to its present location and q_1 brought later, the potential energy U would be the same.

Case 2 : Potential Energy due to three charges :
Let us calculate the potential energy of a system of three charges q_1, q_2 and q_3 located at $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$, respectively. To bring q_1 first from infinity to $\vec{r}_1$, no work is required. Next we bring q_2 from infinity to $\vec{r}_2$. As before, work done in this step is

$$q_2 V_1(\mathbf{r}_2) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} \quad \text{--------- (4)}$$



The charges q_1 and q_2 produce a potential, which at any point P is given by

$$V_{1,2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}}\right) \quad \text{---------} \quad (5)$$

Work done next in bringing q_3 from infinity to the point $\vec{r}_3$ is q_3 times $V_{1,2}$ at $\vec{r}_3$

$$q_3 V_{1,2}(\mathbf{r}_3) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \quad \text{---------} \quad (6)$$

The total work done in assembling the charges at the given locations is obtained by adding the work done in different steps [Eq. (4) and Eq. (6)],

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \quad \text{---------} \quad (7)$$

The potential energy is characteristic of the present state of configuration, and not the way the state is achieved.

2.10 Derivation of electric potential energy of a system of two point charges in the absence of external electric field -

Refer Case 1 above

2.11 Mention of expression for electric potential energy of a system of two point charges in presence of external electric field.

Case 1 : Potential energy of a single charge in presence of external electric field :

The external electric field **E** and the corresponding external potential V may vary from point to point. By definition, V at a point P is the work done in bringing a unit positive charge from infinity to the point P. Thus, work done in bringing a charge q from infinity to the point P in the external field is qV. This work is stored in the form of potential energy of q. If the point P has position vector **r** relative to some origin, we can write:

Potential energy of q at **r** in an external field

$= qV(\mathbf{r})$ --------- (1)

where $V(\mathbf{r})$ is the external potential at the point **r**.

Thus, if an electron with charge $q = e = 1.6\times10^{-19}$ C is accelerated by a potential difference of $\Delta V = 1$ volt, it would gain energy of $q\Delta V = 1.6 \times 10^{-19}$J. This unit of energy is defined as 1 *electron volt* or 1eV, i.e., 1 eV=1.6×10^{-19}J.

Case 2 : Potential energy of a system of two point charges in presence of external electric field :

The potential energy of a system of two charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$, respectively, in an external field can be calculated by knowing the work done in bringing the charge q_1 from infinity to $\vec{r}_1$.

Work done in this step is $q_1\, V(\vec{r}_1)$, using Eq. (1). Next, we consider the work done in bringing q_2 to $\vec{r}_2$. In this step, work is done not only against the external field $\vec{E}$ but also against the field due to q_1.

Work done on q_2 against the external field $= q_2\, V(\vec{r}_2)$

Work done on q_2 against the field due to q_1

$$= \frac{q_1 q_2}{4\pi\varepsilon_o r_{12}} \quad \text{---------} \quad (2)$$

where r_{12} is the distance between q_1 and q_2. We have made use of Eqs. (2.27) and (2.22). By the superposition principle for fields, we add up the work done on q_2 against the two fields ($\vec{E}$ and that due to q_1):

Work done in bringing q_2 to $\vec{r}_2$

$$= q_2 V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_o r_{12}} \quad \text{-----------} \quad (3)$$

Thus,

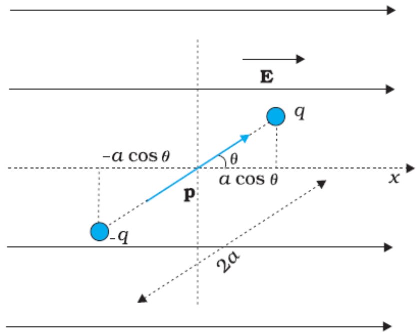
Potential energy of the system = the total work done in assembling the configuration

$$\boxed{= q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}} \quad \text{-----------} \quad (4)$$

2.12 Mention of the expression for the electric potential energy of an electric dipole placed in a uniform electric field.

Consider a dipole with charges $q_1 = +q$ and $q_2 = -q$ placed in a uniform electric field $\vec{E}$, as shown in Fig. In a uniform electric field, the dipole experiences no net force; but experiences a torque τ given by

$\tau = \vec{p} \times \vec{E}$ ---------- (1)



which will tend to rotate it (unless $\vec{p}$ is parallel or antiparallel to E). Suppose an external torque τ_{ext} is applied in such a manner that it just neutralises this torque and rotates it in the plane of paper from angle θ_0 to angle θ_1 at an infinitesimal angular speed and *without angular acceleration*. The amount of work done by the external torque will be given by

$$W = \int_{\theta_0}^{\theta_1} \tau_{ext}(\theta) d\theta = \int_{\theta_0}^{\theta_1} pE \sin\theta \, d\theta$$

$$= pE(\cos\theta_0 - \cos\theta_1) \quad \text{----------} \quad (2)$$

This work is stored as the potential energy of the system. We can then associate potential energy $U(\theta)$ with an inclination θ of the dipole. By choosing the angle $\theta_0 = \pi / 2$, the potential energy U is made to be zero.

$$U(\theta) = pE\left(\cos\frac{\pi}{2} - \cos\theta\right) = -pE\cos\theta \quad = -\vec{p} \cdot \vec{E} \quad \text{--------} \quad (3)$$

If we consider a system of two charges $+q$ and $-q$. The potential energy expression then reads

$$U'(\theta) = q[V(\mathbf{r}_1) - V(\mathbf{r}_2)] - \frac{q^2}{4\pi\varepsilon_0 \times 2a} \quad \text{------------} \quad (4)$$

Here, $\vec{r}_1$ and $\vec{r}_2$, denote the position vectors of $+q$ and $-q$. Now, the potential difference between positions $\vec{r}_1$ and $\vec{r}_2$ equals the work done in bringing a unit positive charge against field from $\vec{r}_2$ to $\vec{r}_1$. The displacement parallel to the force is $2a\cos\theta$. Thus, $[V(\vec{r}_1) - V(\vec{r}_2)] = -E \times 2a\cos\theta$. We thus obtain,

$$U'(\theta) = -pE\cos\theta - \frac{q^2}{4\pi\varepsilon_0 \times 2a} \quad = -\vec{p} \cdot \vec{E} \quad - \frac{q^2}{4\pi\varepsilon_0 \times 2a}$$

Note that $U'(\theta)$ differs from $U(\theta)$ by

a quantity which is just a constant for a given dipole.
Since a constant is insignificant for potential energy, we can drop the second term in above Eq. and it then reduces to Eq. (3).

2.13 Electrostatics of conductors -

Conductors contain mobile charge carriers. In metallic conductors, these charge carriers are electrons. In a metal, the outer (valence) electrons part away from their atoms and are free to move. These electrons are free within the metal but not free to leave the metal. The free electrons form a kind of 'gas'; they collide with each other and with the ions, and move randomly in different directions. In an external electric field, they drift against the direction of the field. The positive ions made up of the nuclei and the bound electrons remain held in their fixed positions. In electrolytic conductors, the charge carriers are both positive and negative ions; but the situation in this case is more involved – the movement of the charge carriers is affected both by the external electric field as also by the so-called chemical forces. The following points are important regarding behaviour of conductors in electrostatics field in case of metallic solid conductors :

(1) Inside a conductor, electrostatic field is zero

When a conductor is placed in an electrostatic field, the charges (free electrons) drift towards the positive plate leaving the + ve core behind. At an equilibrium, the electric field due to the polarisation becomes equal to the applied field. So, the net electrostatic field inside the conductor is zero.

(2) At the surface of a charged conductor, electrostatic field must be normal to the surface at every point

Suppose the electric field is acting at an angle other than 90°, then there will be a component E cos θ acting along the tangent at that point to the surface which will tend to accelerate the charge on the surface leading to 'surface current'. But there is no surface current in electrostatics. So, θ = 90°and cos 90°= 0.

(3) The interior of a conductor can have no excess charge in the static situation (Net charge in the interior of a conductor is zero.)

A neutral conductor has equal amounts of positive and negative charges in every small volume or surface element. When the conductor is charged, the excess charge can reside only on the surface in the static situation.
This follows from the Gauss's law. Consider any arbitrary volume element v inside a conductor. On the closed surface S bounding the volume element v, electrostatic field is zero. Thus the total electric flux through S is zero. Hence, by Gauss's law, there is no net charge enclosed by S.

(4) Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface

Since **E** = 0 inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface. That is, there is no potential difference between any two points inside or on the surface of the conductor. Hence, the result.

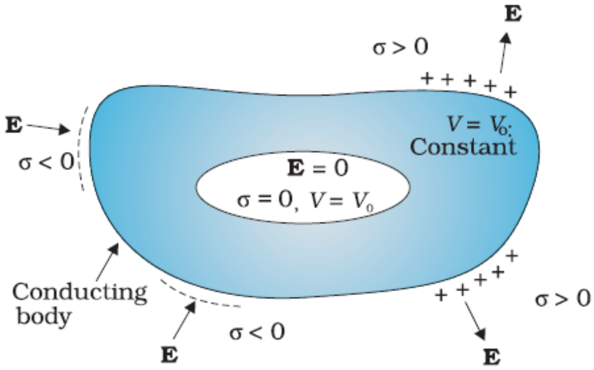
(5) Electric field at the surface of a charged conductor

$$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$$

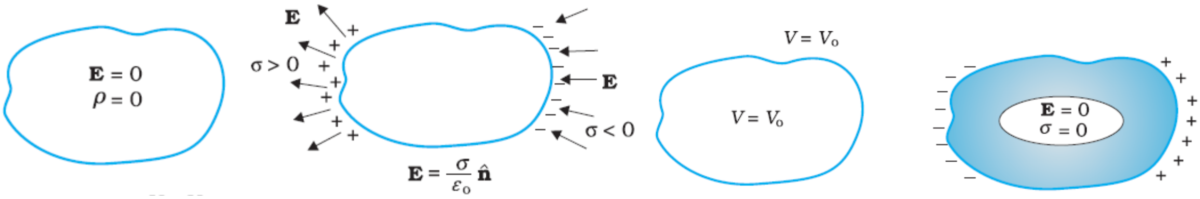
where σ is the surface charge density and ˆ**n** is a unit vector normal to the surface in the outward direction.

(6) Electrostatic shielding

The electric field inside a cavity of any conductor is zero. All charges reside only on the outer surface of a conductor with cavity. (There are no charges placed in the cavity). Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence: *the field inside the cavity is always zero*. This is known as *electrostatic shielding*. The effect can be made use of in protecting sensitive instruments from outside electrical influence.



The following figure gives a summary of the important electrostatic properties of a conductor.



2.14 Dielectrics and electric polarisation: Polar and nonpolar dielectrics and their behavior in the absence and presence of an external electric field.

Dielectrics are non-conducting substances. In contrast to conductors, they have no (or negligible number of) charge carriers.

When a conductor is placed in an external electric field, the free charge carriers move and charge distribution in the conductor adjusts itself in such a way that the electric field due to induced charges opposes the external field within the conductor. This happens until, in the static situation, the two fields cancel each other and the net electrostatic field in the conductor is zero.

In a dielectric, this free movement of charges is not possible. It turns out that the external field induces dipole moment by stretching or re-orienting molecules of the dielectric. The collective effect of all the molecular dipole moments is net charges on the surface of the dielectric which produce a field that opposes the external field. Unlike in a conductor, however, the opposing field so induced does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of the dielectric.

To understand the effect in dielectrics, we need to look at the charge distribution of a dielectric at the molecular level.
The molecules of a substance may be polar or non-polar. In a non-polar molecule, the centres of positive and negative charges coincide. The molecule then has no permanent (or intrinsic) dipole moment. Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2) molecules which, because of their symmetry, have no dipole moment. On the other hand, a polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). Such molecules have a permanent dipole moment. An ionic molecule such as HCl or a molecule of water (H_2O) are examples of polar molecules.

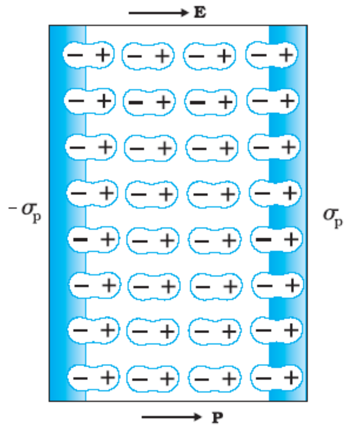
(a) Non-polar molecules in External Electric Field :
In an external electric field, the positive and negative charges of a nonpolar molecule are displaced in opposite directions. The displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force (due to internal fields in the molecule). The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarised by the external field.
If we consider only the simple situation when the induced dipole moment is in the direction of the field and is proportional to the field strength. The induced dipole moments of different molecules add up giving a net dipole moment of the dielectric in the presence of the external field.

(b) Polar molecules in External Electric Field :
A dielectric with polar molecules also develops a net dipole moment in an external field, but for a different reason. In the absence of any external field, the different permanent dipoles are oriented randomly due to thermal agitation; so the total dipole moment is zero. When an external field is applied, the individual dipole moments tend to align with the field. When summed over all the molecules, there is then a net dipole moment in the direction of the external field, i.e., the dielectric is polarised. The extent of polarisation depends on the relative strength of two mutually opposite factors: the dipole potential energy in the external field tending to align the dipoles with the field and thermal energy tending to disrupt the alignment. There may be, in addition, the 'induced dipole moment' effect as for non-polar molecules, but generally the alignment effect is more important for polar molecules.

Thus in above either case, whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called *polarisation* and is denoted by **P**. For linear isotropic dielectrics,
$\mathbf{P} = \chi_c \mathbf{E}$ --------- (1)
where χ_c is a constant characteristic of the dielectric and is known as the *electric susceptibility* of the dielectric medium.
It is possible to relate χ_c to the molecular properties of the substance,

The polarised dielectric modifies the original external field inside it. To prove this, let us consider a rectangular dielectric slab placed in a uniform external field $\mathbf{E}_0$ parallel to two of its faces. The field causes a uniform polarisation **P** of the dielectric. Thus every volume element Δv of the slab has a dipole moment $\mathbf{P}\,\Delta v$ in the direction of the field. The volume element Δv is

macroscopically small but contains a very large number of molecular dipoles. Anywhere inside the dielectric, the volume element Δv has no net charge (though it has net dipole moment). This is, because, the positive charge of one dipole sits close to the negative charge of the adjacent dipole.
However, at the surfaces of the dielectric normal to the electric field, there is evidently a net charge density. As seen in Fig, the positive ends of the dipoles remain unneutralised at the right surface and the negative ends at the left surface. The unbalanced charges are the induced charges due to the external field.



Thus the polarised dielectric is equivalent to two charged surfaces with induced surface charge densities, say σ_p and $-\sigma_p$. Clearly, the field produced by these surface charges opposes the external field. The total field in the dielectric is, thereby, reduced from the case when no dielectric is present. We should note that the surface charge density $\pm\sigma_p$ arises from bound charges (not free charges) in the dielectric.

If E_0 is applied field and E_P is the induced field in the dielectric, the net field in the dielectric = $E_N = E_0 - E_P$.

2.15 Capacitors and capacitance :

A capacitor is a system of two conductors separated by an insulator. The two conductors have charges Q and $-Q$, with potential difference $V = V_1 - V_2$ between them.
The electric field in the region between the conductors is proportional to the charge Q. That is, if the charge on the capacitor is, say doubled, the electric field will also be doubled at every point.
The potential difference *V between two plates* V is also proportional to Q, and the ratio Q/V is a constant: $Q \propto V$ or $Q = CV$

$C = Q/V$ ------- (1)

The constant C is called the *capacitance* of the capacitor. C is independent of Q or V, as stated above. The capacitance C depends only on the geometrical configuration (shape, size, separation) of the system of two conductors.

The SI unit of capacitance is 1 farad (=1 coulomb volt^{-1}) or 1 F = 1 C V^{-1}. A capacitor with fixed capacitance is symbolically shown as --||--.
The charge of the capacitor leaks away due to the reduction in insulating power of the intervening medium.
The maximum electric field that a dielectric medium can withstand without break-down (of its insulating property) is called its *dielectric strength*; for air it is about 3×10^6 Vm^{-1}. For a separation between conductors of the order of 1 cm or so, this field corresponds to a potential difference of 3×10^4 V between the conductors. Thus, for a capacitor to store a large amount of charge without leaking, its capacitance should be high enough so that the potential difference and hence the electric field do not exceed the break-down limits. Put differently, there is a limit to the amount of charge that can be stored on a given capacitor without significant leaking. In practice, a farad is a very big unit; the most common units are its sub-multiples 1 μF = 10^{-6} F, 1 nF = 10^{-9} F, 1 pF = 10^{-12} F, etc.

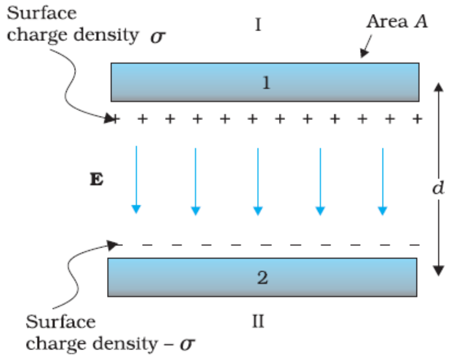
2.16 Derivation of the capacitance of a capacitor without dielectric medium :

Parallel plate capacitor :

A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance (Fig.). We first take the intervening medium between the plates to be vacuum. Let A be the area of each plate and d the separation between them. The two plates have charges Q and $-Q$. Since d is much smaller than the linear dimension of the plates ($d^2 << A$),

Plate 1 has surface charge density $\sigma = Q/A$ and plate 2 has a surface charge density $-\sigma$. Using earlier result of the electric field in different regions is: Outer region I (region above the plate 1),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$



Outer region II (region below the plate 2),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \qquad \text{-----------} \quad (1)$$

The direction of electric field is from the positive to the negative plate.

Thus, the electric field is localised between the two plates and is uniform throughout. For plates with finite area, this will not be true near the outer boundaries of the plates. The field lines bend outward at the edges – an effect called **'fringing of the field'**. Hence σ will not be strictly uniform on the entire plate.

However, for $d^2 << A$, these effects can be ignored in the regions sufficiently far from the edges, and the field there is given by Eq. (1). Now for uniform electric field, potential difference is simply the electric field times the distance between the plates, that is,

$$V = E\,d = \frac{1}{\varepsilon_0}\frac{Qd}{A} \qquad \text{------------} \quad (2)$$

The capacitance C of the parallel plate capacitor is then

$$\boxed{C = \frac{Q}{V} = = \frac{\varepsilon_0 A}{d}} \qquad \text{------------} \quad (3)$$

which, as expected, depends only on the geometry of the system. For typical values like $A = 1$ m^2, $d = 1$ mm, we get

$$C = \frac{8.85\times10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}\times 1\,\mathrm{m^2}}{10^{-3}\,\mathrm{m}} = 8.85\times10^{-9}\,\mathrm{F}$$

To show that 1F is too big unit,

$$A = \frac{Cd}{\varepsilon_0} = \frac{1\mathrm{F}\times10^{-2}\,\mathrm{m}}{8.85\times10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}} = 10^9\,\mathrm{m^2}$$

which is a plate about 30 km in length and breadth!

2.17 Mention of expression for capacitance of a capacitor with dielectric medium -

Effect of Dielectric on Capacitance :

If we consider a capacitor made of two large plates, each of area A, separated by a distance d. The charge on the plates is $\pm Q$, corresponding to the charge density $\pm\sigma$ (with $\sigma = Q/A$). When there is vacuum between the plates,

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

and the potential difference V_0 is

$V_0 = E_0 d$

The capacitance C_0 in this case is

$$\boxed{C_0 = \frac{Q}{V_0} = \varepsilon_0 \frac{A}{d}} \qquad \text{----------} \quad (1)$$

Consider next a dielectric inserted between the plates fully occupying the intervening region. The dielectric is polarised by the field and the effect is equivalent to two charged sheets (at the surfaces of the dielectric normal to the field) with surface charge densities σ_p and $-\sigma_p$. The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$.

That is,

$$E = \frac{\sigma - \sigma_P}{\varepsilon_0}$$

so that the potential difference across the plates is

$$V = E\,d = \frac{\sigma - \sigma_P}{\varepsilon_0}d \qquad \text{--------} \quad (2)$$

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e., to σ. Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write

$$\sigma - \sigma_P = \frac{\sigma}{K} \qquad \text{---------} \quad (3)$$

where K is a constant characteristic of the dielectric. Clearly, $K > 1$. We then have

$$V = \frac{\sigma d}{\varepsilon_0 K} = \frac{Qd}{A\varepsilon_0 K} \qquad \text{----------} \quad (4)$$

The capacitance C, with dielectric between the plates, is then

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon_0 KA}{d}} \qquad \text{--------} \quad (5)$$

The product $\varepsilon_0 K$ is called the *permittivity* of the medium and is denoted by ε

$$\varepsilon = \varepsilon_0 K$$

For vacuum $K = 1$ and $\varepsilon = \varepsilon_0$; ε_0 is called the *permittivity of the vacuum*.

2.18 Dielectric constant :

The dimensionless ratio

$$\boxed{K = \frac{\varepsilon}{\varepsilon_0}}$$

is called the *dielectric constant* of the substance. From Eq. (3), it is clear that K is greater than 1. From Eqs. (1) and (5),

$$K = \frac{C}{C_0}$$

Thus, the **dielectric constant** of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.

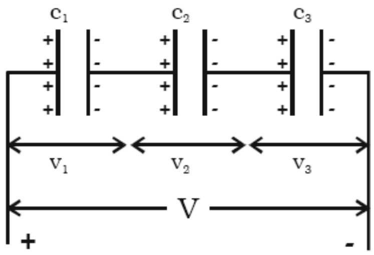
2.19 Combination of capacitors: Derivation of effective capacitance of two capacitors (a) in series combination and (b) in parallel combination :

We can combine several capacitors of capacitance C_1, C_2,…, C_n to obtain a system with some effective capacitance C. The effective capacitance depends on the way the individual capacitors are combined.

1. Capacitors in series :

Figure (a) shows capacitors C_1 and C_2 combined in series. The left plate of C_1 and the right plate of C_2 are connected to two terminals of a battery and have charges Q and $–Q$, respectively. It then follows that the right plate of C_1 has charge $–Q$ and the left plate of C_2 has charge Q.

This would result in an electric field in the conductor connecting $C_1$and C_2. Charge would flow until the net charge on both C_1 and C_2 is zero and there is no electric field in the conductor connecting C_1 and C_2. Thus, in the series combination, charges on the two plates ($\pm Q$) are the same on each capacitor. The total potential drop V across the combination is the sum of the potential drops V_1 and V_2 across C_1 and C_2, respectively.



$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} \qquad \text{---------} \quad (1)$$

$$\text{i.e.,} \quad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}, \qquad \text{----------} \quad (2)$$

If we consider the combination as an effective capacitor with charge Q and potential difference V. The *effective capacitance* of the combination is

$$C = \frac{Q}{V} \qquad \text{----------} \quad (3)$$

By comparing Eq. (2) with Eq. (3), and obtain

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}} \qquad \text{--------} \quad (4)$$

If we connect n number of capacitors in Series the effective capacitance of combination of *n* capacitors:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}}$$

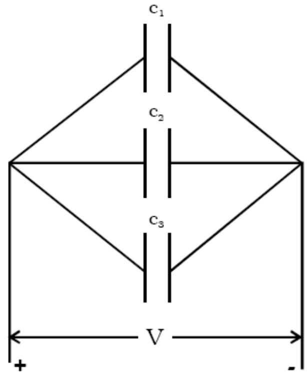
2. Capacitors in parallel

Figure (a) shows two capacitors arranged in parallel. In this case, the same potential difference is applied across both the capacitors. But the plate charges ($\pm Q_1$) on capacitor 1 and the plate charges ($\pm Q_2$) on the capacitor 2 are not necessarily the same:

$Q_1 = C_1 V$, $Q_2 = C_2 V$ --------- (1)

The equivalent capacitor is one with charge

$Q = Q_1 + Q_2$ ---------- (2)



and potential difference V.

$Q = CV = C_1V + C_2V$ ---------- (3)

The effective capacitance C is,

$\boldsymbol{C = C_1 + C_2}$ ---------- (4)

The general formula for effective capacitance C for parallel combination of n capacitors is as follows :

$Q = Q_1 + Q_2 + ... + Q_n$ ----------- (5)

i.e., $CV = C_1V + C_2V + ... C_nV$ --------- (6)

which gives

$\boldsymbol{C = C_1 + C_2 + ... + C_n}$ ---------- (7)

2.20 Energy stored in a capacitor.

The capacitor is a charge storage device. Work has to be done to store the charges in a capacitor. This work done is stored as electrostatic potential energy in the capacitor.

Let q be the charge and V be the potential difference between the plates of the capacitor. If dq is the additional charge given to the plate, then work done is, $dw = Vdq$

$$dw = \frac{q}{C}dq \qquad \left(\because V = \frac{q}{C}\right)$$

Total work done to charge a capacitor is

$$w = \int dw = \int_0^q \frac{q}{C}\,dq = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electrostatic potential energy (U) in the capacitor.

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \qquad (\because q = CV)$$

This energy is recovered if the capacitor is allowed to discharge. When the capacitor discharges, this stored-up energy is released. It is possible to view the potential energy of the capacitor as 'stored' in the electric field between the plates. To see this, consider for simplicity, a parallel plate capacitor [of area A(of each plate) and separation d between the plates].

Energy stored in the capacitor

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{(A\sigma)^2}{2} \times \frac{d}{\varepsilon_0 A}$$

The surface charge density σ is related to the electric field E between the plates, $\boldsymbol{E = \sigma/\varepsilon_0}$

From above two Eqs., we get, Energy stored in the capacitor $\boldsymbol{U = (1/2)\,\varepsilon_0 E^2 \times Ad}$

Note that Ad is the volume of the region between the plates (where electric field alone exists). If we define *energy density as energy stored per unit volume of space*, then above Eq. shows that Energy density of electric field, $\boldsymbol{U = (1/2)\,\varepsilon_0 E^2}$

2.21 Van de Graaff generator: Principle, labeled diagram and use,

Van de Graaff generator is a machine that can build up high voltages of the order of a few million volts (10^7 V). The resulting large electric fields are used to accelerate charged particles (electrons, protons, ions) to high energies needed for experiments to probe the small scale structure of matter.

Principle : The working of Van de Graaff generator is based on the principle of electrostatic induction and action of points.

Suppose we have a large spherical conducting shell of radius *R*, on which we place a charge *Q*. This charge spreads itself uniformly all over the sphere. The field outside the sphere is just that of a point charge *Q* at the centre; while the field inside the sphere vanishes. So the potential outside is that of a point charge; and inside it is constant, namely the value at the radius *R*. We thus have: Potential inside conducting spherical shell of radius *R* carrying charge *Q* = constant.

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \qquad \text{------------} \quad (1)$$

Now, let us suppose that in some way we introduce a small sphere of radius *r*, carrying some charge *q*, into the large one, and place it at the centre (fig. (a). The potential due to this new charge clearly has the following values at the radii indicated: Potential due to small sphere of radius *r* carrying charge *q*

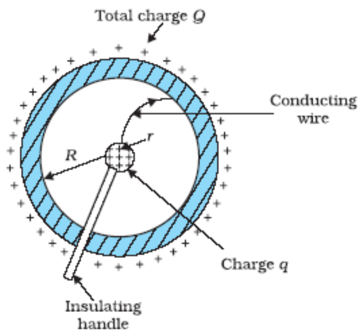
$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ at surface of small sphere $\qquad = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ at large shell of radius R.

Taking both charges *q* and *Q* into account we have for the total potential *V* and the potential difference the values

$$V(R) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

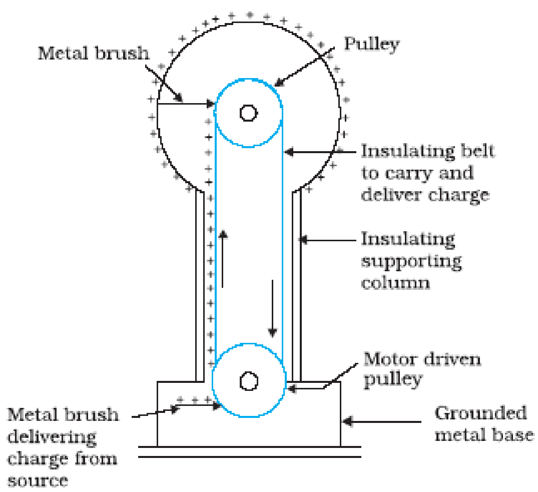
$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

$$V(r) - V(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$



Assume now that *q* is positive. We see that, independent of the amount of charge *Q* that may have accumulated on the larger sphere and even if it is positive, the inner sphere is always at a higher potential: the difference *V*(*r*)–*V*(*R*) is positive. The potential due to *Q* is constant upto radius *R* and so cancels out in the difference! This means that if we now connect the smaller and larger sphere by a wire, the charge *q* on the former will immediately flow onto the matter, even though the charge *Q* may be quite large. The natural tendency is for positive charge to move from higher to lower potential. Thus, provided we are somehow able to introduce the small charged sphere into the larger one, we can in this way keep piling up larger and larger amount of charge on the latter. The potential (Eq. 1) at the outer sphere would also keep rising, at least until we reach the breakdown field of air.

Using this principle, the van de Graaff generator capable of building up potential difference of a few million volts, and fields close to the breakdown field of air which is about 3×10^6 V/m. A schematic diagram of the van de Graaff generator is given in Fig.



Construction :

A large spherical conducting shell (of few metres radius) is supported at a height several meters above the ground on an insulating column. A long narrow endless belt insulating material, like rubber or silk, is wound around two pulleys –one at ground level, one at the centre of the shell. To avoid the leakage of charges from the sphere, the generator is enclosed in the steel tank filled with air or nitrogen at very high pressure (15 atmospheres).

Working :
The belt is kept continuously moving by a motor driving the lower pulley. It continuously carries positive charge, sprayed on to it by a brush at ground level, to the top. There it transfers its positive charge to another conducting brush connected to the large shell. Thus positive charge is transferred to the shell, where it spreads out uniformly on the outer surface. In this way, voltage differences of as much as 6 or 8 million volts (with respect to ground) can be built up.

Uses :
Van de Graaff Generator is used to produce very high potential difference (of the order of several million volts) for accelerating charged particles.
(1) The beam of accelerated charged particles are used to trigger nuclear reactions.
(2) The beam is used to break atoms for various experiments in Physics.
(3) In medicine, such beams are used to treat cancer.
(4) It is used for research purposes.

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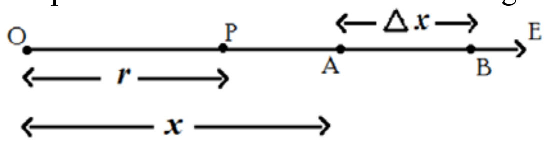
Question Bank :

1) What do you mean by the conservative nature of electric field?
The conservative nature of electric field means that the work done to move a charge from one point to another point in electric field is independent of path, but it depends only on the initial and final positions of the charge.

2) Define Electrostatic Potential.
Electrostatic potential at a point in field is defined as the work done to move a unit positive charge, without any acceleration from infinity to that point at consideration. If is the work done to move a charge from infinity to a point, then the electric potential at that point is V = W/q.
SI Unit is J C^{-1} = volt (V)

3) What is the SI unit of Electric Potential?
SI Unit of Electric Potential is J C^{-1} = volt (V)

4) Derive the expression for electric potential at a point due to a point charge.
Consider a point charge Q at origin O. Let P be a point at a distance *r* from it. Let A and B be two points at a distance and from O along the line OP.



The force experienced by unit positive charge (+1C) at A

$$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{QX1}{x^2}\hat{x} \qquad \dots\dots \quad (1)$$

Where $\hat{x}$ is unit vector along OA
The work done to move a unit positive charge (+1C) from B to A is

$$\Delta W = \vec{F}\,.\,\overrightarrow{\Delta x} = F\,\Delta x\cos\theta$$

Here, $\vec{F}$ and $\overrightarrow{\Delta x}$ are opposite, therefore θ = 180º

$$\Delta W = F\,\Delta x\cos 180 = F\,\Delta x\,(-1)$$
$$\Delta W = -F\,\Delta x$$

From eq (1), we get

$$\Delta W = -\frac{1}{4\pi\varepsilon_o}\frac{QX1}{x^2}\Delta x$$

The potential at a point P is work done to move unit positive charge (+1C) from infinity to P, therefore

$$V = \int_\infty^r -\frac{1}{4\pi\varepsilon_o}\frac{QX1}{x^2}\,dx = -\frac{Q}{4\pi\varepsilon_o}\int_\infty^r \frac{1}{x^2}\,dx$$
$$V = -\frac{Q}{4\pi\varepsilon_o}\left(\frac{-1}{x}\right)_\infty^r = \frac{Q}{4\pi\varepsilon_o}\,\frac{1}{r}$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$$

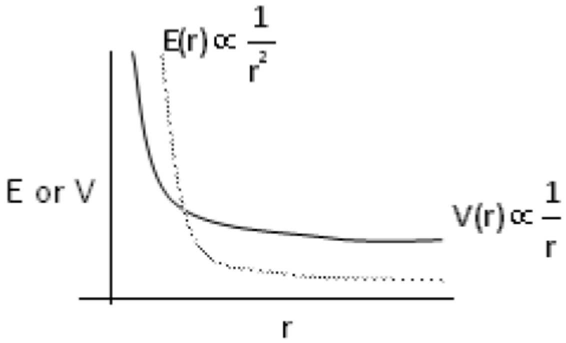
5) How does the electric field and electric potential vary with distance from a point charge?

The electric field

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\hat{r} \Rightarrow E\ \alpha\ \frac{1}{r^2}$$

The electric potential

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} \Rightarrow V\ \alpha\ \frac{1}{r}$$



Electric field is defined as negative potential gradient. **E = - dV/dl**

6) Write the expression for electric potential at a point due to an electric dipole and hence obtain the expression for the same at any point on its axis and any point on its equatorial plane.

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{p\cos\theta}{r^2}\right)$$

Where p = 2 aq is the electric dipole moment.
r = distance of the point from the centre of the dipole.
θ = the angle between $\vec{p}$ and $\vec{r}$.
For any point on the dipole axis, θ = 0 or π => cos θ = ± 1, we get

$$V = \pm\frac{1}{4\pi\varepsilon_o}\left(\frac{p}{r^2}\right)$$

For any point on the equatorial plane, θ = π/2, => cos θ = 0, we get **V = 0**

7) How does the electric potential at a point due to an electric dipole vary with distance measured from its centre? Compare the same for a point charge.

For an electric dipole (at large distances), we have

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{p\cos\theta}{r^2}\right) \Rightarrow V\ \alpha\ \frac{1}{r^2}$$

The electric potential varies inversely with the square of the distance.
For a point charge,

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r}\right) \Rightarrow V\ \alpha\ \frac{1}{r}$$

The electric potential varies inversely with the distance.

8) Using superposition principle, write the expression for electric potential at a point due to a system of charges.

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots \frac{q_n}{r_n}\right)$$

Where q_1, q_2, q_3 ------ are the point charges and r_1, r_2, r_3 ------ are the distances of the point from the respective point charges.

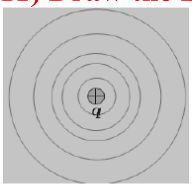
9) What is an equipotential surface? Give an example.

An Equipotential surface is a surface with same potential at all points on it. The surface of a charged conductor is an example.

10) What are the equipotential surfaces of a point charge?

The Equipotential surfaces of a point charge are the concentric spheres with centre at the point charge.

11) Draw the Equipotential surfaces for a point charge.



12) Give the condition for equipotential surface in terms of the direction of the electric field.

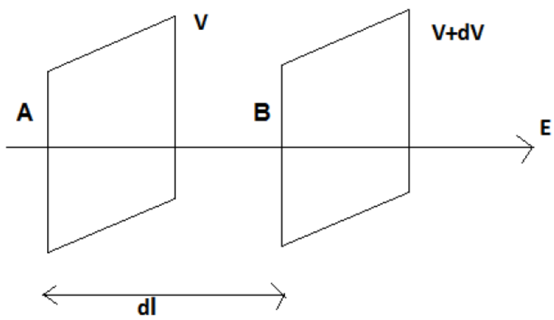
The electric field is always perpendicular to the equipotential surface.

13) Explain why the equipotential surface is normal to the direction of the electric field at that point.

If the Equipotential surface is not normal to the direction of the electric field at a point, then electric field will have a non- zero components along the surface and due to this work must be done to move a unit positive charge against this field component. This means that there is potential difference between two points on the surface. This contradicts the definition of equipotential surface.

14) Obtain the relation between the electric field and potential. OR Show that electric field is in the direction in which the potential decreases steepest.

Consider two equipotential surfaces A and B with the potential difference between them dV as shown in figure. Let dl be the perpendicular distance between them and $\vec{E}$ be the electric field normal to these surfaces.



The work done to move a unit positive charge from B to A against the field $\vec{E}$ through a displacement $\vec{dl}$ is

$$dW = \vec{E}\,.\vec{dl} = E\,dl\cos\pi = -E\,dl$$

This is equal to the potential difference , therefore

$$dV = dW \Rightarrow dV = -E\,dl$$

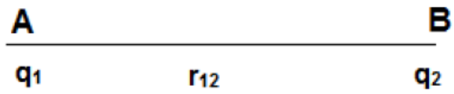
$$E = -\frac{dV}{dl}$$

15) Define Electrostatic Potential energy of a system of charges.

Electrostatic potential energy of a system of charges is defined as the work done to move the charges from infinity to their present configuration.

16) Derive the expression for potential energy of two point charges in the absence of external electric field.

Consider a system of two point charges q_1, q_2 separated by a distance r_{12} as shown in figure.



The work done to move q_1 from infinity to A is,
$W_1 = 0$
($\because$There is no initial electric field)
The work done to move q_2 from infinity to B is,
$W_2 = V_{1B}\, q_2$
Where V_{1B} is the electric potential at B due to q_1, it is given by

$$V_{1B} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_{12}}\right)$$

$$\Rightarrow W_2 = V_{1B} q_2 = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_{12}}\right) q_2$$

$$W_2 = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1 q_2}{r_{12}}\right)$$

The potential energy of this system of charge is equal to total work done to build this configuration. Therefore

$$U = W_1 + W_2 = 0 + \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1 q_2}{r_{12}}\right)$$

$$\Rightarrow U = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1 q_2}{r_{12}}\right)$$

17) Write the expression for potential energy of two point charges in the absence of external electric field.

Electrostatic potential energy is

$$U = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1 q_2}{r_{12}}\right)$$

Where q_1, q_2 are the point charges and r_{12} is the distance between them.

18) Write the expression for potential energy of two point charges in the presence of external electric field.

Electrostatic potential energy is

$$\Rightarrow U = q_1V_1 + q_2V_2 + \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1q_2}{r_{12}}\right)$$

Where q_1, q_2 are the point charges, V1 and V2 are potentials at the positions of q_1, q_2 respectively and r_{12} is the distance between them.

19) Mention the expression for potential energy of an electric dipole placed in an uniform electric field. Discuss its maximum and minimum values.

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Where p = dipole moment. E = Electric field. θ = Angle between dipole axis and electric field.
When the dipole axis is perpendicular to the field
θ = 90° ⇨ $U = 0$ (minimum)
P E is minimum (zero).

When the dipole axis is parallel to the field
θ = 90° ⇨ $U = 0$ (maximum)
P E is maximum.

20) Explain why Electric field inside a conductor is always zero. Otherwise free electrons would experience force and drift causing electric current.

21) Explain why Electrostatic field is always normal to the surface of charged conductor.
If $\vec{E}$ is not normal, it will have component parallel to the surface causing surface currents.

22) Explain why Electric charges always reside on the surface of a charge conductor.
Because, If there are static charges inside the conductor, Electric field can be present inside it which is not true.

23) **Explain why Electrostatic potential is constant throughout the volume.**
Because the Electric field inside the conductor is zero, therefore no work is done to move a charge against field and there is no potential difference between any two points. This means Electrostatic potential is constant throughout the volume.

24) **Write the expression for Electric field near the surface of a charge conductor.**
Electric field at the surface of a charged conductor is

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$\hat{n}$ is unit vector normal to the surface.

25) What is Electrostatic shielding? Mention one use of it.
The field inside the cavity of a conductor is always zero and it remains shielded from outside electric influence. This is known as electrostatic shielding. This property is used in protecting sensitive instruments from outside electrical influence.

26) What are Dielectrics? Mention the types of Dielectrics.

Dielectrics are non-conducting substances. They have no charge carriers. There are two types of Dielectrics, polar Dielectrics and non-polar Dielectrics.

27) What are non-polar Dielectrics? Give examples.
The non-polar Dielectrics are those in which the centers of positive and negative charges coincide. These molecules then have no permanent (or intrinsic) dipole moment. Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2).

28) What are polar Dielectrics? Give examples.
The non-polar Dielectrics are those in which the centers of positive and negative charges are separated (even when there is no external field). Such molecules have a permanent (or intrinsic) dipole moment. Examples of non-polar molecules are HCl and molecule of water (H_2O).

29) What happens when Dielectrics are placed in an electric field?
Both polar and non-polar dielectrics develop a net dipole moment along the external field when placed in it.

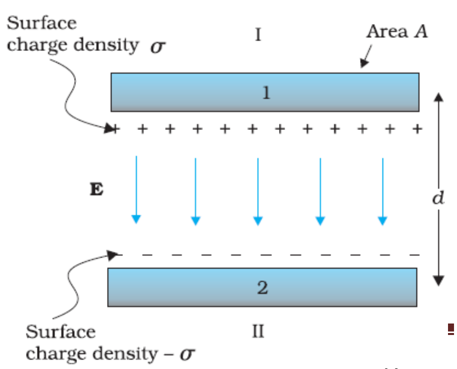
30) What is electric polarisation?
When Dielectric is placed in an external electric field, a net dipole moment is developed along it. The dielectric is now said to be polarised. The dipole moment acquired per unit volume is known as polarisation $\vec{p}$.

31) Define capacitance of a capacitor. What is SI unit?
Capacitance of a capacitor is defined as ratio of the charge Q on it to the potential difference V across its plates. C = Q/V
S.I. Unit = CV^{-1} = Farad (F)

32) What is Parallel plate capacitor?
Parallel plate capacitor is a capacitor with two identical plane parallel plates separated by a small distance and the space between them is filled by an dielectric medium.

33) Derive the expression for capacitance of a Parallel plate capacitor without any dielectric medium between the plates. *(or Parallel plate air capacitor).*
Consider a parallel plate capacitor without any dielectric medium (vacuum) between the plates. Let A be the area of the plates and d be the plate separation. The two plates have charges Q and $-Q$. Since d is much smaller than the linear dimension of the plates ($d^2 << A$),
Plate 1 has surface charge density σ = Q/A and plate 2 has a surface charge density – σ. Using earlier result of the electric field in different regions is: Outer region I (region above the plate 1),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$



Outer region II (region below the plate 2),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \text{-----------} \quad (1)$$

The direction of electric field is from the positive to the negative plate.
Thus, the electric field is localised between the two plates and is uniform throughout. For plates with finite area, this will not be true near the outer boundaries of the plates. The field lines bend outward at the edges – an effect called **'fringing of the field'**. Hence σ will not be strictly uniform on the entire plate.
However, for $d^2 << A$, these effects can be ignored in the regions sufficiently far from the edges, and the field there is given by Eq. (1). Now for uniform electric field, potential difference is simply the electric field times the distance between the plates, that is,

$$V = Ed = \frac{1}{\varepsilon_0}\frac{Qd}{A} \quad \text{------------} \quad (2)$$

The capacitance C of the parallel plate capacitor is then

$$\boxed{C = \frac{Q}{V} = = \frac{\varepsilon_0 A}{d}} \quad \text{------------} \quad (3)$$

which, as expected, depends only on the geometry of the system.

34) Mention the expression for capacitance of a Parallel plate capacitor without any dielectric medium between the plates.

$$\boxed{C = \frac{Q}{V} = = \frac{\varepsilon_0 A}{d}}$$

Where ε_0 = Permittivity of free space. A = Area of the plates. D = Plate separation.

35) Mention the expression for capacitance of a Parallel plate capacitor with a dielectric medium between the plates.

$$C = \frac{\varepsilon_0 KA}{d}$$

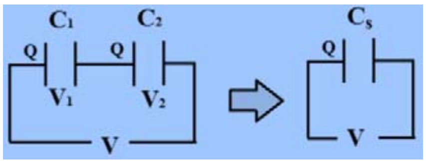
Where ε_0 = Permittivity of free space.
K = dielectric constant of the medium between the plates.
A = Area of the plates.
D = Plate separation.

36) Define Dielectric constant of a substance.

Dielectric constant of a substance is defined as the ratio of the permittivity of the medium to the permittivity of free space.
$K = \frac{\varepsilon}{\varepsilon_0}$

37) Derive the expression for effective capacitance of two capacitors connected in series.
Consider two capacitors of capacitance C_1 and C_2 connected in series.

In series combination charge Q is same on each capacitor and potential across the combination is equal to sum of the potential across each. Therefore $V = V_1 + V_2$



$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} \quad \text{---------} \quad (1)$$

i.e., $\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$, ---------- (2)

If we consider the combination as an effective capacitor with charge Q and potential difference V. The *effective capacitance* of the combination is

$$C = \frac{Q}{V} \quad \text{----------} \quad (3)$$

By comparing Eq. (2) with Eq. (3), and obtain

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}} \quad \text{--------} \quad (4) \qquad \text{or} \qquad C_S = \frac{C_1 C_2}{C_1 + C_2}$$

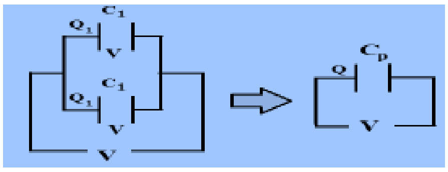
If we connect n number of capacitors in Series the effective capacitance of combination of n capacitors:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}}$$

38) Derive the expression for effective capacitance of two capacitors connected in parallel.

Fig. shows two capacitors arranged in parallel. In this case, the same potential difference is applied across both the capacitors. But the plate charges ($\pm Q_1$) on capacitor 1 and the plate charges ($\pm Q_2$) on the capacitor 2 are not necessarily the same:

$Q_1 = C_1 V$, $Q_2 = C_2 V$ --------- (1)



The equivalent capacitor is one with charge

$Q = Q_1 + Q_2$ ---------- (2)

and potential difference V.

$Q = CV = C_1 V + C_2 V$ ---------- (3)

The effective capacitance C is,

$C = C_1 + C_2$ ---------- (4)

The general formula for effective capacitance C for parallel combination of n capacitors is as follows :

$Q = Q_1 + Q_2 + \ldots + Q_n$ ----------- (5)

i.e., $CV = C_1 V + C_2 V + \ldots C_n V$ --------- (6)

which gives

$C = C_1 + C_2 + \ldots + C_n$ ---------- (7)

39) Derive the expression for energy stored in a capacitor.

The capacitor is a charge storage device. Work has to be done to store the charges in a capacitor. This work done is stored as electrostatic potential energy in the capacitor.

Let q be the charge and V be the potential difference between the plates of the capacitor. If dq is the additional charge given to the plate, then work done is, $dw = V dq$

$$dw = \frac{q}{C} dq \qquad \left(\because V = \frac{q}{C}\right)$$

Total work done to charge a capacitor is

$$w = \int dw = \int_0^q \frac{q}{C} dq = \frac{1}{2}\frac{q^2}{C}$$

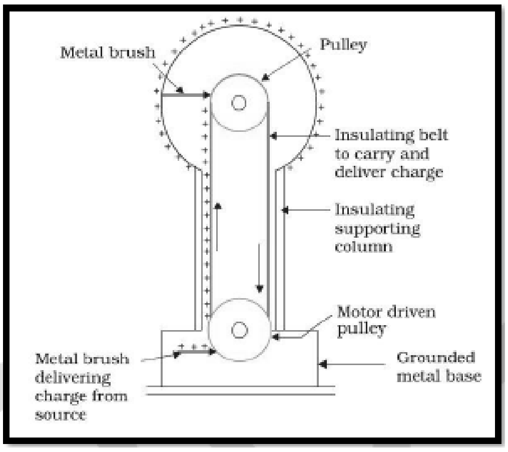
This work done is stored as electrostatic potential energy (U) in the capacitor.

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \qquad (\because\ q = CV)$$

40) What is Van De Graff generator? Write its labeled diagram. What is the principle of its working? Mention its use.

Van De Graff generator is a machine which generates very high voltages of the order of 10^6 V.

Labeled diagram:



Principle : A small conducting sphere when placed inside a large spherical shell is always at higher potential irrespective of the charge on the outer shell. Thus the charge supplied to the inner sphere always rushes to the outer shell building very high voltages.

Use : The high voltage generated in Van De Graff generator is used to accelerate charged particles *(Particle accelerators).*

Numerical Problems.

EXERCISES

2.1 Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

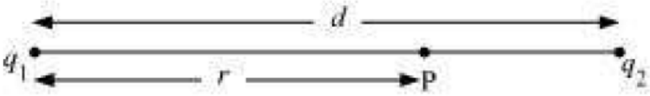
Answer

There are two charges,

$$q_1 = 5\times10^{-8}\text{C} \qquad q_2 = -3\times10^{-8}\text{C}$$

Distance between the two charges, d = 16 cm = 0.16 m

Consider a point P on the line joining the two charges, as shown in the given figure.



r = Distance of point P from charge q1

Let the electric potential (V) at point P be zero.

Potential at point P is the sum of potentials caused by charges q1 and q2 respectively.

$$\therefore V = \frac{q_1}{4\pi \in_0 r} + \frac{q_2}{4\pi \in_0 (d-r)}$$

Where,

ε_0 = Permittivity of free space

For V = 0, equation (i) reduces to

$$\frac{q_1}{4\pi \in_0 r} = -\frac{q_2}{4\pi \in_0 (d-r)}$$

$$\frac{q_1}{r} = \frac{-q_2}{d-r}$$

$$\frac{5\times10^{-8}}{r} = -\frac{(-3\times10^{-8})}{(0.16-r)}$$

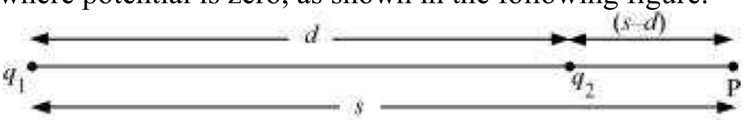
$$\frac{0.16}{r} - 1 = \frac{3}{5}$$

$$\frac{0.16}{r} = \frac{8}{5}$$

$$\therefore r = 0.1\text{m} = 10\text{cm}$$

Therefore, the potential is zero at a distance of 10 cm from the positive charge between the charges.

Suppose point P is outside the system of two charges at a distance s from the negative charge, where potential is zero, as shown in the following figure.



For this arrangement, potential is given by,

$$V = \frac{q_1}{4\pi \in_0 s} + \frac{q_2}{4\pi \in_0 (s-d)}$$

For V = 0, equation (ii) reduces to

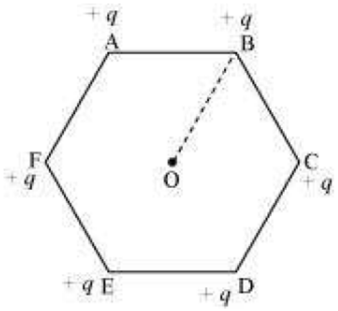
$$\frac{q_1}{4\pi \in_0 s} = -\frac{q_2}{4\pi \in_0 (s-d)} \qquad \text{or} \qquad \frac{q_1}{s} = \frac{-q_2}{s-d}$$

$$\frac{5\times10^{-8}}{s}=-\frac{\left(-3\times10^{-8}\right)}{\left(s-0.16\right)}$$
$$1-\frac{0.16}{s}=\frac{3}{5}$$
$$\frac{0.16}{s}=\frac{2}{5}$$
$$\therefore s=0.4\,\text{m}=40\,\text{cm}$$
Therefore, the potential is zero at a distance of 40 cm from the positive charge outside the system of charges.

2.2 A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Answer

The given figure shows six equal amount of charges, q, at the vertices of a regular hexagon.



Where,

Charge, $q = 5\ \mu C = 5 \times 10^{-6}$ C

Side of the hexagon, l = AB = BC = CD = DE = EF = FA = 10 cm

Distance of each vertex from centre O, d = 10 cm

Electric potential at point O,
$$V=\frac{6\times q}{4\pi\epsilon_0 d}$$
Where,

ε_0 = Permittivity of free space
$$\frac{1}{4\pi\epsilon_0}=9\times10^{9}\,\text{N C}^{-2}\,\text{m}^{-2}$$
$$\therefore V=\frac{6\times9\times10^{9}\times5\times10^{-6}}{0.1}\quad=2.7\times10^{6}\ \text{V}$$
Therefore, the potential at the centre of the hexagon is 2.7×10^6 V.

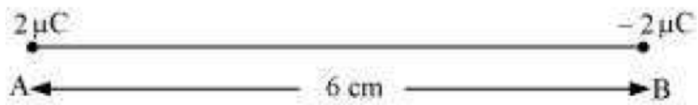
2.3 Two charges 2 μC and –2 μC are placed at points A and B 6 cm apart.

(a) Identify an equipotential surface of the system.

(b) What is the direction of the electric field at every point on this surface?

Answer

(a) The situation is represented in the given figure.



An equipotential surface is the plane on which total potential is zero everywhere. This plane is normal to line AB. The plane is located at the mid-point of line AB because the magnitude of charges is the same.

(b) The direction of the electric field at every point on this surface is normal to the plane in the direction of AB.

2.4 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field

(a) inside the sphere

(b) just outside the sphere

(c) at a point 18 cm from the centre of the sphere?

Answer

(a) Radius of the spherical conductor, r = 12 cm = 0.12 m

Charge is uniformly distributed over the conductor, $q = 1.6 \times 10^{-7}$ C

Electric field inside a spherical conductor is zero. This is because if there is field inside the conductor, then charges will move to neutralize it.

(b) Electric field E just outside the conductor is given by the relation,

Where,

ε_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore E = \frac{1.6 \times 10^{-7} \times 9 \times 10^{-9}}{(0.12)^2} \quad = 10^5 \text{ N C}^{-1}$$

Therefore, the electric field just outside the sphere is 10^5 NC^{-1} .

(c) Electric field at a point 18 m from the centre of the sphere = E_1

Distance of the point from the centre, d = 18 cm = 0.18 m

$$E_1 = \frac{q}{4\pi \epsilon_0 d^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{\left(18 \times 10^{-2}\right)^2} = 4.4 \times 10^4 \text{ N/C}$$

Therefore, the electric field at a point 18 cm from the centre of the sphere is 4.4×10^4 N/C

2.5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Answer

Capacitance between the parallel plates of the capacitor, C = 8 pF

Initially, distance between the parallel plates was d and it was filled with air. Dielectric constant of air, k = 1

Capacitance, C, is given by the formula,

$$C = \frac{k \epsilon_0 A}{d}$$

$$=\frac{\epsilon_0 A}{d}$$

Where,
A = Area of each plate
ε_0 = Permittivity of free space
If distance between the plates is reduced to half, then new distance, d' = d/2
Dielectric constant of the substance filled in between the plates, = 6
Hence, capacitance of the capacitor becomes

$$C' = \frac{k' \epsilon_0 A}{d'} = \frac{6 \epsilon_0 A}{\frac{d}{2}}$$

Taking ratios of equations (i) and (ii), we obtain

$$C' = 2 \times 6C$$
$$= 12C$$
$$= 12 \times 8 = 96 \text{ pF}$$

Therefore, the capacitance between the plates is 96 pF.

2.6 Three capacitors each of capacitance 9 pF are connected in series.
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?
Answer
(a) Capacitance of each of the three capacitors, C = 9 pF
Equivalent capacitance (C') of the combination of the capacitors is given by the relation,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \quad = \quad = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3} \qquad \therefore C' = 3\mu\text{F}$$

Therefore, total capacitance of the combination is 3 μF.
(b) Supply voltage, V = 100 V
Potential difference (V') across each capacitor is equal to one-third of the supply voltage.

$$\therefore V' = \frac{V}{3} = \frac{120}{3} = 40 \text{ V}$$

Therefore, the potential difference across each capacitor is 40 V.

2.7 Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.
(a) What is the total capacitance of the combination?
(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.
Answer
(a) Capacitances of the given capacitors are C1 = 2 pF, C2= 3 pF and 4 pF
For the parallel combination of the capacitors, equivalent capacitor C_T is given by the algebraic sum,

$$C' = 2 + 3 + 4 = 9 \text{ pF}$$

Therefore, total capacitance of the combination is 9 pF.
(b) Supply voltage, V = 100 V
The voltage through all the three capacitors is same = V = 100 V

Charge on a capacitor of capacitance C and potential difference V is given by the relation,
q = VC … (i)
For C = 2 pF,

$$\text{Charge} = VC = 100 \times 2 = 200 \text{ pC} = 2 \times 10^{-10} \text{ C}$$

For C = 3 pF,

$$\text{Charge} = VC = 100 \times 3 = 300 \text{ pC} = 3 \times 10^{-10} \text{ C}$$

For C = 4 pF,

$$\text{Charge} = VC = 100 \times 4 = 200 \text{ pC} = 4 \times 10^{-10} \text{ C}$$

2.8 In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Answer

Area of each plate of the parallel plate capacitor, $A = 6 \times 10^{-3}$ m^2
Distance between the plates, $d = 3$ mm $= 3 \times 10^{-3}$ m
Supply voltage, V = 100 V
Capacitance C of a parallel plate capacitor is given by,

$$C = \frac{\epsilon_0 A}{d}$$

Where,
= Permittivity of free space
$= 8.854 \times 10^{-12}$ N^{-1} m^{-2} C^{-2}

$$\therefore C = \frac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} = 17.71 \times 10^{-12} \text{F} \quad = 17.71 \text{ pF}$$

Potential V is related with the charge q and capacitance C as

$$V = \frac{q}{C} \qquad \therefore q = VC \quad = 100 \times 17.71 \times 10^{-12} \quad = 1.771 \times 10^{-9} \text{ C}$$

Therefore, capacitance of the capacitor is 17.71 pF and charge on each plate is 1.771×10^{-9} C.

2.9 Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,
(a) while the voltage supply remained connected.
(b) after the supply was disconnected.

Answer

(a) Dielectric constant of the mica sheet, k = 6
Initial capacitance, $C = 1.771 \times 10^{-11}$ F

$$\text{New capacitance, } C' = kC = 6 \times 1.771 \times 10^{-11} = 106 \text{ pF}$$

Supply voltage, V = 100 V

$$\text{New charge, } q' = C'V = 6 \times 1.771 \times 10^{-9} = 1.06 \times 10^{-8} \text{C}$$

Potential across the plates remains 100 V.
(b) Dielectric constant, k = 6
Initial capacitance, $C = 1.771 \times 10^{-11}$ F

$$\text{New capacitance, } C' = kC = 6 \times 1.771 \times 10^{-11} = 106 \text{ pF}$$

If supply voltage is removed, then there will be no effect on the amount of charge in the

plates.
Charge = 1.771×10^{-9} C
Potential across the plates is given by,

$$\therefore V' = \frac{q}{C'} \qquad = \frac{1.771\times10^{-9}}{106\times10^{-12}} \qquad = 16.7 \text{ V}$$

2.10 A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Answer
Capacitor of the capacitance, C = 12 pF = 12×10^{-12} F
Potential difference, V = 50 V
Electrostatic energy stored in the capacitor is given by the relation,
Therefore, the electrostatic energy stored in the capacitor is

$$E = \frac{1}{2}CV^2 \qquad = \frac{1}{2}\times12\times10^{-12}\times(50)^2 \qquad = 1.5\times10^{-8} \text{ J}$$

Therefore, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J

2.11 A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Answer
Capacitance of the capacitor, C = 600 pF
Potential difference, V = 200 V
Electrostatic energy stored in the capacitor is given by,

$$E = \frac{1}{2}CV^2 \qquad = \frac{1}{2}\times(600\times10^{-12})\times(200)^2 \qquad = 1.2\times10^{-5} \text{ J}$$

If supply is disconnected from the capacitor and another capacitor of capacitance C = 600 pF is connected to it, then equivalent capacitance (C') of the combination is given by,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} \qquad = \frac{1}{600} + \frac{1}{600} = \frac{2}{600} = \frac{1}{300} \qquad \therefore C' = 300 \text{ pF}$$

New electrostatic energy can be calculated as

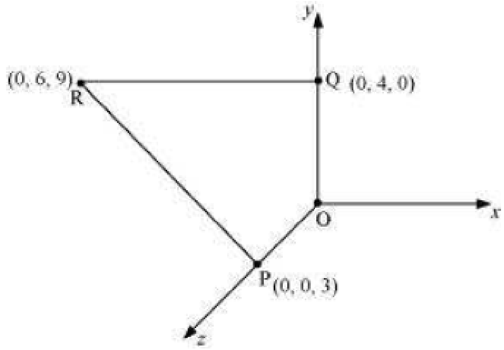
$$E' = \frac{1}{2}\times C'\times V^2 \qquad = \frac{1}{2}\times300\times(200)^2 \qquad = 0.6\times10^{-5} \text{ J}$$

Loss in electrostatic energy = E - E' $= 1.2\times10^{-5} - 0.6\times10^{-5} \quad = 0.6\times10^{-5} \quad = 6\times10^{-6}$ J

Therefore, the electrostatic energy lost in the process is 6×10^{-6} J.

2.12 A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Answer
Charge located at the origin, q = 8 mC= 8×10^{-3} C
Magnitude of a small charge, which is taken from a point P to point R to point Q,
$q_1 = -2 \times 10^{-9}$ C
All the points are represented in the given figure.
Point P is at a distance, d1 = 3 cm, from the origin along z-axis.
Point Q is at a distance, d2 = 4 cm, from the origin along y-axis.



Potential at point P,

$$V_1 = \frac{q}{4\pi \in_0 \times d_1}$$

Potential at point Q,

$$V_2 = \frac{q}{4\pi \in_0 d_2}$$

Work done (W) by the electrostatic force is independent of the path.

$$\therefore W = q_1 [V_2 - V_1]$$

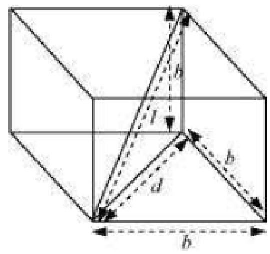
$$= q_1 \left[\frac{q}{4\pi \in_0 d_2} - \frac{q}{4\pi \in_0 d_1}\right] \qquad = \frac{qq_1}{4\pi \in_0}\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

Where, $\frac{1}{4\pi \in_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}$ $\qquad \therefore W = 9\times10^9 \times 8\times10^{-3} \times (-2\times10^{-9})\left[\frac{1}{0.04} - \frac{1}{0.03}\right]$

$$= -144\times10^{-3} \times \left(\frac{-25}{3}\right) \qquad = 1.27 \text{ J}$$

Therefore, work done during the process is 1.27 J.

2.13 A cube of side *b* has a charge *q* at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.



Answer

Length of the side of a cube = b

Charge at each of its vertices = q

A cube of side b is shown in the following figure.

d = Diagonal of one of the six faces of the cube

$$d^2 = \sqrt{b^2 + b^2} = \sqrt{2b^2}$$

$$d = b\sqrt{2}$$

l = Length of the diagonal of the cube

$$l^2 = \sqrt{d^2 + b^2}$$

$$= \sqrt{\left(\sqrt{2}b\right)^2 + b^2} = \sqrt{2b^2 + b^2} = \sqrt{3b^2}$$

$$l = b\sqrt{3}$$

$r = \frac{l}{2} = \frac{b\sqrt{3}}{2}$ is the distance between the centre of the cube an d one of the eight vertices.

The electric potential (V) at the centre of the cube is due to the presence of eight charges at the vertices.

$$V = \frac{8q}{4\pi \in_0}$$

$$= \frac{8q}{4\pi \in_0 \left(b\frac{\sqrt{3}}{2}\right)} \qquad = \frac{4q}{\sqrt{3}\pi \in_0 b}$$

Therefore, the potential at the centre of the cube is $\frac{4q}{\sqrt{3}\pi \in_0 b}$

The electric field at the centre of the cube, due to the eight charges, gets cancelled. This is because the charges are distributed symmetrically with respect to the centre of the cube. Hence, the electric field is zero at the centre.

2.14 Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:
(a) at the mid-point of the line joining the two charges, and
(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Answer
Two charges placed at points A and B are represented in the given figure. O is the midpoint of the line joining the two charges.

Magnitude of charge located at A, q_1 = 1.5 μC
Magnitude of charge located at B, q_2 = 2.5 μC
Distance between the two charges, d = 30 cm = 0.3 m
(a) Let V_1 and E_1 are the electric potential and electric field respectively at O.
V_1 = Potential due to charge at A + Potential due to charge at B

$$V_1 = \frac{q_1}{4\pi\in_0\left(\frac{d}{2}\right)} + \frac{q_2}{4\pi\in_0\left(\frac{d}{2}\right)} = \frac{1}{4\pi\in_0\left(\frac{d}{2}\right)}(q_1+q_2)$$

Where, $\in_0$ = Permittivity of free space

$$\frac{1}{4\pi\in_0} = 9\times10^9\ \text{NC}^2\ \text{m}^{-2}$$

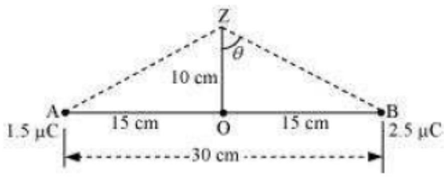
$$\therefore V_1 = \frac{9\times10^9\times10^{-6}}{\left(\frac{0.30}{2}\right)}(2.5+1.5) = 2.4\times10^5\ \text{V}$$

E_1 = Electric field due to q_2 − Electric field due to q_1

$$= \frac{q_2}{4\pi\in_0\left(\frac{d}{2}\right)^2} - \frac{q_1}{4\pi\in_0\left(\frac{d}{2}\right)^2} \qquad = \frac{9\times10^9}{\left(\frac{0.30}{2}\right)^2}\times10^6\times(2.5-1.5)$$

$$= 4\times10^5\ \text{V m}^{-1}$$

Therefore, the potential at mid-point is 2.4×10^5 V and the electric field at mid-point is 4×10^5 V m^{-1}. The field is directed from the larger charge to the smaller charge.
(b) Consider a point Z such that normal distance OZ = 10 cm = 0.1 m, as shown in the following figure.



V_2 and E_2 are the electric potential and electric field respectively at Z.
It can be observed from the figure that distance,

$$BZ = AZ = \sqrt{(0.1)^2+(0.15)^2} = 0.18\ \text{m}$$

V_2 = Electric potential due to A + Electric Potential due to B

$$= \frac{q_1}{4\pi\in_0(AZ)} + \frac{q_1}{4\pi\in_0(BZ)} \qquad = \frac{9\times10^9\times10^{-6}}{0.18}(1.5+2.5) \qquad = 2\times10^5\ \text{V}$$

Electric field due to q at Z,

$$E_A = \frac{q_1}{4\pi \in_0 (AZ)^2} = \frac{9\times10^9\times1.5\times10^{-6}}{(0.18)^2} = 0.416\times10^6 \text{ V/m}$$

Electric field due to q_2 at Z,

$$E_B = \frac{q_2}{4\pi \in_0 (BZ)^2} = \frac{9\times10^9\times2.5\times10^{-6}}{(0.18)^2} = 0.69\times10^6 \text{ V m}^{-1}$$

The resultant field intensity at Z,

$$E = \sqrt{E_A^2 + E_B^2 + 2E_A E_B \cos 2\theta}$$

Where, 2θ is the angle, $\angle AZB$

From the figure, we obtain

$$\cos\theta = \frac{0.10}{0.18} = \frac{5}{9} = 0.5556 \qquad \theta = \cos^{-1}0.5556 = 56.25 \qquad \therefore 2\theta = 112.5°$$

$$\cos 2\theta = -0.38$$

$$E = \sqrt{(0.416\times10^6)^2 \times (0.69\times10^6)^2 + 2\times0.416\times0.69\times10^{12}\times(-0.38)}$$

$$= 6.6\times10^5 \text{ V m}^{-1}$$

Therefore, the potential at a point 10 cm (perpendicular to the mid-point) is 2.0×10^5 V and electric field is 6.6×10^5 V m^{-1}.

2.15 A spherical conducting shell of inner radius *r*1 and outer radius *r*2 has a charge *Q*.
(a) A charge *q* is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?
(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Answer

(a) Charge placed at the centre of a shell is +q. Hence, a charge of magnitude −q will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is −q.
Surface charge density at the inner surface of the shell is given by the relation,

$$\sigma_1 = \frac{\text{Total charge}}{\text{Inner surface area}} = \frac{-q}{4\pi r_1^2} \qquad \ldots (i)$$

A charge of +q is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is Q + q.
Surface charge density at the outer surface of the shell,

$$\sigma_2 = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{Q+q}{4\pi r_2^2} \qquad \ldots (ii)$$

(b) Yes

The electric field intensity inside a cavity is zero, even if the shell is not spherical and has any irregular shape. Take a closed loop such that a part of it is inside the cavity along a field line while the rest is inside the conductor. Net work done by the field in carrying a test charge over a closed loop is zero because the field inside the conductor is zero. Hence, electric field is zero, whatever is the shape.

2.16 (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$(\mathbf{E}_2 - \mathbf{E}_1)\cdot\hat{\mathbf{n}} = \frac{\sigma}{\varepsilon_0}$$

where ˆ**n** is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of ˆ**n** is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is σ ˆ**n** / ε_0.
(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]
Answer
(a) Electric field on one side of a charged body is E1 and electric field on the other side of the same body is E2. If infinite plane charged body has a uniform thickness, then electric field due to one surface of the charged body is given by,

$$\overrightarrow{E_1} = -\frac{\sigma}{2\epsilon_0}\hat{n}$$

Where,
$\hat{n}$ = Unit vector normal to the surface at a point
σ = Surface charge density at that point
Electric field due to the other surface of the charged body,

$$\overrightarrow{E_2} = -\frac{\sigma}{2\epsilon_0}\hat{n}$$

Electric field at any point due to the two surfaces,

$$\overrightarrow{E_2} - \overrightarrow{E_1} = \frac{\sigma}{2\epsilon_0}\hat{n} + \frac{\sigma}{2\epsilon_0}\hat{n} = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$\left(\overrightarrow{E_2} - \overrightarrow{E_1}\right).\hat{n} = \frac{\sigma}{\epsilon_0}$$

Since inside a closed conductor, $\overrightarrow{E_1} = 0$ ∴ $\vec{E} = \overrightarrow{E_2} = -\frac{\sigma}{2\epsilon_0}\hat{n}$

Therefore, the electric field just outside the conductor is = $\frac{\sigma}{\epsilon_0}\hat{n}$

(b) When a charged particle is moved from one point to the other on a closed loop, the work done by the electrostatic field is zero. Hence, the tangential component of electrostatic field is continuous from one side of a charged surface to the other.

2.17 A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?
Answer
Charge density of the long charged cylinder of length L and radius r is λ.
Another cylinder of same length surrounds the pervious cylinder. The radius of this cylinder is R.
Let E be the electric field produced in the space between the two cylinders.
Electric flux through the Gaussian surface is given by Gauss's theorem as,

$$\phi = E(2\pi d)L$$

Where, d = Distance of a point from the common axis of the cylinders
Let q be the total charge on the cylinder.
It can be written as

$$\therefore \phi = E(2\pi dL) = \frac{q}{\epsilon_0}$$

Where,
q = Charge on the inner sphere of the outer cylinder
ϵ_0 = Permittivity of free space

$$E(2\pi dL) = \frac{\lambda L}{\epsilon_0} \qquad E = \frac{\lambda}{2\pi \epsilon_0 d}$$

Therefore, the electric field in the space between the two cylinders is $\frac{\lambda}{2\pi \epsilon_0 d}$

2.18 In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.
(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?
(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

Answer
The distance between electron-proton of a hydrogen atom, d = 0.53 Å
Charge on an electron, $q_1 = -1.6 \times 10^{-19}$ C
Charge on a proton, $q_2 = +1.6 \times 10^{-19}$ C
(a) Potential at infinity is zero.
Potential energy of the system, p-e = Potential energy at infinity − Potential energy at distance, d

$$= 0 - \frac{q_1 q_2}{4\pi \epsilon_0 d}$$

Where,
ϵ_0 is the permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

$$\therefore \text{Potential energy} = 0 - \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{0.53 \times 10^{10}} = -43.7 \times 10^{-19} \text{ J}$$

Since 1.6×10^{-19} J = 1 eV,

$$\therefore \text{ Potential energy} = -43.7 \times 10^{-19} = \frac{-43.7 \times 10^{-19}}{1.6 \times 10^{-19}} = -27.2 \text{ eV}$$

Therefore, the potential energy of the system is −27.2 eV.
(b) Kinetic energy is half of the magnitude of potential energy.

$$\text{Kinetic energy} = \frac{1}{2} \times (-27.2) = 13.6 \text{ eV}$$

Total energy = 13.6 − 27.2 = 13.6 eV
Therefore, the minimum work required to free the electron is 13.6 eV.
(c) When zero of potential energy is taken, d_1 = 1.06 Å
∴ Potential energy of the system = Potential energy at d_1 − Potential energy at d

$$= \frac{q_1 q_2}{4\pi \in_0 d_1} - 27.2 \text{ eV}$$

$$= \frac{9\times10^9 \times \left(1.6\times10^{-19}\right)^2}{1.06\times10^{-10}} - 27.2 \text{ eV} \qquad = 21.73\times10^{-19} \text{J} - 27.2 \text{ eV}$$

$$= 13.58 \text{ eV} - 27.2 \text{ eV} \qquad = -13.6 \text{ eV}$$

2.19 If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H^+_2. In the ground state of an H^+_2, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Answer

The system of two protons and one electron is represented in the given figure.

Charge on proton 1, $q_1 = 1.6 \times10^{-19}$ C

Charge on proton 2, $q_2 = 1.6 \times10^{-19}$ C

Charge on electron, $q_3 = -1.6 \times10^{-19}$ C

Distance between protons 1 and 2, $d_1 = 1.5 \times10^{-10}$ m

Distance between proton 1 and electron, $d_2 = 1 \times10^{-10}$ m

Distance between proton 2 and electron, $d_3 = 1 \times 10^{-10}$ m

The potential energy at infinity is zero.

2.20 Two charged conducting spheres of radii *a* and *b* are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

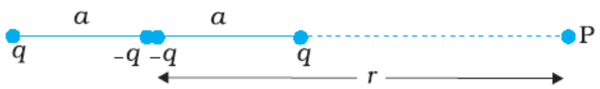
2.21 Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$, respectively.

(a) What is the electrostatic potential at the points $(0, 0, z)$ and $(x, y, 0)$?

(b) Obtain the dependence of potential on the distance r of a point from the origin when $r/a >> 1$.

(c) How much work is done in moving a small test charge from the point (5,0,0) to (–7,0,0) along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

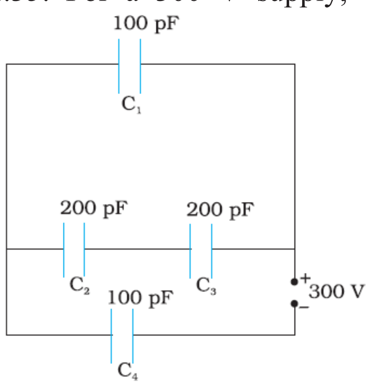
2.22 Figure 2.34 shows a charge array known as an *electric quadrupole*. For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a >> 1$, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



2.23 An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

2.24 What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

2.25 Obtain the equivalent capacitance of the network in Fig. 2.35. For a 300 V supply, determine the charge and voltage across each capacitor.



2.26 The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.
(a) How much electrostatic energy is stored by the capacitor?
(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

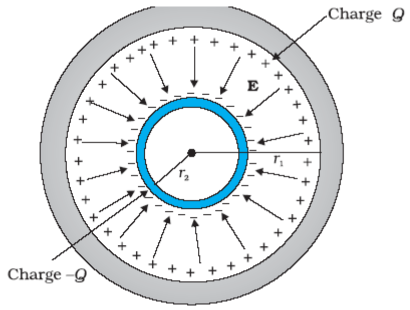
2.27 A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

2.28 Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (½) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor ½.

2.29 A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.36). Show that the capacitance of a spherical capacitor is given by

$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.



2.30 A spherical capacitor has an inner sphere of radius 12 cm and an
outer sphere of radius 13 cm. The outer sphere is earthed and the
inner sphere is given a charge of 2.5 μC. The space between the
concentric spheres is filled with a liquid of dielectric constant 32.
(a) Determine the capacitance of the capacitor.
(b) What is the potential of the inner sphere?
(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

2.31 Answer carefully:
(a) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2/4\pi \varepsilon_0 r^2$, where r is the distance between their centres?
(b) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true ?
(c) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?
(d) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?
(e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?
(f) What meaning would you give to the capacitance of a single conductor?
(g) Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

2.32 A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

2.33 A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength.
What minimum area of the plates is required to have a capacitance of 50 pF?

2.34 Describe schematically the equipotential surfaces corresponding to

(a) a constant electric field in the z-direction,
(b) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,
(c) a single positive charge at the origin, and (d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

2.35 In a Van de Graaff type generator a spherical metal shell is to be a 15 × 106 V electrode. The dielectric strength of the gas surrounding the electrode is 5 × 107 Vm–1. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

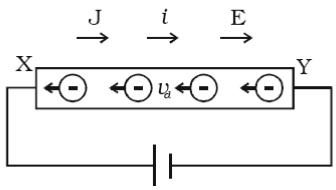
2.36 A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

2.37 Answer the following:
(a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)
(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area $1m^2$. Will he get an electric shock if he touches the metal sheet next morning?
(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?
(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?
(Hint: The earth has an electric field of about 100 Vm–1 at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} C m^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)

Chapter 3:
CURRENT ELECTRICITY
13 M (1M - 1Q; 2M-1Q; 5M(LA) - 1Q; 5M(NP) - 1Q) or 3M-1Q, 5M-2Q(LA)

3.1 Definition of electric current - Electric currents in a conductor -
The current is defined as the rate of flow of charges across any cross sectional area of a conductor. If a net charge q passes through any cross section of a conductor in time t, then the current $I = q / t$, where q is in coulomb and t is in second. The current I is expressed in ampere. If the rate of flow of charge is not uniform, the current varies with time and the instantaneous value of current i is given by,
$i = dq/dt$
Current is a scalar quantity. The direction of conventional current is taken as the direction of flow of positive charges or opposite to the direction of flow of electrons.

Electric currents in a conductor - Drift velocity and mobility
Consider a conductor XY connected to a battery (Fig). A steady electric field E is established in the conductor in the direction X to Y. In the absence of an electric field, the free electrons in the conductor move randomly in all possible directions. They do not produce current. But, as soon as an electric field is applied, the free electrons at the end Y experience a force F = eE in a direction opposite to the electric field. The electrons are accelerated and in the process they collide with each other and with the positive ions in the conductor.
Thus due to collisions, a backward force acts on the electrons and they are slowly drifted with a constant average drift velocity v_d in a direction opposite to electric field.
Drift velocity is defined as the velocity with which free electrons get drifted towards the positive terminal, when an electric field is applied.
If τ is the average time between two successive collisions and the acceleration experienced by the electron be a, then the drift velocity is given by, $v_d = a\tau$
The force experienced by the electron of mass m is $F = ma$
Hence $a = eE/m$



$$\therefore v_d = \frac{eE}{m}\tau = \mu E$$

where $\mu = e\tau/m$ is the **mobility** and is defined as the drift velocity acquired per unit electric field. It takes the unit $m^2V^{-1}s^{-1}$. The drift velocity of electrons is proportional to the electric field intensity. It is very small and is of the order of 0.1 cm s^{-1}.

3.2 Definition of current density - Ohm's law: Statement and explanation -
Current density at a point is defined as the quantity of charge passing per unit time through unit area, taken perpendicular to the direction of flow of charge at that point.
The current density **J** for a current I flowing across a conductor having an area of cross section A is

$$\mathbf{J} = \frac{(q/t)}{A} = \frac{I}{A}$$

Current density is a vector quantity. It is expressed in A m^{-2}

Relation between current and drift velocity :

Consider a conductor XY of length L and area of cross section A (Fig). An electric field E is applied between its ends. Let n be the number of free electrons per unit volume. The free electrons move towards the left with a constant drift velocity v_d. The number of conduction electrons in the conductor = nAL The charge of an electron = e
The total charge passing through the conductor $q = (nAL)\, e$
The time in which the charges pass through the conductor, t = L/v_d
The current flowing through the conductor,

$$I = \frac{q}{t} = \frac{(nAL)e}{(L/v_d)}$$

$$I = nAev_d \qquad \ldots(1)$$

The current flowing through a conductor is directly proportional to the drift velocity.
From equation (1),

$$\frac{I}{A} = nev_d$$

$$\boldsymbol{J} = nev_d \qquad \left[\because \boldsymbol{J} = \frac{I}{A}, \text{current density}\right]$$

We know that,

$$\therefore v_d = \frac{eE}{m}\tau = \mu E$$

$\therefore J = (ne^2/m)\,\tau E$ and $\sigma = (ne^2/m)\tau$

Hence $\vec{J} = \sigma\,\vec{E}$

Ohm's law: Statement and explanation :

George Simon Ohm established the relationship between potential difference and current, which is known as Ohm's law. The current flowing through a conductor is,

$$I = nAev_d$$

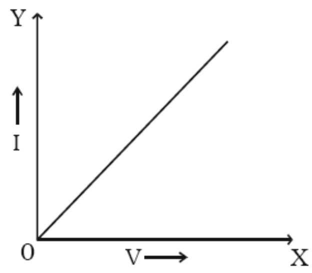
$$\text{But} \quad v_d = \frac{eE}{m}\cdot\tau \qquad \therefore \quad I = nAe\frac{eE}{m}\tau$$

$$I = \frac{nAe^2}{mL}\tau V \qquad \left[\because E = \frac{V}{L}\right]$$

where V is the potential difference. The quantity $mL/nAe^2\tau$, is a constant for a given conductor, called electrical resistance (R).
$\therefore I \alpha V$
The law states that, at a constant temperature, the steady current flowing through a conductor is directly proportional to the potential difference between the two ends of the conductor.

$$\text{(i.e)} \quad I \;\alpha\; V \quad \text{or} \quad I = \frac{1}{R}V$$

$$\therefore \quad V = IR \quad \text{or} \quad R = \frac{V}{I}$$



Resistance of a conductor is defined as the ratio of potential difference across the conductor to the current flowing through it. The unit of resistance is ohm (Ω)
The reciprocal of resistance is conductance. Its unit is mho (Ω^{-1}).

Since, potential difference V is proportional to the current I, the graph (Fig V-I graph) between V and I is a straight line for a conductor. Ohm's law holds good only when a steady current flows through a conductor.

3.3 Electrical resistivity and conductivity - Dependence of electrical resistance on the dimensions of conductor and mention of $R = \rho l/A$ -

Electrical Resistivity and Conductivity :

The resistance of a conductor R is directly proportional to the length of the conductor l and is inversely proportional to its area of cross section A.

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \frac{\rho l}{A}$$

ρ is called specific resistance or electrical resistivity of the material of the conductor.

If $l = 1$ m, A = 1 m^2, then $\rho = R$

The electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section. The unit of ρ is ohm−m (Ω m). It is a constant for a particular material.

The reciprocal of electrical resistivity, is called electrical conductivity, $\sigma = 1/\rho$

The unit of conductivity is mho m^{-1} (Ω^{-1} m^{-1})

3.4 Derivation of the relation $\vec{J} = \sigma \vec{E}$ (equivalent form of Ohm's law) -

Consider a conductor XY of length L and area of cross section A (Fig). An electric field E is applied between its ends. Let n be the number of free electrons per unit volume. The free electrons move towards the left with a constant drift velocity v_d. The number of conduction electrons in the conductor = nAL The charge of an electron = e

The total charge passing through the conductor $q = (nAL)\, e$

The time in which the charges pass through the conductor, t = L/v_d

The current flowing through the conductor,

$$I = \frac{q}{t} = \frac{(nAL)e}{(L/v_d)}$$

$$I = nAev_d \qquad \ldots(1)$$

The current flowing through a conductor is directly proportional to the drift velocity.

From equation (1),

$$\frac{I}{A} = nev_d$$

$$\boldsymbol{J} = nev_d \qquad \left[\because \boldsymbol{J} = \frac{I}{A}, \text{current density}\right]$$

We know that,

$$\therefore v_d = \frac{eE}{m}\tau = \mu E$$

$\therefore J = (ne^2/m)\, \tau E \qquad and \;\; \sigma = (ne^2/m)\tau$

Hence $\vec{J} = \sigma \vec{E}$

3.5 Limitations of Ohm's law.

Although Ohm's law has been found valid over a large class of materials, there do exist materials and devices used in electric circuits where the proportionality of V and I does not hold. The deviations broadly are one or more of the following types:
(a) V ceases to be proportional to I (Fig.).
(b) The relation between V and I depends on the sign of V. In other words, if I is the current for a certain V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction.
(c) The relation between V and I is not unique, i.e., there is more than one value of V for the same current I. A material exhibiting such behaviour is GaAs.

3.6 Derivation of expression for conductivity of a material ($\sigma = ne^2 \tau/m$).

Consider a conductor XY of length L and area of cross section A (Fig). An electric field E is applied between its ends. Let n be the number of free electrons per unit volume. The free electrons move towards the left with a constant drift velocity v_d. The number of conduction electrons in the conductor = nAL The charge of an electron = e
The total charge passing through the conductor $q = (nAL)\ e$
The time in which the charges pass through the conductor, t = L/v_d
The current flowing through the conductor,

$$I = \frac{q}{t} = \frac{(nAL)e}{(L/v_d)}$$

$$I = nAev_d \qquad \ldots(1)$$

The current flowing through a conductor is directly proportional to the drift velocity.
From equation (1),

$$\frac{I}{A} = nev_d$$

$$\boldsymbol{J} = nev_d \qquad \left[\because \boldsymbol{J} = \frac{I}{A}, current\ density\right]$$

We know that,

$$\therefore v_d = \frac{eE}{m}\tau = \mu E$$

$\therefore J = (ne^2/m)\ \tau E \qquad and\ \ \sigma = (ne^2/m)\tau$

Hence $\vec{J} = \sigma \vec{E}$

where σ is called conductivity of conductor.

3.7 Color code of carbon resistors;

Carbon resistors

The wire wound resistors are expensive and huge in size. Hence, carbon resistors are used. Carbon resistor consists of a ceramic core, on which a thin layer of crystalline carbon is deposited. These resistors are cheaper, stable and small in size. The resistance of a carbon resistor is indicated by the colour code drawn on it (Table). A three colour code carbon resistor is discussed here. The silver or gold ring at one end corresponds to the tolerance. It is a tolerable range (±) of the resistance. The tolerance of silver, gold, red and brown rings is 10%, 5%, 2% and 1% respectively. If there is no coloured ring

Colour	Number
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9

at this end, the tolerance is 20%. The first two rings at the other end of tolerance ring are significant figures of resistance in ohm. The third ring indicates the powers of 10 to be multiplied or number of zeroes following the significant figure. (B B ROY Grate Britain has Very Good Wife)

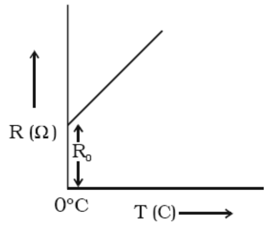
Example :

The first yellow ring in Fig. (a) corresponds to 4. The next violet ring corresponds to 7. The third orange ring corresponds to 10^3. The silver ring represents 10% tolerance. The total resistance is $47 \times 10^3 \pm 10\%$ i.e. 47 k Ω, ±10%. Fig (b) shows 1 k Ω, ±5% carbon resistor.

Presently four colour code carbon resistors are also used. For certain critical applications 1% and 2% tolerance resistors are used.

3.8. Temperature dependence of resistivity of metals and semiconductors :

The resistivity of substances varies with temperature. For conductors the resistance increases with increase in temperature. If R_0 is the resistance of a conductor at 0 °C and R_t is the resistance of same conductor at t °C, then



$$R_t = R_o (1 + \alpha t)$$

where α is called the temperature coefficient of resistance.

$$\alpha = \frac{R_t - R_o}{R_o t}$$

The temperature coefficient of resistance is defined as the ratio of increase in resistance per degree rise in temperature to its resistance at 0° C. Its unit is per °C.

The variation of resistance with temperature is shown in Fig. Metals have positive temperature coefficient of resistance, i.e., their resistance increases with increase in temperature. Insulators and semiconductors have negative temperature coefficient of resistance, i.e., their resistance decreases with increase in temperature. A material with a negative temperature coefficient is called a thermistor. The temperature coefficient is low for alloys.

3. 9 Electrical energy and power:

If I is the current flowing through a conductor of resistance R in time t, then the quantity of charge flowing is, $q = It$. If the charge q, flows between two points having a potential difference V, then the work done in moving the charge is $= V. q = V It$.

Then, electric power is defined as the rate of doing electric work.

$$\therefore \quad \text{Power} = \frac{\text{Work done}}{\text{time}} = \frac{VIt}{t} = VI$$

Electric power is the product of potential difference and current strength.

Since V = IR, Power = I^2R

Electric energy is defined as the capacity to do work. Its unit is joule. In practice, the electrical energy is measured by watt hour (Wh) or kilowatt hour (kWh). 1 kWh is known as one unit of electric energy. (1 kWh = 1000 Wh = 1000 × 3600 J = 36×10^5 J)

3.10 Mention of expression for power loss.

Power Loss :

In case charges moving freely through the conductor under the action of electric field, their kinetic energy would increase as they move with a steady drift velocity. This is because of the collisions with ions and atoms during transit. During collisions, the energy gained by the charges thus is shared with the atoms. The atoms vibrate more vigorously, i.e., the conductor heats up. Thus, in an actual conductor, an amount of energy dissipated as heat in the conductor during the time interval Δt is,

$\Delta W = I\,V\,\Delta t$

The energy dissipated per unit time is the power dissipated

$P = \Delta W/\Delta t$ and we have,

$P = I\,V$

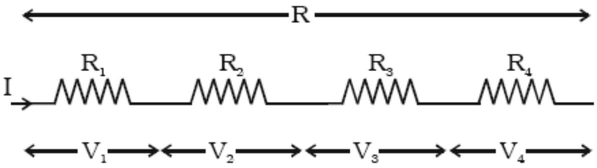
Using Ohm's law $V = IR$, we get $P = I^2 R = V^2/R$

as the power loss ("ohmic loss") in a conductor of resistance R carrying a current I. It is this power which heats up, for example, the coil of an electric bulb to incandescence, radiating out heat and light.

3.11 Combination of resistors: Derivation of effective resistance of two resistors (a) in series combination and (b) in parallel combination.

Resistors in series :

Let us consider the resistors of resistances R_1, R_2, R_3 and R_4 connected in series as shown in Fig. When resistors are connected in series, the current flowing through each resistor is the same. If the potential difference applied between the ends of the combination of resistors is V, then the potential difference across each resistor R_1, R_2, R_3 and R_4 is V_1, V_2, V_3 and V_4 respectively.



The net potential difference $V = V_1 + V_2 + V_3 + V_4$

By Ohm's law

$V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$, $V_4 = IR_4$ and $V = IR_S$

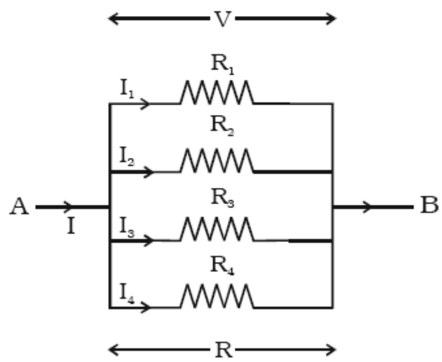
where R_S is the equivalent or effective resistance of the series combination.

Hence, $IR_S = IR_1 + IR_2 + IR_3 + IR_4$ or $R_S = R_1 + R_2 + R_3 + R_4$

Thus, the equivalent resistance of a number of resistors in series connection is equal to the sum of the resistance of individual resistors.

Resistors in parallel :

Consider four resistors of resistances R_1, R_2, R_3 and R_4 are connected in parallel as shown in Fig. A source of emf V is connected to the parallel combination. When resistors are in parallel, the potential difference (V) across each resistor is the same.



A current I entering the combination gets divided into I_1, I_2, I_3 and I_4 through R_1, R_2, R_3 and R_4 respectively, such that $I = I_1 + I_2 + I_3 + I_4$.

By Ohm's law

$$I_1 = \frac{V}{R_1},\quad I_2 = \frac{V}{R_2},\quad I_3 = \frac{V}{R_3},\quad I_4 = \frac{V}{R_4} \text{ and } I = \frac{V}{R_P}$$

where R_P is the equivalent or effective resistance of the parallel combination.

$$\therefore \frac{V}{R_P} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the resistance of the individual resistors is equal to the reciprocal of the effective resistance of the combination.

If two resistors R_1 & R_2 are connected in parallel, then $\mathbf{R_p = R_1R_2/(R_1 + R_2)}$

If three resistors R_1, R_2 & R3 are connected in parallel, then

$\mathbf{R_p = R_1R_2 R_3 /(R_1R_2 + R_2R_3 + R_3R_1)}$

3.12 Cells: Definitions of internal resistance of a cell, terminal potential difference and emf of a cell -

The opposition offered by the electrolyte of the cell to the flow of electric current through it is called the ***internal resistance*** of the cell.

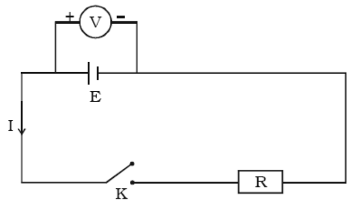
The ***electro motive force*** is the maximum potential difference between the two electrodes of the cell when no current is drawn from the cell.

The electric current in an external circuit flows from the positive terminal to the negative terminal of the cell, through different circuit elements. In order to maintain continuity, the current has to flow through the electrolyte of the cell, from its negative terminal to positive terminal. During this process of flow of current inside the cell, a resistance is offered to current flow by the electrolyte of the cell. This is termed as the ***internal resistance*** of the cell.

A freshly prepared cell has low internal resistance and this increases with ageing.

Determination of internal resistance of a cell using voltmeter :

The circuit connections are made as shown in Fig. With key K open, the emf of cell E is found by connecting a high resistance voltmeter across it. Since the high resistance voltmeter draws only a very feeble current for deflection, the circuit may be considered as an open circuit. Hence the voltmeter reading gives the emf of the cell. A small value of resistance R is included in the external circuit and key K is closed. The potential difference across R is equal to the potential difference across cell (V).



The potential drop across R, $V = IR$ ... (1)

Due to internal resistance r of the cell, the voltmeter reads a value V, less than the emf of cell.

Then $V = E - Ir$ or $Ir = E-V$ ... (2)

Dividing equation (2) by equation (1)

$$\frac{Ir}{IR} = \frac{E-V}{V} \quad \text{or} \quad r = \left(\frac{E-V}{V}\right)R$$

Since E, V and R are known, the internal resistance r of the cell can be determined.

Factors affecting internal resistance of a cell :

(1) Larger the separation between the electrodes of the cell, more the length of the electrolyte through which current has to flow and consequently a higher value of internal resistance.

(2) Greater the conductivity of the electrolyte, lesser is the internal resistance of the cell. i.e., Internal resistance depends on the nature of the electrolyte.

(3) The internal resistance of an electrolyte is inversely proportional to common area of the electrodes dipping in the electrolyte.

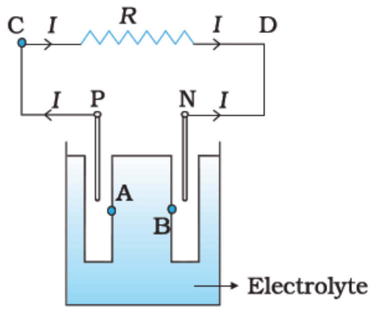
(4) Internal resistance of a cell depends on the nature of the electrolytes.

Comparison of emf and potential difference :

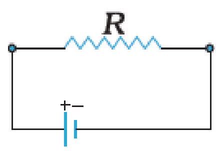
1. The difference of potentials between the two terminals of a cell in an open circuit is called the electromotive force (emf) of a cell. The difference in potentials between any two points in a closed circuit is called potential difference.

2. The emf is independent of external resistance of the circuit, whereas potential difference is proportional to the resistance between any two points.

3.13 Derivation of current drawn by external resistance.



(a)



Symbol

(b)

The electrolyte through which a current flows has a finite resistance r, called the *internal resistance*. Consider first the situation when R is infinite so that $I = V/R = 0$, where V is the potential difference between Positive terminal and Negative terminal. Now,

V = Potential difference between P and A

+ Potential difference between A and B

+ Potential difference between B and N

= e -------- (1)

Thus, emf e is the potential difference between the positive and negative electrodes in an open circuit, i.e., when no current is flowing through the cell.

If however R is finite, I is not zero. In that case the potential difference between P and N is

$V = V+ + V- - I r$

$= e - I r$ -------- *(2)*

Note the negative sign in the expression ($I r$) for the potential difference between A and B. This is because the current I flows from B to A in the electrolyte.

We also observe that since V is the potential difference across R, we have from Ohm's law

$V = I R$ ---------- (3)

Combining Eqs. (2) and (3), we get

$I R = e - I r$ -------- *(4)*

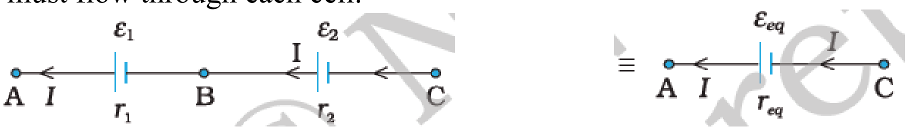
or,
The maximum current that can be drawn from a cell is for R = 0 and it is $I_{max} = e/r$. However, in most cells the maximum allowed current is much lower than this to prevent permanent damage to the cell.

3.14 Combination of cells: Derivation of expressions for equivalent emf and equivalent internal resistance (a) in series and (b) in parallel combination.

Like resistors, cells can be combined together in electric circuits to get more current or voltages.

(a) Cells in Series :

Cells are connected in series when they are joined end to end so that the same quantity of electricity must flow through each cell.



Consider two cells in series (Fig.), where one terminal of the two cells is joined together leaving the other terminal in either cell free. ε_1, ε_2 are the emf's of the two cells and r_1, r_2 their internal resistances, respectively.

Let V (A), V (B), V (C) be the potentials at points A, B and C shown in Fig. Then V (A) – V (B) is the potential difference between the positive and negative terminals of the first cell.

$V_{AB} = V(A) - V(B) = \varepsilon_1 - I\, r_1$

Similarly,

$V_{BC} = V(B) - V(C) = \varepsilon_2 - I\, r_2$

Hence, the potential difference between the terminals A and C of the combination is

$V_{AC} = V(A) - V(C) = V(A) - V(B) + V(B) - V(C) \;=\; (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$

If we replace the combination by a single cell between A and C of emf ε_{eq} and internal resistance r_{eq}, we get

$\boldsymbol{V_{AC} = \varepsilon_{eq} - I\, r_{eq}}$

Comparing the last two equations, we get,

$\boldsymbol{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2}$ and $\boldsymbol{r_{eq} = r_1 + r_2}$

If we connected the negative electrode of the first to the positive electrode of the second. If instead we connect the two negatives, Eq. () would change to $V_{BC} = -\varepsilon_2 - I\, r_2$ and we will get,

$\boldsymbol{\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2}$ $(\varepsilon_1 > \varepsilon_2)$

The rule for series combination clearly can be extended to any number of cells:

(i) The equivalent emf of a series combination of n cells is just the sum of their individual emf's, and

(ii) The equivalent internal resistance of a series combination of n cells is just the sum of their internal resistances.

Thus in series combination of cells (1) The emf of the battery is the sum of the individual emfs. (2) The current in each cell is the same and is identical with the current in the entire arrangement. (3) The total internal resistance of the battery is the sum of the individual internal resistances.

Total emf of the battery = nE (for n no. of identical cells)

Total Internal resistance of the battery = nr

Total resistance of the circuit = nr + R

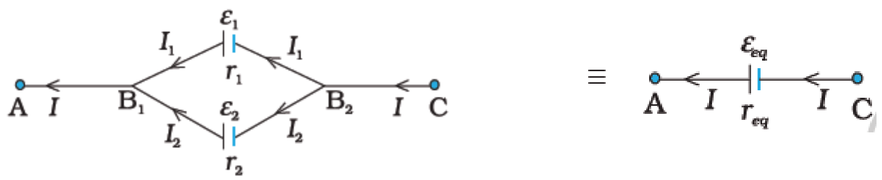
Current $I = \dfrac{nE}{nr + R}$

(i) If R << nr, then I = E / r (ii) If nr << R, then I = n (E / R)

Conclusion: When internal resistance is negligible in comparison to the external resistance, then the cells are connected in series to get maximum current.

(b) Cells in Parallel :

Cells are said to be connected in parallel when they are joined positive to positive and negative to negative such that current is divided between the cells.



Consider a parallel combination of the cells (Fig.). I_1 and I_2 are the currents leaving the positive electrodes of the cells. At the point B_1, I_1 and I_2 flow in whereas the current I flows out. Since as much charge flows in as out, we have

$I = I_1 + I_2$ ------------ (1)

Let $V(B_1)$ and $V(B_2)$ be the potentials at B_1 and B_2, respectively. Then, considering the first cell, the potential difference across its terminals is $V(B_1) - V(B_2)$. Hence,

$\mathbf{V} \equiv V(B_1) - V(B_2) = \varepsilon_1 - I_1 r_1$ ------- (2)

Points B_1 and B_2 are connected exactly similarly to the second cell. Hence considering the second cell, we also have

$\mathbf{V} \equiv V(B_1) - V(B_2) = \varepsilon_2 - I_2 r_2$ -------- (3)

Combining the three equations

$I = I_1 + I_2$

$$= \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Hence, V is given by,

$$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\frac{r_1 r_2}{r_1 + r_2} \quad \text{--------} \quad (1)$$

Since $I + V\left(\dfrac{r_1 + r_2}{r_1 r_2}\right) = \left(\dfrac{\varepsilon_1 r_2 - \varepsilon_2 r_1}{r_1 r_2}\right)$ or $V\left(\dfrac{r_1 + r_2}{r_1 r_2}\right) = \left(\dfrac{\varepsilon_1 r_2 - \varepsilon_2 r_1}{r_1 r_2}\right) - I$

$$V = \frac{\left(\dfrac{\varepsilon_1 r_2 - \varepsilon_2 r_1}{r_1 r_2}\right) - I}{\left(\dfrac{r_1 + r_2}{r_1 r_2}\right)}$$

or

$$V = \left(\frac{\varepsilon_1 r_2 - \varepsilon_2 r_1}{\cancel{r_1 r_2}}\right)\left(\frac{\cancel{r_1 r_2}}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

$$V = \left(\frac{\varepsilon_1 r_2 - \varepsilon_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \quad \text{--------} \quad (1)$$

If the parallel combination of cells is replaced by a single cell between B_1 and B_2 of emf ε_{eq} and internal resistance r_{eq}

$$V_{AC} = \varepsilon_{eq} - I\, r_{eq} \quad \text{----------} \quad (2)$$

Comparing equation (1) and (2)

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \quad \text{--------} \quad (3)$$

and

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad \text{----------} \quad (4) \qquad \text{Also} \quad \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

Dividing (3) by (4)

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{\cancel{r_1 + r_2}}}{\dfrac{r_1 r_2}{\cancel{r_1 + r_2}}} \quad \text{-----------} \quad (5)$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2}$$

$$\boxed{\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}$$

If there an ‘n’ cells of emfs ε_1 , ε_2 , ε_3 , , . . . ε_n and of internal resistances r_1, r_2, r_3,. . . . . r_n respectively, connected in parallel, the combination is equivalent to a single cell of emf ε_{eq} and internal resistance r_{eq}, such that

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \cdots + \frac{\varepsilon_n}{r_n} \qquad \text{and} \qquad \frac{1}{r_{eq}} = \frac{1}{r_1} + \cdots + \frac{1}{r_n}$$

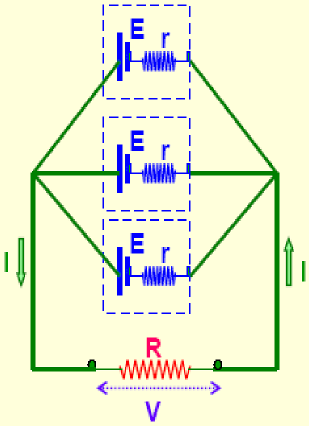
Thus in Parallel combination of cells (1) The ems of the battery is the same as that of a single cell.(2) The current in the external circuit is divided equally among the cells.(3) The reciprocal of the total internal resistance is the sum of the reciprocals of the individual internal resistances.

Cells in Parallel combination:

Cells are said to be connected in parallel when they are joined positive to positive and negative to negative such that current is divided between the cells.

NOTE:

1. The emf of the battery is the same as that of a single cell.
2. The current in the external circuit is divided equally among the cells.
3. The reciprocal of the total internal resistance is the sum of the reciprocals of the individual internal resistances.



Total emf of the battery $= E$

Total Internal resistance of the battery $= r / n$

Total resistance of the circuit $= (r / n) + R$

Current $I = \dfrac{nE}{nR + r}$

(i) If $R << r/n$, then $I = n(E / r)$ (ii) If $r/n << R$, then $I = E / R$

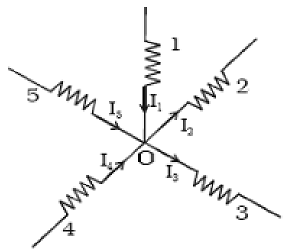
Conclusion: When external resistance is negligible in comparison to the internal resistance, then the cells are connected in parallel to get maximum current.

3.15 Kirchhoff's rules: Statements and explanation.

Kirchhoff's first law (current law or junction rule) :

Kirchhoff's current law states that the algebraic sum of the currents meeting at any junction in a circuit is zero.

The convention is that, the current flowing towards a junction is positive and the current flowing away from the junction is negative. Let 1,2,3,4 and 5 be the conductors meeting at a junction O in an electrical circuit. Let I_1, I_2, I_3, I_4 and I_5 be the currents passing through the conductors respectively.



According to Kirchhoff's first law.

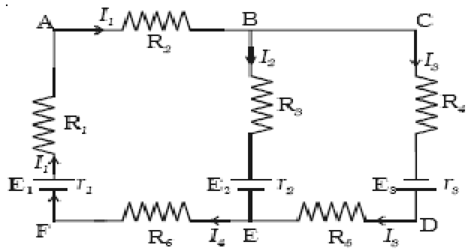
$I_1 + (-I_2) + (-I_3) + I_4 + I_5 = 0$ or $I_1 + I_4 + I_5 = I_2 + I_3$.

The sum of the currents entering the junction is equal to the sum of the currents leaving the junction. This law is a consequence of conservation of charges.

Kirchhoff's second law (voltage law or loop rule) :

Kirchhoff's voltage law states that the algebraic sum of the products of resistance and current in each part of any closed circuit is equal to the algebraic sum of the emf's in that closed circuit.

This law is a consequence of conservation of energy. In applying Kirchhoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the problems, the current will be found to have negative sign. If the result is positive, then the assumed direction is the same as actual direction. In the application of Kirchhoff's second law, we follow that the current in clockwise direction is taken as positive and the current in anticlockwise direction is taken as negative. Let us consider the electric circuit given in Fig.



Considering the closed loop ABCDEFA,

$I_1R_2 + I_3R_4 + I_3r_3 + I_3R_5 + I_4R_6 + I_1r_1 + I_1R_1 = E_1 + E_3$

Both cells E1 and E3 send currents in clockwise direction.

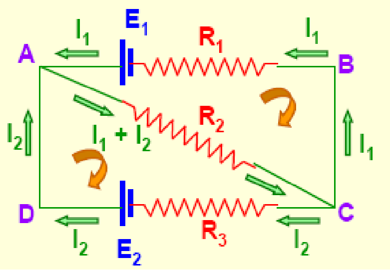
For the closed loop ABEFA

$I_1R_2 + I_2R_3 + I_2r_2 + I_4R_6 + I_1r_1 + I_1R_1 = E_1 – E_2$

Negative sign in E_2 indicates that it sends current in the anticlockwise direction.

II Law or Voltage Law or Loop Rule:

The algebraic sum of all the potential drops and emf's along any closed path in an electrical network is always zero.



Loop ABCA:

$- E_1 + I_1.R_1 + (I_1 + I_2).R_2 = 0$

Loop ACDA:

$- (I_1 + I_2).R_2 - I_2.R_3 + E_2 = 0$

Sign Conventions:

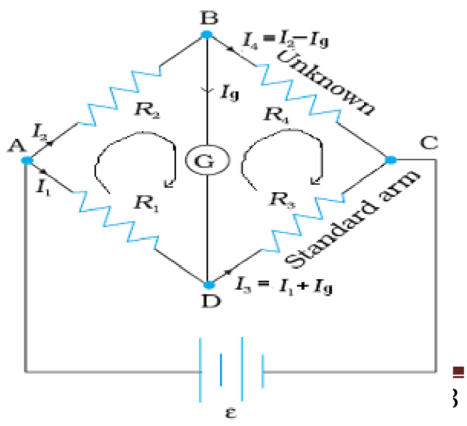
1. The emf is taken negative when we traverse from positive to negative terminal of the cell through the electrolyte.
2. The emf is taken positive when we traverse from negative to positive terminal of the cell through the electrolyte.

The potential falls along the direction of current in a current path and it rises along the direction opposite to the current path.

3. The potential fall is taken negative.
4. The potential rise is taken positive.

Note: The path can be traversed in clockwise or anticlockwise direction of the loop.

3.16 Wheatstone bridge: Derivation of balancing condition –



Wheatstone bridge is an electrical circuit constructed based on the application of Kirchhoff's rules. The bridge has four resistors R_1, R_2, R_3 and R_4. Across one pair of diagonally opposite points 'A' and 'C' a source is connected. (battery arm.) Between the other two vertices, 'B' and 'D', a galvanometer of resistance 'G' is connected. (Galvanometer arm.) Let I_g be the current in the

galvanometer. The branch currents and the directions are as shown in figure.
Apply Kirchhoff's loop rule to closed loop ADBA
$I_2R_2 - I_gG - I_1R_1 = 0$ ------- (1)
Apply Kirchhoff's loop rule to closed loop CBDC
$(I_2 - I_g) R_4 - (I_1 + I_g) R_3 - I_gG = 0$ -------- (2)
The bridge is said to be balanced, when the galvanometer show zero deflection. (i.e. $I_g = 0$, current through the galvanometer is zero.)
Then the equation (1) and (2) becomes
$I_2R_2 - 0 \times G - I_1R_1 = 0$
$I_2R_2 - I_1R_1 = 0$
$I_2R_2 = I_1R_1$ -------- (3)
$(I_2 - 0) R_4 - (I_1 + 0)R_3 - 0 \times G = 0$
$I_2 R_4 - I_1 R_3 = 0$
$I_2 R_4 = I_1 R_3$ ------- (4)
(3) / (4) gives,

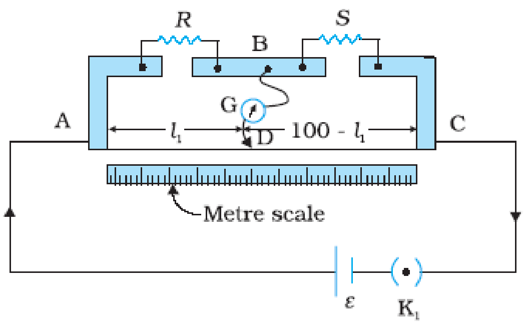
$$\frac{I_2R_2}{I_2R_4} = \frac{I_1R_1}{I_1R_3} \quad \text{or} \quad \frac{R_2}{R_4} = \frac{R_1}{R_3}$$

Hence $\boldsymbol{\frac{R_2}{R_1} = \frac{R_4}{R_3}}$ **or** $\boldsymbol{R_4 = R_3 \frac{R_2}{R_1}}$

This is the balancing condition of Wheatstone network.

3.17 Metre Bridge :

Metre bridge is one form of Wheatstone's bridge. The meter bridge consists of a wire of length 1m and of uniform cross sectional area stretched taut and clamped between two thick metallic strips bent at right angles, as shown. The metallic strip has two gaps across which resistors can be connected. The end points where the wire is clamped are connected to a cell through a key. One end of a galvanometer is connected to the metallic strip midway between the two gaps. The other end of the galvanometer is connected to a 'jockey'. The jockey is essentially a metallic rod whose one end has a knife-edge which can slide over the wire to make electrical connection as Shown in figure.



R is an unknown resistance whose value we want to determine. It is connected across one of the gaps. Across the other gap, we connect a standard known resistance *S*. The jockey is connected to some point D on the wire, a distance *l* cm from the end A. The jockey can be moved along the wire. The portion AD of the wire has a resistance $R_{cm}l$, where R_{cm} is the resistance of the wire per unit centimetre. The portion DC of the wire similarly has a resistance R_{cm} (100-l).

The four arms AB, BC, DA and CD [with resistances R, S, $R_{cm}\, l$ and $R_{cm}(100\text{-}l)$] obviously form a Wheatstone bridge with AC as the battery arm and BD the galvanometer arm. If the jockey is moved along the wire, then there will be one position where the galvanometer will show no current. Let the distance of the jockey from the end A at the balance point be $l= l_1$. The four resistances of the bridge at the balance point then are R, S, $R_{cm}\, l_1$ and $R_{cm}(100–l_1)$. The balance condition, Eq. [3.83(a)] gives

$$\frac{R}{S}=\frac{R_{cm}\, l_1}{R_{cm}\,(100-l_1)}=\frac{l_1}{100-l_1}$$

Thus, once we have found out l_1, the unknown resistance R is known in terms of the standard known resistance S by

$$R=S\,\frac{l_1}{100-l_1}$$

By choosing various values of S, we would get various values of l_1, and calculate R each time. An error in measurement of l_1 would naturally result in an error in R. It can be shown that the percentage error in R can be minimised by adjusting the balance point near the middle of the bridge, i.e., when l_1 is close to 50 cm. (This requires a suitable choice of S.)

3.18 Potentiometer:

The Potentiometer is an instrument used for the measurement of potential difference. It is basically a long piece of uniform wire, sometimes a few meters in length across which a standard cell is connected. In actual design, the wire is sometimes cut in several pieces placed side by side and connected at the ends by thick metal strip. (Fig.). In the figure, the wires run from A to C. The small vertical portions are the thick metal strips connecting the various sections of the wire.
A current I flows through the wire which can be varied by a variable resistance (rheostat, R) in the circuit. Since the wire is uniform, the potential difference between A and any point at a distance l from A is

$\varepsilon\,(l) = \varphi I$ ------- (1)

where φ is the potential drop per unit length.

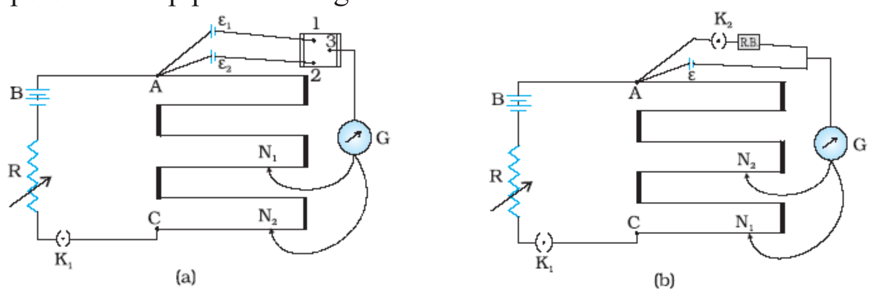


Figure 3.28 (a) shows an application of the potentiometer to compare the emf of two cells of emf ε_1 and ε_2. The points marked 1, 2, 3 form a two way key. Consider first a position of the key where 1 and 3 are connected so that the galvanometer is connected to ε_1. The jockey is moved

along the wire till at a point N_1, at a distance l_1 from A, there is no deflection in the galvanometer. We can apply Kirchhoff's loop rule to the closed loop $AN_1G_{31}A$ and get,

$$\phi\, l_1 + 0 - \varepsilon_1 = 0$$

Similarly, if another emf ε_2 is balanced against l_2 (AN_2)

$$\phi\, l_2 + 0 - \varepsilon_2 = 0$$

From the last two equations

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

This simple mechanism thus allows one to compare the emf's of any two sources. In practice one of the cells is chosen as a standard cell whose emf is known to a high degree of accuracy. The emf of the other cell is then easily calculated from above Eq.

We can also use a potentiometer to measure internal resistance of a cell [Fig. (b)]. For this the cell (emf ε) whose internal resistance (r) is to be determined is connected across a resistance box through a key K_2, as shown in the figure. With key K_2 open, balance is obtained at length l_1 (AN_1). Then,

$$\varepsilon = \phi\, l_1$$

When key K_2 is closed, the cell sends a current (I) through the resistance box (R). If V is the terminal potential difference of the cell and balance is obtained at length l_2 (AN_2),

$$V = \phi\, l_2$$

So, we have **$\varepsilon/V = l_1/l_2$** ------------- (a)

But, $\varepsilon = I(r + R)$ and $V = IR$. This gives

$\varepsilon/V = (r+R)/R$ ----------- (b)

From Eq. (a) and (b) we have

$(R+r)/R = l_1/ l_2$

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

Using Eq. (3.95) we can find the internal resistance of a given cell. The potentiometer has the advantage that it draws *no current* from the voltage source being measured. As such it is unaffected by the internal resistance of the source.

3.19 Mention of applications (a) to compare emf of two cells and (b) to measure internal resistance of a cell,

Numerical Problems.

1. Define of electric current and Explain electric currents in a conductor ?

2. Define of current density ?

3. State and explain Ohm's law ?

4. Deduce the dependence of electrical resistance on the dimensions of conductor ($R= \rho l/A$)

5. Define electrical resistivity and conductivity ? Derive the relation $\vec{J} = \sigma \vec{E}$ (equivalent form of Ohm's law) and list the limitations of Ohm's law ?

6. Explain drift of electrons and origin of resistivity:

7. Define drift velocity, relaxation time and mobility. hence derive an expression for conductivity of a material ($\sigma = ne^2 \tau/m$) ?

8. Explain the method Color coding of carbon resistors;

9. Obtain the expression for temperature dependence of resistivity of metals and semiconductors.

10. Explain electrical energy and power: Mention of expression for power loss.

11. Explain combination of resistors. Derive effective resistance of two resistors (a) in series combination and (b) in parallel combination.

12. What is a cell ? Define internal resistance of a cell, terminal potential difference and emf of a cell ?

13. Derive an expression of current drawn by external resistance ?

14. Explain the various methods of combination of cells: Derive an expressions for equivalent emf and equivalent internal resistance (a) in series and (b) in parallel combination.

15. State and explain Kirchhoff's rules ?

16. What is Wheatstone bridge ? Derive an expression of balancing condition ?

17. What is Metre Bridge. How to measure unknown resistance using it ?

18. What is Potentiometer ? Explain its Principle & working ? Mention its applications (a) to compare emf of two cells and (b) to measure internal resistance of a cell ?

Numerical Problems.

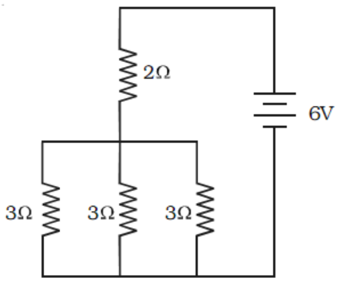
1. What is the drift velocity of an electron in a copper conductor having area 10 × 10−6m2, carrying a current of 2 A. Assume that there are 10 × 1028 electrons / m3.

2. How much time 10^{20} electrons will take to flow through a point, so that the current is 200 mA? ($e = 1.6 \times 10^{-19}$ C)

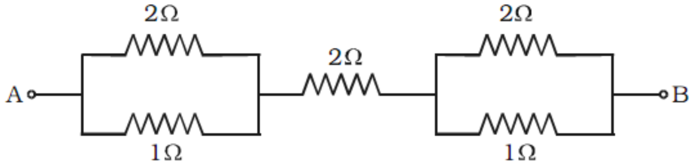
3. A manganin wire of length 2m has a diameter of 0.4 mm with a resistance of 70 Ω. Find the resistivity of the material.

4. The effective resistances are 10Ω, 2.4Ω when two resistors are connected in series and parallel. What are the resistances of individual resistors?

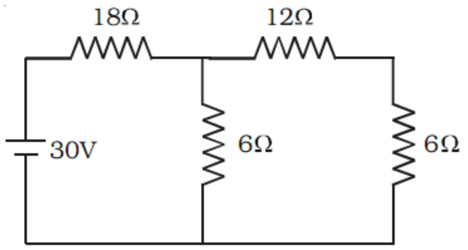
5. In the given circuit, what is the total resistance and current supplied by the battery.



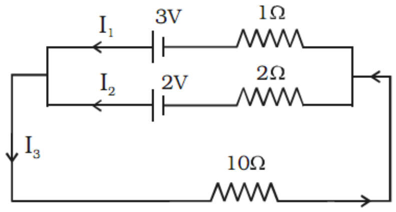
6. Find the effective resistance between A and B in the given circuit



7. Find the voltage drop across 18 Ω resistor in the given circuit



8. Calculate the current I1, I2 and I3 in the given electric circuit.



9. The resistance of a platinum wire at 0^0 C is 4 Ω. What will be the resistance of the wire at 100oC if the temperature coefficient of resistance of platinum is 0.0038 /0 C.

10. A cell has a potential difference of 6 V in an open circuit, but it falls to 4 V when a current of 2 A is drawn from it. Find the internal resistance of the cell.

11. In a Wheatstone's bridge, if the galvanometer shows zero deflection, find the unknown resistance. Given P = 1000Ω Q = 10000 Ω and R = 20 Ω

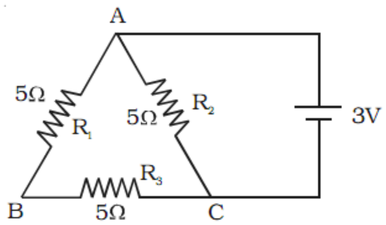
12. An electric iron of resistance 80 Ω is operated at 200 V for two hours. Find the electrical energy consumed.

13. In a house, electric kettle of 1500 W is used everyday for 45 minutes, to boil water. Find the amount payable per month (30 days) for usage of this, if cost per unit is Rs. 3.25

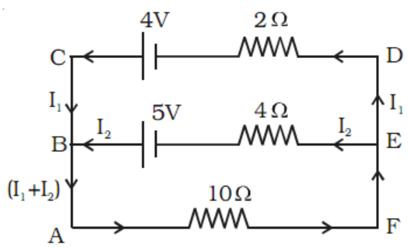
14. A 1.5 V carbon – zinc dry cell is connected across a load of 1000 Ω. Calculate the current and power supplied to it.

15. In a metre bridge, the balancing length for a 10 Ω resistance in left gap is 51.8 cm. Find the unknown resistance and specific resistance of a wire of length 108 cm and radius 0.2 mm.

16. Find the electric current flowing through the given circuit connected to a supply of 3 V.



17. In the given circuit, find the current through each branch of the circuit and the potential drop across the 10 Ω resistor.



ONE MARK QUESTIONS

1. What constitutes an electric current?

Ans. Charges in motion constitutes an electric current.

2. Define electric current.

Ans. The amount of charge flowing across an area held normal to the direction of flow of charge per unit time is called electric current.

3. Give the SI unit of current.

Ans. SI unit of current is ampere(A).

4. Define the unit of electric current. Or Write the relation between coulomb and ampere.

Ans. If one coulomb of charge crosses an area normally in one second, then the current through that area is one ampere.

i.e. 1 ampere = $\frac{1coulomb}{1\ second}$ or 1 A = 1C s^{-1}

5. How many electrons per second constitute a current of one milli ampere?

Ans. We have,

$I = \frac{q}{t} = \frac{ne}{t}$ $\therefore n = \frac{It}{e} = \frac{10^{-3} X 1}{1.6 X 10^{-19}} = $ = 6.25×10^{15} electrons

6. Is electric current is a scalar or vector quantity.

Ans. It is a scalar quantity.

7. What do you mean by steady current?

Ans. A current whose magnitude does not change with time is called steady current.

8. What do you mean by varying current?

Ans. A current whose magnitude changes with time is called varying current.

9. What does the direction of electric current signify in an electric circuit?

Ans. The direction of conventional current in an electric circuit tells the direction of flow of positive charges in that circuit.

10. What is the net flow of electric charges in any direction inside the solid conductor?

Ans. It is zero.

11. Name the current carries in metals (solid conductors), / electrolytic solutions (liquid conductors) and /discharge tubes (gaseous conductors).

Ans. Free electrons in solid conductors ,/ positively and negatively charged ions in liquid conductors and / positive ions and electrons in gaseous conductors.

12. State Ohm's law.

Ans. Ohm's law states that the current (I) flowing through a conductor is directly proportional to the potential difference (V) applied across its ends, provided the temperature and other physical conditions remain constant". i.e. I or V = IR.

13. Define resistance.
Ans. It is defined as the ratio of the potential difference (V) across the ends of the conductor to the electric current (I) through it. or R = V/ I.

14. Define the SI unit of resistance
Ans. The resistance of a conductor is I ohm if one ampere of current flows through it when the potential difference across its ends is one volt.

15. How does the resistance of a conductor depend on length?
Ans. The resistance (R) of a conductor is directly proportional to its length (l) i. e., $R \propto l$

16. How does the resistance of a conductor depend on area of cross section of a conductor?
Ans. The resistance (R) of a conductor is inversely proportional to its area of cross section (A). i.e., $R \propto 1/A$

17. Define electrical conductance.
Ans. The reciprocal of resistance is called electrical conductance (G). G = 1/R

18. Mention the SI unit of conductance.
Ans. Siemen (s) or mho (℧).

19. Define resistivity of a conductor.
Ans. The resistivity of material of a conductor at a given temperature is equal to resistance of unit length of the conductor having unit area of cross section.

20. A wire of resistivity ρ is stretched to three times its length. What will be its new resistivity?
Ans. There will be no change in its resistivity, because resistivity does not depend on length (dimension) of wire.

21. Mention the relation between the resistance and resistivity?
Ans. The resistance R of a conductor is given by $R = \rho\,(l/A)$, where L- length of the conductor, A- area of cross section of the conductor.

22. Mention the SI unit of resistivity?
Ans. The SI unit of resistivity is ohm-meter (Ω - m)

23. Define the term current density (j)
Ans. It is defined as the electric current (I) per unit area (A) taken normal to the direction of current. i.e., j = (I/A)

24. What is the SI unit of current density?
Ans. Ampere / metre^2 (A/m^2)

25. Is current density is a scalar or vector quantity?
Ans. It is a vector quantity.

26. Define electrical conductivity.
Ans. The reciprocal of electrical resistivity of material of a conductor is called conductivity.
i.e. $\sigma = (1/\rho)$

27. Mention the relation between current density and conductivity.
Ans. The current density j and conductivity σ are related by $\vec{J} = \sigma \vec{E}$

28. Define drift velocity.
Ans. It is defined as the average velocity gained by the free electrons of a conductor in the opposite of the externally applied electric field.

29. What is the average velocity of free electrons in a metal at room temperature?
Ans. Zero.

30. What is the effect of temperature on the drift speed of electrons in a metallic conductor?
Ans. The drift speed decreases with increase in temperature.

31. Define relaxation time or mean free time.
Ans. The average time that elapses between two successive collisions of an electron with fixed atoms or ions in the conductor is called relaxation time.

32. What is the effect of relaxation time of electrons in a metal?
Ans. Relaxation time decreases with increase in temperature.

33. Define electron mobility.
Ans. Mobility (μ) is defined as the magnitude of drift velocity (v_d) per unit electric field (E). i.e.
$\mu = (v_d/E)$

34. Mention the SI unit of mobility.
Ans. The SI unit of electron mobility is $m^2\ V^{-1}\ s^{-1}$.

35. Name two materials whose resistivity decreases with the rise of temperature.
Ans. Germanium and Silicon.

36. How does the resistance of an insulator change with temperature?
Ans. The resistance of an insulator decreases with the increase of temperature.

37. What will be the value of resistance of a resistor having four colour bands in the order yellow, violet , orange and silver?
Ans. 47000 ±10%

38. The value of resistance of a resistor is 1 K ±5% Ω. Write the colour sequence of the resistor.
Ans. The colour sequence is Brown, black, red and gold.

39. The value of resistance of a resistor is 0.1 ±10% Ω. Write the colour sequence of the resistor.

Ans. Resistance= 0.1 ±10% = 01 × 10^{-1} ±10% Ω. Thus, the colour sequence is Black, brown and gold. Tolerance of 10% is indicated by silver ring.

40. What is the colour of the third band of a coded resistor of resistance 4.3 x 10^4 Ω ?
Ans. Resistance = 4.3 × 10^4 Ω = 43 × 10^3 Ω. Therefore, the colour of third band of a coded resistance will be related to a number 3, i.e., orange.

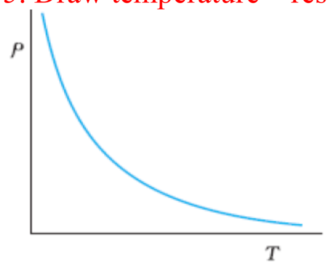
41. How does the resistance of a conductor vary with temperature?
Ans. The resistance of a conductor increases linearly with increase of temperature and vice-versa.

42. How does the resistivity of a conductor vary with temperature?
Ans. The resistivity of a conductor increases linearly with increase of temperature and vice-versa.

43. How does the resistivity of a semi conductor vary with temperature?
Ans. The resistivity of a semi conductor decreases exponentially with increase of temperature.

44.Name a material which exhibit very weak dependence of resistivity with temperature?
Ans. Nichrome , an alloy of nickel, iron and chromium exhibit very weak dependence of resistivity with temperature.

45. Draw temperature—resistivity graph for a semiconductor.



46.When the two resistors are said to be in series?
Ans. Two resistors are said to be in series if only one of their end points is joined.

47. When the two or more resistors are said to be in parallel?
Ans. Two or more resistors are said to be in parallel if one end of all the resistors is joined together and similarly the other ends joined together.

48. R_1 and R_2 are the two resistors in series. The rate of flow of charge through R_1 is I_1. What is the rate of flow through R_2?
Ans. Current is the measure of rate of flow of charge. Therefore the rate of flow of charge through R_2 is also I_1.

49. If V_1 and V_2 be the potential difference across R_1 and R_2 in series. How much is the potential difference across the combination?
Ans. The potential difference across the combination, $V = V_1 + V_2$.

50. What is the equivalent resistance of P resistors each of resistance R connected in series ?
Ans . $R_{eq} = R_1 + R_2 + R_3 + R_4 + \ldots\ldots\ldots\ldots\ldots$
$R_{eq} = R + R + R + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$P times = P(R)

51. What happens to the effective resistance when two or more resistors are connected in series?
Ans. The effective resistance when two or more resistors are connected in series increases and is greater than the greatest of individual resistance.

52. What happens to the effective resistance when two or more resistors are connected in parallel?
Ans. The effective resistance when two or more resistors are connected in parallel decreases and is smaller than the smallest of individual resistance.

53. What is emf of a cell?
Ans. Emf is the potential difference between the positive and negative electrodes in an open circuit. i.e. when no current flowing through the cell.

54. Define internal resistance of a cell.
Ans. The finite resistance offered by the electrolyte for the flow of current through it is called internal resistance.

55. Give the expression for the potential difference between the electrodes of a cell of emf and internal resistance r?
Ans. The potential difference **$V = - I r$.**

56. Write the expression for equivalent emf when two cells of emf's E_1 and E_2 connected in series.
Ans. $E_{eq} = E_1 + E_2$

57. Write the expression for equivalent emf when two cells of emf's E_1 and E_2 connected in series such that negative electrode of E_1 to negative electrode of E_2 ?
Ans. $E_{eq} = E_1 - E_2$

58. Write the expression for equivalent emf of n cells each of emf connected in series.
Ans. $E_{eq} = nE$

59. Write the expression for equivalent internal resistance of n cells each of internal resistance r connected in series.
Ans. $r_{eq} = n\,r$

60. What is an electric network?
Ans. It is a circuit in which several resistors and cells interconnected in a complicated way.

61. What is a node or junction in an electrical network?
Ans. It is a point in a network where more than two currents meet.

62. What is a mesh or loop in an electric network?
Ans. A mesh or loop is a closed path with in the network for the flow of electric current.

63. State Kirchhoff's junction rule.
Ans. At any junction in an electric network the sum of the currents entering the junction is equal to sum of the currents leaving the junction.

64. What is the significance of junction rule?
Ans. Conservation of charge.

65. State Kirchhoff's loop rule?
Ans. The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero.

66. What is the significance of loop rule?
Ans. Conservation of energy.

67. Write the condition for balance of Wheatstone's network.
Ans. At balance of Wheatstone network the resistors are such that the current through the galvanometer is zero.($I_g = 0$) or
$\frac{P}{Q} = \frac{R}{S}$ P, Q, S, R are in cyclic order.

68. On what principle a meter bridge work?
Ans. It works on the principle of balanced condition of whetstone's network.

69. Mention one use of meter bridge.
Ans. It is used to determine the unknown resistance of a given coil.

70.Write the expression for Unknown resistance R interms of standard resistance S and balancing length l,
Ans. $\mathbf{R} = \frac{sl}{1-l}$

71. How the error in finding R in a meter bridge can be minimized?
Ans. The error in finding R in a meter bridge can be minimized by adjusting the balancing point near middle of the bridge (close to 50cm) by suitable choice of standard resistance S.

72. What is a potentiometer?
Ans. It is an instrument consisting of long piece of uniform wire across which a standard cell is connected.

73. Mention the practical use of potentiometer.
Ans. It can be used to determine emf of a one cell knowing emf of the other and also internal resistance of a given cell.

74. Give the equation to compare emf's of two cells in terms of balancing length.
Ans. If l_1 and l_2 are the balancing length's then = $E_1/E_2 = l_1/l_2$

75. Give the formula to determine the internal resistance of the cell using potentiometer.
Ans. r = R $[\frac{l_1}{l_2} - 1]$, l_1 and l_2 are the balancing length's without and with the external resistance respectively.

76. What is the advantage of potentiometer?
Ans. The potentiometer has the advantage that it draws no current from the voltage source being measured.

77. Name the device used for measuring emf of a cell. Ans. potentiometer.
Ans. Potentiometer.
78. What is the direction of conventional current ?
Ans : From positive terminal of battery to negative terminal of battery through the external circuit. It is opposite to electron flow direction.

79.

TWO/THREE MARK QUESTIONS
1. How is the current conducted in metals ? explain
Ans. Every metal conductor has large number of free electrons which move randomly at room temperature. Their average thermal velocity at any instant is zero. When a potential difference is applied across the ends of a conductor, an electric field is set up. Due to it, the free electrons of the conductor experience force due to electric field and drift towards the positive end of the conductor, causing electric current.

2. Define the term (1) drift velocity (2) relaxation time.
Ans. (1) drift velocity :- It is defined as the average velocity gained by the free electrons of a conductor in the opposite of the externally applied electric field.
(2) relaxation time :- The average time that elapses between two successive collisions of an electron with fixed atoms or ions in the conductor is called relaxation time.

3. State and explain Ohm's law.
Ans. Statement:- Ohm's law states that " the current (I) flowing through a conductor is directly proportional to the potential difference (V) applied across its ends, provided the temperature and other physical conditions remain constant".
If I is the current and V is the potential difference between the ends of the conductor, then
i.e. $I \propto V$ or I = (constant) V
But the constant = conductance = 1 / R
Therefore I = V / R or V = IR .

4. Mention the factors on which resistivity of a metal depends.
Ans. Resistivity of a metallic conductor depends on(1) nature of the conductor (2) Temperature

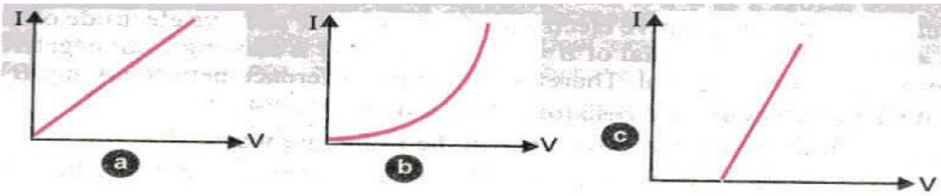
5. Write the expression for resistivity in terms of number density and relaxation time.
Ans. $\rho = \frac{m}{ne^2\tau}$ where n = number density of electrons, τ = relaxation time of free electrons.

6. Mention any two factors on which resistance of a conductor depends.
Ans. Resistance of a conductor depends on (1) length of the conductor (2) Area of cross section of the conductor.

7. Write the relation between current density and conductivity for a conductor.
Ans. $j = \sigma E$ Where σ = conductivity and E = electric field.

8. Why manganin is used to make standard resistance coils?
Ans. For manganin, the temperature coefficient of resistance is very low and its resistivity is quite high. Due to it, the resistance of manganin wire remains almost unchanged with change in temperature. Hence it is used.

9. Draw V-I graph for ohmic and non- ohmic materials.
Ans. V-I graph for ohmic material is a straight line passing through origin.(a) V-I graph for non-ohmic material is a curve i.e. non linear or straight line not passing through origin(b and c)



10. Distinguish between resistance and resistivity.
Ans.

Resistance	Resistivity
1.The opposition offered by a conductor to the flow of electric current through. 2. Resistance depends on dimensions i.e. length and area of cross section. 3. Its SI unit is ohm.	1. The resistance of unit cube of the material of a conductor is called resistivity. 2. Resistivity of a conductor depends on the nature of the material but is independent of the dimensions. 3. Its SI unit is ohm-meter

11. How does the resistance of (1) good conductor, (2) semiconductor change with rise of temperature?
Ans.(1) The resistance of a conductor increases with the increase in the temperature.

(2) The resistance of a semiconductor decreases with the increase in the temperature.

12. Distinguish between terminal potential difference and emf of a cell.

Ans.

Terminal potential difference	emf
1.It is the potential difference between the electrodes of a cell in a closed circuit (When current is drawn from the cell). Represented by V. 2. Its SI unit is volt.	1. It is the potential difference between the electrodes of a cell when no current is drawn from the cell. Represented by E. . .2. Its SI unit is volt

13. Terminal potential difference is less than the emf of a cell. Why?

Ans.When circuit is open,the terminal potential difference is equal to emf of the cell . When current is drawn from the cell, some potential drop takes place due to internal resistance of the cell. Hence terminal potential difference is less than the emf of a cell and is given by

$$V = E - Ir$$

14. Mention the factors on which internal resistance of a cell depend.

Ans. The internal resistance of a cell depend on (1) The nature of the electrolyte (2) nature of electrodes (3) temperature.(4) concentration of electrodes (5) distance between the electrodes (Any two)

15. For what basic purpose the cells are connected (1) in series (2) in parallel

Ans. The cells are connected (1) in series to get maximum voltage, (2) in parallel to get maximum current.

16. State Kirchhoff's laws of electrical network.

Ans. Kirchhoff's junction rule:- At any junction in an electric network the sum of the currents entering the junction is equal to sum of the currents leaving the junction.

Kirchhoff's loop rule:-The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero.

17. What is the cause of resistance of a conductor?

Ans. While drifting, the free electrons collide with the ions and atoms of the conductor i.e., motion of the electrons is opposed during the collisions, this is the basic cause of resistance in a conductor.

18. A large number of free electrons are present in metals. Why there is no current in the absence of electric field across?

Ans.. In the absence of an electric field, the motion of electrons in a metal is random. There is no net flow of charge across any section of the conductor. So no current flows in the metal.

19. State the principle of working of a potentiometer.
Ans. A potentiometer works on the principle that when a steady current flows through a wire of uniform cross section and composition, the potential drop across any length of the wire is directly proportional to that length.

20. Why are the connecting resistors in a meter bridge made of thick copper strips?
Ans. Thick copper strips offer minimum resistance and hence avoid the error due to end resistance which have not been taken in to account in the meter bridge formula.

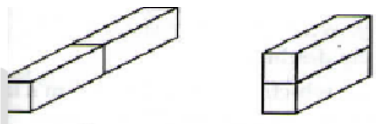
21. A Carbon resistor has three strips of red colour and a gold strip. What is the value of resistor? What is tolerance?
Ans. The value of resistance is 2200 ± 5 %. The percentage of deviation from the rated value is called tolerance.

22. If the emf of the cell be decreased, what will be the effect of zero deflection in a potentiometer? Explain.
Ans. If the emf of the cell is decreased, the potential gradient across the wire will decrease. Due to this the position of zero deflection will be obtained on the longer length.

23. Two identical slabs of given metal joined together in two different ways, as shown in figs. What is the ratio of the resistances of these two combinations?
Ans. For each slab, R = $\rho \times \frac{L}{A}$

$$R_1 = \rho \frac{2L}{A} = 2R ; \ R_2 = \rho \frac{L}{2A} = R/2,$$

$$\therefore \frac{R_1}{R_2} = \frac{2R}{\frac{R}{2}} = 4:1$$


(i) (ii)

24. Define the terms electric energy and electric power. Give their units.
Ans. Electric energy:-The total work done by the source of emf in maintaining an electric current in a circuit for a given time is called electric energy consumed in the circuit. Its SI unit is joule.
Electric power :- The rate at which work is done by a source of emf in maintaining an electric current through a circuit is called electric power of the circuit. Its SI unit is watt.

25. Mention the limitations of Ohm's law.
Ans :
Limitations of Ohm's law:
1. Ohm's law applicable only for good conductors.
2. Ohm's law applicable only, when the physical conditions like temperature, pressure and tension remains constant.
3. Ohm's law is not applicable at very low temperature and very high temperature.
4. Ohm's law is not applicable for semiconductors, thermistors, vacuum tubes, discharge tubes.

FIVE MARKS QUESTIONS

1. Derive the expression for electrical conductivity.
Refer Notes

2. State and deduce Ohm's law. From this law define the resistance of a conductor.
Refer Notes

3. What is meant by equivalent resistance ? Derive expression for equivalent resistance when the resistors are connected in series.
Refer Notes

4. What is meant by equivalent resistance ? Derive expression for equivalent resistance when the resistors are connected in parallel.
Refer Notes

5. Define emf and terminal potential difference of a cell. Derive an expression for main current using Ohm's law.
Refer Notes

6. Discuss the grouping of two unidentical cells in series and find their equivalent emf and internal resistance.
Refer Notes

7. State and explain Kirchhoff's rules with Example
Refer Notes

8. Deduce the condition for balance of Wheatstone's network using Kirchhoff's laws.
Refer Notes

PROBLEMS (5 M)

3.1 The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery?

3.2 A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

3.3 (a) Three resistors 1 Ω, 2 Ω, and 3 Ω are combined in series. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.

3.4 (a) Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.

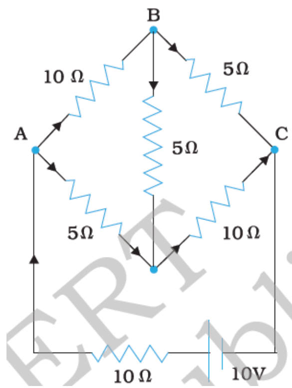
3.5 At room temperature (27.0 °C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is 1.70×10^{-4} °C^{-1}.

3.6 A negligibly small current is passed through a wire of length 15 m and uniform cross-section 6.0×10^{-7} m^2, and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?

3.7 A silver wire has a resistance of 2.1 Ω at 27.5 °C, and a resistance of 2.7 Ω at 100 °C. Determine the temperature coefficient of resistivity of silver.

3.8 A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0 °C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70×10^{-4}°C^{-1}.

3.9 Determine the current in each branch of the network shown in Fig. :



3.10 (a) In a metre bridge [Fig. 3.27], the balance point is found to be at 39.5 cm from the end *A*, when the resistor *Y* is of 12.5 Ω. Determine the resistance of *X*. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips?
(b) Determine the balance point of the bridge above if *X* and *Y* are interchanged.

(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?

3.11 A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

3.12 In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?

3. 13 The number density of free electrons in a copper conductor estimated in Example 3.1 is $8.5 \times 10^{28}\ m^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is $2.0 \times 10^{-6}\ m^2$ and it is carrying a current of 3.0 A.

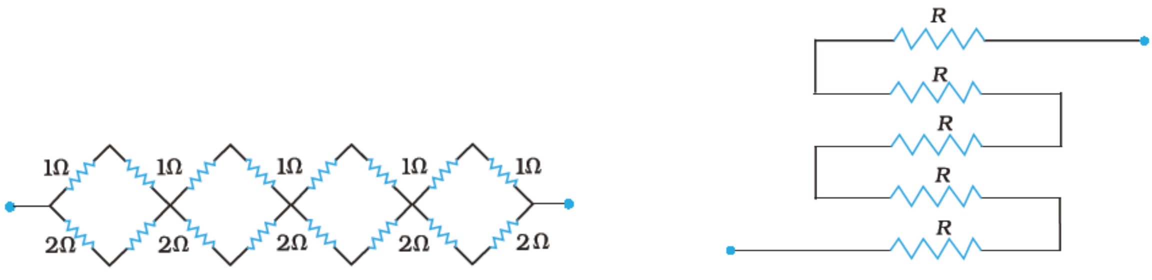
3.18 Answer the following questions:
(a) A steady current flows in a metallic conductor of non-uniform cross-section. Which of these quantities is constant along the conductor: current, current density, electric field, drift speed?
(b) Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.
(c) A low voltage supply from which one needs high currents must have very low internal resistance. Why?
(d) A high tension (HT) supply of, say, 6 kV must have a very large internal resistance. Why?

3.19 Choose the correct alternative:
(a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.
(b) Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.
(c) The resistivity of the alloy manganin is nearly independent of/increases rapidly with increase of temperature.
(d) The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of $(10^{22}/10^{23})$.

3.20 (a) Given *n* resistors each of resistance *R*, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?

(b) Given the resistances of 1 Ω, 2 Ω, 3 Ω, how will be combine them to get an equivalent resistance of (i) (11/3) Ω (ii) (11/5) Ω, (iii) 6 Ω, (iv) (6/11) Ω ?

(c) Determine the equivalent resistance of networks shown in Fig. 3.31.



(a) (b)

H. C. Verma Problems :

1. Suppose you have three resistors each of value 30 Ω. List all the different resistances you can obtain using them.

2. A proton beam is going from east to west. Is there an electric current ? If yes, in what direction ?

3. In an electrolyte, the positive ions move from left to right and the negative ions from right to left. Is there a net current ? If yes, in what direction ?

4. In a TV tube, the electrons are accelerated from the rear to the front. What is the direction of the current ?

5. The drift speed is defined as v_d - $\Delta l/\Delta t$ where Δl is the distance drifted in a long time Δt. Why don't we define the drift speed as the limit of $\Delta l/\Delta t$ as $\Delta t \longrightarrow 0$?

6. One of your friends argues that he has read in previous chapters that there can be no electric field inside a conductor. And hence there can be no current through it. What is the fallacy in this argument ?

7. When a current is established in a wire, the free electrons drift in the direction opposite to the current. Does the number of free electrons in the wire continuously decrease ?

8. A fan with copper winding in its motor consumes less power as compared to an otherwise similar fan having aluminium winding. Explain.

9. The thermal energy developed in a current-carrying resistor is given by $U = i^2 Rt$ and also by $U = Vit$. Should we say that U is proportional to i^2 or to i ?

10. Consider a circuit containing an ideal battery connected to a resistor. Do "work done by the battery" and "the thermal energy developed" represent two names of the same physical quantity ?

11. Is work done by a battery always equal to the thermal energy developed in electrical circuits ? What happens if a capacitor is connected in the circuit ?

12. A nonideal battery is connected to a resistor. Is work done by the battery equal to the thermal energy developed in the resistor ? Does your answer change if the battery is ideal ?

13. Sometimes it is said that "heat is developed" in a resistance when there is an electric current in it. Recall that heat is defined as the energy being transferred due to the temperature difference. Is the statement under quotes technically correct ?

14. We often say "a current is going through the wire". What goes through the wire, the charge or the current ?

15. Would you prefer a voltmeter or a potentiometer to measure the emf of a battery ?

16. Does a conductor become charged when a current is passed through it ?

17. Can the potential difference across a battery be greater than its emf?

OBJECTIVE TYPE QUESTIONS :

1. A metallic resistor is connected across a battery. If the number of collisions of the free electrons with the lattice is somehow decreased in the resistor (for example, by cooling it), the current will (a) increase (b) decrease (c) remain constant (d) become zero.

2. Two resistors A and B have resistances R_A and R_B respectively with $R_A < R_B$. The resistivities of their materials are ρ_A, and ρ_B. (a) $\rho_A > \rho_B$ (b) $\rho_A = \rho_B$ (c) $\rho_A < \rho_B$ (d) The information is not sufficient to find the relation between ρ_A and ρ_B.

3. The product of resistivity and conductivity of a cylindrical conductor depends on (a) temperature (b) material (c) area of cross-section (d) none of these.

4. As the temperature of a metallic resistor is increased, the product of its resistivity and conductivity (a) increases (b) decreases (c) remains constant (d) may increase or decrease.

5. In an electric circuit containing a battery, the charge (assumed positive) inside the battery (a) always goes from the positive terminal to the negative terminal (b) may go from the positive terminal to the negative terminal (c) always goes from the negative terminal to the positive terminal (d) does not move.

6. A resistor of resistance R is connected to an ideal battery. If the value of R is decreased, the power dissipated in the resistor will (a) increase (b) decrease (c) remain unchanged.

7. A current passes through a resistor. Let K_1, and K_2 represent the average kinetic energy of the conduction electrons and the metal ions respectively. (a) $K_1 < K_2$. (b) $K_1 = K_2$. (c) $K_1 > K_2$ (d) Any of these three may occur.

8. Two resistors R and $2R$ are connected in series in an electric circuit. The thermal energy developed in R and 2R are in the ratio (a) 1 :2 (b) 2: 1 (c) 1 : 4 (d) 4 : 1.

9. Two resistances R and $2R$ are connected in parallel in an electric circuit. The thermal energy developed in R and $2R$ are in the ratio (a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1.

10. A uniform wire of resistance 50 Ω is cut into 5 equal parts. These parts are now connected in parallel. The equivalent resistance of the combination is
(a) 2 a (b) 10 a (c) 250 a (d) 6250 a.

11. Consider the following two statements:
(A) Kirchhoffs junction law follows from conservation of charge.
(B) Kirchhoffs loop law follows from conservative nature of electric field.
(a) Both A and *B* are correct, (b) *A* is correct but *B* is wrong.
(c) *B* is correct but A is wrong. (d) Both A and *B* are wrong.

12. Two nonideal batteries are connected in series. Consider the following statements:
(A) The equivalent emf is larger than either of the two emfs.
(B) The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Each of A and *B* is correct. (b) A is correct but *B* is wrong.
(c) *B* is correct but *A* is wrong. (d) Each of A and *B* is wrong.

13. Two nonideal batteries are connected in parallel. Consider the following statements:
(A) The equivalent emf is smaller than either of the two emfs.
(B) The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Both A and *B* are correct. (b) A is correct but *B* is wrong.
(c) *B* is correct but A is wrong. (d) Both A and *B* are wrong.

14. The net resistance of an ammeter should be small to ensure that
(a) it does not get overheated
(b) it does not draw excessive current
(c) it can measure large currents
(d) it does not appreciably change the current to be measured.

15. The net resistance of a voltmeter should be large to ensure that
(a) it does not get overheated
(b) it does not draw excessive current
(c) it can measure large potential differences
(d) it does not appreciably change the potential difference to be measured.

16. Consider a capacitor-charging circuit. Let Q, be the charge given to the capacitor in a time interval of 10 ms and *Q2* be the charge given in the next time interval of 10 ms. Let 10 μC

charge be deposited in a time interval t1, and the next 10 μC charge is deposited in the next time interval t2

(a) Q1 > *Q2, t1 > t2.* (b) *Q1 > Q2, t1 < t2.*
(c) *Q1 < Q2, t1 > t2.* (d) *Q1 < Q2, t1 < t2.*

3. When no current is passed through a conductor, (a) the free electrons do not move (b) the average speed of a free electron over a large period of time is zero (c) the average velocity of a free electron over a large period of time is zero (d) the average of the velocities of all the free electrons at an instant is zero.

4. Which of the following quantities do not change when a resistor connected to a battery is heated due to the current ? (a) drift speed (b) resistivity (c) resistance (d) number of free electrons.

5. As the temperature of a conductor increases, its resistivity and conductivity change. The ratio of resistivity to conductivity (a) increases (b) decreases (c) remains constant (d) may increase or decrease depending on the actual temperature

6. A current passes through a wire of nonuniform cross-section. Which of the following quantities are independent of the cross-section ? (a) the charge crossing in a given time interval (b) drift speed (c) current density (d) free-electron density.

7. Mark out the correct options. (a) An ammeter should have small resistance. (b) An ammeter should have large resistance.
(c) A voltmeter should have small resistance. (d) A voltmeter should have large resistance.

8. A capacitor of capacitance 500 pF is connected to a battery through a 10 kΩ resistor. The charge stored on the capacitor in the first 5 s is larger than the charge stored in the next (a) 5 s (b) 50 s (c) 500 s (d) 500.

9. A capacitor C_1 of capacitance 1 μF and a capacitor C_2 of capacitance 2 μF are separately charged by a common battery for a long time. The two capacitors are then separately discharged through equal resistors. Both the discharge circuits are connected at $t = 0$.
(a) The current in each of the two discharging circuits is zero at $t = 0$.
(b) The currents in the two discharging circuits at t = 0 are equal but not zero.
(c) The currents in the two discharging circuits at $t = 0$ are unequal.
(d) C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge.

EXERCISES

1. The amount of charge passed in time *t* through a cross-section of a wire is *Q{t) = At2 + Bt + C.* (a) Write the dimensional formulae for *A, B* and C. (b) If the numerical values of *A, B* and *C* are 5, 3 and 1 respectively in SI units, find the value of the current at *t* = 5 s.

Ans :

1. $Q(t) = At^2 + Bt + c$

a) $At^2 = Q$

$\Rightarrow A = \frac{Q}{t^2} = \frac{A'T'}{T^{-2}} = A^1T^{-1}$

b) $Bt = Q$

$\Rightarrow B = \frac{Q}{t} = \frac{A'T'}{T} = A$

c) $C = [Q]$

$\Rightarrow C = A'T'$

d) Current t = $\frac{dQ}{dt} = \frac{d}{dt}\left(At^2 + Bt + C\right)$

$= 2At + B = 2 \times 5 \times 5 + 3 = 53$ A.

2. An electron gun emits 2'0 x 10^{19} electrons per second. What electric current does this correspond to?

Ans :

2. No. of electrons per second = 2×10^{16} electrons / sec.

Charge passing per second = $2 \times 10^{16} \times 1.6 \times 10^{-9} \frac{\text{coulomb}}{\text{sec}}$

$= 3.2 \times 10^{-9}$ Coulomb/sec

Current = 3.2×10^{-3} A.

3. The electric current existing in a discharge tube is 2.0 μA. How much charge is transferred across a cross-section of the tube in 5 minutes ?

Ans :

3. $i' = 2\ \mu A$, $t = 5$ min $= 5 \times 60$ sec.

$q = i\,t = 2 \times 10^{-6} \times 5 \times 60$

$= 10 \times 60 \times 10^{-6}\ c = 6 \times 10^{-4}\ c$

4. The current through a wire depends on time as $i = i_0 + at$, where i_0 = 10 A and a = 4 A/s. Find the charge crossed through a section of the wire in 10 seconds.

Ans :

4. $i = i_0 + \alpha t$, $t = 10$ sec, $i_0 = 10$ A, $\alpha = 4$ A/sec.

$q = \int_0^t i dt = \int_0^t (i_0 + \alpha t) dt = \int_0^t i_0 dt + \int_0^t \alpha t dt$

$= i_0 t + \alpha \frac{t^2}{2} = 10 \times 10 + 4 \times \frac{10 \times 10}{2}$

$= 100 + 200 = 300$ C.

5. A current of 1.0 A exists in a copper wire of cross-section 1.0 mm^2. Assuming one free electron per atom calculate the drift speed of the free electrons in the wire. The density of copper is 9000 kg/m^3.

Ans :

5. $i = 1$ A, $A = 1\ mm^2 = 1 \times 10^{-6}\ m^2$
f' cu = 9000 kg/m^3
Molecular mass has N_0 atoms
$= m$ Kg has $(N_0/M \times m)$ atoms $= \frac{N_0 Al9000}{63.5 \times 10^{-3}}$

No.of atoms = No.of electrons

$$n = \frac{\text{No.of electrons}}{\text{Unit volume}} = \frac{N_0 Af}{mAl} = \frac{N_0 f}{M}$$

$$= \frac{6 \times 10^{23} \times 9000}{63.5 \times 10^{-3}}$$

$i = V_d\, n\, A\, e.$

$$\Rightarrow V_d = \frac{i}{nAe} = \frac{1}{\frac{6 \times 10^{23} \times 9000}{63.5 \times 10^{-3}} \times 10^{-6} \times 1.6 \times 10^{-19}}$$

$$= \frac{63.5 \times 10^{-3}}{6 \times 10^{23} \times 9000 \times 10^{-6} \times 1.6 \times 10^{-19}} = \frac{63.5 \times 10^{-3}}{6 \times 9 \times 1.6 \times 10^{26} \times 10^{-19} \times 10^{-6}}$$

$$= \frac{63.5 \times 10^{-3}}{6 \times 9 \times 1.6 \times 10} = \frac{63.5 \times 10^{-3}}{6 \times 9 \times 16}$$

$= 0.074 \times 10^{-3}$ m/s = 0.074 mm/s.

6. A wire of length 1 m and radius 0.1 mm has a resistance of 100 Ω. Find the resistivity of the material.

Ans :

6. $\ell = 1$ m, $r = 0.1$ mm $= 0.1 \times 10^{-3}$ m
$R = 100\ \Omega$, $f = ?$
$\Rightarrow R = f\ell / a$

$$\Rightarrow f = \frac{Ra}{\ell} = \frac{100 \times 3.14 \times 0.1 \times 0.1 \times 10^{-6}}{1}$$

$= 3.14 \times 10^{-6} = \pi \times 10^{-6}\ \Omega$-m.

7. A uniform wire of resistance 100 Ω is melted and recast in a wire of length double that of the original. What would be the resistance of the wire ?

Ans :

7. $\ell' = 2\ell$
volume of the wire remains constant.
$A\ell = A'\ell'$
$\Rightarrow A\ell = A' \times 2\ell$
$\Rightarrow A' = A/2$
f = Specific resistance

$$R = \frac{f\ell}{A}\ ;\ R' = \frac{f\ell'}{A'}$$

$$100\ \Omega = \frac{f2\ell}{A/2} = \frac{4f\ell}{A} = 4R$$

$\Rightarrow 4 \times 100\ \Omega = 400\ \Omega$

8. Consider a wire of length 4 m and cross-sectional area 1 mm carrying a current of 2 A. If each cubic metre of the material contains 10^{29} free electrons, find the average time taken by an electron to cross the length of the wire.

Ans :

8. $\ell = 4$ m, $A = 1\ mm^2 = 1 \times 10^{-6}\ m^2$
$I = 2$ A, $n/V = 10^{29}$, $t = ?$
$i = n\, A\, V_d\, e$
$\Rightarrow e = 10^{29} \times 1 \times 10^{-6} \times V_d \times 1.6 \times 10^{-19}$

$$\Rightarrow V_d = \frac{2}{10^{29} \times 10^{-6} \times 1.6 \times 10^{-19}}$$

$$= \frac{1}{0.8 \times 10^4} = \frac{1}{8000}$$

$$t = \frac{\ell}{V_d} = \frac{4}{1/8000} = 4 \times 8000$$

$= 32000 = 3.2 \times 10^4$ sec.

9. What length of a copper wire of cross-sectional area 0.01 mm will be needed to prepare a resistance of 1 kΩ ? Resistivity of copper $1.7 \times 10^2\ \Omega$-m.

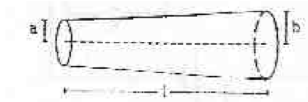
Ans :

9. $f_{cu} = 1.7 \times 10^{-8}\ \Omega$-m
$A = 0.01\ mm^2 = 0.01 \times 10^{-6}\ m^2$
$R = 1\ K\Omega = 10^3\ \Omega$
$R = \frac{f\ell}{a}$

$$\Rightarrow 10^3 = \frac{1.7 \times 10^{-8} \times \ell}{10^{-6}}$$

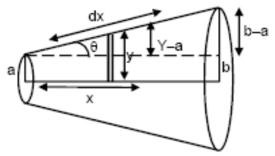
$$\Rightarrow \ell = \frac{10^3}{1.7} = 0.58 \times 10^3 \text{ m} = 0.6 \text{ km}.$$

10. Figure shows a conductor of length l having a circular cross-section. The radius of cross-section varies linearly from *a* to b. The resistivity of the material is ρ. Assuming that *b - a << l,* find the resistance of the conductor.



Ans :

10. dR, due to the small strip dx at a distanc x d = R = $\frac{f dx}{\pi y^2}$ ...(1)



$$\tan\theta = \frac{y-a}{x} = \frac{b-a}{L}$$

$$\Rightarrow \frac{y-a}{x} = \frac{b-a}{L}$$

$$\Rightarrow L(y-a) = x(b-a)$$

$$\Rightarrow Ly - La = xb - xa$$

$$\Rightarrow L\frac{dy}{dx} - 0 = b - a \quad \text{(diff. w.r.t. x)}$$

$$\Rightarrow L\frac{dy}{dx} = b - a$$

$$\Rightarrow dx = \frac{L dy}{b-a} \qquad ...(2)$$

Putting the value of dx in equation (1)

$$dR = \frac{f L dy}{\pi y^2 (b-a)}$$

$$\Rightarrow dR = \frac{fl}{\pi(b-a)} \frac{dy}{y^2}$$

$$\Rightarrow \int_0^R dR = \frac{fl}{\pi(b-a)} \int_a^b \frac{dy}{y^2}$$

$$\Rightarrow R = \frac{fl}{\pi(b-a)} \frac{(b-a)}{ab} = \frac{fl}{\pi ab}.$$

11. A copper wire of radius 01 mm and resistance 1 kΩ is connected across a power supply of 20 V. (a) How many electrons are transferred per second between the supply and the wire at one end ? (b) Write down the current density in the wire.

Ans :

11. $r = 0.1 \text{ mm} = 10^{-4} \text{ m}$

$R = 1 \text{ K}\Omega = 10^3 \ \Omega$, $V = 20 \text{ V}$

a) No.of electrons transferred

$$i = \frac{V}{R} = \frac{20}{10^3} = 20 \times 10^{-3} = 2 \times 10^{-2} \text{ A}$$

$$q = i\,t = 2 \times 10^{-2} \times 1 = 2 \times 10^{-2} \text{ C.}$$

$$\text{No. of electrons transferred} = \frac{2 \times 10^{-2}}{1.6 \times 10^{-19}} = \frac{2 \times 10^{-17}}{1.6} = 1.25 \times 10^{17}.$$

b) Current density of wire

$$= \frac{i}{A} = \frac{2 \times 10^{-2}}{\pi \times 10^{-8}} = \frac{2}{3.14} \times 10^{+6}$$

$$= 0.6369 \times 10^{+6} = 6.37 \times 10^5 \text{ A/m}^2.$$

12. Calculate the electric field in a copper wire of cross-sectional area 2.0 mm^2 carrying a current of 1 A. The resistivity of copper = 1.7 x 10^8 Ω-m.

Ans :

12. $A = 2 \times 10^{-6}\ m^2$, $I = 1\ A$
$f = 1.7 \times 10^{-8}\ \Omega\text{-m}$
$E = ?$

$$R = \frac{f\ell}{A} = \frac{1.7 \times 10^{-8} \times \ell}{2 \times 10^{-6}}$$

$$V = IR = \frac{1 \times 1.7 \times 10^{-8} \times \ell}{2 \times 10^{-6}}$$

$$E = \frac{dV}{dL} = \frac{V}{I} = \frac{1.7 \times 10^{-8} \times \ell}{2 \times 10^{-6}\ \ell} = \frac{1.7}{2} \times 10^{-2}\ V/m$$

$= 8.5$ mV/m.

13. A wire has a length of 2.0 m and a resistance of 5.0 Ω. Find the elecric field existing inside the wire if it carries a current of 10 A.

Ans :

13. $l = 2$ m, $R = 5\ \Omega$, $i = 10$ A, $E = ?$
$V = iR = 10 \times 5 = 50$ V

$$E = \frac{V}{l} = \frac{50}{2} = 25\ V/m.$$

14. The resistances of an iron wire and a copper wire at 20°C are 3.9 Ω and 4.1 Ω respectively. At what temperature will the resistances be equal ? Temperature coefficient of resistivity for iron is 5.0 x 10^{-3} and for copper it is 4.0 x 10^{-3} K Neglect any thermal expansion.

Ans :

14. $R'_{Fe} = R_{Fe}(1 + \alpha_{Fe}\,\Delta\theta)$, $R'_{Cu} = R_{Cu}(1 + \alpha_{Cu}\,\Delta\theta)$
$R'_{Fe} = R'_{Cu}$
$\Rightarrow R_{Fe}(1 + \alpha_{Fe}\,\Delta\theta), = R_{Cu}(1 + \alpha_{Cu}\,\Delta\theta)$
$\Rightarrow 3.9\,[\,1 + 5 \times 10^{-3}\,(20 - \theta)] = 4.1\,[1 + 4 \text{ x } 10^{-3}\,(20 - \theta)]$
$\Rightarrow 3.9 + 3.9 \times 5 \times 10^{-3}\,(20 - \theta) = 4.1 + 4.1 \times 4 \times 10^{-3}\,(20 - \theta)$
$\Rightarrow 4.1 \times 4 \times 10^{-3}\,(20 - \theta) - 3.9 \times 5 \times 10^{-3}\,(20 - \theta) = 3.9 - 4.1$
$\Rightarrow 16.4(20 - \theta) - 19.5(20 - \theta) = 0.2 \times 10^{3}$
$\Rightarrow (20 - \theta)\,(-3.1) = 0.2 \times 10^{3}$
$\Rightarrow \theta - 20 = 200$
$\Rightarrow \theta = 220°C.$

15. The current in a conductor and the potential difference across its ends are measured by an ammeter and a voltmeter. The meters draw negligible currents. The ammeter is accurate but the voltmeter has a zero error (that is, it does not read zero when no potential difference is applied). Calculate the zero error if the readings for two different conditions are 1.75 A, 14.4 V and 2.75 A, 22.4 V.

Ans :

15. Let the voltmeter reading when, the voltage is 0 be X.

$$\frac{I_1R}{I_2R} = \frac{V_1}{V_2}$$

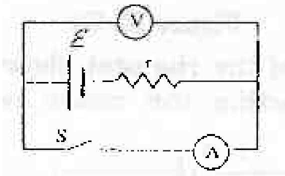
$$\Rightarrow \frac{1.75}{2.75} = \frac{14.4 - V}{22.4 - V} \Rightarrow \frac{0.35}{0.55} = \frac{14.4 - V}{22.4 - V}$$

$$\Rightarrow \frac{0.07}{0.11} = \frac{14.4 - V}{22.4 - V} \Rightarrow \frac{7}{11} = \frac{14.4 - V}{22.4 - V}$$

$\Rightarrow 7(22.4 - V) = 11(14.4 - V) \Rightarrow 156.8 - 7V = 158.4 - 11V$
$\Rightarrow (7 - 11)V = 156.8 - 158.4 \Rightarrow -4V = -1.6$
$\Rightarrow V = 0.4$ V.

16. Figure shows an arrangement to measure the emf ε and internal resistance *r* of a battery. The voltmeter has a very high resistance and the ammeter also has some resistance. The voltmeter

reads 1.52 V when the switch S is open. When the switch is closed the voltmeter reading drops to 1.45 V and the ammeter reads 1.0 A. Find the emf and the internal resistance of the battery.

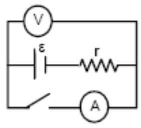


Ans :

16. a) When switch is open, no current passes through the ammeter. In the upper part of the circuit the Voltmenter has ∞ resistance. Thus current in it is 0.

∴ Voltmeter read the emf. (There is not Pot. Drop across the resistor).

b) When switch is closed current passes through the circuit and if its value of i.



The voltmeter reads

$\varepsilon - ir = 1.45$

$\Rightarrow 1.52 - ir = 1.45$

$\Rightarrow ir = 0.07$

$\Rightarrow 1\,r = 0.07 \Rightarrow r = 0.07\ \Omega.$

17. The potential difference between the terminals of a battery of emf 60 V and internal resistance 1 Ω drops to 5.8 V when connected across an external resistor. Find the resistance of the external resistor.

Ans :

17. E = 6 V, r = 1 Ω, V = 5.8 V, R = ?

$I = \frac{E}{R+r} = \frac{6}{R+1},\ V = E - Ir$

$\Rightarrow 5.8 = 6 - \frac{6}{R+1}\times 1 \Rightarrow \frac{6}{R+1} = 0.2$

$\Rightarrow R + 1 = 30 \Rightarrow R = 29\ \Omega.$

18. The potential difference between the terminals of a 6.0 V battery is 7.2 V when it is being charged by a current of 2.0 A. What is the internal resistance of the battery ?

Ans :

18. $V = \varepsilon + ir$

$\Rightarrow 7.2 = 6 + 2\times r$

$\Rightarrow 1.2 = 2r \Rightarrow r = 0.6\ \Omega.$

19. The internal resistance of an accumulator battery of emf 6 V is 10 Ω when it is fully discharged. As the battery gets charged up, its internal resistance decreases to 1 Ω. The battery in its completely discharged state is connected to a charger which maintains a constant potential difference of 9 V. Find the current through the battery (a) just after the connections are made and (b) after a long time when it is completely charged.

Ans :

19. a) net emf while charging

9 – 6 = 3V

Current = 3/10 = 0.3 A

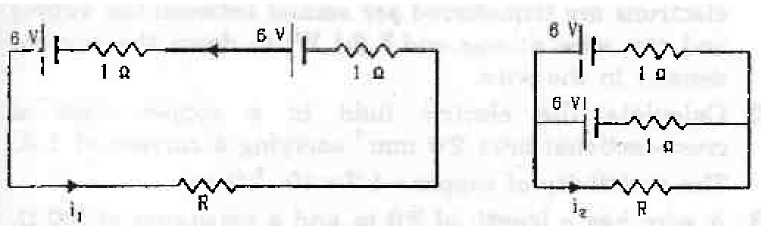
b) When completely charged.

Internal resistance 'r' = 1 Ω

Current = 3/1 = 3 A

20. Find the value of i_1, i_2 in figure if (a) R - 01 Ω, (b) R = 1 Ω (c) R - 10 Ω. Note from your answers that in order to get more current from a combination of two batteries they should be

joined in parallel if the external resistance is small and in series if the external resistance is large as compared to the internal resistances.



Ans :

20. a) $0.1i_1 + 1\, i_1 - 6 + 1i_1 - 6 = 0$

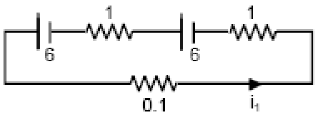
$\Rightarrow 0.1\, i_1 + 1i_1 + 1i_1 = 12$

$\Rightarrow i_1 = \dfrac{12}{2.1}$

ABCDA

$\Rightarrow 0.1i_2 + 1i - 6 = 0$

$\Rightarrow 0.1i_2 + 1i$



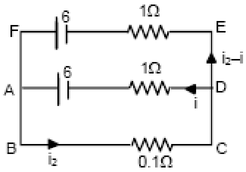
ADEFA,

$\Rightarrow i - 6 + 6 - (i_2 - i)1 = 0$

$\Rightarrow i - i_2 + i = 0$

$\Rightarrow 2i - i_2 = 0 \Rightarrow -2i \pm 0.2i = 0$

$\Rightarrow i_2 = 0.$



b) $1i_1 + 1\, i_1 - 6 + 1i_1 = 0$

$\Rightarrow 3i_1 = 12 \Rightarrow i_1 = 4$

DCFED

$\Rightarrow i_2 + i - 6 = 0 \Rightarrow i_2 + i = 6$

ABCDA,

$i_2 + (i_2 - i) - 6 = 0$

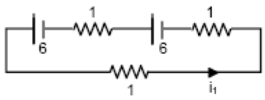
$\Rightarrow i_2 + i_2 - i = 6 \Rightarrow 2i_2 - i = 6$

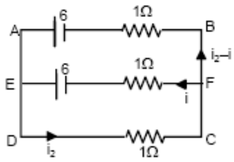
$\Rightarrow -2i_2 \pm 2i = 6 \Rightarrow i = -2$

$i_2 + i = 6$

$\Rightarrow i_2 - 2 = 6 \;\Rightarrow i_2 = 8$

$\dfrac{i_1}{i_2} = \dfrac{4}{8} = \dfrac{1}{2}.$





c) $10i_1 + 1i_1 - 6 + 1i_1 - 6 = 0$

$\Rightarrow 12i_1 = 12 \Rightarrow i_1 = 1$

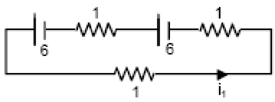
$10i_2 - i_1 - 6 = 0$

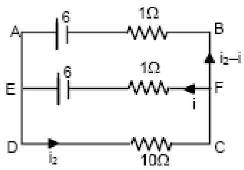
$\Rightarrow 10i_2 - i_1 = 6$

$\Rightarrow 10i_2 + (i_2 - i)1 - 6 = 0$

$\Rightarrow 11i_2 = 6$

$\Rightarrow -i_2 = 0$





21. Consider n_1, n_2 identical cells, each of emf ε and internal resistance *r*. Suppose n_1 cells are joined in series to form a line and n_2 such lines are connected in parallel. The combination drives a current in an external resistance *R*. (a) Find the current in the external resistance, (b) Assuming that n_1 and n_2 can be continuously varied, find the relation between n_1, n_2, *R* and r for which the current in *R* is maximum.

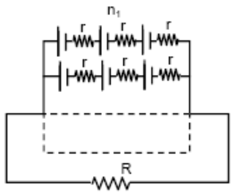
Ans :

21. a) Total emf = n_1E
in 1 row
Total emf in all news = n_1E
Total resistance in one row = n_1r

Total resistance in all rows = $\frac{n_1r}{n_2}$

Net resistance = $\frac{n_1r}{n_2} + R$

Current = $\frac{n_1E}{n_1/n_2r + R} = \frac{n_1n_2E}{n_1r + n_2R}$



b) $I = \frac{n_1n_2E}{n_1r + n_2R}$

for I = max,

$n_1r + n_2R = \min$

$\Rightarrow \left(\sqrt{n_1r} - \sqrt{n_2R}\right)^2 + 2\sqrt{n_1m_2R} = \min$

it is min, when

$\sqrt{n_1r} = \sqrt{n_2R}$

$\Rightarrow n_1r = n_2R$

I is max when $n_1r = n_2R$.

22. A battery of emf 100 V and a resistor of resistance 10 kΩ are joined in series. This system is used as a source to supply current to an external resistance *R*. If *R* is not greater than 100 Ω, the current through it is constant upto two significant digits. Find its value. This is the basic principle of a *constant-current source.*

Ans :

22. E = 100 V, R′ = 100 kΩ = 100000 Ω
R = 1 – 100
When no other resister is added or R = 0.

$i = \frac{E}{R'} = \frac{100}{100000} = 0.001\text{Amp}$

When R = 1

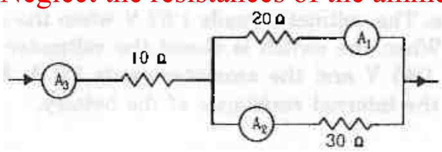
$i = \frac{100}{100000+1} = \frac{100}{100001} = 0.0009\text{A}$

When R = 100

$i = \frac{100}{100000+100} = \frac{100}{100100} = 0.000999 \text{ A}$.

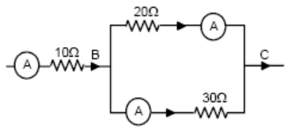
Upto R = 100 the current does not upto 2 significant digits. Thus it proved.

23. If the reading of ammeter A, in figure is 2.4 A, what will the ammeters A_2 and A_3 read ? Neglect the resistances of the ammeters.

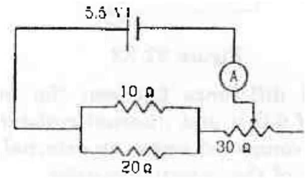


Ans :

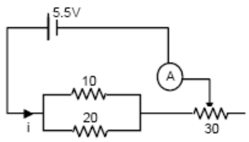
23. $A_1 = 2.4$ A
Since A_1 and A_2 are in parallel,
$\Rightarrow 20 \times 2.4 = 30 \times X$
$\Rightarrow X = \frac{20 \times 2.4}{30} = 1.6$ A.
Reading in Ammeter A_2 is 1.6 A.
$A_3 = A_1 + A_2 = 2.4 + 1.6 = 4.0$ A.

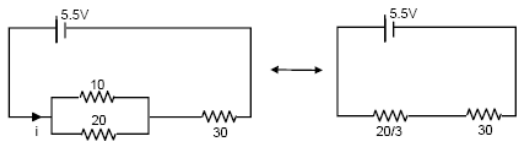


24. The resistance of the rheostat shown in figure is 30 Ω. Neglecting the meter resistance, find the minimum and maximum currents through the ammeter as the rheostat is varied.

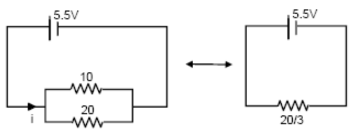


Ans :
24.



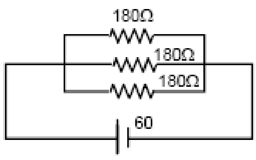


$i_{min} = \frac{5.5 \times 3}{110} = 0.15$



$i_{max} = \frac{5.5 \times 3}{20} = \frac{16.5}{20} = 0.825.$

25. Three bulbs, each having a resistance of 180 Ω, are connected in parallel to an ideal battery of emf 60 V. Find the current delivered by the battery when (a) all the bulbs are switched on, (b) two of the bulbs are switched on and (c) only one bulb is switched on.
Ans :
25. a) $R_{eff} = \frac{180}{3} = 60\ \Omega$
$i = 60 / 60 = 1$ A
b) $R_{eff} = \frac{180}{2} = 90\ \Omega$
$i = 60 / 90 = 0.67$ A
c) $R_{eff} = 180\ \Omega \Rightarrow i = 60 / 180 = 0.33$ A



26. Suppose you have three resistors of 20 Ω, 50 Ω and 100 Ω. What minimum and maximum resistances can you obtain from these resistors ?
Ans :
26. Max. $R = (20 + 50 + 100)\ \Omega = 170\ \Omega$
$$\text{Min } R = \frac{1}{\left(\frac{1}{20} + \frac{1}{50} + \frac{1}{100}\right)} = \frac{100}{8} = 12.5\ \Omega.$$

27. A bulb is made using two filaments. A switch selects whether the filaments are used individually or in parallel. When used with a 15 V battery, the bulb can be operated at 5 W, 10 W or 15 W. What should be the resistances of the filaments ?
Ans :

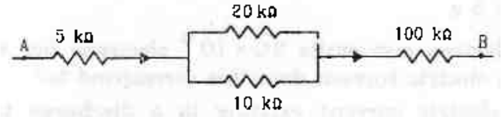
27. The various resistances of the bulbs = $\frac{V^2}{P}$

Resistances are $\frac{(15)^2}{10}, \frac{(15)^2}{10}, \frac{(15)^2}{15}$ = 45, 22.5, 15.

Since two resistances when used in parallel have resistances less than both.

The resistances are 45 and 22.5.

28. Figure shows a part of a circuit. If a current of 12 mA exists in the 5 kΩ resistor, find the currents in the other three resistors. What is the potential difference between the points A and *B ?*



Ans :

28. $i_1 \times 20 = i_2 \times 10$

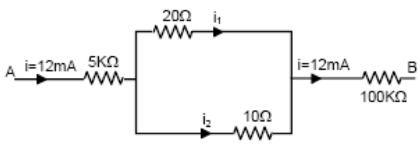
$\Rightarrow \frac{i_1}{i_2} = \frac{10}{20} = \frac{1}{2}$

i_1 = 4 mA, i_2 = 8 mA

Current in 20 KΩ resistor = 4 mA

Current in 10 KΩ resistor = 8 mA

Current in 100 KΩ resistor = 12 mA



$V = V_1 + V_2 + V_3$

= 5 KΩ × 12 mA + 10 KΩ × 8 mA + 100 KΩ × 12 mA

= 60 + 80 + 1200 = 1340 volts.

29. An ideal battery sends a current of 5 A in a resistor. When another resistor of value 10 Ω is connected in parallel, the current through the battery is increased to 6 A. Find the resistance of the first resistor.

Ans :

29. $R_1 = R$, $i_1 = 5$ A

$R_2 = \frac{10R}{10+R}$, $i_2 = 6A$

Since potential constant,

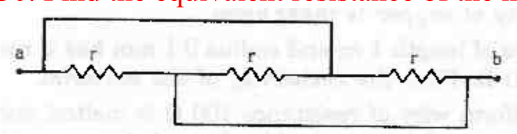
$i_1R_1 = i_2R_2$

$\Rightarrow 5 \times R = \frac{6 \times 10R}{10+R}$

$\Rightarrow (10 + R)5 = 60$

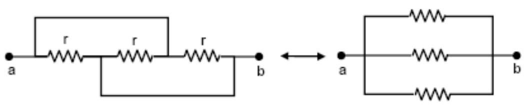
$\Rightarrow 5R = 10 \Rightarrow R = 2\ \Omega$.

30. Find the equivalent resistance of the network shown in figure between the points *a* and *b.*



Ans :

30.



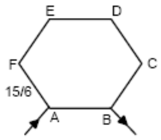
Eq. Resistance = r/3.

31. A wire of resistance 15.0 Ω is bent to form a regular hexagon *ABCDEFA.* Find the equivalent resistance of the loop between the points (a) *A* and *B,* (b) A and *C* and (c) A and *D.*

Ans :

31. a) $R_{eff} = \dfrac{\dfrac{15\times5}{6}\times\dfrac{15}{6}}{\dfrac{15\times5}{6}+\dfrac{15}{6}} = \dfrac{\dfrac{15\times5\times15}{6\times6}}{\dfrac{75+15}{6}}$

$= \dfrac{15\times5\times15}{6\times90} = \dfrac{25}{12} = 2.08\ \Omega.$



b) Across AC,

$R_{eff} = \dfrac{\dfrac{15\times4}{6}\times\dfrac{15\times2}{6}}{\dfrac{15\times4}{6}+\dfrac{15\times2}{6}} = \dfrac{\dfrac{15\times4\times15\times2}{6\times6}}{\dfrac{60+30}{6}}$

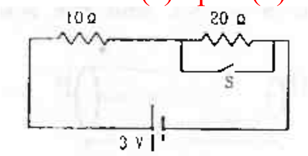
$= \dfrac{15\times4\times15\times2}{6\times90} = \dfrac{10}{3} = 3.33\ \Omega.$

c) Across AD,

$R_{eff} = \dfrac{\dfrac{15\times3}{6}\times\dfrac{15\times3}{6}}{\dfrac{15\times3}{6}+\dfrac{15\times3}{6}} = \dfrac{\dfrac{15\times3\times15\times3}{6\times6}}{\dfrac{60+30}{6}}$

$= \dfrac{15\times3\times15\times3}{6\times90} = \dfrac{15}{4} = 3.75\ \Omega.$

32. Consider the circuit shown in figure. Find the current through the 10 Ω resistor when the switch S is (a) open (b) closed.



Ans :

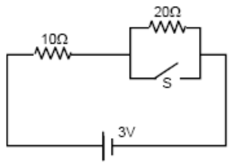
32. a) When S is open

$R_{eq} = (10 + 20)\ \Omega = 30\ \Omega.$

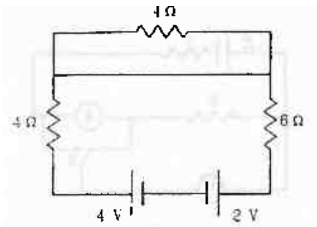
i = When S is closed,

$R_{eq} = 10\ \Omega$

$i = (3/10)\ \Omega = 0.3\ \Omega.$



33. Find the currents through the three resistors shown in figure.



Ans :

33. a) Current through (1) 4 Ω resistor = 0

b) Current through (2) and (3)

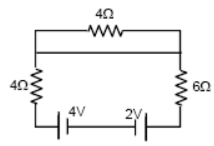
net E = 4V – 2V = 2V

(2) and (3) are in series,

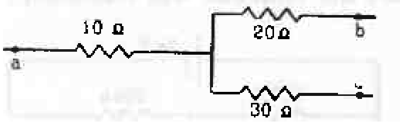
$R_{eff} = 4 + 6 = 10\ \Omega$

$i = 2/10 = 0.2$ A

Current through (2) and (3) are 0.2 A.



34. Figure shows a part of an electric circuit. The potentials at the points *a, b* and c are 30 V, 12 V and 2 V respectively. Find the currents through the three resistors.



Ans :

34. Let potential at the point be xV.

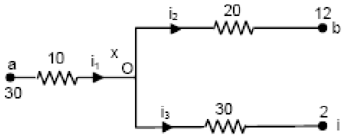
$(30 - x) = 10\, i_1$

$(x - 12) = 20\, i_2$

$(x - 2) = 30\, i_3$

$i_1 = i_2 + i_3$

$\Rightarrow \frac{30 - x}{10} = \frac{x - 12}{20} + \frac{x - 2}{30}$



$\Rightarrow 30 - x = \frac{x - 12}{2} + \frac{x - 2}{3}$

$\Rightarrow 30 - x = \frac{3x - 36 + 2x - 4}{6}$

$\Rightarrow 180 - 6x = 5x - 40$

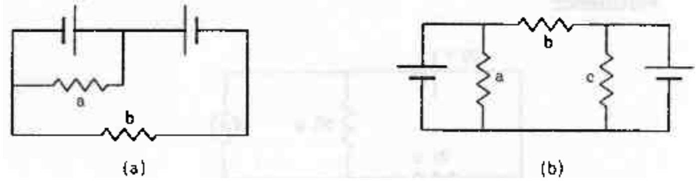
$\Rightarrow 11x = 220 \Rightarrow x = 220 / 11 = 20$ V.

$i_1 = \frac{30 - 20}{10} = 1$ A

$i_2 = \frac{20 - 12}{20} = 0.4$ A

$i_3 = \frac{20 - 2}{30} = \frac{6}{10} = 0.6$ A.

35. Each of the resistors shown in figure has a resistance of 10 Ω and each of the batteries has an emf of 10 V. Find the currents through the resistors *a* and *b* in the two circuits.



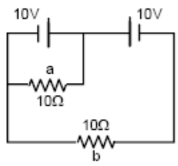
Ans :

35. a) Potential difference between terminals of 'a' is 10 V.

i through a = 10 / 10 = 1A

Potential different between terminals of b is 10 – 10 = 0 V

i through b = 0/10 = 0 A

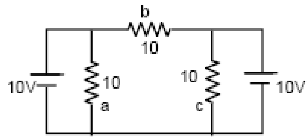


b) Potential difference across 'a' is 10 V

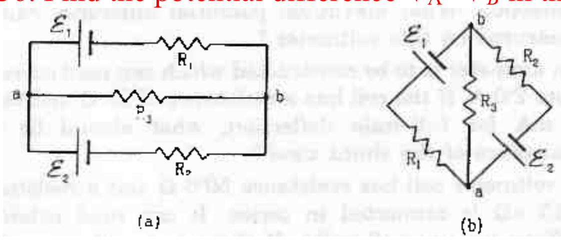
i through a = 10 / 10 = 1A

Potential different between terminals of b is 10 – 10 = 0 V

i through b = 0/10 = 0 A



36. Find the potential difference $V_A - V_B$ in the circuits shown in figure.



Ans :

36. a) In circuit, AB ba A

$E_2 + iR_2 + i_1R_3 = 0$

In circuit, $i_1R_3 + E_1 - (i - i_1)R_1 = 0$

$\Rightarrow i_1R_3 + E_1 - iR_1 + i_1R_1 = 0$

$[iR_2 + i_1R_3 \quad = -E_2]R_1$

$[iR_2 - i_1 (R_1 + R_3) = E_1]\, R_2$

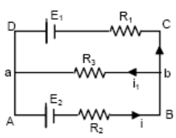
$iR_2R_1 + i_1R_3R_1 \quad = -E_2R_1$

$iR_2R_1 - i_1R_2 (R_1 + R_3) \quad = E_1\, R_2$

$iR_3R_1 + i_1R_2R_1 + i_1R_2R_3 = E_1R_2 - E_2R_1$

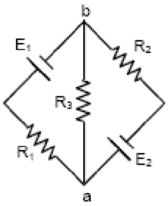
$\Rightarrow i_1(R_3R_1 + R_2R_1 + R_2R_3) = E_1R_2 - E_2R_1$

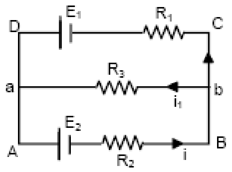
$\Rightarrow i_1 = \frac{E_1R_2 - E_2R_1}{R_3R_1 + R_2R_1 + R_2R_3}$



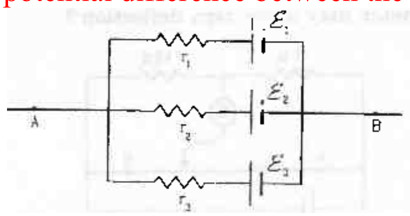
$$\Rightarrow \frac{E_1R_2R_3 - E_2R_1R_3}{R_3R_1 + R_2R_1 + R_2R_3} = \left(\frac{\frac{E_1}{R_1} - \frac{E_2}{R_2}}{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3}}\right)$$

b) $\therefore$ Same as a





37. In the circuit shown in figure, $\varepsilon_1 = 3$ V, $\varepsilon_2 = 2$ V, $\varepsilon_3 = 1$ V and $r_1 = r_2 = r_3 = 1\ \Omega$. Find the potential difference between the points A and B and the current through each branch.



Ans :

37. In circuit ABDCA,

$i_1 + 2 - 3 + i = 0$

$\Rightarrow i + i_1 - 1 = 0$ ...(1)

In circuit CFEDC,

$(i - i_1) + 1 - 3 + i = 0$

$\Rightarrow 2i - i_1 - 2 = 0$ ...(2)

From (1) and (2)

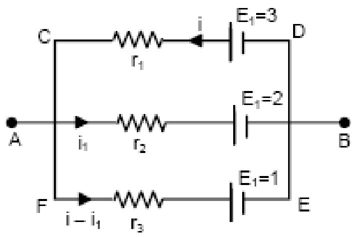
$3i = 3 \Rightarrow i = 1$ A

$i_1 = 1 - i = 0$ A

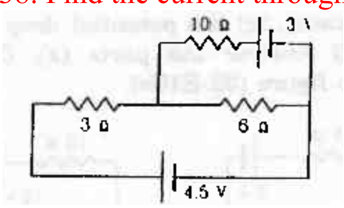
$i - i_1 = 1 - 0 = 1$ A

Potential difference between A and B

$= E - ir = 3 - 1.1 = 2$ V.



38. Find the current through the 10 Ω resistor shown in figure.



Ans :

38. In the circuit ADCBA,

$3i + 6i_1 - 4.5 = 0$

In the circuit GEFCG,

$3i + 6i_1 = 4.5 \quad = \quad 10i - 10i_1 - 6i_1 = -3$

$\Rightarrow [10i - 16i_1 = -3]3$ ...(1)

$[3i + 6i_1 = 4.5]\ 10$ ...(2)

From (1) and (2)

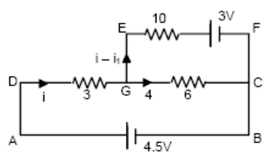
$-108\ i_1 = -54$

$\Rightarrow i_1 = \frac{54}{108} = \frac{1}{2} = 0.5$

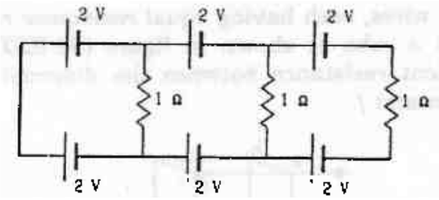
$3i + 6 \times ½ - 4.5 = 0$

$3i - 1.5 = 0 \Rightarrow i = 0.5.$

Current through 10 Ω resistor = 0 A.



39. Find the current in the three resistors shown in figure.



Ans :

39. In AHGBA,

$2 + (i - i_1) - 2 = 0$

$\Rightarrow i - i_1 = 0$

In circuit CFEDC,

$-(i_1 - i_2) + 2 + i_2 - 2 = 0$

$\Rightarrow i_2 - i_1 + i_2 = 0 \Rightarrow 2i_2 - i_1 = 0.$

In circuit BGFCB,

$-(i_1 - i_2) + 2 + (i_1 - i_2) - 2 = 0$

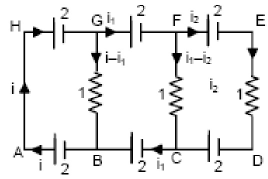
$\Rightarrow i_1 - i + i_1 - i_2 = 0 \qquad \Rightarrow 2i_1 - i - i_2 = 0 \qquad \ldots(1)$

$\Rightarrow i_1 - (i - i_1) - i_2 = 0 \qquad \Rightarrow i_1 - i_2 = 0 \qquad \ldots(2)$

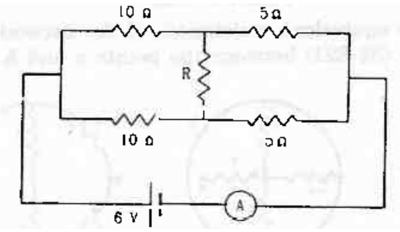
$\therefore i_1 - i_2 = 0$

From (1) and (2)

Current in the three resistors is 0.

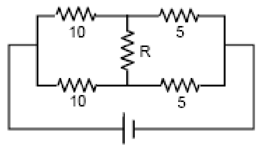


40. What should be the value of *R* in figure for which the current in it is zero ?



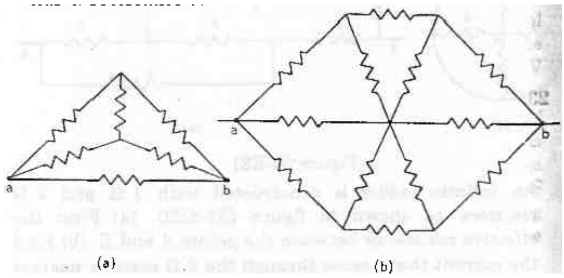
Ans :

40.



For an value of R, the current in the branch is 0.

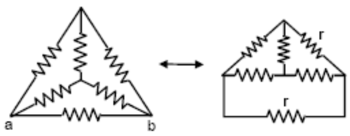
41. Find the equivalent resistance of the circuits shown in figure between the points *a* and *b*. Each resistor has a resistance *r*.



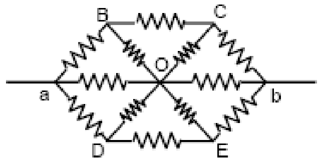
Ans :

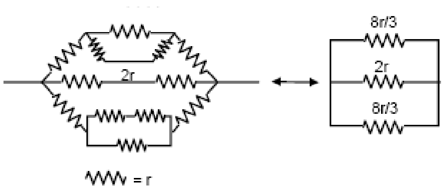
41. a) $R_{eff} = \dfrac{(2r/2)\times r}{(2r/2)+r}$

$= \dfrac{r^2}{2r} = \dfrac{r}{2}$



b) At 0 current coming to the junction is current going from BO = Current going along OE.
Current on CO = Current on OD
Thus it can be assumed that current coming in OC goes in OB.
Thus the figure becomes

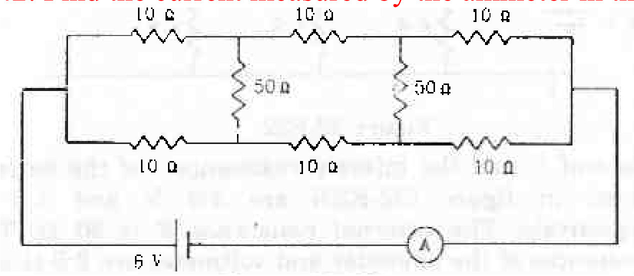




$$\left[r+\left(\frac{2r.r}{3r}\right)+r\right]=2r+\frac{2r}{3}=\frac{8r}{3}$$

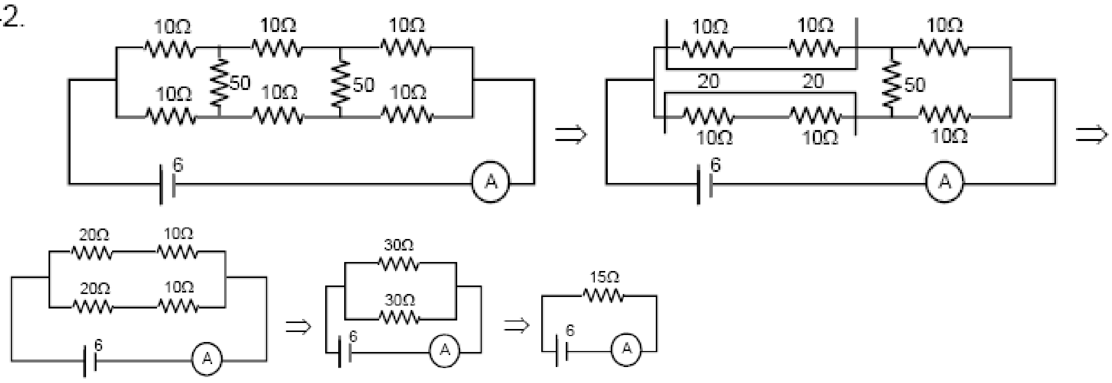
$$R_{eff}=\frac{(8r/6)\times 2r}{(8r/6)+2r}=\frac{8r^2/3}{20r/6}=\frac{8r^2}{3}\times\frac{6}{20}=\frac{8r}{10}=4r.$$

42. Find the current measured by the ammeter in the circuit shown in figure.



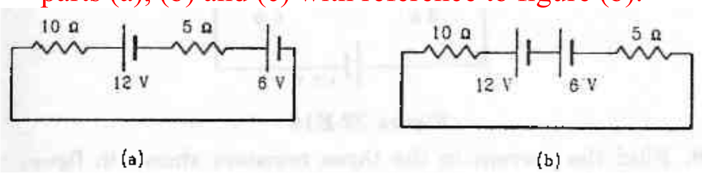
Ans :

42.



$$I=\frac{6}{15}=\frac{2}{5}=0.4\text{ A}.$$

43. Consider the circuit shown in figure (a). Find (a) the current in the circuit, (b) the potential drop across the 5 Ω resistor, (c) the potential drop across the 10 Ω resistor, (d) Answer the parts (a), (b) and (c) with reference to figure (b).



Ans :

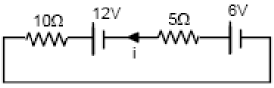
43. a) Applying Kirchoff's law,

$10i - 6 + 5i - 12 = 0$

$\Rightarrow 10i + 5i = 18$

$\Rightarrow 15i = 18$

$\Rightarrow i = \frac{18}{15} = \frac{6}{5} = 1.2$ A.

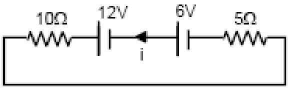


b) Potential drop across 5 Ω resistor,

$i\,5 = 1.2 \times 5$ V = 6 V

c) Potential drop across 10 Ω resistor

$i\,10 = 1.2 \times 10$ V = 12 V



d) $10i - 6 + 5i - 12 = 0$

$\Rightarrow 10i + 5i = 18$

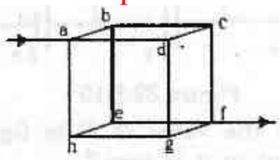
$\Rightarrow 15i = 18$

$\Rightarrow i = \frac{18}{15} = \frac{6}{5} = 1.2$ A.

Potential drop across 5 Ω resistor = 6 V

Potential drop across 10 Ω resistor = 12 V

44. Twelve wires, each having equal resistance r, are joined to form a cube as shown in figure. Find the equivalent resistance between the diagonally opposite points *a* and *f*.



Ans :

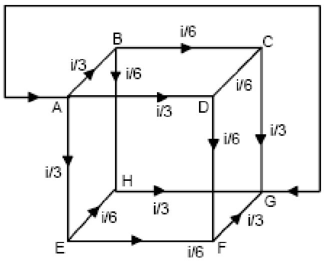
44. Taking circuit ABHGA,

$$\frac{i}{3r} + \frac{i}{6r} + \frac{i}{3r} = V$$

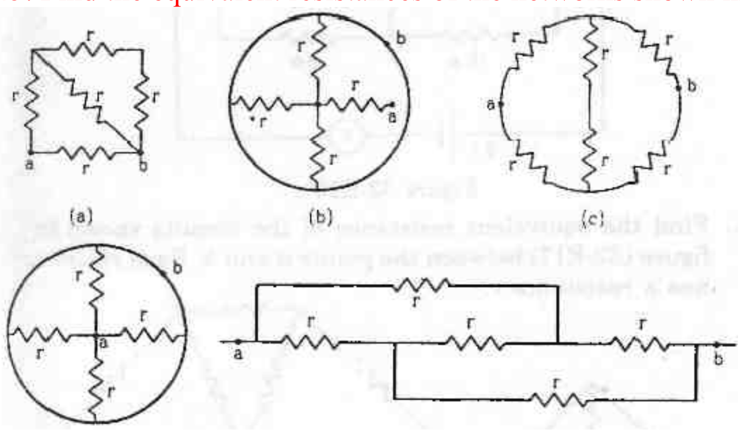
$$\Rightarrow \left(\frac{2i}{3} + \frac{i}{6}\right) r = V$$

$$\Rightarrow V = \frac{5i}{6} r$$

$$\Rightarrow R_{eff} = \frac{V}{i} = \frac{5}{6r}$$

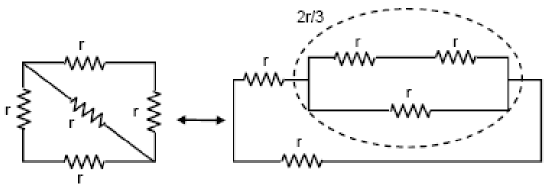


45. Find the equivalent resistances of the networks shown in figure between the points *a* and *b*.

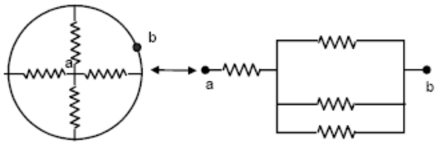


Ans :

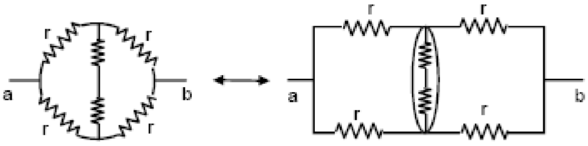
45. $$R_{eff} = \frac{\left(\frac{2r}{3} + r\right) r}{\left(\frac{2r}{3} + r + r\right)} = \frac{5r}{8}$$



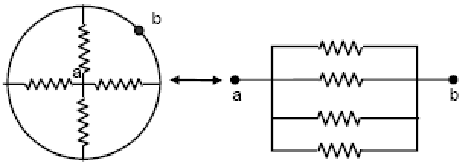
$R_{eff} = \frac{r}{3} + r = \frac{4r}{3}$



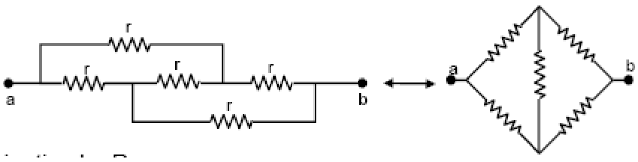
$R_{eff} = \frac{2r}{2} = r$



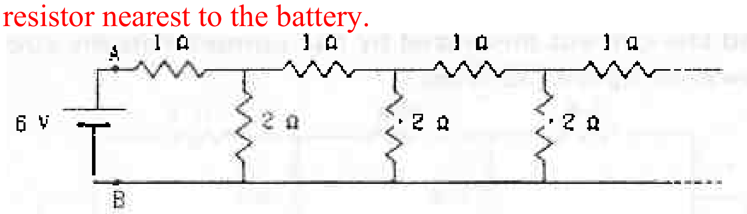
$R_{eff} = \frac{r}{4}$



$R_{eff} = r$



46. An infinite ladder is constructed with 1 Ω and 2 Ω resistors as shown in figure. (a) Find the effective resistance between the points A and *B*. (b) Find the current that passes through the 2 Ω resistor nearest to the battery.



Ans :

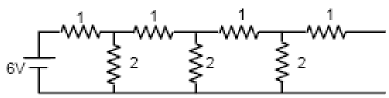
46. a) Let the equation resistance of the combination be R.

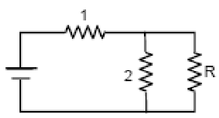
$$\left(\frac{2R}{R+2}\right) + 1 = R$$

$$\Rightarrow \frac{2R + R + 2}{R+2} = R \Rightarrow 3R + 2 = R^2 + 2R$$

$$\Rightarrow R^2 - R - 2 = 0$$

$$\Rightarrow R = \frac{+1 \pm \sqrt{1 + 4.1.2}}{2.1} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} = 2\,\Omega.$$





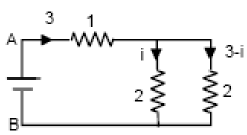
b) Total current sent by battery $= \frac{6}{R_{eff}} = \frac{6}{2} = 3$

Potential between A and B

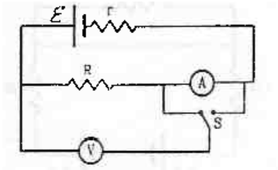
$3.1 + 2.i = 6$

$\Rightarrow 3 + 2i = 6 \Rightarrow 2i = 3$

$\Rightarrow i = 1.5$ a



47. The emf ε and the internal resistance *r* of the battery shown in figure are 4.3 V and 1.0 Ω respectively. The external resistance *R* is 50 Ω. The resistances of the ammeter and voltmeter are 2.0 Ω and 200 Ω respectively, (a) Find the readings of the two meters, (b) The switch is thrown to the other side. What will be the readings of the two meters now ?



Ans :

47. a) In circuit ABFGA,

$i_1 50 + 2i + i - 4.3 = 0$

$\Rightarrow 50i_1 + 3i = 4.3$ ...(1)

In circuit BEDCB,

$50i_1 - (i - i_1)200 = 0$

$\Rightarrow 50i_1 - 200i + 200i_1 = 0$

$\Rightarrow 250\, i_1 - 200i = 0$

$\Rightarrow 50i_1 - 40i = 0$ ...(2)

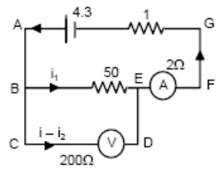
From (1) and (2)

$43i = 4.3 \qquad \Rightarrow i = 0.1$

$5i_1 = 4 \times i = 4 \times 0.1 \qquad \Rightarrow i_1 = \frac{4 \times 0.1}{5} = 0.08$ A.

Ammeter reads a current $= i = 0.1$ A.

Voltmeter reads a potential difference equal to $i_1 \times 50 = 0.08 \times 50 = 4$ V.



b) In circuit ABEFA,

$50i_1 + 2i_1 + 1i - 4.3 = 0$

$\Rightarrow 52i_1 + i = 4.3$

$\Rightarrow 200 \times 52i_1 + 200\, i = 4.3 \times 200$ ...(1)

In circuit BCDEB,

$(i - i_1)200 - i_1 2 - i_1 50 = 0$

$\Rightarrow 200i - 200i_1 - 2i_1 - 50i_1 = 0$

$\Rightarrow 200i - 252i_1 = 0$ ...(2)

From (1) and (2)

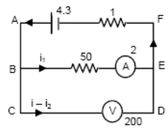
$i_1(10652) = 4.3 \times 2 \times 100$

$\Rightarrow i_1 = \frac{4.3 \times 2 \times 100}{10652} = 0.08$

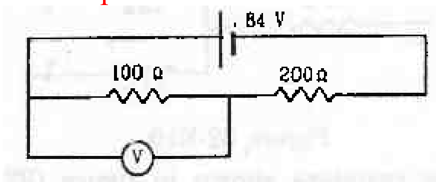
$i = 4.3 - 52 \times 0.08 = 0.14$

Reading of the ammeter = 0.08 a

Reading of the voltmeter $= (i - i_1)200 = (0.14 - 0.08) \times 200 = 12$ V.



48. A voltmeter of resistance 400 Ω is used to measure the potential difference across the 100 Ω resistor in the circuit shown in figure. (a) What will be the reading of the voltmeter ? (b) What was the potential difference across 100 Ω before the voltmeter was connected ?



Ans :

48. a) $R_{eff} = \frac{100 \times 400}{500} + 200 = 280$

$i = \frac{84}{280} = 0.3$

$100i = (0.3 - i)\, 400$

$\Rightarrow i = 1.2 - 4i$

$\Rightarrow 5i = 1.2 \Rightarrow i = 0.24.$

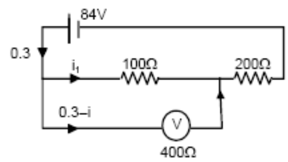
Voltage measured by the voltmeter $= \frac{0.24 \times 100}{24V}$

b) If voltmeter is not connected

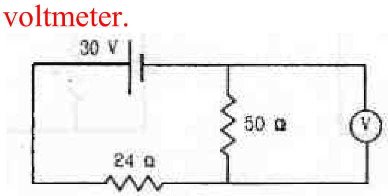
$R_{eff} = (200 + 100) = 300\ \Omega$

$i = \frac{84}{300} = 0.28$ A

Voltage across 100 Ω = (0.28 × 100) = 28 V.



49. The voltmeter shown in figure reads 18 V across the 50 Ω resistor. Find the resistance of the voltmeter.



Ans :

49. Let resistance of the voltmeter be R Ω.

$R_1 = \frac{50R}{50+R}$, $R_2 = 24$

Both are in series.

$30 = V_1 + V_2$

$\Rightarrow 30 = iR_1 + iR_2$

$\Rightarrow 30 - iR_2 = iR_1$

$\Rightarrow iR_1 = 30 - \frac{30}{R_1+R_2}R_2$

$\Rightarrow V_1 = 30\left(1 - \frac{R_2}{R_1+R_2}\right)$

$\Rightarrow V_1 = 30\left(\frac{R_1}{R_1+R_2}\right)$

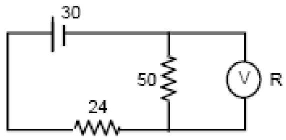
$$\Rightarrow 18 = 30\left(\frac{50R}{50+R\left(\frac{50R}{50+R}+24\right)}\right)$$

$$\Rightarrow 18 = 30\left(\frac{50R\times(50+R)}{(50+R)+(50R+24)(50+R)}\right) = \frac{30(50R)}{50R+1200+24R}$$

$$\Rightarrow 18 = \frac{30\times 50\times R}{74R+1200} = 18(74R + 1200) = 1500\ R$$

$\Rightarrow 1332R + 21600 = 1500\ R \Rightarrow 21600 = 1.68\ R$

$\Rightarrow R = 21600 / 168 = 128.57.$



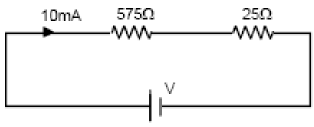
50. A voltmeter consists of a 25 Ω coil connected in series with a 575 Ω resistor. The coil takes 10 mA for full scale deflection. What maximum potential difference can be measured on this voltmeter ?

Ans :

50. Full deflection current = 10 mA = (10×10^{-3})A

$R_{eff} = (575 + 25)\Omega = 600\ \Omega$

$V = R_{eff} \times i = 600 \times 10 \times 10^{-3} = 6$ V.



51. An ammeter is to be constructed which can read currents upto 2.0 A. If the coil has a resistance of 25 Ω and takes 1 mA for full-scale deflection, what should be the resistance of the shunt used ?

Ans :

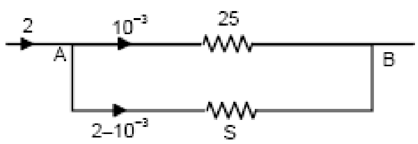
51. G = 25 Ω, Ig = 1 ma, I = 2A, S = ?

Potential across A B is same

$25 \times 10^{-3} = (2 - 10^{-3})S$

$$\Rightarrow S = \frac{25\times 10^{-3}}{2-10^{-3}} = \frac{25\times 10^{-3}}{1.999}$$

$= 12.5 \times 10^{-3} = 1.25 \times 10^{-2}$.



52. A voltmeter coil has resistance 50.0 Ω and a resistor of 1.15 kΩ is connected in series. It can read potential differences upto 12 volts. If this same coil is used to construct an ammeter which can measure currents upto 2.0 A, what should be the resistance of the shunt used ?

Ans :

52. $R_{eff} = (1150 + 50)\Omega = 1200\ \Omega$

i = (12 / 1200)A = 0.01 A.

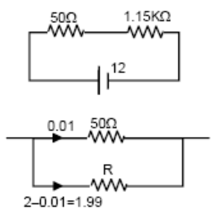
(The resistor of 50 Ω can tolerate)

Let R be the resistance of sheet used.

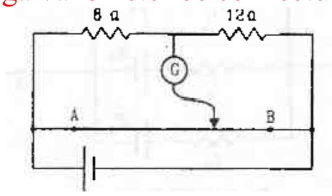
The potential across both the resistors is same.

$0.01 \times 50 = 1.99 \times R$

$$\Rightarrow R = \frac{0.01\times 50}{1.99} = \frac{50}{199} = 0.251\ \Omega.$$



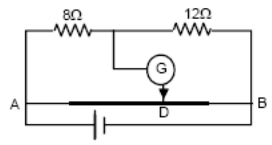
53. The potentiometer wire *AB* shown in figure is 40 cm long. Where should the free end of the galvanometer be connected on *AB* so that galvanometer may show zero deflection ?



Ans :

53. If the wire is connected to the potentiometer wire so that $\frac{R_{AD}}{R_{DB}} = \frac{8}{12}$, then according to wheat stone's bridge no current will flow through galvanometer.

$\frac{R_{AB}}{R_{DB}} = \frac{L_{AB}}{L_B} = \frac{8}{12} = \frac{2}{3}$ (Acc. To principle of potentiometer).



$l_{AB} + l_{DB} = 40$ cm

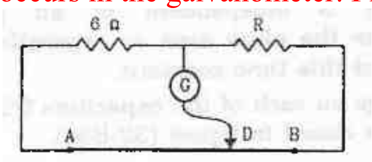
$\Rightarrow l_{DB}\ 2/3 + l_{DB} = 40$ cm

$\Rightarrow (2/3 + 1)l_{DB} = 40$ cm

$\Rightarrow 5/3\ l_{DB} = 40 \Rightarrow L_{DB} = \frac{40 \times 3}{5} = 24$ cm.

$l_{AB} = (40 - 24)$ cm = 16 cm.

54. The potentiometer wire *AB* shown in figure is 50 cm long. When *AD* = 30 cm, no deflection occurs in the galvanometer. Find *R*.



Ans :

54. The deflections does not occur in galvanometer if the condition is a balanced wheatstone bridge.

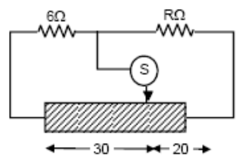
Let Resistance / unit length = r.

Resistance of 30 m length = 30 r.

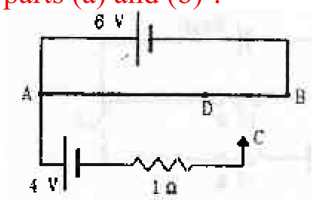
Resistance of 20 m length = 20 r.

For balanced wheatstones bridge = $\frac{6}{R} = \frac{30r}{20r}$

$\Rightarrow 30\ R = 20 \times 6 \Rightarrow R = \frac{20 \times 6}{30} = 4\ \Omega$.

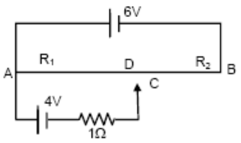


55. A 6 volt battery of negligible internal resistance is connected across a uniform wire *AB* of length 100 cm. The positive terminal of another battery of emf 4 V and internal resistance 1 Ω is joined to the point A as shown in figure. Take the potential at *B* to be zero, (a) What are the potentials at the points A and C ? (b) At which point *D* of the wire *AB,* the potential is equal to the potenial at C ? (c) If the points C and *D* are connected by a wire, what will be the current through it ? (d) If the 4 V battery is replaced by 7.5 V battery, what would be the answers of parts (a) and (b) ?



Ans :

55. a) Potential difference between A and B is 6 V.
B is at 0 potential.
Thus potential of A point is 6 V.
The potential difference between Ac is 4 V.
$V_A - V_C = 0.4$
$V_C = V_A - 4 = 6 - 4 = 2$ V.



b) The potential at D = 2V, $V_{AD} = 4$ V ; V_{BD} = OV
Current through the resisters R_1 and R_2 are equal.

Thus, $\frac{4}{R_1} = \frac{2}{R_2}$

$\Rightarrow \frac{R_1}{R_2} = 2$

$\Rightarrow \frac{l_1}{l_2} = 2$ (Acc. to the law of potentiometer)

$l_1 + l_2 = 100$ cm

$\Rightarrow l_1 + \frac{l_1}{2} = 100$ cm $\Rightarrow \frac{3l_1}{2} = 100$ cm

$\Rightarrow l_1 = \frac{200}{3}$ cm = 66.67 cm.

AD = 66.67 cm

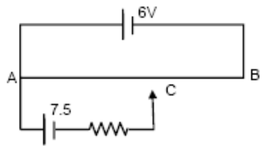
c) When the points C and D are connected by a wire current flowing through it is 0 since the points are equipotential.

d) Potential at A = 6 v

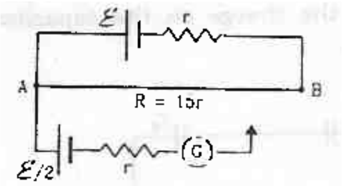
Potential at C = 6 – 7.5 = –1.5 V

The potential at B = 0 and towards A potential increases.

Thus –ve potential point does not come within the wire.



56. Consider the potentiometer circuit arranged as in figure. The potentiometer wire is 600 cm long, (a) At what distance from the point A should the jockey touch the wire to get zero deflection in the galvanometer ? (b) If the jockey touches the wire at a distance of 560 cm from A, what will be the current in the galvanometer ?

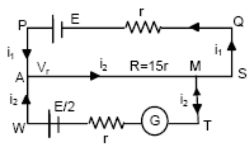


Ans :

56. Resistance per unit length = $\frac{15r}{6}$

For length x, Rx = $\frac{15r}{6} \times x$

a) For the loop PASQ $(i_1 + i_2)\frac{15}{6}rx + \frac{15}{6}(6 - x)i_1 + i_1R = E$ ...(1)



For the loop AWTM, $-i_2.R - \frac{15}{6}rx\,(i_1 + i_2) = E/2$

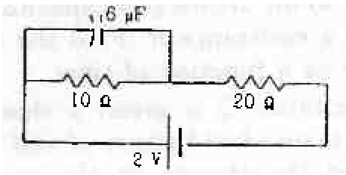
$\Rightarrow i_2R + \frac{15}{6}r \times (i_1 + i_2) = E/2$ ...(2)

For zero deflection galvanometer $i_2 = 0 \Rightarrow \frac{15}{6}rx \,.\, i_1 = E/2 = i_1 = \frac{E}{5x \cdot r}$

Putting $i_1 = \frac{E}{5x \cdot r}$ and $i_2 = 0$ in equation (1), we get x = 320 cm.

b) Putting x = 5.6 and solving equation (1) and (2) we get $i_2 = \frac{3E}{22r}$.

57. Find the charge on the capacitor shown in figure



Ans :

57. In steady stage condition no current flows through the capacitor.

$R_{eff} = 10 + 20 = 30\ \Omega$

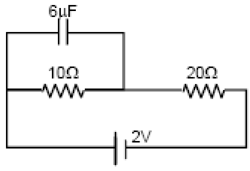
$i = \frac{2}{30} = \frac{1}{15} A$

Voltage drop across 10 Ω resistor = i × R

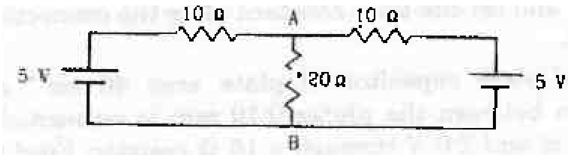
$= \frac{1}{15} \times 10 = \frac{10}{15} = \frac{2}{3} V$

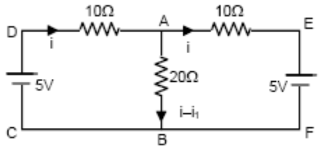
Charge stored on the capacitor (Q) = CV

$= 6 \times 10^{-6} \times 2/3 = 4 \times 10^{-6}\ C = 4\ \mu C.$



58. (a) Find the current in the 20 Ω resistor shown in figure. (b) If a capacitor of capacitance 4 μF is joined between the points *A* and *B,* what would be the electrostatic energy stored in it in steady state ?





Ans :

58. Taking circuit, ABCDA,

$10i + 20(i - i_1) - 5 = 0$

$\Rightarrow 10i + 20i - 20i_1 - 5 = 0$

$\Rightarrow 30i - 20i_1 - 5 = 0$ ...(1)

Taking circuit ABFEA,

$20(i - i_1) - 5 - 10i_1 = 0$

$\Rightarrow 10i - 20i_1 - 10i_1 - 5 = 0$

$\Rightarrow 20i - 30i_1 - 5 = 0$ ...(2)

From (1) and (2)

$(90 - 40)i_1 = 0$

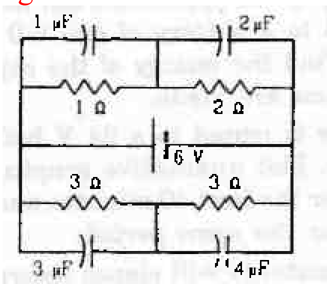
$\Rightarrow i_1 = 0$

$30i - 5 = 0$

$\Rightarrow i = 5/30 = 0.16$ A

Current through 20 Ω is 0.16 A.

59. Find the charges on the four capacitors of capacitances 1 μF, 2 μF, 3μF and 4 μF shown in figure.



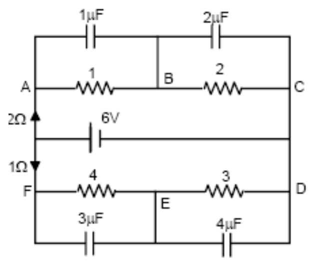
Ans :

59. At steady state no current flows through the capacitor.

$R_{eq} = \frac{3 \times 6}{3 + 6} = 2\ \Omega.$

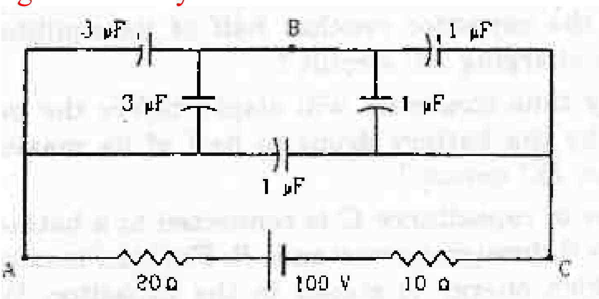
$i = \frac{6}{2} = 3.$

Since current is divided in the inverse ratio of the resistance in each branch, thus 2Ω will pass through 1, 2 Ω branch and 1 through 3, 3Ω branch



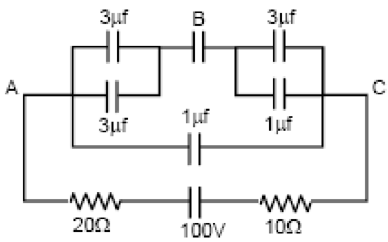
$V_{AB} = 2 \times 1 = 2V.$
Q on 1 μF capacitor = 2 × 1 μc = 2 μC
$V_{BC} = 2 \times 2 = 4V.$
Q on 2 μF capacitor = 4 × 2 μc = 8 μC
$V_{DE} = 1 \times 3 = 2V.$
Q on 4 μF capacitor = 3 × 4 μc = 12 μC
$V_{FE} = 3 \times 1 = V.$
Q across 3 μF capacitor = 3 × 3 μc = 9 μC.

60. Find the potential difference between the points A and *B* and between the points *B* and C of figure in steady state.



Ans :

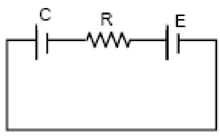
60. C_{eq} = [(3 μf p 3 μf) s (1 μf p 1 μf)] p (1 μf)
= [(3 + 3)μf s (2μf)] p 1 μf
= 3/2 + 1 = 5/2 μf
V = 100 V
Q = CV = 5/2 × 100 = 250 μc
Charge stored across 1 μf capacitor = 100 μc
C_{eq} between A and B is 6 μf = C
Potential drop across AB = V = Q/C = 25 V
Potential drop across BC = 75 V.



61. A capacitance C, a resistance *R* and an emf ε are connected in series at *t* = 0. What is the maximum value of (a) the potential difference across the resistor, (b) the current in the circuit, (c) the potential difference across the capacitor, (d) the energy stored in the capacitor, (e) the power delivered by the battery and (f) the power converted into heat.

Ans :

61. a) Potential difference = E across resistor
b) Current in the circuit = E/R
c) Pd. Across capacitor = E/R
d) Energy stored in capacitor = $\frac{1}{2}CE^2$
e) Power delivered by battery = $E \times I = E \times \frac{E}{R} = \frac{E^2}{R}$
f) Power converted to heat = $\frac{E^2}{R}$



62. A parallel-plate capacitor with plate area 20 cm and plate separation 1.0 mm is connected to a battery. The resistance of the circuit is 10 kΩ. Find the time constant of the circuit.

Ans :

62. $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$
$d = 1\ mm = 1 \times 10^{-3}\ m$; $R = 10\ K\Omega$
$C = \frac{E_0 A}{d} = \frac{8.85 \times 10^{-12} \times 20 \times 10^{-4}}{1 \times 10^{-3}}$
$= \frac{8.85 \times 10^{-12} \times 2 \times 10^{-3}}{10^{-3}} = 17.7 \times 10^{-2}$ Farad.
Time constant $= CR = 17.7 \times 10^{-2} \times 10 \times 10^3$
$= 17.7 \times 10^{-8} = 0.177 \times 10^{-6}\ s = 0.18\ \mu s.$

63. A capacitor of capacitance 10 µF is connected to a battery of emf 2 V. It is found that it takes 50 ms for the charge on the capacitor to become 12.6 µC. Find the resistance of the circuit.

Ans :

63. $C = 10\ \mu F = 10^{-5}\ F$, emf = 2 V
$t = 50\ ms = 5 \times 10^{-2}\ s$, $q = Q(1 - e^{-t/RC})$
$Q = CV = 10^{-5} \times 2$
$q = 12.6 \times 10^{-6}\ F$
$\Rightarrow 12.6 \times 10^{-6} = 2 \times 10^{-5}\ (1 - e^{-5\times10^{-2}/R\times10^{-5}})$
$\Rightarrow \frac{12.6 \times 10^{-6}}{2 \times 10^{-5}} = 1 - e^{-5\times10^{-2}/R\times10^{-5}}$
$\Rightarrow 1 - 0.63 = e^{-5\times10^{3}/R}$
$\Rightarrow \frac{-5000}{R} = \ln 0.37$
$\Rightarrow R = \frac{5000}{0.9942} = 5028\ \Omega = 5.028 \times 10^3 \Omega = 5\ K\Omega.$

64. A 20 µF capacitor is joined to a battery of emf 6.0 V through a resistance of 100 Ω. Find the charge on the capacitor 2.0 ms after the connections are made.

Ans :

64. $C = 20 \times 10^{-6}\ F$, $E = 6\ V$, $R = 100\ \Omega$
$t = 2 \times 10^{-3}$ sec
$q = EC\ (1 - e^{-t/RC})$
$= 6 \times 20 \times 10^{-6} \left(1 - e^{\frac{-2\times10^{-3}}{100\times20\times10^{-6}}}\right)$
$= 12 \times 10^{-5}\ (1 - e^{-1}) = 7.12 \times 0.63 \times 10^{-5} = 7.56 \times 10^{-5}$
$= 75.6 \times 10^{-6} = 76\ \mu c.$

65. The plates of a capacitor of capacitance 10 µF, charged to 60 µ*C*, are joined together by a wire of resistance 10 £2 at *t* 0. Find the charge on the capacitor in the circuit at (a) *t* = 0, (b) *t* - 30 (as, (c) *t* = 120 µs and (d) *t* - 1.0 ms.

Ans :

65. $C = 10\ \mu F$, $Q = 60\ \mu C$, $R = 10\ \Omega$
a) at $t = 0$, $q = 60\ \mu c$
b) at $t = 30\ \mu s$, $q = Qe^{-t/RC}$
$= 60 \times 10^{-6} \times e^{-0.3} = 44\ \mu c$
c) at $t = 120\ \mu s$, $q = 60 \times 10^{-6} \times e^{-1.2} = 18\ \mu c$
d) at $t = 1.0\ ms$, $q = 60 \times 10^{-6} \times e^{-10} = 0.00272 = 0.003\ \mu c.$

66. A capacitor of capacitance 8'0 uF is connected to a battery of emf 6.0 V through a resistance of 24 Ω. Find the current in the circuit (a) just after the connections are made and (b) one time constant after the connections are made.

Ans :

66. $C = 8\ \mu F$, $E = 6V$, $R = 24\ \Omega$
a) $I = \frac{V}{R} = \frac{6}{24} = 0.25A$
b) $q = Q(1 - e^{-t/RC})$
$= (8 \times 10^{-6} \times 6)\ [1 - c^{-1}] = 48 \times 10^{-6} \times 0.63 = 3.024 \times 10^{-5}$
$V = \frac{Q}{C} = \frac{3.024 \times 10^{-5}}{8 \times 10^{-6}} = 3.78$
$E = V + iR$
$\Rightarrow 6 = 3.78 + i24$
$\Rightarrow i = 0.09\ \text{Å}$

67. A parallel-plate capacitor of plate area 40 cm^2 and separation between the plates 0.10 mm is connected to a battery of emf 2 0 V through a 16 Ω resistor. Find the electric field in the capacitor 10 ns after the connections are made.

Ans :

67. $A = 40\ m^2 = 40 \times 10^{-4}$

$d = 0.1\text{ mm} = 1 \times 10^{-4}\text{ m}$

$R = 16\ \Omega$; emf = 2 V

$$C = \frac{E_0 A}{d} = \frac{8.85 \times 10^{-12} \times 40 \times 10^{-4}}{1 \times 10^{-4}} = 35.4 \times 10^{-11}\text{ F}$$

$$\text{Now, } E = \frac{Q}{AE_0}(1 - e^{-t/RC}) = \frac{CV}{AE_0}(1 - e^{-t/RC})$$

$$= \frac{35.4 \times 10^{-11} \times 2}{40 \times 10^{-4} \times 8.85 \times 10^{-12}}(1 - e^{-1.76})$$

$= 1.655 \times 10^{-4} = 1.7 \times 10^{-4}$ V/m.

68. A parallel-plate capacitor has plate area 20 cm^2, plate separation 1.0 mm and a dielectric slab of dielectric constant 5.0 filling up the space between the plates. This capacitor is joined to a battery of emf 6.0 V through a 100 kΩ resistor. Find the energy of the capacitor 8.9 μs after the connections are made.

Ans :

68. $A = 20\ cm^2$, $d = 1$ mm, $K = 5$, $e = 6$ V

$R = 100 \times 10^3\ \Omega$, $t = 8.9 \times 10^{-6}$ s

$$C = \frac{KE_0A}{d} = \frac{5 \times 8.85 \times 10^{-12} \times 20 \times 10^{-4}}{1 \times 10^{-3}}$$

$$= \frac{10 \times 8.85 \times 10^{-3} \times 10^{-12}}{10^{-3}} = 88.5 \times 10^{-12}$$

$$q = EC(1 - e^{-t/RC})$$

$$= 6 \times 88.5 \times 10^{-12}\left(1 - e^{\frac{-89 \times 10^{-6}}{88.5 \times 10^{-12} \times 10^{4}}}\right) = 530.97$$

$$\text{Energy} = \frac{1}{2} \times \frac{500.97 \times 530}{88.5 \times 10^{-12}}$$

$$= \frac{530.97 \times 530.97}{88.5 \times 2} \times 10^{12}$$

69. A 100 μF capacitor is joined to a 24 V battery through a 1.0 MΩ resistor. Plot qualitative graphs (a) between current and time for the first 10 minutes and (b) between charge and time for the same period.

Ans :

69. Time constant $RC = 1 \times 10^6 \times 100 \times 10^6 = 100$ sec

a) $q = VC(1 - e^{-t/CR})$

I = Current = $dq/dt = VC.(-)\ e^{-t/RC}, (-1)/RC$

$$= \frac{V}{R}e^{-t/RC} = \frac{V}{R \cdot e^{t/RC}} = \frac{24}{10^6} \cdot \frac{1}{e^{t/100}}$$

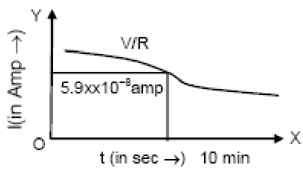
$= 24 \times 10^{-6}\ 1/e^{t/100}$

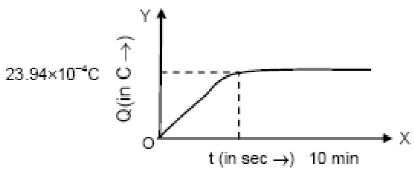
$t = 10$ min, 600 sec.

$Q = 24 \times 10{+}{-}4 \times (1 - e^{-6}) = 23.99 \times 10^{-4}$

$$I = \frac{24}{10^6} \cdot \frac{1}{e^6} = 5.9 \times 10^{-8}\text{ Amp.}.$$

b) $q = VC(1 - e^{-t/CR})$





70. How many time constants will elapse before the current in a charging *RC* circuit drops to half of its initial value ? Answer the same question for a discharging *RC* circuit.

Ans :

70. $Q/2 = Q(1 - e^{-t/CR})$

$$\Rightarrow \frac{1}{2} = (1 - e^{-t/CR})$$

$\Rightarrow e^{-t/CR} = ½$

$$\Rightarrow \frac{t}{RC} = \log 2 \Rightarrow n = 0.69.$$

71. How many time constants will elapse before the charge on a capacitor falls to 0.1% of its maximum value in a discharging *RC* circuit ?

Ans :

71. $q = Qe^{-t/RC}$

$q = 0.1\,\%\,Q \qquad RC \Rightarrow$ Time constant

$= 1 \times 10^{-3} Q$

So, $1 \times 10^{-3} Q = Q \times e^{-t/RC}$

$\Rightarrow e^{-t/RC} = \ln 10^{-3}$

$\Rightarrow t/RC = -(-6.9) = 6.9$

72. How many time constants will elapse before the energy stored in the capacitor reaches half of its equilibrium value in a charging *RC* circuit ?

Ans :

72. $q = Q(1 - e^{-n})$

$\frac{1}{2}\frac{Q^2}{C} =$ Initial value ; $\frac{1}{2}\frac{q^2}{c} =$ Final value

$\frac{1}{2}\frac{q^2}{c} \times 2 = \frac{1}{2}\frac{Q^2}{C}$

$\Rightarrow q^2 = \frac{Q^2}{2} \Rightarrow q = \frac{Q}{\sqrt{2}}$

$\frac{Q}{\sqrt{2}} = Q(1 - e^{-n})$

$\Rightarrow \frac{1}{\sqrt{2}} = 1 - e^{-n} \Rightarrow e^{-n} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow n = \log\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right) = 1.22$

73. How many time constants will elapse before the power delivered by the battery drops to half of its maximum value in an *RC* circuit ?

Ans :

73. Power $= CV^2 = Q \times V$

Now, $\frac{QV}{2} = QV \times e^{-t/RC}$

$\Rightarrow ½ = e^{-t/RC}$

$\Rightarrow \frac{t}{RC} = -\ln 0.5$

$\Rightarrow -(-0.69) = 0.69$

74. A capacitor of capacitance *C* is connected to a battery of emf ε at *t* = 0 through a resistance *R*. Find the maximum rate at which energy is stored in the capacitor. When does the rate has this maximum value ?

Ans :

74. Let at any time t, $q = EC\,(1 - e^{-t/CR})$

$E =$ Energy stored $= \frac{q^2}{2c} = \frac{E^2C^2}{2c}(1-e^{-t/CR})^2 = \frac{E^2C}{2}(1-e^{-t/CR})^2$

$R =$ rate of energy stored $= \frac{dE}{dt} = \frac{-E^2C}{2}\left(\frac{-1}{RC}\right)^2(1-e^{-t/RC})e^{-t/RC} = \frac{E^2}{CR}\cdot e^{-t/RC}(1-e^{-t/CR})$

$\frac{dR}{dt} = \frac{E^2}{2R}\left[\frac{-1}{RC}e^{-t/CR}\cdot(1-e^{-t/CR}) + (-)\cdot e^{-t/CR(1-/RC)}\cdot e^{-t/CR}\right]$

$\frac{E^2}{2R} = \left(\frac{-e^{-t/CR}}{RC} + \frac{e^{-2t/CR}}{RC} + \frac{1}{RC}\cdot e^{-2t/CR}\right) = \frac{E^2}{2R}\left(\frac{2}{RC}\cdot e^{-2t/CR} - \frac{e^{-t/CR}}{RC}\right) \qquad \ldots(1)$

For R_{max} $dR/dt = 0 \Rightarrow 2.e^{-t/RC} - 1 = 0 \Rightarrow e^{-t/CR} = 1/2$

$\Rightarrow -t/RC = -\ln^2 \Rightarrow t = RC \ln 2$

$\therefore$ Putting $t = RC \ln 2$ in equation (1) We get $\frac{dR}{dt} = \frac{E^2}{4R}$.

75. A capacitor of capacitance 12.0 μF is connected to a battery of emf 6.00 V and internal resistance 100 Ω through resistanceless leads. 12 μC as after the connections are made, what

will be (a) the current in the circuit, (b) the power delivered by the battery, (c) the power dissipated in heat and (d) the rate at which the energy stored in the capacitor is increasing.

Ans :

75. $C = 12.0\ \mu F = 12 \times 10^{-6}$

emf = 6.00 V, R = 1 Ω

$t = 12\ \mu c,\ i = i_0\ e^{-t/RC}$

$= \frac{CV}{T} \times e^{-t/RC} = \frac{12 \times 10^{-6} \times 6}{12 \times 10^{-6}} \times e^{-1}$

$= 2.207 = 2.1$ A

b) Power delivered by battery

We known, $V = V_0\ e^{-t/RC}$ (where V and V_0 are potential VI)

$VI = V_0 I\ e^{-t/RC}$

$\Rightarrow VI = V_0 I \times e^{-1} = 6 \times 6 \times e^{-1} = 13.24$ W

c) $U = \frac{CV^2}{T}(e^{-t/RC})^2$ $[\frac{CV^2}{T}$ = energy drawing per unit time]

$= \frac{12 \times 10^{-6} \times 36}{12 \times 10^{-6}} \times (e^{-1})^2 = 4.872.$

76. A capacitance C charged to a potential difference V is discharged by connecting its plates through a resistance *R*. Find the heat dissipated in one time constant after the connections are made. Do this by calculating $\int i\ R\ dt$ and also by finding the decrease in the energy stored in the capacitor.

Ans :

76. Energy stored at a part time in discharging $= \frac{1}{2}CV^2(e^{-t/RC})^2$

Heat dissipated at any time

= (Energy stored at t = 0) – (Energy stored at time t)

$= \frac{1}{2}CV^2 - \frac{1}{2}CV^2(-e^{-1})^2 = \frac{1}{2}CV^2(1-e^{-2})$

77. By evaluating $\int i^2 Rdt$, show that when a capacitor is charged by connecting it to a battery through a resistor, the energy dissipated as heat equals the energy stored in the capacitor.

Ans :

77. $\int i^2 Rdt = \int i_0^2\ Re^{-2t/RC}\ dt = i_0^2 R \int e^{-2t/RC} dt$

$= i_0^2 R(-RC/2)e^{-2t/RC} = \frac{1}{2}Ci_0^2R^2e^{-2t/RC} = \frac{1}{2}CV^2$ (Proved).

78. A parallel-plate capacitor is filled with a dielectric material having resistivity p and dielectric constant K. The capacitor is charged and disconnected from the charging source. The capacitor is slowly discharged through the dielectric. Show that the time constant of the discharge is independent of all geometrical parameters like the plate area or separation between the plates. Find this time constant.

Ans :

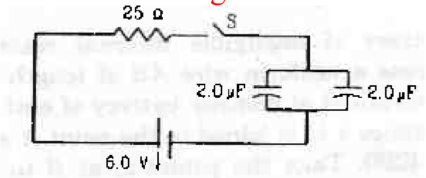
78. Equation of discharging capacitor

$= q_0 e^{-t/RC} = \frac{K \epsilon_0 AV}{d} e^{\frac{-1}{(\rho d K \epsilon_0 A)/Ad}} = \frac{K \epsilon_0 AV}{d} e^{-t/\rho K \epsilon_0}$

$\therefore \tau = \rho K \epsilon_0$

$\therefore$ Time constant is $\rho K \epsilon_0$ is independent of plate area or separation between the plate.

79. Find the charge on each of the capacitors 0.20 ms after the switch S is closed in figure.



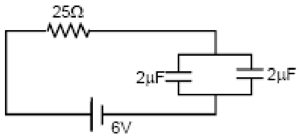
Ans :

79. $q = q_0(1 - e^{-t/RC})$

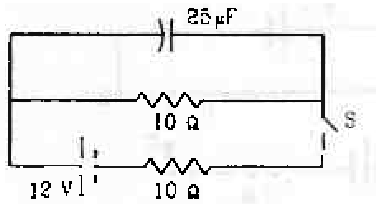
$= 25(2 + 2) \times 10^{-6} \left(1 - e^{\frac{-0.2\times10^{-3}}{25\times4\times10^{-6}}}\right)$

$= 24 \times 10^{-6} (1 - e^{-2}) = 20.75$

Charge on each capacitor = 20.75/2 = 10.3



80. The switch 5 shown in figure is kept closed for a long time and is then opened at *t* - 0. Find the current in the middle 10 ft resistor at *t = 1.0* ms.



Ans :

80. In steady state condition, no current passes through the 25 μF capacitor,

$\therefore$ Net resistance $= \frac{10\Omega}{2} = 5\Omega$.

Net current $= \frac{12}{5}$

Potential difference across the capacitor = 5

Potential difference across the 10 Ω resistor

$= 12/5 \times 10 = 24$ V

$q = Q(e^{-t/RC}) = V \times C(e^{-t/RC}) = 24 \times 25 \times 10^{-6}\left[e^{-1\times10^{-3}/10\times25\times10^{-4}}\right]$

$= 24 \times 25 \times 10^{-6}\, e^{-4} = 24 \times 25 \times 10^{-6} \times 0.0183 = 10.9 \times 10^{-6}$ C

Charge given by the capacitor after time t.

Current in the 10 Ω resistor $= \frac{10.9\times10^{-6}\,C}{1\times10^{-3}\,sec} = 11mA$.

81. A capacitor of capacitance 100 μF is connected across a battery of emf 6.0 V through a resistance of 20 kΩ for 4.0 s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 4.0 s after the battery is disconnected ?

Ans :

81. C = 100 μF, emf = 6 V, R = 20 KΩ, t = 4 S.

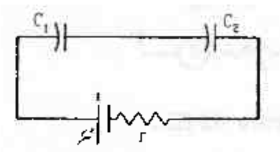
Charging : $Q = CV(1 - e^{-t/RC})$ $\left[\frac{-t}{RC} = \frac{4}{2\times10^{4}\times10^{-4}}\right]$

$= 6 \times 10^{-4}(1 - e^{-2}) = 5.187 \times 10^{-4}$ C = Q

Discharging : $q = Q(e^{-t/RC}) = 5.184 \times 10^{-4} \times e^{-2}$

$= 0.7 \times 10^{-4}$ C = 70 μc.

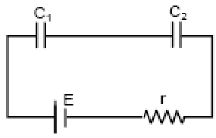
82. Consider the situation shown in figure. The switch is closed at *t* = 0 when the capacitors are uncharged. Find the charge on the capacitor C, as a function of time *t.*



Ans :

82. $C_{eff} = \dfrac{C_1C_2}{C_1+C_2}$

$Q = C_{eff}\,E(1-e^{-t/RC}) = \dfrac{C_1C_2}{C_1+C_2}\,E(1-e^{-t/RC})$



83. A capacitor of capacitance C is given a charge Q. At *t* - 0, it is connected to an uncharged capacitor of equal capacitance through a resistance *R*. Find the charge on - the second capacitor as a function of time.

Ans :

83. Let after time t charge on plate B is +Q.
Hence charge on plate A is Q – q.

$V_A = \dfrac{Q-q}{C},\ V_B = \dfrac{q}{C}$

$V_A - V_B = \dfrac{Q-q}{C} - \dfrac{q}{C} = \dfrac{Q-2q}{C}$

$\text{Current} = \dfrac{V_A - V_B}{R} = \dfrac{Q-2q}{CR}$

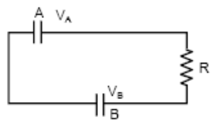
$\text{Current} = \dfrac{dq}{dt} = \dfrac{Q-2q}{CR}$

$\Rightarrow \dfrac{dq}{Q-2q} = \dfrac{1}{RC}\cdot dt \quad \Rightarrow \int_0^q \dfrac{dq}{Q-2q} = \dfrac{1}{RC}\cdot\int_0^t dt$

$\Rightarrow -\dfrac{1}{2}[\ln(Q-2q) - \ln Q] = \dfrac{1}{RC}\cdot t \Rightarrow \ln\dfrac{Q-2q}{Q} = \dfrac{-2}{RC}\cdot t$

$\Rightarrow Q - 2q = Q\,e^{-2t/RC} \Rightarrow 2q = Q(1-e^{-2t/RC})$

$\Rightarrow q = \dfrac{Q}{2}(1-e^{-2t/RC})$



84. A capacitor of capacitance *C* is given a charge *Q*. At *t* = 0, it is connected to an ideal battery of emf ε through a resistance *R*. Find the charge on the capacitor at time *t*.

Ans :

84. The capacitor is given a charge Q. It will discharge and the capacitor will be charged up when connected with battery.
Net charge at time t = $Qe^{-t/RC} + Q(1-e^{-t/RC})$.

Chapter 4
MOVING CHARGES AND MAGNETISM
8M or 9M
3M - 1Q ; 5M - 1Q (LA) or 1M-1Q; 3M-1Q; 5M-1Q(NP)

4.1 Concept of magnetic field -

(a) Electric Field : It can convey energy and momentum and is not established instantaneously but takes finite time to propagate. The concept of a field was specially stressed by Faraday and was incorporated by Maxwell in his unification of electricity and magnetism. In addition to depending on each point in space, it can also vary with time, i.e., be a function of time. In our discussions we will assume that the fields do not change with time.

The field at a particular point can be due to one or more charges. If there are more charges the fields add vectorially and is called the principle of superposition.

Just as static charges produce an electric field, the currents or moving charges produce (in addition) a magnetic field, denoted by B (r), again a vector field. It has several basic properties identical to the electric field. It is defined at each point in space (and can in addition depend on time). Experimentally, it is found to obey the principle of superposition: *the magnetic field of several sources is the vector addition of magnetic field of each individual source.*

(b) Magnetic Field : Lorentz force

Consider a point charge q (moving with a velocity v and, located at r at a given time t) in presence of both the electric field $\vec{E}$ (r) and the magnetic field $\vec{B}$ (r). The force on an electric charge q due to both of them can be written as

$$\mathbf{F} = q\,[\vec{E}(\mathbf{r}) + \mathbf{v} \times \vec{B}(\mathbf{r})] = \mathbf{F}_{electric} + \mathbf{F}_{magnetic} \quad \text{----------} \quad (1)$$

This force was given first by H.A. Lorentz based on the extensive experiments of Ampere and others. It is called the *Lorentz force*.

The interaction of electric field with the magnetic field have following features :

(i) It depends on q, v and B (charge of the particle, the velocity and the magnetic field). *Force on a negative charge is opposite to that on a positive charge.*

(ii) The magnetic force q [v × B] includes a vector product of velocity and magnetic field. The vector product makes the force due to magnetic field vanish (become zero) if velocity and magnetic field are parallel or anti-parallel. The force acts in a (sideways) direction perpendicular to both the velocity and the magnetic field. Its direction is given by the screw rule or right hand rule for vector (or cross) product (Fig.).

(iii) The magnetic force is zero if charge is not moving (as then |v|= 0). Only a moving charge feels the magnetic force.

Unit of magnetic field :

If one takes q, F and v, all to be unity in the force equation, $F = q\,[\,v \times B] = q\,v\,B \sin\theta\,\hat{n}$, where θ is the angle between v and B [see Fig. 2 (a)]. The magnitude of magnetic field B is 1 SI unit, when the force acting on a unit charge (1 C), moving perpendicular to B with a speed 1m/s, is one newton.

Dimensionally, we have $[B] = [F/qv]$ and the unit of B are Newton second / (coulomb metre). This unit is called *tesla* (T) named after Nikola Tesla (1856 – 1943). Tesla is a rather large unit. A smaller unit (non-SI) called *gauss* ($=10^{-4}$ tesla) is also often used. The earth's magnetic field is about 3.6×10^{-5} T.

4.2 Oersted's experiment – Force on a moving charge in uniform magnetic and electric fields:

Magnetic force on a current-carrying conductor : (Derivation of magnetic force on a current carrying conductor $\vec{F} = I\left(\vec{l} \times \vec{B}\right)$).

Consider a rod of a uniform cross-sectional area *A* and length *l*. We shall assume one kind of mobile carriers as in a conductor (here electrons). Let the number density of these mobile charge carriers in it be *n*. Then the total number of mobile charge carriers in it is *nAl*. For a steady current *I* in this conducting rod, we may assume that each mobile carrier has an average drift velocity v_d . In the presence of an external magnetic field $\vec{B}$, the force on these carriers is:

$F = (nAl)q\, \mathbf{v}_d \times \vec{B}$

where *q* is the value of the charge on a carrier. Now $nq\mathrm{v_d}$ is the current density j and $|(nq\, \mathrm{v_d})|A$ is the current *I*. Thus,

$F = [(nqev_d)A\vec{l}\,] \times \vec{B} = [\,\mathbf{j}\, A\vec{l}\,] \times \vec{B}$

$= I\,(\vec{l} \times \vec{B})$ ------------ (2)

where $\vec{l}$ is a vector of magnitude *l*, the length of the rod, and with a direction identical to the current *I*.

4.5 Motion of a charge in a uniform magnetic field: Nature of trajectories -

In the case of motion of a charge in a magnetic field, the magnetic force is perpendicular to the velocity of the particle. So no work is done and no change in the magnitude of the velocity is produced (though the direction of momentum may be changed).

[Notice that this is unlike the force due to an electric field, *q*E, which *can* have a component parallel (or antiparallel) to motion and thus can transfer energy in addition to momentum.]

We shall consider motion of a charged particle in a *uniform* magnetic field. First consider the case of v perpendicular to B.

The perpendicular force, $q\,[v \times B]$, acts as a centripetal force and produces a circular motion perpendicular to the magnetic field. *The particle will describe a circle if* v *and* B *are perpendicular to each other* (Fig. 4.5).

If velocity has a component along B, this component remains unchanged as the motion along the magnetic field will not be affected by the magnetic field. The motion in a plane perpendicular to B is as before a circular one, thereby producing a *helical motion* (Fig. 4.6).

Derivation of radius and angular frequency of circular motion of a charge in uniform magnetic field.

If *r* is the radius of the circular path of a particle, then a force of $m\,v^2 / r$, acts perpendicular to the path towards the centre of the circle, and is called the centripetal force. If the velocity v is perpendicular to the magnetic field B, the magnetic force is perpendicular to both v and B and acts like a centripetal force. It has a magnitude $q\,v\,B$. Equating the two expressions for centripetal force, $m\,v^2/r = q\,v\,B$, which gives $\boldsymbol{r = m\,v / qB}$ --------- (4.5) for the radius of the circle described by the charged particle. The larger the momentum, the larger is the radius and bigger the circle described. If w is the angular frequency, then $v = \omega r$. So,

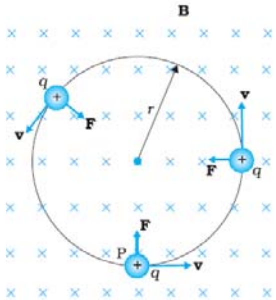
$\omega = 2\pi\upsilon = qB/m$ ---------- [4.6(a)]

which is independent of the velocity or energy . Here n is the frequency of rotation. The independence of n from energy has important application in the design of a cyclotron.

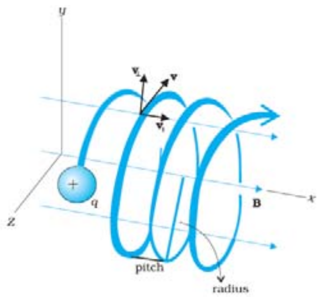
The time taken for one revolution is $T = 2\pi/\omega = 1/\upsilon$. If there is a component of the velocity parallel to the magnetic field (denoted by $v_{||}$), it will make the particle move along the field and the path of the particle would be a helical one. The distance moved along the magnetic field in one rotation is called pitch p. Using Eq. [4.6 (a)], we have

$p = v_{||}T = 2\pi m v_{||} / qB$ --------- [4.6(b)]

The radius of the circular component of motion is called the *radius* of the *helix*.



(a) Circular motion



(b) Helical motion

Ex. 1 : What is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of 3 ×107 m/s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency? Calculate its energy in keV. (1 eV = 1.6×10^{-19} J).

Solution : Using Eq. (4.5) we find

$r = mv/(qB) = 9 \times 10^{-31}$ kg $\times 3 \times 10^{7}$ m s^{-1} / (1.6×10^{-19} C $\times 6 \times 10^{-4}$ T)

$= 26 \times 10^{-2}$ m = 26 cm

n = v / (2 pr) = 2×106 s^{-1} = 2×10^{6} Hz = 2 MHz.

$E = (½)mv^{2} = (½)\ 9 \times 10^{-31}$ kg $\times 9 \times 10^{14}$ m^2/s^2 = 40.5×10^{-17} J

» 4×10^{-16} J = 2.5 keV.

4.6 Velocity selector: Crossed electric and magnetic fields serve as velocity selector.

We know that a charge q moving with velocity v in presence of both electric and magnetic fields experiences a force given by,

$\mathbf{F} = q(\vec{E} + v \times \vec{B}) = \mathbf{F_E} + \mathbf{F_B}$

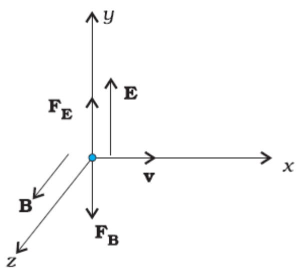
We shall consider the simple case in which electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of the particle, as shown in Fig. We have,

$\mathbf{F}_E = q\mathbf{E} = qE\,\hat{\mathbf{j}},\ \mathbf{F}_B = q\mathbf{v} \times \mathbf{B},\ = q\left(\upsilon\,\hat{\mathbf{i}} \times B\hat{\mathbf{k}}\right) = -qB\,\hat{\mathbf{j}}$

Therefore,

$\mathbf{F} = q(E - \upsilon B)\,\hat{\mathbf{j}}$.

Thus, electric and magnetic forces are in opposite directions as shown in the figure. Suppose, we adjust the value of E and B such that magnitudes of the two forces are equal. Then, total force on



the charge is zero and the charge will move in the fields undeflected.
This happens when,

$$qE = qvB \quad \text{or} \quad v = \frac{E}{B} \qquad \text{-------------} \quad (4.7)$$

This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and mass). The crossed E and B fields, therefore, serve as a *velocity selector*. Only particles with speed E/B pass undeflected through the region of crossed fields. This method was employed by J. J. Thomson in 1897 to measure the charge to mass ratio (e/m) of an electron. The principle is also employed in **Mass Spectrometer** – a device that separates charged particles, usually ions, according to their **charge to mass ratio**.

4.7 Cyclotron: Principle, construction, working and uses.

The cyclotron is a machine to accelerate charged particles or ions to high energies. It was invented by E.O. Lawrence and M.S. Livingston in 1934 to investigate nuclear structure.
The cyclotron uses both electric and magnetic fields in combination to increase the energy of charged particles. As the fields are perpendicular to each other they are called *crossed fields*. Cyclotron uses the fact that the frequency of revolution of the charged particle in a magnetic field is independent of its energy. The particles move most of the time inside two semicircular disc-like metal containers, D_1 and D_2, which are called *dees* as they look like the letter D.

Construction :

Figure shows a schematic view of the cyclotron. There is a source of charged particles or ions at P which move in a circular fashion in the dees, D_1 and D_2, on account of a uniform perpendicular magnetic field B. An alternating voltage source accelerates these ions to high speeds. The ions are eventually 'extracted' at the exit port.

Inside the metal boxes the particle is shielded and is not acted on by the electric field. The magnetic field, however, acts on the particle and makes it go round in a circular path inside a dee. Every time the particle moves from one dee to another it is acted upon by the electric field. The sign of the electric field is changed alternately in tune with the circular motion of the particle. This ensures that the particle is always accelerated by the electric field. Each time the acceleration increases the energy of the particle. As energy increases, the radius of the circular path increases. So the path is a spiral one. The whole assembly is evacuated to minimise collisions between the ions and the air molecules. A high frequency alternating voltage is applied to the dees.

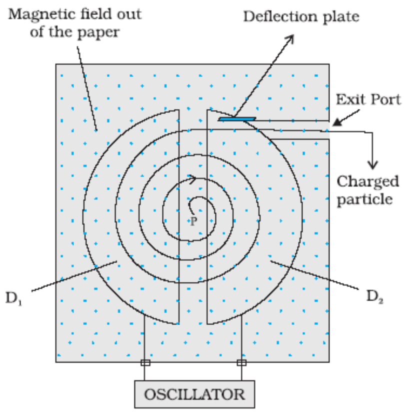
Principle :

In the sketch shown in Fig., positive ions or positively charged particles (e.g., protons) are released at the centre P. They move in a semi-circular path in one of the dees and arrive in the gap between the dees in a time interval $T/2$; where T, the period of revolution, is given by Eq. (4.6),

$$T = \frac{1}{\nu_c} = \frac{2\pi m}{qB} \quad \text{or} \quad \nu_c = \frac{qB}{2\pi m} \qquad \text{---------} \quad (4.8)$$

This frequency is called the *cyclotron frequency* for obvious reasons and is denoted by ν_c. The frequency ν_a of the applied voltage is adjusted so that the polarity of the dees is reversed in the same time that it takes the ions to complete one half of the revolution. The requirement $\nu_a = \nu_c$ is called the *resonance condition*. The phase of the supply is adjusted so that when the positive ions arrive at the edge of D_1, D_2 is at a lower potential and the ions are accelerated across the gap. Inside the dees the particles travel in a region free of the electric field. The increase in their kinetic energy is qV each time they cross from one dee to another (V refers to the voltage across the dees at that time). From Eq. (4.5), it is clear that the radius of their path goes on increasing each time their kinetic energy increases. The ions are repeatedly accelerated across the dees until they have the required energy to have a radius approximately that of the dees. They are then deflected by a magnetic field and leave the system via an exit slit. From Eq. (4.5) we have,

$$v = \frac{qBR}{m} \qquad \text{----------} \quad (4.9)$$



where R is the radius of the trajectory at exit, and equals the radius of a dee.

Hence, the kinetic energy of the ions is,

$$\frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m} \qquad \text{-----------} \quad (4.10)$$

Working :

The operation of the cyclotron is based on the fact that the time for one revolution of an ion is independent of its speed or radius of its orbit.

Uses :

The cyclotron is used (1) To bombard nuclei with energetic particles, so accelerated by it, and study the resulting nuclear reactions.

(2) It is also used to implant ions into solids and modify their properties or even synthesise new materials.

(3) It is used in hospitals to produce radioactive substances which can be used in diagnosis and treatment.

Ex. 2 : A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, what is the kinetic energy (in MeV) of the proton beam produced by the accelerator.

(e =1.60 × 10^{-19} C, m_p = 1.67 × 10^{-27} kg, 1 MeV = 1.6 × 10^{-13} J).

Solution : The oscillator frequency should be same as proton's cyclotron frequency.

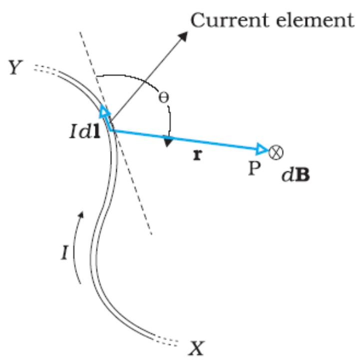
Using Eqs. (4.5) and [4.6(a)] we have $B = 2\pi m\nu /q = 6.3 \times 1.67 \times 10^{-27} \times 10^7 / (1.6 \times 10^{-19}) = 0.66$ T

Final velocity of protons is

$v = r \times 2\pi\nu = 0.6 \text{ m} \times 6.3 \times 10^7 = 3.78 \times 10^7$ m/s.

$E = \frac{1}{2} mv^2 = 1.67 \times 10^{-27} \times 14.3 \times 10^{14} / (2 \times 1.6 \times 10^{-13}) = 7$ MeV.

4.8 Biot–Savart law: Statement, explanation and expression in vector form -



The Biot-Savart's law the relation between current and the magnetic field it produces. Figure 4.9 shows a finite conductor XY carrying current I. Consider an infinitesimal element dl of the conductor. The magnetic field $\overrightarrow{dB}$ due to this element is to be determined at a point P which is at a distance r from it. Let θ be the angle between dl and the displacement vector $\vec{r}$.

According to Biot-Savart's law, the magnitude of the magnetic field $\vec{dB}$ is proportional to the current I, the element length $|\vec{dl}\,|$, and inversely proportional to the square of the distance *r*. Its direction is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$.
Thus, in vector notation,

$$\vec{dB} \propto \frac{I\,\vec{dl}\times\vec{r}}{r^3} \quad = \frac{\mu_0}{4\pi}\,\frac{I\,\vec{dl}\times\vec{r}}{r^3}$$

where $\varepsilon_0/4\pi$ is a constant of proportionality. The above expression holds when the medium is vacuum.
The magnitude of this field is,

$$\boxed{|\mathbf{dB}| = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}l\sin\theta}{r^2}} \quad \text{---------} \quad (4.11(a))$$

where we have used the property of cross-product. Equation [4.11 (a)] constitutes our basic equation for the magnetic field. The proportionality constant in SI units has the exact value,

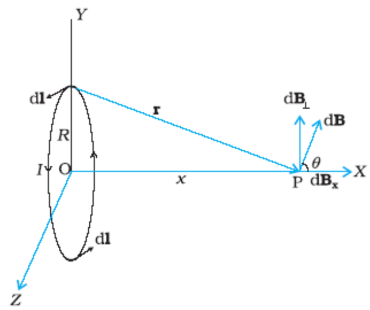
$$\frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{Tm/A}$$

We call μ_0 the *permeability* of free space (or vacuum).

The Biot-Savart law for the magnetic field has certain similarities as well as differences with the Coulomb's law for the electrostatic field. Some of these are :

S.No.	Biot-Savart law for the magnetic field	Coulomb's law for the electrostatic field
1	Long range force, since both depend inversely on the square of distance from the source to the point of interest.	Long range force, since both depend inversely on the square of distance from the source to the point of interest.
2	The principle of superposition applies to magnetic fields.	The principle of superposition applies to electric fields.
3	The magnetic field is produced by a vector source $I\,\vec{dl}$.	The electrostatic field is produced by a scalar source, namely, the electric charge.
4	The magnetic field is produced by a vector source $I\,\vec{dl}$.	The electrostatic field is along the displacement vector joining the source and the field point.
5	There is an angle dependence in the Biot-Savart law. In the above fig., the magnetic field at any point in the direction of dl (the dashed line) is zero.	There is no angle dependence in the Coulombs law.

4.9 Derivation of magnetic field on the axis of a circular current loop - Right hand thumb rule to find direction.



Consider a circular loop carrying a steady current *I as shown in fig*. The loop is placed in the *y-z* plane with its centre at the origin O and has a radius *R*. The *x*-axis is the axis of the loop. We wish to calculate the magnetic field at the point P on this axis. Let *x* be the distance of P from the centre O of the loop.
Consider a conducting element dl of the loop as shown in Fig.

The magnitude dB of the magnetic
field due to dl is given by the Biot-Savart law [Eq. 4.11(a)],

$$dB = \frac{\mu_0}{4\pi} \frac{I\left|d\mathbf{l} \times \mathbf{r}\right|}{r^3}$$

Now $r^2 = x^2 + R^2$. Further, any element of the loop will be perpendicular to the displacement vector from the element to the axial point. For example, the element dl in Fig. 4.11 is in the y-z plane whereas the displacement vector $\vec{r}$ from $\vec{dl}$ to the axial point P is in the x-y plane. Hence $|\vec{dl} \times \vec{r}| = r\,dl$. Thus,

$$\mathrm{d}B = \frac{\mu_0}{4\pi} \frac{I\mathrm{d}l}{\left(x^2 + R^2\right)}$$

The direction of dB is shown in Fig. 4.11. It is perpendicular to the plane formed by dl and r. It has an x-component $\vec{dB}_x$ and a component perpendicular to x-axis, $\vec{dB}_\perp$. When the components perpendicular to the x-axis are summed over, they cancel out and we obtain a null result.
For example, the $\vec{dB}_\perp$ component due to $\vec{dl}$ is cancelled by the contribution due to the diametrically opposite dl element, shown in Fig. 4.11. Thus, only the x-component survives. The net contribution along x-direction can be obtained by integrating dB_x = dB cos θ over the loop. For Fig. 4.11,

$$\cos\theta = \frac{R}{(x^2 + R^2)^{1/2}} \quad \text{--------} \quad (4.14)$$

From Eqs. (4.13) and (4.14),

$$\mathrm{d}B_x = \frac{\mu_0 I\mathrm{d}l}{4\pi} \frac{R}{\left(x^2 + R^2\right)^{3/2}}$$

The summation of elements dl over the loop yields 2πR, the circumference of the loop. Thus, the magnetic field at P due to entire circular loop is

$$\mathbf{B} = B_x\hat{\mathbf{i}} = \frac{\mu_0 I\, R^2}{2\left(x^2 + R^2\right)^{3/2}}\hat{\mathbf{i}} \quad \text{---------} \quad (4.15)$$

As a special case of the above result, we may obtain the field at the centre of the loop. Here x = 0, and we obtain,

$$\mathbf{B}_0 = \frac{\mu_0 I}{2R}\hat{\mathbf{i}}$$

The magnetic field lines due to a circular wire form closed loops and are shown in Fig. 4.12. The direction of the magnetic field is given by (another) *right-hand thumb rule* stated below:
Curl the palm of your right hand around the circular wire with the fingers pointing in the direction of the current. The right-hand thumb gives the direction of the magnetic field.

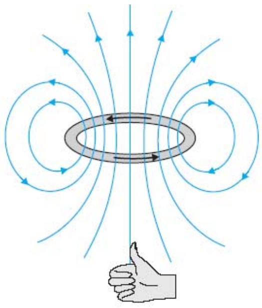


FIGURE 4.12 The magnetic field lines for a current loop.

Example : Consider a tightly wound 100 turn coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil?

Solution : Since the coil is tightly wound, we may take each circular element to have the same radius R = 10 cm = 0.1 m. The number of turns N = 100. The magnitude of the magnetic field is,

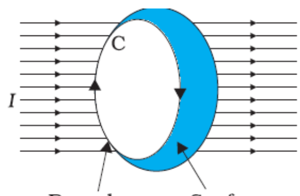
$$B = \frac{\mu_0 NI}{2R} = \frac{4\pi \times 10^{-7} \times 10^2 \times 1}{2 \times 10^{-1}} = 2\pi \times 10^{-4} = 6.28 \times 10^{-4}\ \text{T}$$

4.10 Ampere's circuital law: Statement and explanation -

Ampere's circuital law is an alternative and appealing way to express the Biot-Savart law.

Considers an open surface with a boundary (Fig. 4.14). The surface has current passing through it. We consider the boundary to be made up of a number of small line elements. Consider one such element of length *dl*. We take the value of the tangential component of the magnetic field, *Bt*, at this element and multiply it by the length of that element *dl* [Note: $B_t\, dl = \vec{B}.\vec{dl}$]. All such products are added together. We consider the limit as the lengths of elements get smaller and their number gets larger. The sum then tends to an integral.

Ampere's law states that this integral is equal to μ_0 times the total current passing through the surface, i.e.,



$$\oint \vec{B}.\vec{dl} = \mu_0 I \qquad \text{--------} \quad 4.17\ (a)$$

where I is the total current through the surface. The integral is taken over the closed loop coinciding with the boundary C of the surface. The relation above involves a sign-convention, given by the right-hand rule. Let the fingers of the right-hand be curled in the sense the boundary is traversed in the loop integral $\oint \vec{B}.\vec{dl}$ Then the direction of the thumb gives the sense in which the current I is regarded as positive.

For practical applications, it is possible to choose the loop (called an *amperian loop*) such that at each point of the loop, *either*

(i) B is tangential to the loop and is a non-zero *constant* B, *or*

(ii) B is normal to the loop, *or*

(iii) B vanishes.

4.11. Application of Ampere's circuital law to derive the magnetic field due to an infinitely long straight current carrying wire: Solenoid and toroid -

The solenoid and the toroid are two pieces of equipment which generate magnetic fields. The television uses the solenoid to generate magnetic fields needed. The synchrotron uses a combination of both to generate the high magnetic fields required. In both, solenoid and toroid, we come across a situation of high symmetry where Ampere's law can be conveniently applied.

(1) The solenoid :

Consider a long solenoid where the solenoid's length is large compared to its radius. It consists of a long wire wound in the form of a helix where the neighbouring turns are closely spaced. So each turn can be regarded as a circular loop. The net magnetic field is the vector sum of the fields due to all the turns. Enamelled wires are used for winding so that turns are insulated from each other.

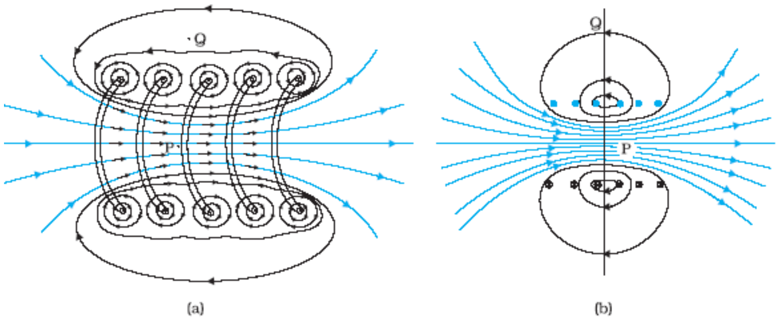


FIGURE 4.17 (a) The magnetic field due to a section of the solenoid. (b) The magnetic field of a finite solenoid.

Figure 4.17 displays the magnetic field lines for a finite solenoid. We show a section of this solenoid in an enlarged manner in Fig. 4.17(a). Figure 4.17(b) shows the entire finite solenoid with its magnetic field. In Fig. 4.17(a), it is clear from the circular loops that the field between two neighbouring turns vanishes. In Fig. 4.17(b), we see that the field at the interior mid-point P is uniform, strong and along the axis of the solenoid.
The field at the exterior mid-point Q is weak and moreover is along the axis of the solenoid with no perpendicular or normal component. As the solenoid is made longer it appears like a long cylindrical metal sheet.
Figure 4.18 represents this idealized picture. The field outside the solenoid approaches zero. We shall assume that the field outside is zero. The field inside becomes everywhere parallel to the axis.

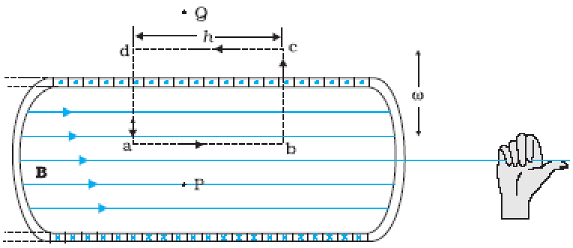


FIGURE 4.18 The magnetic field of a very long solenoid.

Consider a rectangular Amperian loop abcd. Along cd the field is zero as argued above. Along transverse sections bc and ad, the field component is zero. Thus, these two sections make no contribution. Let the field along ab be B. Thus, the relevant length of the Amperian loop is, $L = h$.
Let n be the number of turns per unit length, then the total number of turns is nh. The enclosed current is, $I_e = I\ (n\ h)$, where I is the current in the solenoid. From Ampere's circuital law [Eq. 4.17 (b)]

$BL = \mu_0 I_e,$ $\qquad B\ h = \mu_0 I\ (n\ h)$

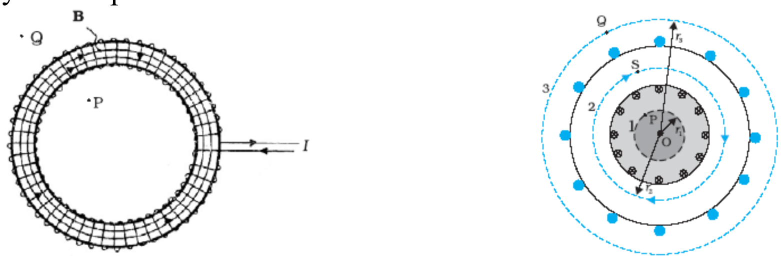
$B = \mu_0 n I$ ---------- (4.20)

The direction of the field is given by the right-hand rule. The solenoid is commonly used to obtain a uniform magnetic field. We shall see in the next chapter that a large field is possible by inserting a soft iron core inside the solenoid.

(2) The toroid :

The toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. It can be viewed as a solenoid which has been bent into a circular shape to close on itself. It is shown in Fig. 4.19(a) carrying a current *I*. We shall see that the magnetic field in the open space inside (point P) and exterior to the toroid (point Q) is zero. The field B inside the toroid is constant in magnitude for the *ideal* toroid of closely wound turns.

Figure 4.19(b) shows a sectional view of the toroid. The direction of the magnetic field inside is clockwise as per the right-hand thumb rule for circular loops. Three circular Amperian loops 1, 2 and 3 are shown by dashed lines. By symmetry, the magnetic field should be tangential to each of them and constant in magnitude for a given loop. The circular areas bounded by loops 2 and 3 both cut the toroid: so that each turn of current carrying wire is cut once by the loop 2 and twice by the loop 3.



(a) (b)

Let the magnetic field along loop l be B_1 in magnitude.

Then in Ampere's circuital law [Eq. 4.17(a)], $L = 2\pi\, r_1$.

However, the loop encloses no current, so $Ie = 0$. Thus, $B_1\,(2\,\pi\, r_1) = \mu_0(0)$, $B_1 = 0$

Thus, the magnetic field at any point P in the open space inside the toroid is zero.

We shall now show that magnetic field at Q is likewise zero. Let the magnetic field along loop 3 be B_3. Once again from Ampere's law $L = 2\,\pi\, r^3$. However, from the sectional cut, we see that the current coming out of the plane of the paper is cancelled exactly by the current going into it. Thus,

$I_e = 0$, and $B_3 = 0$. Let the magnetic field inside the solenoid be B. We shall now consider the magnetic field at S. Once again we employ Ampere's law in the form of Eq. [4.17 (a)]. We find, $L = 2\pi\, r$.

The current enclosed I_e is (for N turns of toroidal coil) $N\,I$.

$B\,(2\pi r) = \mu_0 NI$

$$B = \frac{\mu_0 NI}{2\pi r} \quad \text{--------} \quad (4.21)$$

We shall now compare the two results: for a toroid and solenoid. We re-express Eq. (4.21) to make the comparison easier with the solenoid result given in Eq. (4.20). Let r be the average radius of the toroid and n be the number of turns per unit length. Then

$N = 2\pi r\, n$ = *(average) perimeter of the toroid × number of turns per unit length*

and thus,

$B = \mu_0\, n\, I,$ -------- (4.22)

i.e., the result for the solenoid!
In an ideal toroid the coils are circular. In reality the turns of the toroidal coil form a helix and there is always a small magnetic field external to the toroid.

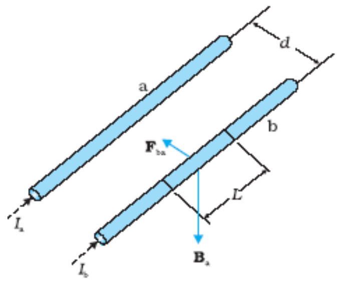
4.12 Mention of expressions for the magnetic field at a point inside a solenoid and a toroid.

Refer above.

4.13 Derivation of the force between two parallel current carrying conductors -

Figure 4.20 shows two long parallel conductors a and b separated by a distance d and carrying (parallel) currents I_a and I_b, respectively. The conductor 'a' produces, the same magnetic field B_a at all points along the conductor 'b'. The right-hand rule tells us that the direction of this field is downwards (when the conductors are placed horizontally). Its magnitude is given by Eq. [4.19(a)] or from Ampere's circuital law,

$$B_a = \frac{\mu_0 I_a}{2\pi d}$$



The conductor 'b' carrying a current Ib will experience a sideways force due to the field Ba. The direction of this force is towards the conductor 'a' (Verify this). We label this force as F_{ba}, the force on a segment L of 'b' due to 'a'. The magnitude of this force is given by Eq. (4.4),

$$F_{ba} = I_b L B_a$$

$$= \frac{\mu_0 I_a I_b}{2\pi d} L$$

It is of course possible to compute the force on 'a' due to 'b'. From considerations similar to above we can find the force F_{ab}, on a segment of length L of 'a' due to the current in 'b'. It is equal in magnitude to F_{ba}, and directed towards 'b'. Thus,

$\mathbf{F}_{ba} = -\mathbf{F}_{ab}$ ----------- (4.24)

Note that this is consistent with Newton's third Law. Thus, at least for parallel conductors and steady currents, we have shown that the Biot-Savart law and the Lorentz force yield results in accordance with Newton's third Law.

We have seen from above that currents flowing in the same direction attract each other. One can show that oppositely directed currents repel each other. Thus,
Parallel currents attract, and antiparallel currents repel.
This rule is the opposite of what we find in electrostatics. Like (same sign) charges repel each other, but like (parallel) currents attract each other.

Let f_{ba} represent the magnitude of the force F_{ba} per unit length. Then, from Eq. (4.23),

$$f_{ba} = \frac{\mu_0 I_a I_b}{2\pi d}$$

The above expression is used to define the ampere (A), which is one of the seven SI base units.
The *ampere* is the value of that steady current which, when maintained in each of the two very long, straight, parallel conductors of negligible cross-section, and placed one metre apart in vacuum, would produce on each of these conductors a force equal to 2×10^{-7} newtons per metre

of length.
The SI unit of charge, namely, the coulomb, can now be defined in terms of the ampere.
When a steady current of 1A is set up in a conductor, the quantity of charge that flows through its cross-section in 1s is one coulomb (1C).

4.14 Definition of ampere.

The *ampere* is the value of that steady current which, when maintained in each of the two very long, straight, parallel conductors of negligible cross-section, and placed one metre apart in vacuum, would produce on each of these conductors a force equal to 2×10^{-7} Newton per metre of length.

4.15.Current loop as a magnetic dipole -

The magnetic field (at large distances) due to current in a circular current loop is very similar in behavior to the electric field of an electric dipole.
We know that the magnetic field on the axis of a circular loop, of a radius *R*, carrying a steady current *I*. The magnitude of this field is

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

and its direction is along the axis and given by the right-hand thumb rule (Fig. 4.12). Here, *x* is the distance along the axis from the centre of the loop. For $x >> R$, we may drop the R^2 term in the denominator. Thus,

$$B = \frac{\mu_0 R^2}{2x^3}$$

Note that the area of the loop $A = \pi R^2$. Thus,

$$B = \frac{\mu_0 IA}{2\pi x^3}$$

We define the magnetic moment **m** to have a magnitude *IA*,
$\boldsymbol{m} = I\,\mathbf{A}$. Hence,

$$\mathbf{B} \simeq \frac{\mu_0 \mathbf{m}}{2\pi x^3} = \frac{\mu_0}{4\pi}\frac{2\mathbf{m}}{x^3} \qquad \text{-------} \quad [4.31(a)]$$

The expression of Eq. [4.31(a)] is very similar to an expression obtained earlier for the electric field of a dipole. The similarity may be seen if we substitute,

$$\mu_0 \rightarrow 1/\varepsilon_0$$

$$\mathbf{m} \rightarrow \mathbf{p}_e \text{ (electrostatic dipole)}$$

$$\mathbf{B} \rightarrow \mathbf{E} \text{ (electrostatic field)}$$

We then obtain,

$$\mathbf{E} = \frac{2\mathbf{p}_e}{4\pi\varepsilon_0 x^3}$$

which is precisely the field for an electric dipole at a point on its axis.

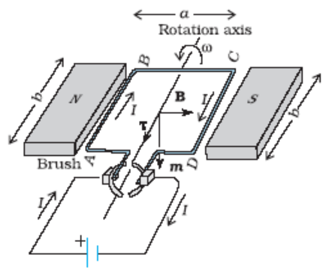
4.16 Qualitative explanation and definition of magnetic dipole moment -

We define the magnetic moment **m** to have a magnitude *IA,*
If the loop has *N* closely wound turns, the expression for torque, Eq. (4.29), still holds, with
$\mathbf{m} = N\,I\,\mathbf{A}$

4.17 Mention of expression for torque experienced by a current loop in a magnetic field -

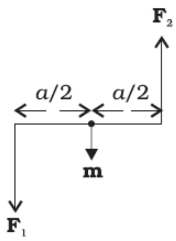
A rectangular loop carrying a steady current *I* and placed in a uniform magnetic field experiences a torque. It does not experience a net force. This behaviour is analogous to that of electric dipole in a uniform electric field.

CASE 1 :
When the rectangular loop is placed such that the uniform magnetic field B is in the plane of the loop. This is illustrated in Fig. 4.21(a).



(a)

The field exerts no force on the two arms AD and BC of the loop. It is perpendicular to the arm AB of the loop and exerts a force F_1 on it which is directed into the plane of the loop. Its magnitude is,
$F_1 = I\,b\,B$
Similarly it exerts a force F_2 on the arm CD and F_2 is directed out of the plane of the paper.
$F_2 = I\,b\,B = F_1$
Thus, the *net force* on the loop *is zero*. There is a torque on the loop due to the pair of forces F_1 and F_2.
Figure 4.21(b) shows a view of the loop from the AD end and is the couple acting on the coil.



(b)

. It shows that the torque on the loop tends to rotate it anti-clockwise.
This torque is (in magnitude),

$$\tau = F_1\frac{a}{2} + F_2\frac{a}{2}$$

$$= IbB\frac{a}{2} + IbB\frac{a}{2} = I(ab)B \qquad \boxed{= I\,A\,B} \qquad \text{--------} \quad (4.26)$$

where $A = ab$ is the area of the rectangle.

CASE 2 :
When the plane of the loop is not along the magnetic field, but makes an angle with it. We take the angle between the field and the normal to the coil to be angle θ (The previous case corresponds to $\theta = \pi/2$) as shown in following figure.

The forces on the arms BC and DA are equal, opposite, and act along the axis of the coil, which connects the centres of mass of BC and DA. Being collinear along the axis they cancel each other, resulting in no net force or torque. The forces on arms AB and CD are $\mathbf{F_1}$ and $\mathbf{F_2}$. They too are equal and opposite, with magnitude, $F_1 = F_2 = I\,b\,B$
But they are not collinear! This results in a couple as before. The torque is, however, less than the earlier case when plane of loop was along the magnetic field. This is because the perpendicular distance between the forces of the couple has decreased. Figure 4.22(b) is a view of the arrangement from the AD end and it illustrates these two forces constituting a couple. The

magnitude of the torque on the loop is,

$$\tau = F_1 \frac{a}{2}\sin\theta + F_2 \frac{a}{2}\sin\theta$$

$$= I\,ab\,B\sin\theta \qquad = I\,A\,B\sin\theta \qquad \text{---------} \quad (4.27)$$

As θ → 0, the perpendicular distance between the forces of the couple also approaches zero. This makes the forces collinear and the net force and torque zero. The torques in Eqs. (4.26) and (4.27) can be expressed as vector product of the magnetic moment of the coil and the magnetic field. We define the *magnetic moment* of the current loop as,

$\mathbf{m} = I\,\mathbf{A}$ ---------- (4.28)

where the direction of the area vector **A** is given by the right-hand thumb rule and is directed into the plane of the paper in Fig. 4.21. Then as the angle between **m** and **B** is θ, Eqs. (4.26) and (4.27) can be expressed by one expression

$\boldsymbol{\tau} = \mathbf{m}\,\mathbf{B}$ -------- (4.29)

This is analogous to the electrostatic case (Electric dipole of dipole moment **pe** in an electric field **E**).

$\boldsymbol{\tau} = \mathbf{p_e} \times \mathbf{E}$

As is clear from Eq. (4.28), the dimensions of the magnetic moment are $[A][L^2]$ and its unit is Am^2.

In eqn. (4.29), the torque τ vanishes when **m** is either parallel or antiparallel to the magnetic field **B**. This indicates a state of equilibrium as there is no torque on the coil (this also applies to any object with a magnetic moment **m**). When **m** and **B** are parallel the equilibrium is a stable one. Any small rotation of the coil produces a torque which brings it back to its original position. When they are antiparallel, the equilibrium is unstable as any rotation produces a torque which increases with the amount of rotation. The presence of this torque is also the reason why a small magnet or any magnetic dipole aligns itself with the external magnetic field.

If the loop has *N* closely wound turns, the expression for torque, Eq. (4.29), still holds, with

$\mathbf{m} = N\,I\,\mathbf{A}$ ---------- (4.30)

4.18 Derivation of magnetic dipole moment of a revolving electron in a hydrogen atom and to obtain the value of Bohr magneton.

(1) Magnetic dipole moment of a revolving electron :

In the Bohr model, the electron (a negatively charged particle) revolves around a positively charged nucleus much as a planet revolves around the sun. The force in the former case is electrostatic (Coulomb force) while it is gravitational for the planet-Sun case.

The electron of charge (*–e*) ($e = +1.6 \times 10^{-19}$ C) performs uniform circular motion around a stationary heavy nucleus of charge *+Ze*. This constitutes a current *I*, where, $I = e/T$

and *T* is the time period of revolution. Let *r* be the orbital radius of the electron, and *v* the orbital speed. Then, $T = 2\pi r/v$

Substituting in Eq. (4.32), we have $I = ev/2\pi r$.

There will be a magnetic moment, usually denoted by μ_l,

associated with this circulating current. From Eq. (4.28) its magnitude is, $\mu_l = I\pi r^2 = evr/2$.

The direction of this magnetic moment is into the plane of the paper in Fig. 4.23.
Multiplying and dividing the right-hand side of the above expression by the electron mass m_e, we have,

$$\mu_l = \frac{e}{2m_e}(m_e v r) \qquad = \frac{e}{2m_e} l$$

Here, l is the magnitude of the angular momentum of the electron about the central nucleus ("orbital" angular momentum). Vectorially,

$$\boldsymbol{\mu}_l = -\frac{e}{2m_e}\mathbf{l}$$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment. Instead of electron with charge (–e), if we had taken a particle with charge (+q), the angular momentum and magnetic moment would be in the same direction. The ratio

$$\frac{\mu_l}{l} = \frac{e}{2m_e}$$

is called the *gyromagnetic ratio* and is a constant. Its value is 8.8 □□1010 C /kg for an electron, which has been verified by experiments.

The fact that even at an atomic level there is a magnetic moment, confirms Ampere's bold hypothesis of atomic magnetic moments. This according to Ampere, would help one to explain the magnetic properties of materials.

Bohr hypothesised that the angular momentum assumes a discrete set of values, namely,

$$l = \frac{nh}{2\pi}$$

where n is a natural number, n = 1, 2, 3, ... and h is a constant named after Max Planck (Planck's constant) with a value h = 6.626 × 10–34 J s. This condition of discreteness is called the *Bohr quantisation condition*.

If we use it to calculate the elementary dipole moment. Take the value n = 1, we have from Eq. (4.34) that,

$$(\mu_l)_{\min} = \frac{e}{4\pi m_e} h$$

$$= \frac{1.60\times10^{-19}\times6.63\times10^{-34}}{4\times3.14\times9.11\times10^{-31}} \quad = 9.27 \times 10^{-24}\ \text{Am}^2 \quad \text{-----------} \quad (4.37)$$

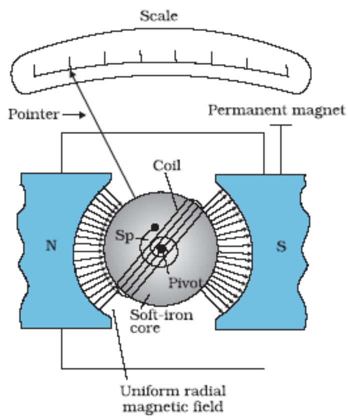
where the subscript 'min' stands for minimum. This value is called the *Bohr magneton*.

Any charge in uniform circular motion would have an associated magnetic moment This dipole moment is labelled as the *orbital magnetic moment*. Hence the subscript 'l' in μ_l . Besides the orbital moment, the electron has an *intrinsic* magnetic moment, which has the same numerical value as given in Eq. (4.37). It is called the *spin magnetic moment*.

4.19 Moving coil galvanometer: Mention of expression for angular deflection -

Moving coil galvanometer is a device used to measure small current and voltages.
Principle : It works using the principle of MCG is that a rectangular loop carrying a steady current and placed in a uniform magnetic field experiences a torque.

The galvanometer consists of a coil, with many turns, free to rotate about a fixed axis (Fig. 4.24), in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. When a current flows through the coil, a torque acts on it. This torque is given by Eq. (4.26) to be $\boldsymbol{\tau} = \boldsymbol{NI\,AB}$
where the symbols have their usual meaning. Since the field is radial by design, we have taken $\sin\theta = 1$ in the above expression for the torque.



The magnetic torque *NIAB* tends to rotate the coil. A spring Sp provides a counter torque $k\,\varphi$ that balances the magnetic torque *NIAB;* resulting in a steady angular deflection φ. In equilibrium
$k\,\varphi = NI\,AB$
where k is the torsional constant of the spring; i.e. the restoring torque per unit twist. The deflection φ is indicated on the scale by a pointer attached to the spring. We have

$$\phi = \left(\frac{NAB}{k}\right)I \qquad \text{----------} \qquad (4.38)$$

The quantity in brackets is a constant for a given galvanometer.
The galvanometer can be used in a number of ways.
It can be used as a detector to check if a current is flowing in the circuit. We have come across this usage in the Wheatstone's bridge arrangement. In this usage the neutral position of the pointer (when no current is flowing through the galvanometer) is in the middle of the scale and not at the left end as shown in Fig.4.24. Depending on the direction of the current, the pointer deflection is either to the right or the left.

4.20 Definitions of current sensitivity and voltage sensitivity -

(1) Current Sensitivity : It is defined as deflection per unit current of Moving coil galvanometer. It is given by

$$\frac{\phi}{I} = \frac{NAB}{k}$$

(2) Voltage Sensitivity : It is defined as *as the deflection per unit voltage* of Moving coil galvanometer. It is given by

$$\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$$

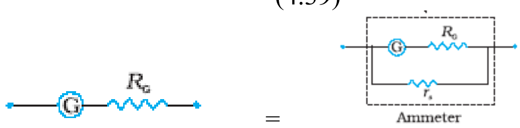
4.21 Conversion of galvanometer to ammeter and voltmeter, Numerical Problems.

(1) Conversion of galvanometer into Ammeter :
The galvanometer cannot as such be used as an ammeter to measure the value of the current in a given circuit. This is for two reasons:
(i) Galvanometer is a very sensitive device, it gives a full-scale deflection for a current of the order of µA. (ii) For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will change the value of the current in the circuit. To overcome these difficulties, one attaches a small resistance r_s, called *shunt resistance*, in parallel with the galvanometer coil; so that most of the current passes through the shunt. The resistance of this arrangement is,

$$R_G\, r_s / (R_G + r_s) \simeq r_s \qquad \text{if} \quad R_G >> r_s$$

If r_s has small value, in relation to the resistance of the rest of the circuit R_c, the effect of introducing the measuring instrument is also small and negligible. This arrangement is schematically shown in Fig. 4.25. The scale of this ammeter is calibrated and then graduated to read off the current value
with ease. We define the *current sensitivity of the galvanometer as the deflection per unit current*. From Eq. (4.38) this **current sensitivity** is,

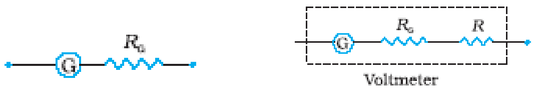
$$\frac{\phi}{I} = \frac{NAB}{k} \qquad \text{--------} \quad (4.39)$$



A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns N. We choose galvanometers having sensitivities of value, required by our experiment.

(2) Conversion of galvanometer into Voltmeter :

The galvanometer can be used as a voltmeter to measure the voltage across a given section of the circuit. For this it must be connected *in parallel* with that section of the circuit. Further, it must draw a very small current, otherwise the voltage measurement will disturb the original set up by an amount which is very large. Usually we like to keep the disturbance due to the measuring device below one per cent. To ensure this, a large resistance R is connected *in series* with the galvanometer.



This arrangement is schematically depicted in Fig.4.26. Note that the resistance of the voltmeter is now,

$RG + R \approx R$: large

The scale of the voltmeter is calibrated to read off the voltage value with ease. We define the ***voltage sensitivity*** *as the deflection per unit voltage*. From Eq. (4.38),

$$\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R} \qquad \text{---------} \quad (4.40)$$

Note that increasing the current sensitivity may not necessarily increase the voltage sensitivity. Let us take Eq. (4.39) which provides a measure of current sensitivity. If $N \rightarrow 2N$, i.e., we double the number of turns, then

$$\frac{\phi}{I} \rightarrow 2\frac{\phi}{I}$$

Thus, the current sensitivity doubles. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. In Eq. (4.40), $N \rightarrow 2N$, and $R \rightarrow 2R$, thus the voltage sensitivity,

$$\frac{\phi}{V} \rightarrow \frac{\phi}{V}$$

remains unchanged. So in general, the modification needed for conversion of a galvanometer to an ammeter will be different from what is needed for converting it into a voltmeter.

****

One marks questions & answers

1. Who concluded that moving charges or current produces magnetic field in the surrounding surface?

Ans : Christian Oersted.

2. Mention the expression for the magnetic force experienced by moving charge.

Ans : F = q ($\vec{v} \times \vec{B}$) or f = qvB sin θ $\hat{n}$

3. In a certain arrangement a proton does not get deflected while passing through a magnetic field region. Under what condition is it possible?

Ans : It is possible if the proton enters the magnetic field along the field direction.

4. What is the trajectory of charged particle moving perpendicular to the direction of uniform magnetic field?

Ans : Circle

5. What is significance of velocity selector?

Ans : Velocity selector is used in accelerator to select charged particle of particular velocity out of a beam containing charges moving with different speeds.

6. Which one of the following will describe the smallest circle when projected with the same velocity v perpendicular to the magnetic field B : (i) α-particle (ii) β- particle ?

Ans : α-particle.

7. What is cyclotron?

Ans : It is a device used to accelerate charged particles or ions.

8. Who invented Cyclotron?

Ans : E .O Lawrence and M. S. Livingston

9. What is resonance condition in cyclotron?

The condition in which the trajectory of the applied voltage is adjusted so that the polarity of the Dee's is reversed in the same time that it takes the ions to complete one half of the revolution.

10. What is solenoid?

Ans : Solenoids consist of a long insulated wire wound in the form of a helix where neighboring turns are closely spaced.

11. What is toroid?

Ans : This is a hallow circular ring on which a large number of turns of a wire are closely wound.

12. What is an ideal toroid?

Ans : The ideal toroid is one in which coils are circular.

13. Define magnetic dipole moment of a current loop.

Ans : The magnetic moment of a current loop is defined as the product of current *I* and the area vector of the loop.

14. What is the value of Bohr magneton?
$\mu_l = 9.27\times10^{-27} Am^2$

15. Define current sensitivity of the galvanometer?
Ans : It is defined as deflection per unit current of Moving coil galvanometer.

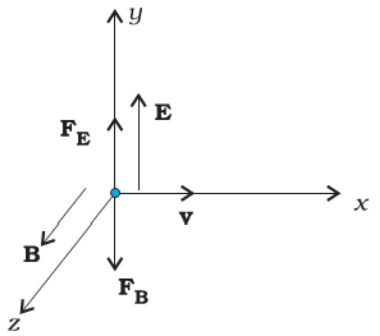
16. An ammeter and a milliammeter are converted from the galvanometer. Out of the two, Which current measuring instrument has higher resistance?
Ans : Higher is the range lower will be the value of shunt, so milliammeter will be having higher resistance.

Two marks questions & answers

1. Mention the expression for Lorentz's force.
In the presence of both electric field, E(r) and magnetic field, B(r) a point charge 'q' is moving with a velocity v. Then the total force on that charge is Lorentz force,

i.e $F = F_{electric} + F_{magnetic} = qE(r) + qvB(r)$
Note: 1. Magnetic force on the charge depends on 'q', 'v' and 'B'
2. $\vec{F}_{magnetic} = q\,(\vec{v} \times \vec{B})$ and is always perpendicular to the plane containing v and B. Also, $F = qvB\sin\theta\,\hat{n}$
3. If $\theta = 0$ or 180^o, then $F = 0$ and if $\theta = 90^o$, then $F = F_{maximum} = qvB$.

2. Show that crossed electric and magnetic fields serves as velocity selector.



Suppose we consider a charged particle 'q' moving with velocity 'v' in presence of both electric and magnetic fields, experiences a force given by $F = F_E + F_B = (qE + qvB)\,\hat{j}$ ($\therefore$ assuming $\theta = 90^o$).
If E is perpendicular to B as shown in the diagram, then $F=(qE - qvB)\,\hat{j}$

Suppose we adjust the values of E and B, such that $qE=qvB$, then $E=vB$
or $v = E/B$
This velocity is that chosen velocity under which the charged particle move undeflected through the fields. The ratio E/B is called velocity selector.

Note : E/B is independent of 'q' and 'm' of the particle under motion.

3. Mention the uses of cyclotron.

It is used to implant ions into solids and modify their properties.

It is used in hospitals to produce radioactive substance. This can be used in diagnosis and treatment.

4. State and explain Ampere's circuital law.

"The line integral of resultant magnetic field along a closed plane curve is equal to μ_0 time the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant" i.e., $\oint B\, dl = \mu_0 I$; where I is the total current through the surface.

5. Mention the expression for angular for deflection produced in Moving Coil Galvanometer?

Ans : $\phi = (NAB/k)\, I$

Where, N is number of turns B Magnetic field

A Area of the coil , K is torsional constant of the spring.

Three marks questions & answers

1. Derive the expression for magnetic force in a current carrying conductor. F=*i*(l × B)

Consider a rod of a uniform cross-sectional area A and length l. Let n be the number density of charge carriers(free electrons) in it.

Then the total number of mobile charge carriers in it is= nAl. Assume that these charge carriers are under motion with a drift velocity, vd.

In the F= (n A l) q vd × B; here q is the charge of each charge carrier.

presence of an external magnetic field B, the force on these charge carriers is

But current density j = n q vd

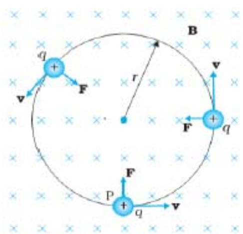
□ F= j A l × B

But, j A = I, the electric current in the conductor, then

∴ F = I *l* × B

i.e. F=(I *l* B sinθ)

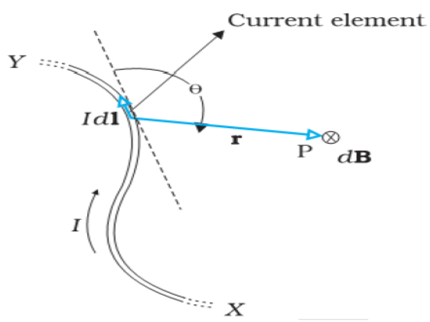
2. Obtain the expression for radius of circular path traversed by a charge in a magnetic field.



Assume that a charged particle 'q' is moving perpendicular to the uniform magnetic field B, i.e. θ=90º. The perpendicular force F = qv x B acts as centripetal force, thus producing a uniform circular motion for the particle in a plane perpendicular to the field.

i.e., $mv^2/r = qvB$ or $mv/r = qB$, here m is the mass the particle, r is the radius of the circular path traced.

∴ $r = mv/qB$



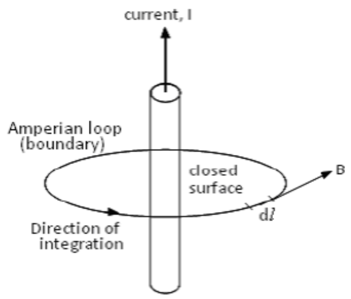
3. State and explain Biot-Savart's law.

Consider a conductor XY carrying current I. There we choose an infinitesimal element dl of the conductor. The magnetic field dB due to this element is to be determined at a point P which is at a distance ‘r’ from it. Let θ be the angle between dl and the position vector ‘r’. According to Biot-Savart’s law, the magnitude of the magnetic field dB at a point p is proportional to the current I, the element length |dl|, and inversely proportional to the square of the distance r and dB is directed perpendicular to the plane containing dl and r .

i.e $\vec{dB} = \frac{\mu_o}{4\pi}\frac{I\vec{dl}x\vec{r}}{r^3}$ or $\left|\vec{dB}\right| = \frac{\mu_o}{4\pi}\frac{I\left|\vec{dl}\right|\left|\vec{r}\right|\sin\theta}{r^3} = \frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2}$

here $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1} is a constant called permeability of vacuum.

4. Using ampere circuital law, obtain an expression for magnetic field due to infinitely long straight current carry wire.



Consider a infinitely long conductor carrying current. Let I_e be the current enclosed by the loop and L be the length of the loop for which B is tangential, then the amperes circuital law

$\oint B.dl = \mu_o I$; becomes $BL=\mu_o Ie$

If we assume a straight conductor and the boundary of the surface surrounding the conductor as a circle, t hen length of the boundary is the circumference, 2πr; where ‘r’ is the radius of the circle. Then $B.2\pi r = \mu_0 I$

$$\boxed{\therefore B = \frac{\mu_o I}{2\pi r}}$$

5. Show that current loop as a magnetic dipole?

When x>>R, (i.e at a long distance from O along the x-axis),

$$B = \frac{\mu_o NIR^2}{2x^3} = \frac{\mu_o NI\pi R^2}{2\pi x^3} = \frac{\mu_o NIA}{2\pi x^3} = \frac{\mu_o}{2\pi}\frac{m}{x^3} = \frac{\mu_o}{4\pi}\frac{2m}{x^3}$$

here m=NIA called Magnetic dipole moment of the loop and A=□R2, the circular area of the loop.

Similarly in electrostatics, for an electric dipole, electric field due to the dipole along its axis, ;

$$E = \frac{1}{4\pi\varepsilon_o}\frac{2p_e}{x^3}$$

here p_e is the electric dipole moment.

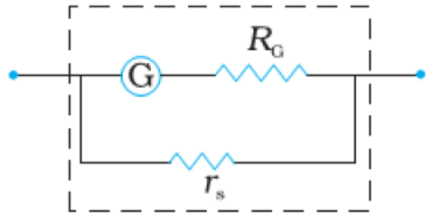
This shows the current carrying circular loop is equivalent to a magnetic dipole.

6. Explain how do you convert moving coile galvanometer into an ammeter.

A small resistance rs , called shunt resistance is connected in parallel with the galvanometer coil; so that most of the current passes through the shunt.

The resistance of this arrangement is

$$\frac{1}{R_G}+\frac{1}{r_s} = \Rightarrow \frac{R_G r_s}{R_G + r_s} \approx \frac{R_G r_s}{R_G} = r_s$$



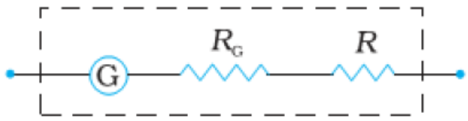
Ammeter

If $R_G >> r_s$, then the resistance of the arrangement,
This arrangement is calibrated to standard values of currents and hence we define, the current sensitivity of the galvanometer as the deflection per unit current, i.e.

$$\frac{\phi}{I} = \frac{NAB}{k}$$

7. Explain how do you convert moving coile galvanometer into voltmeter.

For this the galvanometer must be connected in parallel with a high resistance R. in series



Voltmeter

The resistance of the voltmeter is now, $R_G + R$
Since $R >> R_G$, $R_G + R \approx R$
The scale of the voltmeter is calibrated to read off the p.d across a circuit.

i.e $\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$ [because, $\phi = \left(\frac{NAB}{k}\right) I \Rightarrow \frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V} \Rightarrow \frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$

Five marks questions & answers

1. **Describe the construction and working theory of cyclotron.**

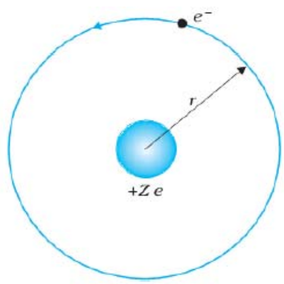
The cyclotron is a machine to accelerate charged particles or ions to high energies.
Refer Previous Answers.

2. **Derive an expression for magnetic field on the axis of a circular current loop.**
Refer Previous Answers.

3. **Obtain the expression for the force per unit length of two parallel conductors carrying current and hence define one ampere.**
Refer Previous Answers.

4. Derive an expression to magnetic dipole moment of a revolving electron in a hydrogen atom and hence deduce Bohr magneton.

An electron revolving around the nucleus possesses a dipole moment and the system acts like a tiny magnet. According to Bohr's model, the magnetic moment of an electron is
$\mu l = I\pi r^2 = (evR)/2$ here 'e' is an electron charge, 'v' is its speed in the orbit and 'r' is the corresponding radius of the orbit.



The direction of this magnetic moment is into the plane of the paper.
We know angular momentum,

$$\mu_l = \frac{evr}{2}$$

Dividing the above expression on RHS by electron mass me, we get

$$\mu_l = \frac{e}{2m_e}(m_e vr)$$

But, $m_e vr = l$, the angular momentum,

$$\mu_l = \frac{e}{2m_e} l$$

Vectorially,

$$\vec{\mu_l} = -\frac{e}{2m_e}\vec{l}$$

here the negative sign indicates that $\vec{\mu_l}$ and $\vec{l}$ are in opposite directions.

Further, $\frac{\mu_l}{l} = \frac{e}{2m_e}$ and $l = n\frac{h}{2\pi}$;

here n=1,2,3… called principal quantum number and h is Planck's constant. Since l is minimum when n=1, we write,

$$\frac{(\mu_l)_{min}}{h/2\pi} = \frac{e}{2m_e} \text{ or } (\mu_l)_{min} = \frac{eh}{4\pi m_e}$$

And on substituting all the values, we get $\mu_{min} = 9.27 \times 10^{-24} Am^2$ and is called Bohr magneton.

MOST LIKELY QUESTIONS :

3M - 1Q :

1. Derive an expression to magnetic dipole moment of a revolving electron in a hydrogen atom and hence deduce Bohr magneton.

5M - 1Q (LA)

1. Describe the construction and working theory of cyclotron.
2. Derive an expression for magnetic field on the axis of a circular current loop.
3. Obtain the expression for the force per unit length of two parallel conductors carrying current and hence define one ampere.
4. Write a note on Conversion of galvanometer to ammeter and voltmeter ?

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Chapter 5:
MAGNETISM AND MATTER
7M
2M - 1Q ; 5M - 1Q (LA) or 1M-1Q; 3M-2Q

5.1 Bar magnet: Properties of magnetic field lines -

(1) Common information about Magnetism :

Some of the commonly known ideas regarding magnetism are:

(i) The earth behaves as a magnet with the magnetic field pointing approximately from the geographic south to the geographic north.

(ii) When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called the north pole and the tip which points to the geographic south is called the south pole of the magnet.

(iii) There is a repulsive force when north poles (or south poles) of two magnets are brought close together. Conversely, there is an attractive force between the north pole of one magnet and the south pole of the other.

(iv) We cannot isolate the north, or south pole of a magnet. If a bar magnet is broken into two halves, we get two similar bar magnets with somewhat weaker properties. Unlike electric charges, isolated magnetic north and south poles known as magnetic monopoles do not exist.

(v) It is possible to make magnets out of iron and its alloys.

(2) Magnetic field lines & their Properties ?

The magnetic field lines are a visual and intuitive realisation of the magnetic field. Their properties are:

(i) The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.

(ii) The tangent to the field line at a given point represents the direction of the net magnetic field B at that point.

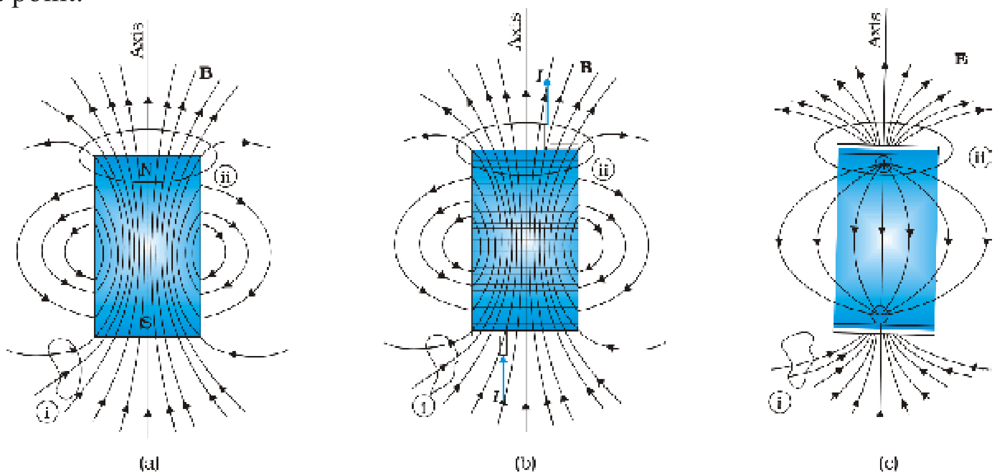


Figure 5.1 : The field lines of (a) a bar magnet, (b) a current-carrying finite solenoid and (c) electric dipole. At large distances, the field lines are very similar. The curves labelled (i) and (ii) are closed Gaussian surfaces.

(iii) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field B. In Fig. 5.1. (a), B is larger around region (ii) than in region (i).

(iv) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

5.2 Bar magnet as an equivalent solenoid with derivation -

(3) Derive the expression for axial magnetic field of a bar magnet as an equivalent solenoid ?

Bar magnet can be treated equivalent to solenoid. We know that the magnetic dipole moment m associated with a current loop is defined to be m = NIA where N is the number of turns in the loop, I the current and A the area vector.
The resemblance of magnetic field lines for a bar magnet and a solenoid suggest that a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid. Cutting a bar magnet in half is like cutting a solenoid. We get two smaller solenoids with weaker magnetic properties. The field lines remain continuous, emerging from one face of the solenoid and entering into the other face. One can test this analogy by moving a small compass needle in the neighbourhood of a bar magnet and a current-carrying finite solenoid and noting that the deflections of the needle are similar in both cases.

To make this analogy more firm we calculate the axial field of a finite solenoid depicted in Fig. 5.2 (a). We shall demonstrate that at large distances this axial field resembles that of a bar magnet.

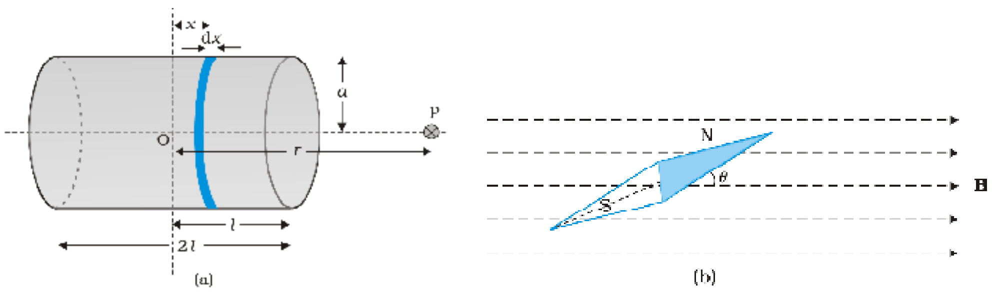


Fig. 5.2 : Calculation of (a) The axial field of a finite solenoid in order to demonstrate its similarity to that of a bar magnet. (b) A magnetic needle in a uniform magnetic field B. The arrangement may be used to determine either B or the magnetic moment m of the needle.

Let the solenoid of Fig. 5.2 (a) consists of n turns per unit length. Let its length be 2l and radius a. We can evaluate the axial field at a point P, at a distance r from the centre O of the solenoid. To do this, consider a circular element of thickness dx of the solenoid at a distance x from its centre. It consists of n dx turns. Let I be the current in the solenoid. We know that in case of the magnetic field on the axis of a circular current loop, the magnitude of the field at point P due to the circular element is

$$dB = \frac{\mu_0 n\, dx\, I\, a^2}{2[(r-x)^2 + a^2]^{3/2}}$$

The magnitude of the total field is obtained by summing over all the elements $x = l$ to $x = +l$. Thus,

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^{l} \frac{dx}{[(r-x)^2 + a^2]^{3/2}}$$

This integration can be done by trigonometric substitutions. This exercise, however, is not necessary for our purpose. Note that the range of x is from l to + l . Consider the far axial field of the solenoid, i.e., r >> a and r >> l . Then the denominator is approximated by

$$[(r-x)^2+a^2]^{3/2} \approx r^3$$ and

$$B = \frac{\mu_0 n I a^2}{2r^3}\int_{-l}^{l} dx \quad = \frac{\mu_0 n I}{2}\frac{2l a^2}{r^3}$$

Note that the magnitude of the magnetic moment of the solenoid is,

$m = n (2 l) I (\pi a^2)$

where a = area. Thus,

$$B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$$

This is also the far axial magnetic field of a bar magnet which one may obtain experimentally. Thus, a bar magnet and a solenoid produce similar magnetic fields. The magnetic moment of a bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

5.3 Dipole in a uniform magnetic field:

(5) Mention of expression for time period of oscillation of small compass needle in a uniform magnetic field ?

The pattern of iron filings, i.e., the magnetic field lines gives us an approximate idea of the magnetic field B. We may at times be required to determine the magnitude of B accurately. This is done by placing a small compass needle of known magnetic moment m and moment of inertia I and allowing it to oscillate in the magnetic field. This arrangement is shown in Fig. 5.2(b).

The torque on the needle is $\tau = m \times B$

In magnitude $\tau = mB \sin\theta$

Here τ is restoring torque and θ is the angle between m and B. Therefore, in equilibrium

$$I\frac{d^2\theta}{dt^2} = -mB\sin\theta$$

Negative sign with mB sinθ implies that restoring torque is in opposition to deflecting torque. For small values of θ in radians, we approximate $\sin\theta \approx \theta$ and get

$$I\frac{d^2\theta}{dt^2} \approx mB\,\theta \qquad \text{or,} \quad \frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$$

This represents a simple harmonic motion. The square of the angular frequency is

$\omega^2 = mB/I$ and the time period is,

$$T = 2\pi\sqrt{\frac{I}{mB}} \qquad \text{or} \qquad B = \frac{4\pi^2 I}{mT^2}$$

An expression for magnetic potential energy can also be obtained on lines similar to electrostatic potential energy.

The magnetic potential energy U_m is given by

$$U_m = \int \tau(\theta)d\theta$$

$$= \int mB\sin\theta = -mB\cos\theta$$

$$= -m \cdot B$$

Example 5.1 : In Fig. 5.2(b), the magnetic needle has magnetic moment 6.7×10^{-6} Am^2 and moment of inertia $I = 7.5 \times 10^{-6}$ kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field ?

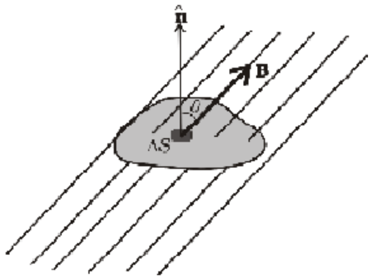
Solution : The time period of oscillation is,

$$T = \frac{6.70}{10} = 0.67s$$

$$B = \frac{4\pi^2 I}{mT^2} = \frac{4 \times (3.14)^2 \times 7.5 \times 10^{-6}}{6.7 \times 10 \quad \times (0.67)} = 0.01 \text{ T}$$

5.4 Gauss law in magnetism: Statement and explanation.

In magnetism, we can visually demonstrate that the number of magnetic field lines leaving the surface is balanced by the number of lines entering it. The net magnetic flux is zero for both the surfaces. This is true for any closed surface.



Consider a small vector area element ΔS of a closed surface S as in Fig. 5.3. The magnetic flux through ΔS is defined as $\Delta\phi_B = B.\Delta S$, where B is the field at ΔS. We divide S into many small area elements and calculate the individual flux through each. Then, the net flux ϕ_B is,

$$\phi_B = \sum_{\text{'all'}} \Delta\phi_B = \sum_{\text{'all'}} B \cdot \Delta S = 0$$

Comparing this with the Gauss law for electrostatics, in that case is given by

$$\sum E \cdot \Delta S = \frac{q}{\varepsilon_0}$$

where q is the electric charge enclosed by the surface.

The difference between the Gauss electrostatics is a reflection of the fact that isolated magnetic poles (also called monopoles) are not known to exist. There are no sources or sinks of B; the simplest magnetic element is a dipole or a current loop. All magnetic phenomena can be explained in terms of an arrangement of dipoles and/or current loops.

Thus, Gauss law in magnetism states that the net magnetic flux through any closed surface is zero.

Comparison between Electrostatic and Magnetic quantities :

Physical Quantity	Electrostatics	Magnetism
Free space constant	$1/\varepsilon_0$	μ_0
Dipole moment	$\vec{P}$	$\vec{M}$
Axial field	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2\vec{P}}{r^3}$	$\frac{\mu_0}{4\pi} \cdot \frac{2\vec{M}}{r^3}$
Equatorial field	$-\frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{P}}{r^3}$	$-\frac{\mu_0}{4\pi} \cdot \frac{\vec{M}}{r^3}$
Torque in external field	$\vec{P} \times \vec{E}$	$\vec{M} \times \vec{B}$
P.E. in external field	$-\vec{P}\,\vec{E}$	$-\vec{M}\,\vec{B}$

5.5 Earth's magnetic field and its elements: Declination, Dip and Earth's horizontal component B_H and their variation -

(1) Write a note on earth's magnetic field ?

The strength of the earth magnetic field value being of the order of 10 T.

The magnetic field is now thought to arise due to electrical currents produced by convective motion of metallic fluids (consisting mostly of molten iron and nickel) in the outer core of the earth. This is known as the dynamo effect.
The magnetic field lines of the earth resemble that of a (hypothetical) magnetic dipole located at the centre of the earth. The axis of the dipole does not coincide with the axis of rotation of the earth but is presently titled by approximately 11º.3" with respect to the later. In this way of looking at it, the magnetic poles are located where the magnetic field lines due to the dipole enter or leave the earth. The location of the north magnetic pole is at a latitude of 79º.74” N and a longitude of 71.8” W, a place somewhere in north Canada. The magnetic south pole is at 79º.74” S, 108º.22” E in the Antarctica.
The pole near the geographic north pole of the earth is called the north magnetic pole. Likewise, the pole near the geographic south pole is called the south magnetic pole. There is some confusion in the nomenclature of the poles. If one looks at the magnetic field lines of the earth (Fig. 5.8), one sees that unlike in the case of a bar magnet, the field lines go into the earth at the north magnetic pole (N_m) and come out from the south magnetic pole (S_m). The convention arose because the magnetic north was the direction to which the north pole of a magnetic needle pointed; the north pole of a magnet was so named as it was the north seeking pole. Thus, in reality, the north magnetic pole behaves like the south pole of a bar magnet inside the earth and vice versa.

(2) Write a note on Declination, Dip and Earth’s horizontal component B_H and their variation.

In case of earth, the vertical plane containing the longitude circle and the axis of rotation of the earth is called the geographic meridian. In a similar way, one can define magnetic meridian of a place as the vertical plane which passes through the imaginary line joining the magnetic north and the south poles. This plane would intersect the surface of the earth in a longitude like circle. A magnetic needle, which is free to swing horizontally, would then lie in the magnetic meridian and the north pole of the needle would point towards the magnetic north pole. Since the line joining the magnetic poles is titled with respect to the geographic axis of the earth, the magnetic meridian at a point makes angle with the geographic meridian.

(i) The angle between the true geographic north and the north pole shown by a compass needle is called the **magnetic declination** (D) or simply **declination** (Fig. 5.9).
The declination is greater at higher latitudes and smaller near the equator. The declination in India is small, it being 0”41' E at Delhi and 0”58' W at Mumbai. Thus, at both these places a magnetic needle shows the true north quite accurately.

(ii) The angle that the total magnetic field (B_E) of the earth makes with the surface of the earth and horizontal component of earth's magnetic field (H_E) is called **dip or inclination (I)** at that place.
In most of the northern hemisphere, the north pole of the dip needle tilts downwards. Likewise in most of the southern hemisphere, the south pole of the dip needle tilts downwards.

(iii) To describe the magnetic field of the earth at a point on its surface, we need to specify three quantities, viz., the declination (D), the angle of dip or the inclination (I) and the horizontal component of the earth HE. These are known as the element of the earth magnetic field.

Representing the verticle component by Z_E, we have

$Z_E = B_E \sin I$

$H_E = B_E \cos I$

which gives,

$$\tan I = \frac{Z_E}{H_E}$$

5.6 Definitions of magnetisation (M), magnetic intensity (H), magnetic susceptibility (χ) and permeability (μ, μ₀ and μᵣ).

We know that a circulating electron in an atom has a magnetic moment. In a bulk material, these moments add up vectorially and they can give a net magnetic moment which is non-zero. We define **magnetisation M** of a sample to be equal to its net magnetic moment per unit volume:

$M = m_{net}/V$

M is a vector with dimensions L A and is measured in a units of Am^{-1} .

Consider a long solenoid of n turns per unit length and carrying a current I. The magnetic field in the interior of the solenoid was shown to be given by

$B_0 = \mu_0 nI$

If the interior of the solenoid is filled with a material with non-zero magnetisation, the field inside the solenoid will be greater than B_0. The net B field in the interior of the solenoid may be expressed as

$B = B_0 + B_m$

where B_m is the field contributed by the material core. It turns out that this additional field B_m is proportional to the **magnetisation M** of the material and is expressed as

$B_m = \mu_0 M$

where μ_0 is the same constant (permittivity of vacuum) that appears in Biot-Savart law.

It is convenient to introduce another vector field H, called the magnetic intensity, which is defined by

$$H = \frac{B}{\mu_0} - M$$

where H has the same dimensions as M and is measured in units of Am^{-1}. Thus, the total magnetic field B is written as

$\mathbf{B = \mu_0 (H + M)}$

If we partition the contribution to the total magnetic field inside the sample into two parts: one, due to external factors such as the current in the solenoid. This is represented by H. The other is due to the specific nature of the magnetic material, namely M. The latter quantity can be influenced by external factors. This influence is mathematically expressed as

$\mathbf{M = \chi H}$

where χ, a dimensionless quantity, is appropriately called the magnetic susceptibility. It is a measure of how a magnetic material responds to an external field. Table 5.2 lists χ for some elements. It is small and positive for materials, which are called paramagnetic. It is small and negative for materials, which are termed diamagnetic. In the latter case M and H are opposite in direction. From Eqs. (5.16) and (5.17) we obtain,

$\mathbf{B = \mu_0 (1 + \chi H) \quad = \mu_0 \mu_r H \quad = \mu H}$

where $\mu_r = 1 + \chi$, is a dimensionless quantity called the relative magnetic permeability of the substance. It is the analog of the dielectric constant in electrostatics. The magnetic permeability of the substance is m and it has the same dimensions and units as μ_0;

$\mathbf{\mu = \mu_0 \mu_r = \mu_0 (1 + \chi)}$

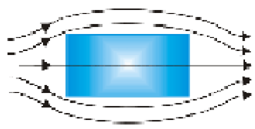
The three quantities χ, μ_r and μ are interrelated and only one of them is independent. Given one, the other two may be easily determined.

5.7 Magnetic properties of materials: Paramagnetic, diamagnetic and ferromagnetic substances, examples and properties -

Materials are classified as diamagnetic, paramagnetic or ferromagnetic. In terms of the susceptibility χ, a material is diamagnetic if χ is negative, para- if χ is positive and small, and ferro- if χ is large and positive.

(1) Diamagnetism :
Diamagnetic substances are those which have tendency to move from stronger to the weaker part of the external magnetic field. In other words, unlike the way a magnet attracts metals like iron, it would repel a diamagnetic substance.

If a bar of diamagnetic material placed in an external magnetic field, the field lines are repelled or expelled and the field inside the material is reduced as shown in fig. (a).


(a)

Origin of Diamagnetism : Electrons in an atom orbiting around nucleus possess orbital angular momentum. These orbiting electrons are equivalent to current-carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in the opposite direction speed up. Thus, the substance develops a net magnetic moment in direction opposite to that of the applied field and hence repulsion of magnetic field lines takes place.

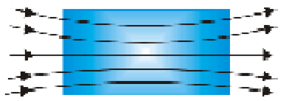
Example : Some diamagnetic materials are bismuth, copper, lead, silicon, nitrogen (at STP), water and sodium chloride. Diamagnetism is present in all the substances. However, the effect is so weak in most cases that it gets shifted by other effects like paramagnetism, ferromagnetism, etc. The most exotic diamagnetic materials are superconductors. These are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism. Here the field lines are completely expelled! $\chi = (\mu_r - 1) = 0$. A superconductor repels a magnet and diamagnetism in superconductors is called the Meissner effect, after the name of its discoverer. Superconducting magnets can be gainfully exploited in variety of situations, for example, for running magnetically levitated superfast trains.

Properties :
(1) Diamagnetic substance is feebly repelled by a strong magnet.
(2) When a diamagnetic substance is placed in magnetic field, the magnetic lines of force prefer to pass through the surrounding air rather than through the substance. This is because the induced magnetic field in the diamagnetic substance opposes the external field.
(3) When a rod of diamagnetic substance , is suspended in a uniform magnetic field the rod comes to rest with its longest axis at right angles to the direction of the field.
(4) When placed in a non-uniform magnetic field, a diamagnetic substance moves from stronger to weaker parts of the field.

(5) The relative permeability (μ_r) of a diamagnetic substance is always less than 1.
(6) The magnetic susceptibility (χ) of a diamagnetic substance has a small negative value.
(7) The magnetic susceptibility (χ) of a diamagnetic substance does not change with temperature.

(2) Paramagnetism :
Paramagnetic substances are those which get weakly magnetised when placed in an external magnetic field. They have tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get weakly attracted to a magnet.

Origin of Paramagnetism : The individual atoms (or ions or molecules) of a paramagnetic material possess a permanent magnetic dipole moment of their own. On account of the ceaseless random thermal motion of the atoms, no net magnetisation is seen. In the presence of an external field B_0, which is strong enough, and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as B_0. Figure (b) shows a bar of paramagnetic material placed in an external field. The field lines gets concentrated inside the material, and the field inside is enhanced. In most cases, this enhancement is slight, being one part in 10^5. When placed in a non-uniform magnetic field, the bar will tend to move from weak field to strong.


(b)

Example : Some paramagnetic materials are aluminium, sodium, calcium, oxygen (at STP) and copper chloride.

Experimentally, one finds that the magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T,

$$M = C\frac{B_0}{T}$$

or equivalently, using Eqs. (5.12) and (5.17)

$$\chi = C\frac{\mu_0}{T}$$

This is known as Curie law, after its discoverer Pieree Curie (1859- 1906). The constant C is called Curie constant. Thus, for a paramagnetic material both χ and μ_r depend not only on the material, but also (in a simple fashion) on the sample temperature. As the field is increased or the temperature is lowered, the magnetisation increases until it reaches the saturation value M_S, at which point all the dipoles are perfectly aligned with the field. Beyond this, Curie longer valid.

Properties of paramagnetic Materials :
(1) A paramagnetic substance is feebly attracted by a strong magnet.
(2) When a paramagnetic substance is placed in a magnetic field, the magnetic lines of force prefer to pass through the substance rather than through air. Therefore, the resultant field B inside the substance is more than the external field B_0.
(3) When a rod of paramagnetic substance is suspended freely in a uniform magnetic field, the rod comes to rest with its longest axis along the direction of external magnetic field.
(4) When placed in a non-uniform magnetic field, a paramagnetic substance moves from weaker to stronger parts of the field.
(5) The relative permeability (μ_r) of paramagnetic substance has small positive value.

(6) The magnetic susceptibility of paramagnetic substance varies inversely as the absolute temperature.

(3) Ferromagnetism :

Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field. They have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get strongly attracted to a magnet.

Origin of Ferromagnetism : The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. The explanation of this cooperative effect requires quantum mechanics and is beyond the scope of this textbook. Each domain has a net magnetisation. Typical domain size is 1mm and the domain contains about 10^{11} atoms. In the first instant, the magnetisation varies randomly from domain to domain and there is no bulk magnetisation. This is shown in Fig. 5.13(a). When we apply an external magnetic field B_0, the domains orient themselves in the direction of B0 and simultaneously the domain oriented in the direction of B_0 grow in size. This existence of domains and their motion in B_0 are not speculations. One may observe this under a microscope after sprinkling a liquid suspension of powdered ferromagnetic substance of samples. This motion of suspension can be observed. Figure 5.12(b) shows the situation when the domains have aligned and amalgamated to form a single sample.
Thus, in a ferromagnetic material the field lines are highly concentrated. In non-uniform magnetic field, the sample tends to move towards the region of high field. We may wonder as to what happens when the external field is removed. In some ferromagnetic materials the magnetisation persists. Such materials are called hard magnetic materials or hard ferromagnets.

Example : Alnico, an alloy of iron, aluminium, nickel, cobalt and copper, is one such material. The naturally occurring lodestone is another. Such materials form permanent magnets to be used among other things as a compass needle. On the other hand, there is a class of ferromagnetic materials in which the magnetisation disappears on removal of the external field. Soft iron is one such material. Appropriately enough, such materials are called soft ferromagnetic materials.

There are a number of elements, which are ferromagnetic: iron, cobalt, nickel, gadolinium, etc. The relative magnetic permeability is >1000!

The ferromagnetic property depends on temperature. At high enough temperature, a ferromagnet becomes a paramagnet. The domain structure disintegrates with temperature. This disappearance of magnetisation with temperature is gradual. It is a phase transition reminding us of the melting of a solid crystal. The temperature of transition from ferromagnetic to paramagnetism is called the Curie temperature T_c . Table 5.4 lists the Curie temperature of certain ferromagnets. The susceptibility above the Curie temperature, i.e., in the paramagnetic phase is described by,

$$\chi = \frac{C}{T - T_c} \quad (T > T_c)$$

Properties of Ferromagnetic Materials :

(1) A ferromagnetic substance is strongly attracted by a magnet.

(2) When a ferromagnetic substance is placed in a magnetic field, the magnetic field lines tends to crowd in to the substance.
(3) When a rod of ferromagnetic substance is suspended in a uniform magnetic field, it quickly aligns itself in the direction of the field.
(4) When placed in a non-uniform magnetic field, a ferromagnetic substance moves from weaker to stronger parts of the magnetic field.
(5) The relative permeability μ_r of ferromagnetic substance is very large. For example, relative permeability of soft iron is about 8,000.
(6) The magnetic susceptibility of ferromagnetic substance is positive having a very high value.

5.8 Curie's law and Curie temperature -

According to Curie law, the magnetisation (M) of a paramagnetic material is inversely proportional to the absolute temperature T, and directly proportional to the external magnetic field.

$$M = C\frac{B_0}{T}$$

or equivalently, using Eqs. (5.12) and (5.17)

$$\chi = C\frac{\mu_0}{T}$$

This is known as Curie law and C is called Curie constant.

5.9 Hysteresis, Hysteresis loop, definitions of retentively and coercively -

The relation between B and H in ferromagnetic materials is complex. It is often not linear and it depends on the magnetic history of the sample. Figure 5.14 depicts the behaviour of the material as we take it through one cycle of magnetisation. Let the material be unmagnetised initially. We place it in a solenoid and increase the current through the solenoid. The magnetic field B in the material rises and saturates as depicted in the curve Oa. This behaviour represents the alignment and merger of domains until no further enhancement is possible. It is pointless to increase the current (and hence the magnetic intensity H) beyond this. Next, we decrease H and reduce it to zero. At H = 0, B ≠ 0. This is represented by the curve ab. The value of B at H = 0 is called **retentivity** or remanence. In Fig., $B_R \sim 1.2$ T, where the subscript R denotes retentivity. The domains are not completely randomised even though the external driving field has been removed. Next, the current in the solenoid is reversed and slowly increased. Certain domains are flipped until the net field inside stands nullified. This is represented by the curve bc. The value of H at c is called **coercivity**. In Fig. $H_c \sim -80$ A/m. As the reversed current is increased in magnitude, we once again obtain saturation. The curve cd depicts this. The saturated magnetic field $B_s \sim 1.5$ T. Next, the current is reduced (curve de) and reversed (curve ea). The cycle repeats itself. Note that the curve Oa does not retrace itself as H is reduced. For a given value of H, B is not unique but depends on previous history of the sample. This phenomenon is called **hysterisis**. The word hysterisis means lagging behind.

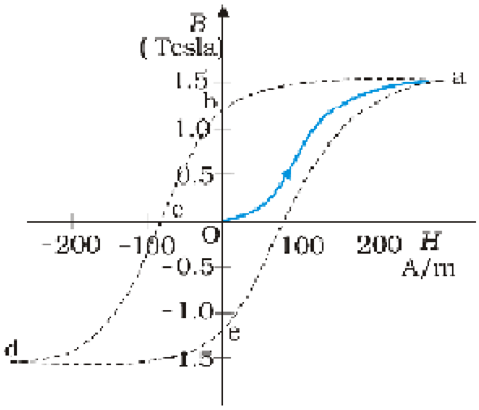


Fig. : The magnetic hysteresis loop is the B-H curve for ferromagnetic materials.

5.10 Permanent magnets and electromagnets.

Substances which at room temperature retain their ferromagnetic property for a long period of time are called **permanent magnets**.

Permanent magnets can be made in a variety of ways.

(1) One can hold an iron rod in the north-south direction and hammer it repeatedly.

(2) One can also hold a steel rod and stroke it with one end of a bar magnet a large number of times, always in the same sense to make a permanent magnet.

(3) An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and pass a current. The magnetic field of the solenoid magnetises the rod.

(4) The hysteresis curve allows us to select suitable materials for permanent magnets. The material should have high retentivity so that the magnet is strong and high coercivity so that the magnetisation is not erased by stray magnetic fields, temperature fluctuations or minor mechanical damage. Further, the material should have a high permeability. Steel is one-favoured choice. It has a slightly smaller retentivity than soft iron but this is outweighed by the much smaller coercivity of soft iron. Other suitable materials for permanent magnets are alnico, cobalt steel and ticonal.

(5) Core of electromagnets are made of ferromagnetic materials which have high permeability and low retentivity. Soft iron is a suitable material for electromagnets. On placing a soft iron rod in a solenoid and passing a current, we increase the magnetism of the solenoid by a thousand fold. When we switch off the solenoid current, the magnetism is effectively switched off since the soft iron core has a low retentivity.

(6) Electromagnets are used in electric bells, loudspeakers and telephone diaphragms. Giant electromagnets are used in cranes to lift machinery, and bulk quantities of iron and steel.

Question Bank

5.1. What are the properties of magnetic field lines?
i. Magnetic field lines are continuous closed loops.
ii. Tangent to the field line represents the direction of net field B .
iii. The larger the number of field lines crossing unit area normally, the stronger is the magnitude of the magnetic field B.
iv. Magnetic field lines do not intersect.

5.2 Is a bar magnet an equivalent current carrying solenoid?
Yes. Each turn of solenoid behaves as a small magnetic dipole. Therefore solenoid can be considered as arrangement of small magnetic dipoles placed in line with each other. The magnetic field produced by solenoid is identical to that produced by the magnet.

5.3. What is the force acting on a bar magnet placed in a uniform magnetic field?
Zero.

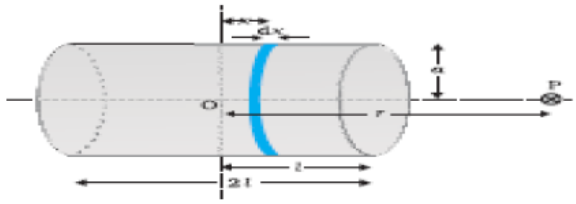
5.4. What is the torque when a bar magnet of dipole moment m is placed in a uniform magnetic field? When is torque maximum and minimum.
$\tau = m \times B = m. B. \sin\theta$
Torque is maximum if $\theta = 90$ i.e. torque is maximum if the magnetic dipole is at right angles to applied magnetic field.
Torque is minimum if $\theta = 0$ i.e. torque is minimum if the magnetic dipole is along the direction of applied magnetic field.

5.5. Give an expression for time period of oscillation when a magnetic needle placed in uniform magnetic field.
A small compass magnetic needle of magnetic moment m and moment of inertia I is made to oscillate in the magnetic field, B .
Time period of oscillation is given by T = 2π Im B .

5.6. State and explain Gauss law in magnetism.
The net magnetic flux through any closed surface is zero.
Consider a small vector area element Δs of closed surface S. According to Gauss law in magnetism, net flux though closed surface, ∅B= B .Δs all areaelement=0.
The implication of Gauss law is that isolated magnetic poles do not exist.

5.7. What is the cause of earth's magnetism?
Earths magnetism is due to electrical currents produced by the convective motion of mainly molten iron and nickel in the outer core of the earth.

5.8. Show that a current carrying solenoid behaves as a magnet.
Let 'r' be radius of solenoid of length 2l.



To calculate magnetic field at a point on axis of solenoid, consider a small element of thickness ‘dx’ of solenoid at a distance ‘x’ from ‘o’.
Number of turns in this element = n.dx
If current ‘i’ flows through element ‘ndx’ the magnitude of magnetic field at P due to this element is

$$dB = \frac{\mu_0}{4\pi} \frac{2\pi (n\,dx) i\, a^2}{[(r-x)^2 + a^2]^{3/2}}$$

If point ‘p’ is at large distance from ‘o’ i.e. r >> l and r>> a then $[(r-x)^2 + a^2] = r^2$

$$dB = \frac{\mu_0}{4\pi} \frac{2\pi (n\,dx) i\, a^2}{[\,r^2]^{3/2}} = dB = \frac{\mu_0}{4\pi} \frac{2\pi (n\,dx) i\, a^2}{[r]^3}$$

The total magnetic field at ‘p’ due to the current ‘i’ in solenoid is

$$B = \int_{-l}^{l} dB = \int_{-l}^{l} \frac{\mu_0}{4\pi} \frac{2\pi\, ni\, a^2 dx}{[r]^3} = \frac{\mu_0}{4\pi} \frac{2\pi\, ni\, a^2}{[r]^3} [x]_{-l}^{l}$$

$$B = \frac{\mu_0}{4\pi} \frac{2\pi\, ni\, a^2}{[r]^3} [l + l] = \frac{\mu_0}{4\pi} \frac{2\, i\, n\, \pi\, a^2\, .2l}{[r]^3}$$

$$B = \frac{\mu_0}{4\pi} \frac{2\, i\, n\;\; A\, .2l}{[r]^3} = \frac{\mu_0}{4\pi} \frac{2\;\, (n\, .2l) iA}{[r]^3}$$

$$B = \frac{\mu_0}{4\pi} \frac{2\, N\, i\;\, A}{[r]^3}$$ (N = No of turns of solenoid = n x 2l)

$$B = \frac{\mu_0}{4\pi} \frac{2\;\, m}{r^3}$$

This equation gives the magnitude of magnetic field at a point on axis of a solenoid.
This equation is similar to the expression for magnetic field on axis of a short bar magnet. Hence a solenoid carrying current behaves as a bar magnet.

5.9. What is magnetic declination?
Declination at the place is the angle between true geographic north direction and the north shown by the magnetic compass needle.

5.10. What is magnetic dip or inclination?
Magnetic dip at a place is the angle between the earth’s total magnetic field at a place and horizontal drawn in magnetic meridian?

5.11. What are elements of earth’s magnetic field? Mention them.
They are: (1) Magnetic declination (θ) at that place

(2) Magnetic inclination (I) dip at that place
(3) Horigontal comp of earths magnetic field (B_H) at that place.

5.12. What is the relation between horizontal component of earth's field H_E, vertical component of earth's field Z_E and inclination, I?

$Z_E = B_E \sin I$ or $H_E = B_E \cos I$ or $\tan I = Z_E/H_E$

5.13. Define magnetisation of a sample.
Magnetisation of a sample is its net magnetic dipole moment per unit volume.

$$\vec{M} = \frac{\vec{m}_{net}}{V}.$$

5.14. What is the unit of magnetisation?
Am^{-1}.

5.15. Define magnetic intensity.
The degree to which a magnetic field can magnetise a material is represented in terms of magnetic intensity.

Magnetic intensity of a material is the ratio of external magnetic field to the permeability of free space.

Magnetic intensity of a material, $H = B_0/\mu_0$, where B_0 - external magnetic field, μ_0 - permeability of free space.

5.16. What is the unit of magnetic intensity, H ?
Am^{-1}.

5.17. What is the relation between magnetic field $\vec{B}$, magnetic intensity $\vec{H}$ and Magnatisation $\vec{M}$ of a specimen?

$B = B_0 + B_m = \mu0 (H + M)$, where $B_0 = \mu_0 H$ due to current in the solenoid, $B_m = \mu_0 M$ is due to nature of the material inside the solenoid.

5.18. What is magnetic susceptibility?
The ratio of magnetisation developed in the material to the magnetic intensity is called magnetic susceptibility.

$$\chi = \frac{\vec{M}}{\vec{H}}.$$

5.19. Write the relation between magnetic intensity, magnetic field and susceptibility.

$$\vec{B} = \mu_0(1 + \chi)\vec{H} = \mu H.$$

5.20. What is the relation between magnetic relative permeability and susceptibility?

$\mu_r = 1 + \chi.$

5.21. What is the relation between magnetic relative permeability and permeability of the medium?

$\mu=\mu_0\mu_r$. μ is the permeability of medium, μ_0 is permeability of free space, and μ_r is relative permeability of the medium.

5.22. Define permeability.

Permeability of a substance is the ability of the substance to allow magnetic field lines to pass through it.

5.23. Distinguish between dia, para and ferro magnetism with examples.

	Diamagnetic material	Paramagnetic material	Ferromagnetic material
1	Weakly repelled by magnetic	Weakly attracted by magnetic	Strongly attracted by magnetic
2	When placed in a magnetic field it is weakly magnetized in a direction opposite to that of applied field.	When placed in a magnetic field it is weakly magnetized in the direction of applied field.	When placed in a magnetic field it is strongly magnetized in the direction of applied field.
3	μ_r is slightly less than 1	μ_r is slightly more than 1	
4	When placed in a magnetic field flux density (B) inside the material is less than in air.	When placed in a external magnetic field, flux density (B) than in air.	When placed in a magnetic field the flux density (B) inside the material is much large than in air.
5	X_m (susceptibility) doesnot change with temperature	X_m varies inversely as the temperature of substance	X_m decreases with rise temperature
6	Intensity of magnetization a small and negative value	I has small and positive value	I has large positive value
7	X_m has small negative value	X_m has small positive value.	X_m has very high positive value.

5.24. State and explain Curie's law for paramagnetism.

The magnetisation of a paramagnetic material is inversely proportional to the absolute temperature until saturation. If T is the absolute temperature of a paramagnet then magnetisation

$M \propto 1/T$ or $M = C\frac{B_0}{T}$ or $\chi = C\frac{\mu_0}{T}$,

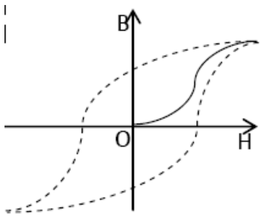
where C – Curie's constant.

At saturation all the dipoles orient in the direction of external field.

5.25. Distinguish between hard and soft ferromagnetic materials with examples.
After removal of external magnetic field if magnetisation remains, they are called hard ferromagnetic materials. Ex: Alnico. After removal of external magnetic field if magnetisation disappears, they are called soft ferromagnetic materials. Ex: Iron.

5.26. Define Curie temperature.
Curie temperature is the temperature above which a ferromagnetic substance becomes a paramagnetic substance.

5.27. Mention the expression for susceptibility in the paramagnetic phase of ferromagnetic material at absolute temperature T above Curie temperature T_C.
Magnetic susceptibility $= \frac{C}{T - T_C}$, where C – constant.



5.28. What is magnetic hysteresis?
The phenomenon of lagging of flux density (B) behind the magnetizing force (H) in a ferromagnetic material subjected to cycles of magnetization is known as hysteresis.

5.29. What is magnetic hysteresis loop?
The magnetic hysteresis loop is the closed B – H curve for cycle of magnetisation of ferromagnetic material.

5.30. What is retentivity or remanence of ferromagnetic material?
The value of B at H = 0 in a B – H loop is called retentivity or remanence.

5.31. What is coercivity?
The value of H at B = 0 in a B – H loop is called coercivity.

5.32. What are permanent magnets?
Substances which retain their ferromagnetic property for a long period of time at room temperature are called permanent magnets.

5.33. Which type of materials are required for permanent magnets? Give examples.
Materials having high retentivity, high coercivity and high permeability are required for permanent magnets. Ex: Steel, Alnico.

5.34. Which type of materials are required for electromagnets? Give example.
Materials having high retentivity and low coercivity are required for electromagnets. Ex: Iron.

5.35. What does area of hysteresis loop represent?
Area represents energy dissipated or heat produced.

5.36. How does angle of dip vary as one move from the magnetic equator to magnetic pole ?
From 0^o to 90^o.

5.37. Where on earth's surface the value of vertical component of magnetic field is zero ?
At magnetic equator

5.38. Define neutral point in magnetic field of a bar magnet ?
It is a point near the magnet where the magnetic field due to the magnet is equal and opposite to the horizontal component of magnetic field. The resultant magnetic field at the neutral point is zero.

MOST LIKELY QUESTIONS : 2M - 1Q ; 5M - 1Q (LA) or 1M-1Q; 3M-2Q

2M - 1Q :
1. Define any two of following : (i) magnetisation (M), (ii) magnetic intensity (H), (iii) magnetic susceptibility (χ) and (iv) permeability.
2. A magnetic needle has magnetic moment 6.7×10^{-6} Am^2 and moment of inertia $I = 7.5 \times 10^{-6}$ kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field ?
3. State and explain Gauss law in magnetism ?
4. Obtain an expression for time period of oscillation of small compass needle in a uniform magnetic field ?
5. Explain retentivity and coercivity in case of Magnetic hysteresis.

3M-2Q:
1. Show that a solenoid carrying current behaves as a bar magnet.
2. Write a note on Declination, Dip and Earth's horizontal component B_H and their variation.
3. List four of properties the magnetic field.
4. Compare Gauss law in electrostatics & magnetism
5. Explain origin, properties and examples for diamagnetic material
6. Explain origin, properties and examples for ferromagnetic material

5M - 1Q (LA) :
1. What are the properties of current carrying solenoid? Obtain an expression for the magnitude of magnetic field at a point on axis of a solenoid. Hence show that a solenoid carrying current behaves as a bar magnet.
2. Write a note on earth's magnetic field and explain Declination, Dip and Earth's horizontal component B_H and their variation.
3. Distinguish between dia, para and ferro magnetism with examples.
4. What is Hysteresis. Draw and explain hysteresis loop for ferromagnetic materials.

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Chapter 5:
MAGNETISM AND MATTER
7M
2M - 1Q ; 5M - 1Q (LA) or 1M-1Q; 3M-2Q

5.1 Bar magnet: Properties of magnetic field lines -

(1) Common information about Magnetism :

Some of the commonly known ideas regarding magnetism are:

(i) The earth behaves as a magnet with the magnetic field pointing approximately from the geographic south to the geographic north.

(ii) When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called the north pole and the tip which points to the geographic south is called the south pole of the magnet.

(iii) There is a repulsive force when north poles (or south poles) of two magnets are brought close together. Conversely, there is an attractive force between the north pole of one magnet and the south pole of the other.

(iv) We cannot isolate the north, or south pole of a magnet. If a bar magnet is broken into two halves, we get two similar bar magnets with somewhat weaker properties. Unlike electric charges, isolated magnetic north and south poles known as magnetic monopoles do not exist.

(v) It is possible to make magnets out of iron and its alloys.

(2) Magnetic field lines & their Properties ?

The magnetic field lines are a visual and intuitive realisation of the magnetic field. Their properties are:

(i) The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.

(ii) The tangent to the field line at a given point represents the direction of the net magnetic field B at that point.

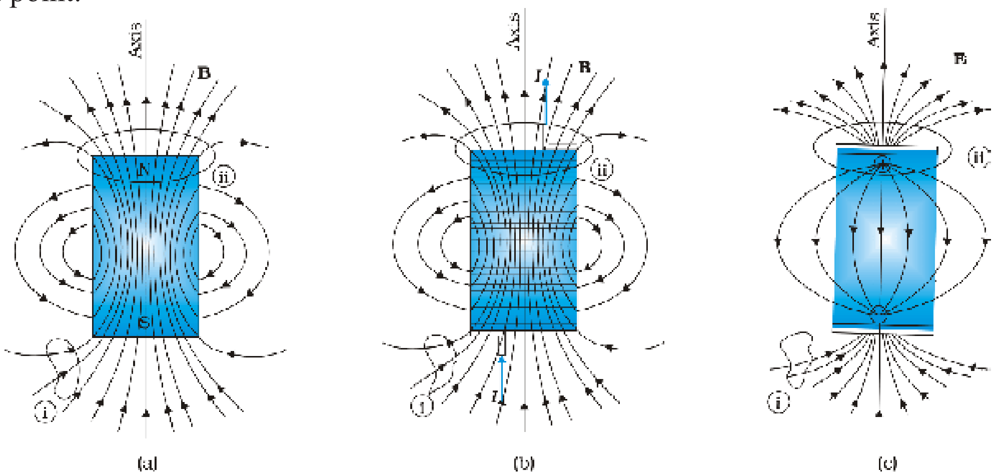


Figure 5.1 : The field lines of (a) a bar magnet, (b) a current-carrying finite solenoid and (c) electric dipole. At large distances, the field lines are very similar. The curves labelled (i) and (ii) are closed Gaussian surfaces.

(iii) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field B. In Fig. 5.1. (a), B is larger around region (ii) than in region (i).

(iv) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

5.2 Bar magnet as an equivalent solenoid with derivation -

(3) Derive the expression for axial magnetic field of a bar magnet as an equivalent solenoid ?

Bar magnet can be treated equivalent to solenoid. We know that the magnetic dipole moment m associated with a current loop is defined to be m = NIA where N is the number of turns in the loop, I the current and A the area vector.

The resemblance of magnetic field lines for a bar magnet and a solenoid suggest that a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid. Cutting a bar magnet in half is like cutting a solenoid. We get two smaller solenoids with weaker magnetic properties. The field lines remain continuous, emerging from one face of the solenoid and entering into the other face. One can test this analogy by moving a small compass needle in the neighbourhood of a bar magnet and a current-carrying finite solenoid and noting that the deflections of the needle are similar in both cases.

To make this analogy more firm we calculate the axial field of a finite solenoid depicted in Fig. 5.2 (a). We shall demonstrate that at large distances this axial field resembles that of a bar magnet.

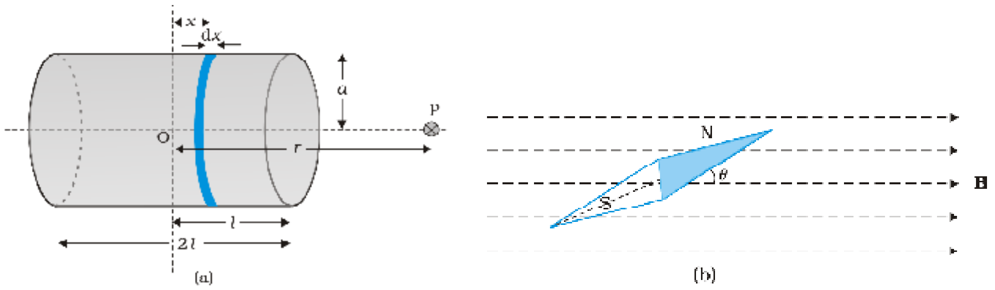


Fig. 5.2 : Calculation of (a) The axial field of a finite solenoid in order to demonstrate its similarity to that of a bar magnet. (b) A magnetic needle in a uniform magnetic field B. The arrangement may be used to determine either B or the magnetic moment m of the needle.

Let the solenoid of Fig. 5.2 (a) consists of n turns per unit length. Let its length be 2l and radius a. We can evaluate the axial field at a point P, at a distance r from the centre O of the solenoid. To do this, consider a circular element of thickness dx of the solenoid at a distance x from its centre. It consists of n dx turns. Let I be the current in the solenoid. We know that in case of the magnetic field on the axis of a circular current loop, the magnitude of the field at point P due to the circular element is

$$dB = \frac{\mu_0 n\, dx\, I\, a^2}{2[(r-x)^2 + a^2]^{3/2}}$$

The magnitude of the total field is obtained by summing over all the elements x = l to x = + l . Thus,

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^{l} \frac{dx}{[(r-x)^2 + a^2]^{3/2}}$$

This integration can be done by trigonometric substitutions. This exercise, however, is not necessary for our purpose. Note that the range of x is from l to + l . Consider the far axial field of the solenoid, i.e., r >> a and r >> l . Then the denominator is approximated by

$$[(r-x)^2 + a^2]^{3/2} \approx r^3 \quad \text{and}$$

$$B = \frac{\mu_0 n I a^2}{2r^3}\int_{-l}^{l} dx \qquad = \frac{\mu_0 n I}{2}\frac{2 l a^2}{r^3}$$

Note that the magnitude of the magnetic moment of the solenoid is,

$m = n\,(2\,l\,)\,I\;(\pi a^2)$

where a = area. Thus,

$$B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$$

This is also the far axial magnetic field of a bar magnet which one may obtain experimentally. Thus, a bar magnet and a solenoid produce similar magnetic fields. The magnetic moment of a bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

5.3 Dipole in a uniform magnetic field:

(5) Mention of expression for time period of oscillation of small compass needle in a uniform magnetic field ?

The pattern of iron filings, i.e., the magnetic field lines gives us an approximate idea of the magnetic field B. We may at times be required to determine the magnitude of B accurately. This is done by placing a small compass needle of known magnetic moment m and moment of inertia I and allowing it to oscillate in the magnetic field. This arrangement is shown in Fig. 5.2(b).

The torque on the needle is $\tau = m \times B$

In magnitude $\tau = mB \sin\theta$

Here τ is restoring torque and θ is the angle between m and B. Therefore, in equilibrium

$$I\frac{d^2\theta}{dt^2} = -mB\sin\theta$$

Negative sign with mB sinθ implies that restoring torque is in opposition to deflecting torque. For small values of θ in radians, we approximate $\sin\theta \approx \theta$ and get

$$I\frac{d^2\theta}{dt^2} \approx mB\,\theta \qquad \text{or,} \quad \frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$$

This represents a simple harmonic motion. The square of the angular frequency is

$\omega^2 = mB/I$ and the time period is,

$$T = 2\pi\sqrt{\frac{I}{mB}} \qquad \text{or} \qquad B = \frac{4\pi^2 I}{mT^2}$$

An expression for magnetic potential energy can also be obtained on lines similar to electrostatic potential energy.

The magnetic potential energy U_m is given by

$$U_m = \int \tau(\theta)d\theta$$
$$= \int mB\sin\theta = -mB\cos\theta$$
$$= -m\cdot B$$

Example 5.1 : In Fig. 5.2(b), the magnetic needle has magnetic moment 6.7×10^{-6} Am^2 and moment of inertia $I = 7.5 \times 10^{-6}$ kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field ?

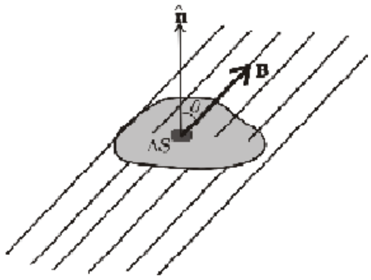
Solution : The time period of oscillation is,

$$T = \frac{6.70}{10} = 0.67s$$

$$B = \frac{4\pi^2 I}{mT^2} = \frac{4\times(3.14)^2\times7.5\times10^{-6}}{6.7\times10\ \ \times(0.67)} = 0.01\ T$$

5.4 Gauss law in magnetism: Statement and explanation.

In magnetism, we can visually demonstrate that the number of magnetic field lines leaving the surface is balanced by the number of lines entering it. The net magnetic flux is zero for both the surfaces. This is true for any closed surface.



Consider a small vector area element ΔS of a closed surface S as in Fig. 5.3. The magnetic flux through ΔS is defined as $\Delta\phi_B = B.\Delta S$, where B is the field at ΔS. We divide S into many small area elements and calculate the individual flux through each. Then, the net flux ϕ_B is,

$$\phi_B = \sum_{'all'} \Delta\phi_B = \sum_{'all'} B\cdot\Delta S = 0$$

Comparing this with the Gauss law for electrostatics, in that case is given by

$$\sum E\cdot\Delta S = \frac{q}{\varepsilon_0}$$

where q is the electric charge enclosed by the surface.

The difference between the Gauss electrostatics is a reflection of the fact that isolated magnetic poles (also called monopoles) are not known to exist. There are no sources or sinks of B; the simplest magnetic element is a dipole or a current loop. All magnetic phenomena can be explained in terms of an arrangement of dipoles and/or current loops.

Thus, Gauss law in magnetism states that the net magnetic flux through any closed surface is zero.

Comparison between Electrostatic and Magnetic quantities :

Physical Quantity	**Electrostatics**	**Magnetism**
Free space constant	$1/\varepsilon_0$	μ_0
Dipole moment	$\vec{P}$	$\vec{M}$
Axial field	$\frac{1}{4\pi\varepsilon_0}\cdot\frac{2\vec{P}}{r^3}$	$\frac{\mu_0}{4\pi}\cdot\frac{2\vec{M}}{r^3}$
Equatorial field	$-\frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{P}}{r^3}$	$-\frac{\mu_0}{4\pi}\cdot\frac{\vec{M}}{r^3}$
Torque in external field	$\vec{P}\times\vec{E}$	$\vec{M}\times\vec{B}$
P.E. in external field	$-\vec{P}\ \vec{E}$	$-\vec{M}\ \vec{B}$

5.5 Earth's magnetic field and its elements: Declination, Dip and Earth's horizontal component B_H and their variation -

(1) Write a note on earth's magnetic field ?

The strength of the earth magnetic field value being of the order of 10 T.

The magnetic field is now thought to arise due to electrical currents produced by convective motion of metallic fluids (consisting mostly of molten iron and nickel) in the outer core of the earth. This is known as the dynamo effect.
The magnetic field lines of the earth resemble that of a (hypothetical) magnetic dipole located at the centre of the earth. The axis of the dipole does not coincide with the axis of rotation of the earth but is presently titled by approximately 11º.3" with respect to the later. In this way of looking at it, the magnetic poles are located where the magnetic field lines due to the dipole enter or leave the earth. The location of the north magnetic pole is at a latitude of 79º.74" N and a longitude of 71.8" W, a place somewhere in north Canada. The magnetic south pole is at 79º.74" S, 108º.22" E in the Antarctica.
The pole near the geographic north pole of the earth is called the north magnetic pole. Likewise, the pole near the geographic south pole is called the south magnetic pole. There is some confusion in the nomenclature of the poles. If one looks at the magnetic field lines of the earth (Fig. 5.8), one sees that unlike in the case of a bar magnet, the field lines go into the earth at the north magnetic pole (N_m) and come out from the south magnetic pole (S_m). The convention arose because the magnetic north was the direction to which the north pole of a magnetic needle pointed; the north pole of a magnet was so named as it was the north seeking pole. Thus, in reality, the north magnetic pole behaves like the south pole of a bar magnet inside the earth and vice versa.

(2) Write a note on Declination, Dip and Earth's horizontal component B_H and their variation.

In case of earth, the vertical plane containing the longitude circle and the axis of rotation of the earth is called the geographic meridian. In a similar way, one can define magnetic meridian of a place as the vertical plane which passes through the imaginary line joining the magnetic north and the south poles. This plane would intersect the surface of the earth in a longitude like circle. A magnetic needle, which is free to swing horizontally, would then lie in the magnetic meridian and the north pole of the needle would point towards the magnetic north pole. Since the line joining the magnetic poles is titled with respect to the geographic axis of the earth, the magnetic meridian at a point makes angle with the geographic meridian.

(i) The angle between the true geographic north and the north pole shown by a compass needle is called the **magnetic declination** (D) or simply **declination** (Fig. 5.9).
The declination is greater at higher latitudes and smaller near the equator. The declination in India is small, it being 0"41' E at Delhi and 0"58' W at Mumbai. Thus, at both these places a magnetic needle shows the true north quite accurately.

(ii) The angle that the total magnetic field (B_E) of the earth makes with the surface of the earth and horizontal component of earth's magnetic field (H_E) is called **dip or inclination (I)** at that place.
In most of the northern hemisphere, the north pole of the dip needle tilts downwards. Likewise in most of the southern hemisphere, the south pole of the dip needle tilts downwards.

(iii) To describe the magnetic field of the earth at a point on its surface, we need to specify three quantities, viz., the declination (D), the angle of dip or the inclination (I) and the horizontal component of the earth HE. These are known as the element of the earth magnetic field.

Representing the verticle component by Z_E, we have

$Z_E = B_E \sin I$

$H_E = B_E \cos I$

which gives,

$$\tan I = \frac{Z_E}{H_E}$$

5.6 Definitions of magnetisation (M), magnetic intensity (H), magnetic susceptibility (χ) and permeability *(μ, μ₀ and μᵣ)*.

We know that a circulating electron in an atom has a magnetic moment. In a bulk material, these moments add up vectorially and they can give a net magnetic moment which is non-zero. We define **magnetisation M** of a sample to be equal to its net magnetic moment per unit volume:

$M = m_{net}/V$

M is a vector with dimensions L A and is measured in a units of Am^{-1} .

Consider a long solenoid of n turns per unit length and carrying a current I. The magnetic field in the interior of the solenoid was shown to be given by

$B_0 = \mu_0 nI$

If the interior of the solenoid is filled with a material with non-zero magnetisation, the field inside the solenoid will be greater than B_0. The net B field in the interior of the solenoid may be expressed as

$B = B_0 + B_m$

where B_m is the field contributed by the material core. It turns out that this additional field B_m is proportional to the **magnetisation M** of the material and is expressed as

$B_m = \mu_0 M$

where μ_0 is the same constant (permittivity of vacuum) that appears in Biot-Savart law.

It is convenient to introduce another vector field H, called the magnetic intensity, which is defined by

$H = \frac{B}{\mu_0} - M$

where H has the same dimensions as M and is measured in units of Am^{-1}. Thus, the total magnetic field B is written as

$\mathbf{B = \mu_0 (H + M)}$

If we partition the contribution to the total magnetic field inside the sample into two parts: one, due to external factors such as the current in the solenoid. This is represented by H. The other is due to the specific nature of the magnetic material, namely M. The latter quantity can be influenced by external factors. This influence is mathematically expressed as

$\mathbf{M = \chi H}$

where χ, a dimensionless quantity, is appropriately called the magnetic susceptibility. It is a measure of how a magnetic material responds to an external field. Table 5.2 lists χ for some elements. It is small and positive for materials, which are called paramagnetic. It is small and negative for materials, which are termed diamagnetic. In the latter case M and H are opposite in direction. From Eqs. (5.16) and (5.17) we obtain,

$\mathbf{B = \mu_0 (1 + \chi H) \quad = \mu_0 \mu_r H \quad = \mu H}$

where $\mu_r = 1 + \chi$, is a dimensionless quantity called the relative magnetic permeability of the substance. It is the analog of the dielectric constant in electrostatics. The magnetic permeability of the substance is m and it has the same dimensions and units as μ_0;

$\mathbf{\mu = \mu_0 \mu_r = \mu_0 (1 + \chi)}$

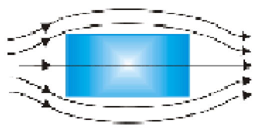
The three quantities χ, μ_r and μ are interrelated and only one of them is independent. Given one, the other two may be easily determined.

5.7 Magnetic properties of materials: Paramagnetic, diamagnetic and ferromagnetic substances, examples and properties -

Materials are classified as diamagnetic, paramagnetic or ferromagnetic. In terms of the susceptibility χ, a material is diamagnetic if χ is negative, para- if χ is positive and small, and ferro- if χ is large and positive.

(1) Diamagnetism :
Diamagnetic substances are those which have tendency to move from stronger to the weaker part of the external magnetic field. In other words, unlike the way a magnet attracts metals like iron, it would repel a diamagnetic substance.

If a bar of diamagnetic material placed in an external magnetic field, the field lines are repelled or expelled and the field inside the material is reduced as shown in fig. (a).


(a)

Origin of Diamagnetism : Electrons in an atom orbiting around nucleus possess orbital angular momentum. These orbiting electrons are equivalent to current-carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in the opposite direction speed up. Thus, the substance develops a net magnetic moment in direction opposite to that of the applied field and hence repulsion of magnetic field lines takes place.

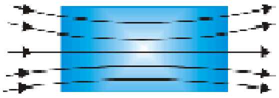
Example : Some diamagnetic materials are bismuth, copper, lead, silicon, nitrogen (at STP), water and sodium chloride. Diamagnetism is present in all the substances. However, the effect is so weak in most cases that it gets shifted by other effects like paramagnetism, ferromagnetism, etc. The most exotic diamagnetic materials are superconductors. These are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism. Here the field lines are completely expelled! $\chi = (\mu_r - 1) = 0$. A superconductor repels a magnet and diamagnetism in superconductors is called the Meissner effect, after the name of its discoverer. Superconducting magnets can be gainfully exploited in variety of situations, for example, for running magnetically levitated superfast trains.

Properties :
(1) Diamagnetic substance is feebly repelled by a strong magnet.
(2) When a diamagnetic substance is placed in magnetic field, the magnetic lines of force prefer to pass through the surrounding air rather than through the substance. This is because the induced magnetic field in the diamagnetic substance opposes the external field.
(3) When a rod of diamagnetic substance , is suspended in a uniform magnetic field the rod comes to rest with its longest axis at right angles to the direction of the field.
(4) When placed in a non-uniform magnetic field, a diamagnetic substance moves from stronger to weaker parts of the field.

(5) The relative permeability (μ_r) of a diamagnetic substance is always less than 1.
(6) The magnetic susceptibility (χ) of a diamagnetic substance has a small negative value.
(7) The magnetic susceptibility (χ) of a diamagnetic substance does not change with temperature.

(2) Paramagnetism :
Paramagnetic substances are those which get weakly magnetised when placed in an external magnetic field. They have tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get weakly attracted to a magnet.

Origin of Paramagnetism : The individual atoms (or ions or molecules) of a paramagnetic material possess a permanent magnetic dipole moment of their own. On account of the ceaseless random thermal motion of the atoms, no net magnetisation is seen. In the presence of an external field B_0, which is strong enough, and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as B_0. Figure (b) shows a bar of paramagnetic material placed in an external field. The field lines gets concentrated inside the material, and the field inside is enhanced. In most cases, this enhancement is slight, being one part in 10^5. When placed in a non-uniform magnetic field, the bar will tend to move from weak field to strong.


(b)

Example : Some paramagnetic materials are aluminium, sodium, calcium, oxygen (at STP) and copper chloride.

Experimentally, one finds that the magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T,

$$M = C\frac{B_0}{T}$$

or equivalently, using Eqs. (5.12) and (5.17)

$$\chi = C\frac{\mu_0}{T}$$

This is known as Curie law, after its discoverer Pieree Curie (1859- 1906). The constant C is called Curie constant. Thus, for a paramagnetic material both χ and μ_r depend not only on the material, but also (in a simple fashion) on the sample temperature. As the field is increased or the temperature is lowered, the magnetisation increases until it reaches the saturation value M_S, at which point all the dipoles are perfectly aligned with the field. Beyond this, Curie longer valid.

Properties of paramagnetic Materials :
(1) A paramagnetic substance is feebly attracted by a strong magnet.
(2) When a paramagnetic substance is placed in a magnetic field, the magnetic lines of force prefer to pass through the substance rather than through air. Therefore, the resultant field B inside the substance is more than the external field B_0.
(3) When a rod of paramagnetic substance is suspended freely in a uniform magnetic field, the rod comes to rest with its longest axis along the direction of external magnetic field.
(4) When placed in a non-uniform magnetic field, a paramagnetic substance moves from weaker to stronger parts of the field.
(5) The relative permeability (μ_r) of paramagnetic substance has small positive value.

(6) The magnetic susceptibility of paramagnetic substance varies inversely as the absolute temperature.

(3) Ferromagnetism :

Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field. They have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get strongly attracted to a magnet.

Origin of Ferromagnetism : The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. The explanation of this cooperative effect requires quantum mechanics and is beyond the scope of this textbook. Each domain has a net magnetisation. Typical domain size is 1mm and the domain contains about 10^{11} atoms. In the first instant, the magnetisation varies randomly from domain to domain and there is no bulk magnetisation. This is shown in Fig. 5.13(a). When we apply an external magnetic field B_0, the domains orient themselves in the direction of B0 and simultaneously the domain oriented in the direction of B_0 grow in size. This existence of domains and their motion in B_0 are not speculations. One may observe this under a microscope after sprinkling a liquid suspension of powdered ferromagnetic substance of samples. This motion of suspension can be observed. Figure 5.12(b) shows the situation when the domains have aligned and amalgamated to form a single sample.
Thus, in a ferromagnetic material the field lines are highly concentrated. In non-uniform magnetic field, the sample tends to move towards the region of high field. We may wonder as to what happens when the external field is removed. In some ferromagnetic materials the magnetisation persists. Such materials are called hard magnetic materials or hard ferromagnets.

Example : Alnico, an alloy of iron, aluminium, nickel, cobalt and copper, is one such material. The naturally occurring lodestone is another. Such materials form permanent magnets to be used among other things as a compass needle. On the other hand, there is a class of ferromagnetic materials in which the magnetisation disappears on removal of the external field. Soft iron is one such material. Appropriately enough, such materials are called soft ferromagnetic materials.

There are a number of elements, which are ferromagnetic: iron, cobalt, nickel, gadolinium, etc. The relative magnetic permeability is >1000!

The ferromagnetic property depends on temperature. At high enough temperature, a ferromagnet becomes a paramagnet. The domain structure disintegrates with temperature. This disappearance of magnetisation with temperature is gradual. It is a phase transition reminding us of the melting of a solid crystal. The temperature of transition from ferromagnetic to paramagnetism is called the Curie temperature T_c . Table 5.4 lists the Curie temperature of certain ferromagnets. The susceptibility above the Curie temperature, i.e., in the paramagnetic phase is described by,

$$\chi = \frac{C}{T - T_c} \quad (T > T_c)$$

Properties of Ferromagnetic Materials :

(1) A ferromagnetic substance is strongly attracted by a magnet.

(2) When a ferromagnetic substance is placed in a magnetic field, the magnetic field lines tends to crowd in to the substance.
(3) When a rod of ferromagnetic substance is suspended in a uniform magnetic field, it quickly aligns itself in the direction of the field.
(4) When placed in a non-uniform magnetic field, a ferromagnetic substance moves from weaker to stronger parts of the magnetic field.
(5) The relative permeability μ_r of ferromagnetic substance is very large. For example, relative permeability of soft iron is about 8,000.
(6) The magnetic susceptibility of ferromagnetic substance is positive having a very high value.

5.8 Curie's law and Curie temperature -

According to Curie law, the magnetisation (M) of a paramagnetic material is inversely proportional to the absolute temperature T, and directly proportional to the external magnetic field.

$$M = C\frac{B_0}{T}$$

or equivalently, using Eqs. (5.12) and (5.17)

$$\chi = C\frac{\mu_0}{T}$$

This is known as Curie law and C is called Curie constant.

5.9 Hysteresis, Hysteresis loop, definitions of retentively and coercively -

The relation between B and H in ferromagnetic materials is complex. It is often not linear and it depends on the magnetic history of the sample. Figure 5.14 depicts the behaviour of the material as we take it through one cycle of magnetisation. Let the material be unmagnetised initially. We place it in a solenoid and increase the current through the solenoid. The magnetic field B in the material rises and saturates as depicted in the curve Oa. This behaviour represents the alignment and merger of domains until no further enhancement is possible. It is pointless to increase the current (and hence the magnetic intensity H) beyond this. Next, we decrease H and reduce it to zero. At H = 0, B ≠ 0. This is represented by the curve ab. The value of B at H = 0 is called **retentivity** or remanence. In Fig., $B_R \sim 1.2$ T, where the subscript R denotes retentivity. The domains are not completely randomised even though the external driving field has been removed. Next, the current in the solenoid is reversed and slowly increased. Certain domains are flipped until the net field inside stands nullified. This is represented by the curve bc. The value of H at c is called **coercivity**. In Fig. $H_c \sim -80$ A/m. As the reversed current is increased in magnitude, we once again obtain saturation. The curve cd depicts this. The saturated magnetic field $B_s \sim 1.5$ T. Next, the current is reduced (curve de) and reversed (curve ea). The cycle repeats itself. Note that the curve Oa does not retrace itself as H is reduced. For a given value of H, B is not unique but depends on previous history of the sample. This phenomenon is called **hysterisis**. The word hysterisis means lagging behind.

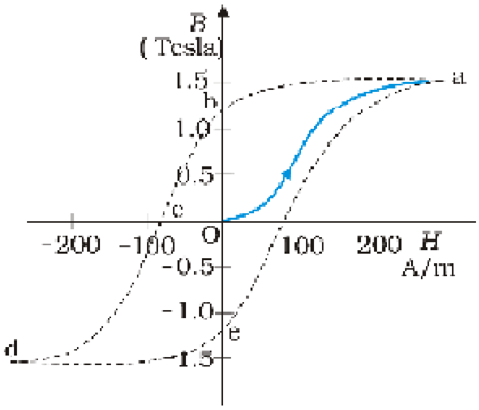


Fig. : The magnetic hysteresis loop is the B-H curve for ferromagnetic materials.

5.10 Permanent magnets and electromagnets.

Substances which at room temperature retain their ferromagnetic property for a long period of time are called **permanent magnets**.

Permanent magnets can be made in a variety of ways.

(1) One can hold an iron rod in the north-south direction and hammer it repeatedly.

(2) One can also hold a steel rod and stroke it with one end of a bar magnet a large number of times, always in the same sense to make a permanent magnet.

(3) An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and pass a current. The magnetic field of the solenoid magnetises the rod.

(4) The hysteresis curve allows us to select suitable materials for permanent magnets. The material should have high retentivity so that the magnet is strong and high coercivity so that the magnetisation is not erased by stray magnetic fields, temperature fluctuations or minor mechanical damage. Further, the material should have a high permeability. Steel is one-favoured choice. It has a slightly smaller retentivity than soft iron but this is outweighed by the much smaller coercivity of soft iron. Other suitable materials for permanent magnets are alnico, cobalt steel and ticonal.

(5) Core of electromagnets are made of ferromagnetic materials which have high permeability and low retentivity. Soft iron is a suitable material for electromagnets. On placing a soft iron rod in a solenoid and passing a current, we increase the magnetism of the solenoid by a thousand fold. When we switch off the solenoid current, the magnetism is effectively switched off since the soft iron core has a low retentivity.

(6) Electromagnets are used in electric bells, loudspeakers and telephone diaphragms. Giant electromagnets are used in cranes to lift machinery, and bulk quantities of iron and steel.

Question Bank

5.1. What are the properties of magnetic field lines?
i. Magnetic field lines are continuous closed loops.
ii. Tangent to the field line represents the direction of net field B .
iii. The larger the number of field lines crossing unit area normally, the stronger is the magnitude of the magnetic field B.
iv. Magnetic field lines do not intersect.

5.2 Is a bar magnet an equivalent current carrying solenoid?
Yes. Each turn of solenoid behaves as a small magnetic dipole. Therefore solenoid can be considered as arrangement of small magnetic dipoles placed in line with each other. The magnetic field produced by solenoid is identical to that produced by the magnet.

5.3. What is the force acting on a bar magnet placed in a uniform magnetic field?
Zero.

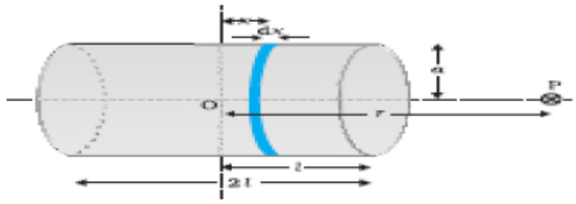
5.4. What is the torque when a bar magnet of dipole moment m is placed in a uniform magnetic field? When is torque maximum and minimum.
$\tau = m \times B = m. B. \sin \theta$
Torque is maximum if $\theta = 90$ i.e. torque is maximum if the magnetic dipole is at right angles to applied magnetic field.
Torque is minimum if $\theta = 0$ i.e. torque is minimum if the magnetic dipole is along the direction of applied magnetic field.

5.5. Give an expression for time period of oscillation when a magnetic needle placed in uniform magnetic field.
A small compass magnetic needle of magnetic moment m and moment of inertia I is made to oscillate in the magnetic field, B .
Time period of oscillation is given by $T = 2\pi$ Im B .

5.6. State and explain Gauss law in magnetism.
The net magnetic flux through any closed surface is zero.
Consider a small vector area element Δs of closed surface S. According to Gauss law in magnetism, net flux though closed surface, $\emptyset B= B .\Delta s$ all areaelement=0.
The implication of Gauss law is that isolated magnetic poles do not exist.

5.7. What is the cause of earth's magnetism?
Earths magnetism is due to electrical currents produced by the convective motion of mainly molten iron and nickel in the outer core of the earth.

5.8. Show that a current carrying solenoid behaves as a magnet.
Let 'r' be radius of solenoid of length 2l.



To calculate magnetic field at a point on axis of solenoid, consider a small element of thickness 'dx' of solenoid at a distance 'x' from 'o'.
Number of turns in this element = n.dx
If current 'i' flows through element 'ndx' the magnitude of magnetic field at P due to this element is

$$dB = \frac{\mu_0}{4\pi}\frac{2\pi(n\,dx)i\,a^2}{[(r-x)^2 + a^2]^{3/2}}$$

If point 'p' is at large distance from 'o' i.e. r >> l and r>> a then $[(r-x)^2 + a^2] = r^2$

$$dB = \frac{\mu_0}{4\pi}\frac{2\pi(n\,dx)i\,a^2}{[r^2]^{3/2}} = dB = \frac{\mu_0}{4\pi}\frac{2\pi(n\,dx)i\,a^2}{[r]^3}$$

The total magnetic field at 'p' due to the current 'i' in solenoid is

$$B = \int_{-l}^{l} dB = \int_{-l}^{l}\frac{\mu_0}{4\pi}\frac{2\pi ni\,a^2 dx}{[r]^3} = \frac{\mu_0}{4\pi}\frac{2\pi ni\,a^2}{[r]^3}[x]_{-l}^{l}$$

$$B = \frac{\mu_0}{4\pi}\frac{2\pi ni\,a^2}{[r]^3}[l+l] = \frac{\mu_0}{4\pi}\frac{2\,i\,n\,\pi\,a^2\,.2l}{[r]^3}$$

$$B = \frac{\mu_0}{4\pi}\frac{2\,i\,n\;A\,.2l}{[r]^3} = \frac{\mu_0}{4\pi}\frac{2\;(n\,.2l)iA}{[r]^3}$$

$$B = \frac{\mu_0}{4\pi}\frac{2\,N\,i\;A}{[r]^3}$$ (N = No of turns of solenoid = n x 2l)

$$B = \frac{\mu_0}{4\pi}\frac{2\;m}{r^3}$$

This equation gives the magnitude of magnetic field at a point on axis of a solenoid.
This equation is similar to the expression for magnetic field on axis of a short bar magnet. Hence a solenoid carrying current behaves as a bar magnet.

5.9. What is magnetic declination?
Declination at the place is the angle between true geographic north direction and the north shown by the magnetic compass needle.

5.10. What is magnetic dip or inclination?
Magnetic dip at a place is the angle between the earth's total magnetic field at a place and horizontal drawn in magnetic meridian?

5.11. What are elements of earth's magnetic field? Mention them.
They are: (1) Magnetic declination (θ) at that place

(2) Magnetic inclination (I) dip at that place
(3) Horigontal comp of earths magnetic field (B_H) at that place.

5.12. What is the relation between horizontal component of earth's field H_E, vertical component of earth's field Z_E and inclination, I?

$Z_E = B_E\sin I$ or $H_E = B_E\cos I$ or $\tan I = Z_E/H_E$

5.13. Define magnetisation of a sample.
Magnetisation of a sample is its net magnetic dipole moment per unit volume.

$$\vec{M} = \frac{\vec{m}_{net}}{V}.$$

5.14. What is the unit of magnetisation?
Am^{-1}.

5.15. Define magnetic intensity.
The degree to which a magnetic field can magnetise a material is represented in terms of magnetic intensity.

Magnetic intensity of a material is the ratio of external magnetic field to the permeability of free space.

Magnetic intensity of a material, $H = B_0/\mu_0$, where B_0 - external magnetic field, μ_0 - permeability of free space.

5.16. What is the unit of magnetic intensity, H ?
Am^{-1}.

5.17. What is the relation between magnetic field $\vec{B}$, magnetic intensity $\vec{H}$ and Magnatisation $\vec{M}$ of a specimen?

$B = B_0 + B_m = \mu 0\ (H + M)$, where $B_0 = \mu_0 H$ due to current in the solenoid, $B_m = \mu_0 M$ is due to nature of the material inside the solenoid.

5.18. What is magnetic susceptibility?
The ratio of magnetisation developed in the material to the magnetic intensity is called magnetic susceptibility.

$$\chi = \frac{\vec{M}}{\vec{H}}.$$

5.19. Write the relation between magnetic intensity, magnetic field and susceptibility.

$$\vec{B} = \mu_0(1 + \chi)\vec{H} = \mu H.$$

5.20. What is the relation between magnetic relative permeability and susceptibility?

$$\mu_r = 1 + \chi.$$

5.21. What is the relation between magnetic relative permeability and permeability of the medium?

$\mu = \mu_0 \mu_r$. μ is the permeability of medium, μ_0 is permeability of free space, and μ_r is relative permeability of the medium.

5.22. Define permeability.

Permeability of a substance is the ability of the substance to allow magnetic field lines to pass through it.

5.23. Distinguish between dia, para and ferro magnetism with examples.

	Diamagnetic material	Paramagnetic material	Ferromagnetic material
1	Weakly repelled by magnetic	Weakly attracted by magnetic	Strongly attracted by magnetic
2	When placed in a magnetic field it is weakly magnetized in a direction opposite to that of applied field.	When placed in a magnetic field it is weakly magnetized in the direction of applied field.	When placed in a magnetic field it is strongly magnetized in the direction of applied field.
3	μ_r is slightly less than 1	μ_r is slightly more than 1	
4	When placed in a magnetic field flux density (B) inside the material is less than in air.	When placed in a external magnetic field, flux density (B) than in air.	When placed in a magnetic field the flux density (B) inside the material is much large than in air.
5	X_m (susceptibility) doesnot change with temperature	X_m varies inversely as the temperature of substance	X_m decreases with rise temperature
6	Intensity of magnetization a small and negative value	I has small and positive value	I has large positive value
7	X_m has small negative value	X_m has small positive value.	X_m has very high positive value.

5.24. State and explain Curie's law for paramagnetism.

The magnetisation of a paramagnetic material is inversely proportional to the absolute temperature until saturation. If T is the absolute temperature of a paramagnet then magnetisation

$M \propto {}^{1}/_{T}$ or $M = C\frac{B_0}{T}$ or $\chi = C\frac{\mu_0}{T}$,

where C – Curie's constant.

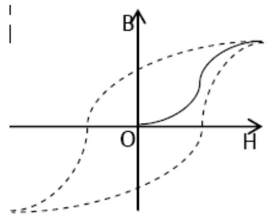
At saturation all the dipoles orient in the direction of external field.

5.25. Distinguish between hard and soft ferromagnetic materials with examples.
After removal of external magnetic field if magnetisation remains, they are called hard ferromagnetic materials. Ex: Alnico. After removal of external magnetic field if magnetisation disappears, they are called soft ferromagnetic materials. Ex: Iron.

5.26. Define Curie temperature.
Curie temperature is the temperature above which a ferromagnetic substance becomes a paramagnetic substance.

5.27. Mention the expression for susceptibility in the paramagnetic phase of ferromagnetic material at absolute temperature T above Curie temperature T_C.
Magnetic susceptibility $= \frac{C}{T - T_C}$, where C – constant.

5.28. What is magnetic hysteresis?
The phenomenon of lagging of flux density (B) behind the magnetizing force (H) in a ferromagnetic material subjected to cycles of magnetization is known as hysteresis.



5.29. What is magnetic hysteresis loop?
The magnetic hysteresis loop is the closed B – H curve for cycle of magnetisation of ferromagnetic material.

5.30. What is retentivity or remanence of ferromagnetic material?
The value of B at H = 0 in a B – H loop is called retentivity or remanence.

5.31. What is coercivity?
The value of H at B = 0 in a B – H loop is called coercivity.

5.32. What are permanent magnets?
Substances which retain their ferromagnetic property for a long period of time at room temperature are called permanent magnets.

5.33. Which type of materials are required for permanent magnets? Give examples.
Materials having high retentivity, high coercivity and high permeability are required for permanent magnets. Ex: Steel, Alnico.

5.34. Which type of materials are required for electromagnets? Give example.
Materials having high retentivity and low coercivity are required for electromagnets. Ex: Iron.

5.35. What does area of hysteresis loop represent?
Area represents energy dissipated or heat produced.

5.36. How does angle of dip vary as one move from the magnetic equator to magnetic pole ?
From 0^{o} to 90^{o}.

5.37. Where on earth's surface the value of vertical component of magnetic field is zero ?
At magnetic equator

5.38. Define neutral point in magnetic field of a bar magnet ?
It is a point near the magnet where the magnetic field due to the magnet is equal and opposite to the horizontal component of magnetic field. The resultant magnetic field at the neutral point is zero.

MOST LIKELY QUESTIONS : 2M - 1Q ; 5M - 1Q (LA) or 1M-1Q; 3M-2Q

2M - 1Q :
1. Define any two of following : (i) magnetisation (M), (ii) magnetic intensity (H), (iii) magnetic susceptibility (χ) and (iv) permeability.
2. A magnetic needle has magnetic moment 6.7×10^{-6} Am^2 and moment of inertia $I = 7.5 \times 10^{-6}$ kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field ?
3. State and explain Gauss law in magnetism ?
4. Obtain an expression for time period of oscillation of small compass needle in a uniform magnetic field ?
5. Explain retentivity and coercivity in case of Magnetic hysteresis.

3M-2Q:
1. Show that a solenoid carrying current behaves as a bar magnet.
2. Write a note on Declination, Dip and Earth's horizontal component B_H and their variation.
3. List four of properties the magnetic field.
4. Compare Gauss law in electrostatics & magnetism
5. Explain origin, properties and examples for diamagnetic material
6. Explain origin, properties and examples for ferromagnetic material

5M - 1Q (LA) :
1. What are the properties of current carrying solenoid? Obtain an expression for the magnitude of magnetic field at a point on axis of a solenoid. Hence show that a solenoid carrying current behaves as a bar magnet.
2. Write a note on earth's magnetic field and explain Declination, Dip and Earth's horizontal component B_H and their variation.
3. Distinguish between dia, para and ferro magnetism with examples.
4. What is Hysteresis. Draw and explain hysteresis loop for ferromagnetic materials.

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Chapter 6:
ELECTROMAGNETIC INDUCTION
6M
1M - 1Q; 2M - 1Q ; 3M - 1Q or 1M -1Q; 5M-1Q (LA)

6.1 Experiments of Faraday and Henry - Magnetic flux $\emptyset_B = \vec{B}\,.\vec{A}$,

The phenomenon in which electric current is generated by varying magnetic fields is appropriately called *electromagnetic induction*.

The discovery and understanding of electromagnetic induction are based on a long series of experiments carried out by Faraday and Henry.

Experiment 1 : The relative motion between the magnet and the coil that is responsible for generation (induction) of electric current in the coil.

Experiment 2 : It is the relative motion between the coils that induces the electric current.

Experiment 3 : It is also observed that the deflection increases dramatically when an iron rod is inserted into the coils along their axis.

Magnetic flux is defined as flux through a plane of area A placed in a uniform magnetic field can be written as

$\phi_B = B\,.\,A = BA\cos\theta$ ------- (6.1)

where θ is angle between B and A. Equation (6.1) can be extended to curved surfaces and non-uniform fields.
If the magnetic field has different magnitudes and directions at various parts of a surface as shown in Fig. 6.1 (b), then the magnetic flux through the surface is given by

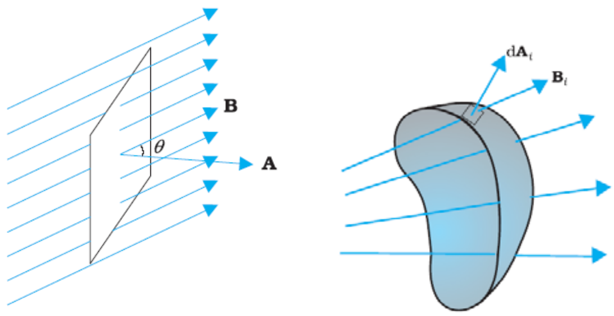


Fig. 6.1 (a) : A plane of surface area A placed in a uniform magnetic field B. (b) Magnetic field B_i at the ith area element. dA_i represents area vector of the ith area element.

$$\Phi_B = \mathbf{B}_1\cdot d\mathbf{A}_1 + \mathbf{B}_2\cdot d\mathbf{A}_2 + \cdots = \sum_{all} \mathbf{B}_i\cdot d\mathbf{A}_i \quad \text{----------} \quad (6.2)$$

where 'all' stands for summation over all the area elements dA_i comprising the surface and B_i is the magnetic field at the area element dA_i. The SI unit of magnetic flux is weber (Wb) or tesla meter squared (T m^2). Magnetic flux is a scalar quantity.

6.2 Faraday's law of electromagnetic induction: Statement and explanation -

Faraday stated experimental observations in the form of a law called *Faraday's law of electromagnetic induction*. The law is stated below.

The magnitude of the induced emf *in a circuit is equal to the time rate of change of magnetic flux through the circuit.*
Mathematically, the induced emf is given by

$$\varepsilon = -\frac{d\Phi_B}{dt} \quad \text{----------} \quad (6.3)$$

The negative sign indicates the direction of e and hence the direction of current in a closed loop.

In the case of a closely wound coil of *N* turns, change of flux associated with each turn, is the same. Therefore, the expression for the total induced emf is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} \quad \text{-----------} \quad (6.4)$$

The induced emf can be increased by increasing the number of turns *N* of a closed coil.

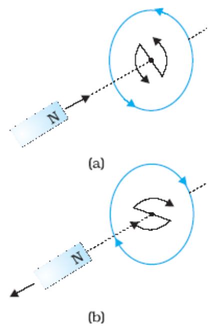
From Eqs. (6.1) and (6.2), we see that the flux can be varied by changing any one or more of the terms B, A and θ.

6.3 Lenz's law: Statement, explanation and its significance as conservation of energy.

In 1834, German physicist Heinrich Friedrich Lenz (1804-1865) deduced a rule, known as *Lenz's law* which gives the polarity of the induced emf.

Statement of the law :
The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.

The negative sign shown in Eq. (6.3) represents this effect.



The direction of the induced emf can be found using Lenz's law.

Explanation and its significance :
Consider Figs. (a) and (b). The direction shown by and indicate the directions of the induced currents. A little reflection on this matter should convince us on the correctness of Lenz's law. Suppose that the induced current was in the direction opposite to the one depicted in Fig. 6.6(a). In that case, the South-pole due to the induced current will face the approaching North-pole of the magnet. The bar magnet will then be attracted towards the coil at an ever increasing acceleration. A gentle push on the magnet will initiate the process and its velocity and kinetic energy will continuously increase without expending any energy. If this can happen, one could construct a perpetual-motion machine by a suitable arrangement. This violates the law of conservation of energy and hence cannot happen.

6.4 Motional emf - Derivation of motional emf - Eddy currents -Advantages of eddy currents with common practical applications.

(1) What is motional emf ? Obtain an expression for it ?

Consider a straight conductor moving in a uniform and time-independent magnetic field. Figure 6.10 shows a rectangular conductor PQRS in which the conductor PQ is free to move. The rod PQ is moved towards the left with a constant velocity v as shown in the figure. Assume that there is no loss of energy due to friction. PQRS forms a closed circuit enclosing an area that changes as PQ moves. It is placed in a uniform magnetic field B which is perpendicular to the plane of this system. If the length RQ = x and RS = l, the magnetic flux ϕ_B enclosed by the loop PQRS will be $\phi_B = Blx$

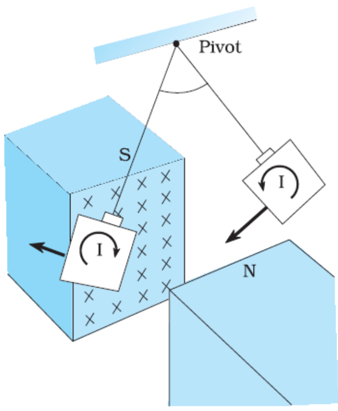
$$\varepsilon = \frac{-\mathrm{d}\Phi_B}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}(Blx) \qquad = -Bl\frac{\mathrm{d}x}{\mathrm{d}t} = Blv \qquad \text{--------} \quad (6.5)$$

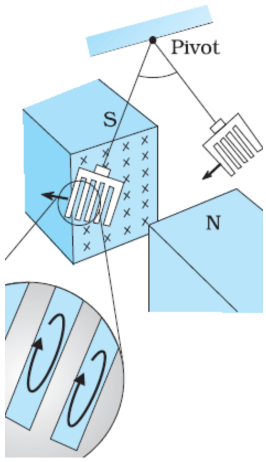
where we have used dx/dt = $-v$ which is the speed of the conductor PQ. The induced emf Blv is called ***motional emf***. Thus, we are able to produce induced emf by moving a conductor instead of varying the magnetic field, that is, by changing the magnetic flux enclosed by the circuit.

(2) Eddy currents -Advantages of eddy currents with common practical applications ?

When bulk pieces of conductors are subjected to changing magnetic flux, induced currents are produced in them. Their flow patterns resemble swirling eddies in water. This effect was discovered by physicist Foucault (1819-1868) and these currents are called ***eddy currents***.

Consider the apparatus shown in Fig. 6.13. A copper plate is allowed to swing like a simple pendulum between the pole pieces of a strong magnet. It is found that the motion is damped and in a little while the plate comes to a halt in the magnetic field. Magnetic flux associated with the plate keeps on changing as the plate moves in and out of the region between magnetic poles. The flux change induces eddy currents in the plate. Directions of eddy currents are opposite when the plate swings into the region between the poles and when it swings out of the region.
If rectangular slots are made in the copper plate as shown in Fig. 6.14, area available to the flow of eddy currents is less. Thus, the pendulum plate with holes or slots reduces electromagnetic damping and the plate swings more freely.





This fact is helpful in reducing eddy currents in the metallic cores of transformers, electric motors and other such devices in which a coil is to be wound over metallic core. Eddy currents are undesirable since they heat up the core and dissipate electrical energy in the form of heat. Eddy currents are minimised by using laminations of metal to make a metal core. The laminations are separated by an insulating material like lacquer. The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths. This arrangement reduces the strength of the eddy currents. Since the dissipation of electrical energy into heat depends on the square of the strength of electric current, heat loss is substantially reduced.

Eddy currents are used to advantage in certain applications like:
(i) *Magnetic braking in trains*: Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are no mechanical linkages, the braking effect is smooth.
(ii) *Electromagnetic damping*: Certain galvanometers have a fixed core made of nonmagnetic metallic material. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.
(iii) *Induction furnace*: Induction furnace can be used to produce high temperatures and can be utilised to prepare alloys, by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt it.
(iv) *Electric power meters*: The shiny metal disc in the electric power meter (analogue type) rotates due to the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil.

6.5 Inductance - Mutual inductance: Mention of expression for mutual inductance of two coaxial solenoids – Mention of expression for induced emf $E = -M\frac{dI}{dt}$.

(1) Explain inductance of a coil ?
An electric current can be induced in a coil by flux change produced by another coil in its vicinity or flux change produced by the same coil. The flux through a coil is proportional to the current. That is, $\phi_B \propto I$.
Further, if the geometry of the coil does not vary with time then,

$$\frac{d\Phi_B}{dt} \propto \frac{dI}{dt}$$

For a closely wound coil of N turns, the same magnetic flux is linked with all the turns. When the flux ϕ_B through the coil changes, each turn contributes to the induced emf. Therefore, a term called *flux linkage* is used which is equal to $N\phi_B$ for a closely wound coil and in such a case
$N\phi B \propto I$

The constant of proportionality, in this relation, is called *inductance*. We shall see that inductance depends only on the geometry of the coil and intrinsic material properties.
Inductance is a scalar quantity. It has the dimensions of $[M L^2 T^{-2} A^{-2}]$ given by the dimensions of flux divided by the dimensions of current. The
SI unit of inductance is *henry* and is denoted by H.

(2) Define mutual inductance. Obtain an expression for mutual inductance of two co-axial solenoids and mention expression for induced emf ?
Consider two long co-axial solenoids each of length l *as shown in figure*. We denote the radius of the inner solenoid S_1 by r_1 and the number of turns per unit length by n_1. The corresponding quantities for the outer solenoid S_2 are r_2 and n_2, respectively. Let N_1 and N_2 be the total number of turns of coils S_1 and S_2, respectively. When a current I_2 is set up through S_2, it in turn sets up a

magnetic flux through S_1. Let us denote it by ϕ_1. The corresponding flux linkage with solenoid S_1 is

$N_1 \phi_1 = M_{12} I_2$ (6.9)

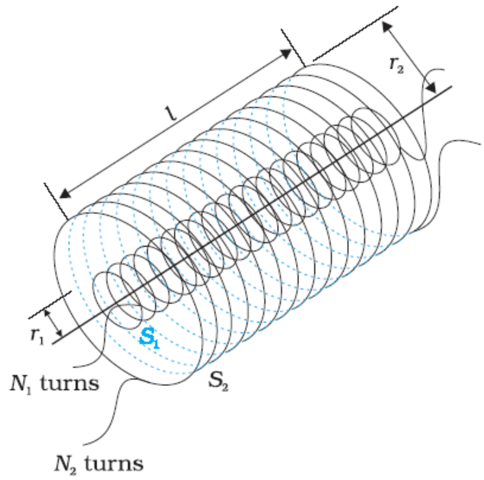
M_{12} is called the *mutual inductance* of solenoid S_1 with respect to solenoid S_2. It is also referred to as the *coefficient of mutual induction*.

Expression for mutual inductance of two co-axial solenoids :

For these simple co-axial solenoids it is possible to calculate M_{12}. The magnetic field due to the current I_2 in S_2 is $\mu_0 n_2 I_2$. The resulting flux linkage with coil S_1 is,

$$N_1\Phi_1 = (n_1 l)(\pi r_1^2)(\mu_0 n_2 I_2)$$

$$= \mu_0 n_1 n_2 \pi r_1^2 l I_2 \qquad (6.10)$$



where $n_1 l$ is the total number of turns in solenoid S_1.

Thus, from Eq. (6.9) and Eq. (6.10),

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l \qquad \text{-------} \quad (6.11)$$

We now consider the reverse case. A current $I1$ is passed through the solenoid $S1$ and the flux linkage with coil S_2 is,

$N_2\phi_2 = M_{21} I_1$ ----- (6.12)

M_{21} is called the *mutual inductance* of solenoid S_2 with respect to solenoid S_1.

The flux due to the current I_1 in S_1 can be assumed to be confined solely inside S_1 since the solenoids are very long. Thus, flux linkage with solenoid S_2 is

$$N_2\Phi_2 = (n_2 l)(\pi r_1^2)(\mu_0 n_1 I_1)$$

where $n_2 l$ is the total number of turns of S2. From Eq. (6.12),

$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$ ---------- (6.13)

Using Eq. (6.11) and Eq. (6.12), we get

$M_{12} = M_{21} = M$ (say)

if a medium of relative permeability μ_r had been present, the mutual inductance would be

$\boldsymbol{M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l}$

It is also important to know that the mutual inductance of a pair of coils, solenoids, etc., depends on their separation as well as their relative orientation.

Expression for induced emf :

Consider an experiment with two coils C_1 & C_2. Let the emf induced in coil C_1 wherever there was any change in current through coil C_2. Let ϕ_1 be the flux through coil C_1 (say of N_1 turns) when current in coil C_2 is I_2.

Then, from Eq. (6.9), we have

$N_1\phi_1 = MI_2$

For currents varrying with time,

$$\frac{d(N_1\Phi_1)}{dt} = \frac{d(MI_2)}{dt}$$

Since induced emf in coil C_1 is given by

$$\varepsilon_1 = -\frac{\mathrm{d}(N_1\Phi_1)}{\mathrm{d}t}$$

We get,

$$\varepsilon_1 = -M\frac{\mathrm{d}I_2}{\mathrm{d}t}$$

It shows that varying current in a coil can induce emf in a neighbouring coil. The magnitude of the induced emf depends upon the rate of change of current and mutual inductance of the two coils.

6.6 Self-inductance: Mention of expression for self-inductance of solenoid - Mention of expression for induced emf $E = -L\frac{dI}{dt}$. Derivation of energy stored in the coil.

(1) What is self-inductance ? Obtain an expression for self-inductance of solenoid ?

(a) Self-inductance :

It is possible that emf is induced in a single isolated coil due to change of flux through the coil by means of varying the current through the same coil. This phenomenon is called *self-induction*. In this case, flux linkage through a coil of N turns is proportional to the current through the coil and is expressed as

$N\phi_B \propto \mathrm{I}$

$N\phi_B = \mathrm{L\,I}$ -------- (6.15)

where constant of proportionality L is called *self-inductance* of the coil. It is also called the *coefficient of self-induction* of the coil. When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil. Using Eq. (6.15), the induced emf is given by

$$\varepsilon = -\frac{\mathrm{d}(N\Phi_B)}{\mathrm{d}t}$$

$$\varepsilon = -L\frac{\mathrm{d}I}{\mathrm{d}t} \quad \text{----------} \quad (6.16)$$

Thus, the self-induced emf always opposes any change (increase or decrease) of current in the coil.

(b) Self-inductance of a long solenoid :

To calculate the self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length. The magnetic field due to a current I flowing in the solenoid is $B = \mu_0\, n\, I$ (neglecting edge effects, as before). The total flux linked with the solenoid is

$$N\Phi_B = (nl)(\mu_0 n I)(A)$$
$$= \mu_0 n^2 A l I$$

where nl is the total number of turns. Thus, the self-inductance is,

$$L = \frac{N\Phi_B}{I} \quad = \mu_0 n^2 A l$$

If we fill the inside of the solenoid with a material of relative permeability μ_r (for example soft iron, which has a high value of relative permiability), then,

$$L = \mu_r\, \mu_0\, n^2 A l$$

The self-inductance of the coil depends on its geometry and on the permeability of the medium.

The self-induced emf is also called the *back emf* as it opposes any change in the current in a circuit.

(c) Expression for energy stored in the coil :

Physically, the *self-inductance plays the role of inertia*. It is the electromagnetic analogue of mass in mechanics.

So, work needs to be done against the back emf (ε) in establishing the current. This work done is stored as magnetic potential energy. For the current *I* at an instant in a circuit, the rate of work done is

$$\frac{dW}{dt} = |\varepsilon| I$$

If we ignore the resistive losses and consider only inductive effect, then using Eq. (6.16),

$$\frac{dW}{dt} = L\, I \frac{dI}{dt}$$

Total amount of work done in establishing the current *I* is

$$W = \int dW = \int_0^I L\, I\, dI$$

Thus, the energy required to build up the current *I* is,

$$W = \frac{1}{2} L I^2 \qquad \text{---------} \quad (6.19)$$

This expression reminds us of $mv^2/2$ for the (mechanical) kinetic energy of a particle of mass *m*, and shows that *L* is analogus to *m* (i.e., *L* is electrical inertia and opposes growth and decay of current in the circuit).

(d) Total inductance due to two coils :

Consider the general case of currents flowing simultaneously in two nearby coils. The flux linked with one coil will be the sum of two fluxes which exist independently. Equation (6.9) would be modified into

$$N_1 \Phi_1 = M_{11} I_1 + M_{12} I_2$$

where M_{11} represents inductance due to the same coil.

Therefore, using Faraday's law,

$$\varepsilon_1 = -M_{11} \frac{dI_1}{dt} - M_{12} \frac{dI_2}{dt}$$

M_{11} is the *self-inductance* and is written as L_1. Therefore,

$$\varepsilon_1 = -L_1 \frac{dI_1}{dt} - M_{12} \frac{dI_2}{dt}$$

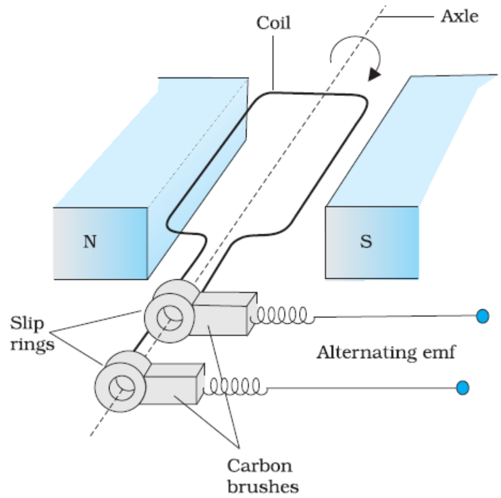
6.7 AC generator: Labeled diagram - Derivation of instantaneous emf in an ac generator

The Yugoslav inventor Nicola Tesla is invented AC generator.

An ac generator converts mechanical energy into electrical energy by producing a flux change.

Principle :

The basic elements of an ac generator are shown in Fig. It consists of a coil mounted on a rotor shaft. The axis of rotation of the coil is perpendicular to the direction of the magnetic field. The coil (called armature) is mechanically rotated in the uniform magnetic field by some external means. The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil. The ends of the coil are connected to an external circuit by means of slip rings and brushes.



When the coil is rotated with a constant angular speed ω, the angle θ between the magnetic field vector B and the area vector A of the coil at any instant t is $\theta = \omega t$ (assuming $\theta = 0^{\circ}$ at $t = 0$).

As a result, the effective area of the coil exposed to the magnetic field lines changes with time, and from Eq. (6.1), the flux at any time t is

$\phi_B = BA\cos\theta = BA\cos\omega t$

From Faraday's law, the induced emf for the rotating coil of N turns is then,

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t)$$

Thus, the instantaneous value of the emf is

$\varepsilon = NBA\omega \sin\omega t$ ------- (6.21)

where NBA is the maximum value of the emf, which occurs when $\sin\omega t = \pm 1$. If we denote $NBA\omega$ as ε_0, then

$\varepsilon = \varepsilon_0 \sin\omega t$ --------- (6.22)

Since the value of the sine function varies between +1 and –1, the sign, or polarity of the emf changes with time. Note from Fig. 6.17 that the emf has its extremum value when $\theta = 90^{\circ}$ or $\theta = 270^{\circ}$, as the change of flux is greatest at these points.

The direction of the current changes periodically and therefore the current is called *alternating current* (ac). Since $\omega = 2\pi\nu$, The instantaneous emf in Eq. (6.22) can be written as

$\varepsilon = \varepsilon_0 \sin 2\pi\nu t$ ---------- (6.23)

where ν is the frequency of revolution of the generator's coil.

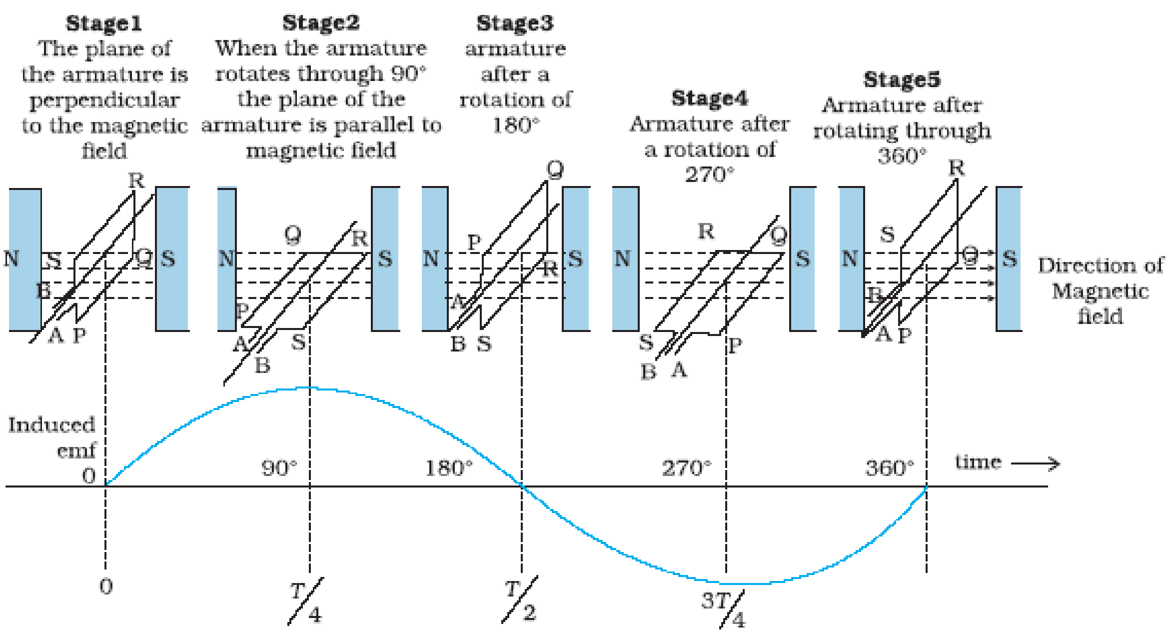


Fig. 1 : An alternating emf is generated by a loop of wire rotating in a magnetic field.

In commercial generators, the mechanical energy required for rotation of the armature is provided by water falling from a height, for example, from dams. These are called *hydro-electric generators*. Alternatively, water is heated to produce steam using coal or other sources. The steam at high pressure produces the rotation of the armature. These are called *thermal generators*. Instead of coal, if a nuclear fuel is used, we get *nuclear power generators*. Modern day generators produce electric power as high as 500 MW, i.e., one can light up 5 million 100 W bulbs! In most generators, the coils are held stationary and it is the electromagnets which are rotated. The frequency of rotation is 50 Hz in India. In certain countries such as USA, it is 60 Hz.

6.8 Numerical Problems.

Question Bank

1. Name the phenomena in which a current induced in coil due to change in magnetic flux linked with it.
Answer: Electromagnetic Induction

2. Define electromagnetic induction.
Answer: The phenomena of induction of an emf in a circuit due to change in magnetic flux linked with it is called electromagnetic induction.

3. What is magnetic flux? Explain.
Answer: Magnetic flux through a surface is the scalar product of the magnetic field and the area. The magnetic flux through an area kept in a magnetic field is given by:
$\phi_B = \vec{B} \cdot d\vec{S} = B\, dS \cos\theta$
Magnetic flux is a scalar quantity. Its SI unit is weber (Wb).

4. What does magnetic flux measure?
Answer: Magnetic flux through a surface is a measure of the number of lines of magnetic field lines passing through the surface.

5. Is magnetic flux scalar or a vector?
Answer: Scalar

6. What is the SI unit of magnetic flux?
Answer: it is weber (Wb) or T m^2

7. When is the flux through a surface a) maximum? B) zero?
Answer: a) when the plane of the surface is perpendicular to the magnetic field ($\theta = 0^o$)
b) when the plane of the surface is kept parallel to the magneti field ($\theta = 90^o$)

8. What is the value of the magnetic flux through a closed surface:
Answer: Zero

9. Does a magnet kept near a coil induce current in it?
Answer: No. EMF is induced in the coil only when the magnet is moving relative the coil.

10. Why does a galvanometer connected to a coil show deflection when a magnet is moved near it?
Answer: Moving a magnet near the coil changes the magnetic field at the coil which in turn changes the magnetic flux linked with the coil. Therefore an emf is induced in the circuit hence a current.

11. What happens to the induced emf if an iron bar is introduced into the coils in Faraday' experiment ?
Answer: The emf increases

12. State and explain Faraday' law of electromagnetic induction.

Answer: “The magnitude of the induced emf in a circuit equal to the time rate of change of magnetic flux through the circuit”.

If ϕB is the varying magnetic flux linked with a circuit then the magnitude of the induced emf in the circuit is:

$$|\varepsilon| = \left|\frac{d\phi_B}{dt}\right|$$

13. Give the mathematical from Faraday’ law of electromagnetic induction.
Induced emf

$$|\varepsilon| = N\left|\frac{d\phi_B}{dt}\right|$$

where N is the number of turns in the coil and ϕ_B is the magnetic flux linked with the coil.

14. Magnetic flux linked with a closed loop at a certain instant of time is zero. Does it imply that that induced emf at that instant is also zero?
Answer: No. The emf does not depend on the magnetic flux but on the change of magnetic flux.

15. Can you induce an emf in an open circuit by electromagnetic induction?
Answer: Yes.

16. Does the electromagnetically induced emf in a coil depend on the resistance of the coil?
Answer: No. But the current does.

17. If the number of turns in a coil subjected to a varying magnetic flux is increased, what happens to the induced emf?
Answer: EMF also increases (directly proportional to the number of turns)

18. How can magnetic flux linked with a surface be changed?
Answer: By changing a) the magnetic field b) area of the surface or c) by changing the orientation of the area with the magnetic field

19. Why did Faraday’ law need a correction by Lenz?
Answer: Because Faraday’ law was incompatible with the law of conservation of energy.

20. State Lenz’ law.
Answer: “The polarity of the induced emf is such that it tend to produce a current which opposes the change in magnetic flux that produced it.

21. Write Faraday’ law with Lenz’s correction.

$$\text{Induced emf}, \varepsilon = -\frac{d\phi_B}{dt}$$

22. What does the negative sign in the following expression imply ?

$$\varepsilon = -\frac{d\phi_B}{dt}$$

Answer: The negative sign implies that the direction of induced emf opposes its cause, the change in magnetic flux.

23. The magnetic flux linked with a coil changes from 12 x 10^{-3} Wb to 6 x 10^{-3} Wb in 0.01 second. Calculate the induced emf.

Answer:

Induced emf =

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{(6 \text{ x } 10^{-3} - 12 \text{ x } 10^{-3})}{0.01} = \frac{6 \text{ x } 10^{-3}}{10^{-2}} = 0.6\, V$$

24. If you bring the North Pole of a magnet near a face of a coil, what is the direction of the current induced in that side?

Answer: Anticlockwise. This makes that side of the coil magnetically North which repels the magnet coming towards it.

25. If the area of a coil kept in a magnetic field is changed, is there any induced current in it? Answer: Yes. By changing the area we change the magnetic flux linked with the coil. The current induced is in a direction to counteract this change in magnetic flux.

26. Lenz's law consistent with the law of conservation of energy?

Answer: Yes

27. Why does not the induced current in a coil flow in clockwise direction if the south pole of a magnet is moved away from it?

Answer: If the induced current flows in clockwise direction, that face of the coil becomes magnetic south. This repels the away – moving magnet and the magnet flies off without spending energy anymore. This would be inconsistent with the law of conservation of energy.

28. Use Lenz' law to find the direction of induced emf in a coil when (a) a north pole is brought towards the coil (b) north pole taken away from the coil (c) A south pole is brought towards the coil and (d) a south pole is taken away from the coil.

Answer: a) Anticlockwise b) Clockwise c) Clockwise d) Anticlockwise

29. What is motional emf?

Answer: The emf induced in a conductor moving in a plane perpendicular to a magnetic field is called motional emf.

30. What happens to the magnitude of the motional emf if the a) velocity of the rod b) length of the rod c) the applied magnetic field are increased?

Answer: increases (in all of the three cases)

31. Is there an induced emf (motional emf) in a conductor if it moves in a plane parallel to a magnetic field?

Answer: No

32. A wire pointing north-south is dropped freely towards earth. Will any potential difference be induced across its ends?
Answer: No. (If it is made to fall in E –W direction, there is an emf across its ends)

33. When a glass rod moves perpendicular to a magnetic field, is there any emf induced in it?
Answer: No. Because glass is an insulator.

34. What are eddy currents?
Answer: When a bulk conductor is placed in a varying magnetic field, circulating currents are induced in it. These currents are called eddy currents.

35. What happens to a velocity of a conductor when it moves in a varying magnetic field?
Answer: Decreases. The eddy currents induced in the conductor damp the motion of the conductor.

36. Why are the oscillations of a copper disc in a magnetic field damped?
Answer: Because of the eddy currents produced in the disc.

37. Why are eddy currents undesirable?
Answer: Because they produce heating effect and damping effect.

38. Mention applications of eddy currents.
Answer: a. Magnetic braking in trains b. Magnetic damping in galvanometers c. Induction furnaces d. Electric power meters

39. How does the magnetic braking in train work?
Answer: Strong electromagnets placed over the rails are activated. This produces eddy current in the rails which produce braking effect.

40. What is principle behind induction furnaces?
Answer: Eddy currents. In an induction furnace, a high frequency AC is passed through a coil which surrounds the metal to be melted. The eddy currents produced in the metal heats it to high temperatures and melts it.

41. How can eddy currents be minimized?
Answer: Eddy currents can be minimized by slicing the conductor into pieces and laminating them so that the area for circulating currents decreases.

42. What is inductance?.
Answer: Inductance of a coil is the magnetic flux linked with the coil per unit current producing it. $L = \phi_B / I$

43. On what factors does the inductance of a coil depend?
Answer: The inductance of coil depends on the geometry of the coil and intrinsic material properties.

44. What is the SI unit of inductance? Define it.
Answer: Henry (H). One henry is defined as the inductance of a coil for which there is a magnetic flux of 1 Wb is linked with it when a current of 1 A is causing it.

45. What is mutual induction?
Answer: Mutual induction is the phenomena of production of emf induced in a coil due to a change in current in a nearby coil.

46. Define mutual inductance (coefficient of mutual inductance). Mention its SI unit.
Answer: Mutual inductance is the ratio of the magnetic flux linked with a coil due to a current in a nearby coil. Its SI unit is henry.

47. Mention the factors on which mutual inductance depends.
Answer: a. The number of turns per unit length in each coil, b. Area of the coils, c. Length of the coils, d. permeability of medium inside the coils, e. separation between the coils, f. the relative orientation of the coils.

48. Give the expression for mutual inductance between two coils which are wound one over another.
Answer: $M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 L$
where μ_r is the relative permeability of the medium inside the coils, μ_0 is the permeability of free space, n_1 and n_2 are the number turns per unit length of each coils, $r^2{}_1$ is the radius of the inner coil and L is the length of the coil.

49. Mention one device which works on the principle of mutual induction.
Answer: Transformer

50. How can mutual inductance be increased without changing the geometry of the coils?
Answer: By inserting a ferromagnetic material inside the coils

51. Mention the expression for the emf induced in a coil of a mutual inductance due to the change in current through another.
Answer:
$$\varepsilon_1 = -M \frac{dI_2}{dt}$$

52. What is self-induction?
Answer: Self – inductance is the ratio of the magnetic flux linked with a coil to the current flowing through it.

53. What is self – inductance? Mention its SI unit.
Answer: Self – inductance is the ratio of the magnetic flux linked with a coil to the current flowing through it. Its SI unit is henry.

54. Mention the expression for the emf induced in a solenoid in terms of change in current through it.

Answer:

$$\varepsilon = -L\frac{dI}{dt}$$

55. What is electrical analogue of mass in mechanics? OR Which electrical device plays the role of electrical inertia?
Answer: Self – inductance.

56. What is back emf?
Answer: The emf induced in a coil which opposes the rise of current through a coil is called back emf.

57. Why does a bulb connected in series with a self – inductance glows brilliantly for a moment when the current in the circuit is switched off?
Answer: Because of the forward emf produced.
58. Mention the expression for the self – inductance.
Answer:

$$L = \mu_o\mu_r\, n^2 Al$$

where μ_r is the relative permeability of the medium inside the coil, n is the number of turns per unit length of the coil, A is the area of the coil and l is the length of the coil.

59. Does emf rise instantaneously after the battery connected to it is switched on?
Answer: No. Because the back emf produced opposes the growth of current through the coil.

60. Can a thin wire act as an inductor?
Answer: No. Because a thin wire does not enclose a significant magnetic flux.

61. On what factors does the coefficient of self – inductance of a coil depend?
Answer: a) The length of the solenoid (α l), b) the number of turns per unit length in the solenoid (α n^2) c) the area of the coil (α A) d) the permeability of the medium in ide the len id (α μ_r)

62. What happens to the self – induction of a coil if a soft – iron rod is inserted into it?
Answer: Increases. Since iron has large permeability, the inductance increases.

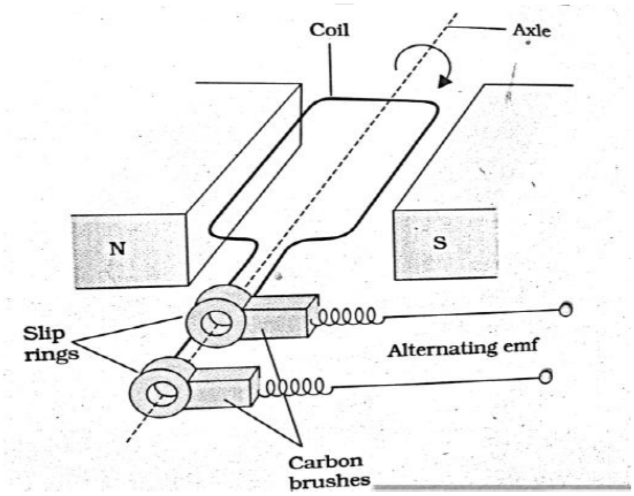
63. Mention the expression for the magnetic potential energy stored in an inductor.
Answer:

$$Energy, E = \frac{1}{2}LI^2$$

where L is the self – inductance of the inductor and I is the current flowing through it.

64. What is an AC generator? What is its principle?
Answer: An AC generator is which converts mechanical energy into electrical energy (alternating emf). It works on the principle of electromagnetic induction.

65. Draw a neat labeled diagram of an AC generator.



66. What is the frequency of AC in India?
Answer: 50 Hz

Long Answer Questions:

67. Explain coil and magnet experiment performed by Faraday to discover electromagnetic induction.
Answer: When the North-pole of a magnet is moved towards a coil connected to a galvanometer, the galvanometer in the circuit shows a deflection indicating a current (and hence an emf) in the circuit. The deflection continues as long as the magnet is in motion. A deflection can be observed if and only if the coil and the magnet are in relative motion. When the magnet is moved away from the coil, the galvanometer shows a deflection in the opposite direction.
Bringing the South-pole towards the coil produces the opposite deflection as bringing the North-pole. Faster the magnet or the coil is moved, larger is the deflection produced.
By this experiment we can conclude that: the relative motion between the coil and the magnet generates an emf (current) in the coil.

68. Explain the coil and coil experiment of Faraday.
Answer: If we replace the magnet by a current carrying coil, a similar observation can be made. When the current carrying coil is brought near the coil connected to the galvanometer, the galvanometer shows deflection indicating a current and hence an emf in the coil. Thus, the relative motion between a coil and another coil carrying current induces an emf (current).

69. Derive an expression for motional emf induced in a conductor moving in a magnetic field.
Answer: Consider metallic frame MSRN placed in a uniform constant magnetic field. Let the magnetic field be perpendicular to the plane of the coil. Let a metal rod PQ of length l placed on it be moving towards left with a velocity towards left as shown in the figure. Let the distance of PQ from SR be x.

The magnetic flux linked with the area SPQR is:
$\phi_B = B\,A\cos\theta = B\,lx\cos 0 = B\,lx$

As the rod PQ is moving towards left with a velocity , x is changing and
$\vec{v} = -\frac{dx}{dt}$.
Hence:
$$\text{Induced emf, } \varepsilon = -\frac{d\phi_B}{dt} = -\frac{d(Blx)}{dt} = -Bl\frac{dx}{dt} \qquad \Longrightarrow \varepsilon = Blv$$

This emf is induced in the rod because of the motion of the rod in the magnetic field. Therefore this emf is called motional emf.

70. Derive an expression for magnetic potential energy stored in a self – inductor.
Answer: When a current is established in a solenoid (coil), work has to be done against the back emf. This work done is stored in the form of magnetic energy in the coil. For a current I in the coil, the rate of work done (power) is:
$$\frac{dW}{dt} = \varepsilon I$$
But we know that:
$$\varepsilon = L\frac{dI}{dt}.$$
Therefore:
$$\frac{dW}{dt} = LI\frac{dI}{dt} \Longrightarrow dW = LIdI$$

Therefore the work done in establishing a current I is given by:
$$W = \int dW = \int LIdI = \frac{1}{2}LI^2$$
This work is stored in the coil in the form of energy. Therefore the energy stored in a solenoid is given by:
$$U = \frac{1}{2}LI^2$$

71. Explain the construction and working of an AC generator.
Answer: Fig (refer the figure of the AC generator) An AC generator consists of a coil (armature) placed in a magnetic field as shown. The coil can be rotated about an axis perpendicular to the magnetic field. When the coil is rotated the angle (θ) between the magnetic field and the area changes. Therefore, the flux linked with the coil changes which induces an emf in the coil. The ends of the armature are connected to an external circuit.

72. Give the theory of an AC generator.

Answer: Let the area of the coil be A and the magnetic field be B. Let N be the number of turn in the armature. Let θ be the angle between the area and the magnetic field. If ω is the constant angular velocity of the rotation of the coil then $\theta = \omega t$.
The magnetic flux linked with the coil is:

$$\phi_B = NBA\cos\theta = NBA\cos\omega t$$

From Faraday' law the induced emf by rotating coil is given by:

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d(NBA\cos\omega t)}{dt} = -NBA\frac{d(\cos\omega t)}{dt}$$

$$\Longrightarrow \varepsilon = -NBA(-\omega\sin\omega t) = NBA\omega\sin\omega t$$

$$\Longrightarrow \varepsilon = \varepsilon_0\sin\omega t$$

where $\varepsilon_0 = NBA\omega$ is called the peak value of the emf or the maximum emf.
If ν is the frequency of rotation of the armature then: $\omega = 2\pi\nu$. Therefore
$\varepsilon = \varepsilon_0 \sin 2\pi\nu t$
This is the expression for the alternating emf produced by a generator. As the time increases the emf ε increases from zero to $+\varepsilon_0$ and then falls to zero. Then it becomes negative and reaches $-\varepsilon_0$. Then gradually it increases to become zero. This completes one cycle of AC.

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MOST LIKELY QUESTIONS :
1M - 1Q; 2M - 1Q ; 3M - 1Q or 1M -1Q; 5M-1Q (LA)

2M - 1Q :
1. State and explain Faraday's law of electromagnetic induction ?
2. State Lenz's law and explain its significance as conservation of energy.
3. What is motional emf ? Obtain an expression for it ?

3M - 1Q :
1. Explain inductance of a coil ? Define (i) Self inductance (ii) Mutual inductance ?
2. What is self-inductance ? Obtain an expression for energy stored in a coil ?
3. What is eddy current ? Explain its two practical applications ?

5M-1Q (LA) :
1. With principle, theory, and functioning diagram, explain the working of AC generator.
2. Define mutual inductance. Obtain an expression for mutual inductance of two co-axial solenoids and mention expression for induced emf ?

CHAPTER 7
ALTERNATING CURRENT

8M [1Q - 3M, 1Q - 5M (NP)] **or 7M** [2M-1Q; 5M-1Q(NP)]

1. Mention of expression for instantaneous, peak and rms values of alternating current and voltage :

(1) Mention of expression for instantaneous, peak and rms values of alternating current and voltage.

Ans :

(a) Expression for instantaneous value of alternating current = $i = i_m \sin \omega t$

(b) Expression for instantaneous value of alternating voltage = $v = v_m \sin \omega t$

(c) Expression for peak value of alternating current = $i_m = v_m/x$

(d) Expression for peak value of alternating voltage = $v_m = NAB\omega$

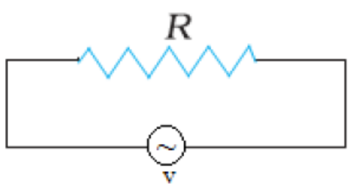
(e) Expression for rms value of alternating current = effective current = $I= i_{rms} = i_m/\sqrt{2} = 0.707\ i_m$

(f) Expression for rms value of alternating voltage = $v_{rms} = V = v_m/\sqrt{2} = 0.707\ v_m$

Note : Root mean square of alternating current or emf can be calculated over any period of the cycle since it is based on the heat energy produced when pass through a resistor.

2. AC voltage applied to a resistor: Derivation of expression for current, mention of phase relation between voltage and current, phasor representation.

(2) Derive the expression for current when AC voltage applied to a resistor. What is the phase relation between voltage and current. Represent in phasor diagram.

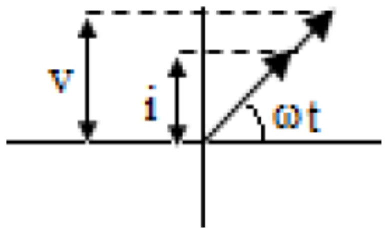


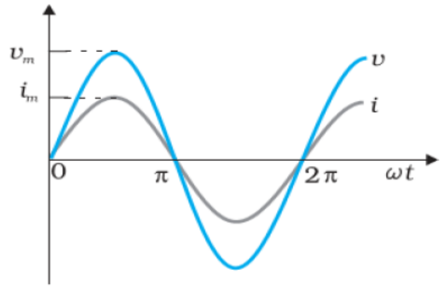
Consider pure resistor of resistance R connected to sinusoidal AC.
Let $v = v_m \sin \omega t$ --- (1) be the instantaneous voltage.
According to Kirchoff's loop rule, $v_m \sin\omega t = iR$; here 'i' is the AC current.

$\therefore\ i = \frac{v_m}{R}\sin\omega t \quad \Rightarrow i = i_m \sin\omega t$ ---(2)

$\Rightarrow i_m = \frac{v_m}{R}$ here i_m is called current amplitude (or peak current)

From (1) and (2), **voltage-current are in phase with each other** and the phasor diagram is as shown.





3. AC voltage applied to an inductor: Derivation of expression for current, mention of phase relation between voltage and current, phasor representation and mention of expression for inductive reactance.

(3) Derive the expression for current when AC voltage applied to a inductor. Mention the expression for inductive reactance.

Consider an inductor of inductance L is connected across an AC source,
Let $v = v_m \sin \omega t$ --------(1), here v - the source voltage, v_m – peak voltage , ω - angular frequency of AC.

The self induced emf in the inductor is $\varepsilon = -L\frac{di}{dt}$

According to Kirchoff's loop rule, $v - L\frac{di}{dt} = 0$

$\Rightarrow v_m \sin\omega t - L\frac{di}{dt} = 0$ $\Rightarrow L\frac{di}{dt} = v_m \sin\omega t.$

This indicates the current in an inductor is a function of time.

$\Rightarrow$ $di = \frac{v_m}{L}$ [sin ωt] dt

To obtain the current at any instant, we integrate the above expression.

$$\text{i.e } i = \int di = \frac{v_m}{L}\int \sin\omega t \, dt \qquad \Rightarrow i = \frac{v_m}{L}\left[\frac{-\cos\omega t}{\omega} + \text{constant}\right]$$

$$\Rightarrow i = \frac{v_m}{L\omega}[-\cos\omega t]$$

[because, we can show the integration constant over a cycle is zero]

If we take $\frac{v_m}{L\,\omega} = i_m$, the amplitude of the current, then $i = i_m$ [-cos ωt]

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right) \text{------(2)}$$

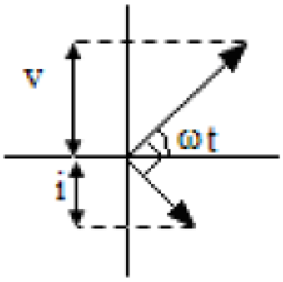
Inductive reactance is given by $X_L = \omega L = 2\pi\nu L = 2\pi f L$

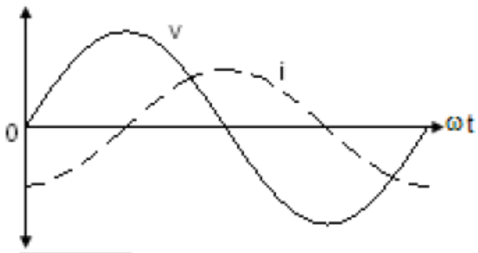
The SI unit of X_L is ohm (Ω)

$$\text{Definition of } X_L = \frac{v_{rms}}{i_{rms}} = \frac{\text{RMS value of voltage across inductor}}{\text{RMS value of current through inductor}}$$

(4) What is the phase relation between voltage and current in inductor. Represent in phasor diagram.

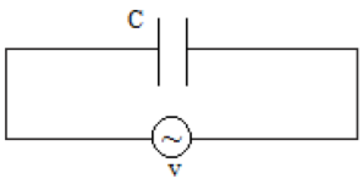
The current is lagging the applied emf by an angle π/2
The phasor diagram is as shown.





4. AC voltage applied to a capacitor: Derivation of expression for current, mention of phase relation between voltage and current, phasor representation and mention of expression for capacitive reactance.

(5) Derive the expression for current when AC voltage applied to a capacitor. mention the expression for capacitive reactance.



Consider a capacitor of capacitance C is connected across an AC source,
Let $v = v_m \sin \omega t$ --------(1), here v - the source voltage, v_m – peak voltage , ω - angular frequency of AC.

The p.d. across the capacitor at any instant of time is $v = q/C$

According to Kirchoff's loop rule, $v_m \sin \omega t - q/C = 0$

$$\Rightarrow v_m \sin\omega t = \frac{q}{C} \qquad \Rightarrow q = v_m C \sin\omega t$$

∴ Instantaneous current,

$$i = \frac{dq}{dt} = v_m C \frac{d(\sin\omega t)}{dt} \qquad \Rightarrow i = v_m C(\omega \cos\omega t)$$

Let $\omega v_m C = i_m$ be the amplitude of the current, then $i = i_m \cos \omega t$

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{--------(2)}$$

Capacitive Reactance : X_C :

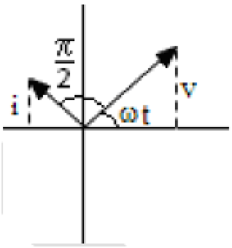
Capacitive reactance is given by $= X_C = 1/\omega C = 1/2\pi\nu C = 1/2\pi f C$
The SI unit of X_C is ohm (Ω)
Defination of X_C :

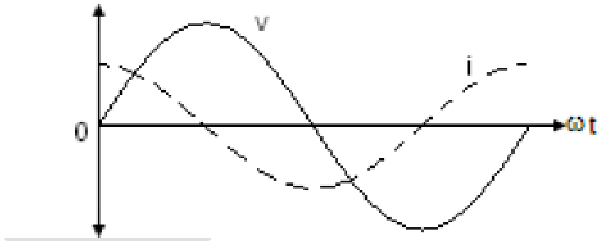
$$X_C = \frac{v_{rms}}{i_{rms}} = \frac{\text{RMS value of voltage across capacitor}}{\text{RMS value of current through capacitor}}$$

6. What is the phase relation between voltage and current in capacitor. Represent in phasor diagram.

The current in the circuit is leading the voltage by an angle $\pi/2$.

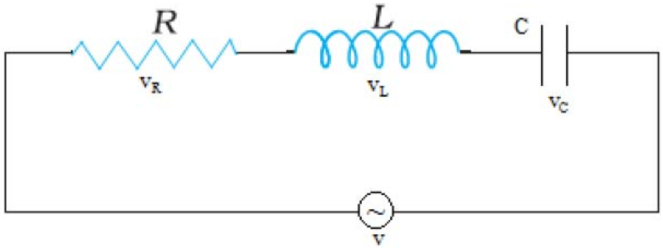
The phasor diagram is as shown.





5. AC voltage applied to series LCR circuit: Derivation of expression for impedance, current and phase angle using phasor diagram - Electrical resonance - Derivation of expression for resonant frequency - Mention of expressions for bandwidth and sharpness (quality factor).

(7) Derive the expression for impedance, current and phase angle in a series LCR circuit using phasor diagram.



Consider a series LCR circuit connected to an AC source $v = v_m \sin\omega t$ ---- (1)
Let $i = i_m \sin(\omega t+\phi)$ ---- (2) be the instantaneous current through the circuit and ϕ is the phase difference between the applied voltage and the current.
Voltage equation at any instant

$$\vec{V_R} + \vec{V_L} + \vec{V_C} = \vec{V}$$

Its magnitude of v is the phasor sum of v_R, v_L and v_C.
And the phasor diagram for the circuit is as shown below. The symbols in the diagram are having usual meaning.

We know, $v_{Rm} = i_m R$, $v_{Cm} = I_m X_C$ and $v_{Lm} = i_m X_L$

From the diagram, $v_m^2 = v_{Rm}^2 + (v_{Cm} - V_{Lm})^2$

$$\Rightarrow v_m^2 = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$

$$\Rightarrow v_m^2 = i_m^2 \left[R^2 + (X_C - X_L)^2\right]$$

$$\Rightarrow i_m^2 = \frac{v_m^2}{R^2 + (X_C - X_L)^2}$$

$$\Rightarrow i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

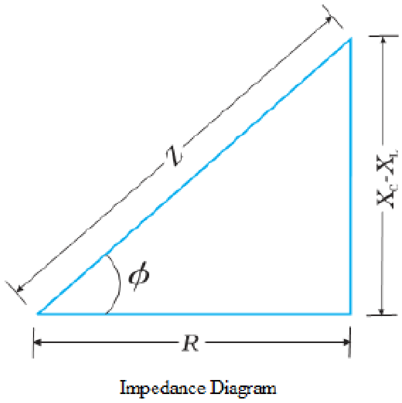
Here,

$\sqrt{R^2 + (X_C - X_L)^2}$ is analogous to resistance in DC called impedance, Z.

$$\therefore Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\Rightarrow i_m = \frac{v_m}{Z}.$$

If ϕ is the phase angle between i and v,



Impedance Diagram

$$\tan\phi = \frac{v_{Cm} - v_{Lm}}{v_{Rm}} \qquad \therefore \tan\phi = \frac{X_C - X_L}{R}$$

$$\Rightarrow \phi = \tan^{-1}\left[\frac{X_C - X_L}{R}\right]$$

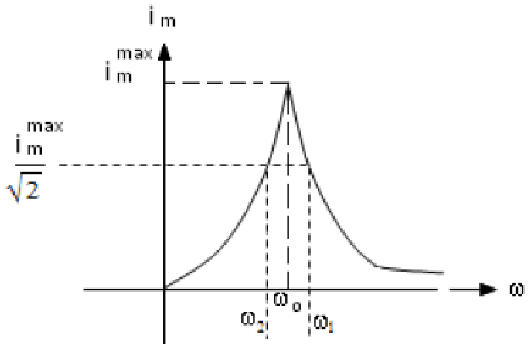
(8) What is electrical resonance ? Derive the expression for resonant frequency.

Series LCR circuit is said to be in resonance when current through the circuit is maximum In an series LCR circuit current amplitude is given by

$$i_m = \frac{V_m}{Z}$$

$$\therefore \quad i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}.$$

where $X_C = 1/\omega C$ and $X_L = \omega L$



If frequency is varied, at particular angular frequency ω_o the condition $X_C = X_L$ is achieved, this condition is called **resonance**,

$$\frac{1}{\omega_o C} = \omega_o L$$

$$\therefore \omega_o = \frac{1}{\sqrt{LC}} \qquad \text{or} \qquad 2\pi\nu_o = \frac{1}{\sqrt{LC}}$$

$$\nu_o = \frac{1}{2\pi\sqrt{LC}}$$ is called resonant frequency

(9) Mention the expressions for bandwidth and sharpness (quality factor) ?

Let ω_1 and ω_2 are two applied frequencies for which the current amplitude is $1/\sqrt{2}$ times the maximum value, Then $\omega_1 - \omega_2 = 2\Delta\omega$ is called bandwidth of the circuit.

Also, Band width = 2Δω = R/L
Sharpness of resonance is denoted by quality factor(Q-factor),

$$Q = \frac{\omega_o}{2\Delta\omega} = \frac{\text{resonacne frequency}}{\text{band width}}$$

Also $Q = \frac{\omega_o L}{R}$ & $Q = \frac{1}{\omega_o CR}$

6. Mention of expression for power in ac circuit - Power factor and qualitative discussion in the case of resistive, inductive and capacitive circuit-Meaning of wattless current.

(10) Mention the expression for power and power factor in ac circuit. What are their values in the case of resistive, inductive and capacitive circuit.

In an series LCR circuit, average power over a full cycle of AC,

$$p = \frac{v_m i_m}{2}\cos\phi \quad \Rightarrow p = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi \quad \Rightarrow p = VI\cos\phi \quad \Rightarrow p = I^2 Z\cos\phi$$

Where V and I are RMS values of voltage and current and the term **cosϕ** is called power factor.
Power factor is given by **$\cos\phi = R/Z$**
Case (1): In purely resistive circuit ϕ=0. Power factor, **$\cos\phi = 1$**

Power $p = v\,i = i^2R$.

Case (2) In purely inductive circuit or capacitive circuit $\phi = \pi/2$
Power factor, **$\cos\phi = 0$**
Power p = 0.

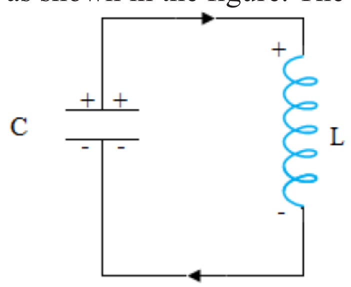
(11) What is meant by wattless current.

The AC current through pure L and C circuit is called wattles current. *This is because power factor of pure L and C circuits is zero.*

7. LC oscillations: Qualitative explanation - Mention of expressions for frequency of LC oscillations and total energy of LC circuit.

(12) Explain LC oscillations qualitatively and mention expressions for frequency of LC oscillations and total energy of LC circuit.

Let a capacitor be charged q_m (at $t = 0$) and connected to an inductor as shown in the figure. The charge oscillates from one plate of capacitor to another plate through the inductor. This results in electric oscillations called LC oscillation.



The moment the circuit is completed, the charge on the capacitor starts decreasing, giving rise to current in the circuit. As q decreases , energy stored in the capacitor decreases and the energy transferred from capacitor to inductor.

Once the capacitor is fully discharged, magnetic field begin to decrease produces an opposing emf. Now capacitor is begin to but in opposite direction(acc to Lenz's law). Charge oscillates simple harmonically with natural frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

Charge varies sinusoidally with time as $q = q_m \cos(\omega_o t)$. And current varies sinusoidally with time as $i = \omega_o q_m \sin\omega t = i_m \sin\omega t$.

At time t = 0, electrical energy stored in the capacitor, $U_E = \frac{1}{2}\frac{q_m^2}{C}$ and magnetic energy in the inductor $U_B = 0$

Similarly, when $U_B = \frac{1}{2} L i_m^2$ then $U_E = 0$

$\therefore$ Total energy of the LC circuit at any instant of time,

$$U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \frac{1}{2}\frac{q_m^2}{C} = \frac{1}{2}Li_m^2$$

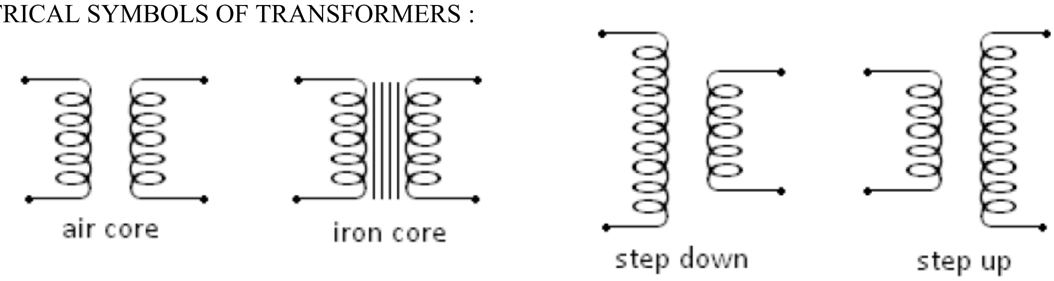
8. Transformer: Principle, construction and working - Mention of expression for turns ratio - Sources of energy losses, Numerical Problems.

(13) Write a note on transformer with special reference to principle, construction and working.

It is a device used to increase or decrease the AC. It works on the principle of mutual induction. A transformer consists of two sets of coils, insulated from each other. They are wound on a soft-iron core. One of the coil is called primary (input) with N_P turns and the other is called secondary (output) with N_S turns. When an alternating voltage v_p is applied to the primary, the induced magnetic flux is linked to the secondary through the core. So an voltage vs is induced in secondary If $N_s > N_p$ then transformer is called Step up transformer; where $v_s > v_p$, and if $N_p > N_s$, then the transformer is called Step down transformer; where $v_s < v_p$. If the transformer is ideal, $p_{in} = p_{out}$, i.e., $i_p v_p = i_s v_s$.

$$\Rightarrow \frac{i_p}{i_s} = \frac{v_s}{v_p} = \frac{N_s}{N_p}$$

ELECTRICAL SYMBOLS OF TRANSFORMERS :



(14) Mention the sources of energy losses in transformer. How they can be minimised?

(i) Magnetic flux leakage - It can be reduced by winding the primary and secondary coils one over the another.

(ii) Ohmic loss due to the resistance of the windings (wires) - It can be reduced by using thick copper wires.

(iii) Eddy current loss- It can be minimised by laminating and insulating the core of the transformer.

(iv) Hysteresis loss - It can be minimised by using material(soft iron) which has a low hysteresis loss.

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One Mark Questions :

(1) What is an alternating voltage ?

Alternating voltage is one whose magnitude changes continuously with time and direction reverses periodically.

(2) What is an alternating current ?

An alternating current is one whose magnitude changes continuously withtime and direction reverses periodically.

(3) What is the average value of ac over a cycle and why ?

Ans : Zero. It is because positive half cycle is exactly equal to the negative half cycle.

(4) What is the peak value of 220 volt ac ?

Here, 220 volt is rms value. Peak value $V_0 = \sqrt{2}\ V_{rms} = \sqrt{2} \times 220 = 311$ volt.

(5) Why r.m.s value of a.c. defined on heating effect ?

Ans : It is because the heating effect of electric current does not depend on the direction of current.

(6) Which value of current is read by an a.c. ammeter ?

Ans : r.m.s. value of current.

(7) The divisions marked on scale of a.c. ammeter are not equally spaced. Why ?

The working of a.c. ammeter is based on heating effect of electric current and heat produced is directly proportional to l2 (and not l). Therefore, an a.c. ammeter has non-linear scale.

(8) What is the frequency of direct current ?

Ans : Zero.

(9) What is the reactance of an inductor for d.c. ?

Zero. The reactance of an inductor is $X_L = 2\pi fL$ and for d.c., $f = 0$. Therefore, an inductor acts as a conductor for d.c.

(10) What is the phase difference between current through and voltage across (1) resistor, (2) inductor, (3) Capacitor ?

Ans : (1) In resistor, current is in phase with the voltage. (2) In inductor, current lags the applied voltage by 90° or $\pi/2$ (3) In capacitor, current leads the applied voltage by 90° or $\pi/2$.

(11)

PROBLEMS :

1. An alternating current is given by I = 5 sin 100πt. Find (1) maximum value of current (2) Frequency, (3) Time period

Ans : Comparing with $I = I_0 \sin \omega t$, (1) $I_0 = 5$ Amp, (2) $f = \omega/2\pi = 100\pi/2\pi = 50$ Hz, (3) $T = 1/f = 1/50 = 0.02$ sec.

2. An a.c. circuit consists of a pure resistance of 10 Ω and is connected across an a.c. supply of 230V, 50 Hz. Calculate (i) circuit current (ii) power dissipated.

Ans : Given Vm =230V, R = 10 Ω, f = 50 Hz
(i) Circuit current, Irms = Vrms/R =230/10 = 23A
(ii) Power dissipated, P = $V_{rms}.I_{rms}$ = 230 x 23 = 5290 W

3. Calculate resistance peak voltage and rms current in a 100W bulb connected to 220V supply.

Ans : Given Vrms = 220V, P = 100W; Irms = ? ; V_0 = ?
P = V^2rms/R
R = V^2rms/P = $(220)^2/100$ = 484 Ω
Peak voltage = V_0 = Vrms x √2 = 220 x √2 = 311.126 V
Irms = Vrms/R = 220/484 = 0.45 A.

4. A pure inductive coil allows a current of 10A to flow from a 230 V, 50 Hz supply. Find (i) Inductive reactance (ii) inductance of the coil (iii) power consumed ?

Ans :

Homework

10. A series LCR circuit is connected to 220 V ac source of variable frequency. The inductance of the coil is 5 H, capacitance of the capacitor is 5μF and resistance is 40 Ω. At resonance, calculate a) The resonant frequency, b) current in the circuit and c) the inductive reactance. (Sample QP).

Ans :

(a) $f_0 = 1/(2\pi\sqrt{LC})$ = 1000/31.4 = 31.8 Hz.
(b) I = V/R = 230/40 = 5.75 Amp
(c) $X_L = 2\pi fL$ = 998.5 Ω

11. A resistor, an inductor and a capacitor are connected in series with a 120V, 100Hz ac source. Voltage leads the current by 35° in the circuit. If the resistance of the resistor is 10Ω and the sum of inductive and capacitive reactance is 17Ω, calculate the self-inductance of the inductor. (Sample QP).

Ans :

$$\tan\phi = \frac{X_L - X_C}{R} \; or \; \cos\phi = \frac{R}{Z} \; and \; X_L = 2\pi fL$$

$$X_L - X_C = 7$$

$$X_L = 12\Omega$$

Calculation of *L= 19 mH* with unit

12. Calculate resonant frequency and Q-factor of a series LCR circuit containing pure inductor of 3H, capacitor of 27μF and resistor of 7.4Ω (2014 QP)

Ans :

IMPORTANT FORMULE :

(1) AC CURRENT & VOLTAGE

(a) Expression for instantaneous value of alternating current = $i = i_m \sin \omega t$

(b) Expression for instantaneous value of alternating voltage = $v = v_m \sin \omega t$

(c) Expression for peak value of alternating current = $i_m = v_m/x$

(d) Expression for peak value of alternating voltage = $v_m = NAB\omega$

(e) Expression for rms value of alternating current = effective current = $I = i_{rms} = i_m/\sqrt{2} = 0.707\ i_m$

(f) Expression for rms value of alternating voltage = $v_{rms} = V = v_m/\sqrt{2} = 0.707\ v_m$

(2) PURE RESISTIVE CIRCUIT :

$v = v_m \sin \omega t$ & $i = i_m \sin \omega t$

Both ac voltage and ac current in pure resistive circuit are in-phase to each other.

(3) PURE INDUCTIVE CIRCUIT :

$v = v_m \sin \omega t$ & $i = i_m \sin (\omega t - \pi/2)$

The current is lagging the applied emf by an angle $\pi/2$ or $90°$.

Inductive reactance $X_L = \omega L = 2\pi\nu L = 2\pi f L$

(4) PURE CAPACITIVE CIRCUIT :

$v = v_m \sin \omega t$ & $i = i_m \sin (\omega t + \pi/2)$

In capacitive circuit, the current is leads the applied emf by an angle $\pi/2$ or $90°$.

Capacitive reactance = $X_C = 1/\omega C = 1/2\pi\nu C = 1/2\pi f C$

(5) Impedance in LCR circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

If ϕ is the phase angle between i and v,

$$\tan\phi = \frac{V_{Cm} - V_{Lm}}{V_{Rm}} \qquad \therefore \tan\phi = \frac{X_C - X_L}{R} \qquad \Rightarrow \phi = \tan^{-1}\left[\frac{X_C - X_L}{R}\right]$$

(6) Electrical Resonance in LCR circuit :

The condition $X_C = X_L$ is called **resonance condition**,

$$\frac{1}{\omega_o C} = \omega_o L \qquad \therefore \omega_o = \frac{1}{\sqrt{LC}} \qquad \text{or} \qquad 2\pi\nu_o = \frac{1}{\sqrt{LC}}$$

$\nu_o = \frac{1}{2\pi\sqrt{LC}}$ is called resonant frequency

(7) Sharpness of resonance of RC or RL circuit is denoted by quality factor(Q-factor),

$$Q = \frac{\omega_o}{2\Delta\omega} = \frac{\text{resonacne frequency}}{\text{band width}} \qquad \text{Also} \qquad Q = \frac{\omega_o L}{R} \quad \& \quad Q = \frac{1}{\omega_o CR}$$

(8) Power and power factor in ac circuit :

Power factor of pure L and C circuits for ac is zero

$\Rightarrow$ p= VI $\cos\phi$ $\Rightarrow$ $p = I^2 Z \cos\phi$ since $\phi = 0$, Power consumed by ac circuit in L & C is zero.

(ii) The AC current through pure L and C circuit is called wattles current. *This is because power factor of pure L and C circuits is zero.*

(9) Total energy of the LC circuit at any instant of time :

$$U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \frac{1}{2}\frac{q_m^2}{C} = \frac{1}{2}Li_m^2$$

(10) Transformer :

If the transformer is ideal, $p_{in} = p_{out}$, i.e., $i_p v_p = i_s v_s$.

$$\Rightarrow \frac{i_p}{i_s} = \frac{v_s}{v_p} = \frac{N_s}{N_p}$$

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Chapter 8:
ELECTROMAGNETIC WAVES

2M

1M - 2Q **or** 2M-1Q

1. Displacement current - Mention the need for displacement current (inconsistency of Ampere's circuital law) -Mention of expression for displacement current - Mention of expression for Ampere-Maxwell law.

(1) Write a note on Electro-magnetic field.

James Clerk Maxwell (1831-1879), argued that time-varying electric field generates magnetic field.

Maxwell formulated a set of equations involving electric and magnetic fields, and their sources, the charge and current densities. These equations are known as Maxwell's equations. The most important prediction to emerge from Maxwell's equations is the existence of electromagnetic waves, which are (coupled) timevarying electric and magnetic fields that propagate in space. The speed of the waves, according to these equations, turned out to be very close to the speed of light(3 ×108 m/s), obtained from optical measurements. This led to the remarkable conclusion that light is an electromagnetic wave. Maxwell's work thus unified the domain of electricity, magnetism and light.

Hertz, in 1885, experimentally demonstrated the existence of electromagnetic waves. Its technological use by Marconi by inventing Radio Communication.

(2) What is the wavelength range of Broad E. M. Spectrum ?

The broad spectrum of electromagnetic waves, stretching from γ rays (wavelength $\sim 10^{-12}$ m) to long radio waves (wavelength $\sim 10^{6}$ m).

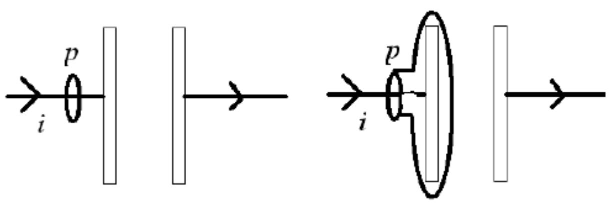
(3) Mention the need for displacement current (inconsistency of Ampere's circuital law) and hence define displacement current ?

Maxwell found an inconsistency in the Ampere's law and suggested the existence of an additional current, called displacement current, to remove this inconsistency. This displacement current is due to time-varying electric field and is given by $\mathbf{i_d = \varepsilon_0 \frac{d\phi_E}{dt}}$ and acts as a source of magnetic field in exactly the same way as conduction current.

(4) Discuss the inconsistency in Ampere's circuital law. How did Maxwell modify this law?

According to Ampere's circuital law, $\oint \vec{B}\,\vec{dl} = \mu_0\, i$

To understand the inconsistency of this law, let us consider the process of charging of a capacitor. Let S_1 and S_2 be the two surfaces bounded by the same perimeter and let P be a point on them.



When we apply Ampere's law to S_1,we have , $\oint \vec{B}\,\vec{dl} = \mu_0\, i = B\,(2\pi r) = \mu_0\, i$ ------- (1)
When we apply S_2,we have, $\oint \vec{B}\,\vec{dl} = 0$ ………………(2)
Calculated one way, there is a magnetic field at P ; calculated another way there is no magnetic field at P. It follows that Ampere's law is not consistent when the circuit includes a capacitor.

(5) What modification was made by Maxwell in ampere's circuital law?
In order to remove inconsistency, Maxwell suggested the existence of an additional current called displacement current. It is due to time-varying electric field It is given by $\mathbf{i_d = \varepsilon_0 \frac{d\phi_E}{dt}}$.
Therefore Ampere's circuital is restated as

$$\oint \vec{B}\,.\vec{dl} = \mu_0\,[i_c + i_d]$$

Where $i_c \rightarrow$ conduction current and $i_d = [\varepsilon_0 \frac{d\phi_E}{dt}] \rightarrow$ displacement current

$$\oint \vec{B}\,.\vec{dl} = \mu_0\,[i_c + \varepsilon_0 \frac{d\phi_E}{dt}] \quad = \quad \oint \vec{B}\,.\vec{dl} = \mu_0\, i_c + \mu_0\,\varepsilon_0 \frac{d\phi_E}{dt}$$

this is known as **Ampere-Maxwell law**.

(6) Explain clearly how Maxwell was led to predict the existence of electromagnetic waves
On the basis of Faraday's law of electro-magnetic induction and modified Ampere's law, Maxwell ,theoretically predicted the existence of electromagnetic wave The magnetic field changing with time , gives rise to electric field [$\oint \vec{E}\,\vec{dl} = \frac{d\phi_B}{dt}$] and an electric field changing with time gives rise to magnetic field $\oint \vec{B}\,.\vec{dl} = \mu_0 i_c + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$, it means laws of electricity and magnetism are symmetrical. The consequence of this symmetry is the existence of electromagnetic wave. According to Maxwell an accelerating charge produces electromagnetic waves. An electric charge oscillating harmonically with frequency ν, produces electromagnetic waves of same frequency ν.

(7) Represent electric and magnetic fields of an electromagnetic wave mathematically by suitable wave equations. Express c in terms of μ_0 and ε_0 .

Mathematically, for a wave of angular frequency ω wavelength propagating along z-direction we can write E_x and B_y as follows :

$E_x = E_0 \sin(kz - \omega t)$ and $B_y = B_0 \sin(kz - \omega t)$, where ω is the angular frequency and $k = 2\pi/\lambda$ is the magnitude of the wave vector (or propagation vector) and $\omega = ck$, where, $c = 1/\sqrt{\mu_0\varepsilon_0}$ = speed of light in vacuum.
The amplitude of the electric and magnetic field are related as $\mathbf{B_0 = E_0/c}$.

(8) Write any four properties of electromagnetic waves.
Ans: (1) They are transverse in nature.
(2) They are produced by accelerated charges.
(3) In an electromagnetic wave, Electric $\vec{E}$ and $\vec{B}$ magnetic fields oscillate sinusoidally in space and time The oscillating $\vec{E}$ and $\vec{B}$ are perpendicular to each other, and to the direction of propagation.

(4) The oscillations of $\vec{E}$ and $\vec{B}$ are in same phase.
(5) All electromagnetic waves travel in vacuum with the same speed c = $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ = c =3x10^8 m s^{-1}.
(6) They transport energy and momentum as they travel through space.
(7) When these waves strike the surface, a pressure is exerted on the surface.
(8) They show the properties of reflection, refraction, interference, diffraction and polarization .
(9) Electric field is responsible for optical effects of em waves.

(9) Name the main parts of the electromagnetic spectrum giving their wavelength range or frequency range ?

Ans: Types of em waves	Wavelength range	frequency range
* γ-ray,	$< 10^{-3}$m	10^{18}to10^{22}Hz
* X-rays	10^{-3}nm to 1 nm	10^{16} to 10^{19}Hz
*UV-rays	1nm to 400 nm	10^{16}to10^{17}Hz
* visible rays	400 nm to 700 nm	4× $10^{14}$to 7× 10^{14}Hz
* IR-rays	700 nm to 1mm	$10^{11}$to 5×10^{14}Hz
*Microwaves	1 mm to 0.1m	10^{9}to10^{12}Hz
*Radio waves	0.1m to 3o km	10^{4} to 10^{8}Hz

(10) Write Maxwell's electro-magnetic equations ?

1. $\oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$ **(Gauss's Law for electricity)**
2. $\oint \vec{B}.\vec{dA} = 0$ **(Gauss's Law for magnetism)**
3. $\oint \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$ **(Faraday's Law)**
4. $\oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$ **(Ampere – Maxwell Law)**

(11) In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0 × 10^{10} Hz and amplitude 48 Vm^{-1}. (a) What is the wave length of the wave (b) What is the amplitude of the oscillating magnetic field (c) Show that the average energy density of the Electric field equals the average energy density of the magnetic field
λ=c/ν= (3x10^8) / (2.0x10^{10}) = 1.5 × 10^{-2}m

$B_0 = E_0/c = 48/(3 \times 10^8) = 16 \times 10^{-8}$T

$$\frac{u_E}{u_m} = \frac{\frac{\varepsilon_0 E^2}{2}}{\frac{B^2}{2\mu_0}} = \varepsilon_0\mu_0\left(\frac{E}{B}\right)^2 = \frac{1}{c^2} \text{x} c^2 = 1$$

Therefore $\mu_E = \mu_m$

1. Distinguish between conduction current and displacement current.

S. No.	Conduction current	Displacement current
1	The electric current carried by conductors due to flow of charges is called conduction current.	The electric current due to changing electric field is called displacement current.
2	$i_C = V/R$	$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$

2. What is displacement current? write the expression for displacement current

The electric current due to changing electric field is called displacement current

$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$

where ε_0 is called absolute permittivity of vacuum.

3. State Ampere-Maxwell law. Write its mathematical form

The total current passing through any surface of which the closed loop is the Perimeter" is the sum of the conduction current and the displacement current.

$\oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$

4. Briefly explain, how does an accelerating charge act as a source of an electromagnetic wave?

Ans: Consider a charge oscillating with some frequency. This is an example of accelerating charge. This charge produces an oscillating electric field in space. This field, in turn, produces an oscillating magnetic field in the neighborhood.

The process continues because the oscillating electric and magnetic fields regenerate each other. Hence an electromagnetic wave originates from the accelerating charges.

5. Write down the expression for the velocity of electromagnetic wave in a) vacuum and b) material medium

Ans: In vacuum, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ where absolute permeability and absolute permittivity of vacuum

In material medium, $v = 1/\sqrt{\mu\varepsilon}$ where ε is called permittivity and μ is called magnetic permeability of material medium.

6. What are the contributions of Hertz to electromagnetic wave theory?

Ans:* Hertz confirms the existence of electromagnetic waves

* He produced stationary electromagnetic waves

* Using $v = c\lambda$, he found that the em-waves travelled with the same speed as the speed of light.

7. The amplitude of the magnetic field part of a electromagnetic wave in vacuum is 510 nT. What is the amplitude of the electric field part of the wave?

Ans: $B_0 = \frac{E_0}{c} = \frac{510\times10^{-9}}{3\times10^8} = 170\times 10^{-17}$T

8. Give any two uses of microwaves.

Ans: Microwaves are used in aircraft navigation, (speed guns to time fast balls, tennis serves, and automobiles). Microwaves are also used in microwave ovens

9. Give any two uses of IR-waves
Ans: IR-waves from the sun keep the earth warm and hence help to sustain life on the earth IR-rays photographs are used for weather forecasting. (They are used in detectors, remote switches)

10. Mention any two uses of UV waves
Ans: Highly focused UV-rays are used in eye surgery (LASIK-Laser Assisted in situ ketatomileusis). UV-lamps are used to kill germs in water purifiers.

1. What is the source of an electromagnetic wave?
Ans:An accelerated charged particle is the source of e.m. waves

2. Who proposed electromagnetic wave theory?
Ans:James clerk Maxwell proposed electromagnetic wave theory

3 What is displacement current?
Ans: The electric current due to changing electric field is called displacement current

4. Is displacement current a source of magnetic field?
Ans: Yes, it is a source of magnetic field.

5 Write an expression for the displacement current.
Ans: $i_d = \varepsilon_0 \frac{d\phi_E}{dt}$
Where, ε_0 is called absolute permittivity of air.

6 Give the mathematical form of Ampere-Maxwell law.
Ans:
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 [i_c + i_d]$$
or
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 [i_c + \varepsilon_0 \frac{d\phi_E}{dt}]$$

7. What are electromagnetic waves?
Ans: Waves radiated by accelerated charges and consist of time varying, transverse electric and magnetic fields are called electromagnetic waves.

8. Name the scientist who first predicted the existence of e.m waves
Ans: Hertz

2. Electromagnetic waves: Sources and nature of electromagnetic waves – Characteristics - Mention of expression of speed of light.

(1) Give different sources of electro-magnetic waves

(1) Accelerated charges radiate electromagnetic waves. The frequency of the electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source – the accelerated charge.

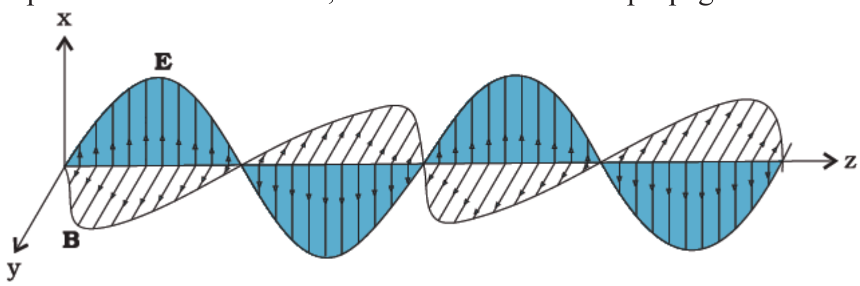
(2) Hertz's experiment (1887) produced low frequency region (the radio wave region).

(3) Jagdish Chandra Bose, working at Calcutta, produced electromagnetic waves of much shorter wavelength (25 mm to 5 mm).

(4) Guglielmo Marconi in Italy succeeded in transmitting electromagnetic waves over distances of many kilometres.

(2) Explain the nature of electromagnetic waves ?

It can be shown from Maxwell's equations that electric and magnetic fields in an electromagnetic wave are
perpendicular to each other, *and* to the direction of propagation as shown in figure.



In the above fig., a plane electromagnetic wave propagating along the z direction with time t. The electric field E_x is along the x-axis, and varies sinusoidally with z, at a given time. The magnetic field B_y is along the y-axis, and again varies sinusoidally with z. The electric and magnetic fields E_x and B_y are perpendicular to each other, and to the direction z of propagation. We can write E_x and B_y as follows:

$E_x = E_0 \sin(kz - \omega t)$ --------- (1)

$B_y = B_0 \sin(kz - \omega t)$ --------- (2)

Here k is related to the wave length λ of the wave by the usual equation. $k = 2\pi/\lambda$ and ω is the angular frequency. k is the magnitude of the wave vector (or propagation vector) **k** and its direction describes the direction of propagation of the wave. The speed of propagation of the wave is $c = (\omega / k)$.

We can also prove that $\omega = ck$, where, $c = 1/\sqrt{\mu_0 \varepsilon_0}$

This relation is often written in terms of frequency, ν $(= \omega/2\pi)$ and wavelength, λ $(=2\pi/k)$ as

$2\pi\nu = c\left(\frac{2\pi}{\lambda}\right)$ or => $\nu\lambda = c$

It is also seen from Maxwell's equations that the magnitude of the electric and the magnetic fields in an electromagnetic wave are related as $\mathbf{B_0 = (E_0/c)}$.

In material medium, the description to electric and magnetic fields in Maxwell's equations with the result that in a material medium of permittivity ε and magnetic permeability μ, the velocity of light becomes,

$$\mathbf{v = 1/\sqrt{\mu\varepsilon} \quad = 1/\sqrt{\mu_0\mu_r\, \varepsilon_0\varepsilon_r}}$$

Thus, the velocity of light depends on electric and magnetic properties of the medium.

As electromagnetic wave contains both electric and magnetic fields, there is a non-zero energy density associated with it. An electromagnetic wave (like other waves) carries energy and momentum. Since it carries momentum, an electromagnetic wave also exerts pressure, called *radiation pressure*.

If the total energy transferred to a surface in time t is U, it can be shown that the magnitude of the total momentum delivered to this surface (*for complete absorption*) is, p = U/c.

In 1903, the American scientists Nicols and Hull succeeded in measuring radiation pressure of visible light and verified eqn. p = U/c . It was found to be of the order of 7×10^{-6} N/m^2. Thus, on a surface of area 10 cm^2, the force due to radiation is only about 7×10^{-9} N.

3. Electromagnetic spectrum: Wavelength range and their uses.

(2) Write a note on different waves in EM spectrum ?

(a) Radio waves :

Radio waves are produced by the accelerated motion of charges in conducting wires. They are used in radio and television communication systems. They are generally in the frequency range from 500 kHz to about 1000 MHz. The AM (amplitude modulated) band is from 530 kHz to 1710 kHz. Higher frequencies upto 54 MHz are used for *short wave* bands. TV waves range from 54 MHz to 890 MHz. The FM (frequency modulated) radio band extends from 88 MHz to 108 MHz. Cellular phones use radio waves to transmit voice communication in the ultrahigh frequency (UHF) band.

(b) Microwaves :

Microwaves (short-wavelength radio waves), with frequencies in the gigahertz (GHz) range, are produced by special vacuum tubes (called klystrons, magnetrons and Gunn diodes). Due to their short wavelengths, they are suitable for the **radar systems** used in aircraft navigation. Radar also provides the basis for the speed guns used to time fast balls, tennisserves, and automobiles. Microwave ovens are an interesting domestic application of these waves. In such ovens, the frequency of the microwaves is selected to match the resonant frequency of water molecules so that energy from the waves is transferred efficiently to the kinetic energy of the molecules. This raises the temperature of any food containing water.

(c) Infrared waves :

Infrared waves are produced by hot bodies and molecules. This band lies adjacent to the low-frequency or long-wave length end of the visible spectrum. Infrared waves are sometimes referred to as *heat waves*. This is because water molecules present in most materials readily absorb infrared waves (many other molecules, for example, CO_2, NH_3, also absorb infrared waves). After absorption, their thermal motion increases, that is, they heat up and heat their surroundings. Infrared lamps are used in physical therapy. Infrared radiation also plays an

important role in maintaining the earth's warmth or average temperature through the greenhouse effect. Incoming visible light (which passes relatively easily through the atmosphere) is absorbed by the earth's surface and reradiated as infrared (longer wavelength) radiations. This radiation is trapped by greenhouse gases such as carbon dioxide and water vapour.
Infrared detectors are used in Earth satellites, both for military purposes and to observe growth of crops. Electronic devices (for example semiconductor light emitting diodes) also emit infrared and are widely used in the remote switches of household electronic systems such as TV sets, video recorders and hi-fi systems.

(d) Visible rays :

It is the most familiar form of electromagnetic waves. It is the part of the spectrum that is detected by the human eye. It runs from about 4×10^{14} Hz to about 7×10^{14} Hz or a wavelength range of about 700 – 400 nm. Visible light emitted or reflected from objects around us provides us information about the world. Our eyes are sensitive to this range of wavelengths. Different animals are sensitive to different range of wavelengths. For example, snakes can detect infrared waves, and the 'visible' range of many insects extends well into the utraviolet.

(e) Ultraviolet rays :

It covers wavelengths ranging from about 4×10^{-7} m (400 nm) down to 6×10^{-10}m (0.6 nm). Ultraviolet (UV) radiation is produced by special lamps and very hot bodies. The sun is an important source of ultraviolet light. But fortunately, most of it is absorbed in the ozone layer in the atmosphere at an altitude of about 40 – 50 km. UV light in large quantities has harmful effects on humans. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin. UV radiation is absorbed by ordinary glass. Hence, one cannot get tanned or sunburn through glass windows.
Welders wear special glass goggles or face masks with glass windows to protect their eyes from large amount of UV produced by welding arcs. Due to its shorter wavelengths, UV radiations can be focussed into very narrow beams for high precision applications such as LASIK (*Laserassisted in situ keratomileusis*) eye surgery. UV lamps are used to kill germs in water purifiers. Ozone layer in the atmosphere plays a protective role, and hence its depletion by chlorofluorocarbons (CFCs) gas (such as freon) is a matter of international concern.

(f) X-rays :

Beyond the UV region of the electromagnetic spectrum lies the X-ray region. We are familiar with X-rays because of its medical applications. It covers wavelengths from about 10^{-8} m (10 nm) down to 10^{-13} m (10^{-4} nm). One common way to generate X-rays is to bombard a metal target by high energy electrons. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because X-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary or over exposure.

(g) Gamma rays :

They lie in the upper frequency range of the electromagnetic spectrum and have wavelengths of from about 10^{-10}m to less than 10^{-14}m. This high frequency radiation is produced in nuclear reactions and also emitted by radioactive nuclei. They are used in medicine to destroy cancer cells.

DIFFERENT TYPES OF ELECTROMAGNETIC WAVES :

Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1mm to 700 nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye Photocells Photographic film
Ultraviolet	400 nm to 1nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	$<10^{-3}$ nm	Radioactive decay of the nucleus	-do-

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Chapter 9:
RAY OPTICS AND OPTICAL INSTRUMENTS

8 M [1M -1Q, 2M- 1Q, 5M - 1Q (LA)] or [3M-1Q; 5M-1Q(NP)]

(1) State and explain laws of reflection ?

First Law : The angle of reflection (i.e., the angle between reflected ray and the normal to the reflecting surface or the mirror) equals the angle of incidence (angle between incident ray and the normal).

Second Law : The incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane.

(2) Explain Reflection of light by spherical mirrors

the geometric centre of a spherical mirror is called its pole while that of a spherical lens is called its optical centre. The line joining the pole and the centre of curvature of the spherical mirror is known as the *principal axis*.

(3) Cartesian sign convention rule :

(1) All distances are measured from the pole of the mirror or the optical centre of the lens.

(2) The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of incident light are taken as negative.

(3) The heights measured upwards with respect to *x*-axis and normal to the principal axis (*x*-axis) of the mirror/lens are taken as positive. The heights measured downwards are taken as negative.

(4) Define focal length of spherical mirror ?

The distance between the focus F and the pole P of the mirror is called the *focal length* of the mirror, denoted by *f*.

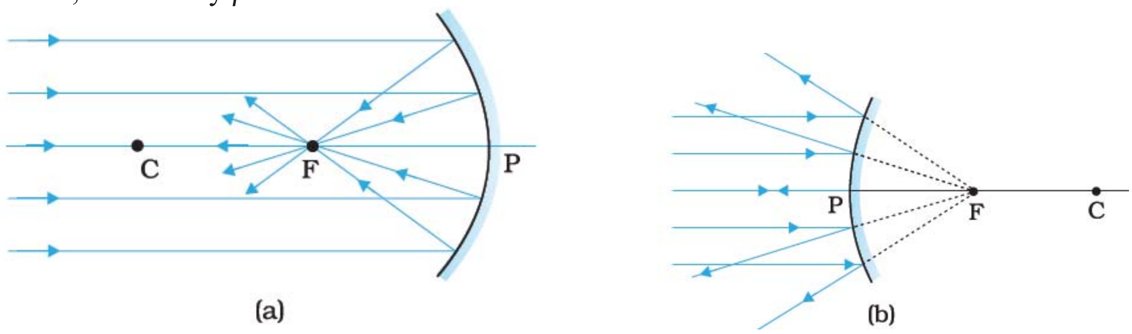


Fig. 1 : Focal length of mirrors.

(5) Derive the relation *f* = *R/2* in the case of a concave mirror ? (2M)

Let C be the centre of curvature of the mirror. Consider a ray parallel to the principal axis striking the mirror at M. Then CM will be perpendicular to the mirror at M. Let θ be the angle of incidence, and MD be the perpendicular from M on the principal axis. Then, ∠MCP = θ and ∠MFP = 2θ.

Now, $\tan\theta = \frac{MD}{CD} = \tan 2\theta = \frac{MD}{FD}$ --------- (1)

For small θ, which is true for paraxial rays, $\tan\theta \approx \theta$, $\tan 2\theta \approx 2\theta$. Therefore, Eq. (1) gives

$$\frac{MD}{FD} = 2\,\frac{MD}{CD} \qquad \text{or, } FD = \frac{CD}{2} \qquad \text{-------------} \quad (2)$$

Now, for small θ, the point D is very close to the point P. Therefore, FD = f and CD = R. Equation (2) then gives $f = R/2$ ---------- (3)

(6). Explain reflection in the case of concave mirror producing a real image ?

If rays emanating from a point actually meet at another point after reflection and/or refraction, that point is called the *image* of the first point. The image is *real* if the rays actually converge to the point; it is *virtual* if the rays do not actually meet but appear to diverge from the point when produced backwards. An image is thus a point-to-point correspondence with the object established through reflection and/or refraction.

In case of spherical mirror,
(i) The ray from the point which is parallel to the principal axis. The reflected ray goes through the focus of the mirror.
(ii) The ray passing through the centre of curvature of a concave mirror or appearing to pass through it for a convex mirror. The reflected ray simply retraces the path.
(iii) The ray passing through (or directed towards) the focus of the concave mirror or appearing to pass through (or directed towards) the focus of a convex mirror. The reflected ray is parallel to the principal axis.
(iv) The ray incident at any angle at the pole. The reflected ray follows laws of reflection.

Fig. 2 shows the ray diagram considering three rays. It shows the image A′B′ (in this case, real) of an object AB formed by a concave mirror. It does not mean that only three rays emanate from the point A.
An infinite number of rays emanate from any source, in all directions. Thus, point A′ is image point of A if every ray originating at point A and falling on the concave mirror after reflection passes through the point A′.

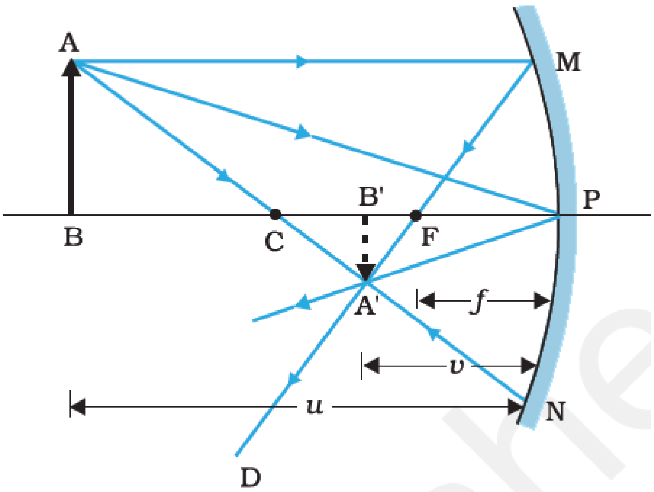


Fig. 2 : Ray diagram for image formation by a concave mirror.

(7) Derive mirror formula/Equation for real image formation by a concave mirror? (2M)

We now derive the mirror equation or the relation between the object distance (u), image distance (v) and the focal length (f).

From Fig. 2, the two right-angled triangles A′B′F and MPF are similar. (For paraxial rays, MP can be considered to be a straight line perpendicular to CP.) Therefore,

$$\frac{B'A'}{PM}=\frac{B'F}{FP} \quad \text{or} \quad \frac{B'A'}{BA}=\frac{B'F}{FP} \quad (\because PM = AB) \qquad \text{------------------} \quad (4)$$

Since ∠ APB = ∠ A′PB′, the right angled triangles A′B′P and ABP are also similar. Therefore,

$$\frac{B'A'}{BA}=\frac{B'P}{BP} \qquad \text{--------------------} \quad (5)$$

Comparing Eqns.(4) and (5) we get,

$$\frac{B'F}{FP}=\frac{B'P-FP}{FP}=\frac{B'P}{BP} \qquad \text{-----------} \quad (6)$$

Equation (6) is a relation involving magnitude of distances. We now apply the sign convention. We note that light travels from the object to the mirror MPN. Hence this is taken as the positive direction. To reach the object AB, image A′B′ as well as the focus F from the pole P, we have to travel opposite to the direction of incident light. Hence, all the three will have negative signs. Thus,

$B'P = -v$, $FP = -f$, $BP = -u$

Using these in Eq. (6), we get,

$$\frac{-v+f}{-f}=\frac{-v}{-u} \quad \text{or} \quad \frac{v-f}{f}=\frac{v}{u} \qquad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

This relation is known as the *mirror equation.*

(8) Define linear magnification and obtain an expression for it?

The size of the image relative to the size of the object is another important quantity to consider. We define linear *magnification* (*m*) as the ratio of the height of the image (*h*′) to the height of the object (*h*):

$$m=\frac{h'}{h}$$

h and *h* ′ will be taken positive or negative in accordance with the accepted sign convention. In triangles A′B′P and ABP, we have,

$$\frac{B'A'}{BA}=\frac{B'P}{BP}$$

With the sign convention, this becomes

$$\frac{-h'}{h}=\frac{-v}{-u}$$

so that

$$m=\frac{h'}{h}=-\frac{v}{u}$$

(9) Problems I

An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.

Solution : The focal length $f = -15/2$ cm $= -7.5$ cm (i) The object distance $u = -10$ cm. Then Eq. (9.7) gives

$$\frac{1}{v}+\frac{1}{-10}=\frac{1}{-7.5} \quad \text{or} \quad v=\frac{10\times 7.5}{-2.5}=-30\ \text{cm}$$

The image is 30 cm from the mirror on the same side as the object.
Also, magnification m = -u/v = - (-30/-10) = -3. The image is magnified, real and inverted.

(ii) The object distance u = –5 cm. Then from Eq. (9.7),

$$\frac{1}{v}+\frac{1}{-5}=\frac{1}{-7.5} \quad \text{or} \quad v=\frac{5\times 7.5}{(7.5-5)}=15\ \text{cm}$$

This image is formed at 15 cm behind the mirror. It is a virtual image.
Magnification m = - (v/u) = - (15/-5) = 3. The image is magnified, virtual and erect.

(10) Explanation of phenomenon of refraction ?

Visible light in Electromagnetic spectrum has wavelength of about 400 nm to 750 nm. The speed of light in vacuum is the highest speed attainable in nature and is equal to c = 3×10^8 m/s. The path of light is called a *ray* of light, and a bundle of such rays constitutes a *beam* of light.

The phenomenon of change in direction of propagation of an obliquely incident ray of light that enters from one medium to the other medium, at the interface of the two media is called **refraction** of light.

(11) Define laws of refraction ? Explain their consequences ?

Laws of refraction : Snell experimentally obtained the following laws of refraction:
(i) The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.
(ii) The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant.
The angles of incidence (i) and refraction (r) are the angles that the incident and its refracted ray make with the normal, respectively.
We have

$$\frac{\sin i}{\sin r}=n_{21} \qquad \text{---------} \quad (7)$$

where n_{21} is a constant, called the *refractive index* of the second medium with respect to the first medium. Equation (7) is the well-known Snell's law of refraction.

$$\frac{\sin i}{\sin r}=n_{21}=\frac{1}{n_{12}}=\frac{n_2}{n_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2} \qquad \text{---------} \quad (8)$$

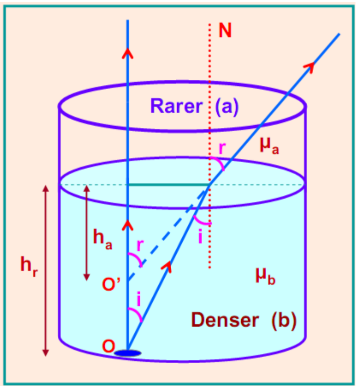
if $n_{21} > 1$, $r < i$, i.e., the refracted ray bends towards the normal. In such a case medium 2 is said to be *optically denser* (or *denser*, in short) than medium 1. On the other hand, if $n_{21} < 1$, $r > i$, the refracted ray bends away from the normal. This is the case when incident ray in a denser medium refracts into a rarer medium.

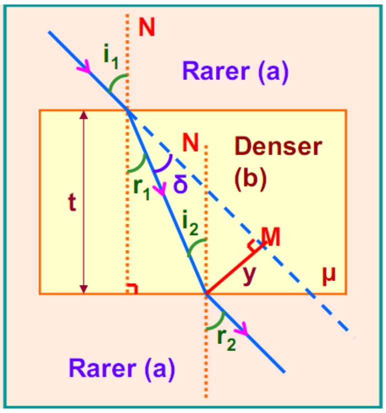
Consequences :

(1) Lateral Shift : **y** = $\frac{t\sin\delta}{\cos r_1}=\frac{t\sin(i_1-r_1)}{\cos r_1}$

(2) Normal Shift :

$n_{21} = h_r/h_a$ = Real depth/Apparent depth,
Apparent shift = $h_r - h_a = h_r - (h_r/n_{21}) = h_r\,[1- 1/n_{21}]$



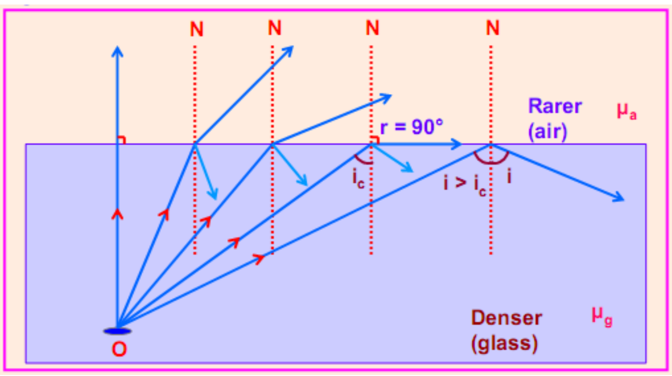


(12) Explain total internal Reflection ? Mention various conditions ?

When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the *internal reflection.*

Total Internal Reflection (TIR) is the phenomenon of complete reflection of light back into the same medium for angles of incidence greater than the critical angle of that medium.

Explanation :



Conditions for TIR:

1. The incident ray must be in optically denser medium.
2. The angle of incidence in the denser medium must be greater than the critical angle for the pair of media in contact.

(13) Define critical angle ? Obtain relation between Critical Angle and Refractive Index ?

Critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.

Relation between Critical Angle and Refractive Index :

The angle of incidence corresponding to an angle of refraction 90º, say ∠AON, is called the *critical angle* (i_c) for the given pair of media. Using Snell's law, if the relative refractive index is less than one then, since the maximum value of sin r is unity, there is an upper limit to the value of sin i for which the law can be satisfied, that is, $i = i_c$ such that

sin $i_c = n_{21}$

The refractive index of denser medium 2 with respect to rarer medium 1 will be $n_{12} = 1/\sin i_c$

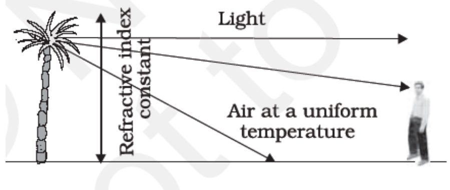
Red colour has maximum value of critical angle and Violet colour has minimum value of critical angle since,

$$\sin i_c = \frac{1}{{}_a\mu_g} = \frac{1}{a + (b/\lambda^2)}$$

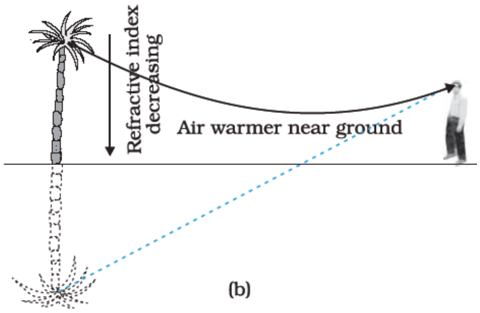
(14) Explain various Applications of T I R ?

1. Mirage formation

On hot summer days, the air near the ground becomes hotter than the air at higher levels. The refractive index of air increases with its density. Hotter air is less dense, and has smaller refractive index than the cooler air. If the air currents are small, that is, the air is still, the optical density at different layers of air increases with height. As a result, light from a tall object such as a tree, passes through a medium whose refractive index decreases towards the ground. Thus, a ray of light from such an object successively bends away from the normal and undergoes total internal reflection, if the angle of incidence for the air near the ground exceeds the critical angle. This is shown in Fig. (b). To a distant observer, the light appears to be coming from somewhere below the ground. The observer naturally assumes that light is being reflected from the ground, say, by a pool of water near the tall object. Such inverted images of distant tall objects cause an optical illusion to the observer. This phenomenon is called *mirage*. This type of mirage is especially common in hot deserts. We have noticed that while moving in a bus or a car during a hot summer day, a distant patch of road, especially on a highway, appears to be wet. But, we do not find any evidence of wetness when reach that spot. This is also due to mirage.



(a)

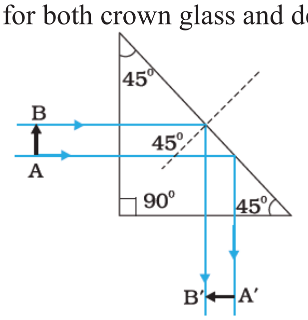


(b)

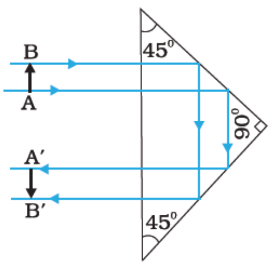
2. Looming

3. Totally reflecting Prisms :

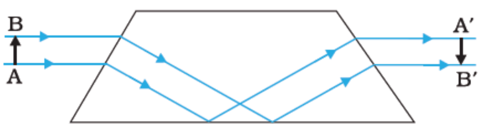
Prisms designed to bend light by 90º or by 180º make use of total internal reflection [Fig. (a) and (b)]. Such a prism is also used to invert images without changing their size [Fig. (c)]. In the first two cases, the critical angle *ic* for the material of the prism must be less than 45º. We see from Table 9.1 that this is true for both crown glass and dense flint glass.



(a)



(b)



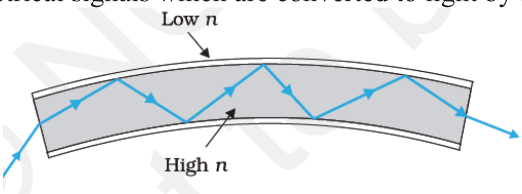
(c)

4. Optical Fibres :

Optical fibres are extensively used for transmitting audio and video signals through long distances. Optical fibres too make use of the phenomenon of total internal reflection. Optical fibres are fabricated with high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding.

When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out at the other end (Fig.).

Since light undergoes total internal reflection at each stage, there is no appreciable loss in the intensity of the light signal. Optical fibres are fabricated such that light reflected at one side of inner surface strikes the other at an angle larger than the critical angle. Even if the fibre is bent, light can easily travel along its length. Thus, an optical fibre can be used to act as an optical pipe. In silica glass fibres, it is possible to transmit more than 95% of the light over a fibre length of 1 km. Optical fibres are extensively used for transmitting and receiving electrical signals which are converted to light by suitable transducers.



5. Sparkling of Diamonds

Diamonds are known for their spectacular brilliance. Their brilliance is mainly due to the total internal reflection of light inside them. The critical angle for diamond-air interface (≈ 24.4°) is very small, therefore once light enters a diamond, it is very likely to undergo total internal reflection inside it. Diamonds found in nature rarely exhibit the brilliance for which they are known. It is the technical skill of a diamond cutter which makes diamonds to sparkle so brilliantly. By cutting the diamond suitably, multiple total internal reflections can be made to occur.

(15) Explain refraction at spherical surfaces ? Derivation of the relation between *u, v, n* and *R*.

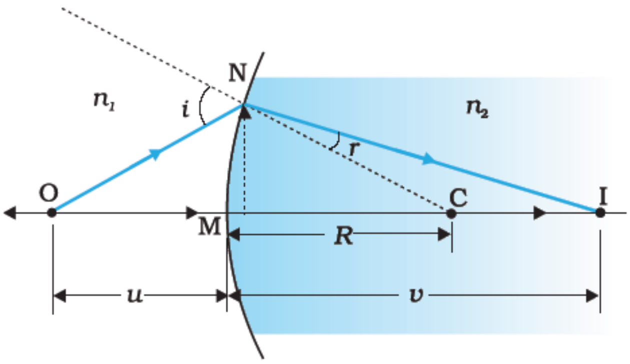


Fig. 7 : Refraction at a spherical surface separating two media.

Figure 7 shows the geometry of formation of image *I* of an object *O* on the principal axis of a spherical surface with centre of curvature C, and radius of curvature *R*. The rays are incident from a medium of refractive index n_1, to another of refractive index n_2. As before, we take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. In particular, NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis. We have, for small angles,

$\tan \angle NOM = \dfrac{MN}{OM}$ $\tan \angle NCM = \dfrac{MN}{MC}$ $\tan \angle NIM = \dfrac{MN}{MI}$

Now, for ΔNOC, i is the exterior angle. Therefore, i = ∠NOM + ∠NCM

$$i = \frac{MN}{OM} + \frac{MN}{MC}$$

Similarly,

$r = \angle NCM - \angle NIM$

$$\text{i.e., } r = \frac{MN}{MC} - \frac{MN}{MI}$$

Now, by Snell's law

$n_1 \sin i = n_2 \sin r$

or for small angles $\boldsymbol{n_1 i = n_2 r}$

Substituting i and r from Eqs. (9.13) and (9.14), we get

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \quad \text{--------} \quad (15)$$

Here, OM, MI and MC represent magnitudes of distances. Applying the Cartesian sign convention,

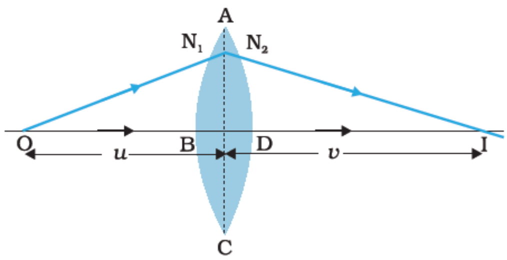
OM = $-u$, MI = $+v$, MC = $+R$

Substituting these in Eq. (15), we get

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad \text{--------} \quad (16)$$

Equation (9.16) gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface. It holds for any curved spherical surface.

(16) Describe Refraction by a Lens: Derive an expression for lens-maker's formula ?



(a)

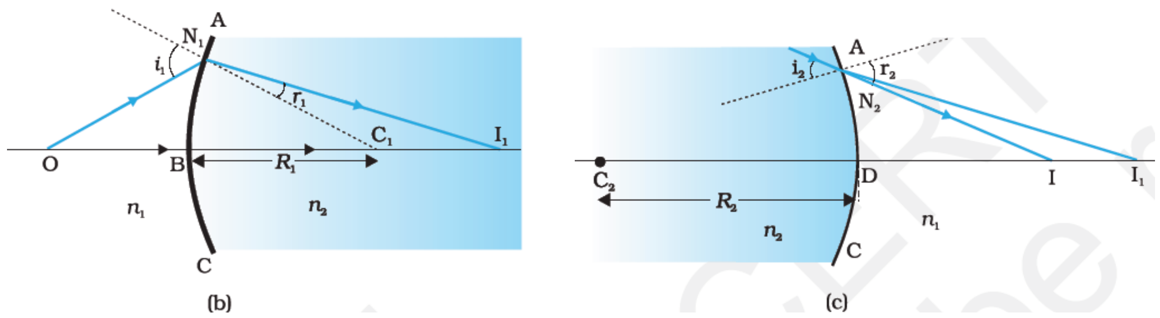


Figure .(a) shows geometry of image formation by a double convex lens. The image formation can be seen in terms of two steps:

(i) The first refracting surface forms the image I_1 of the object O [Fig. (b)]. The image I_1 acts as a virtual object for the second surface that forms the image at I [Fig. (c)]. Applying Eq. (15) to the first interface ABC, we get,

$$\frac{n_1}{OB}+\frac{n_2}{BI_1}=\frac{n_2-n_1}{BC_1} \quad \text{-------} \quad (1)$$

A similar procedure applied to the second interface ADC gives,

$$-\frac{n_2}{DI_1}+\frac{n_1}{DI}=\frac{n_2-n_1}{DC_2} \quad \text{----------} \quad (2)$$

For a thin lens, $BI_1 = DI_1$. Adding Eqs. (1) and (2), we get,

$$\frac{n_1}{OB}+\frac{n_1}{DI}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right)$$

Suppose the object is at infinity, i.e., OB → ∞ and DI = *f*, Eq. (2) gives

$$\frac{n_1}{f}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right) \quad \text{--------------} \quad (3)$$

The point where image of an object placed at infinity is formed is called the *focus* F, of the lens and the distance *f* gives its *focal length*. A lens has two foci, F and F′, on either side of it (Fig.). By the sign convention,

$$BC_1 = +R_1,$$

$$DC_2 = -R_2$$

So Eq. (3) can be written as

$$\frac{1}{f}=(n_{21}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \left(\because n_{21}=\frac{n_2}{n_1}\right) \quad \text{----------} \quad (4)$$

Eqn. (3) is known as the *lens maker's formula*. It is useful to design lenses of desired focal length using surfaces of suitable radii of curvature.

(17) Definition and expression for linear magnification.

Magnification (*m*) produced by a lens is defined, like that for a mirror, as the ratio of the size of the image to that of the object. Proceeding in the same way as for spherical mirrors, it is easily seen that for a lens

$$m=\frac{h'}{h}=\frac{v}{u}$$

(18) Using Lens Makers formula, Derive thin lens formula ? Draw image of an object ?

Consider Lens Makers formula

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \left(\because n_{21} = \frac{n_2}{n_1}\right)$$

From Eqs. (1) and (3), we get

$$\frac{n_1}{OB} + \frac{n_1}{DI} = \frac{n_1}{f}$$

where BO = u and DI = v

Applying sign convention, for convex lens, we write, 1/u + 1/v = 1/f

Applying sign convention, for concave lens, we write 1/v - 1/u = 1/f

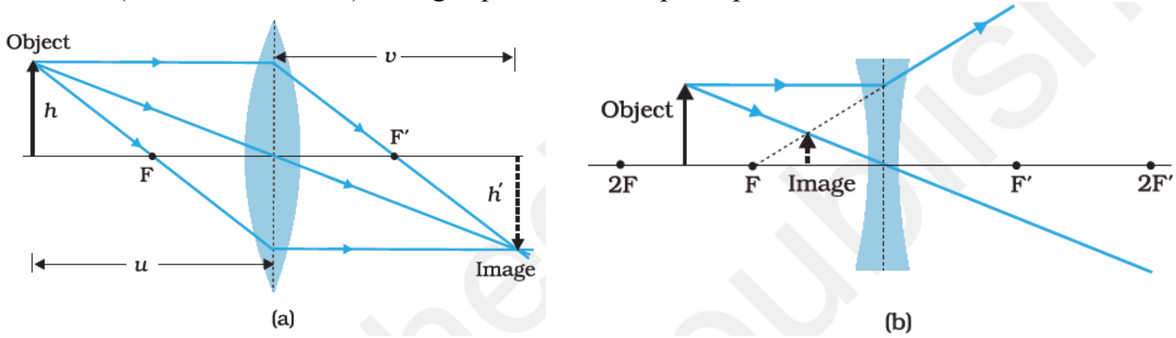
Which is lens formula for thin lens.

To find the image of an object by a lens, we can, in principle, take any two rays emanating from a point on an object; trace their paths using the laws of refraction and find the point where the refracted rays meet (or appear to meet). In practice, however, it is convenient to choose any two of the following rays:

(i) A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the second principal focus F′ (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus F.

(ii) A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction.

(iii) A ray of light passing through the first principal focus (for a convex lens) or appearing to meet at it (for a concave lens) emerges parallel to the principal axis after refraction.



(19) Explain power of a lens and mention of expression for it.

The *power P* of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre.

$$\tan\delta = \frac{h}{f}; \text{ if } h = 1 \quad \tan\delta = \frac{1}{f} \quad \text{or} \quad \delta = \frac{1}{f}$$

for small value of δ. Thus, P = 1/f.

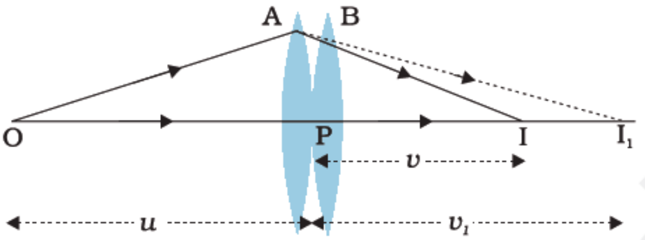
The SI unit for power of a lens is dioptre (D): 1D = 1m^{-1}. The power of a lens of focal length of 1 metre is one dioptre. Power of a lens is positive for a converging lens and negative for a diverging lens. Thus, when an optician prescribes a corrective lens of power + 2.5 D, the required lens is a convex lens of focal length + 40 cm. A lens of power of – 4.0 D means a concave lens of focal length – 25 cm.

(20) Derive an expression for the effective (equivalent) focal length of two thin lenses in contact.

Consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other. Let the object be placed at a point O beyond the focus of the first lens A (Fig.). The first lens produces an image at I_1. Since image I_1 is real, it serves as a virtual object for the second lens B, producing the final image at I. It must, however, be borne in mind that formation of image by the first lens is presumed only to facilitate determination of the position of the final image. In fact, the direction of rays emerging from the first lens gets modified in accordance with the angle at which they strike the second lens. Since the lenses are thin, we assume the optical centres of the lenses to be coincident. Let this central point be denoted by P.

For the image formed by the first lens A, we get

$$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1} \quad \text{------ (1)}$$



For the image formed by the second lens B, we get

$$\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2} \quad \text{------- (2)}$$

Adding Eqs. (1) and (2), we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}$$

If the two lens-system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

so that we get

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

The derivation is valid for any number of thin lenses in contact. If several thin lenses of focal length $f1$, $f2$, $f3$,... are in contact, the effective focal length of their combination is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots \quad \text{------- (3)}$$

In terms of power, Eq. (3) can be written as

$$P=P_1+P_2+P_3+\ldots \quad \text{--------- (4)}$$

where P is the net power of the lens combination. Note that the sum in Eq. (4) is an algebraic sum of individual powers, so some of the terms on the right side may be positive (for convex lenses) and some negative (for concave lenses). Combination of lenses helps to obtain diverging or converging lenses of desired magnification.

(21) Discuss refraction of monochromatic light through a prism and Derive prism formula ? Also obtain an expression for refractive index of the material of the prism ?

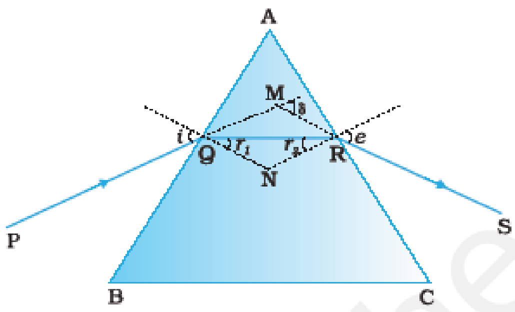


Figure shows the passage of light through a triangular Prism ABC. The angles of incidence and refraction at the first face AB are i and $r1$, while the angle of incidence (from glass to air) at the second face AC is r_2 and the angle of refraction or emergence e. The angle between the emergent ray RS and the direction of the incident ray PQ is called the *angle of deviation*, δ.

In the quadrilateral AQNR, two of the angles (at the vertices Q and R) are right angles.
Therefore, the sum of the other angles of the quadrilateral is 180º.
$\angle A + \angle QNR = 180^o$
From the triangle QNR,
$r_1 + r_2 + \angle QNR = 180^o$
Comparing these two equations, we get $r_1 + r_2 = A$ -------- (1)
The total deviation δ is the sum of deviations at the two faces,
$\delta = (i - r_1) + (e - r_2)$
that is, $\delta = i + e - A$ ---------- (2)
Thus, the angle of deviation depends on the angle of incidence. A plot between the angle of deviation and angle of incidence is shown in Fig. (2). We can see that, in general, any given value of δ, except for $i = e$, corresponds to two values i and hence of e. This, in fact, is expected from the symmetry of i and e in Eq. (2), i.e., δ remains the same if i and e are interchanged. Physically, this is related to the fact that the path of ray in Fig. (1) can be traced back, resulting in the same angle of deviation. At the minimum deviation D_m, the refracted ray inside the prism becomes parallel to its base. We have
$\delta = D_m$, $i = e$ which implies $r_1 = r_2$.
Equation (1) gives, $2r = A$ or $r = A/2$ -------- (3)

In the same way equation (2) gives $D_m = 2i - A$ or $i = (A + D_m)/2$ ------- (4)

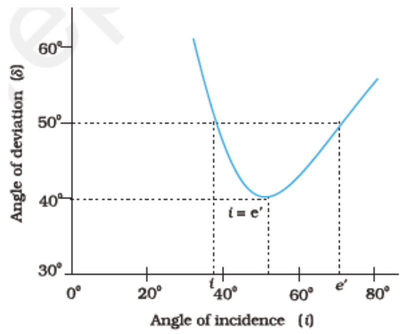
The refractive index of the prism is $n_{21} = \frac{n_2}{n_1} = \frac{Sin\,[\frac{A+D_m}{2}]}{\sin[\frac{A}{2}]}$ -------- (5)

The angles A and D_m can be measured experimentally.
Equation (5) thus provides a method of determining refractive index of the material of the prism.
For a small angle prism, i.e., a thin prism, D_m is also very small, and we get,

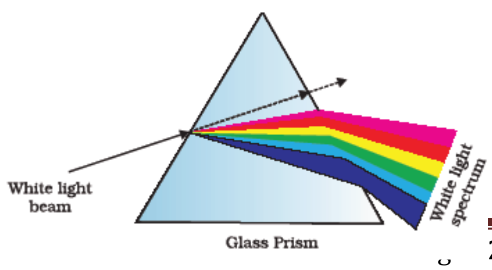
$$n_{21} = \frac{\sin[(A + D_m)/2]}{\sin[A/2]} \simeq \frac{(A + D_m)/2}{A/2}$$

$$D_m = (n_{21} - 1)A$$

It implies that, thin prisms do not deviate light much.



(22) Write a note on Dispersion by prism.
When a white light beam (sun light) passes through a prism, a continuous sequence of colours : **v**iolet, **i**ndigo, **b**lue, **g**reen, **y**ellow, **o**range and **r**ed (given by the acronym VIBGYOR). The red light bends the least, while the violet light bends the most as shown in Fig.



The phenomenon of splitting of light into its component colours is known as *dispersion*. The pattern of colour components of light is called the spectrum of light.

We know that colour is associated with wavelength of light. In the visible spectrum, red light is at the long wavelength end (~700 nm) while the violet light is at the short wavelength end (~ 400 nm). Dispersion takes place because the refractive index of medium for different wavelengths (colours) is different. For example, the bending of red component of white light is least while it is most for the violet. Equivalently, red light travels faster than violet light in a glass prism.

In vacuum, of course, the speed of light is independent of wavelength. Thus, vacuum (or air approximately) is a **non-dispersive medium** in which all colours travel with the same speed. This also follows from the fact that sunlight reaches us in the form of white light and not as its components. On the other hand, glass is a **dispersive medium**.

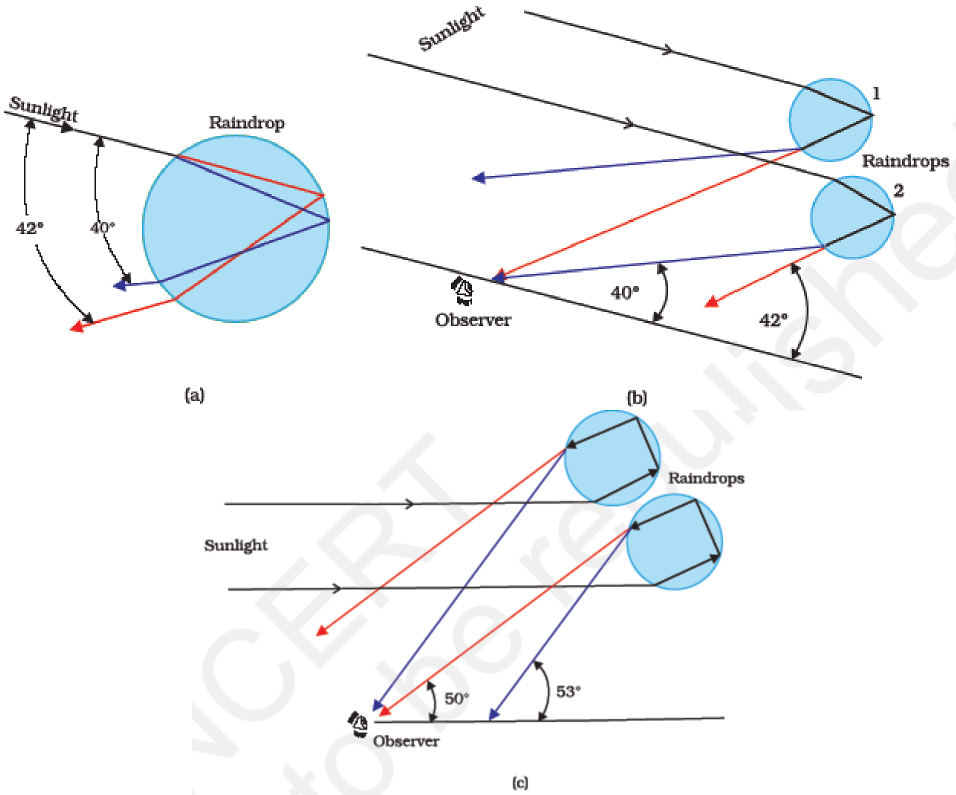
(23) Explain the formation of rainbow ?

The rainbow is an example of the dispersion of sunlight by the water drops in the atmosphere. This is a phenomenon due to combined effect of dispersion, refraction and reflection of sunlight by spherical water droplets of rain. The conditions for observing a rainbow are that the sun should be shining in one part of the sky (say near western horizon) while it is raining in the opposite part of the sky (say eastern horizon). An observer can therefore see a rainbow only when his back is towards the sun.

In order to understand the formation of rainbows, consider Fig. (a). Sunlight is first refracted as it enters a raindrop, which causes the different wavelengths (colours) of white light to separate. Longer wavelength of light (red) are bent the least while the shorter wavelength (violet) are bent the most. Next, these component rays strike the inner surface of the water drop and get internally reflected if the angle between the refracted ray and normal to the drop surface is greater then the critical angle (48º, in this case). The reflected light is refracted again as it comes out of the drop as shown in the figure. It is found that the violet light emerges at an angle of 40º related to the incoming sunlight and red light emerges at an angle of 42º. For other colours, angles lie in between these two values.

Figure (b) explains the formation of primary rainbow. We see that red light from drop 1 and violet light from drop 2 reach the observers eye. The violet from drop 1 and red light from drop 2 are directed at level above or below the observer. Thus the observer sees a rainbow with red colour on the top and violet on the bottom. Thus, the primary rainbow is a result of three-step process, that is, refraction, reflection and refraction.

When light rays undergoes *two* internal reflections inside a raindrop, instead of *one* as in the primary rainbow, a secondary rainbow is formed as shown in Fig. (c). It is due to four-step process. The intensity of light is reduced at the second reflection and hence the secondary rainbow is fainter than the primary rainbow. Further, the order of the colours is reversed in it as is clear from Fig. (c).

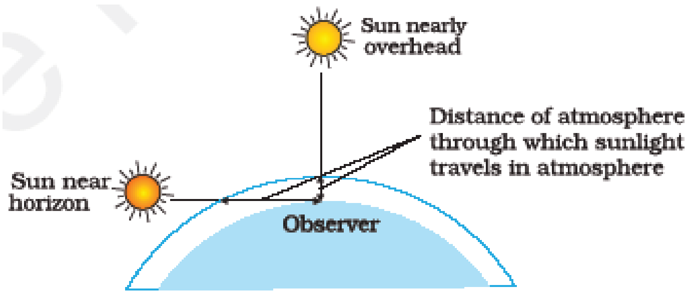


(24) Write a note Scattering of light. State Rayleigh's scattering law and hence explain Blue colour of the sky and reddish appearance of the sun at sunrise and sunset ?

As sunlight travels through the earth's atmosphere, it gets *scattered* (changes its direction) by the atmospheric particles. Light of shorter wavelengths is scattered much more than light of longer wavelengths. (The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as Rayleigh scattering). Hence, the bluish colour predominates in a clear sky, since blue has a shorter wavelength than red and is scattered much more strongly. In fact, violet gets scattered even more than blue, having a shorter wavelength. But since our eyes are more sensitive to blue than violet, we see the sky blue.

Large particles like dust and water droplets present in the atmosphere behave differently. The relevant quantity here is the relative size of the wavelength say, a). For $a \ll \lambda$, one has Rayleigh scattering which is proportional to $1/\lambda^4$. For $a \gg \lambda$, i.e., large scattering objects (for example, raindrops, large dust or ice particles) this is not true; all wavelengths are scattered nearly equally. Thus, clouds which have droplets of water with $a \gg \lambda$ are generally white.

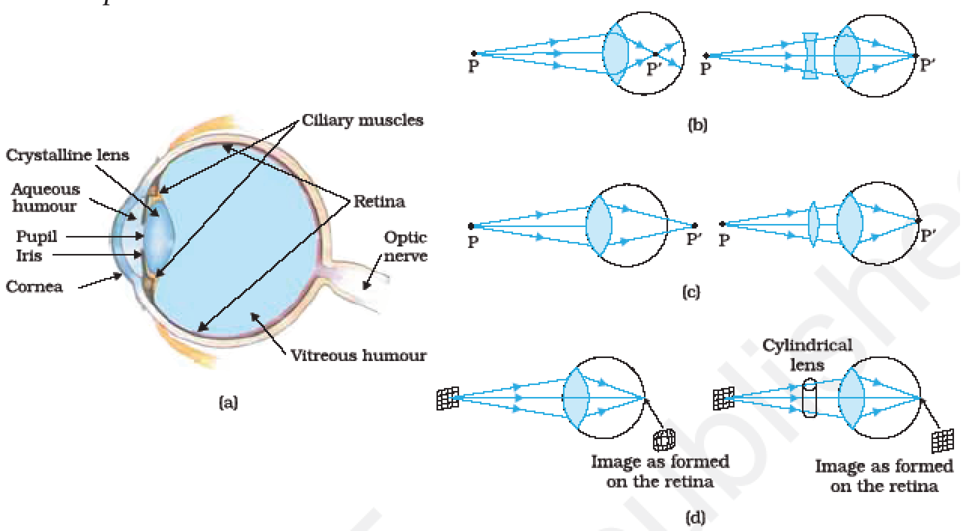
At sunset or sunrise, the sun's rays have to pass through a larger distance in the atmosphere (Fig.). Most of the blue and other shorter wavelengths are removed by scattering. The least scattered light reaching our eyes, therefore, the sun looks reddish. This explains the reddish appearance of the sun and full moon near the horizon.



(25) Write a note on Eye: Accommodation and least distance of distinct vision - Correction of eye defects (myopia and hypermetropia) using lenses.

Figure (a) shows the eye. Light enters the eye through a curved front surface, the cornea. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information. The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles. For example, when the muscle is relaxed, the focal length is about 2.5 cm and objects at infinity are in sharp focus on the retina.

When the object is brought closer to the eye, in order to maintain the same image-lens distance (= 2.5 cm), the focal length of the eye lens becomes shorter by the action of the ciliary muscles. This property of the eye is called *accommodation*. If the object is too close to the eye, the lens cannot curve enough to focus the image on to the retina, and the image is blurred. The closest distance for which the lens can focus light on the retina is called the *least distance of distinct vision*, or the *near point*.



The standard value for normal vision is taken as 25 cm. (Often the near point is given the symbol D.) This distance increases with age, because of the decreasing effectiveness of the ciliary muscle and the loss of flexibility of the lens. The near point may be as close as about 7 to 8 cm in a child ten years of age, and may increase to as much as 200 cm at 60 years of age. Thus, if an elderly person tries to read a book at about 25 cm from the eye, the image appears blurred. This condition (defect of the eye) is called *presbyopia*. It is corrected by using a converging lens for reading. Thus, our eyes are marvellous organs that have the capability to interpret incoming electromagnetic waves as images through a complex process. These are our greatest assets and we must take proper care to protect them. Imagine the world without a pair of functional eyes. Yet many amongst us bravely face this challenge by effectively overcoming their limitations to lead a normal life. They deserve our appreciation for their courage and conviction. In spite of all precautions and proactive action, our eyes may develop some defects due to various reasons. We shall restrict our discussion to some common optical defects of the eye. For example, the light from a distant object arriving at the eye-lens may get converged at a point in front of the retina. This type of defect is called *nearsightedness* or *myopia*. This means that the eye is producing too much convergence in the incident beam. To compensate this, we interpose a concave lens between the eye and the object, with the diverging effect desired to get the image focussed on the retina [Fig. (b)].

Similarly, if the eye-lens focusses the incoming light at a point behind the retina, a convergent lens is needed to compensate for the defect in vision. This defect is called *farsightedness* or *hypermetropia* [Fig. (c)].

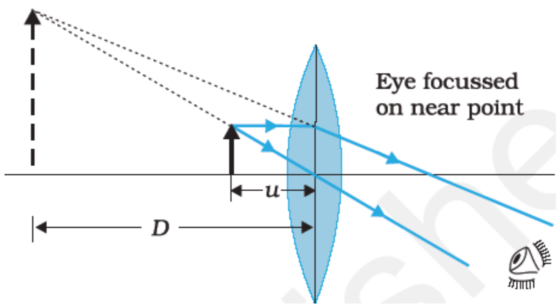
(26) Explain Simple microscope. Draw Ray diagram for image formation and Mention of expression for the magnifying power ?

A simple magnifier or microscope is a converging lens of small focal length (Fig.). In order to use such a lens as a microscope, the lens is held near the object, one focal length away or less, and the eye is positioned close to the lens on the other side. The idea is to get an erect, magnified and virtual image of the object at a distance so that it can be viewed comfortably, i.e., at 25 cm or more.

If the object is at a distance *f*, the image is at infinity. However, if the object is at a distance slightly less than the focal length of the lens, the image is virtual and closer than infinity. Although the closest comfortable distance for viewing the image is when it is at the near point (distance D = 25 cm), it causes some strain on the eye.

Therefore, the image formed at infinity is often considered most suitable for viewing by the relaxed eye.

Ray diagram for image formation :



The linear magnification m, for the image formed at the near point D, by a simple microscope can be obtained by using the relation

$$m = \frac{v}{u} = v\left(\frac{1}{v} - \frac{1}{f}\right) = \left(1 - \frac{v}{f}\right)$$

Now according to sign convention, v is negative, and is equal in magnitude to D. Thus, the magnification is

$$m = \left(1 + \frac{D}{f}\right)$$

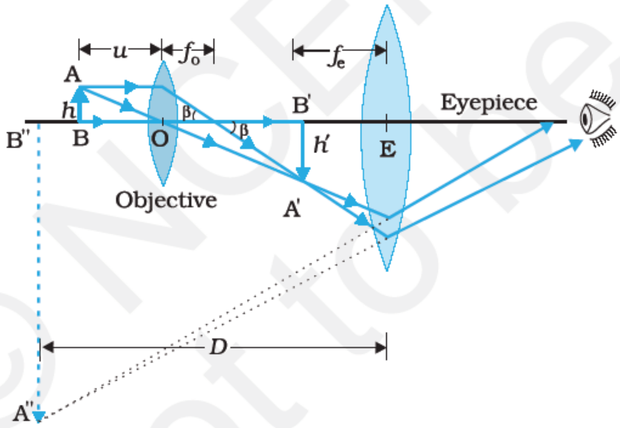
The angle subtended at the eye by the image when the object is at u is called angular magnification.
The angular magnification

$$m = \frac{\theta_i}{\theta_0} = \frac{h}{f}\frac{D}{h} = \frac{D}{f}$$

(27) What is Compound microscope ? Draw a ray diagram showing the formation of image by a compound microscope and derive expression for its magnifying power when the final image is at (a) least distance of distinct vision and (b) infinity.

A simple microscope has a limited maximum magnification (≤ 9) for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a *compound microscope*. A schematic diagram of a compound microscope is shown in Fig. 1.

The lens nearest the object, called the *objective*, forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the *eyepiece*, which functions essentially like a simple microscope or magnifier, produces the final image, which is enlarged and virtual. The first inverted image is thus near (at or within) the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity, or a little closer for image formation at the near point. Clearly, the final image is inverted with respect to the original object.



The total magnification [(according to Eq. (9.33)], when the image is formed at infinity, is
Magnification =

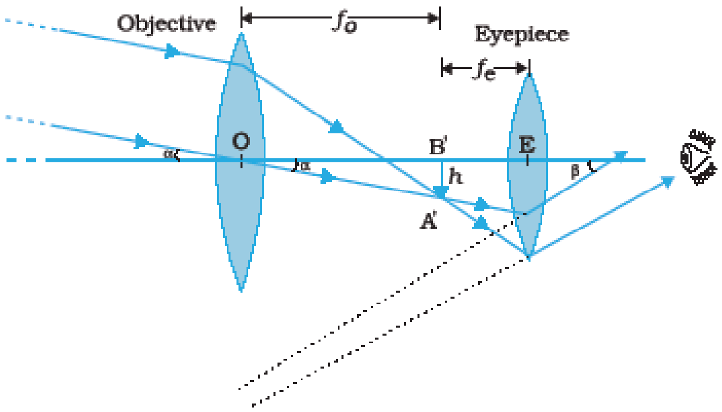
$$m = m_o m_e = \left(\frac{L}{f_o}\right)\left(\frac{D}{f_e}\right)$$

(28) Explain Telescope with Ray diagram for image formation. Mention of expression for the magnifying power and length of the telescope ($L = f_o + f_e$) ? (Refracting Telescope)

The telescope is used to provide angular magnification of distant objects (Fig. 9.32). It also has an objective and an eyepiece. But here, the objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image. The magnifying power *m* is the ratio of the angle β subtended at the eye by the final image to the angle α which the object subtends at the lens or the eye. Hence

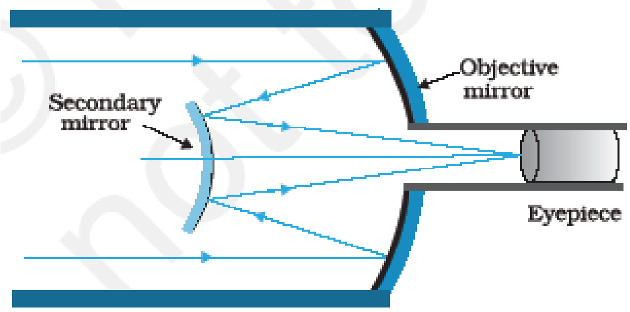
$$m \approx \frac{\beta}{\alpha} \approx \frac{h}{f_e} \cdot \frac{f_o}{h} = \frac{f_o}{f_e}$$

In this case, the length of the telescope tube is $f_o + f_e$.



(29) Draw Schematic ray diagram of reflecting telescope & explain its parts ?

Telescopes with mirror objectives are called *reflecting* telescopes. They have several advantages. First, there is no chromatic aberration in a mirror. Second, if a parabolic reflecting surface is chosen, spherical aberration is also removed. Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim. One obvious problem with a reflecting telescope is that the objective mirror focusses light inside the telescope tube. One must have an eyepiece and the observer right there, obstructing some light (depending on the size of the observer cage). This is what is done in the very large 200 inch (~5.08 m) diameters, Mt. Palomar telescope, California.



The viewer sits near the focal point of the mirror, in a small cage. Another solution to the problem is to deflect the light being focussed by another mirror. One such arrangement using a convex secondary mirror to focus the incident light, which now passes through a hole in the objective primary mirror, is shown in following figure. This is known as a *Cassegrain* telescope, after its inventor. It has the advantages of a large focal length in a short telescope. The largest telescope in India is in Kavalur, Tamil Nadu. It is a 2.34 m diameter reflecting telescope (Cassegrain). It was ground, polished, set up, and is being used by the Indian Institute of Astrophysics, Bangalore. The largest reflecting telescopes in the world are the pair of Keck telescopes in Hawaii, USA, with a reflector of 10 metre in diameter.

Numerical Problems.:

PU Board Questions

1. Define the terms (a) ray of light & (b) beam of light

A ray is defined as the straight line path joining the two points by which light is travelling. A beam is defined as the bundle of number of rays

2. State laws of reflection

I law:- the incident ray the reflected ray and the normal drawn at the point of incidence all lie in the same plane II law:- angle of incidence is equal to angle of reflection

3. Write the sign conventions used for measuring distances in case of spherical surfaces

a) All the distances are measured from the pole or optical center of the lens b) The distances measured along the direction of incident light are taken as positive and negative in a direction opposite to it. c) The heights measured upwards with respect to X-axis are positive and negative downwards

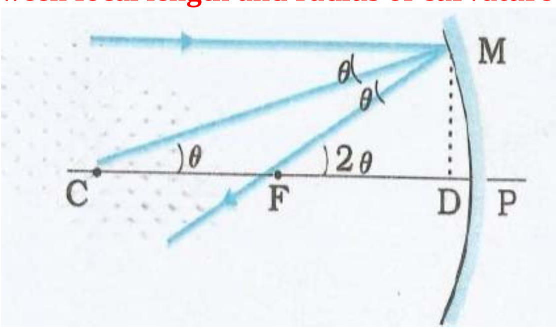
4. Define principal focus of a mirror.

It is a point on the principal axis where the parallel beams of light converge or appear to diverge after reflection

5. Define focal length of a mirror.

It is the distance between the principal focus and the pole of the mirror.

6. Derive the relation between focal length and radius of curvature of a spherical mirror



C = center of curvature, F = focal point or principal focus
θ = angle of incidence = angle of reflection
PF = f= focal length PC= R = radius of curvature
MD = perpendicular to PC $M\hat{F}P = 2\theta$

Consider the ΔMCD $tan\theta = \frac{MD}{CD}$ & in ΔMFD $tan2\theta = \frac{MD}{FD}$

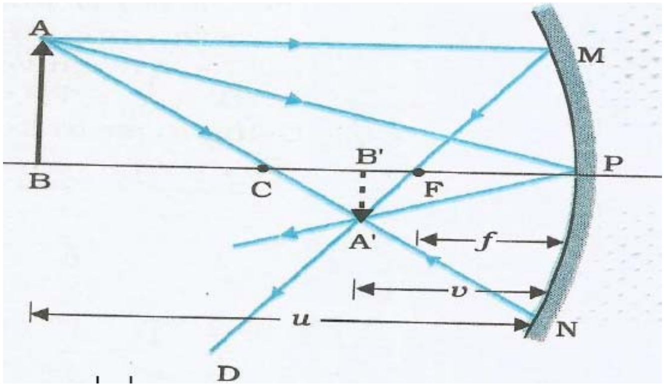
Since θ is very small tan θ ≈ θ and tan 2θ = 2θ

$\theta = \frac{MD}{CD}$ & $2\theta = \frac{MD}{FD}$ dividing, we get CD = 2 FD

D is a point very close to P. Therefore FD = FP = f CD =CP = R

$\therefore R = 2f \quad f = \frac{R}{2}$

7. Derive mirror equation.



MPN = spherical mirror,
AB = linear size of the object,
$A^{I}B^{I}$ = linear size of the image,
BP = u = object distance
$B^{I}P$ = v = image distance
FP = f =focal length
CP= R = radius of curvature

Triangles $A^{I}B^{I}F$ & MPF are similar

Therefore $\frac{B^{I}A^{I}}{PM} = \frac{B^{I}F}{FP} = \frac{B^{I}A^{I}}{BA} - - - - - - - - -(1)$

Triangles $A^{I}B^{I}P$ & ABP are similar

Therefore $\frac{B^{I}P}{BP} = \frac{B^{I}A^{I}}{BA} - - - - - - - -(2)$

From (1) & (2) $\frac{B^{I}P}{BP} = \frac{B^{I}F}{FP} = \frac{B^{I}P - FP}{FP}$

Applying sign convention $B^{I}P$ = -v, FP = -f, BP = -u

$\frac{-v+f}{-f} = \frac{-v}{-u} \Rightarrow \frac{v-f}{f} = \frac{v}{u}$ $\quad fv = u(v-f) = uv - uf$

Divide throughout by uvf

$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$ $\quad \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

8. Define linear magnification.

It is the ratio of the height of the image to the height of the object

9. Write the expression for the magnification in terms of object and image distance.

$m = -\frac{v}{u}$

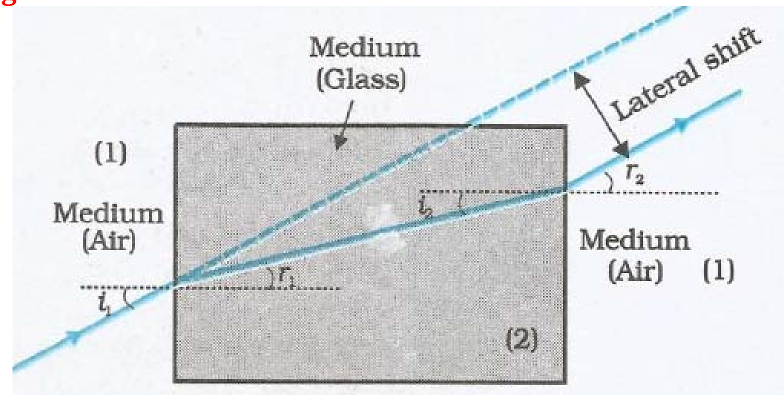
10. What is refraction of light?

The phenomenon of bending of light when it travels from one optical medium to the other is called refraction

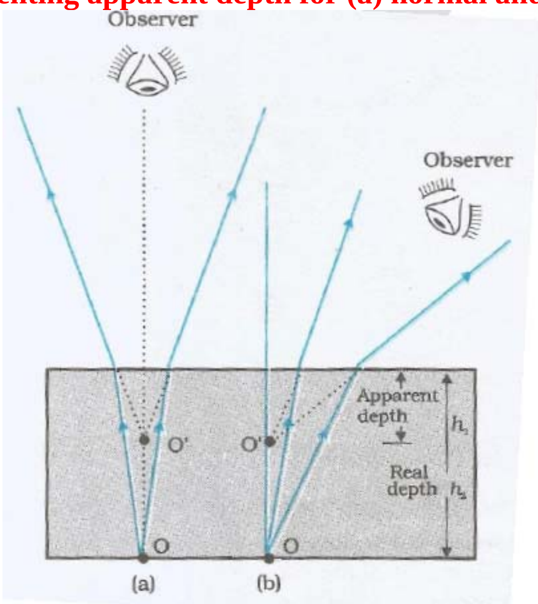
11. State laws of refraction.

I law: the incident ray, the refracted ray and the normal drawn at the point of incidence all lie in the same plane
II law: the ratio of the sine of the angle of the incidence to the sine of the angle of refraction is constant for a given pair of media and given wavelength (color) of light

12. Draw diagram representing lateral shift (lateral displacement) of a ray passing through a parallel sided glass slab.



13. Draw diagram representing apparent depth for (a) normal and (b) oblique viewing



14. Mention a few illustrations that occur in nature due to refraction of light.
A) The apparent shift in the direction of the sun and hence 2 minute apparent delays between actual sun set and apparent sun set
B) The apparent flattening of the sun at sunrise and sunset

15. Write the formula for refractive index for normal refraction.

$$n = \frac{\text{Real depth}}{\text{Apparent depth}}$$

16. What happens to the direction of the incident ray when it travels from (a) optically denser medium to rarer medium & (b) optically rarer medium to denser medium?

(a) It bends away from the normal
(b) It bends towards the normal

17. Define critical angle.

It is a particular angle of incidence in denser medium for which the refracted ray grazes the surface of separation OR the angle of refraction is 90° .

18. Write the relation between refractive index and critical angle of a material

$n_{12} = \frac{1}{\sin i_c}$ Where n_{12} = refractive index of denser medium with respect to rarer medium and i_c =critical angle

19. What is total internal reflection?

When a ray of light travels from denser to rarer medium and if the angle of incidence is greater than the critical angle then the light gets totally internally reflected to the same medium. This phenomena is called total internal reflection

20. Write the conditions to have total internal reflection

(a) A ray of light should travel from denser to rarer medium
(b) Angle of incidence must be greater than the critical angle

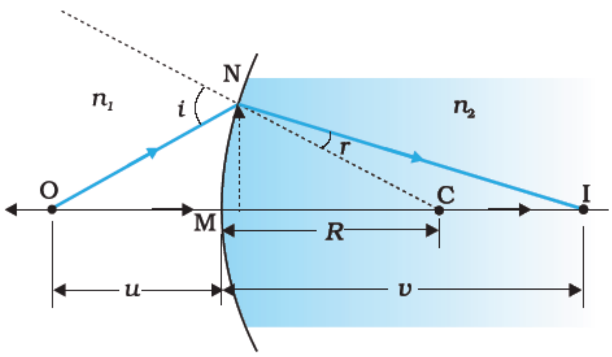
21. Mention a few illustrations of total internal reflection Mirage, sparkling of diamond, total internal reflecting prisms, optical fibers

22. **On what principle optical fiber does works?** It works on the principle of total internal reflection.

23. What is a lens? It is an optical medium bounded by two spherical surfaces.

24. Derive the relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the spherical surface OR derive the relation between n, u, v, & R

OM = u = object distance
MI = v = image distance
MC = R = radius of curvature
Angle i = angle of incidence
Angle r = angle of refraction
ON = incident ray NI = refracted ray
NC = normal & n_1, n_2 are the refractive indices
From the figure for small angles



$$\tan N\hat{O}M = \frac{MN}{OM} \quad \tan N\hat{C}M = \frac{MN}{MC} \quad \tan N\hat{I}M = \frac{MN}{MI}$$

in the triangle NOC, $\hat{\imath}$ = exterior angle = sum of the interior opposite angle

$$\hat{\imath} = N\hat{O}C + N\hat{C}O = N\hat{O}M + N\hat{C}M = \frac{MN}{OM} + \frac{MN}{MC}$$

Similarly $N\hat{C}M = C\hat{N}I + N\hat{I}C = C\hat{N}I + N\hat{I}M$

$$\hat{r} = C\hat{N}I = M\hat{C}N - N\hat{I}M = \frac{MN}{MC} - \frac{MN}{MI}$$

From Snell's law $n_1 \sin i = n_2 \sin r$

For small angles $n_1 i = n_2 r$

Substituting the values of i & r we get

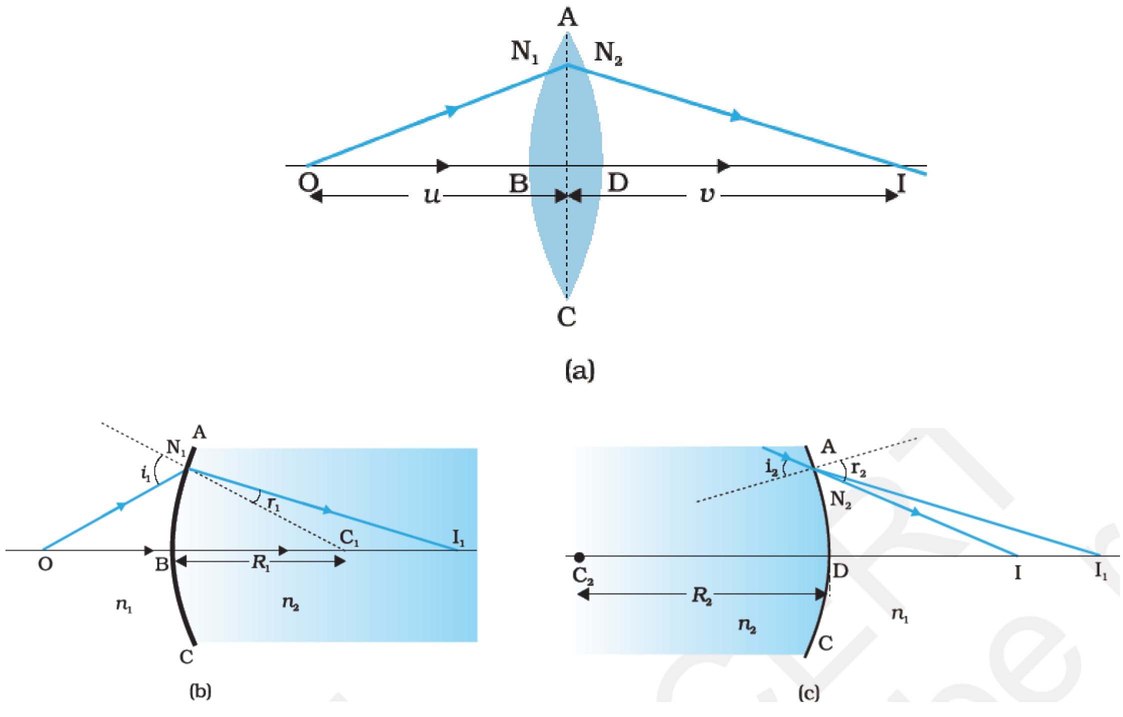
$$n_1\left(\frac{MN}{OM} + \frac{MN}{MC}\right) = n_2\left(\frac{MN}{MC} - \frac{MN}{MI}\right)$$

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC}$$

Applying sign convention, OM = -u , MI = v MC = R

$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_2 - n_1}{R}$$

25. Derive lens maker's formula



Let u = object distance, v = image distance, R_1 & R_2 are the radii of curvatures of surface I and surface II. Consider the I spherical surface ABC, applying the formula

$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{n_1}{OB} - \frac{n_2}{BI_1} = \frac{n_2 - n_1}{BC_1} ------(1)$$

For the second surface ADC

$$\frac{n_1}{DI} - \frac{n_2}{DI_1} = \frac{n_2 - n_1}{DC_2} ------(2)$$

For a thin lens $BI_1 = DI_1$

Adding equations (1) & (2)

$$\frac{n_1}{OB} + \frac{n_1}{DI} = (n_2 - n_1)\left(\frac{1}{BC_1} - \frac{1}{DC_2}\right)$$

But OB = u, DI = v, $BC_1 = R_1$, $BC_2 = R_2$ and

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{n_1}{u} + \frac{n_1}{v} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{n_1}{f} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

26. Define power of a lens. Write its S.I unit.

It is the ability of a lens to converge or diverge a beam of light falling on it.

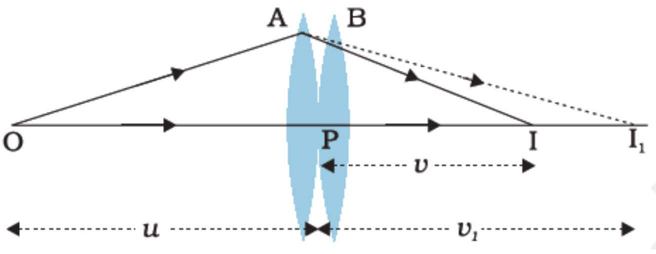
S.I unit of power is diopter (D)

27. Write the expression for power of a lens

P = 1/f

28. Derive the expression for effective focal length of two thin lenses in contact.

OP = u = object distance PI = v = image distance due to the combination $PI_1 = V_1$ = image distance due to first lens For the lens A,



$$-\frac{1}{u} + \frac{1}{v_1} = \frac{1}{f_1} ----(1)$$

For the lens B,

$$\frac{1}{v} + \frac{1}{v_1} = \frac{1}{f_2} ----(2)$$

Adding equations (1) & (2)

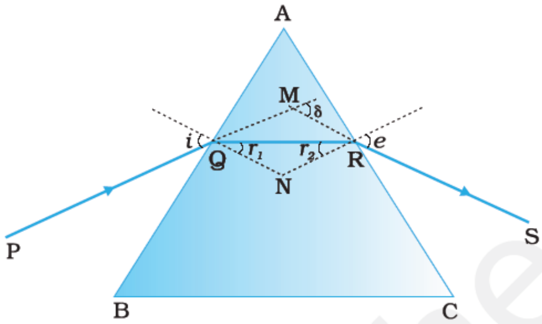
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}$$

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

29. Write the expression for the power of a combination of number of thin lenses

$P = P_1 + P_2 + P_3 + \ldots\ldots\ldots\ldots\ldots$

30. Arrive at the expression for refractive index of material of prism in terms of angle of the prism and angle of minimum deviation.



ABC = principal section of the prism

A = angle of the prism PQ = incident ray

QR = refracted ray RS = emergent ray

I = angle of incidence

e=angle of emergence

r_1 & r_2 angles of refraction

δ = angle of deviation

In the quadrilateral AQNR, $\hat{A}+Q\hat{N}R=180^0$

In the triangle QNR $r_1+r_2+Q\hat{N}R=180^0$

$$\therefore \hat{A}=r_1+r_2$$

Exterior angle δ = sum of interior opposite angles

$$\delta=(i-r_1)+(e-r_2)$$

$$\delta=(i+e)-(r_1+r_2)=(i+e)-A$$

A graph of angle of incidence with angle of deviation is as shown in the figure At minimum deviation $\delta=D_m,\quad i=e,\quad r_1=r_2=r$

$\therefore A=2r \qquad r={}^{A}/_{2} \qquad D_m=2i-A \qquad i=\frac{A+D_m}{2}$

$$n_{21}=\frac{\sin i}{\sin r}=\frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left({}^{A}/_{2}\right)}$$

31. What is dispersion of light?

The phenomenon of splitting of white light into its component colors is known as dispersion

32. State Rayleigh's law of scattering.

The intensity of the scattered light (the amount of scattering) is inversely proportional to the fourth power of the wavelength.

33. Why sky is blue in color?

Blue has a shorter wavelength than red and is scattered much more strongly than any other color. Violet scatters more than that of blue, but our eyes are more sensitive to blue than violet. Therefore sky appears blue.

34. Why sun is red at rise and set?
At sunset and sun rise, sun is at horizon. Sun rays have to pass through larger distance in the atmosphere. Most of the blue and other shorter wavelengths are removed by scattering. The least scattered red light reaches our eyes. Hence sun is red at rise and set.

35. What is accommodation of eye?
The modification of the focal length of the eye lens by the ciliary muscles to see the objects at all possible distances is called accommodation.

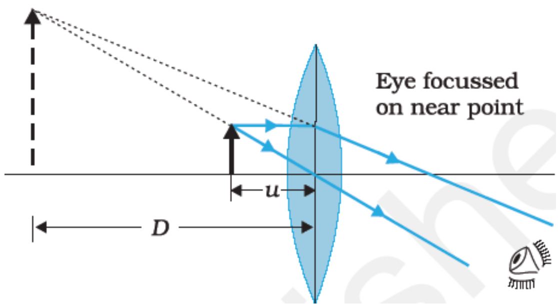
36. What is least distance of distinct vision? Write its value.
The closest distance for which the eye lens can focus light on the retina is called least distance of distinct vision
For normal vision it is 25 cm

37. Which are the common defects of human eye?
a) Myopia or near sightedness
b) Hypermetropia or far sightedness
c) Astigmatism

38. What is myopia? Why it occurs? How to correct it?
It is a defect in human eye where the image of the object is formed in front of the retina. This is due to too much of convergence produced by the eye lens. It can be corrected using a concave lens.

39. What is hypermetropia? Why it occurs? How to correct it?
It is a defect in human eye where the image of the object is formed behind the retina. This is due to too much of divergence produced by the eye lens. It can be corrected using a convex lens.

40. Draw ray diagram of a simple microscope.



41. Mention the expression for linear magnification of a simple microscope.

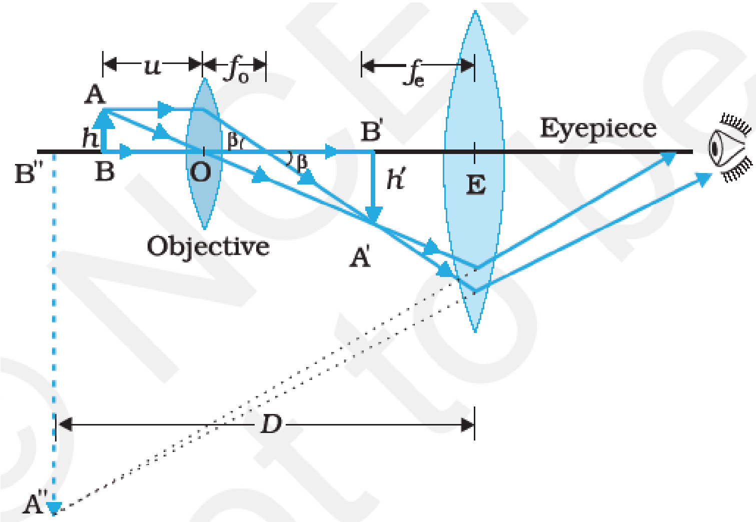
$$m = 1 + \frac{D}{f}$$

42. Mention the expression for angular magnification of a simple microscope.

The angular magnification

$$m = \frac{\theta_i}{\theta_0} = \frac{h}{f}\frac{D}{h} = \frac{D}{f}$$

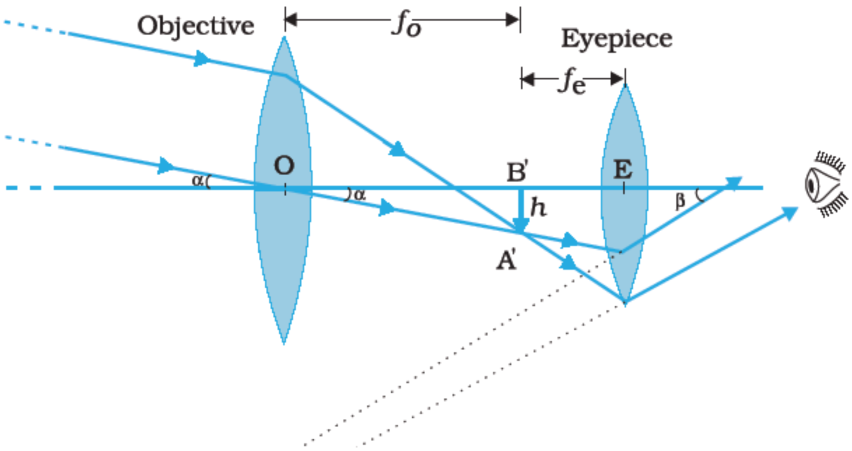
43. Draw ray diagram showing the image formation in a compound microscope and label the parts.



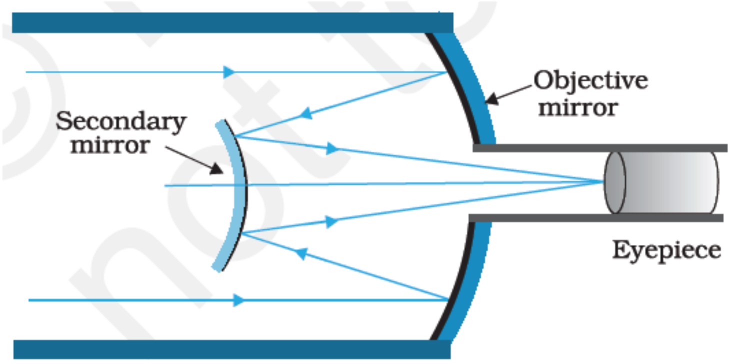
44. Mention the expression for magnification of a compound microscope.

Magnification . $= m_o m_e = \frac{L}{fo}\frac{D}{fe}$.

45. Draw the ray diagram of a refracting telescope and label the parts.



46. Draw schematic diagram of a reflecting telescope



Most Likely Questions : (1 M - 1Q; 2M-1Q; 5M-1Q)

1 Mark Question

(1) Write the expression for power of lens and its S.I. Unit.

2 Mark Question

(1) Define dispersion and its cause. Explain the formation of rainbow.

(2) Draw a ray diagram showing the formation of image by a simple microscope and derive expression for its magnifying power.

(3) Draw a ray diagram showing the formation of image by a compound microscope and derive expression for its magnifying power.

(4) Draw a ray diagram showing the formation of image by an astronomical telescope in normal adjustment and derive expression for its magnifying power.

(5) Which are the defects of human eye? How are they corrected?

5 Mark Question

(1) Derive mirror formula

(2) Derive an expression for refraction at a spherical surface and hence derive lens makers formula

(3) Derive thin lens formula

(4) Derive an expression for the effective focal length of two thin lenses in contact.

(5) Discuss refraction of monochromatic light through a prism and Derive prism formula. Also obtain an expression for refractive index of the material of the prism.

(6) Derive the relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the spherical surface OR derive the relation between n, u, v, & R ?

(7) Define focal length of a mirror. Derive the relation between focal length and radius of curvature of a spherical mirror.

Chapter 10
WAVE OPTICS
8M
(3M - 1Q, 5M - 1Q(NP))

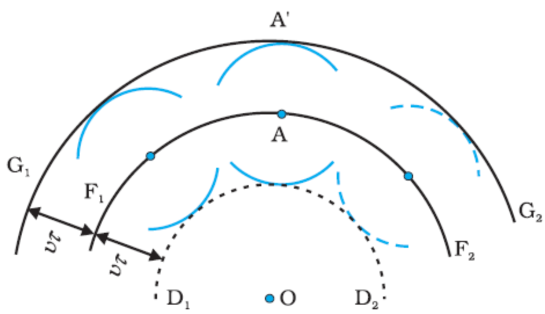
(1) Define Huygens principle. Explain wave front: plane, spherical and cylindrical type. ?

A locus of points, which oscillate in phase is called a *wavefront* ; thus *a wavefront is defined as a surface of constant phase*. The speed with which the wavefront moves outwards from the source is called the speed of the wave. The energy of the wave travels in a direction perpendicular to the wavefront.

If we have a point source emitting waves uniformly in all directions, then the locus of points which have the same amplitude and vibrate in the same phase are spheres and we have what is known as a *spherical wave* as shown in Fig. 1(a).

At a large distance from the source, a small portion of the sphere can be considered as a plane and we have what is known as a *plane wave Fig. 1 (b).*

A according to **Huygens principle**, *each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront are usually referred to as secondary wavelets and if we draw a common tangent to all these spheres, we obtain the new position of the wavefront at a later time.*



A wavefront is the long edge that moves, for example, the crest or the trough. Each point on the wavefront emits a semicircular wave that moves at the propagation speed v . These are drawn at a time t later, so that they have moved a distance $s = vt$. The new wavefront is a line tangent to the wavelets and is where we would expect the wave to be a time t later. Huygens's principle works for all types of waves, including water waves, sound waves, and light waves.

2. Refraction of plane wave (rarer to denser), derivation of Snell's law -

3. Reflection of a plane wave by a plane surface, derivation of the law of reflection.

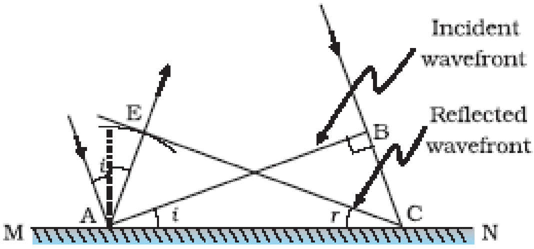
Consider a plane wave AB incident at an angle i on a reflecting surface MN. If v represents the speed of the wave in the medium and if t represents the time taken by the wavefront to advance from the point B to C then the distance

BC = v t

In order the construct the reflected wavefront we draw a sphere of radius v t from the point A as shown in following Fig. Let CE represent the tangent plane drawn from the point C to this sphere. Obviously

AE = BC = v t

If we now consider the triangles EAC and BAC in following figure, we will find that they are congruent and therefore, the angles i and r would be equal. This is the *law of reflection*.



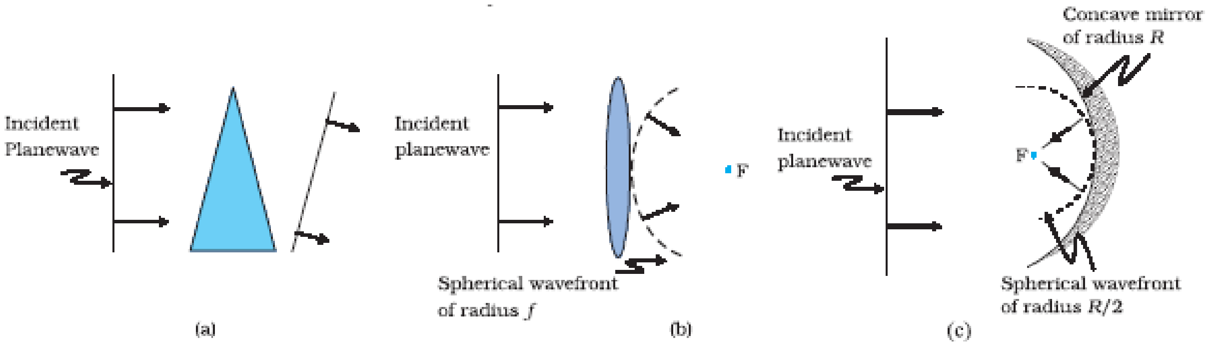
4. Explain of refraction of a plane wave by (a) a thin prism, (b) by a convex lens and (c) by a concave mirror, using diagrams.

Here we described the behaviour of the wavefronts as they undergo reflection or refraction. In Fig. (a) we consider a plane wave passing through a thin prism. Clearly, since the speed of light waves is less in glass, the lower portion of the incoming wavefront (which travels through the greatest thickness of glass) will get delayed resulting in a tilt in the emerging wavefront as shown in the figure.

In Fig. (b) we consider a plane wave incident on a thin convex lens; the central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most. The emerging wavefront has a depression at the centre and therefore the wavefront becomes spherical and converges to the point F which is known as the *focus*.

In Fig. (c) a plane wave is incident on a concave mirror and on reflection we have a spherical wave converging to the focal point F. In a similar manner, we can understand refraction and reflection by concave lenses and convex mirrors.

Thus the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray.



5. Coherent sources - Theory of interference, (with equal amplitude) arriving at the conditions for constructive and destructive interference.

Interference is based on the *superposition principle* according to which *at a particular point in the medium, the resultant displacement produced by a number of waves is the vector sum of the displacements produced by each of the waves*.

Two sources producing waves are said to be *coherent* if at a particular point, the phase difference between the displacements produced by each of the waves does not change with time;

Consider two needles S_1 and S_2 moving periodically up and down in an identical fashion in a trough of water [Fig. 10.8(a)]. They produce two water waves, and at a particular point, the phase difference between the displacements produced by each of the waves does not change with time; hence they are coherent.

Figure 10.8(b) shows the position of crests (solid circles) and troughs (dashed circles) at a given instant of time. Consider a point P for which $S_1 P = S_2 P$
Since the distances S_1 P and S_2 P are equal, waves from S_1 and S_2 will take the same time to travel to the point P and waves that emanate from S_1 and S_2 in phase will also arrive, at the point P, in phase. Thus, if the displacement produced by the source $S1$ at the point P is given by
$y_1 = a \cos wt$
then, the displacement produced by the source S_2 (at the point P) will also be given by
$y_2 = a \cos wt$
Thus, the resultant of displacement at P would be given by
$y = y_1 + y_2 = 2\, a \cos wt$
Since the intensity is the proportional to the square of the amplitude, the resultant intensity will be given by
$I = 4\, I_0$
where I_0 represents the intensity produced by each one of the individual sources; I_0 is proportional to a_2. In fact at any point on the perpendicular bisector of S_1S_2, the intensity will be $4I_0$. The two sources are said to interfere constructively and we have what is referred to as *constructive interference*.

We next consider a point Q [Fig. 10.9(a)] for which
$S_2Q - S_1Q = 2\,\lambda$

The waves emanating from S1 will arrive exactly two cycles earlier than the waves from S_2 and will again be in phase [Fig. 10.9(a)]. Thus, if the displacement produced by S_1 is given by
$y_1 = a \cos wt$
then the displacement produced by S_2 will be given by
$y_2 = a \cos (wt - 4p) = a \cos wt$
where we have used the fact that a path difference of 2λ corresponds to a phase difference of 4p. The two displacements are once again in phase and the intensity will again be 4 I_0 giving rise to constructive interference.
In the above analysis we have assumed that the distances S_1Q and S_2Q are much greater than d (which represents the distance between S_1 and S_2) so that although S_1Q and S_2Q are not equal, the amplitudes of the displacement produced by each wave are very nearly the same.

We next consider a point R [Fig. 10.9(b)] for which
$S_2R - S_1R = -2.5\,\lambda$
The waves emanating from S_1 will arrive exactly two and a half cycles later than the waves from S_2 [Fig. 10.10(b)]. Thus if the displacement produced by S_1 is given by
$y_1 = a \cos wt$
then the displacement produced by S_2 will be given by
$y_2 = a \cos (wt + 5\phi) = - a \cos wt$
where we have used the fact that a path difference of 2.5 λ corresponds to a phase difference of 5p. The two displacements are now out of phase and the two displacements will cancel out to give zero intensity. This is referred to as *destructive interference*.

To summarise:
(1) If we have two coherent sources S_1 and S_2 vibrating in phase, then for an arbitrary point P whenever the path difference,
$S_1P \sim S_2P = n\lambda \quad (n = 0, 1, 2, 3,...)$ -------- (10.10)
we will have constructive interference and the resultant intensity will be $4I_0$; the sign ~ between S_1P and S_2 P represents the difference between S_1P and S_2 P.

On the other hand, if the point P is such that the path difference,
$S_1P \sim S_2P = (n + ½)\,\lambda \quad (n = 0, 1, 2, 3, ...)$ --------- (10.11)
we will have *destructive interference* and the resultant intensity will be zero.
Now, for any other arbitrary point G (Fig. 10.10) let the phase difference between the two displacements be ϕ. Thus, if the displacement produced by S_1 is given by
$y_1 = a \cos wt$
then, the displacement produced by S_2 would be
$y_2 = a \cos (wt + \phi)$
and the resultant displacement will be given by
$y = y_1 + y_2$
$= a [\cos wt + \cos (wt + \phi)]$
$= 2\,a \cos (\phi/2) \cos (wt + \phi/2)$
The amplitude of the resultant displacement is $2a \cos (\phi/2)$ and therefore the intensity at that point will be

$I = 4\, I_0 \cos^2 (\phi/2)$ ---------- (10.12)

If $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$ which corresponds to the condition given by Eq. (10.10) we will have constructive interference leading to maximum intensity.

On the other hand, if $\phi = \pm\pi, \pm 3\pi, \pm 5\pi \ldots$ [which corresponds to the condition given by Eq. (10.11)] we will have destructive interference leading to zero intensity.

(2) Now if the two sources are coherent (i.e., if the two needles are going up and down regularly) then the phase difference f at any point will not change with time and we will have a stable interference pattern; i.e., the positions of maxima and minima will not change with time. However, if the two needles do not maintain a constant phase difference, then the interference pattern will also change with time and, if the phase difference changes very rapidly with time, the positions of maxima and minima will also vary rapidly with time and we will see a "time-averaged" intensity distribution. When this happens, we will observe an average intensity that will be given by

$I = 4\, I_0\, (\cos^2 \phi /2)$

(3) When the phase difference between the two vibrating sources changes rapidly with time, we say that the two sources are incoherent and when this happens the intensities just add up.

The resultant intensity will be given by

$\boldsymbol{I = 2\, I_0}$ ----------- (10.14)

at all points.

6. Young's experiment: Brief description - Derivation of fringe width.

(1) If we use two sodium vapour lamps to illuminate two pinholes we will not observe any interference fringes. This is because of the fact that the light wave emitted from an ordinary source (like a sodium lamp) undergoes abrupt phase changes in times of the order of 10^{-10} seconds. Thus the light waves coming out from two independent sources of light will not have any fixed phase relationship and would be incoherent, when this happens, the intensities on the screen will add up.

(2) British physicist Thomas Young used an ingenious technique to "lock" the phases of the waves emanating from S_1 and S_2. He made two pinholes S_1 and S_2 (very close to each other) on an opaque screen [Fig. 10.12(a)]. These were illuminated by another pinholes that was in turn, lit by a bright source. Light waves spread out from S and fall on both S_1 and S_2.

S_1 and S_2 then behave like two coherent sources because light waves coming out from S_1 and S_2 are derived from the same original source and any abrupt phase change in S will manifest in exactly similar phase changes in the light coming out from S_1 and S_2. Thus, the two sources S_1 and S_2 will be *locked* in phase; i.e., they will be coherent.

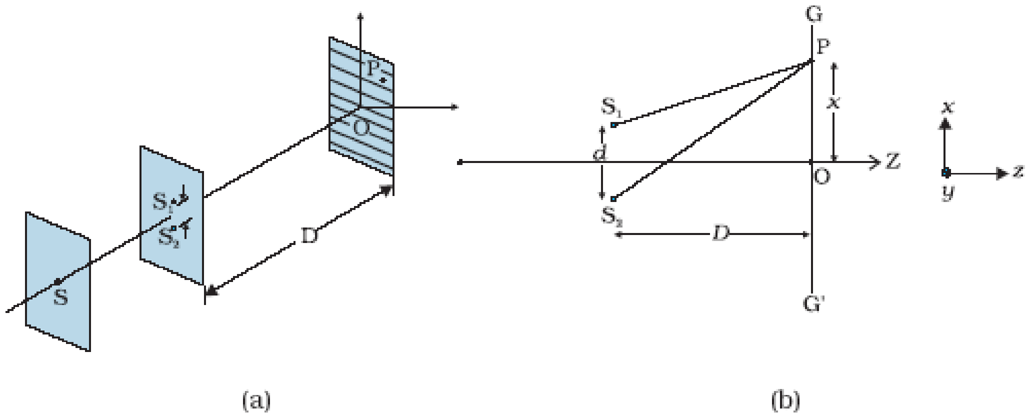


FIGURE 10.12 Young's arrangement to produce interference pattern.

Thus spherical waves emanating from S1 and S2 will produce interference fringes on the screen GG¢, as shown in Fig. 10.12(b).

(3) The positions of maximum and minimum intensities can be calculated by using an arbitrary point P on the line GG¢ [Fig. 10.12(b)] to correspond to a maximum, we must have

$S_2P - S_1P = n\lambda \quad n = 0, 1, 2 \ldots$ --------- (10.15)

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right] = 2xd$$

where $S_1S_2 = d$ and $OP = x$. Thus

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

If x, $d \ll D$ then negligible error will be introduced if $S_2P + S_1P$ (in the denominator) is replaced by $2D$. For example, for $d = 0.1$ cm, $D = 100$ cm, OP = 1 cm (which correspond to typical values for an interference experiment using light waves), we have

$S_2P + S_1P = [(100)^2 + (1.05)^2]^{½} + [(100)^2 + (0.95)^2]^{½}$

$\approx$ 200.01 cm

Thus if we replace $S_2P + S_1P$ by 2 D, the error involved is about 0.005%. In this approximation, Eq. (10.16) becomes

$S_2P - S_1P \approx xd/D$ --------- (10.17)

Hence we will have constructive interference resulting in a bright region when

$x = x_n = n\,\lambda D/d \;;\; n = 0, \pm 1, \pm 2, \ldots$ -------- (10.18)

On the other hand, we will have a dark region near

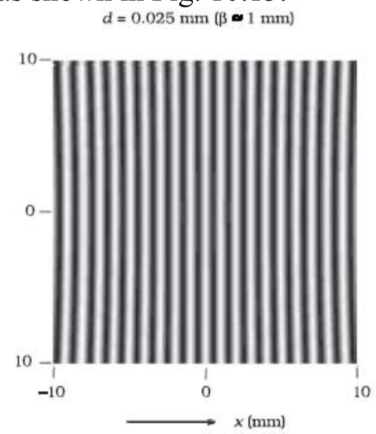
$x = x_n = (n + ½)\,\lambda D/d \;;\quad n = 0, \pm1, \pm2$ -------- (10.19)

Thus dark and bright bands appear on the screen, as shown in Fig. 10.13. Such bands are called *fringes*. Equations (10.18) and (10.19) show that dark and bright fringes are equally spaced and the distance between two consecutive bright and dark fringes is given by

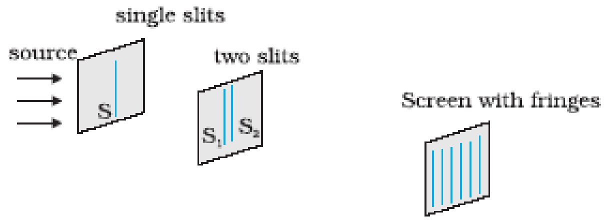
$\beta = x_{n+1} - x_n$ or $\beta = \lambda D/d$ ---------- (10.20)

which is the expression for the *fringe width*. Obviously, the central point O (in Fig. 10.12) will be bright because $S_1O = S_2O$ and it will correspond to $n = 0$. If we consider the line perpendicular to the plane of the paper and passing through O [i.e., along the y-axis] then all points on this line will be equidistant from S_1 and S_2 and we will have a bright central fringe which is a straight line as shown in Fig. 10.13.

(4) In order to determine the shape of the interference pattern on the screen we note that a particular fringe would correspond to the locus of points with a constant value of $S_2P - S_1P$. Whenever this constant is an integral multiple of l, the fringe will be bright and whenever it is an odd integral multiple of $\lambda/2$ it will be a dark fringe. Now, the locus of the point P lying in the x-y plane such that $S_2P - S_1P$ ($= \Delta$) is a constant, is a hyperbola. Thus the fringe pattern will strictly be a hyperbola; however, if the distance D is very large compared to the fringe width, the fringes will be very nearly straight lines as shown in Fig. 10.13.



(5) We should mention here that the fringes are straight lines although S_1 and S_2 are point sources. If we had slits instead of the point sources (Fig. 10.14), each pair of points would have produced straight line fringes resulting in straight line fringes with increased intensities.



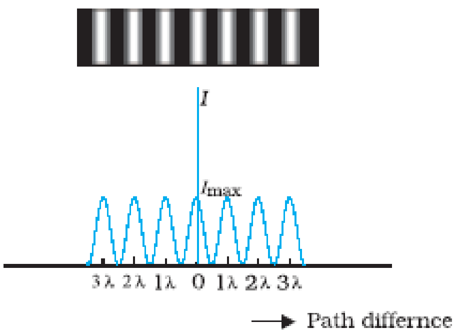


FIGURE 10.14 Photograph and the graph of the intensity distribution in Young's double-slit experiment.

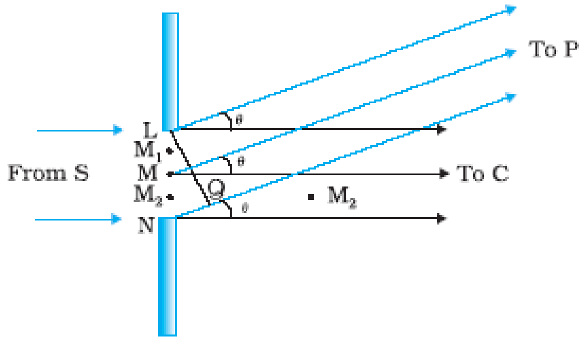
7. Diffraction: Explanation of the phenomenon - Diffraction due to a single slit -Mention of the conditions for diffraction minima and maxima - Intensity distribution curve.

(1) Explain the phenomenon of diffraction ?

The phenomena of spreading of light from narrow holes or slits when the edges of holes/slits are of the order of wavelength of light is called diffraction.

(2) Explain Diffraction due to a single slit. Obtain the conditions for diffraction minima and maxima ?

When the double slit in Young's experiment is replaced by a single narrow slit (illuminated by a monochromatic source), a broad pattern with a central bright region is seen. On both sides, there are alternate dark and bright regions, the intensity becoming weaker away from the centre (Fig. 10.16).



Consider a parallel beam of light falling normally on a single slit LN of width *a*. The diffracted light goes on to meet a screen. The midpoint of the slit is M.

A straight line through M perpendicular to the slit plane meets the screen at C. We want the intensity at any point P on the screen. As before, straight lines joining P to the different points L,M,N, etc., can be treated as parallel, making an angle q with the normal MC.

The basic idea is to divide the slit into much smaller parts, and add their contributions at P with the proper phase differences. We are treating different parts of the wavefront at the slit as secondary sources. Because the incoming wavefront is parallel to the plane of the slit, these sources are in phase.

The path difference NP – LP between the two edges of the slit can be calculated exactly as for Young's experiment. From Fig. 10.15,

NP – LP = NQ

$= a \sin\theta \quad \approx a\,\theta$ -------- (10.21)

Similarly, if two points M_1 and M_2 in the slit plane are separated by y, the path difference $M_2P - M_1P \approx y\theta$. We now have to sum up equal, coherent contributions from a large number of sources, each with a different phase.

At the central point C on the screen, the angle θ is zero. All path differences are zero and hence all the parts of the slit contribute in phase. This gives maximum intensity at C. Experimental observation shown in Fig. 10.15 indicates that the intensity has a central maximum at θ = 0 and other secondary maxima at $\theta = (n + 1/2)\,\lambda/a$, and has minima (zero intensity) at $\theta \approx n\,\lambda/a$, $n = \pm 1, \pm 2, \pm 3, ...$ It is easy to see why it has minima at these values of angle. Consider first the angle θ where the path difference $a\theta$ is λ. Then,

$\theta = \lambda/a$ ---------- (10.22)

Now, divide the slit into two equal halves LM and MN each of size $a/2$. For every point M_1 in LM, there is a point M_2 in MN such that $M_1M_2 = a/2$. The path difference between M_1 and M_2 at $P = M_2P - M_1P = \theta a/2 = \lambda/2$ for the angle chosen. This means that the contributions from M_1 and M_2 are 180º out of phase and cancel in the direction $\theta = \lambda/a$. Contributions from the two halves of the slit LM and MN, therefore, cancel each other. Equation (10.22) gives the angle at which the intensity falls to zero.

One can similarly show that the intensity is zero for $\theta = n\lambda/a$, with n being any integer (except zero!). Notice that the angular size of the central maximum increases when the slit width a decreases.

It is also easy to see why there are maxima at $\theta \approx (n + 1/2)\,\lambda/a$ and why they go on becoming weaker and weaker with increasing n. Consider an angle $\theta = 3\,\lambda/2a$ which is midway between two of the dark fringes.

Divide the slit into three equal parts. If we take the first two thirds of the slit, the path difference between the two ends would be

$$\frac{2}{3}a \times \theta = \frac{2a}{3} \times \frac{3\lambda}{2a} = \lambda$$

Thus in diffraction the path difference between two fringes = λ/2.

(3) Intensity distribution curve :

The first two-thirds of the slit can therefore be divided into two halves which have a λ/2 path difference. The contributions of these two halves cancel in the same manner as described earlier. Only the remaining one-third of the slit contributes to the intensity at a point between the two minima. Clearly, this will be much weaker than the central maximum (where the entire slit contributes in phase). One can similarly show that there are maxima at $(n + 1/2)\,\lambda/a$ with $n = 2$, 3, etc. These become weaker with increasing n, since only one-fifth, one-seventh, etc., of the slit contributes in these cases. The photograph and intensity pattern corresponding to it is shown in Fig. 10.16.

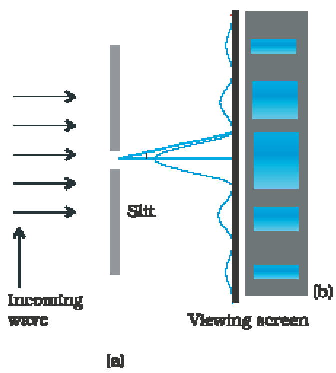


FIGURE 10.16 Intensity distribution and photograph of fringes due to diffraction at single slit.

(4) Compare & contrast Interference & Diffraction ?

(i) The interference pattern has a number of equally spaced bright and dark bands. The diffraction pattern has a central bright maximum which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre, on either side.

(ii) We calculate the interference pattern by superposing two waves originating from the two narrow slits. The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.

(iii) For a single slit of width a, the first null of the interference pattern occurs at an angle of l/a. At the same angle of l/a, we get a maximum (not a null) for two narrow slits separated by a distance a.

(iv) In interference and diffraction, light energy is redistributed. If it reduces in one region, producing a dark fringe, it increases in another region, producing a bright fringe. There is no gain or loss of energy, which is consistent with the principle of conservation of energy.

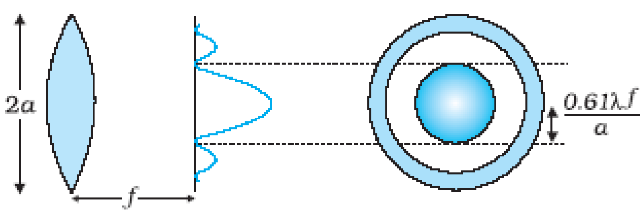
(v) As per Richard Feynman, there is no *important physical difference between* interference and diffraction. *when there are only a few sources, say two interfering sources, then the result is usually called interference, but if there is a large number of them, it seems that the word diffraction is more often used.*

8. Resolving power of optical instruments: Mention of expressions for limit of resolution of (a) microscope and (b) telescope - Methods of increasing resolving power of microscope and telescope.

(1) The angular resolution of the telescope is determined by the objective lens of the telescope.

(2) A parallel beam of light is incident on a convex lens. Because of diffraction effects, the beam gets focused to a spot of radius of the central bright region is approximately given by

$$r_0 \approx \frac{1.22\lambda f}{2a} = \frac{0.61\lambda f}{a} =$$



where f is the focal length of the lens and $2a$ is the diameter of the circular aperture or the diameter of the lens, whichever is smaller. Typically if $\lambda \approx 0.5$ µm, $f \approx 20$ cm and $a \approx 5$ cm, we have

$r_0 \approx 1.2$ µm

Although the size of the spot is very small, it plays an important role in determining the limit of resolution of optical instruments like a **telescope or a microscope**. For the two stars to be just resolved

$$f\Delta\theta \approx r_0 \approx \frac{0.61\lambda f}{a}$$

implying

$$\Delta\theta \approx \frac{0.61\lambda}{a}$$

where $\Delta\theta$ is called limit of resolution.

(3) Explain the Method of increasing resolving power of microscope and telescope

In optical instruments the limit of resolution $\Delta\theta$ will be small if the diameter of the objective is large. This implies that the telescope will have better resolving power if *a* is large. It is for this reason that for better resolution, a telescope must have a large diameter objective.

9. Polarisation: Explanation of the phenomenon - Plane polarised light - Polaroid and its uses - Pass axis – Malus' law - Polarisation by reflection: Brewster's angle - Arriving at Brewster's law - Statement of Brewster's law, Numerical Problems.

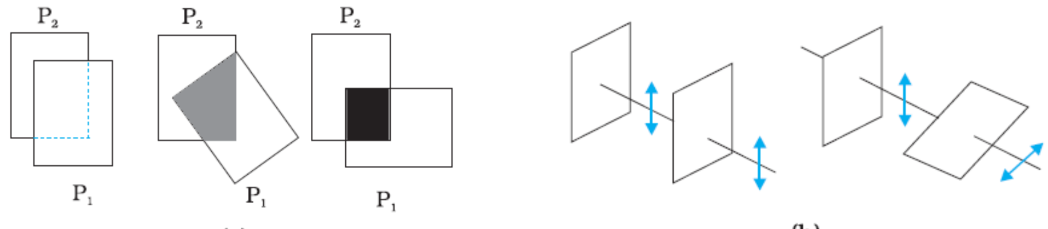
1. Explain the phenomenon of polarization ? What is plane polarized light ?

In unpolarised wave the displacement will be randomly changing with time though it will always be perpendicular to the direction of propagation. Natural light, e.g., from the sun is unpolarised. This means the electric vector takes all possible directions in the transverse plane, rapidly and randomly, during a measurement. A polaroid transmits only one component (parallel to a special axis). The resulting light is called linearly polarised or plane polarised.

2. Explain Polaroids ? Pass axis – Malus' law & uses of polaroids ?

Light waves are transverse in nature; i.e., the electric field associated with a propagating light wave is always at right angles to the direction of propagation of the wave. This can be easily demonstrated using a simple polaroid, which is a thin plastic like sheets. A polaroid consists of long chain molecules aligned in a particular direction. The electric vectors (associated with the propagating light wave) along the direction of the aligned molecules get absorbed. Thus, if an unpolarised light wave is incident on such a polaroid then the light wave will get linearly polarised with the electric vector oscillating along a direction perpendicular to the aligned molecules; this direction is known as the *pass-axis* of the polaroid.

Thus, if the light from an ordinary source (like a sodium lamp) passes through a polaroid sheet P_1, it is observed that its intensity is reduced by half. Rotating P_1 has no effect on the transmitted beam and transmitted intensity remains constant. Now, let an identical piece of polaroid P_2 be placed before P_1. As expected, the light from the lamp is reduced in intensity on passing through P_2 alone. But now rotating P_1 has a dramatic effect on the light coming from P_2. In one position, the intensity transmitted by P_2 followed by P_1 is nearly zero. When turned by 90º from this position, P_1 transmits nearly the full intensity emerging from P_2 as shown in figure..



(a)
(b)

The above experiment can be easily understood by assuming that light passing through the polaroid P_2 gets polarised along the pass-axis of P_2. If the pass-axis of P_2 makes an angle θ with the pass-axis of P_1, then when the polarised beam passes through the polaroid P_2, the component $E \cos\theta$ (along the pass-axis of P_2) will pass through P_2. Thus, as we rotate the polaroid P_1 (or P_2), the intensity will vary as:

$\boldsymbol{I = I_0 \cos^2\theta}$ --------- (1)

where I_0 is the intensity of the polarized light after passing through P_1. This is known as ***Malus' law***.

The above discussion shows that the intensity coming out of a single polaroid is half of the incident intensity. By putting a second polaroid, the intensity can be further controlled from 50% to zero of the incident intensity by adjusting the angle between the pass-axes of two polaroids.

Uses of Polaroids :

(1) Polaroids can be used to control the intensity, in sunglasses, windowpanes, etc.

(2) Polaroids are also used in photographic cameras and 3D movie cameras.

Thus a polaroid transmits only one component (parallel to a special axis). The resulting light is called linearly polarised or plane polarised. When this kind of light is viewed through a second polaroid whose axis turns through 2π, two maxima and minima of intensity are seen.

4. Polarisation by reflection: Brewster's angle - Arriving at Brewster's law - Statement of Brewster's law ?

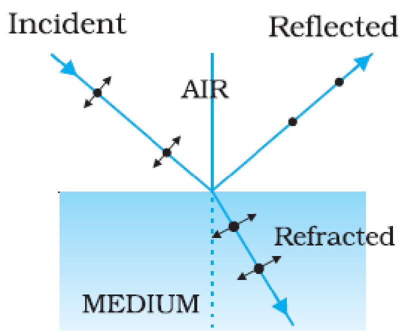


Figure shows light reflected from a transparent medium, say, water. As before, the dots and arrows indicate that both polarisations are present in the incident and refracted waves. We have drawn a situation in which the reflected wave travels at right angles to the refracted wave.

The oscillating electrons in the water produce the reflected wave. These move in the two directions transverse to the radiation from wave in the medium, i.e., the *refracted wave*. The arrows are parallel to the direction of the *reflected* wave. Motion in this direction does not

contribute to the reflected wave. As the figure shows, the reflected light is therefore linearly polarised perpendicular to the plane of the figure (represented by dots).
This can be checked by looking at the reflected light through an analyser. The transmitted intensity will be zero when the axis of the analyser is in the plane of the figure, i.e., the plane of incidence.

When unpolarised light is incident on the boundary between two transparent media, the reflected light is polarised with its electric vector perpendicular to the plane of incidence when the refracted and reflected rays make a right angle with each other. Thus we have seen that when reflected wave is perpendicular to the refracted wave, the reflected wave is a totally polarised wave. The angle of incidence in this case is called *Brewster's angle* and is denoted by i_B. We can see that i_B is related to the refractive index of the denser medium. Since we have $i_B + r = \pi/2$, we get from Snell's law

$$\mu = \frac{\sin i_B}{\sin r} = \frac{\sin i_B}{\sin(\pi/2 - i_B)} = \frac{\sin i_B}{\cos i_B} = \tan i_B \qquad \text{----------} \quad (2)$$

This is known as *Brewster's law*.

Polarised light can also be produced by reflection at a special angle (called the Brewster angle) and by scattering through $\pi/2$ in the earth's atmosphere.

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Additional Questions :

1. State and explain Huygens' wave theory of light and explain the laws of reflection and refraction using it
2. Explain the principle of superposition and deduce the conditions for constructive and destructive interference when two waves superpose each other.
3. What are coherent sources? Describe any one method to obtain coherent sources with neat labeled diagram.
4. Describe Young's double slit experiment and draw a graph showing the distribution of light intensity on the screen.
5. Derive an expression for the bandwidth in Young's double slit experiment and hence show that bright fringe and dark fringe are of equal width.
6. Draw a diagram showing the experimental arrangement for Fraunhoffer diffraction at a single slit.
7. Deduce the conditions for maxima and minima in diffraction at a single slit.
8. Show that the width of central maximum is double the width of secondary maxima in diffraction at a single slit.
9. State an prove Brewster's law of polarisation
10. What is Fresnel distance? derive an expression for it.
11. Explain how diffraction effects limit the resolving power of microscopes and telescopes.
12. State and explain Huygens' wave theory of light and explain the laws of reflection and refraction using it
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19. Show that the width of central maximum is double the width of secondary maxima in diffraction at a single slit.
20. State an prove Brewster's law of polarisation
21. What is Fresnel distance? derive an expression for it.
22. Explain how diffraction effects limit the resolving power of microscopes and telescopes.

ONE MARK QUESTIONS

1. Define wavefront.

Ans : A locus of points, which oscillate in phase is called a wavefront.
OR A surface of constant phase is called wavefront.

2. What is the shape of wavefront obtained from a point source at a (i) small distance (ii) large distance?

(i) Spherical wavefront (ii) Plane wavefront.

3. Under what conditions a cylindrical wavefront is obtained?

A cylindrical wavefront is obtained at a small distance from a linear source of light.

4. What type of wavefront is obtained when a plane wave is reflected by a concave mirror?

Spherical wavefront (converging).

5. Who proposed the wave theory of light?

Christiaan Huygens.

6. Name the physicist who experimentally studied the interference of light for the first time.
Thomas Young.

7. What is interference of light?

The modification in the distribution of light energy due to the superposition of two or more waves of light is called interference of light.

8. What is the maximum intensity of light in Young's double slit experiment if the intensity of light emerging from each slit is Io?
OR What is the intensity of light due to constructive interference in Young's double slit experiment if the intensity of light emerging from each slit is Io?

Maximum intensity of light in Young's double slit experiment is 4Io

9. Define fringe width.
The distance between two consecutive bright (or two consecutive dark) fringes is called fringe width.

10. Instead of using two slits as in Young's experiment, if two separate but identical sodium lamps are used, what is the result on interference pattern?
Ans : Interference pattern disappears.

11. What is the effect on interference fringes when yellow light is replaced by blue light in Young's double slit experiment?
The fringe width decreases. Since □□□□λ and λ is smaller for blue light than yellow light.

12. How does the fringe width in interference pattern vary with the wavelength of incident light?
The fringe width is directly proportional to the wavelength of incident light.

13. What is the effect on the interference fringes in a Young's double-slit experiment when the monochromatic source is replaced by a source of white light?
The central fringe is white. The fringe closest on either side of the central white fringe is red and the farthest will appear blue.

14. How does the fringe width in interference vary with the intensity of incident light?
The fringe width is not affected by the intensity of incident light.

15. Which colour of light undergoes diffraction to maximum extent?
Red.

16. Name a factor which affects the resolving power of a microscope.
The wave length of light or refractive index of medium between objective lens and the object.

17. How will the diffraction pattern due single slit change when violet light replaces green light?
The diffraction bands become narrower.

18. Do all waves exhibit diffraction or only light?
All the waves exhibit the phenomenon of diffraction.

19. We do not encounter diffraction effects of light in everyday observations. Why?
Since the wavelength of light is much smaller than the dimensions of most of the obstacles.

20. Why are diffraction effects due to sound waves more noticeable than those due to light waves?
The wavelength of sound waves is comparable with the size of the obstacles whereas for light, the wavelength is much smaller than the dimensions of most of the obstacles.

21. Is the width of all secondary maxima in diffraction at slit same? If not how does it vary?

No. As the order of the secondary maximum increases its width decreases.

22. What is resolving power of microscope?
The resolving power of the microscope is defined as the reciprocal of the minimum separation of two points which are seen as distinct.

23. What about the consistency of the principle of conservation of energy in interference and in diffraction? OR Does the law of conservation of energy holds good in interference and in diffraction?
Interference and diffraction are consistent with the principle of conservation of energy.

24. How can the resolving power of a telescope be increased?
Using objective of larger diameter.

25. Which phenomenon confirms the transverse nature of light?
Polarisation.

26. What is meant by plane polarised light?
Plane polarised light is one which contains transverse linear vibrations in only one direction perpendicular to the direction of propagation.

27. What is pass axis?
When an unpolarised light wave is incident on a polaroid, the light wave will get linearly polarised with the electric vector oscillating along a direction perpendicular to the aligned molecules. This direction is known as the pass-axis of the polaroid.

28. By what percentage the intensity of light decreases when an ordinary unpolarised (like from sodium lamp) light is passed through a polaroid sheet?
50%.

29. Let the intensity of unpolarised light incident on P1 be I. What is the intensity of light crossing polaroid P2 , when the pass-axis of P2 makes an angle 90o with the pass-axis of P1?
Zero. Since no light passes through the polaroids when they are crossed.

30. What should be the angle between the pass axes of two polaroids so that the intensity of transmitted light form the second polaroid will be maximum?
0o .

31. State Brewster's Law.
Brewster's Law: The refractive index of a reflector is equal to tangent of the polarising angle.

32. Write the relation between refractive index of a reflector and polarising angle.
n = tan iB, where n - refractive index of the reflector and iB - polarising angle/Brewster's angle.

33. Define Brewster's angle (OR Polarising angle).

Brewster's angle/Polarising angle(iB): The angle of incidence for which the reflected light is completely plane polarised is called Brewster's angle.

TWO MARKS QUESTIONS

1. State Huygens' principle.

Each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront are called as secondary wavelets and if we draw a common tangent to all these spheres, we obtain the new position of the wavefront at a later time.

2. Name the wavefront obtained when a plane wave passed through (i) a thin convex lens (ii) thin prism.

(i) Spherical converging wavefront; (ii) Plane wavefront.

3. What is the shape of the wavefront in each of the following cases:
(a) Light emerging out of a convex lens when a point source is placed at its focus.
(b) The portion of the wavefront of light from a distant star intercepted by the Earth.

(a) Plane wavefront; (b) Plane wavefront.

4. What are coherent sources? Give an example.

The two sources are said to be coherent if the phase difference between the waves emitted by them at any point will not change with time. OR Any two sources continuously emitting waves having zero or constant phase difference are called coherent sources. Example: In Young's double slit experiment the two slits behave like coherent sources.

5. Can two sodium vapour lamps be considered as coherent sources? Why?

No, because the phase difference between light coming from two independent sources continuously change.

6. Write the expression for fringe width in Young's double slit experiment.

The fringe width: β□ $\lambda D/d$; where □ – wavelength of light, d – distance between the slits and D – distance between the screen and the slits.

7. What are the factors which affect the fringe width in Young's double slit experiment?

The wavelength of light, distance between the slits and the screen or slit separation. [any two]

8. Let the fringe width in Young's double slit experiment be □. What is the fringe width if the distance between the slits and the screen is doubled and slit separation is halved?

Initial fringe width: : $\beta = \lambda D/d$; **and the new fringe width:**

$$\beta' = \frac{\lambda(2D)}{\left(d/2\right)} = 4\left(\frac{\lambda D}{d}\right) = 4\beta$$

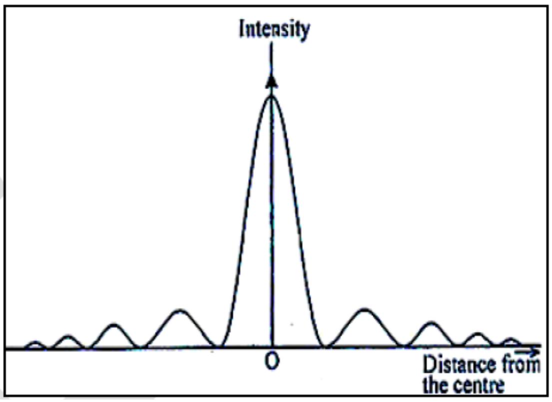
Thus, the new fringe width becomes four times the initial.

9. What is diffraction of light? Give an example.

The phenomenon of bending of light waves around the edges (or corners) of the obstacles and entering into the expected geometrical shadow of the obstacle is called diffraction of light. Example: Colours observed when a CD (Compact Disc) is viewed is due to diffraction of light.

10. Mention the conditions for diffraction minima and maxima.
Condition for secondary maxima is
Angle of diffraction $\theta \approx (n + 1/2)\lambda/a$ where $n = \pm1, \pm2, \pm3$.......
Condition for diffraction minima:
Angle of diffraction $\theta \approx n\lambda/a$ where $n = \pm1, \pm2, \pm3$.......
where λ is wavelength of light used and a is slit width.

11. Give the graphical representation to show the variation of intensity of light in single slit diffraction.
GRAPHICAL REPRESENTATION for the variation of intensity of light in single slit diffraction is as shown in the adjacent diagram.



12. Mention the expression for limit of resolution of microscope.
Minimum separation OR Limit of resolution:
$d_{min} = 1.22\lambda / 2n\sin\beta$
where λ – wavelength of light, n – refractive index of the medium between the object and the objective lens and 2β – angle subtended by the object at the diameter of the objective lens at the focus of the microscope.

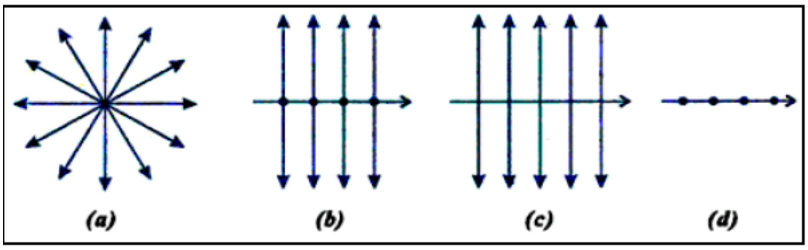
13. Write the expression for limit of resolution of telescope.
Expression for limit of resolution of telescope: Limit of resolution:
$\Delta\theta = 0.61\lambda/a = 1.22\lambda/2a$
where λ – wavelength of light and 2a – diameter of aperture of the objective.

14. Give the two methods of increasing the resolving power of microscope.
Resolving power of a microscope can be increased (i) by choosing a medium of higher refractive index and (ii) by using light of shorter wavelength.

15. Write the mathematical expression for Malus law. Explain the terms.
$I = I_0 \cos^2\theta$, Where I is the intensity of the emergent light from second polaroid (analyser), I_0 is the intensity of plane polarised light incident on second polaroid after passing through first polaroid (polariser) and θ is the angle between the pass-axes of two polaroids (analyser and polariser).

16. Represent polarised light and unpolarised light.

Unpolarized light is represented as shown in figure(a) and figure (b). [any one] Plane polarized light with vibrations parallel to the plane of the paper is shown in figure(c). Plane polarized light with vibrations perpendicular to the plane of the paper is as shown in figure(d).



17. Unpolarised light is incident on a plane glass surface. What should be the angle of incidence so that the reflected and refracted rays are perpendicular to each other? (For glass refractive index = 1.5).
OR What is the Brewster angle for air to glass transition? (For glass refractive index = 1.5).
$n = \tan i_B$ => Brewster's angle for glass: $i_B = \tan^{-1}(n) = \tan^{-1}(1.5) = 56^o\ 19'$.

18. In a Young's double slit experiment, the angular width of a fringe formed on distant screen is 0.10. The wavelength of light used is 6000 Å. What is the spacing between the slits?
Given $\theta = 0.1^o = 0.1(\pi/180) = 1.745 \times 10^{-3}$ rad and wavelength of light $= \lambda = 6000$ Å $= 6 \times 10^{-7}$m.
$\therefore$ Spacing between the slits is $d = \lambda/\theta = (6 \times 10^{-7} / 1.745 \times 10^{-3}) = 3.438 \times 10^{-4}$ m.

19. A beam of unpolarised is incident on an arrangement of two polaroids successively. If the angle between the pass axes of the two polaroids is 600, then what percentage of light intensity emerges out of the second polaroid sheet?
Given $\theta = 60^o$, Intensity of light incident on the polaroid $= I_o$, Intensity of light transmitted through the polaroid I = ?
$I = I_0 \cos^2\theta$ => $I = I_0 \cos^2 60^o = I_0/4$. Thus 25% of the light intensity is transmitted through the polaroids.

20. Assume that light of wavelength 5000Å is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 5.08m ?

Given wavelength of light = 5000A, = 5×10^{-7}m, Diameter of the objective = 5.08 m.
Limit of resolution: $\Delta\theta = 1.22\lambda/2a = (1.22 \times 5 \times 10^{-7}\text{m} / 5.08) = 1.2 \times 10^{-7}$ radians.

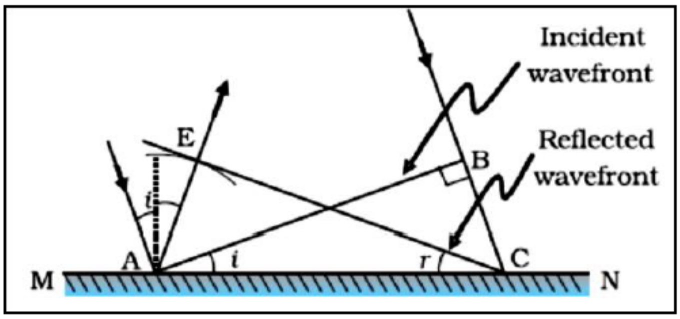
THREE MARKS QUESTIONS

1. Using Huygen's wave theory of light, show that the angle of incidence is equal to angle of reflection in case of reflection of a plane wave by a plane surface.

Consider a plane wave AB incident at an angle *i* on a reflecting surface MN. If v represents the speed of the wave in the medium and if τ represents the time taken by the wavefront to advance from the point B to C then the distance BC = $v\tau$, In order to construct the reflected wavefront, a sphere of radius = $v\tau$, is drawn from the point A as shown in the adjacent figure. Let CE represent the tangent plane drawn from the point C to this sphere.

$\therefore$ AE = BC = $v\tau$, $\angle ABC = \angle CEA = 90^o$, AC is common.

Triangles EAC and BAC are congruent. $\therefore$ i = r.



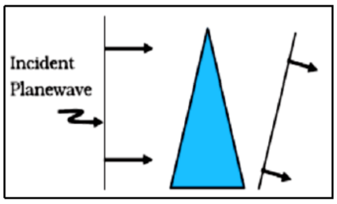
2. Illustrate with the help of suitable diagram, action of the following when a plane wavefront incidents. (i) a prism (ii) a convex lens and (iii) a concave mirror. (each three marks)

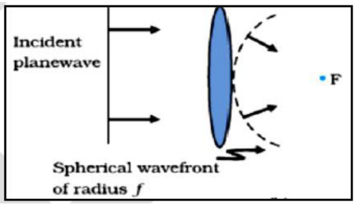
(i) Action of the prism when a plane wavefront incident on it:

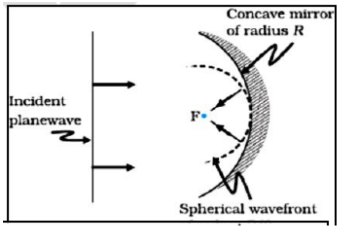
In adjacent figure, consider a plane wave passing through a thin prism. Since the speed of light waves is less in glass, the lower portion of the incoming wavefront which travels through the greatest thickness of glass will get delayed resulting in a tilt in the emerging plane wavefront.

(ii) Action of the convex lens when a plane wavefront incident on it: In the adjacent figure, a plane wave incident on a thin convex lens; the central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most. The emerging wavefront has a depression at the centre and therefore the wavefront becomes spherical (radius = f, focal length) and converges to the point focus F.

(iii) Action of the concave mirror when a plane wavefront incident on it: In adjacent figure, a plane wave is incident on a concave mirror and on reflection we have a spherical wave converging to the focus F.



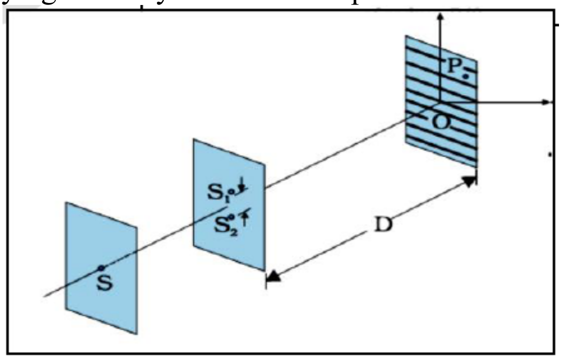




3. Briefly describe Young's experiment with the help of a schematic diagram.

Young's experiment : Description with a schematic diagram

S represents a pin hole illuminated by sunlight. The spherical wave front from S is incident on two pin holes S_1 and S_2 which are very close to each other and equidistant from S. Then the pin holes S_1 and S_2 act as two coherent sources of light of same intensity. The two sets of spherical wave fronts coming out of S_1 and S_2 interfere with each other in such a way as to produce a symmetrical pattern of varying intensity on the screen placed at a suitable distance D.



4. Distinguish between interference of light and diffraction of light.

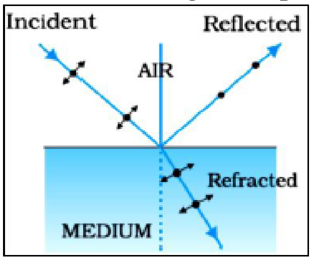
Differences between interference of light and diffraction of light:

	INTERFERENCE	DIFFRACTION
1	Interference fringes have equal width.	Diffraction bands have unequal width. (width of secondary maxima decreases with increase in order)
2	Interference is due to the superposition of two waves originating from two coherent sources	It is due to the superposition of secondary wavelets originating from different parts of single slit.
3	Intensity of all bright fringes is equal and Intensity of dark fringes is zero.	Intensity of central maximum is highest, Intensity of secondary maxima decreases with increase in order.
4	At an angle of λ/a, maximum intensity for two narrow slits separated by a distance 'a' is found.	At an angle of λ/a, the first minimum of the diffraction pattern occurs for a single slit of width a.
5	In an interference pattern there is a good contrast between dark and bright fringes.	In a diffraction pattern the contrast between the bright band and dark band is comparatively lesser.

5. Briefly explain Polarisation by reflection with the help of a diagram.

POLARIZATION BY REFLECTION:

It is found that when a beam of ordinary light is reflected by the surface of a transparent medium like glass or water, the reflected light is partially polarized.
The degree of polarization depends on the angle of incidence.
As the angle of incidence is gradually increased from a small value, the degree of polarization also increases. At a particular angle of incidence the reflected light is completely plane polarized. This angle of incidence is called Brewster's angle or polarizing angle (i_B).
If the angle of incidence is further increased, the degree of polarization decreases.



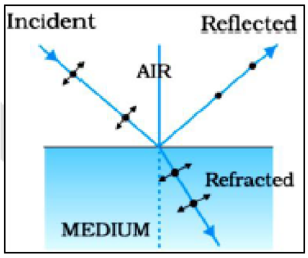
6. Show that the refractive index of a reflector is equal to tangent of the polarising angle. OR Arrive at Brewster's law.

From Snell's law, $n = \dfrac{\sin i}{\sin r} = \dfrac{\sin i_B}{\sin r}$, since $i = i_B$.

For Brewster's angle of incidence, $r + i_B = 90° \Rightarrow r = 90° - i_B$

$\Rightarrow n = \dfrac{\sin i_B}{\sin(90^o - i_B)} = \dfrac{\sin i_B}{\cos i_B}$

$\Rightarrow$ Refractive index of reflector: $n = \tan i_B$



7. What are Polaroids? Mention any two uses of polaroids.
Polaroids are the devices used to produce plane polarised light.
Uses of polaroids: 1) To control the intensity of light in sunglasses, windowpanes, etc..
2) In photographic cameras and 3D movie cameras.

FIVE MARKS QUESTIONS

1. Using Huygen's wave theory of light, derive Snell's law of refraction.

2. Obtain the expressions for resultant displacement and amplitude when two waves having same amplitude and a phase difference ϕ superpose. Hence give the conditions for constructive and destructive interference. OR Give the theory of interference. Hence arrive at the conditions for constructive and destructive interferences.

3. Derive an expression for the width of interference fringes in a double slit experiment.

4. Explain the phenomenon of diffraction of light due to a single slit and mention of the conditions for diffraction minima and maxima.

FIVE MARKS NUMERICAL PROBLEMS

1. A monochromatic yellow light of wavelength 589nm is incident from air on a water surface. What are the wavelength, frequency and speed of a refracted light? Refractive index of water is 1.33 .

2. In a double slit experiment angular width of a fringe is found to be 0.2^{o} on a screen placed 80 cm away. The wave length of light used is 600 nm. Find the fringe width. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.

3. A beam of light consisting of two wavelengths 650 nm and 520 nm, is used to obtain interference fringes in Young's double slit experiment with D = 60 cm and d = 1 mm. a) Find the distance of third bright fringe on the screen from central maximum for wavelength 650nm. b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

4. In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is (i) λ/3 (ii) λ/2?

5. A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1.25 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find (i) the width of the slit and (ii) angular position of the first secondary maximum.

6. In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment. Also find the distance of fifth dark fringe from the central bright fringe.

7. In Young's double slit experiment with monochromatic light and slit separation of 1mm, the fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5cm towards the slits, the change in fringe width is 30 μm. Calculate the wavelength of the light used.

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Chapter 11:

DUAL NATURE OF RADIATION AND MATTER

5M

(1M-2Q, 3M-1Q) OR 5M-1Q(NP)

11.1 Electron emission: Definition of electron volt (*eV*) -

Free electrons (negatively charged particles) in metals are responsible for their conductivity. However, the free electrons cannot normally escape out of the metal surface. If an electron attempts to come out of the metal, the metal surface acquires a positive charge and pulls the electron back to the metal. The free electron is thus held inside the metal surface by the attractive forces of the ions. Consequently, the electron can come out of the metal surface only if it has got sufficient energy to overcome the attractive pull. A certain minimum amount of energy is required to be given to an electron to pull it out from the surface of the metal. This minimum energy required by an electron to escape from the metal surface is called the *work function* of the metal. It is generally denoted by f0 and measured in eV (electron volt).
One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt, so that 1 eV = 1.602×10^{-19} J.

11.2 Types of electron emission.

The minimum energy required for the electron emission from the metal surface can be supplied to the free electrons by any one of the following physical processes:

(i) *Thermionic emission:* By suitably heating, sufficient thermal energy can be imparted to the free electrons to enable them to come out of the metal.
(ii) *Field emission*: By applying a very strong electric field (of the order of 10^8 V m^{-1}) to a metal, electrons can be pulled out of the metal, as in a spark plug.
(iii) *Photo-electric emission*: When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface. These photo(light)-generated electrons are called p*hotoelectrons*.

11.3 Photoelectric effect: Mention of Hertz's observations - Mention of Hallwachs' and Lenard's observations -

The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz (1857-1894), during his electromagnetic wave experiments. In his experimental investigation on the production of electromagnetic waves by means of a spark discharge, Hertz observed that high voltage sparks across the detector loop were enhanced when the emitter plate was illuminated by ultraviolet light from an arc lamp.

When light falls on a metal surface, some electrons near the surface absorb enough energy from the incident radiation to overcome the attraction of the positive ions in the material of the surface. After gaining sufficient energy from the incident light, the electrons escape from the surface of the metal into the surrounding space.

Hallwachs' and Lenard's observations :
Wilhelm Hallwachs and Philipp Lenard investigated the phenomenon of photoelectric emission in detail during 1886-1902.

Lenard (1862-1947) observed that when ultraviolet radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit (Fig. 11.1). As soon as the ultraviolet radiations were stopped, the current flow also stopped. These observations indicate that when ultraviolet radiations fall on the emitter plate C, electrons are ejected from it which are attracted towards the positive, collector plate A by the electric field. The electrons flow through the evacuated glass tube, resulting in the current flow. Thus, light falling on the surface of the emitter causes current in the external circuit. Hallwachs and Lenard studied how this photo current varied with collector plate potential, and with frequency and intensity of incident light.

Hallwachs, in 1888, undertook the study further and connected a negatively charged zinc plate to an electroscope. He observed that the zinc plate lost its charge when it was illuminated by ultraviolet light.
Further, the uncharged zinc plate became positively charged when it was irradiated by ultraviolet light. Positive charge on a positively charged zinc plate was found to be further enhanced when it was illuminated by ultraviolet light. From these observations he concluded that negatively charged particles were emitted from the zinc plate under the action of ultraviolet light.

11.4 Explanation of the phenomenon of Photoelectric effect –

After the discovery of the electron in 1897, it became evident that the incident light causes electrons to be emitted from the emitter plate. Due to negative charge, the emitted electrons are pushed towards the collector plate by the electric field. Hallwachs and Lenard also observed that when ultraviolet light fell on the emitter plate, no electrons were emitted at all when the frequency of the incident light was smaller than a certain minimum value, called the *threshold frequency*. This minimum frequency depends on the nature of the material of the emitter plate.
It was found that certain metals like zinc, cadmium, magnesium, etc., responded only to ultraviolet light, having short wavelength, to cause electron emission from the surface. However, some alkali metals such as lithium, sodium, potassium, caesium and rubidium were sensitive even to visible light. All these *photosensitive substances* emit electrons when they are illuminated by light. After the discovery of electrons, these electrons were termed as *photoelectrons.* The phenomenon is called *photoelectric effect.*

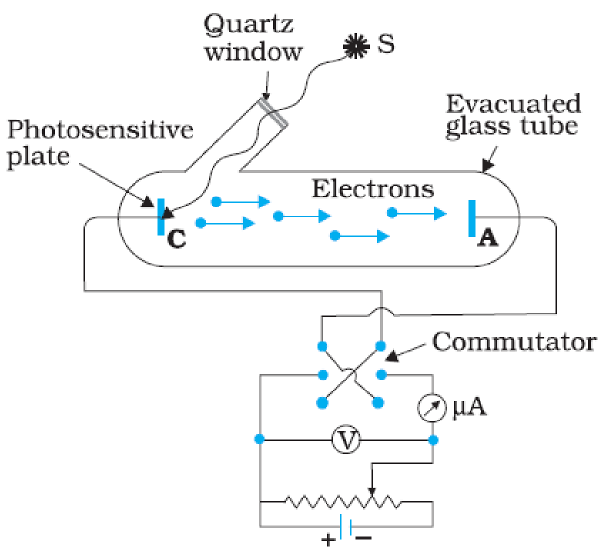
11.5 Definition of work function, threshold frequency and stopping potential -

(a) Work function :
A certain minimum amount of energy is required to be given to an electron to pull it out from the surface of the metal. This minimum energy required by an electron to escape from the metal surface is called the *work function* of the metal. It is generally denoted by f_0 and measured in eV (electron volt).
(b) Threshold Frequency :
In a photoelectric experiment, it is observed that when ultraviolet light fell on the emitter metal plate, no electrons were emitted at all when the frequency of the incident light was smaller than a certain minimum value, called the *threshold frequency*. This minimum frequency depends on the nature of the material of the emitter plate.
(c) Stopping Potential :

In a photoelectric experiment, for a particular frequency of incident radiation, *the minimum negative (retarding) potential V_0 given to the plate A for which the photocurrent stops or becomes zero is called the cut-off or stopping potential.*

11.6 Experimental setup to study Photoelectric effect: Observations - Mention of effect of (a)intensity of light on photocurrent, (b) potential on photocurrent and (c) frequency of incident radiation on stopping potential.

Figure 11.1 depicts a schematic view of the arrangement used for the experimental study of the photoelectric effect. It consists of an evacuated glass/quartz tube having a photosensitive plate C and another metal plate A. Monochromatic light from the source S of sufficiently short wavelength passes through the window W and falls on the photosensitive plate C (emitter). A transparent quartz window is sealed on to the glass tube, which permits ultraviolet radiation to pass through it and irradiate the photosensitive plate C. The electrons are emitted by the plate C and are collected by the plate A (collector), by the electric field created by the battery. The battery maintains the potential difference between the plates C and A, that can be varied. The polarity of the plates C and A can be reversed by a commutator. Thus, the plate A can be maintained at a desired positive or negative potential with respect to emitter C. When the collector plate A is positive with respect to the emitter plate C, the electrons are attracted to it. The emission of electrons causes flow of electric current in the circuit. The potential difference between the emitter and collector plates is measured by a voltmeter (V) whereas the resulting photo current flowing in the circuit is measured by a microammeter (μA). The photoelectric current can be increased or decreased by varying the potential of collector plate A with respect to the emitter plate C. The intensity and frequency of the incident light can be varied, as can the potential difference V between the emitter C and the collector A.



We can use the experimental arrangement of Fig. 11.1 to study the variation of photocurrent with
(a) intensity of radiation, (b) frequency of incident radiation, (c) the potential difference between the plates A and C, and (d) the nature of the material of plate C. Light of different frequencies can be used by putting appropriate coloured filter or coloured glass in the path of light falling on the emitter C. The intensity of light is varied by changing the distance of the light source from the emitter.

(a) Effect of intensity of light on photocurrent :

The collector A is maintained at a positive potential with respect to emitter C so that electrons ejected from C are attracted towards collector A. Keeping the frequency of the incident radiation and the accelerating potential fixed, the intensity of light is varied and the resulting photoelectric current is measured each time. It is found that the photocurrent increases linearly with intensity of incident light as shown graphically in Fig. 11.2. The photocurrent is directly proportional to the number of photoelectrons emitted per second. This implies that *the number of photoelectrons emitted per second is directly proportional to the intensity of incident radiation.*

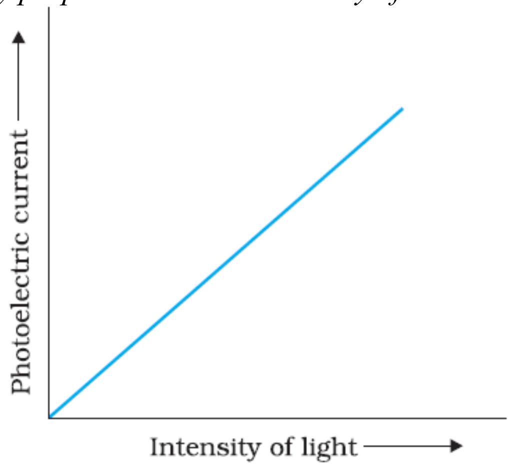


Fig. 11.2 : Variation of Photoelectric current with intensity of light.

(b) Effect of potential on photoelectric current :

We first keep the plate A at some positive accelerating potential with respect to the plate C and illuminate the plate C with light of fixed frequency n and fixed intensity I_1. We next vary the positive potential of plate A gradually and measure the resulting photocurrent each time. It is found that the photoelectric current increases with increase in accelerating (positive) potential. At some stage, for a certain positive potential of plate A, all the emitted electrons are collected by the plate A and the photoelectric current becomes maximum or saturates. If we increase the accelerating potential of plate A further, the photocurrent does not increase. This maximum value of the photoelectric current is called *saturation current.* Saturation current corresponds to the case when all the photoelectrons emitted by the emitter plate C reach the collector plate A.

We now apply a negative (retarding) potential to the plate A with respect to the plate C and make it increasingly negative gradually. When the polarity is reversed, the electrons are repelled and only the most energetic electrons are able to reach the collector A. The photocurrent is found to decrease rapidly until it drops to zero at a certain sharply defined, critical value of the negative potential V_0 on the plate A. For a particular frequency of incident radiation, *the minimum negative (retarding) potential V_0 given to the plate A for which the photocurrent stops or becomes zero is called the cut-off or stopping potential*.

The interpretation of the observation in terms of photoelectrons is straightforward. All the photoelectrons emitted from the metal do not have the same energy. Photoelectric current is zero when the stopping potential is sufficient to repel even the most energetic photoelectrons, with the maximum kinetic energy (K_{max}), so that

$K_{max} = e\, V_0$ ----------------- (11.1)

We can now repeat this experiment with incident radiation of the same frequency but of higher intensity I_2 and I_3 ($I_3 > I_2 > I_1$). We note that the saturation currents are now found to be at higher values. This shows that more electrons are being emitted per second, proportional to the intensity of incident radiation. But the stopping potential remains the same as that for the incident radiation of intensity I_1, as shown graphically in Fig. 11.3. Thus, *for a given frequency of the incident radiation, the stopping potential is independent of its intensity*. In other words, the maximum kinetic energy of photoelectrons depends on the light source and the emitter plate material, but is independent of intensity of incident radiation.

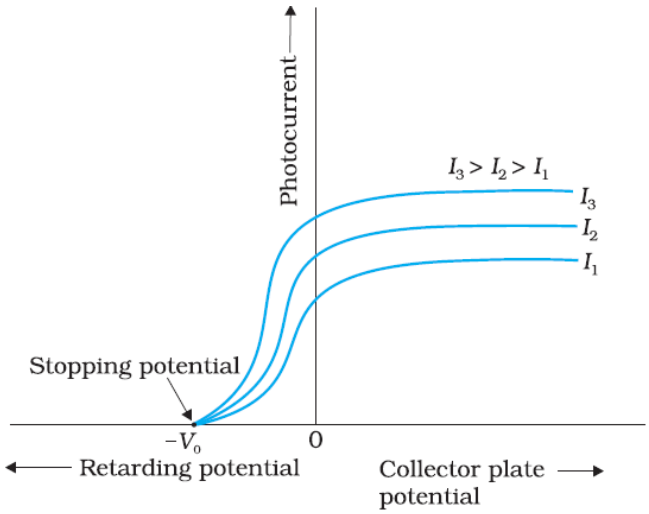


Fig. 11.3 : Variation of photocurrent with collector plate potential for different intensity of incident radiation.

(c) Effect of frequency of incident radiation on stopping potential :

We now study the relation between the frequency n of the incident radiation and the stopping potential V_0. We suitably adjust the same intensity of light radiation at various frequencies and study the variation of photocurrent with collector plate potential. The resulting variation is shown in Fig. 11.4. We obtain different values of stopping potential but the same value of the saturation current for incident radiation of different frequencies. The energy of the emitted electrons depends on the frequency of the incident radiations. The stopping potential is more negative for higher frequencies of incident radiation. Note from Fig. 11.4 that the stopping potentials are in the order $V_{03} > V_{02} > V_{01}$ if the frequencies are in the order $n_3 > n_2 > n_1$. This implies that greater the frequency of incident light, greater is the maximum kinetic energy of the photoelectrons. Consequently, we need greater retarding potential to stop them completely. If we plot a graph between the frequency of incident radiation and the corresponding stopping potential for different metals we get a straight line, as shown in Fig. 11.5.

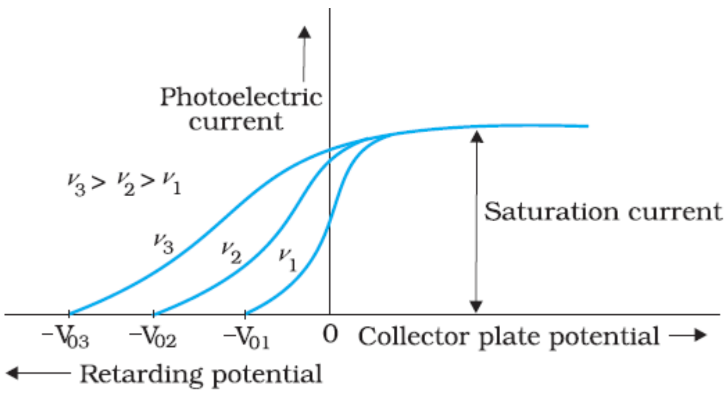


Fig. 11.4 : Variation of photoelectric current with collector plate potential for different frequencies of incident radiation.

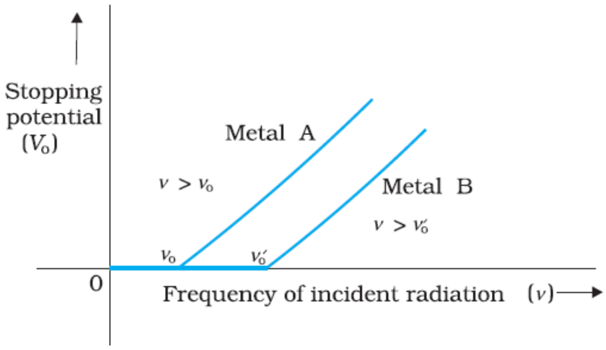


Fig. 11.5 : Variation of stopping potential V0 with frequency n of incident radiation for a given photosensitive material.

The graph shows that (i) the stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.
(ii) there exists a certain minimum cut-off frequency n0 for which the stopping potential is zero.
These observations have two implications:
(i) *The maximum kinetic energy of the photoelectrons varies linearly with the frequency of incident radiation, but is independent of its intensity.*
(ii) *For a frequency* n *of incident radiation, lower than the cut-off frequency* n_0*, no photoelectric emission is possible even if the intensity is large.* This minimum, cut-off frequency n0, is called the *threshold frequency*. It is different for different metals.
Different photosensitive materials respond differently to light. Selenium is more sensitive than zinc or copper. The same photosensitive substance gives different response to light of different wavelengths. For example, ultraviolet light gives rise to photoelectric effect in copper while green or red light does not.

Note that in all the above experiments, it is found that, if frequency of the incident radiation exceeds the threshold frequency, the photoelectric emission starts instantaneously without any apparent time lag, even if the incident radiation is very dim. It is now known that emission starts in a time of the order of 10^{-9} s or less.

We now summarise the experimental features and observations described in this section :
(i) For a given photosensitive material and frequency of incident radiation (above the threshold frequency), the photoelectric current is directly proportional to the intensity of incident light (Fig. 11.2).
(ii) For a given photosensitive material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation whereas the stopping potential is independent of its intensity (Fig. 11.3).
(iii) For a given photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation, called the *threshold frequency*, below which no emission of photoelectrons takes place, no matter how intense the incident light is. Above the threshold frequency, the stopping potential or equivalently the maximum kinetic energy of the emitted photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity (Fig. 11.5).
(iv) The photoelectric emission is an instantaneous process without any apparent time lag ($\sim 10^{-9}$s or less), even when the incident radiation is made exceedingly dim.

11.7 Einstein's photoelectric equation: Explanation of experimental results.

In 1905, Albert Einstein (1879-1955) proposed a radically new picture of electromagnetic radiation to explain photoelectric effect. In this picture, photoelectric emission does not take place by continuous absorption of energy from radiation. Radiation energy is built up of discrete units – the so called *quanta of energy of radiation*. Each quantum of radiant energy has energy $h\nu$, where h is Planck's constant and ν the frequency of light. In photoelectric effect, an electron absorbs a quantum of energy ($h\nu$) of radiation. If this quantum of energy absorbed exceeds the minimum energy needed for the electron to escape from the metal surface (work function ϕ_0), the electron is emitted with maximum kinetic energy

$$K_{max} = h\nu - \phi_0 \qquad \text{-------------} \quad (11.2)$$

More tightly bound electrons will emerge with kinetic energies less than the maximum value. Note that the intensity of light of a given frequency is determined by the number of photons incident per second. Increasing the intensity will increase the number of emitted electrons per second. However, the maximum kinetic energy of the emitted photoelectrons is determined by the energy of each photon.

Equation (11.2) is known as *Einstein's photoelectric equation*.

• According to Eq. (11.2), K_{max} depends linearly on n, and is independent of intensity of radiation, in agreement with observation. This has happened because in Einstein's picture, photoelectric effect arises from the absorption of a single quantum of radiation by a single electron. The intensity of radiation (that is proportional to the number of energy quanta per unit area per unit time) is irrelevant to this basic process.

• Since K_{max} must be non-negative, Eq. (11.2) implies that photoelectric emission is possible only if $h\nu > \phi_0$ or $\nu > \nu_0$, where

$$\nu_0 = \frac{\phi_0}{h} \qquad \text{---------} \quad (11.3)$$

Equation (11.3) shows that the greater the work function ϕ_0, the higher the minimum or threshold frequency ν_0 needed to emit photoelectrons. Thus, there exists a threshold frequency ν_0 (= ϕ_0/h) for the metal surface, below which no photoelectric emission is possible, no matter how intense the incident radiation may be or how long it falls on the surface.

• The intensity of radiation is proportional to the number of energy quanta per unit area per unit time. The greater the number of energy quanta available, the greater is the number of electrons absorbing the energy quanta and greater, therefore, is the number of electrons coming out of the metal (for $\nu > \nu_0$). This explains why, for $\nu > \nu_0$, photoelectric current is proportional to intensity.

• In Einstein's picture, the basic elementary process involved in photoelectric effect is the absorption of a light quantum by an electron. This process is instantaneous. Thus, whatever may be the intensity i.e., the number of quanta of radiation per unit area per unit time, photoelectric emission is instantaneous. Low intensity does not mean delay in emission, since the basic elementary process is the same. Intensity only determines how many electrons are able to participate in the elementary process (absorption of a light quantum by a single electron) and, therefore, the photoelectric current.

Using Eq. (11.1), the photoelectric equation, Eq. (11.2), can be written as

$$e V_0 = h \nu - \phi_0; \text{ for } \nu \geq \nu_0$$

or $$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e} \qquad \text{---------} \quad (11.4)$$

The above equation predicts that the V_0 versus ν curve is a straight line with slope = (h/e), independent of the nature of the material.

During 1906-1916, Millikan performed a series of experiments on photoelectric effect, aimed at disproving Einstein's photoelectric equation. He measured the slope of the straight line obtained for sodium, similar to that shown in Fig. 11.5. Using the known value of e, he determined the value of Planck's constant h. This value was close to the value of Planck's constant (= 6.626×10^{-34} J s) determined in an entirely different context. In this way, in 1916, Millikan proved the validity of Einstein's photoelectric equation. Millikan verified photoelectric equation with great precision, for a number of alkali metals over a wide range of radiation frequencies.

11.8 Particle nature of light: Characteristics of photon.

Photoelectric effect thus gave evidence to the strange fact that light in interaction with matter behaved as if it was made of quanta or packets of energy, each of energy $h\nu$.

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.
(ii) Each photon has energy E (=hn) and momentum p (= $h\ \nu/c$), and speed c, the speed of light.
(iii) All photons of light of a particular frequency ν, or wavelength l, have the same energy E (=$h\nu$ = hc/λ) and momentum p (= $h\ \nu/c$ = h/λ), whatever the intensity of radiation may be. By increasing the intensity of light of given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.
(iv) Photons are electrically neutral and are not deflected by electric and magnetic fields.
(v) In a photon-particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

11.9 Wave nature of matter: de-Broglie hypothesis - Mention of de-Broglie relation- Mention of expression for de-Broglie wavelength in terms of kinetic energy and acceleration potential -

Light behaves like both wave and particle. The wave nature of light shows up in the phenomena of interference, diffraction and polarisation. On the other hand, in photoelectric effect and Compton effect which involve energy and momentum transfer, radiation behaves as if it is made up of a bunch of particles – the photons. Whether a particle or wave description is best suited for understanding an experiment depends on the nature of the experiment.

The gathering and focussing mechanism of light by the eye-lens is well described in the wave picture. But its absorption by the rods and cones (of the retina) requires the photon picture of light.

In 1924, the French physicist Louis Victor de Broglie (pronounced as de Broy) put forward a hypothesis that moving particles of matter should display wave-like properties under suitable conditions. He reasoned that nature was symmetrical and that the two basic physical entities –

matter and energy, must have symmetrical character. If radiation shows dual aspects, so should matter. De Broglie proposed that the wave length λ associated with a particle of momentum *p* is given as

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \text{--------------} \quad (11.5)$$

where *m* is the mass of the particle and *v* its speed. Equation (11.5) is known as the *de Broglie relation* and the wavelength λ of the *matter wave* is called *de Broglie wavelength*. The dual aspect of matter is evident in the de Broglie relation. On the left hand side of Eq. (11.5), λ is the attribute of a wave while on the right hand side the momentum *p* is a typical attribute of a particle. Planck's constant *h* relates the two attributes.

For a photon, the equation becomes,

$p = h\,\nu / c$ -------- *(11.6)*

Therefore,

$$\frac{h}{p} = \frac{c}{\nu} = \lambda \qquad \text{----------} \quad (11.7)$$

That is, the de Broglie wavelength of a photon given by Eq. (11.5) equals the wavelength of electromagnetic radiation of which the photon is a quantum of energy and momentum.

From Eq. (11.5), λ is smaller for a heavier particle (large *m*) or more energetic particle (large *v*). For example, the de Broglie wavelength of a ball of mass 0.12 kg moving with a speed of 20 m s^{-1} is easily calculated:

$$p = mv = 0.12 \text{ kg} \times 20 \text{ m s}^{-1} = 2.40 \text{ kg m s}^{-1}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J s}}{2.40 \text{ kg m s}^{-1}} = 2.76 \times 10^{-34} \text{ m}$$

This wavelength is so small that it is beyond any measurement. This is the reason why macroscopic objects in our daily life do not show wave-like properties. On the other hand, in the sub-atomic domain, the wave character of particles is significant and measurable.

Expression for de-Broglie wavelength in terms of kinetic energy and acceleration potential : Consider an electron (mass *m,* charge *e*) accelerated from rest through a potential *V*. The kinetic energy *K* of the electron equals the work done (*eV*) on it by the electric field:

$K = e\,V$ ------- *(11.8)*

Now, $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ so that $p = \sqrt{2mk} = \sqrt{2m\,eV}$ ---------- (11.9)

The de Broglie wavelength λ of the electron is then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\,mK}} = \frac{h}{\sqrt{2\,meV}} \qquad \text{------------} \quad (11.10)$$

Substituting the numerical values of *h, m, e*, we get,

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm} \qquad \text{------------} \quad (11.11)$$

where V is the magnitude of accelerating potential in volts. For a 120 V accelerating potential, Eq. (11.11) gives l = 0.112 nm. This wavelength is of the same order as the spacing between the atomic planes in crystals. This suggests that matter waves associated with an electron could be verified by crystal diffraction experiments analogous to X-ray diffraction.

In 1929, de Broglie was awarded the Nobel Prize in Physics for his discovery of the wave nature of electrons.

Heisenberg's *uncertainty principle* : According to the principle, it is not possible to measure *both* the position and momentum of an electron (or any other particle) *at the same time* exactly. There is always some uncertainty (Δx) in the specification of position and some uncertainty (Δp) in the specification of momentum. The product of Δx and Δp is of the order of ħ (with $\hbar = h/2\pi$),

i.e., $\Delta x\, \Delta p \approx \hbar$ ---------------- (11.12)

Equation (11.12) allows the possibility that Δx is zero; but then Δp must be infinite in order that the product is non-zero. Similarly, if Δp is zero, Δx must be infinite. Ordinarily, both Δx and Δp are non-zero such that their product is of the order of ħ.

11.10 Davisson and Germer experiment: (No experimental details)

The wave nature of electrons was first experimentally verified by C.J. Davisson and L.H. Germer in 1927 and independently by G.P. Thomson, in 1928, who observed diffraction effects with beams of electrons scattered by crystals.

Davisson and Thomson shared the Nobel Prize in 1937 for their experimental discovery of diffraction of electrons by crystals.

They studied the variation of the intensity (I) of the scattered electrons with the angle of scattering θ is obtained for different accelerating voltages. The experiment was performed by varying the accelarating voltage from 44 V to 68 V. It was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelarating voltage of 54 V at a scattering angle $\theta = 50^o$

The appearance of the peak in a particular direction is due to the constructive interference of electrons scattered from different layers of the regularly spaced atoms of the crystals. From the electron diffraction measurements, the wavelength of matter waves was found to be 0.165 nm. The de Broglie wavelength l associated with electrons, using Eq. (11.11), for V = 54 V is given by

$$\lambda = h/p = \frac{1.227}{\sqrt{V}} \text{ nm}$$

$$\lambda = \frac{1.227}{\sqrt{54}} \text{ nm} = 0.167 \text{ nm}$$

Thus, there is an excellent agreement between the theoretical value and the experimentally obtained value of de Broglie wavelength. Davisson- Germer experiment thus strikingly confirms the wave nature of electrons and the de Broglie relation.

11.11 Brief explanation of conclusion - wave nature of electrons on the basis of electron diffraction :

Electron diffraction experiments by Davisson and Germer, and by G. P. Thomson, as well as many later experiments, have verified and confirmed the wave-nature of electrons. The de Broglie hypothesis of matter waves supports the Bohr's concept of stationary orbits.

The de Broglie hypothesis has been basic to the development of modern quantum mechanics. It has also led to the field of electron optics. The wave properties of electrons have been utilised in the design of electron microscope which is a great improvement, with higher resolution, over the optical microscope.

Important Formula :

1. 1 eV = 1.602×10^{-19} J.
2. Maximum kinetic energy (K_{max}), so that $K_{max} = e\,V_0$
3. Maximum kinetic energy $\boldsymbol{K_{max} = h\,\nu - \phi_0}$
4. Photoelectric emission is possible only if $h\nu > \phi_0$ or $\nu > \nu_0$, where $\boldsymbol{\nu_0 = \frac{\phi_0}{h}}$
5. $e\,V_0 = h\,\nu - \phi_0$; for $\nu \geq \nu_0$ or $V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e}$
6. De Broglie proposed that the wave length λ associated with a particle of momentum p is given as

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

7. For a photon, the equation becomes, $p = h\,\nu/c$ Therefore,

$$\frac{h}{p} = \frac{c}{\nu} = \lambda$$

8. To calculate wavelength of any particle, use p = mv; and λ = h/p
9. De-Broglie wavelength in terms of kinetic energy and acceleration potential =

$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ so that $p = \sqrt{2mk} = \sqrt{2m\,eV}$

The de Broglie wavelength λ of the electron is then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\,mK}} = \frac{h}{\sqrt{2\,meV}}$$

Substituting the numerical values of h, m, e, for electron, we get,

$$\lambda = \frac{1.227}{\sqrt{V}}\ \text{nm}$$

10. Heisenberg's *uncertainty principle* $\Delta x\,\Delta p \approx \hbar = h/2\pi$
11. Davisson and Germer used the formula to calculate wavelength of electron

$$\lambda = h/p = \frac{1.227}{\sqrt{V}}\ \text{nm}$$

1. Define work function of a metal?
The minimum energy required for an electron to escape from the metal surface is called the work function of the metal

2. Define 1eV
1eV is the energy gained by an electron when it is accelerated through a potential difference of one volt.

3. Define thermionic emission?
Emission of electron from a metal surface when it is heated to sufficiently high temperature is called thermionic emission.

4. Define field emission?
Emission of electron from metal surface when it is subjected to high electric field (of the order of 108V) is called field emission.

5. Define photoelectric emission?
Emission of electrons from metal surface, when it is illuminated with light of suitable frequency is called photoelectric effect.

6. Who discovered photoelectric effect?
Henrich Hertz discovered photoelectric effect.

7. Define threshold frequency of a metal?
Threshold frequency of a metal is the minimum cut-off frequency of incident light below which no photoelectric emission takes place irrespective of intensity of incident light.

8. How photo electric current depends on intensity of incident light?
Above threshold frequency, photoelectric current is directly proportional to intensity of incident light.

9. What do you mean by saturation current?
As the potential of collector is increased for a radiation of certain high frequency and intensity, photoelectric current increases and reaches to a maximum constant value. This constant current is called saturation current.

10. Define stopping potential of a given photosensitive metal?
Stopping potential of a photosensitive metal is defined as the minimum negative potential applied to the collector at which the photoelectric current just drops zero.

11. Give the mathematical relation between stopping potential and maximum kinetic energy of photoelectron.

$$K_{max} = eV_0$$

12. Give the graphical representation of the variation of photoelectric current with collector plate potential.

13. Represent the variation of stopping potential with frequency of incident light graphically.

14. Give the graphical representation of effect of frequency of incident radiation on stopping potential .

15. Define quanta?
Radiation energy is made up of discrete unit of energy called quanta.

16. What is a de-Broglie wave?
A wave associated with moving particle is called de-Broglie wave.

17. What is the experimental outcome of Davisson and Germer experiment?
Davisson and Germer provided experimental proof for the wave nature of matter particle and verified the de-Broglie's expression for wavelength of matter wave.

18. What happens to the kinetic energy of photoelectrons if the intensity of incident radiation is increased?
Kinetic energy remains same as kinetic energy is independent of intensity of incident radiation

19. Why sufficiently powerful AM radio signal cannot produce photoelectric effect?
The energy of radio photon is less than the work function of any metal so even sufficiently powerful AM radio signal cannot produce photoelectric effect.

20. Give the labeled schematic representation of experimental arrangement for the study of photoelectric effect.

21. Name the factors on which maximum kinetic energy of photoelectrons depends.
Maximum kinetic energy of photoelectrons depends on the nature of the emitter and the frequency of incident radiation.

22. Give the Einstein's photoelectric equation and explain the terms.
Einstein's photoelectric equation is given by
$K_{max} = h\nu - \phi_0$
where Kmax = Maximumkineticenergy
ϕ_0 *= Work function*
h = plank's constant
ν *= Frequency of incident radiation*

23. What is the threshold frequency of a photon for photoelectric emission from a metal of work function 1eV

$$Threshold\ frequency = \frac{\phi_o}{h} = \frac{1x1.6x10^{-19}}{6.625x10^{-34}} = 2.41x10^{14}\,Hz$$

24. Why the photoelectrons emitted from a metal surface for a certain radiation have different energies even if work function of metal is a constant?

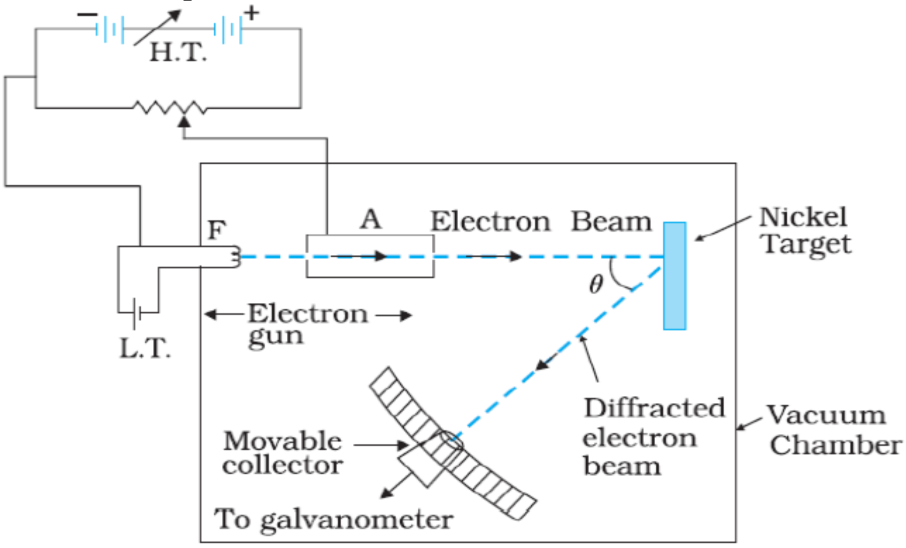
Work function is the minimum energy required for the electron in the highest level of the conduction band to get out of the metal. Not all electrons in the metal belong to this level. They occupy a continuous band of levels. Consequently, for the same incident radiation, electrons knocked off from different levels come out with different energies.

25. What is the significance of the slope of graph of stopping potential of an emitter verses frequency of incident radiation?

The slope of graph of stopping potential of an emitter verses frequency of incident radiation is observed to be a constant.

The value of slope is measured to be h/e which is independent of nature of emitter. Millikan calculated the value of h with the help of experimental value of slope and known value of e. The calculated value observed to be matching with Plank's constant exactly.

26. Draw labeled schematics diagram to show the experimental arrangement of Davisson and Germer experiment.



27. Mention the relation for de-Broglie wavelength.

According to de-Broglie theory, wavelength of matter wave associated with particle of momentum p (p = mv) is given by λ = h/p.

28. Give the relationship between the accelerating potential and the de-Broglie wavelength associated with a charged particle.

de-Broglie wavelength associated with a charged particle is given $\lambda = \frac{h}{\sqrt{2mqV}}$

Where q = charge of the particle V = potential through particle is accelerated m = mass of the particle.

1. Define 1 eV ?
Ans : One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt, so that 1 eV = 1.602×10^{-19} J.
2. What is the order of potential required to full the electron from metal as per field emission.
Ans :
3. What are the three types of electron emission
Ans :
4. Who discovered Photo-electric emission
Ans : Heinrich Hertz
5. Who made detailed study of photo-electric effect
Ans : Wilhelm Hallwachs and Philipp Lenard
6. Name a metal which produces photoelectrons when illuminated by UV light ?
Ans : Zinc, cadmium, magnesium etc.
7. Name a metal which produces photoelectrons when illuminated by visible light ?
Ans : Alkali metals like Li, Na, K, caesium and rubidium.
8. What is photoelectric effect ?

9. What is work function

10. What is threshold frequency in case of Photoelectric effect ?

11. What is stopping potential in case of Photoelectric effect ?

12. How intensity of light effects photocurrent ?
Ans : The photocurrent is directly proportional to the number of photoelectrons emitted per second. This implies that *the number of photoelectrons emitted per second is directly proportional to the intensity of incident radiation.*
13. The maximum kinetic energy of photoelectrons depends on the light source and the emitter plate material, but is independent of intensity of incident radiation.

14. *The maximum kinetic energy of the photoelectrons varies linearly with the frequency of incident radiation, but is independent of its intensity.*

15. *For a frequency* n *of incident radiation, lower than the cut-off frequency* n_0*, no photoelectric emission is possible even if the intensity is large.* This minimum, cut-off frequency n0, is called the *threshold frequency*. It is different for different metals.

16. The photoelectric emission is an instantaneous process without any apparent time lag ($\sim 10^{-9}$s or less), even when the incident radiation is made exceedingly dim.
17. *Einstein's photoelectric equation*
Ans : $K_{max} = h\nu - \phi_0$
18. Who verified Einsteins equation experimentally ?

Ans : Millikan verified photoelectric equation with great precision, for a number of alkali metals over a wide range of radiation frequencies.

19. What is dual nature of photon/light ?

20. In a photon-particle collision, which one of following is not conserved ?
(a) total energy (b) total momentum (c) number of photons (d) All of these Ans : (c)

21. Who proposed the concept of wave-matter dualism ?

22. Give expression for de-Broglie wavelength in terms of kinetic energy and acceleration potential ?
Ans :

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\,mK}} = \frac{h}{\sqrt{2\,meV}}$$

23. Give Heisenberg's *uncertainty principle*

24. Which theory of modern physics led to the development of modern quantum mechanics and the field of electron optics.
Ans : De-Broglie Hypothesis.

25. **How does the de-Broglie wavelength of a charged particle changes when accelerating potential increases? (MQP)**
Ans :

26. What is the rest mass of photon? (MQP)
Ans : Zero
27. What is the outcome of Davisson Germer Experiment? (MQP)

28. A proton and an electron have same kinetic energy. Which one has smaller de-Broglie wavelength?
Ans : Proton (note : wavelength decreases with increase in particle size)
29. Write any two types of electron emission ? (MQP) 2M
Ans :
30. Is matter wave is an electro-magnetic wave ? (MQP)
Ans : No. Because e.m wave is always emitted by an accelerating electric charge.

31. Does photoelectric current depends on frequency of incident radiation
Ans :
32. If the frequency of incident photon is equal to threshold frequency, what is the speed of ejected electrons.
Ans : Zero
33. Radiation of 6800 A° is incident on a material of threshold wavelength 5000 A°. Does it produce photoelectric effect ?
Ans : No.
34. A photo surface just emits photoelectrons when it is illuminated with blue light. Will there be a photoemission if red light is used ?
Ans : No.

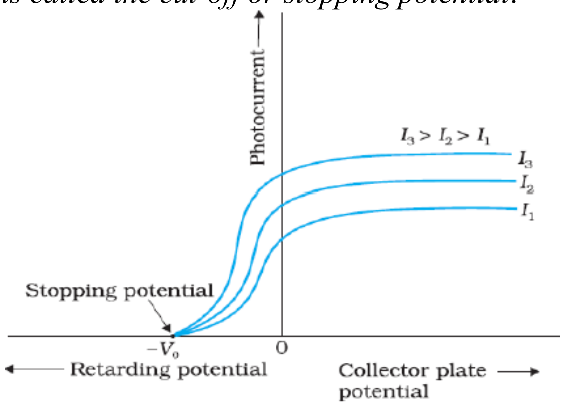
35. What is the principle of electron microscope ?
Ans : Wave nature of electron.
36. Which experiment confirms the wave nature of matter ?
Ans : Davision and Germer experiment.

1. Explain Hallwachs' and Lenard's experimental observations.

Lenard observed that when ultraviolet radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit. As soon as the ultraviolet radiation is stopped, the current flow also stops. Thus, light falling on the surface of the emitter causes current in the external circuit. Hallwachs observed that the uncharged zinc plate became positively charged when it is irradiated by ultraviolet light. Also positive charge on a positively charged zinc plate gets enhanced when it is illuminated by ultraviolet light. From the experimental observations he concluded that zinc plate emits negatively charged particles under the action of ultraviolet light.

2. Explain the effect of photoelectric current with collector plate potential

Photoelectric current increases with increase in accelerating (positive) potential. At some stage, for a certain positive potential of plate A, the photoelectric current becomes maximum or saturates. If potential of plate A is further increased, the photocurrent remains same. This maximum value of the photoelectric current is called *saturation current.*
When the potential of the collector plate is made more and more negative (retarding) with respect to the plate emitter, the electrons are repelled and only the most energetic electrons reach the collector. The photocurrent decreases rapidly until it drops to zero at a certain sharply defined, critical value of the negative potential V_0. For a particular frequency of incident radiation, *the minimum negative (retarding) potential V_0 given to the collector plate for which the photoelectrons are completely stopped from reaching collector or photocurrent becomes zero is called the cut-off or stopping potential.*



3. Mention the experimental observations of photoelectric effect.

(i) For a given photosensitive material and frequency of incident radiation(above the threshold frequency), the photoelectric current is directly proportional to the intensity of incident light . (ii) For a given photosensitive material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation whereas the stopping potential is independent of its intensity. (iii) For a given photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation, called the *threshold frequency*, below which no emission of photoelectrons takes place, no matter how intense the incident light is.

Above the threshold v frequency, the stopping potential and the maximum kinetic energy of the emitted photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity. (iv) The photoelectric emission is an instantaneous process, irrespective of intensity of the incident radiation.

4. Explain the experimental observations with the help of Einstein's photoelectric equation.

a) According to Einstein's theory, the basic elementary process involved in photoelectric effect is the absorption of a light quantum by an electron. This process is instantaneous. Thus, irrespective of the intensity, photoelectric emission is instantaneous.

b) According to Einstein's equation, $K_{max} = h\nu - \phi_0$, where K_{max} depends linearly on ν as ϕ_0 is a constant for a given metal. Also K_{max} is independent of intensity of radiation. Above concepts are in good agreement with the experimental observation. This is due to the fact that according to Einstein's theory, photoelectric effect arises from the absorption of a single quantum of radiation by a single electron..

c) K_{max} is always non-negative, => Photoelectric emission is possible only if $h\nu > \phi_0$

$\nu > \nu_0$ where $\nu_0 = \frac{\phi_0}{h}$

Thus, there exists a threshold frequency for the metal surface, below which no photoelectric emission possible, no matter how intense the incident radiation may be or how long it falls on the surface.

d) Intensity of radiation is proportional to the number of energy quanta per unit area per unit time. The greater the number of energy quanta available, the greater is the number of electrons absorbing the energy. Hence the number of electrons coming out of the metal is also higher. This explains why, for $\nu > \nu_0$, photoelectric current is proportional to intensity.

5. Give the characteristics of photon.

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.

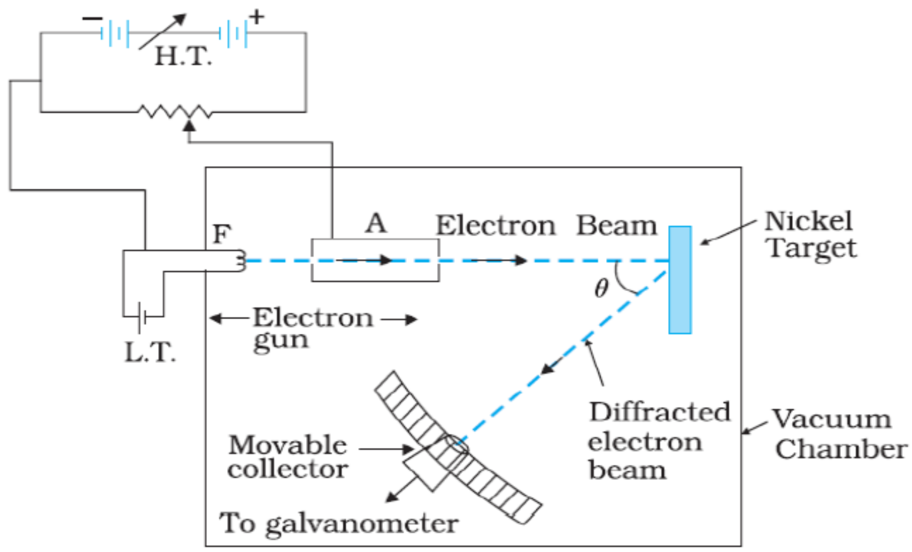
(ii) Each photon has energy $E = h\nu$ where ν is the frequency, momentum $p = h\nu/c$ where *c is* the speed of light.

(iii) All photons of light of a particular frequency ν, or wavelength λ, have the same energy and momentum, whatever the intensity of radiation may be. By increasing the intensity of light of given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.

(iv) Photons are electrically neutral and are not deflected by electric and magnetic fields.

(v) In a photon-particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

6. Explain Davisson and Germer experiment.



The experiment is performed by varying the accelerating voltage from 44 V to 68 V. A strong peak observed in the intensity (I) of the scattered electron for an accelerating voltage of 54V at a scattering angle 50º. The appearance of the peak in a particular direction is due to the constructive interference of electrons scattered from different layers of the regularly spaced atoms of the crystals. From the electron diffraction theory, the wavelength of matter waves producing maxima at50º is calculated to be λ= 0.165 nm.

$$\lambda = h/p = \frac{1.227}{\sqrt{V}} \text{ nm}$$

For V = 54 Volt,

$$\lambda = \frac{1.227}{\sqrt{54}} \text{ nm} = 0.167 \text{ nm}$$

Thus, there is an excellent agreement between the theoretical value and the experimentally obtained value of de Broglie wavelength. Davisson- Germer experiment thus strikingly confirms the wave nature of electrons, particles in general and the de Broglie relation.

7. **Write the five experimental observations of photoelectric effect. (MQP) 5M**

8. **What is photoelectric effect? Using Einstein's photoelectric equation, explain three experimental results. (MQP) 5M**

9. Explain three facts of photoelectric effect using Einstein's photoelectric equation. (MQP) 3M

Ans :

10. Mention three experimental observations of photoelectric effect. (MQP) 3M

Ans :

11. Give three characteristics of photon ? (MQP) 3M

Ans :

12. What are matter waves ? Obtain an expression for de-Broglie wavelength of a particle. ? (MQP)(3M)
Ans :

PROBLEMS :
1. The work function of Ce metal is 2.14 eV. When light of frequency 6 x 10^{14} Hz is incident on the metal surface, photo emission of eletrons occuts. Find (a) Energy of incident photon. (b) Maximum kinetic energy of incident photons ? Given : h = 6.63 x 10^{-34} JS, 1 eV = 1.6 x 10^{-19} J. (MQP) 5M
Ans :

2. A monocromatic radiation of wavelength 640.2 nm neon lamp radiates on cesium. The stopping potential is measured to be 0.54 volt. The source is replaced by an iron source with 427.2 nm line. What is the new stopping voltage ? (MQP) 5M
Ans :

3. The threshold wavelength of a photosensative metal is 5000 A°. Find the kinetic energy of the photoelectron emitted by it when radiation of wavelength 4000 A° is incident on it. Express it in eV. Given h = 6.62 x 10^{14} J/s. (MQP) 5M
Ans :

4. Light of wavelength 4000 A and intensity 100 Wm-2 incident on a plate of threshold frequency 5.5 x 1014 Hz. Find maximum kinetic energy of the photoelectron and number of photons incident per metre square per second. (MQP) 5M
Ans :

Chapter 12
Atoms
5M
(1Q - 5M (NP) **OR** 2M-1Q; 3M-1Q)

(1) Explain Alpha particle scattering with Schematic diagram of Geiger-Marsden experiment, observations and conclusion ?

At the suggestion of Ernst Rutherford, in 1911, H. Geiger and E. Marsden performed some experiments in which they directed a beam of 5.5 MeV α-particles emitted from a 214 83Bi radioactive source at a thin metal foil made of gold. Figure 12.2 shows a schematic diagram of this experiment.

Alpha-particles emitted by a 214 83 Bi radioactive source were collimated into a narrow beam by their passage through lead bricks. The beam was allowed to fall on a thin foil of gold of thickness 2.1×10^{-7} m. The scattered alpha-particles were observed through a rotatable detector consisting of zinc sulphide screen and a microscope. The scattered alpha-particles on striking the screen produced brief light flashes or scintillations. These flashes may be viewed through a microscope and the distribution of the number of scattered particles may be studied as a function of angle of scattering.

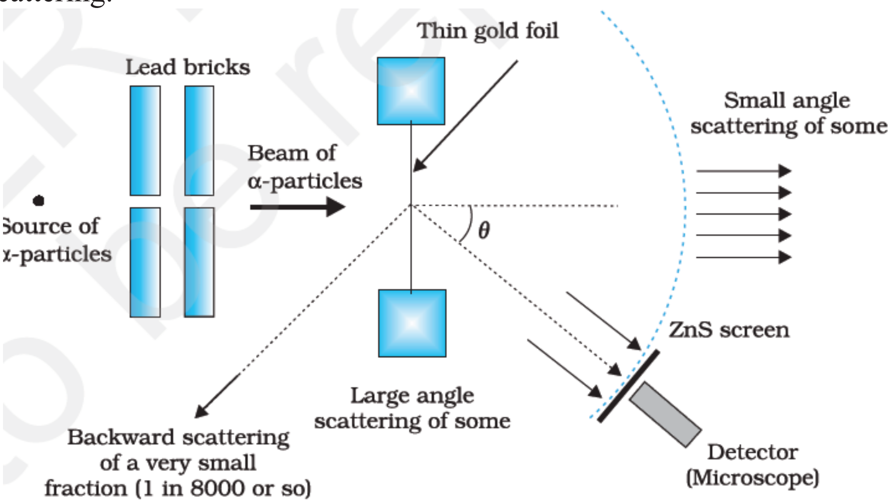


Fig. Schematic arrangement of the Geiger-Marsden experiment.

A typical graph of the total number of α-particles scattered at different angles, in a given interval of time, is shown in Fig. 12.3. The dots in this figure represent the data points and the solid curve is the theoretical prediction based on the assumption that the target atom has a small, dense, positively charged nucleus. Many of the α-particles pass through the foil. It means that they do not suffer any collisions. Only about 0.14% of the incident α-particles scatter by more than 1º; and about 1 in 8000 deflect by more than 90º. Rutherford argued that, to deflect the α-particle backwards, it must experience a large repulsive force. This force could be provided if the greater part of the mass of the atom and its positive charge were concentrated tightly at its centre. Then the incoming α-particle could get very close to the positive charge without penetrating it, and such a close encounter would result in a large deflection. This agreement supported the hypothesis of the nuclear atom. This is why Rutherford is credited with the *discovery* of the nucleus.

2. Explain Rutherford's model of an atom - Derivation of total energy of electron in hydrogen atom in terms of orbit radius.

In Rutherford's nuclear model of the atom, the entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away. The electrons would be moving in orbits about the nucleus just as the planets do around the sun. Rutherford's experiments suggested the size of the nucleus to be about 10^{-15} m to 10^{-14} m. From kinetic theory, the size of an atom was known to be 10^{-10} m, about 10,000 to 100,000 times larger than the size of the nucleus. Thus, the electrons would seem to be at a distance from the nucleus of about 10,000 to 100,000 times the size of the nucleus itself. Thus, most of an atom is empty space. With the atom being largely empty space, it is easy to see why most α-particles go right through a thin metal foil. However, when α-particle happens to come near a nucleus, the intense electric field there scatters it through a large angle. The atomic electrons, being so light, do not appreciably affect the α-particles.

The Rutherford's nuclear model of the atom which involves classical concepts, pictures the atom as an electrically neutral sphere consisting of a very small, massive and positively charged nucleus at the centre surrounded by the revolving electrons in their respective dynamically stable orbits. The electrostatic force of attraction, *Fe* between the revolving electrons and the nucleus provides the requisite centripetal force (*Fc*) to keep them in their orbits. Thus, for a dynamically stable orbit in a hydrogen atom

$$F_e = F_c$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Thus the relation between the orbit radius and the electron velocity is

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

The kinetic energy (*K*) and electrostatic potential energy (*U*) of the electron in hydrogen atom are

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \text{ and } U = -\frac{e^2}{4\pi\varepsilon_0 r}$$

(The negative sign in *U* signifies that the electrostatic force is in the –*r* direction.) Thus the total energy *E* of the electron in a hydrogen atom is

$$E = K + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} \quad = -\frac{e^2}{8\pi\varepsilon_0 r}$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If *E* were positive, an electron will not follow a closed orbit around the nucleus.

(3) Explain Atomic spectra: Spectral series of hydrogen - Mention of empirical formulae for 1/λ (wave number) of different series.

When an atomic gas or vapour is excited at low pressure, usually by passing an electric current through it, the emitted radiation has a spectrum which contains certain specific wavelengths only. A spectrum of this kind is termed as emission line spectrum and it consists of bright lines on a dark background.

The spectrum emitted by atomic hydrogen is shown in Fig. Study of emission line spectra of a material can therefore serve as a type of "fingerprint" for identification of the gas. When white light passes through a gas and we analyse the transmitted light using a spectrometer we find some dark lines in the spectrum. These dark lines correspond precisely to those wavelengths which were found in the emission line spectrum of the gas. This is called the *absorption spectrum* of the material of the gas.

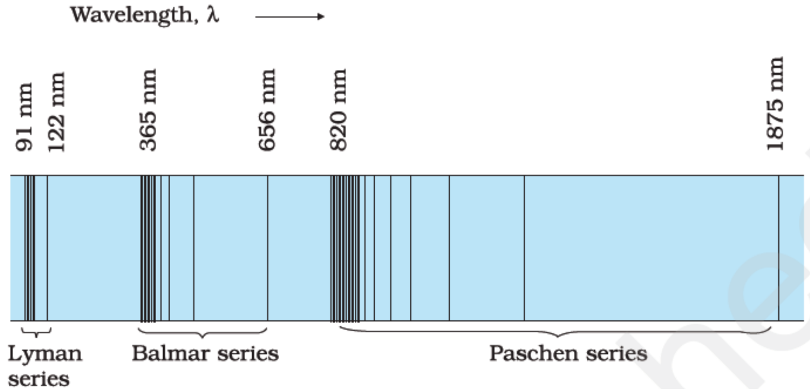
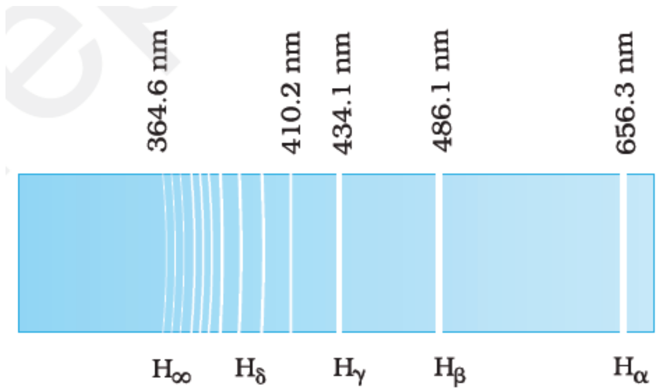


Fig. Emission lines in the spectrum of hydrogen.

Spectral series of Hydrogen :

In 1885, Jakob Balmer observed hydrogen spectrum in the visible region as shown in fig.



This series is called *Balmer series* (Fig.). The line with the longest wavelength, 656.3 nm in the red is called Hα; the next line with wavelength 486.1 nm in the bluegreen is called Hβ, the third line 434.1 nm in the violet is called Hγ; and so on. As the wavelength decreases, the lines appear closer together and are weaker in intensity. Balmer found a simple empirical formula for the observed wavelengths

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where λ is the wavelength, *R* is a constant called the *Rydberg constant*, and *n* may have integral values 3, 4, 5, etc. The value of *R* is 1.097×10^7 m^{-1}. This equation is also called Balmer formula.

Other series of spectra for hydrogen were subsequently discovered. These are known, after their discoverers, as Lyman, Paschen, Brackett, and Pfund series. These are represented by the formulae:

Lyman series:

$$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right) \qquad n=2,3,4\ldots$$

Paschen series:

$$\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n^2}\right) \qquad n=4,5,6\ldots$$

Brackett series:

$$\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right) \qquad n=5,6,7\ldots$$

Pfund series:

$$\frac{1}{\lambda}=R\left(\frac{1}{5^2}-\frac{1}{n^2}\right) \qquad n=6,7,8\ldots$$

The Lyman series is in the ultraviolet, and the Paschen and Brackett series are in the infrared region.

Since $c = \nu\lambda \rightarrow 1/\lambda = \nu/c$

Then Balmer series formula becomes

$$\nu = Rc\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$$

(4) Explain drawbacks of Rutherford Model ?

(a) According to Rutherford, an atom consisting of a central nucleus and revolving electron is stable much like sun-planet system with electro-static force of attraction and object (electron) which moves in a circle is being constantly accelerated – the acceleration being centripetal in nature.

According to classical electromagnetic theory, an accelerating charged particle emits radiation in the form of electromagnetic waves. The energy of an accelerating electron should therefore, continuously decrease. The electron would spiral inward and eventually fall into the nucleus. Thus, such an atom can not be stable. But most of the atoms in nature are stable which is against Rutherford model.

(b) According to the classical electromagnetic theory, the frequency of the electromagnetic waves emitted by the revolving electrons is equal to the frequency of revolution. As the electrons spiral inwards, their angular velocities and hence their frequencies would change continuously, and so will the frequency of the light emitted. Thus, they would emit a continuous spectrum, in contradiction to the line spectrum actually observed.

Rutherford model which depends on the classical ideas are not sufficient to explain the atomic structure.

(5) Explain Bohr model of hydrogen atom: Bohr's postulates - Derivation of Bohr radius - Derivation of energy of electron in stationary states of hydrogen atom.

In 1913, Niels Bohr Bohr, combined classical and early quantum concepts and gave his theory in the form of three postulates. These are :
i) Bohr's first postulate was that *an electron in an atom could revolve in certain stable orbits without the emission of radiant energy*, contrary to the predictions of electromagnetic theory. According to this postulate, each atom has certain definite stable states in which it can exist, and each possible state has definite total energy. These are called the stationary states of the atom.

(ii) Bohr's second postulate defines these stable orbits. This postulate states that the *electron* revolves around the nucleus *only in those orbits for which the angular momentum is some integral multiple of* $h/2\pi$ where h is the Planck's constant (= 6.6×10^{-34} J s). Thus the angular momentum (L) of the orbiting electron is quantised. That is
$L = nh/2\pi$

(iii) Bohr's third postulate incorporated into atomic theory the early quantum concepts that had been developed by Planck and Einstein.
It states that *an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is then given by*
$h\nu = E_i - E_f$ ----------- (2)
where E_i and E_f are the energies of the initial and final states and $E_i > E_f$.
For a hydrogen atom, Eq. (12.4) gives the expression to determine the energies of different energy states. But then this equation requires the radius r of the electron orbit. To calculate r, Bohr's second postulate about the angular momentum of the electron–the quantisation condition – is used. The angular momentum L is given by
$L = mvr$
Bohr's second postulate of quantisation [Eq. (12.11)] says that the allowed values of angular momentum are integral multiples of $h/2\pi$.

$$L_n = mv_n r_n = \frac{nh}{2\pi}$$

where n is an integer, r_n is the radius of nth possible orbit and v_n is the speed of moving electron in the nth orbit. The allowed orbits are numbered 1, 2, 3 ..., according to the values of n, which is called the *principal quantum number* of the orbit.
From Eq. (12.3), the relation between v_n and r_n is

$$v_n = \frac{e}{\sqrt{4\pi\varepsilon_0 m r_n}}$$

Combining it with Eq. (12.13), we get the following expressions for v_n and r_n,

$$v_n = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{(h/2\pi)}$$

and

$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\varepsilon_0}{e^2}$$

Eq. (12.14) depicts that the orbital speed in the nth orbit falls by a factor of *n*. Using Eq. (12.15), the size of the innermost orbit (n = 1) can be obtained as

$$r_1 = \frac{h^2\varepsilon_0}{\pi m e^2}$$

This is called the *Bohr radius*, represented by the symbol *a*0. Thus,

$$a_0 = \frac{h^2\varepsilon_0}{\pi m e^2}$$

Substitution of values of *h*, *m*, ε0 and *e* gives *a*0 = 5.29 × 10–11 m. From Eq. (12.15), it can also be seen that the radii of the orbits increase as *n*2. The total energy of the electron in the stationary states of the hydrogen atom can be obtained by substituting the value of orbital radius in Eq. (12.4) as

$$E_n = -\left(\frac{e^2}{8\pi\varepsilon_0}\right)\left(\frac{m}{n^2}\right)\left(\frac{2\pi}{h}\right)^2\left(\frac{e^2}{4\pi\varepsilon_0}\right)$$

or

$$E_n = -\frac{me^4}{8n^2\varepsilon_0^2h^2}$$

Substituting the values yields,

$$E_n = -\frac{2.18\times10^{-18}}{n^2}\,\text{J}$$

Atomic energies are often expressed in electron volts (eV) rather than joules. Since 1 eV = 1.6 × 10^{-19} J, Eq. (12.18) can be rewritten as

$$E_n = -\frac{13.6}{n^2}\ \text{eV}$$

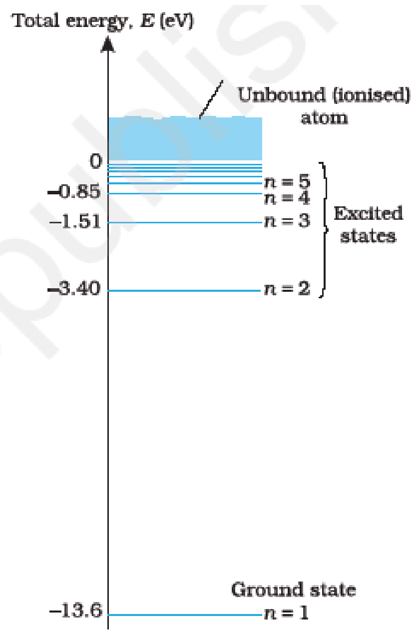
The negative sign of the total energy of an electron moving in an orbit means that the electron is bound with the nucleus. Energy will thus be required to remove the electron from the hydrogen atom to a distance infinitely far away from its nucleus (or proton in hydrogen atom).

The derivation of above Eqs. involves the assumption that the electronic orbits are circular, though orbits under inverse square force are, in general elliptical. (Planets move in elliptical orbits under the inverse square gravitational force of the sun.)

(5) Draw & Explain Energy level diagram of Hydrogen atom ?

The energy of an atom is the *least* (largest negative value) when its electron is revolving in an orbit closest to the nucleus i.e., the one for which n = 1. For n = 2, 3, ... the absolute value of the energy E is smaller, hence the energy is progressively larger in the outer orbits. The *lowest* state of the atom, called the *ground state*, is that of the lowest energy, with the electron revolving in the orbit of smallest radius, the Bohr radius, a_0. The energy of this state (n = 1), E_1 is –13.6 eV. Therefore, the minimum energy required to free the electron from the ground state of the hydrogen atom is 13.6 eV. It is called the *ionisation energy* of the hydrogen atom. This prediction of the Bohr's model is in excellent agreement with the experimental value of ionisation energy.

At room temperature, most of the hydrogen atoms are in *ground state*. When a hydrogen atom receives energy by processes such as electron collisions, the atom may acquire sufficient energy to raise the electron to higher energy states. The atom is then said to be in an *excited state*. From Eq. (12.19), for $n = 2$; the energy E_2 is –3.40 eV. It means that the energy required to excite an electron in hydrogen atom to its first excited state, is an energy equal to $E_2 - E_1 = -3.40$ eV – (–13.6) eV = 10.2 eV.



Similarly, $E_3 = -1.51$ eV and $E_3 - E_1 = 12.09$ eV, or to excite the hydrogen atom from its ground state ($n = 1$) to second excited state ($n = 3$), 12.09 eV energy is required, and so on. From these excited states the electron can then fall back to a state of lower energy, emitting a photon in the process. Thus, as the excitation of hydrogen atom increases (that is as n increases) the value of minimum energy required to free the electron from the excited atom decreases.

(6) Explain Line spectra of hydrogen atom: Derivation of frequency of emitted radiation - Mention of expression for Rydberg constant.

According to the third postulate of Bohr's model, when an atom makes a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number n_f ($n_f < n_i$), the difference of energy is carried away by a photon of frequency ν_{if} such that

$$h\,\nu_{if} = E_{ni} - E_{nf} \qquad \text{------------} \quad (1)$$

Using Eq. (1), for E_{nf} and E_{ni}, we get

$$h\nu_{if} = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad \text{---------} \quad \textbf{(2)}$$

or

$$\nu_{if} = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad \text{------------} \quad \textbf{(3)}$$

Equation (2) is the Rydberg formula, for the spectrum of the hydrogen atom. In this relation, if we take $n_f = 2$ and $n_i = 3, 4, 5...$, it reduces to a form similar for the Balmer series. The Rydberg constant R is readily identified to be

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c} \qquad \text{-------------} \quad (4)$$

If we insert the values of various constants in Eq. (4), we get $R = 1.03 \times 10^7\ \text{m}^{-1}$

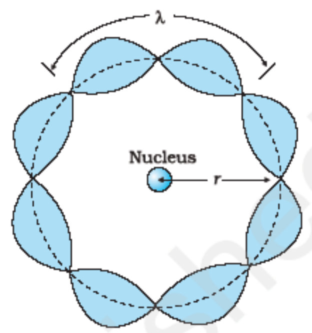
This is a value very close to the value (1.097×10^7 m^{-1}) obtained from the empirical Balmer formula. This agreement between the theoretical and experimental values of the Rydberg constant provided a direct and striking confirmation of the Bohr's model.

(6) Give de-Broglie's explanation of Bohr's second postulate and explain Limitations of Bohr model ?

The second postulate of bohr states that the angular momentum of the electron orbiting around the nucleus is quantised (that is, $L_n = nh/2\pi$; $n = 1, 2, 3 \ldots$). Why should the angular momentum have only those values that are integral multiples of $h/2\pi$? The French physicist Louis de Broglie explained this puzzle in 1923, ten years after Bohr proposed his model.

The de Broglie's hypothesis that material particles, such as electrons, also have a wave nature. C. J. Davisson and L. H. Germer later experimentally verified the wave nature of electrons in 1927. Louis de Broglie argued that the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave.

Accordingly when a stretched string is plucked, a vast number of wavelengths are excited. However only those wavelengths survive which have nodes at the ends and form the standing wave in the string. It means that in a string, standing waves are formed when the total distance travelled by a wave down the string and back is one wavelength, two wavelengths, or any integral number of wavelengths. Waves with other wavelengths interfere with themselves upon reflection and their amplitudes quickly drop to zero. For an electron moving in nth circular orbit of radius r_n, the total distance is the circumference of the orbit, $2\pi r_n$.



Thus $\mathbf{2\pi\, r_n = n\, \lambda}$, *where* $n = 1, 2, 3...$ --------- (1)

de Broglie's hypothesis that electrons have a wavelength $\lambda = h/mv$ gave an explanation for Bohr's quantised orbits by bringing in the waveparticle duality. The orbits correspond to circular standing waves in which the circumference of the orbit equals a whole number of wavelengths.

Figure illustrates a standing particle wave on a circular orbit for $n = 4$, i.e., $2\pi rn = 4\,\lambda$, where λ is the de Broglie wavelength of the electron moving in nth orbit. We know that $\lambda = h/p$, where p is the magnitude of the electron's momentum. If the speed of the electron is much less than the speed of light, the momentum is mv_n.

Thus, $\lambda = h/mv_n$. From Eq. (1), we have $2\pi\, r_n = n\, h/mv_n$ or $mv_n\, r_n = nh/2\pi$.

This is the quantum condition proposed by Bohr for the angular momentum of the electron.

Limitations of Bohr model :

Bohr model involves classical trajectory picture (planet-like electron orbiting the nucleus), correctly predicts the gross features of the hydrogenic atoms*, in particular, the frequencies of the radiation emitted or selectively absorbed. This model however has many limitations.

Some are:

(i) The Bohr model is applicable to hydrogenic atoms. It cannot be extended even to mere two electron atoms such as helium. The analysis of atoms with more than one electron was attempted on the lines of Bohr's model for hydrogenic atoms but did not meet with any success.
Difficulty lies in the fact that each electron interacts not only with the positively charged nucleus but also with all other electrons. The formulation of Bohr model involves electrical force between positively charged nucleus and electron. It does not include the electrical forces between electrons which necessarily appear in multi-electron atoms.
(ii) While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogenic atoms, the model is unable to explain the relative intensities of the frequencies in the spectrum. In emission spectrum of hydrogen, some of the visible frequencies have weak intensity, others strong. Why? Experimental observations depict that some transitions are more favoured than others. Bohr's model is unable to account for the intensity variations.

Bohr's model presents an elegant picture of an atom and cannot be generalised to complex atoms. For complex atoms we have to use a new and radical theory based on Quantum Mechanics, which provides a more complete picture of the atomic structure.

(7) Numerical Problems :

ONE MARK QUESTIONS

1. Who discovered electrons?
Ans: Electrons were discovered by J.J Thomason in the year 1897.

2. What is the electric charge on an atom?
Ans: At atom of an element is electrically neutral.

3. Who proposed the first model of an atom?
Ans: J.J. Thomson proposed the first model of an atom in the year 1898.

4. Name the sources which emit electromagnetic radiations forming a continuous emission spectrum.
Ans: Condensed matter like solids and liquids and non-condensed matter lie dense gases at all temperatures emit electromagnetic radiations of several wavelengths as a continuous spectrum.

5. How does the spectrum emitted by rarefied gases different from those dense gases?
Ans: In the rarefied gases ,the separation between atoms or molecules are farther apart. Hence the atoms give discrete wavelengths without any interaction with the neighbouring atoms.

6. Give any one difference between Thomson's model and Rutherford's model of an atom.
Ans: In the Thomson's atom model, electrons are in stable equilibrium while in the Rutherford's atom model electrons always experience a net force due to electrostatic force of attraction between electron and nucleus.

7. In which model atoms become unstable?
Ans: In Rutherford atom model. (An accelerating electron radiates energy and spiral around the nucleus. Ultimately electrons should fall inside the nucleus.)

8. What is a stationary orbit?
Ans: A stationary orbit is one in which the revolving electron does not radiate energy.

9. Give the relation between radius and principle Quantum number of an atom.
Ans: . $r_n \propto n^2$

10. Are the electron orbits equally spaced?
Ans: No. Electron orbits are unequally spaced.

11. What is the relation between the energy of an electron and the principle Quantum number?
Ans: $E_n \propto 1/n^2$

12. **What is excited state of an atom?**

Ans: When atom is given sufficient energy, the transition takes place to an orbit of higher energy. The atom is then said to be in an excited state.

13. What is wave number of spectral line?
Ans: Wave number represents number of waves present in one metre length of the medium.

14. What is the value of Rydberg's constant?
Ans: R = 1.097×10^{7} m^{-1}.

TWO MARKS QUESTIONS

1. Name the two quantised conditions proposed by Bohr in the atom model.
Ans: Bohr proposed i) quantised of energy states related to the transition of electrons from one orbit to another. ii) quantisation of orbit or angular momentum.

2. Write the mathematical conditions for quantisation of orbits and energy states.
Ans: i) $mvr = nh/2\pi \rightarrow$ quantisation of angular momentum
ii) $E_2 - E_1 = h\nu$ $\rightarrow$ quantisation of energy states and resulting transition.

3. Write the expression for the radius of n_{th} orbit. Give the meaning of symbols used.
Ans:
$$r = \frac{\varepsilon_o n^2 h^2}{\pi m Z e^2},$$
where 'r' is radius, n is principal Quantum number, h is Planck's constant, m is the mass of electron, Z atomic number and e is quantised unit of charge and ε_0 absolute permittivity for air or free space.

4. Give the expression for velocity of an electron in the nth orbit. Give the meaning of symbols used.
Ans:
$$v = \frac{Ze^2}{2\varepsilon_o nh},$$
where Z atomic number , e is charge, n is principle quantum number, *h* is Planck constant, ε_0 is absolute permittivity.

5. Write the formula for the wave number of a spectral line.
Ans:
$$\bar{\upsilon} = \frac{me^4}{8\varepsilon_0^2 ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{ or } \bar{\upsilon} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

6. What is the expression for the Rydberg's constant ? Give the meaning of the symbols used.
Ans:
$$R = \frac{me^4}{8\varepsilon_0^2 ch^3}$$
where m-mass of the electron, e-charge on the electron, c-speed of light and h- Planck's constant.

7. Write the formula for wave number of the spectral lines of Lyman series.

Ans: Lyman Series consists of spectral lines corresponding to the transition of an electron from higher energy orbits to the first orbit . i.e., $n_1 = 1$ and $n_2 = 2, 3, 4, 5……\infty$

$$\bar{\upsilon} = R\left(\frac{1}{1} - \frac{1}{n_2^2}\right) = R\left(\frac{n_2^2-1}{n_2^2}\right)$$

8. Write the formula for wave number of the spectral lines of Balmer series.

Ans: Balmer Series consists of spectral lines emitted during transitions of electron from higher energy orbits to the second orbit. i.e., $n_1 = 2$ and $n_2 = 3,4,5….. \infty$.

$$\bar{\upsilon} = R\left(\frac{1}{4} - \frac{1}{n_2^2}\right) = R\left(\frac{n_2^2-4}{4n_2^2}\right)$$

9. Mention any two demerits of Bohr's Theory.

Ans: i) The theory is applicable only for hydrogen atom.

ii) The relativistic variation of mass is not taken into account in the theory.

iii) The fine structure of spectral lines cannot be accounted for.

iv) The theory fails to account for relative intensities of spectral lines.

10. How does Rydberg's constant vary with atomic number?

Ans: $R = Z^2R_H$, where Z = atomic number. R is directly proportional to Z^2 R_H = Rydberg's constant for hydrogen atom.

11. What is the value of ionization potential of ${}^4_2He$ atom ?

Ans: I.E = -(13.6 eV)Z^2 put Z = 2 I.E = $-13.6(2)^2$ eV = -54.4 eV.

Ionisation potential = -54.4 V.

12. Name the physicists who for the first time verified the wave nature of electrons.

Ans: C.J. Davisson and L.H. Germer verified the wave nature of electrons.

THREE MARK QUESTIONS

1. Explain briefly 1) Bohr's Quantisation rule and 2) Bohr's frequency condition.

Ans: 1) The radius of the allowed electron orbits is determined by Quantum condition which states that the orbit angular momentum of the electron about the nucleus is an integral multiple of h/2π. According to Bohr's postulate

$mvr = nh/2\pi$ ------ (1)

2) The atom radiates energy only when an electron jumps from one stationary orbit of higher energy to another of lower energy $E_2 - E_1 = h\nu$ or $\nu = (E_2 - E_1)/h$.

2. Write de-Broglie wavelength associated with 3rd and 4th orbit in Bohr's atom model .

Ans: According to quantisation rule of Bohr's model

$$mvr = \left(\frac{nh}{2\pi}\right)$$

i.e., linear momentum $p=\left(\frac{nh}{2\pi}\right)$

but from de-Broglie's wave concept of moving matter p = h/λ, i.e., λ = h/mv

or **$\lambda = (h/nh)(2\pi r)$** or **$\lambda = (2\pi r)/n$**
For 3^{rd} and 4^{th} orbit,

$\lambda_3 = 2\pi r_3/3$ & $\lambda_4 = 2\pi r_4/4$

3. Give de-Broglie's explanation of quantisation of angular momentum as proposed by Bohr.
Ans: The condition for stationary wave formation is that the total distance travelled between the nodes(two) up and down or given path is integral multiple of 'λ'

i.e ., $2\pi r_n = n\lambda$ where, n = 1,2,3,…….
But $\lambda = h/mv$ (from de-Broglie's hypothesis)

$2\pi r_n = nh/mv_n$

$mv_n r_n = n(h/2\pi)$

i.e. integral multiple of $(h/2\pi)$ should be equal to the angular momentum of electron in the orbit.

4. What are hydrogenic atoms?
Ans: Hydrogenic (Hydrogen like) atoms are the atoms consisting of a nucleus with positive charge '+Ze' and a single electron. Here 'Z' is atomic mass number and 'e' is the quantised unit of charge. E.g., singly ionised helium, doubly ionised lithium.

5. Relate KE, PE and total energy of electron of an hydrogenic atom.
Ans: i) Potential Energy = 2 times the total energy. ii) Kinetic Energy of an electron = Minus of total energy.
where, total energy is negative and

$$E = -\frac{Z^2 m e^2}{8\varepsilon_o^2 n^2 h^2}$$

For H_2 atom , Z=1 For H_2 like atoms , charge on the nucleus = +Ze

6. How is frequency of radiation different from that of frequency of electron in its orbit?
Ans: i) For radiation frequency

$$\nu = \frac{E_1 - E_2}{h} \quad \text{or} \quad \nu = Z^2 Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where n_2 = higher orbit and n_1= lower orbit.

7. Why do we use gold in Rutherford's α-particle scattering experiment?
Ans: The reasons are i) gold is malleable ii) Gold nucleus is heavy and produce large deflections of α – particles.

8. Using Balmer empirical formula, obtain the wavelengths of H_α, H_β, H_γ, H_δ......... H_∞
Ans: Balmer empirical formula is given by $1/\lambda = R(1/2^2 - 1/n^2)$

For $H\alpha$ line, put n=3, $1/\lambda_\alpha = R\,(5/36)$ where $R = 1.097 \times 10^7\ m^{-1}$ this gives $\lambda_\alpha = 656.3$ nm.

For H_β line, put n=4, $1/\lambda_\beta = R(3/16)$, this gives line = 486.1nm

For Hγ line, put n = 5, then $\lambda_\gamma = 434.1$nm

For H_δ line put n = 6, then $\lambda_\delta = 434.1$nm

For H_∞ line is called **series limit**, put n= ∞; $\lambda_\infty = 364.6$nm

FIVE MARKS QUESTIONS

1. State the postulates of a Bohr's theory of hydrogen atom.

Ans: i) An electron cannot revolve round the nucleus in any arbitrary orbit. Only certain orbits are permitted. Electron does not radiate energy in those stationary orbits.

ii) The radius of the allowed electron orbits is determined by the quantum condition. It stated that the orbital angular momentum of electron about the nucleus is integral multiple of h/2π. According to Bohr's postulate **mvr = nh/2π**.

iii) The atom radiation the energy only when an electron Jumps from higher energy to lower energy. If E_1 and E_2 are lower and higher energies and

$\nu = (E_2 - E_1) / h$

2. Derive an expression for the radius of nth Bohr's orbit of H_2 atom.

Ans: Consider an atom of atomic number Z. The charge on its nucleus is +Ze. Let an electron of mass m and charge '-e' revolve round the nucleus in a circular orbit. Let v be its velocity. The coulomb's electrostatic force of attraction between the electron and the nucleus is =

$$\frac{1}{4\pi\varepsilon_o}\left[\frac{Ze^2}{r^2}\right]$$

This provides the centripetal force mv^2/r needed for orbital motion of electron.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\left[\frac{Ze^2}{r^2}\right] \quad \text{or} \quad mv^2r = \frac{Ze^2}{4\pi\varepsilon_o}$$

From Bohr's Quantization rule, for the n^{th} orbit,
We have, $mvr = nh/2\pi$ ------- *(2)*

Squaring Equation (2) $m^2v^2r^2 = n^2h^2/4\pi^2$

Dividing Equation (3) by Equation (1)

$$\frac{m^2v^2r^2}{mv^2r} = \frac{n^2h^2}{4\pi^2} \text{ X } \frac{4\pi\varepsilon_o}{Ze^2}$$

$$\text{i.e., } mr = \frac{\varepsilon_o n^2h^2}{\pi Ze^2} \qquad \therefore \qquad r = \frac{\varepsilon_o n^2h^2}{\pi mZe^2}$$

For H_2 atom put Z = 1 and for I[st] orbit put n = 1 and $r = \frac{\varepsilon_0 h^2}{\pi m e^2}$

3. Obtain an expression for the energy of an electron in the nth orbit of hydrogen atom in terms of the radius of the orbit and absolute constants.

Ans: Consider an electron of mass *m* and charge –e revolving round the nucleus of an atom of atomic number Z in the nth orbit of radius 'r'. Let v be the velocity of the electron. The electron possess potential energy because it is in the electrostatic field of the nucleus it also possess kinetic energy by virtue of its motion.

Potential energy of the electron is given by Ep = (potential at a distance *r* from the nucleus)(-e)

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{Ze^2}{r}\right](-e)$$

$$E_p = -\frac{Ze^2}{4\pi\varepsilon_o r} \quad \text{------ (1)}$$

Kinetic energy of the electron is given by

$E_k = 1/2\ mv^2$ --------- (2)

From Bohr's postulate

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\left[\frac{Ze^2}{r}\right] \qquad \therefore \qquad mv^2 = \frac{Ze^2}{4\pi\varepsilon_o r}$$

Substituting this value of mv^2 in equation (2)

$$E_k = \frac{1}{2}\left(\frac{Ze^2}{4\pi\varepsilon_o r}\right)$$

Total energy of the electron resolving in the *n*th orbit is given by $E_n = E_p + E_k$

$$E_n = -\frac{Ze^2}{4\pi\varepsilon_o r} + \frac{1}{2}\left(\frac{Ze^2}{4\pi\varepsilon_o r}\right) \qquad \text{Using (1) and (2)}$$

$$= \frac{Ze^2}{4\pi\varepsilon_o r}\left[\frac{-1}{1} + \frac{1}{2}\right] = \frac{Ze^2}{4\pi\varepsilon_o r}\left[\frac{-1}{2}\right]$$

$$E_n = -\frac{Ze^2}{8\pi\varepsilon_o r}$$

The radius of n^{th} permitted orbit of the electron is given by

$$r = \frac{\varepsilon_o n^2 h^2}{\pi m Z e^2}$$

Substituting this value of *r* in equation (4).

$$E_n = -\frac{Ze^2}{8\pi\varepsilon_o} \times \frac{\pi m Z e^2}{\varepsilon_o n^2 h^2}$$

$$E_n = \frac{-Z^2 m e^4}{8\varepsilon_0^2 n^2 h^2} \quad \text{for hydrogen like atoms.}$$

For hydrogen atom Z = 1.

Total energy of the electron in the nth orbit of hydrogen atom is

$$E_n = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2}$$

4. Give an account of the spectral series of hydrogen atom.

Ans: Hydrogen atom has a single electron. Its spectrum consists of series of spectral lines.

Lyman series : Lyman Series consists of spectral lines corresponding to the transition of an electron from higher energy orbits n = 1 and n_2 = 2,3,4…

These lines belong to Ultraviolet region.

Balmer series: Balmer series consists of spectral lines emitted during transitions of electrons from higher energy orbits to the second orbit. n_1 = 2 and n_2 = 3,4,5… .These lines lie is the visible region.

Paschen series : Paschen series consists of spectral lines emitted when electron jumps higher energy orbits to the third orbit n_1 = 3 and n_2 = 4,5,6 … .These lines lie in the infrared region.

Brackett series : Brackett series consists of spectral lines emitted during transitions of electrons from higher energy orbits to fourth orbit. n_1 = 4 and n_2 = 5,6,7…

Pfund series : Pfund series consists spectral lines emitted during transition of electrons from higher energy orbits to the fifth orbit. . n_1 = 5 and n_2 = 6,7,8… .

These lines lie in infrared region . The transition from (n_1 + 1) to n_1 corresponding to I^{st} member or longest wavelength of the series. The transition from (infinity) state to 'n_1' state corresponds to the last number or series limit or shortest wavelength of the series.

5. Explain energy level diagram of hydrogen atom.

Ans: The total energy of an electron in its orbit for an hydrogen atom

$$E = \frac{-13.6}{n^2}\,eV$$

Where 'n' is known as the principle quantum number whose value n = 1,2,3….

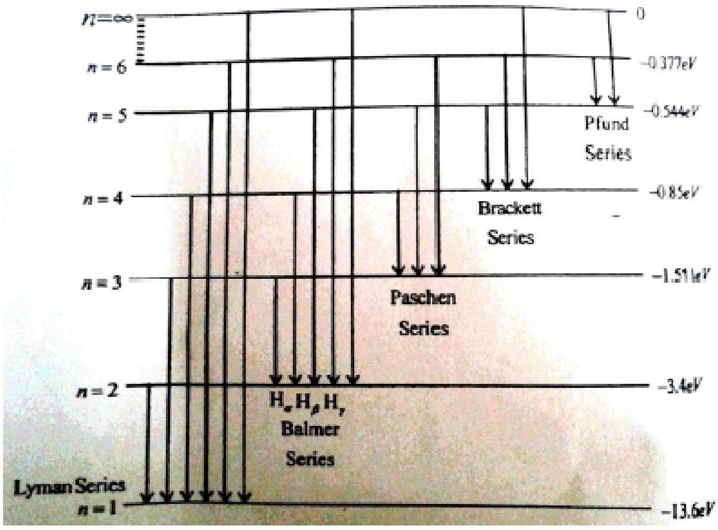
$E_1 = \frac{-13.6}{1^2}\,eV = -13.6\ eV$ $\quad$ $E_2 = \frac{-13.6}{2^2}\,eV = -3.4\ eV$ $\quad$ $E_3 = \frac{-13.6}{3^2}\,eV = -1.511\ eV$

$E_4 = \frac{-13.6}{4^2}\,eV = -0.85\ eV$

$E_5 = \frac{-13.6}{5^2}\,eV = -0.544\ eV$

$E_6 = \frac{-13.6}{6^2}\,eV = -0.377\ eV$ $\quad$ and $E_\infty = 0$

The energy level diagram represents different energy states with an increasing energy. The transition of electrons from higher energy states to lower energy states results in various levels of radiations classified into ultra violet, visible and invisible spectrum of radiation is illustrated below.



6. Derive an expression for the frequency of spectral series by assuming the expression for energy.

Ans: When atom in the excited state returns to normal state , results in transition.

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda}$$

i.e, $\frac{1}{\lambda} = \left(\frac{1}{hc}\right)(E2 - E1)$

where

$$E_n = \frac{-Z^2 m e^4}{8\varepsilon_0^2 n^2 h^2}$$

hence $\frac{1}{\lambda} = \frac{Z^2 m e^4}{8\varepsilon_0^2 h^3 c}\left(\frac{-1}{n_2^2} - \left(-\frac{1}{n_1^2}\right)\right)$

i.e., $\frac{1}{\lambda} = \frac{Z^2 m e^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ i.e., $\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

where, $R = \frac{m e^4}{8\varepsilon_0^2 h^3 c}$ is Rydberg's constant.

For H_2 atom, Z = 1

Wave number $\bar{\upsilon} = \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and frequency $\gamma = \frac{c}{\lambda} = Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Where $n_2 = n_1 + 1, n_1 + 2, n_1 + 3, \ldots\ldots\ldots \infty$

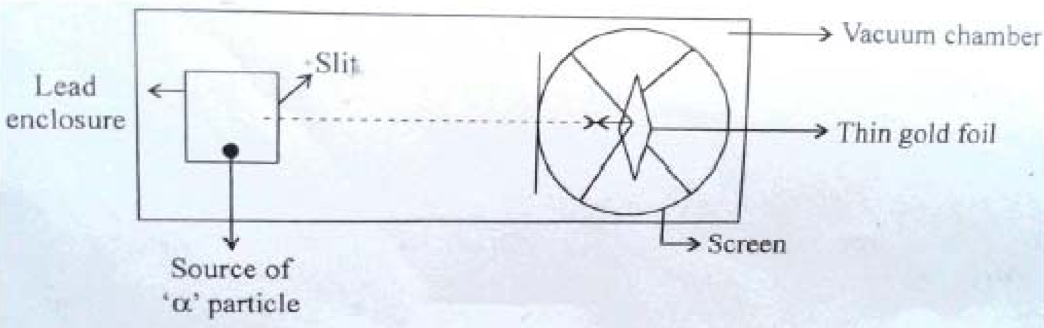
7. Outline the experiment study of - scattering by a gold foil.

Ans:

The source of α- particles are taken from α- decay of $^{214}_{83}Bi$. Gold foil used is of thickness 0.21 micron. The scattering α– particles were discovered by rotatable detector with a fluorescent flashes observed through the microscope. The intensity of α- particles is studied as the function of 'θ'.

For a range value of I_p , α- particles travel straight because of small deflection.

For θ=180º, $I_p = 0$, the α- particles when directed towards the centre (for a head – on collision) , it retraces the path.



Geiger - Marsden α-scattering experiment

8. Give the experimental conclusions arrived by Rutherford in the - scattering experiment.

Ans: conclusions:

i) The entire mass of the atom is concentrated in the nucleus of an atom.

ii) The entire charge is concentrated in the nucleus rather than distributing throughout the volume of the atom.

iii) The size of the nucleus is estimated to be of the order of 10^{-15} m and atom of the order 10^{-10} m.

iv) The size of the electron is negligibly small and space between the electron and nucleus is almost void.

v) Atom as a whole is electrically neutral.

vi) Electron is acted upon by a force and hence it is not in the state of static equilibrium.

vii) To explain the H_2 – spectrum Rutherford proposed radiating circular orbits and for the stability he assumed that the centripetal force is balanced by electrostatic force of attraction.

i.e., $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Ze^2}{r^2}\right]$

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Chapter 13:
NUCLEI
6M
1M - 1Q, 5M - 1Q (LA) OR (1M-1Q ; 2M-1Q ; 3M-1Q)

Definition of atomic mass unit (u) - Isotopes, isobars and isotones - Composition, size, mass and density of the nucleus -

1. Define AMU ?

On the atomic scale, mass is measured in atomic mass units (u). By definition, 1 atomic mass unit (1u) is 1/12th mass of one atom of ^{12}C; $1u = 1.660563 \times 10^{-27}$ kg.

2. Compare isotopes, isobars and isotones with examples ?

Nuclides with the same atomic number Z, but different neutron number N are called *isotopes*.
Ex :
Nuclides with the same A are *isobars* and those with the same N are *isotones*.
Ex : (i) Isobars : (ii) Isotones :
Most elements are mixtures of two or more isotopes. The atomic mass of an element is a weighted average of the masses of its isotopes. The masses are the relative abundances of the isotopes.

3. Write a note on Nucleus ?

An atom has a nucleus. The nucleus is positively charged. The radius of the nucleus is smaller than the radius of an atom by a factor of 104. More than 99.9% mass of the atom is concentrated in the nucleus.

A nucleus contains a neutral particle called neutron. Its mass is almost the same as that of proton.

The atomic number Z is the number of protons in the atomic nucleus of an element. The mass number A is the total number of protons and neutrons in the atomic nucleus; $A = Z+N$; Here N denotes the number of neutrons in the nucleus. A nuclear species or a nuclide is represented as $^{A}_{Z}X$, where X is the chemical symbol of the species.

A nucleus can be considered to be spherical in shape and assigned a radius. Electron scattering experiments allow determination of the nuclear radius; it is found that radii of nuclei fit the formula $R = R_0 A^{1/3}$,
where R_0 = a constant = 1.2 fm. This implies that the nuclear density is independent of A. It is of the order of 10^{17} kg/m^3.

Neutrons and protons are bound in a nucleus by the short-range strong nuclear force. The nuclear force does not distinguish between neutron and proton.

Einstein's mass energy relation - Nuclear binding energy: Brief explanation of mass defect and binding energy - Binding energy per nucleon - Binding energy curve -

1. Einstein's mass energy relation ?

Einstein gave the famous mass-energy equivalence relation $E = mc^2$ -------- (1)

Here the energy equivalent of mass m is related by the above equation and c is the velocity of light in vacuum and is approximately equal to 3×10^{8} m s^{-1}.

2. Calculate the energy equivalent of 1 g of substance.

Solution :

Energy, $E = 10^{-3} \times (3 \times 10^{8})^{2}$ J

$E = 10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13}$ J

Thus, if one gram of matter is converted to energy, there is a release of enormous amount of energy.

3. Explain mass defect and binding energy with example?

The nuclear mass M is always less than the total mass of its constituents. The difference in mass of a nucleus and its constituents ΔM is called the *mass defect*,

$\Delta M = (Z\, m_p + (A - Z)m_n) - M$

Using Einstein's mass energy relation, we express this mass difference in terms of energy as

$\Delta E_b = \Delta M\, c^2$

The energy ΔE_b represents the *binding energy* of the nucleus. In the mass number range A = 30 to 170, the binding energy per nucleon is nearly constant, about 8 MeV/nucleon.

Example :

The atomic mass of nucleus of ${}^{16}_{8}O$ found from mass spectroscopy experiments is seen to be 15.99053 u. The expected mass of ${}^{16}_{8}O$ nucleus is by adding 8 protons mass and 8 neutrons mass = 16.12744 u.

The difference between these two masses is mass defect = 0.13691u.

4. Explain the Binding energy per nucleon - Binding energy curve

If a certain number of neutrons and protons are brought together to
form a nucleus of a certain charge and mass, an energy E_b will be released in the process. The energy E_b is called the *binding energy* of the nucleus.

If we separate a nucleus into its nucleons, we would have to supply a total energy equal to E_b, to those particles.

A more useful measure of the binding between the constituents of the nucleus is the *binding energy per nucleon*, E_{bn}, which is the ratio of the binding energy E_b of a nucleus to the number of the nucleons, A, in that nucleus:

$E_{bn} = E_b / A$ ----------- (2)

We can think of binding energy per nucleon as the average energy per nucleon needed to separate a nucleus into its individual nucleons.

A plot of the binding energy per nucleon E_{bn} versus the mass number A for a large number of nuclei is called binding energy curve.

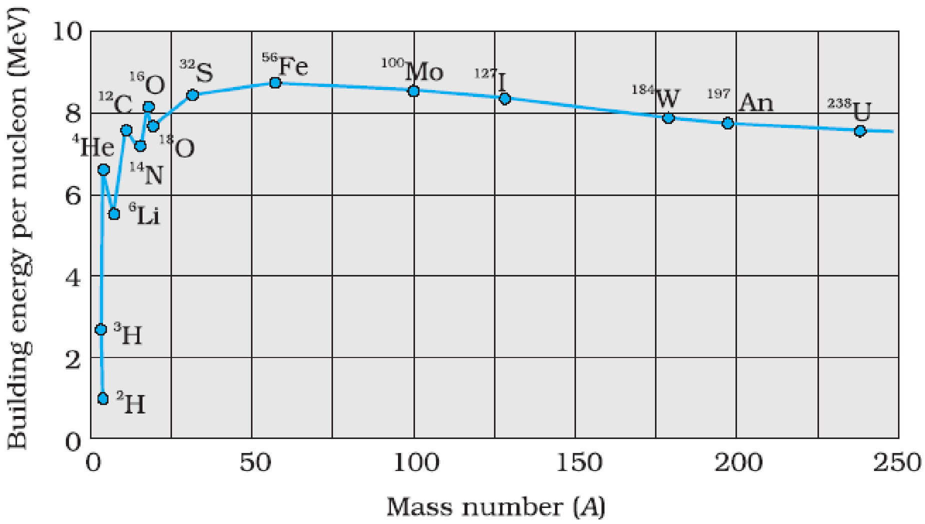


Fig. 1 : The binding energy per nucleon as a function of mass number.

Main features of the Binding energy plot :
(i) the binding energy per nucleon, E_{bn}, is practically constant, i.e. practically independent of the atomic number for nuclei of middle mass number (30 < A < 170).
The curve has a maximum of about 8.75 MeV for A = 56 and has a value of 7.6 MeV for A = 238.
(ii) E_{bn} is lower for both light nuclei (A<30) and heavy nuclei (A>170).
We can draw some conclusions from these two observations:
(i) The force is attractive and sufficiently strong to produce a binding energy of a few MeV per nucleon.
(ii) The constancy of the binding energy in the range 30 < A < 170 is a consequence of the fact that the nuclear force is short-ranged. Consider a particular nucleon inside a sufficiently large nucleus. It will be under the influence of only some of its neighbours, which come within the range of the nuclear force. If any other nucleon is at a distance more than the range of the nuclear force from the particular nucleon it will have no influence on the binding energy of the nucleon under consideration. If a nucleon can have a maximum of p neighbours within the range of nuclear force, its binding energy would be proportional to p.

(iii) A very heavy nucleus, say A = 240, has lower binding energy per nucleon compared to that of a nucleus with A = 120. Thus if a nucleus A = 240 breaks into two A = 120 nuclei, nucleons get more tightly bound. This implies energy would be released in the process. It has very important implications for energy production through ***fission***.

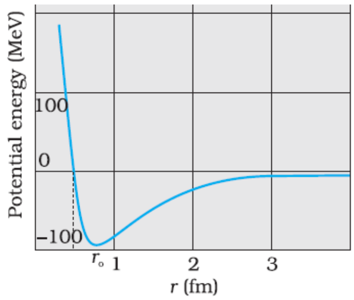
(iv) Consider two very light nuclei ($A \leq 10$) joining to form a heavier nucleus. The binding energy per nucleon of the fused heavier nuclei is more than the binding energy per nucleon of the lighter nuclei. This means that the final system is more tightly bound than the initial system. Again energy would be released in such a process of ***fusion***.

5. Explain Nuclear force and its characteristics.
For an average mass nuclei, the binding energy per nucleon is approximately 8 MeV, which is much larger than the binding energy in atoms. Therefore, to bind a nucleus together there must

be a strong attractive force which should be strong enough to overcome the repulsion between the (positively charged) protons and to bind both protons and neutrons into the tiny nuclear volume.

Some of the characteristics of nuclear binding force are :
(i) The nuclear force is much stronger than the Coulomb force acting between charges or the gravitational forces between masses. The nuclear binding force has to dominate over the Coulomb repulsive force between protons inside the nucleus. This happens only because the nuclear force is much stronger than the coulomb force. The gravitational force is much weaker than even Coulomb force.
(ii) The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres. This leads to *saturation of forces* in a medium or a large-sized nucleus, which is the reason for the constancy of the binding energy per nucleon.

(iii) A rough plot of the potential energy between two nucleons as a function of distance is shown in the Fig. The potential energy is a minimum at a distance r_0 of about 0.8 fm. This means that the force is attractive for distances larger than 0.8 fm and repulsive if they are separated by distances less than 0.8 fm.



(iv) The nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same. The nuclear force does not depend on the electric charge.

Radioactivity: Law of radioactive decay - Derivation of $N = N_0 e^{-\lambda t}$:

(1) Explain Radioactivity

Radioactivity is the phenomenon in which nuclei of a given species transform by giving out a or b or g rays; a-rays are helium nuclei; b-rays are electrons. g-rays are electromagnetic radiation of wavelengths shorter than *X*-rays;

A. H. Becquerel discovered radioactivity in 1896 purely by accident. Experiments performed subsequently showed that radioactivity was a nuclear phenomenon in which an unstable nucleus undergoes a decay. This is referred to as *radioactive decay*. Three types of radioactive decay occur in nature :
(i) α-decay in which a helium nucleus ${}_2^4He$ is emitted;
(ii) β-decay in which electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) are emitted;
(iii) γ-decay in which high energy (hundreds of keV or more) photons are emitted.
Each of these decay will be considered in subsequent sub-sections.

(2) Give Law of radioactive decay ?

In any radioactive sample, which undergoes a, b or g-decay, it is found that the number of nuclei undergoing the decay per unit time is proportional to the total number of nuclei in the sample. If N is the number of nuclei in the sample and ΔN undergo decay in time Δt then

$$\frac{\Delta N}{\Delta t} \propto N$$

or, $\Delta N/\Delta t = \lambda N$,

where λ is called the radioactive *decay constant* or *disintegration constant*.

The change in the number of nuclei in the sample is dN = – ΔN in time Δt. Thus the rate of change of N is (in the limit $\Delta t \rightarrow 0$)

$$\frac{dN}{dt} = -\lambda N \qquad \text{or,} \quad \frac{dN}{N} = -\lambda dt$$

Now, integrating both sides of the above equation, we get,

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt \qquad \text{--------------} \quad (1)$$

$$\text{or,} \; \ln N - \ln N_0 = -\lambda (t - t_0) \qquad \text{------------} \quad (2)$$

Here N_0 is the number of radioactive nuclei in the sample at some arbitrary time t_0 and N is the number of radioactive nuclei at any subsequent time t. Setting t_0 = 0 and rearranging Eq. (13.12) gives us

$$\ln \frac{N}{N_0} = -\lambda t \qquad \text{----------} \quad (3)$$

which gives

$$N(t) = N_0 e^{-\lambda t} \qquad \text{---------} \quad (4)$$

Eqn. (4) represents **law of radio-active decay**.

The total decay rate R of a sample is the number of nuclei disintegrating per unit time. Suppose in a time interval dt, the decay count measured is ΔN. Then dN = – ΔN.
The positive quantity R is then defined as

$$R = -\frac{dN}{dt}$$

Differentiating Eq. (4), we get,

$$R = \lambda N_0 e^{-\lambda t} \qquad \text{or,} \; R = R_0 e^{-\lambda t} \qquad \text{---------} \quad (5)$$

This is equivalent to the law of radioactivity decay,

Activity (decay rate) and its units - becquerel and curie -

The decay rate R at a certain time t and the number of undecayed nuclei N at the same time are related by $R = \lambda N$

The decay rate of a sample is experimentally measurable quantity and is given a specific name: *activity*. The SI unit for activity is **becquerel**, named after the discoverer of radioactivity, Henry Becquerel.

1 becquerel is simply equal to 1 disintegration or decay per second.

There is also another unit named "curie" that is widely used and is related to the SI unit as:

1 curie = 1 Ci = 3.7×10^{10} decays per second

= 3.7×10^{10} Bq.

Definition and derivation of half-life of radioactive element - Definition of mean life and mention its expression.

(1) Define half-life of radio-active element and derive an expression for it ?

Half-life of a radionuclide (denoted by $T_{1/2}$) is the time it takes for a sample that has initially, say N_0 radionuclei to reduce to $N_0/2$.

Putting $N = N_0/2$ and $t = T_{1/2}$ in Eq. $N(t) = N_0\, e^{-\lambda t}$, we get

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

if N_0 reduces to half its value in time $T_{1/2}$, R_0 will also reduce to half its value in the same time.

The *average* or *mean life* τ. The number of nuclei which decay in the time interval t to $t + \Delta t$ is $R(t)\Delta t$ (= $\lambda N_0\, e^{-\lambda t}\, \Delta t$). Each of them has lived for time t. Thus the total life of all these nuclei would be $t\, \lambda N_0 e^{-\lambda t}\, \Delta t$. It is clear that some nuclei may live for a short time while others may live longer. Therefore to obtain the mean life, we have to sum (or integrate) this expression over all times from 0 to ∞ , and divide by the total number N_0 of nuclei at $t = 0$. Thus,

$$\tau = \frac{\lambda N_0 \int_0^{\infty} t e^{-\lambda t} \mathrm{d}t}{N_0} = \lambda \int_0^{\infty} t e^{-\lambda t} \mathrm{d}t$$

One can show by performing this integral that

$\tau = 1/\lambda$

We summarize these results with the following:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2$$

Radioactive elements (e.g., tritium, plutonium) which are short-lived i.e., have half-lives much less than the age of the universe (≈15 billion years) have obviously decayed long ago and are not found in nature. They can, however, be produced artificially in nuclear reactions.

Problem : The half-life of 238 92 U undergoing a-decay is 4.5 × 109 years. What is the activity of 1g sample of 238 92U ?

Solution

$T_{1/2} = 4.5 \times 10^{9}$ y $= 4.5 \times 10^{9}$ y x 3.16×10^{7} s/y $= 1.42 \times 10^{17}$s

One k mol of any isotope contains Avogadro's number of atoms, and so 1g of 238 92 U contains 1/ 238× 10-3 kmol × 6.025 × 10^{26} atoms/kmol
= 25.3 × 10^{20} atoms.
The decay rate *R* is
$R = \lambda N$

$$= \frac{0.693}{T_{1/2}} N = \frac{0.693 \times 25.3 \times 10^{20}}{1.42 \times 10^{17}} s^{-1}$$
$$= 1.23 \times 10^4 \text{ s}^{-1}$$
$$= 1.23 \times 10^4 \text{ Bq}$$

Alpha decay, beta decay (negative and positive) and gamma decay with examples -

(2) Write a note on Alpha decay, beta decay (negative and positive) and gamma decay with examples ?

Energies associated with nuclear processes are about a million times larger than chemical process.

(a) **Alpha Decay :**

The example of alpha decay is the decay of uranium $^{238}_{92}U$ to thorium $^{234}_{92}Th$ with the emission of a helium nucleus $^{4}_{2}He$.

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} \qquad (\alpha\text{-decay})$$

In a-decay, the mass number of the product nucleus (daughter nucleus) is four less than that of the decaying nucleus (parent nucleus), while the atomic number decreases by two. In general, α-decay of a parent nucleus $^{A}_{Z}X$ results in a daughter nucleus $^{A-4}_{Z-2}Y$.

$$^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$$

From Einstein's mass-energy equivalance relation E = mc^2 and energy conservation, it is clear that this spontaneous decay is possible only when the total mass of the decay products is less than the mass of the initial nucleus. This difference in mass appears as kinetic energy of the products. By referring to a table of nuclear masses, one can check that the total mass of $^{234}_{90}Th$ and $^{4}_{2}He$ is indeed less than that of $^{238}_{92}U$.

The disintegration energy or the *Q*-value of a nuclear reaction is the difference between the initial mass energy and the total mass energy of the decay products.

For α-decay

$Q = (m_X - m_Y - m_{He})\, c^2$

Q is also the net kinetic energy gained in the process or, if the initial nucleus X is at rest, the kinetic energy of the products. Clearly, ***Q*> 0 for exothermic processes such as a-decay**.

(b) Beta Decay :

In beta decay, a nucleus spontaneously emits an electron (β^- decay) or a positron (β^+ decay). A common example of β^- decay is

$$^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + e^- + \overline{\nu}$$

and that of β^+ decay is

$$^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne} + e^{+} + \nu$$

The decays are governed by the Eqs. $N(t) = N_0\, e^{-\lambda t}$ and $R = R_0\, e^{-\lambda t}$, so that one can never predict *which* nucleus will undergo decay, but one can characterize the decay by a half-life $T_{1/2}$. For example, $T_{1/2}$ for the decays above is respectively 14.3 d and 2.6y. The emission of electron in β^- decay is accompanied by the emission of an antineutrino ($\overline{\vartheta}$); in β^+ decay, instead, a neutrino (ϑ) is generated. Neutrinos are neutral particles with very small (possiblly, even zero) mass compared to electrons. They have only weak interaction with other particles. They are, therefore, very difficult to detect, since they can penetrate large quantity of matter (even earth) without any interaction.

In both β^- and β^+ decay, the mass number *A* remains unchanged. In β^- decay, the atomic number *Z* of the nucleus goes up by 1, while in β^+ decay Z goes down by 1. The basic nuclear process underlying β^- decay is the conversion of neutron to proton

$$n \rightarrow p + e^{-} + \overline{\nu}$$

while for β^+ decay, it is the conversion of proton into neutron

$$p \rightarrow n + e^{+} + \nu$$

Note that while a free neutron decays to proton, the decay of proton to neutron is possible only inside the nucleus, since proton has smaller mass than neutron.

(c) Gamma Decay :

Like an atom, a nucleus also has discrete energy levels - the ground state and excited states. The scale of energy is, however, very different. Atomic energy level spacings are of the order of eV, while the difference in nuclear energy levels is of the order of MeV. When a nucleus in an excited state spontaneously decays to its ground state (or to a lower energy state), a photon is emitted with energy equal to the difference in the two energy levels of the nucleus. This is the so-called *gamma decay*. The energy (MeV) corresponds to radiation of extremely short wavelength, shorter than the hard X-ray region.

Typically, a gamma ray is emitted when a a or b decay results in a daughter nucleus in an excited state. This then returns to the ground state by a single photon transition or successive transitions involving more than one photon.

Example is the successive emmission of gamma rays of energies 1.17 MeV and 1.33 MeV from the de-excitation of 60 28 Ni nuclei formed from b– decay of 60 27Co .

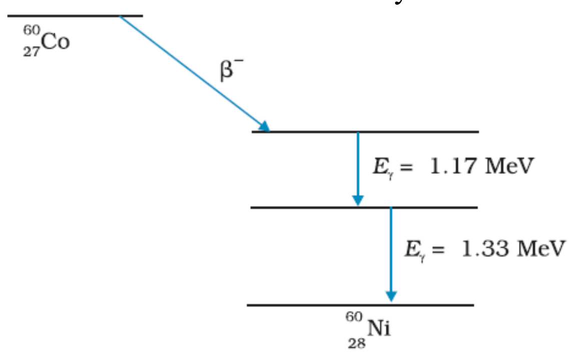


Fig. : β^-decay of $_{28}{}^{60}$Ni nucleus followed by emission of two g rays from deexcitation of the daughter nucleus $_{28}{}^{60}$Ni .

Q value of nuclear reaction :

1. What is Q value of nuclear reaction

The Q-value of a nuclear process is
Q = final kinetic energy – initial kinetic energy.
Due to conservation of mass-energy, this is also,
Q = (sum of initial masses – sum of final masses) c^2

Numerical Problems.

One Mark Questions

1. How many neutrons will be there in the nucleus of an element with mass number A and atomic number Z?

A-Z

2. Mention the commonly used unit to measure the nuclear mass.

Atomic Mass Unit denoted by amu or u.

3. Which type of radioactive emission produces a daughter nucleus which is an isobar of the parent?

Beta particle

4. Mention the SI unit of activity.

Becquerel (Bq)

5. What are isotones?

Nuclei of different elements having same number of neutrons

6. How does the radius of the nucleus vary with respect to mass number?

R = $R_0\, A^{1/3}$

Two Mark Questions

7. What is mass defect? Write the formula for the mass defect for the nucleus of an element ^A_ZX

The difference between the sum of the masses of the constituent particles and the actual mass of the nucleus is known as mass defect. Mass defect

$$\Delta M = \left\{ Zm_P + \left(A - Z\right)m_N \right\} - M$$

8. Mention any two characteristics of nuclear forces.

Strongest force in nature / short range / non-central /charge independent / spin dependent / saturated – any two

9. Mention the order of nuclear density. How does the nuclear density vary as we move from the centre to the surface?

$\rho = 10^{17}$ Kg/m^3 nuclear density remains constant.

10. Define nuclear fission and give an example for it.

Process of splitting up of heavy nucleus into two or more fragments of comparable masses along with the liberation of energy is known as nuclear fission.

$$_{92}U^{235} + {}_0n^1 \rightarrow 3\ {}_0n^1 + {}_{36}Kr^{89} + {}_{56}Ba^{144} + \text{ENERGY}$$

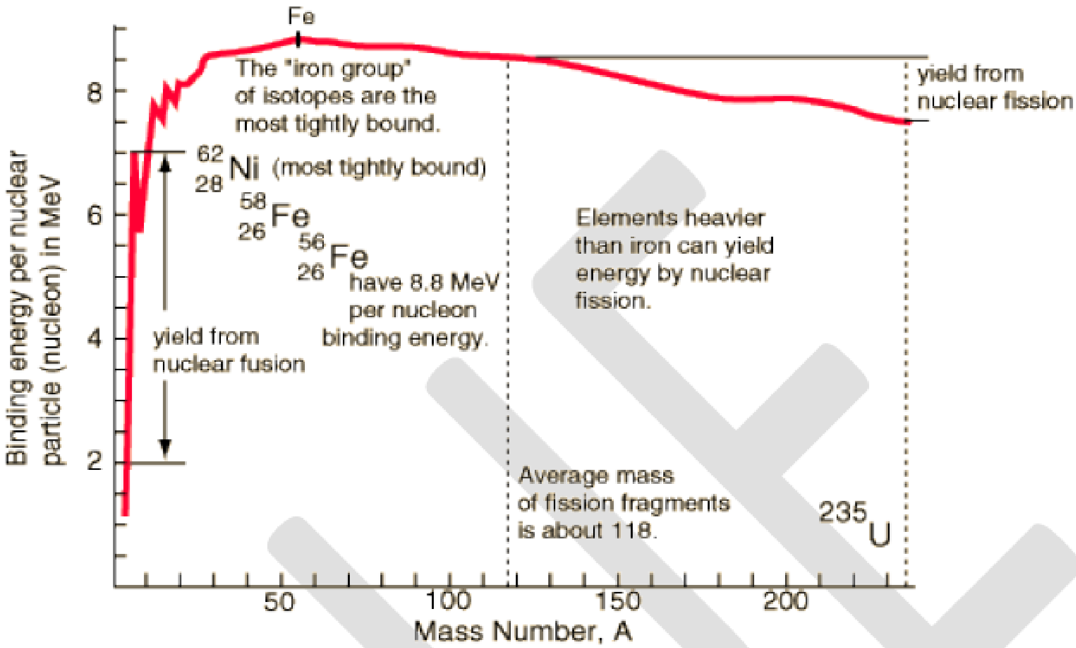
11. Define half-life and mean-life of a radioactive nucleus.

Time during which the number of radioactive atoms will reduce to half the original number is known as half-life of radioactive element.

Average life expectancy of the nucleus is called mean-life. It is the average life of all the atoms which will disintegrate anywhere between zero and infinity. It is numerically equal to time during which number of atoms reduced to about 37% of the original number.

Three Mark Questions

12. Draw the graph of binding energy per nucleon with respect to mass number. What is the significance of the graph?



It represents the stability of the nucleus.

13. Write the equation representing nuclear reaction corresponding to α, β and γ emission.

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$
$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + \beta + \overline{\upsilon}$$
$$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$

14. What is Q-value of a nuclear reaction? Write the formula for Q-value for β-emission and explain the terms.

During alpha or beta emission total mass of the products is slightly less than the reactant nucleus. The difference in the mass will be converted into energy which will appear in the form of kinetic energy of products- the sum of the kinetic energies of the emitted particle(s) and the recoil nucleus. The energy equivalent of mass difference is known as Q-value.
For β-emission

$$_Z^AX \rightarrow {}_{Z+1}^{A}Y + \beta + \overline{\upsilon}$$

$$Q = \left(m_X - m_y - m_\beta - m_{\overline{\upsilon}}\right) \cdot C^2 ;$$

Where,

$m_X \rightarrow$ *mass of the parent nucleus*

$m_y \rightarrow$ *mass of the daughter nucleus*

$m_\beta \rightarrow$ *mass of the* $\beta -$ *particle*

$m_{\overline{\upsilon}} \rightarrow$ *mass of the antineutrino*

and C is the speed of light

15. Define Atomic Mass Unit. Mention Einstein's mass energy relation.

1 / 12th the mass of one atomic nucleus of carbon-12 is known as Atomic Mass Unit denoted by amu of u. This unit is normally used to measure the mass of the nuclei. According to Einstein's mass energy relation, when a mass 'm' is converted into energy, the energy equivalent E is given by $E = mC^2$.

NOTE; Energy equivalent of one amu : We know that Mass of one mole of C-12 = 12g. = 12×10^{-3} kg.

Therefore, mass of 1 atom of C-12 = $\frac{12\times10^{-3}}{6.023\times10^{23}}$ kg.

Therefore the energy equivalent of 1 amu = $\Delta m.c^2$

$$= \frac{12\times10^{-3}}{6.023\times10^{23}} \cdot \left(3\times10^8\right)^2 \text{J}$$

$$= \frac{12\times10^{-3} \cdot \left(3\times10^8\right)^2}{6.023\times10^{23}\times1.6\times10^{-19}} \text{eV}$$

$$\cong 931\times10^6 \text{eV} = 931 \text{MeV}$$

16. Prove that $N = N_0\, e^{-\lambda t}$ where the symbols have their usual meaning.

According to radioactive decay law, rate of disintegration of a radioactive substance is directly proportional to the number of radioactive atoms present at that instant of time. Therefore,

$$\frac{dN}{dt} = -\lambda N$$

$$\Rightarrow \frac{dN}{N} = -\lambda dt$$

$$\Rightarrow \int \frac{dN}{N} = \int -\lambda dt$$

$$\Rightarrow \log N = -\lambda t + C$$

$$At\ t = 0,\ N = N_0$$
$$\Rightarrow \log N = -\lambda t + \log N_0$$
$$\Rightarrow \log \frac{N}{N_0} = -\lambda t$$
$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$
$$\therefore N = N_0 e^{-\lambda t}$$

17. Define mean-life. Write the expression for mean-life in terms of decay constant..

The average life or mean-life of a radioactive sample is the ratio of total life time of all N0 number of atoms in the sample to the total number of atoms which will disintegrate anywhere between zero and infinity. Therefore,

$$mean - life\ \tau = \frac{life - time\ of\ all\ atoms}{N_0} \qquad \tau = \frac{1}{\lambda}$$

18. Obtain the relation between half-life and decay constant.

Relation between half-life and decay constant We know that,

$$N = N_0 e^{-\lambda t}$$
$$When\ \ t \to T \quad then \quad N \to \frac{N_0}{2}$$
$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T} \qquad \Rightarrow e^{-\lambda T} = \frac{1}{2}$$
$$\Rightarrow e^{\lambda T} = 2 \Rightarrow \lambda T = \ln 2 = 0.693$$
$$\Rightarrow T = \frac{0.693}{\lambda} = 0.693\tau$$

19. Calculate the binding energy and binding energy per nucleon in MeV for carbo-12 nucleus. Given that mass of the proton is 1.00727amu while the mass of the neutron is 1.00866 amu.

Mass defect

$$\Delta M = Zm_p + (A - Z)m_N - M$$
$$For\ carbon - 12\ nucleus$$
$$Z = 6,\ A = 12\ and\ M = 12amu$$
$$\Rightarrow \Delta M = 6 \times 1.00727 + 6 \times 1.00866 - 12 = 0.09558amu$$
$$E = \Delta M \times 931MeV = 0.09558 \times 931 = 88.985MeV$$

(21) Half-life of $^{90}_{38}Sr$ is 28 years. Calculate the activity in Ci of 30mg of $^{90}_{38}Sr$

$$90g\ of\ ^{90}_{38}Sr\ contains\ 6.023\times10^{23}\ atoms$$

$$\therefore 30mg\ of\ ^{90}_{38}Sr\ contains\ \frac{30\times10^{-3}}{90}\times6.023\times10^{23}=2.0077\times10^{20}\ atoms$$

$$Activity\ A=\lambda N=\frac{0.693}{T}\cdot N$$

$$\Rightarrow A=\frac{0.693}{28\times365\times24\times3600}\times2.0077\times10^{20}=1.5757\times10^{11}\ Bq$$

$$1Ci=3.7\times10^{10}\ Bq$$

$$\therefore A=\frac{1.5757\times10^{11}}{3.7\times10^{10}}=4.26Ci$$

(22) Calculate the Q-value of the emitted α-particle in the α-decay of $^{22}_{08}Ra$

Given

Mass of 220 $^{220}_{86}Rn$ = 220.01137amu

Mass of residual nucleus $^{216}_{84}Po$ = 216.00189 amu.

Mass of α-particle = 4.002603 amu.

$$Q=\left(m_{Rn}-m_{Po}-m_{\alpha}\right)\cdot C^2J=\left(m_{Rn}-m_{Po}-m_{\alpha}\right)\cdot931MeV$$

$$=\left(220.01137-216.00189-4.002603\right)\times931=6.402MeV$$

MOST LIKELY QUESTIONS :

1. Compare isotopes, isobars and isotones with examples ?
2. What is Q-value of a nuclear reaction? Write the formula for Q-value for β-emission and explain the terms.

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Chapter 14:
SEMICONDUCTOR ELECTRONICS
10 M
2M - 1Q, 3M - 1Q, 5M - 1Q (LA) OR 1M-2Q ; 3M-1Q ; 5M-1Q(LA)

14.1 Energy bands in solids: Valance band, conduction band and energy gap -

(1) Based on band theory how energy bands are formed in solids and explain valance band, conduction band and energy gap ?

Consider a solid in crystalline form. Inside the crystal each electron has a unique position and no two electrons see exactly the same pattern of surrounding charges. Because of this, each electron will have a different *energy level*. These different energy levels with continuous energy variation form what are called *energy bands*. The energy band which includes the energy levels of the valence electrons is called the ***valence band***. The energy band above the valence band is called the ***conduction band***. With no external energy, all the valence electrons will reside in the valence band.

If there is some energy gap between the conduction band and the valence band, electrons in the valence band all remain bound and no free electrons are available in the conduction band. This makes the material an insulator. But some of the electrons from the valence band may gain external energy to cross the gap between the conduction band and the valence band. Then these electrons will move into the conduction band.
At the same time they will create vacant energy levels in the valence band where other valence electrons can move. Thus the process creates the possibility of conduction due to electrons in conduction band as well as due to vacancies in the valence band.

The gap between the top of the valence band and bottom of the conduction band is called the *energy band gap* (Energy gap E_g). It may be large, small, or zero, depending upon the material.

14.3 Classification of solids on the basis of energy bands :

(2) Write a note on classification of solids on the basis of energy bands.

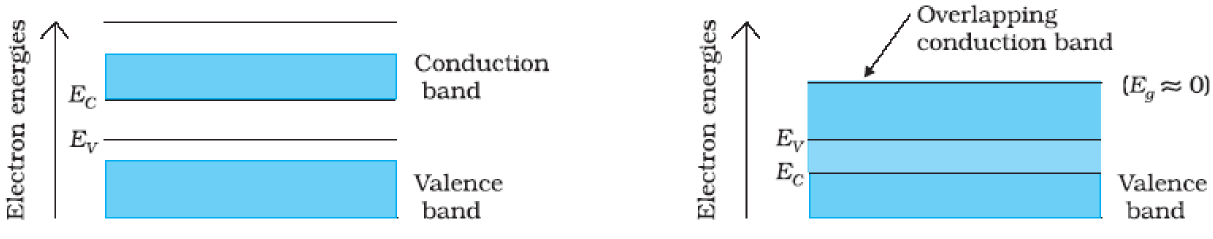
Various semiconductors available in practice could be:
(i) *Elemental semiconductors*: Si and Ge
(ii) *Compound semiconductors*: Examples are:
• Inorganic: CdS, GaAs, CdSe, InP, etc.
• Organic: anthracene, doped pthalocyanines, etc.
• Organic polymers: polypyrrole, polyaniline, polythiophene, etc.
Most of the currently available semiconductor devices are based on elemental semiconductors Si or Ge and compound *inorganic* semiconductors. However, after 1990, a few semiconductor devices using organic semiconductors and semiconducting polymers have been developed signaling the birth of a futuristic technology of polymer electronics and molecular-electronics.

Inside the crystal each electron has a unique position and no two electrons see exactly the same pattern of surrounding charges. Because of this, each electron will have a different *energy level*.

These different energy levels with continuous energy variation form what are called *energy bands*. The energy band which includes the energy levels of the valence electrons is called the *valence band*. The energy band above the valence band is called the *conduction band*. With no external energy, all the valence electrons will reside in the valence band.

(a) Metallic Conductors :

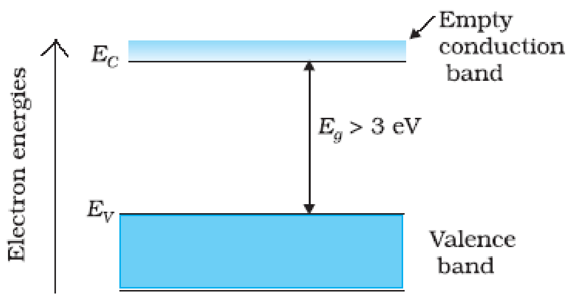
In case with metallic conductors, the lowest level in the conduction band happens to be lower than the highest level of the valence band, the electrons from the valence band can easily move into the conduction band. Normally the conduction band is empty. But when it overlaps on the valence band electrons can move freely into it. Some conductors, the conduction band is partially filled and the valence band is partially empty which allows electrons from its lower level to move to higher level making conduction possible.



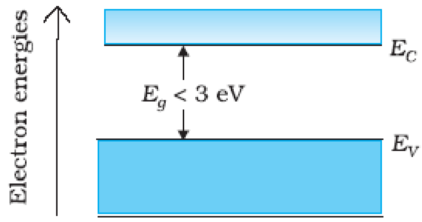
(i) (a) (ii)

(b) Insulators :

If there is some gap between the conduction band and the valence band, electrons in the valence band all remain bound and no free electrons are available in the conduction band. This makes the material an insulator (Fig. (b)).



(b)



(c)

(c) Semiconductors :

If there is some gap between the conduction band and the valence band, electrons in the valence band all remain bound and no free electrons are available in the conduction band. This makes the material an insulator. But some of the electrons from the valence band may gain external energy to cross the gap between the conduction band and the valence band. Then these electrons will move into the conduction band (Fig. (c)).

At the same time they will create vacant energy levels in the valence band where other valence electrons can move. Thus the process creates the possibility of conduction due to electrons in conduction band as well as due to vacancies in the valence band. Such materials are called semiconductors.

14.3 Semiconductors: Intrinsic semiconductors -

(3) Write a note on intrinsic semiconductors ?

Ge and Si are examples for semiconductors and have diamond like lattice structure. In its crystalline structure, every Si or Ge atom tends *to share* one of its four valence electrons with each of its four nearest neighbour atoms, and also *to take share* of one electron from each such neighbour. These shared electron pairs are referred to as forming a *covalent bond* or simply a *valence bond*.

As the temperature increases, more thermal energy becomes available to these electrons and some of these electrons may break–away (becoming *free* electrons contributing to conduction). The thermal energy effectively ionises only a few atoms in the crystalline lattice and creates a *vacancy* in the bond. The neighbourhood, from which the free electron (with charge $-q$) has come out leaves a vacancy with an effective charge ($+q$). This *vacancy* with the effective positive electronic charge is called a *hole*. The hole behaves as an *apparent free particle* with effective positive charge.

In intrinsic semiconductors, the number of free electrons, n_e is equal to the number of holes, n_h. That is $n_e = n_h = n_i$ where n_i is called intrinsic carrier concentration.

Under the action of an electric field, these holes move towards negative potential giving the hole current, I_h. The total current, I is thus the sum of the electron current Ie and the hole current I_h:

$I = I_e + I_h$ ----------- (14.2)

It may be noted that apart from the *process of generation* of conduction electrons and holes, a simultaneous *process of recombination* occurs in which the electrons *recombine* with the holes. At equilibrium, the rate of generation is equal to the rate of recombination of charge carriers. The recombination occurs due to an electron colliding with a hole.

14.4 Extrinsic semiconductors (p-type and n-type); p-n junction: p-n junction formation.

(4) Write a note on extrinsic semiconductors ?

The conductivity of an intrinsic semiconductor depends on its temperature, but at room temperature its conductivity is very low. As such, no important electronic devices can be developed using these semiconductors. Hence there is a necessity of improving their conductivity. This can be done by making use of impurities.

When a small amount, say, a few parts per million (ppm), of a suitable impurity is added to the pure semiconductor, the conductivity of the semiconductor is increased manifold. Such materials are known as e*xtrinsic semiconductors* or *impurity semiconductors*. The deliberate addition of a desirable impurity is called *doping* and the impurity atoms are called *dopants*. Such a material is also called a *doped semiconductor*.

The dopant has to be such that it does not distort the original pure semiconductor lattice. It occupies only a very few of the original semiconductor atom sites in the crystal. A necessary condition to attain this is that the sizes of the dopant and the semiconductor atoms should be nearly the same.

There are two types of dopants used in doping the tetravalent Si or Ge:

(i) Pentavalent (valency 5); like Arsenic (As), Antimony (Sb), Phosphorous (P), etc.

(ii) Trivalent (valency 3); like Indium (In), Boron (B), Aluminium (Al), etc.

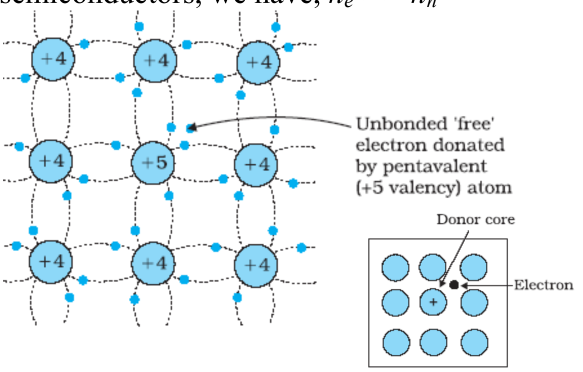
(i) n-type semiconductor

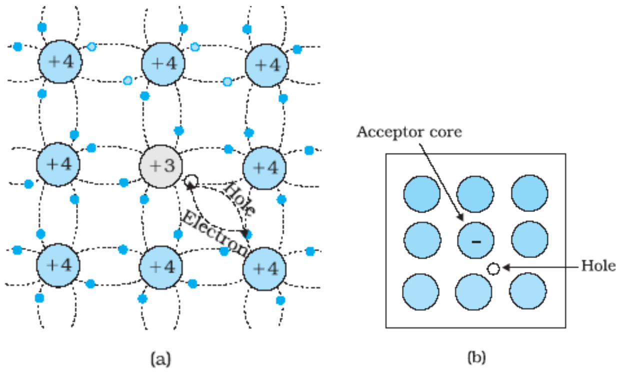
Suppose we dope Si or Ge with a pentavalent element When an atom of +5 valency element occupies the position of an atom in the crystal lattice of Si, four of its electrons bond with the four silicon neighbours while the fifth remains very weakly bound to its parent atom. This is because the four electrons participating in bonding are seen as part of the effective core of the atom by the fifth electron. As a result the ionisation energy required to set this electron free is very small and even at room temperature it will be free to move in the lattice of the semiconductor. For example, the energy required is ~ 0.01 eV for germanium, and 0.05 eV for silicon, to separate this electron from its atom. This is in contrast to the energy required to jump the forbidden band (about 0.72 eV for germanium and about 1.1 eV for silicon) at room temperature in the intrinsic semiconductor. Thus, the pentavalent dopant is donating one extra electron for conduction and hence is known as *donor* impurity. The number of electrons made available for conduction by dopant atoms depends strongly upon the doping level and is independent of any increase in ambient temperature.

On the other hand, the number of free electrons (with an equal number of holes) generated by Si atoms, increases weakly with temperature. In a doped semiconductor the total number of conduction electrons n_e is due to the electrons contributed by donors and those generated intrinsically, while the total number of holes n_h is only due to the holes from the intrinsic source. But the rate of recombination of holes would increase due to the increase in the number of electrons. As a result, the number of holes would get reduced further.

Thus, with proper level of doping the number of conduction electrons can be made much larger than the number of holes. Hence in an extrinsic semiconductor doped with pentavalent impurity, electrons become the *majority carriers* and holes the *minority carriers*.

These semiconductors are, therefore, known as n-*type semiconductors*. For n-type semiconductors, we have, $n_e >> n_h$





(ii) p-type semiconductor

This is obtained when Si or Ge is doped with a trivalent impurity like Al, B, In, etc. The dopant has one valence electron less than Si or Ge and, therefore, this atom can form covalent bonds with neighbouring three Si atoms but does not have any electron to offer to the fourth Si atom. So the bond between the fourth neighbour and the trivalent atom has a vacancy or hole as shown in Fig. 14.8. Since the neighbouring Si atom in the lattice wants an electron in place of a hole, an electron in the outer orbit of an atom in the neighbourhood may jump to fill this vacancy, leaving a vacancy or hole at its own site. Thus the *hole* is available for conduction. Note that the trivalent foreign atom becomes effectively negatively charged when it shares fourth electron with neighbouring Si atom. Therefore, the dopant atom of p-type material can be treated as *core of one negative charge* along with its associated hole as shown in Fig. 14.8(b). It is obvious that one *acceptor* atom gives one *hole*. These holes are in addition to the intrinsically generated holes while the source of conduction electrons is only intrinsic generation. Thus, for such a material, the holes are the majority carriers and electrons are minority carriers. Therefore, extrinsic semiconductors doped with trivalent impurity are called p-*type semiconductors*. For p-type semiconductors, the recombination process will reduce the number (n_i) of intrinsically generated electrons to n_e. We have, for p-type semiconductors

$n_h >> n_e$

Note that *the crystal maintains an overall charge neutrality as the charge of additional charge carriers is just equal and opposite to that of the ionised cores in the lattice.*

In extrinsic semiconductors, because of the abundance of majority current carriers, the minority carriers produced thermally have more chance of meeting majority carriers and thus getting destroyed. Hence, the dopant, by adding a large number of current carriers of one type, which become the majority carriers, indirectly helps to reduce the intrinsic concentration of minority carriers.

Energy bands in Extrinsic Semiconductors :

The semiconductor's energy band structure is affected by doping. In the case of extrinsic semiconductors, additional energy states due to donor impurities (E_D) and acceptor impurities (E_A) also exist.

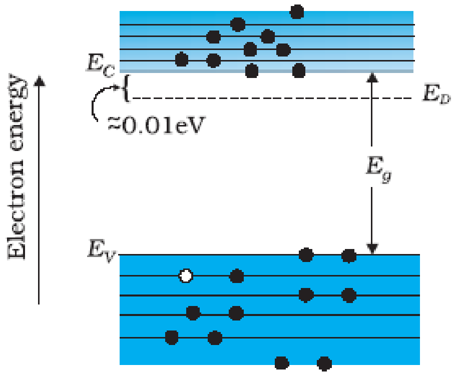
In n-type Si semiconductor, the donor energy level E_D is slightly below the bottom E_C of the conduction band and electrons from this level move into the conduction band with very small supply of energy. At room temperature, most of the donor atoms get ionised but very few atoms of Si get ionised. So the conduction band will have most electrons coming from the donor impurities, as shown in Fig. (a).

Similarly, for p-type semiconductor, the acceptor energy level E_A is slightly above the top E_V of the valence band as shown in Fig. (b).

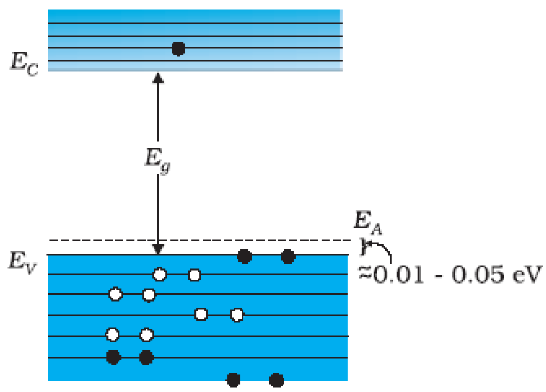
With very small supply of energy an electron from the valence band can jump to the level E_A and ionise the acceptor negatively, leaving holes in valance band. At room temperature, most of the acceptor atoms get ionised leaving holes in the valence band. Thus at room temperature the density of holes in the valence band is predominantly due to impurity in the extrinsic semiconductor.

The electron and hole concentration in a semiconductor *in thermal equilibrium* is given by

$n_e n_h = n_i^2$



(a) $T > 0K$
one thermally generated electron-hole pair + 9 electrons from donor atoms



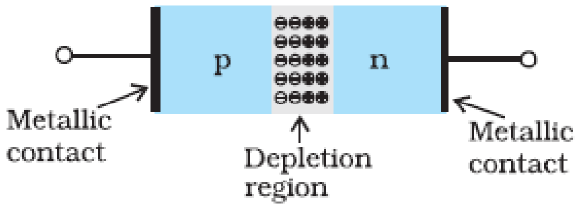
(b) $T > 0K$

Note that for C (diamond), Si and Ge, the energy gaps are 5.4 eV, 1.1 eV and 0.7 eV, respectively. Sn also is a group IV element but it is a metal because the energy gap is 0 eV.

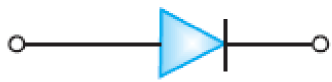
14.5 Semiconductor diode: Forward and reverse bias - I-V characteristics - Definitions of cut-in-voltage, breakdown voltage and reverse saturation current.

(5) Write a note on semiconductor diode ? Explain its I-V characteristics ?

A semiconductor diode [Fig. (a)] is basically a p-n junction with metallic contacts provided at the ends for the application of an external voltage. It is a two terminal device. A p-n junction diode is symbolically represented as shown in Fig. (b).
The direction of arrow indicates the conventional direction of current (when the diode is under forward bias). The equilibrium barrier potential can be altered by applying an external voltage V across the diode.



(a)

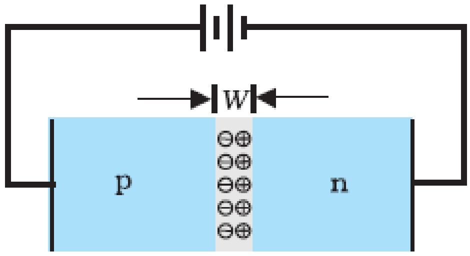

(b)

(1) p-n junction diode under forward bias :

When an external voltage V is applied across a semiconductor diode such that p-side is connected to the positive terminal of the battery and n-side to the negative terminal [Fig. 14.13(a)], it is said to be *forward biased.*
The applied voltage mostly drops across the depletion region and the voltage drop across the p-side and n-side of the junction is negligible. (This is because the resistance of the depletion region – a region where there are no charges – is very high compared to the resistance of n-side and p-side.) The direction of the applied voltage (V) is opposite to the built-in potential V_O. As a result, the depletion layer width decreases and the barrier height is reduced [Fig. 14.13(b)]. The effective barrier height under forward bias is ($V_O - V$).
If the applied voltage is small, the barrier potential will be reduced only slightly below the equilibrium value, and only a small number of carriers in the material—those that happen to be in the uppermost energy levels—will possess enough energy to cross the junction. So the current will be small. If we increase the applied voltage significantly, the barrier height will be reduced and more number of carriers will have the required energy. Thus the current increases. This applied voltage at which the diode current increases

significantly is called the ***threshold voltage*** or **cut-in voltage** (~0.2V for germanium diode and ~0.7 V for silicon diode).

Due to the applied voltage, electrons from n-side cross the depletion region and reach p-side (where they are minority carries). Similarly, holes from p-side cross the junction and reach the n-side (where they are minority carries). This process under forward bias is known as minority carrier injection. At the junction boundary, on each side, the minority carrier concentration increases significantly compared to the locations far from the junction.
Due to this concentration gradient, the injected electrons on p-side diffuse from the junction edge of p-side to the other end of p-side. Likewise, the injected holes on n-side diffuse from the junction edge of n-side to the other end of n-side (Fig. (c)). This motion of charged carriers on either side gives rise to current. The total diode forward current is sum of hole diffusion current and conventional current due to electron diffusion. The magnitude of this current is usually in mA.



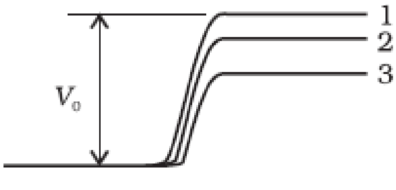
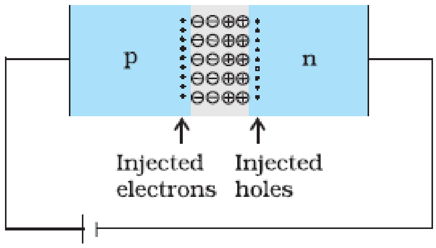
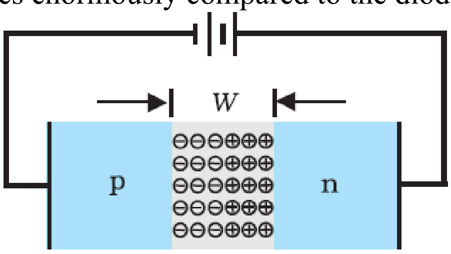


Fig. (a) p-n junction diode under forward bias, (b) Barrier potential (1) without battery, (2) Low battery voltage, and (3) High voltage battery.

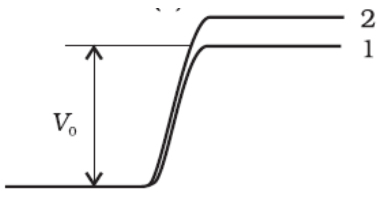
Fig. (c)



(2) p-n junction diode under reverse bias :
When an external voltage (V) is applied across the diode such that n-side is positive and p-side is negative, it is said to be *reverse biased* [Fig.14.15(a)]. The applied voltage mostly drops across the depletion region. The direction of applied voltage is same as the direction of barrier potential. As a result, the barrier height increases and the depletion region widens due to the change in the electric field. The effective barrier height under reverse bias is ($V_O + V$), [Fig. (b)].
This suppresses the flow of electrons from n → p and holes from p → n. Thus, diffusion current, decreases enormously compared to the diode under forward bias.



(a)



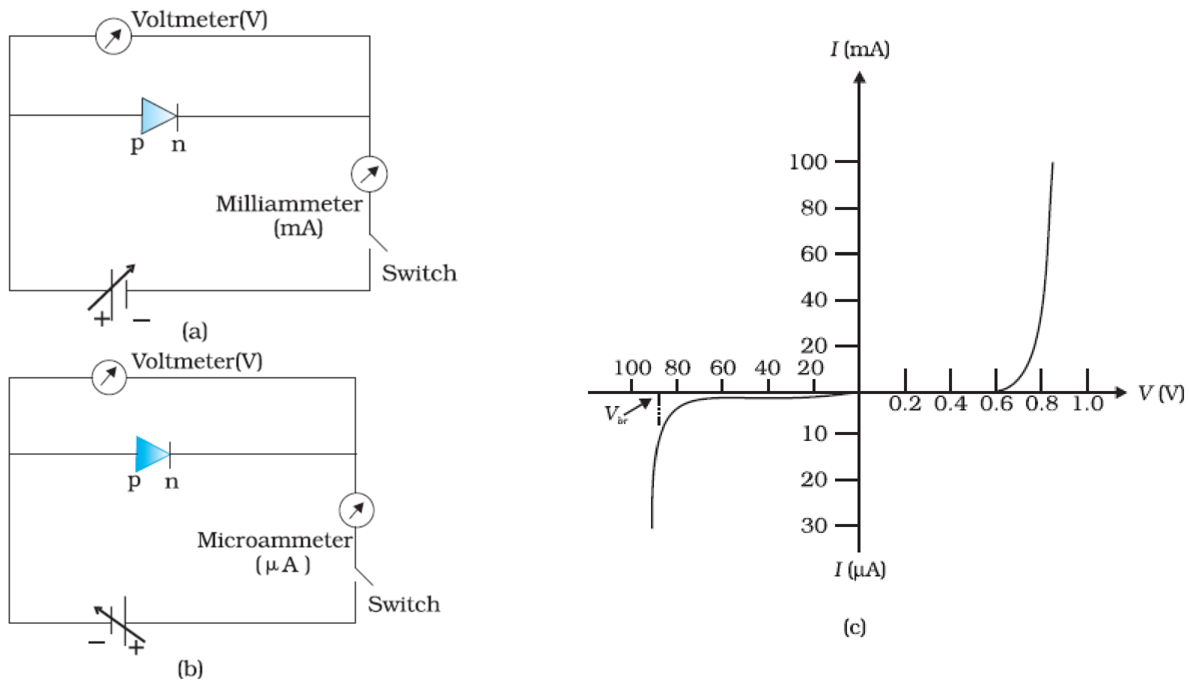
(b)

The electric field direction of the junction is such that if electrons on p-side or holes on n-side in their random motion come close to the junction, they will be swept to its majority zone. This

drift of carriers gives rise to current. The drift current is of the order of a few µA. This is quite low because it is due to the motion of carriers from their minority side to their majority side across the junction. The drift current is also there under forward bias but it is negligible (µA) when compared with current due to injected carriers which is usually in mA.
The diode reverse current is not very much dependent on the applied voltage. Even a small voltage is sufficient to sweep the minority carriers from one side of the junction to the other side of the junction. The current is not limited by the magnitude of the applied voltage but is limited due to the concentration of the minority carrier on either side of the junction.

The current under reverse bias is essentially voltage independent upto a critical reverse bias voltage, known as **breakdown voltage** (V_{br}). When $V = V_{br}$, the diode reverse current increases sharply. Even a slight increase in the bias voltage causes large change in the current. If the reverse current is not limited by an external circuit below the rated value (specified by the manufacturer) the p-n junction will get destroyed. Once it exceeds the rated value, the diode gets destroyed due to overheating. This can happen even for the diode under forward bias, if the forward current exceeds the rated value.

The circuit arrangement for studying the *V-I* characteristics of a diode, (i.e., the variation of current as a function of applied voltage) are shown in Fig. 14.16(a) and (b). The battery is connected to the diode through a potentiometer (or reheostat) so that the applied voltage to the diode can be changed. For different values of voltages, the value of the current is noted. A graph between *V* and *I* is obtained as in Fig. 14.16(c). Note that in forward bias measurement, we use a milliammeter since the expected current is large (as explained in the earlier section) while a micrometer is used in reverse bias to measure the current.



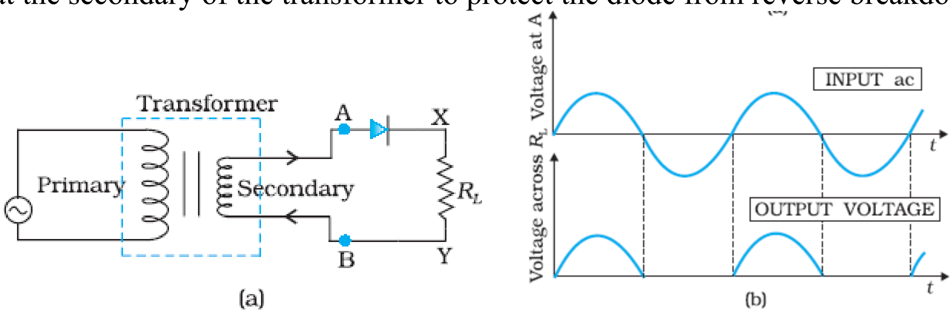
14.6 Diode as a rectifier: Circuit diagram, working, input and output waveforms of a) half-wave rectifier and (b) full-wave rectifier.

(6) With circuit diagrams explain the working of half-wave and full-wave rectifiers ?

A P-N Junction diode allows current to pass only when it is forward biased. So if an alternating voltage is applied across a diode the current flows only in that part of the cycle when the diode is forward biased. This property is used *to rectify* alternating voltages and the circuit used for this purpose is called a *rectifier*.

(a) Half-wave Rectifier :

If an alternating voltage is applied across a diode in series with a load, a pulsating voltage will appear across the load only during the half cycles of the ac input during which the diode is forward biased. Such rectifier circuit, as shown in Fig. (a), is called a *half-wave rectifier*. The secondary of a transformer supplies the desired ac voltage across terminals A and B. When the voltage at A is positive, the diode is forward biased and it conducts. When A is negative, the diode is reverse-biased and it does not conduct. The reverse saturation current of a diode is negligible and can be considered equal to zero for practical purposes.
(The reverse breakdown voltage of the diode must be sufficiently higher than the peak ac voltage at the secondary of the transformer to protect the diode from reverse breakdown.)



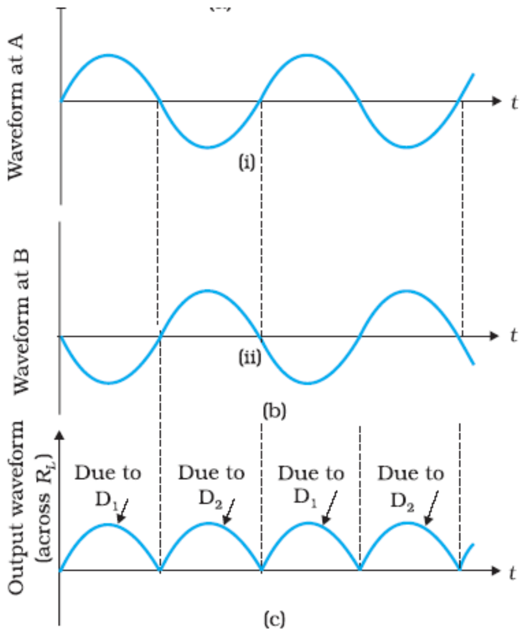
(a) (b)

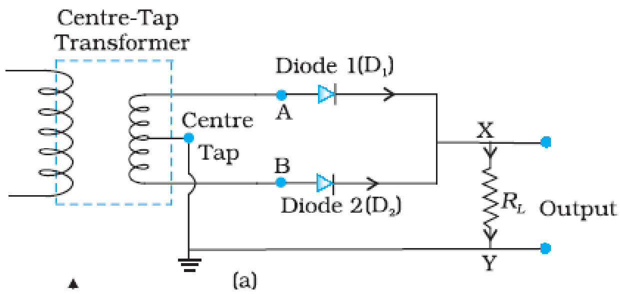
Therefore, in the positive *half-cycle* of ac there is a current through the load resistor R_L and we get an output voltage, as shown in Fig. (b), whereas there is no current in the negative halfcycle. In the next positive half-cycle, again we get the output voltage. Thus, the output voltage, though still varying, is restricted to *only one direction* and is said to be *rectified*. Since the rectified output of this circuit is only for half of the input ac wave it is called as *half-wave rectifier*.

(b) Full-wave Rectifier :

Full-wav rectifier is the circuit that uses two diodes, shown in Fig. (a), gives output rectified voltage corresponding to both the positive as well as negative half of the ac cycle.

Here the p-side of the two diodes are connected to the ends of the secondary of the transformer. The n-side of the diodes are connected together and the output is taken between this common point of diodes and the midpoint of the secondary of the transformer. So for a full-wave rectifier the secondary of the transformer is provided with a centre tapping and so it is called *centre-tap transformer*. As can be seen from Fig. (c) the voltage rectified by each diode is only half the total secondary voltage.





Each diode rectifies only for half the cycle, but the two do so for alternate cycles. Thus, the output between their common terminals and the centretap of the transformer becomes a full-wave rectifier output. (Note that there is another circuit of full wave rectifier which does not need a centretap transformer but needs four diodes.) Suppose the input voltage to A with respect to the centre tap at any instant is positive. It is clear that, at that instant, voltage at B being out of phase will be negative as shown in Fig.(b). So, diode D_1 gets forward biased and conducts (while D_2 being reverse biased is not conducting). Hence, during this positive half cycle we get an output current (and a output voltage across the load resistor R_L) as shown in Fig.(c). In the course of the ac cycle when the voltage at A becomes negative with respect to centre tap, the voltage at B would be positive. In this part of the cycle diode D_1 would not conduct but diode D_2 would, giving an output current and output voltage (across R_L) during the negative half cycle of the input ac. Thus, we get output voltage during both the positive as well as the negative half of the cycle. Obviously, this is a more efficient circuit for getting rectified voltage or current than the halfwave rectifier.

The rectified voltage is in the form of pulses of the shape of half sinusoids. Though it is unidirectional it does not have a steady value. To get steady dc output from the pulsating voltage normally a capacitor is connected across the output terminals (parallel to the load R_L). One can also use an inductor in series with R_L for the same purpose. Since these additional circuits appear to *filter* out the *ac ripple* and give a *pure dc* voltage, so they are called filters.

A capacitor can be used to filter the ripple in output of rectifier. When the voltage across the capacitor is rising, it gets charged. If there is no external load, it remains charged to the peak voltage of the rectified output. When there is a load, it gets discharged through the load and the voltage across it begins to fall. In the next half-cycle of rectified output it again gets charged to the peak value (Fig.). The rate of fall of the voltage across the capacitor depends upon the inverse product of capacitor C and the effective resistance R_L used in the circuit and is called the *time constant*. To make the time constant large value of C should be large. So capacitor input filters use large capacitors. The *output voltage* obtained by using capacitor input filter is nearer to the *peak voltage* of the rectified voltage. This type of filter is most widely used in power supplies.

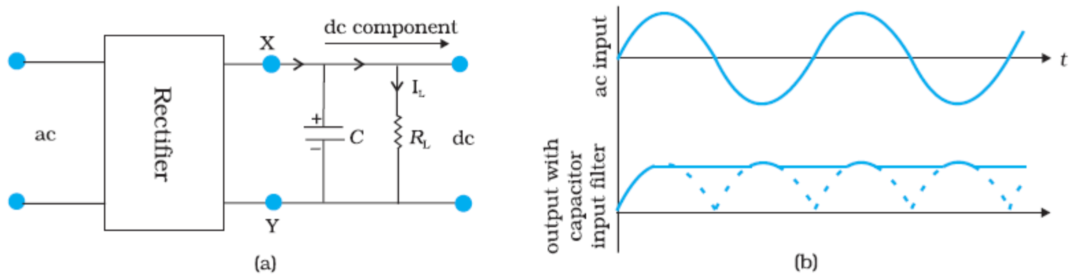


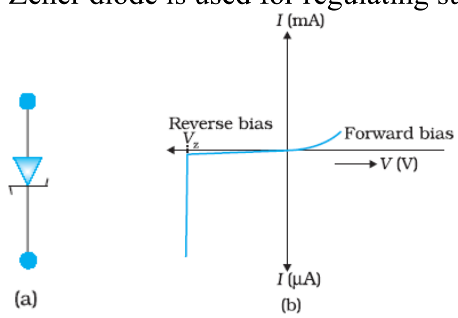
Fig. (a) A full-wave rectifier with capacitor filter, (b) Input and output voltage of rectifier in (a).

14.7 Zener diode: I-V characteristics - Zener diode as a voltage regulator.

(7) Explain the fabrication and I-V characteristics of a Zener diode. How it is used for voltage regulation ?

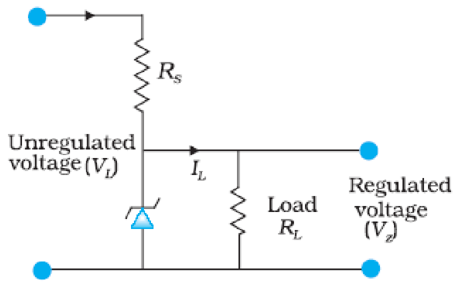
It is a special purpose semiconductor diode, named after its inventor C. Zener. It is designed to operate under reverse bias in the breakdown region and used as a voltage regulator. The symbol for Zener diode is shown in Fig. 14.21(a).

Zener diode is fabricated by heavily doping both p-, and n- sides of the junction. Due to this, depletion region formed is very thin ($<10^{-6}$ m) and the electric field of the junction is extremely high ($\sim5\times10^{6}$ V/m) even for a small reverse bias voltage of about 5V. The I-V characteristics of a Zener diode is shown in Fig. 14.21(b). It is seen that when the applied reverse bias voltage(V) reaches the breakdown voltage (Vz) of the Zener diode, there is a large change in the current. Note that after the breakdown voltage Vz, a large change in the current can be produced by almost insignificant change in the reverse bias voltage. In other words, Zener voltage remains constant, even though current through the Zener diode varies over a wide range. This property of the Zener diode is used for regulating supply voltages so that they are constant.



Let us understand how reverse current suddenly increases at the breakdown voltage. We know that reverse current is due to the flow of electrons (minority carriers) from p → n and holes from n → p. As the reverse bias voltage is increased, the electric field at the junction becomes significant. When the reverse bias voltage V = V_z, then the electric field strength is high enough to pull valence electrons from the host atoms on the p-side which are accelerated to n-side. These electrons account for high current observed at the breakdown. The emission of electrons from the host atoms due to the high electric field is known as internal field emission or field ionisation. The electric field required for field ionisation is of the order of 10^{6} V/m.

Zener diode as a voltage regulator



When the ac input voltage of a rectifier fluctuates, its rectified output also fluctuates. To get a constant dc voltage from the dc unregulated output of a rectifier, we use a Zener diode. The circuit diagram of a voltage regulator using a Zener diode is shown in Fig.

The unregulated dc voltage (filtered output of a rectifier) is connected to the Zener diode through a series resistance R_s such that the Zener diode is reverse biased. If the input voltage increases, the current through R_s and Zener diode also increases. This increases the voltage drop across R_s without any change in the voltage across the Zener diode. This is because in the breakdown region, Zener voltage remains constant even though the current through the Zener diode changes. Similarly, if the input voltage decreases, the current through R_s and Zener diode also decreases. The voltage drop across R_s decreases without any change in the voltage across the Zener diode. Thus any increase/decrease in the input voltage results in, increase/decrease of the voltage drop across R_s without any change in voltage across the Zener diode. Thus the Zener diode acts as a voltage regulator. We have to select the Zener diode according to the required output voltage and accordingly the series resistance R_s.

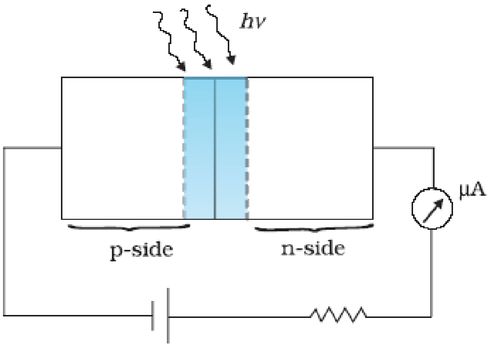
14.8 Optoelectronic junction devices: Working principles and mention of applications of photodiode, LED and solar cell.

(8) Write a note on construction, working and applications of following opto-electronic devices : (i) photodiode, (ii) LED and (iii) solar cell ?

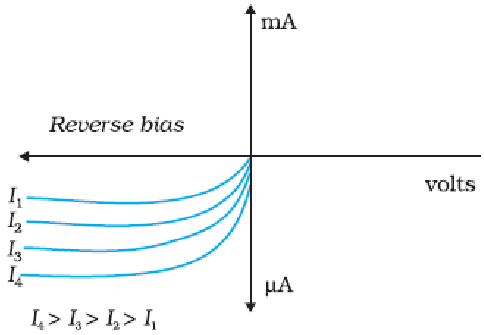
(i) *Photodiode*

A Photodiode is again a special purpose p-n junction diode fabricated with a transparent window to allow light to fall on the diode. It is operated under reverse bias. When the photodiode is illuminated with light (photons) with energy (h_n) greater than the energy gap (E_g) of the semiconductor, then electron-hole pairs are generated due to the absorption of photons. The diode is fabricated such that the generation of *e-h* pairs takes place in or near the depletion region of the diode. Due to electric field of the junction, electrons and holes are separated before they recombine. The direction of the electric field is such that electrons reach n-side and holes reach p-side.

Electrons are collected on n-side and holes are collected on p-side giving rise to an emf. When an external load is connected, current flows. The magnitude of the photocurrent depends on the intensity of incident light (photocurrent is proportional to incident light intensity).



(a)



(b)

Uses : It is easier to observe the change in the current with change in the light intensity, if a reverse bias is applied. Thus photodiode can be used as a photodetector to detect optical signals. The circuit diagram used for the measurement of *I-V* characteristics of a photodiode is shown in Fig. (a) and a typical *I-V* characteristics in Fig. (b).

(ii) *Light emitting diode*

It is a heavily doped p-n junction which under forward bias emits spontaneous radiation. The diode is encapsulated with a transparent cover so that emitted light can come out.

When the diode is forward biased, electrons are sent from n → p (where they are minority carriers) and holes are sent from p → n (where they are minority carriers). At the junction boundary the concentration of minority carriers increases compared to the equilibrium concentration (i.e., when there is no bias). Thus at the junction boundary on either side of the junction, excess minority carriers are there which recombine with majority carriers near the junction. On recombination, the energy is released in the form of photons. Photons with energy equal to or slightly less than the band gap are emitted. When the forward current of the diode is small, the intensity of light emitted is small. As the forward current increases, intensity of light increases and reaches a maximum. Further increase in the forward current results in decrease of light intensity. LEDs are biased such that the light emitting efficiency is maximum.

The *V-I* characteristics of a LED is similar to that of a Si junction diode. But the threshold voltages are much higher and slightly different for each colour. The reverse breakdown voltages of LEDs are very low, typically around 5V. So care should be taken that high reverse voltages do not appear across them.

LEDs that can emit red, yellow, orange, green and blue light are commercially available. The semiconductor used for fabrication of visible LEDs must at least have a band gap of 1.8 eV (spectral range of visible light is from about 0.4 μm to 0.7 μm, i.e., from about 3 eV to 1.8 eV). The compound semiconductor Gallium Arsenide – Phosphide ($GaAs_{1-x}P_x$) is used for making LEDs of different colours. $GaAs_{0.6}\,P_{0.4}$ ($E_g \sim 1.9$ eV) is used for red LED. GaAs ($E_g \sim 1.4$ eV) is used for making infrared LED.

Uses : LEDs find extensive use in remote controls, burglar alarm systems, optical communication, etc. Extensive research is being done for developing white LEDs which can replace incandescent lamps.

LEDs have the following advantages over conventional incandescent low power lamps:

(i) Low operational voltage and less power.
(ii) Fast action and no warm-up time required.
(iii) The bandwidth of emitted light is 100 Å to 500 Å or in other words it is nearly (but not exactly) monochromatic.
(iv) Long life and ruggedness.
(v) Fast on-off switching capability.

(iii) *Solar cell*

A solar cell is basically a p-n junction which generates emf when solar radiation falls on the p-n junction. It works on the same principle (photovoltaic effect) as the photodiode, except that no external bias is applied and the junction area is kept much larger for solar radiation to be incident because to get more power.
A simple p-n junction solar cell is shown in Fig.
A p-Si wafer of about 300 µm is taken over which a thin layer (~0.3 µm) of n-Si is grown on one-side by diffusion process. The other side of p-Si is coated with a metal (back contact). On the top of n-Si layer, metal finger electrode (or metallic grid) is deposited. This acts as a front contact. The metallic grid occupies only a very small fraction of the cell area (<15%) so that light can be incident on the cell from the top.

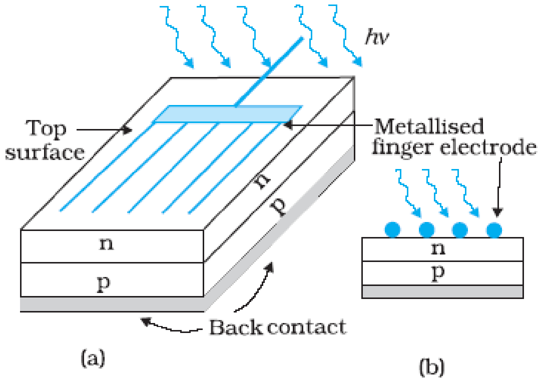
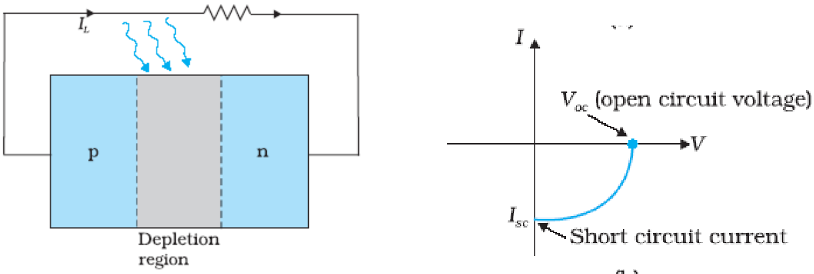


Fig. (a) Typical p-n junction solar cell; (b) Cross-sectional view.

The generation of emf by a solar cell, when light falls on, it is due to the following three basic processes: generation, separation and collection—
(i) generation of e-h pairs due to light (with $h_n > E_g$) close to the junction;
(ii) separation of electrons and holes due to electric field of the depletion region. Electrons are swept to n-side and holes to p-side;
(iii) the electrons reaching the n-side are collected by the front contact and holes reaching p-side are collected by the back contact. Thus p-side becomes positive and n-side becomes negative giving rise to *photovoltage*.

When an external load is connected as shown in the Fig. 14.25(a) a photocurrent *IL* flows through the load. A typical *I-V* characteristics of a solar cell is shown in the Fig. 14.25(b).
Note that the $I - V$ characteristics of solar cell is drawn in the fourth quadrant of the coordinate axes. This is because a solar cell does not draw current but supplies the same to the load.



(a) (b)

Semiconductors with band gap close to 1.5 eV are ideal materials for solar cell fabrication. Solar cells are made with semiconductors like Si (Eg = 1.1 eV), GaAs (E_g = 1.43 eV), CdTe (E_g = 1.45 eV), $CuInSe_2$ (E_g = 1.04 eV), etc. The important criteria for the selection of a material for solar cell fabrication are (i) band gap (~1.0 to 1.8 eV), (ii) high optical absorption (~104 cm–1), (iii) electrical conductivity, (iv) availability of the raw material, and (v) cost. Note that sunlight is not always required for a solar cell. *Any* light with photon energies greater than the bandgap will do.

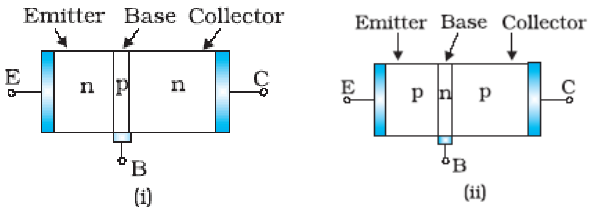
Uses : Solar cells are used to power electronic devices in satellites and space vehicles and also as power supply to some calculators. Production of low-cost photovoltaic cells for large-scale solar energy is a topic for research.

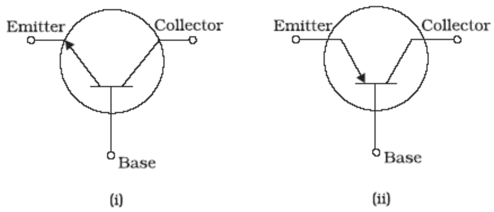
14.9 Junction transistor: Types of transistor :

(9) What is a transistor ? Explain different types of transistors ?

Transistor is invented in the year 1947 by J. Bardeen and W.H. Brattain of Bell Telephone Laboratories, U.S.A. That transistor was a point-contact transistor. The first junction transistor consisting of two back-to-back p-n junctions was invented by William Schockley in 1951. The junction transistor is also called Bipolar Junction Transistor (BJT).

A transistor has three doped regions forming two p-n junctions between them. Obviously, there are two types of transistors, as shown in Fig.





(i) n-p-n transistor **:** Here two segments of n-type semiconductor (emitter and collector) are separated by a segment of p-type semiconductor (base).
(ii) p-n-p transistor **:** Here two segments of p-type semiconductor (termed as emitter and collector) are separated by a segment of n-type semiconductor (termed as base).
The schematic representations of an n-p-n and a p-n-p configuration are shown in Fig. (a). All the three segments of a transistor have different thickness and their doping levels are also different. In the schematic symbols used for representing p-n-p and n-p-n transistors [Fig. (b)] the arrowhead shows the direction of conventional current in the transistor. A brief description of the three segments of a transistor is given below:

• ***Emitter*** **:** This is the segment on one side of the transistor shown in Fig. (a). It is of *moderate size* and *heavily doped*. It supplies a large number of majority carriers for the current flow through the transistor.
• ***Base*** **:** This is the central segment. *It is very thin* and *lightly doped*.
• ***Collector*** **:** This segment collects a *major* portion of the majority carriers supplied by the emitter. The collector side is *moderately doped* and *larger* in size as compared to the *emitter*.

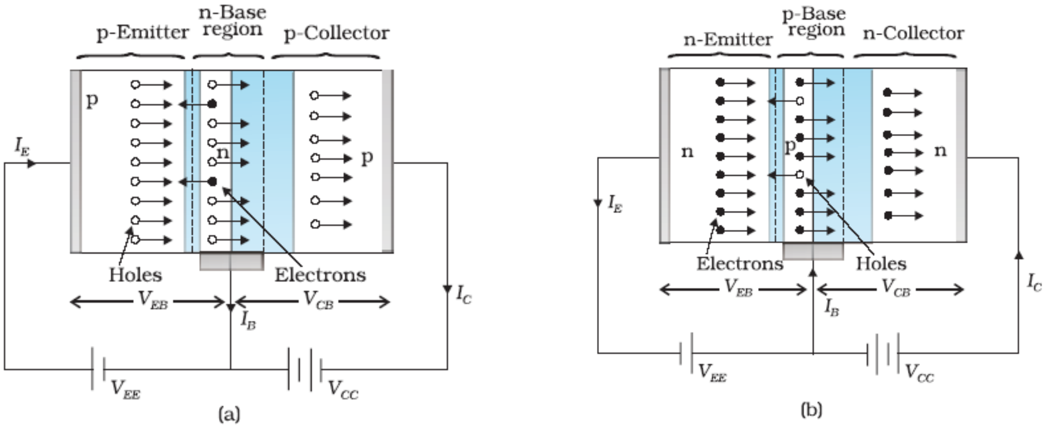
Depletion regions are formed at the emitter-base junction and the base-collector junction of a transistor.

14.10 Transistor action - Common emitter characteristics of a transistor: Drawing of input and output characteristics :

(10) Explain transistor action of an NPN type.

In case of a transistor depletion regions are formed at the emitter-base junction and the base-collector junction. For understanding the action of a transistor, we have to consider the nature of depletion regions formed at these junctions. The charge carriers move across different regions of the transistor when proper voltages are applied across its terminals. By means of two distinct ways of biasing a transistor can be used for two purposes. It can be used as an amplifier, a device which produces a enlarged copy of an ac signal mainly used in communication electronics. Also it can be used as a switch in digital electronics.

The transistor works as an amplifier, with its emitter-base junction forward biased and the base-collector junction reverse biased. This situation is shown in Fig. 14.28, where V_{CC} and V_{EE} are used for creating the respective biasing. When the transistor is biased in this way it is said to be in *active* state. We represent the voltage between emitter and base as V_{EB} and that between the collector and the base as V_{CB}. In Fig. 14.28, base is a common terminal for the two power supplies whose other terminals are connected to emitter and collector, respectively. So the two power supplies are represented as V_{EE}, and V_{CC}, respectively. In circuits, where emitter is the common terminal, the power supply between the base and the emitter is represented as V_{BB} and that between collector and emitter as V_{CC}.



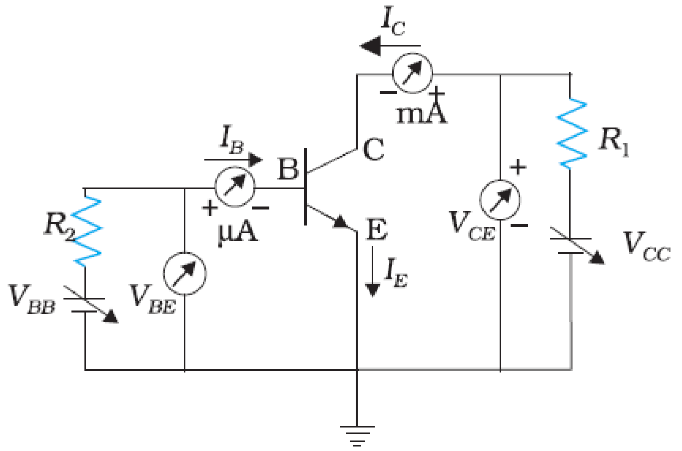
Let us see now the paths of current carriers in the transistor with emitter-base junction forward biased and base-collector junction reverse biased. The heavily doped emitter has a high concentration of majority carriers, which will be holes in a p-n-p transistor and electrons in an n-p-n transistor.

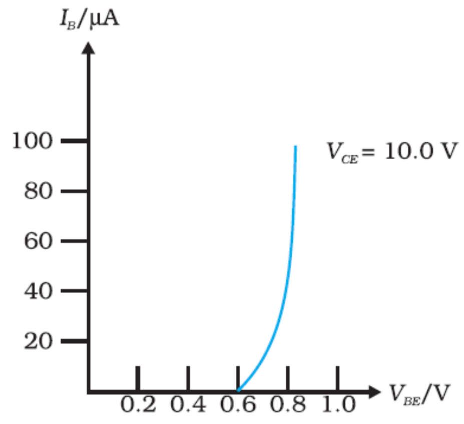
These majority carriers enter the base region in large numbers. The base is thin and lightly doped. So the majority carriers there would be few. In a p-n-p transistor the majority carriers in the base are electrons since base is of n-type semiconductor. The large number of holes entering the base from the emitter swamps the small number of electrons there. As the base collector-junction is reverse biased, these holes, which appear as minority carriers at the junction, can easily cross the junction and enter the collector. The holes in the base could move either towards the base terminal to combine with the electrons entering from outside or cross the junction to enter into the collector and reach the collector terminal. The base is made thin so that most of the holes find themselves near the reverse-biased base-collector junction and so cross the junction instead of moving to the base terminal.

It is interesting to note that due to forward bias a large current enters the emitter-base junction, but most of it is diverted to adjacent reverse-biased base-collector junction and the current coming out of the base becomes a very small fraction of the current that entered the junction. If we represent the hole current and the electron current crossing the forward biased junction by I_h and I_e respectively then the total current in a forward biased diode is the sum $I_h + I_e$. We see that the emitter current $I_E = I_h + I_e$ but the base current $I_B << I_h + I_e$, because a major part of I_E goes to collector instead of coming out of the base terminal. The base current is thus a small fraction of the emitter current.
The current entering into the emitter from outside is equal to the emitter current I_E. Similarly the current emerging from the base terminal is I_B and that from collector terminal is I_C. It is obvious from the above description and also from a straight forward application of Kirchhoff's law to Fig. 14.28(a) that the emitter current is the sum of collector current and base current:
$I_E = I_C + I_B$ ------------ (14.7)
We also see that $I_C \gg I_E$.

(11) With circuit diagram draw Common emitter characteristics of a transistor ?
When a transistor is used in CE configuration, the input is between the base and the emitter and the output is between the collector and the emitter. The variation of the base current I_B with the base-emitter voltage V_{BE} is called the *input characteristic*. Similarly, the variation of the collector current I_C with the collector-emitter voltage V_{CE} is called the *output characteristic*.

The input and the output characteristics of an n-p-n transistors can be studied by using the circuit shown in Fig. 14.29.

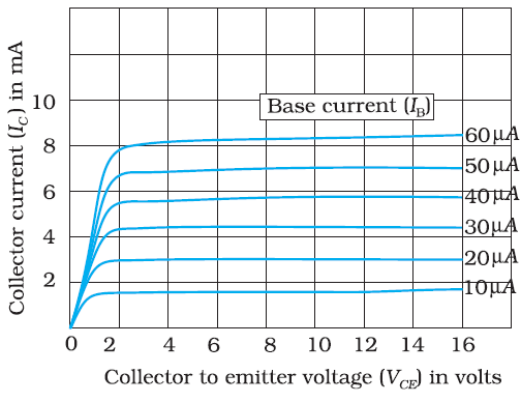




(a)

To study the input characteristics of the transistor in *CE* configuration, a curve is plotted between the base current I_B against the base-emitter voltage V_{BE}. The collector-emitter voltage V_{CE} is kept fixed while studying the dependence of I_B on V_{BE}. We are interested to obtain the input characteristic when the transistor is in active state. So the collector-emitter voltage V_{CE} is kept large enough to make the base collector junction reverse biased. Since $V_{CE} = V_{CB} + V_{BE}$ and for Si transistor V_{BE} is 0.6 to 0.7 V, V_{CE} must be sufficiently larger than 0.7 V. Since the transistor is operated as an amplifier over large range of V_{CE}, the reverse bias across the base-collector junction is high most of the time.

Therefore, the input characteristics may be obtained for V_{CE} somewhere in the range of 3 V to 20 V. Since *the increase in* V_{CE} *appears as increase in* V_{CB}, *its effect on* I_B *is negligible*. As a consequence, input characteristics for various values of V_{CE} will give almost identical curves. Hence, *it is enough to determine only one input characteristics*. The input characteristics of a transistor is as shown in Fig. 14.30(a).



(b)

The output characteristic is obtained by observing the variation of I_C as V_{CE} is varied keeping I_B constant. It is obvious that if V_{BE} is increased by a small amount, both hole current from the emitter region and the electron current from the base region will increase. As a consequence both I_B and I_C will increase proportionately. This shows that when I_B increases I_C also increases. The plot of I_C versus V_{CE} for different fixed values of I_B gives one output characteristic. So there will be different output characteristics corresponding to different values of I_B as shown in Fig. (b).

14.11 Definitions of input resistance, output resistance and current amplification factor.

(12) Define input resistance, output resistance and current amplification factor of an amplifier ?

The linear segments of both the input and output characteristics can be used to calculate some important ac parameters of transistors as shown below.

(i) *Input resistance* (r_i) : This is defined as the ratio of change in base-emitter voltage (ΔV_{BE}) to the resulting change in base current (ΔI_B) at constant collector-emitter voltage (V_{CE}). This is dynamic (ac resistance) and as can be seen from the input characteristic, its value varies with the operating current in the transistor:

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$$

The value of r_i can be anything from a few hundreds to a few thousand ohms.

(ii) *Output resistance* (r_o): This is defined as the ratio of change in collector-emitter voltage (ΔV_{CE}) to the change in collector current (ΔI_C) at a constant base current I_B.

$$r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B}$$

The output characteristics show that initially for very small values of V_{CE}, I_C increases almost linearly. This happens because the base-collector junction is not reverse biased and the transistor is not in active state. In fact, the transistor is in the saturation state and the current is controlled by the supply voltage V_{CC} (=V_{CE}) in this part of the characteristic. When V_{CE} is more than that required to reverse bias the base-collector junction, I_C increases very little with V_{CE}. The reciprocal of the slope of the linear part of the output characteristic gives the values of r_o. The output resistance of the transistor is mainly controlled by the bias of the base-collector junction. The high magnitude of the output resistance (of the order of 100 kW) is due to the reverse-biased state of this diode. This also explains why the resistance at the initial part of the characteristic, when the transistor is in saturation state, is very low.

(iii) *Current amplification factor* (β): This is defined as the ratio of the change in collector current to the change in base current at a constant collector-emitter voltage (V_{CE}) when the transistor is in active state.

$$\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}}$$

This is also known as *small signal current gain* and its value is very large.
If we simply find the ratio of I_C and I_B we get what is called dc b of the transistor. Hence,

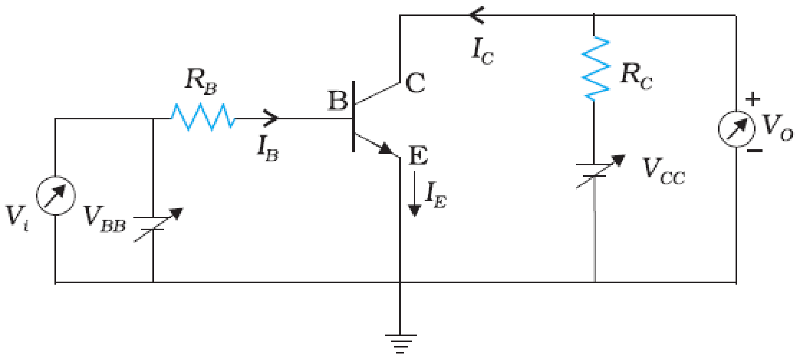
$$\beta_{dc} = \frac{I_C}{I_B}$$

Since I_C increases with I_B almost linearly and $I_C = 0$ when $I_B = 0$, the values of both β_{dc} and β_{ac} are nearly equal. So, for most calculations β_{dc} can be used. Both β_{ac} and β_{dc} vary with V_{CE} and I_B (or I_C) slightly.

14.12 Transistor as a switch: Circuit diagram and working :

(13) With circuit diagram, explain the function of a transistor as switch ?

When the transistor is used in the cutoff or saturation state it acts as a *switch*. On the other hand for using the transistor as an *amplifier*, it has to operate in the active region.
When transistor is in cut-off region it acts like open switch and when it is in saturation region, it acts like closed switch.



Let us understand the operation of the transistor as a switch by analysing the behaviour of the base-biased transistor in CE configuration as shown in Fig. (a).

Applying Kirchhoff's voltage rule to the input and output sides of this circuit, we get

$V_{BB} = I_B R_B + V_{BE}$ ---------- (14.12)

and

$V_{CE} = V_{CC} - I_C R_C.$ ------------ (14.13)

We shall treat V_{BB} as the dc input voltage V_i and V_{CE} as the dc output voltage V_O. So, we have

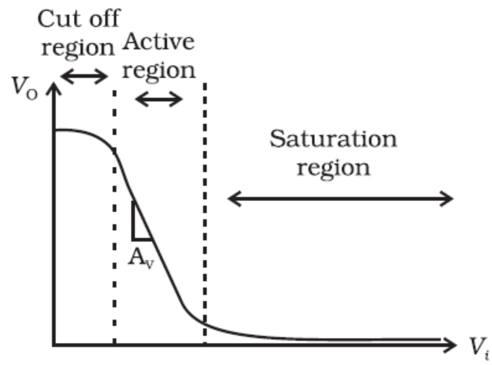
$V_i = I_B R_B + V_{BE}$ and

$V_o = V_{CC} - I_C R_C.$

Let us see how Vo changes as V_i increases from zero onwards. In the case of Si transistor, as long as input Vi is less than 0.6 V, the transistor will be in cut off state and current I_C will be zero. Hence $V_o = V_{CC}$

When V_i becomes greater than 0.6 V the transistor is in active state with some current I_C in the output path and the output V_o decrease as the term $ICRC$ increases. With increase of V_i , I_C increases almost linearly and so V_0 decreases linearly till its value becomes less than about 1.0 V. Beyond this, the change becomes non linear and transistor goes into saturation state. With further increase in Vi the output voltage is found to decrease further towards zero though it may never become zero.

If we plot the V_o vs V_i curve, we see that between cut off state and active state and also between active state and saturation state there are regions of non-linearity showing that the transition from cutoff state to active state and from active state to saturation state are not sharply defined.



(b)

Let us see now how the transistor is operated as a switch. As long as V_i is *low* and unable to forward-bias the transistor, Vo is *high* (at V_{CC}). If Vi is *high* enough to drive the transistor into saturation, then V_o is *low*, very near to zero. When the transistor is not conducting it is said to be *switched off* and when it is driven into saturation it is said to be *switched on*.

14.13 Transistor as an amplifier (CE - configuration): Circuit diagram and working -

(14) With principle, circuit diagram explain the working of a CE amplifier. Obtain expressions for current gain and voltage gain ?

Principle : To operate the transistor as an amplifier it is necessary to bias it in active region and fix its operating point (Q-point) somewhere in the middle of its active region. For this, base-emitter junction should be forward biased and collector - emitter junction should be reverse biased. If we fix the value of V_{BB} corresponding to a point in the middle of the linear part of the transfer curve then the dc base current I_B would be constant and corresponding collector current I_C will also be constant. The dc voltage $V_{CE} = V_{CC} - I_C R_C$ would also remain constant. The operating values of V_{CE} and I_B determine the operating point, of the amplifier.

If a small sinusoidal voltage with amplitude v_s is superposed on the dc base bias by connecting the source of that signal in series with the V_{BB} supply, then the base current will have sinusoidal variations superimposed on the value of I_B. As a consequence the collector current also will have sinusoidal variations superimposed on the value of I_C, producing in turn corresponding change in the value of v_o. We can measure the ac variations across the input and output terminals by blocking the dc voltages by large capacitors.

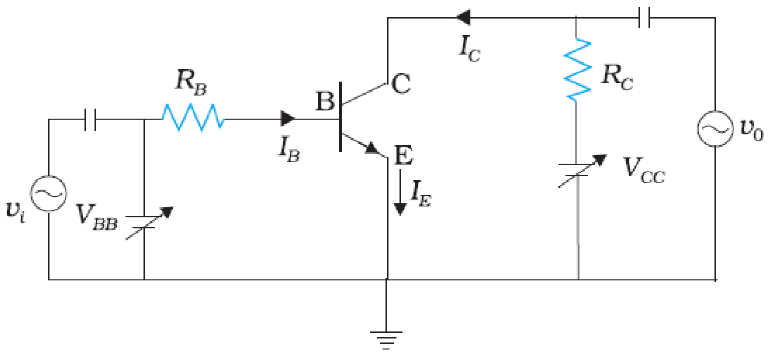


Fig. : A simple circuit of a CE-transistor amplifier.

In general, amplifiers are used to amplify alternating signals. Now let us superimpose an ac input signal v_i (to be amplified) on the bias V_{BB} (dc) as shown in Fig. The output is taken between the collector and the ground.

Working : if we first assume that $v_i = 0$. Then applying Kirchhoff's law to the output loop, we get

$Vcc = V_{CE} + IcR_L$ -------- (14.15)

Likewise, the input loop gives

$V_{BB} = V_{BE} + I_B R_B$ ------ (14.16)

When v_i is not zero, we get

$V_{BE} + v_i = V_{BE} + I_B R_B + \Delta I_B (R_B + r_i)$

The change in V_{BE} can be related to the input resistance r_i [see Eq. (14.8)] and the change in I_B. Hence

$v_i = \Delta I_B (R_B + r_i)$

$= r \ \Delta I_B$

The change in I_B causes a change in Ic.

The change in Ic due to a change in I_B causes a change in V_{CE} and the voltage drop across the resistor R_L because V_{CC} is fixed.

These changes can be given by Eq. (14.15) as

$\Delta V_{CC} = \Delta V_{CE} + R_L \, \Delta I_C = 0$

or $\Delta V_{CE} = -R_L \, \Delta I_C$

The change in V_{CE} is the output voltage v_0.

Expression for Current gain & Voltage gain :

We define a parameter β_{ac}, which is similar to the β_{dc} as

$$\beta_{ac} = \frac{\Delta I_c}{\Delta I_B} = \frac{i_c}{i_b}$$

which is also known as the *ac current gain* A_i. Usually β_{ac} is close to β_{dc} in the linear region of the output characteristics.

From Eq. (14.10), we get

$v_0 = \Delta V_{CE} = -\beta_{ac} R_L \, \Delta I_B$

The voltage gain of the amplifier is

$$A_v = \frac{v_0}{v_i} = \frac{\Delta V_{CE}}{r \Delta I_B}$$

$$= -\frac{\beta_{ac} R_L}{r}$$

The negative sign represents that output voltage is out of phase with the input voltage.

The power gain A_p can be expressed as the product of the current gain and voltage gain. Mathematically $A_p = \beta_{ac} \times A_v$

Since β_{ac} and A_v are greater than 1, we get ac power gain. However it should be realised that transistor is not a power generating device. The energy for the higher ac power at the output is supplied by the battery.

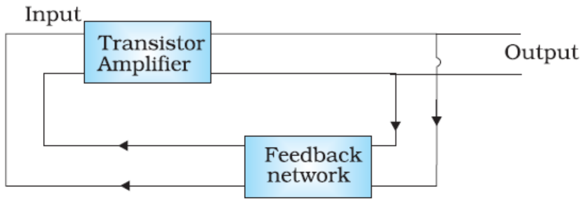
14.14 Transistor as an oscillator: principle and block diagram :

(15) What is an oscillator ? With principle and block diagram explain working of an LC oscillator ?

An oscillator is an electronic circuit in which we get ac output of desired frequency without any external input signal. In other words, the output in an oscillator is *self-sustained*.

Principle : To attain self-sustained oscillation, an amplifier is taken. A portion of the output power of this amplifier is returned back (feedback) to the input *in phase* with the starting power (this process is termed *positive feedback)* as shown in Fig. 14.33(a).

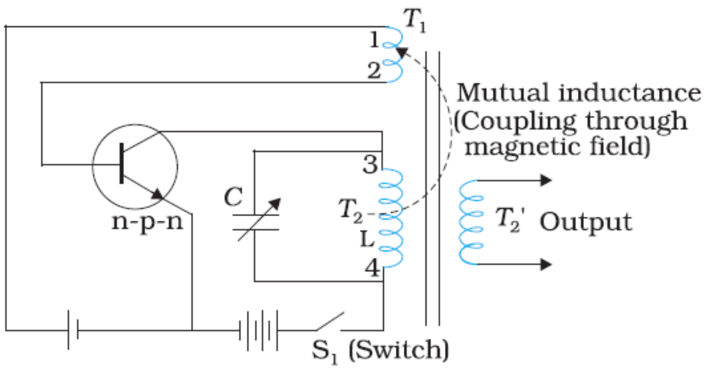
The feedback can be achieved by inductive coupling (through mutual inductance) or *LC* or *RC* networks. Different types of oscillators essentially use different methods of coupling the output to the input (feedback network), apart from the resonant circuit for obtaining oscillation at a particular frequency.



(a)

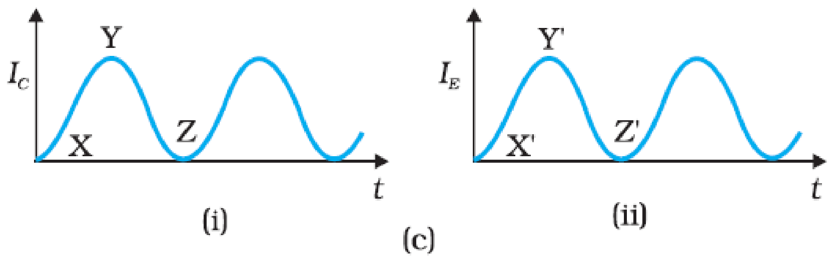
Fig. (a) Principle of a transistor amplifier with positive feedback working as an oscillator.

Circuit : For understanding the oscillator action, we consider the circuit shown in Fig. (b) in which the feedback is accomplished by *inductive coupling* from one coil winding (T_1) to another coil winding (T_2). Note that the coils T_2 and T_1 are wound on the same core and hence are inductively coupled through their mutual inductance. As in an amplifier, the base-emitter junction is forward biased while the base-collector junction is reverse biased.



(b)

(b) Tuned collector oscillator.



(c)

(c) Rise and fall (or built up) of current I_c and I_e due to the inductive coupling.

Working : Let us try to understand how oscillations are built. Suppose switch S1 is put *on* to apply proper bias for the first time. Obviously, a *surge* of collector current flows in the transistor. This current flows through the coil T_2 where terminals are numbered 3 and 4 [Fig. (b)]. This current does not reach full amplitude instantaneously but increases from X to Y, as shown in Fig. [(c)(i)]. The inductive coupling between coil T_2 and coil T_1 now causes a current to flow in the emitter circuit (note that this actually is the 'feedback' from input to output). As a result of this positive feedback, this current (in T_1; emitter current) also increases from X′ to Y′ [Fig. (c)(ii)]. The current in T_2 (collector current) connected in the collector circuit acquires the value Y when the transistor becomes *saturated*. This means that maximum collector current is flowing and can increase no further. Since there is no further change in collector current, the magnetic field around T_2 ceases to grow. As soon as the field becomes static, there will be no further feedback from T_2 to T_1. Without continued feedback, the emitter current begins to fall. Consequently, collector current decreases from Y towards Z [Fig. (c)(i)]. However, a decrease of collector current causes the magnetic field to decay around the coil T_2. Thus, T_1 is now seeing a decaying field in T_2 (opposite from what it saw when the field was growing at the initial *start* operation). This causes a further decrease in the emitter current till it reaches Z' when the transistor is *cut-off*. This means that both I_E and I_C cease to flow. Therefore, the transistor has reverted back to its original state (when the power was first switched on).

The whole process now repeats itself. That is, the transistor is driven to saturation, then to cut-off, and then back to saturation. The time for change from saturation to cut-off and back is determined by the constants of the tank circuit or tuned circuit (inductance L of coil T_2 and C connected in parallel to it). The resonance frequency (ν) of this tuned circuit determines the frequency at which the oscillator will oscillate.

$$\nu = \left(\frac{1}{2\pi\sqrt{LC}}\right)$$

In the circuit of Fig. (b), the tank or tuned circuit is connected in the collector side. Hence, it is known as *tuned collector oscillator*. If the tuned circuit is on the base side, it will be known as *tuned base oscillator*.
There are many other types of tank circuits (say *RC*) or feedback circuits giving different types of oscillators like Colpitt's oscillator, Hartley oscillator, *RC*-oscillator.
Uses : Oscillators are used to produce sinusoidal signal of any frequency including radio frequency signals. Hence they are used in electronic communication both at transmitter and receiver side.

14.16 Logic gates: Logic symbol and truth table of NOT, OR, AND, NAND and NOR gates.

(1) What do you mean by logic gates ?

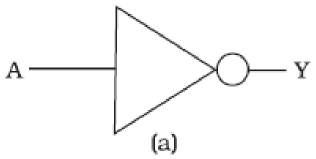
Logic gates are basic building blocks of digital electronics, which process the digital signals in a specific manner. Logic gates are used in calculators, digital watches, computers, robots, industrial control systems, and in telecommunications.

Logic gates follows curtain *logical* relationship between the input and output voltages. Therefore, they are generally known as *logic gates* — gates because they control the flow of information. The five common logic gates used are NOT, AND, OR, NAND, NOR. Each logic gate is indicated by a symbol and its function is defined by a *truth table* that shows all the possible input logic level combinations with their respective output logic levels. Truth tables help understand the behaviour of logic gates. These logic gates can be realised using semiconductor devices.

(2) Give logic symbol and truth table of NOT, OR, AND, NAND and NOR gates ?

(i) NOT gate :

Not gate is also called inverter. It is an electronic circuit with one input and one output. It produces a '1' output if the input is '0' and vice-versa. The fig. shows the logic symbol and truth table of NOT gate.



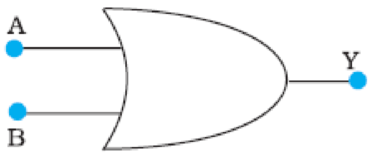
(a)

Input	Output
A	Y
0	1
1	0

(b)

(ii) OR Gate :

An *OR* gate has two or more inputs with one output. The logic symbol and truth table are shown in Fig. The output Y is 1 when either input A *or* input B *or* both are 1s, that is, if any of the input is high, the output is high.



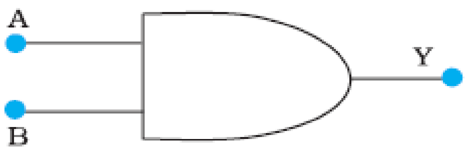
(a)

Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(b)

(iii) AND Gate

An *AND* gate has two or more inputs and one output. The output Y of AND gate is 1 only when input A *and* input B are both 1. The logic symbol and truth table for this gate are given in Fig.

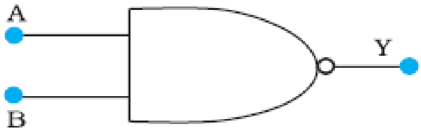


(a)

Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(iv) NAND Gate

This is an AND gate followed by a NOT gate. If inputs A *and* B are both '1', the output Y is *not* '1'. The gate gets its name from this NOT AND behaviour. Figure 14.40 shows the symbol and truth table of NAND gate.

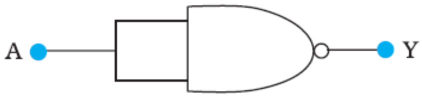


(a)

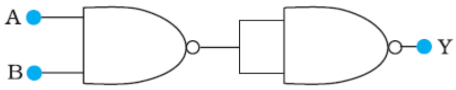
Input		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(b)

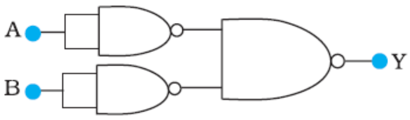
NAND gates are also called *Universal Gates* since by using these gates you can realise other basic gates like OR, AND and NOT. as shown below :



NOT gate



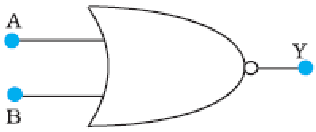
AND gate



OR gate

(v) NOR Gate

It has two or more inputs and one output. A NOT- operation applied *after* OR gate gives a NOT-OR gate (or simply NOR gate). Its output Y is '1' only when both inputs A and B are '0', i.e., neither one input *nor* the other is '1'. The symbol and truth table for NOR gate is given in Fig.

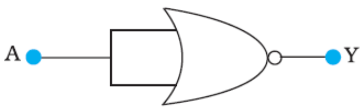


(a)

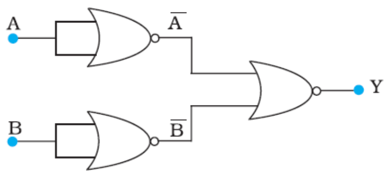
Input		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(b)

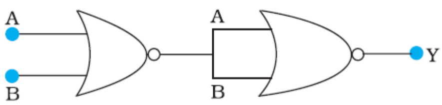
NOR gates are considered as *universal* gates because you can obtain all the gates like AND, OR, NOT by using only NOR gates as shown in following fig.



(a) NOT gate



(b) OR gate



AND gate

****

(ONE MARK QUESTIONS)

1. What is an electronic device?
Ans. It is a device in which controlled flow of electrons takes place either in vacuum or in semiconductors.

2. What is an energy band in a solid?
Ans. Energy band is a group of close by energy levels with continuous energy variation.

3. What is a valence band?
Ans. Valence band is the energy band which includes the energy levels of the valence electrons. It is the range of energies possessed by valence electrons.

4. What is conduction band?
Ans. Conduction band is the energy band which includes the energy levels of conduction electrons or free electrons.

5. What is energy gap or energy band gap?
Ans. The gap (spacing) between the top of the valence band (EV) and the bottom of the conduction band (Ec) is called the energy band gap (Eg) or energy gap.

6. What is the order of energy gap in a semiconductor?
Ans. 1eV

7. At what temperature would an intrinsic semiconductor behave like a perfect insulator?
Ans. 0 K (absolute zero temperature)

8. What is an intrinsic semiconductor?
Ans. It is a pure semiconductor in which electrical conductivity is solely due to the thermally generated electrons and holes.

9. What is doping?
Ans. The process of adding suitable impurity atoms to the crystal structure of pure semiconductor like Ge or Si to enhance their electrical conductivity is called doping.

10. What is a hole?
Ans. The vacancy of an electron(of charge -e) in the covalent bond with an effective positive charge +e is called a hole.

11. What is an extrinsic semiconductor?
Ans. The semiconductor obtained by doping a pure semiconductor like silicon with impurity atoms to enhance its conductivity is called an extrinsic or doped semiconductor.

12. Name one dopant which can be used with germanium to form an n-type semiconductor.
Ans. Phosphorus.

13. What are dopants?
Ans. The impurity atoms added to pure semiconductors like germanium to increase their electrical conductivity are called dopants.

14. Name the majority charge carriers in p-type semiconductors.
Ans. Holes.

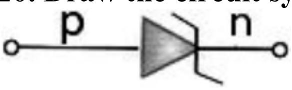
15. What is depletion region in a p-n junction?
Ans. The space charge region at the p-n junction which consists only of immobile ions and is depleted of mobile charge carriers is called depletion region.

16. How does the width of the depletion region of a p-n junction change when it is reverse biased?
Ans. The depletion region width increases.

17. What is the forward resistance of an ideal p-n junction diode?
Ans. Zero.

18. Draw the circuit symbol of a semiconductor diode.
Ans. 

19. Name any one optoelectronic device.
Ans. Photodiode / Light emitting diode / photovoltaic cell or solar cell.

20. Draw the circuit symbol of a Zener diode. Ans.


22. What is rectification?
Ans. The process of converting AC (alternating current) to pulsating DC is called rectification.

23. How does the conductivity of a semiconductor change with rise in its temperature?
Ans. The conductivity increases exponentially with temperature.

24. Is the ionisation energy of an isolated free atom different from the ionization energy Eg for the atoms in a crystalline lattice?
Ans. Yes. It is different since in a periodic crystal lattice each bound electron is influenced by many neighbouring atoms.

25. Which process causes depletion region in a p-n junction?
Ans. The diffusion of majority charge carriers i.e., free electrons and holes across the p–n junction causes the depletion region.

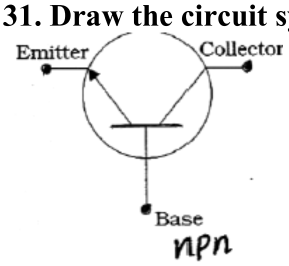
26. What is the order of the thickness of the depletion layer in an unbiased p-n junction?
Ans. micrometer (10-6 m).

27. What is a photodiode?
Ans. It is a special purpose p-n junction diode whose reverse current strength varies with the intensity of incident light.

28. Under which bias condition a Zener diode is used as a voltage regulator?
Ans. Reverse bias.

29. How is the band gap Eg of a photodiode related to the maximum wavelength λ_m that can be detected by it?
$E_g = hC/\lambda_m$

30. What is a solar cell?
Ans. It is a photovoltaic cell which is basically a p-n junction which generates emf when solar radiation falls on it.

31. Draw the circuit symbol of an npn transistor.


32. Define current gain or current amplification factor of transistor in CE mode.
Ans. The current gain (β) is defined as the ratio of change in collector current to corresponding change in base current at constant collector–emitter voltage.

33. What kind of biasing will be required to the emitter and collector junctions when a transistor is used as an amplifier?
Ans. Emitter-base junction is forward biased while collector-base junction is reverse biased.

34. Which region of the transistor is made thin and is lightly doped?
Ans. Base.

35. Under what condition a transistor works as an open switch?
Ans. When the transistor is in cut off state it works as an open switch.

36. What is an oscillator?
Ans. It is an electronic device which is used to produce sustained electrical oscillations of constant frequency and amplitude without any external input.

37. What type of feedback is used in an oscillator?
Ans. Positive feedback.

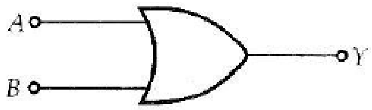
38. What is an analogue signal?

Ans. An electrical signal (current or voltage) which varies continuously with time is called an analogue signal.

39. What is a digital signal?
Ans. A signal (current or voltage) which takes only discrete values is called digital signal.

40. What is a logic gate?
Ans. A logic gate is a digital circuit that follows certain logical relationship between the input and output signals and works on the principles of Boolean algebra.

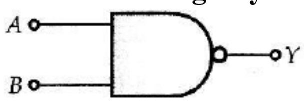
41. Draw the logic symbol of an OR gate.
Ans.



42. Write the truth table for a NOT gate.
Ans.

A	y=Ā
0	1
1	0

43. Draw the logic symbol of NAND gate



(TWO MARKS QUESTIONS)

1. Name the charge carriers in the following at room temperature: (i) conductor (ii) semiconductor.
Ans. (i) conductor: Electrons are charge carriers (ii) semiconductor: electrons and holes are charge carriers.

2. Name the factors on which electrical conductivity of a pure semiconductor depends at a given temperature.
Ans. (i) The width of the forbidden band. (ii) Intrinsic charge carrier concentration.

3. Mention the necessary conditions for doping.
Ans. 1. The dopant (impurity atom) should not distort the original pure semiconductor lattice. 2. The size of the dopant atom should be nearly the same as that of the semiconductor (host) atoms.

4. Name one impurity each, which when added to pure Si produces (i) n-type and (ii) p-type semiconductor.
Ans. (i) n-type impurity to be added – phosphorus / antimony (ii) p-type impurity to be added – aluminium / boron.

5. Give two differences between intrinsic and extrinsic semiconductors.

Ans.

Intrinsic semiconductors	Extrinsic semiconductors
1. Electrical conductivity depends only on temperature	1. Electrical conductivity depends on both temperature and dopant concentration
2. The number of free electrons is equal to the number of holes	2. The number of free electrons is not equal to number of holes

6. Give two differences between n-type and p-type semiconductors.

Ans.

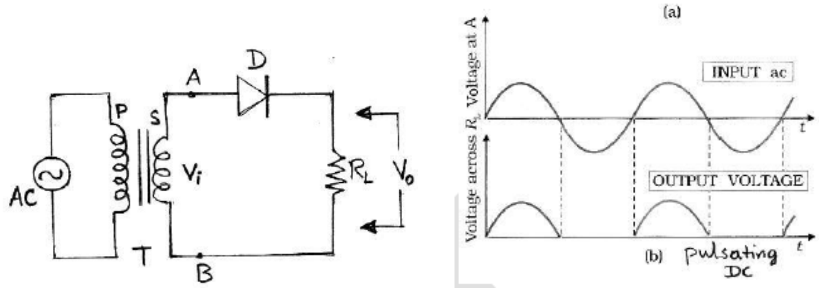
n-type semiconductor	p-type semiconductor
1. These are extrinsic semiconductors obtained by doping Ge or Si crystals with pentavalent dopants like Phosphorus.	1. These are extrinsic semiconductors obtained by doping Ge or Si crystals with trivalent dopants like aluminium.
2. Free electrons are the majority charge carriers and holes are minority charge carriers	2. Free electrons are minority charge carriers and holes are majority charge carriers.

7. What happens to the width of the depletion layer of a p-n junction when it is (i) forward biased? (ii) reverse biased?

Ans. (i) The depletion layer width decreases when p-n junction is forward biased. (ii) The depletion layer width increases when p-n junction is reverse biased.

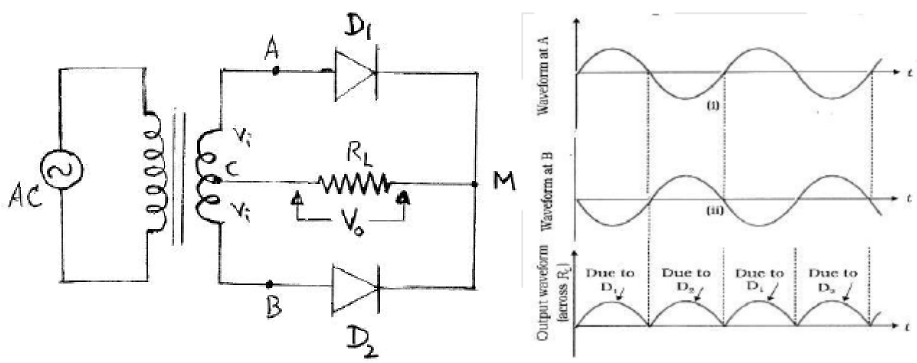
8. Draw a labelled diagram of a half wave rectifier. Draw the input and output waveforms.

Ans.



9. Draw a labelled diagram of a full wave rectifier. Draw the input and output waveforms.

Ans.



10. Zener diodes have higher dopant densities as compared to ordinary p-n junction diodes. How does it effect: (i) the width of the depletion layer? (ii) the junction field?
Ans. (i) The width of the depletion layer becomes small. (ii) The junction electric field becomes large.

11. Explain why a photodiode is usually operated under reverse bias.
Ans. During reverse bias the reverse saturation current due to minority charge carriers is small. When light is incident the fractional increase in minority charge carrier concentration is significant and is early measurable. If the photodiode is forward biased, under illumination the fractional increase in charge carriers is insignificant and is difficult to measure. Hence photodiode is usually operated in reverse bias.

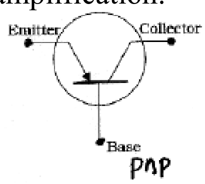
12. What is an LED? Mention two advantages of LED over conventional incandescent lamps.
Ans. LED (Light emitting diode) is a heavily doped p-n junction which under suitable forward bias emits spontaneous radiation. The advantages of LEDs over incandescent lamps. (i) LEDs operate at low voltages and consume less power. (ii) LEDs have long life, are rugged and have fast switching (on-off) capability.

13. Mention the factor which determines the (i) frequency and (ii) intensity of light emitted by LED.
Ans. (i) The frequency of light emitted by an LED depends on the band gap of the semiconductor used in LED. (ii) The intensity of light emitted depends on the doping level of the semiconductor used.

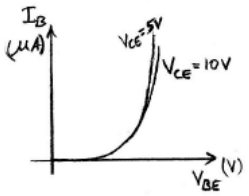
14. Give two operational differences between light emitting diode (LED) and photodiode.
Ans.

LED	PHOTODIODE
1. It is forward biased	1. It is reverse biased
2. Recombination of electrons and holes takes place at the junction and light is emitted ($h\nu$)	2. Light energy ($h\nu$) falling on the p-n junction creates electron-hole pair which increases photocurrent.

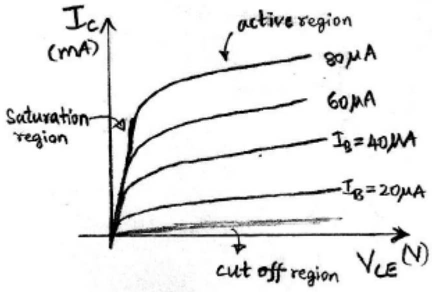
15. What is a transistor? Draw the circuit symbol of pnp transistor.
Ans. Transistor is a three terminal two junction semiconductor device whose basic action is amplification.



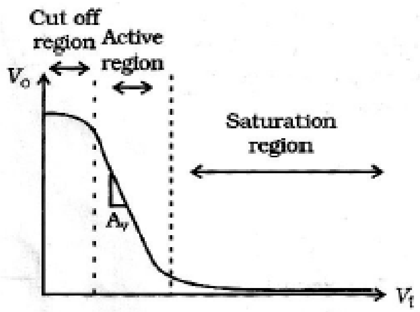
16. Draw input characteristics of a transistor in CE mode and define input resistance.
Ans. The input resistance ri of the transistor in CE mode is defined as the ratio of change in base-emitter voltage to the corresponding change in base current at constant collector-emitter voltage.



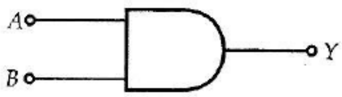
17. Draw output characteristics of a transistor in CE mode and define output resistance.
Ans. Output resistance(r0) is the ratio of the change in collector-emitter voltage to the corresponding change in collector current at a constant base current.



18. Draw the transfer characteristics of base-biased transistor in CE configuration and indicate the regions of operation when transistor is used as (i) an amplifier (ii) a switch.
Ans. (i) Transistor is used as an amplifier in the active region (ii) Transistor is used as a switch in the cut-off region and saturation region



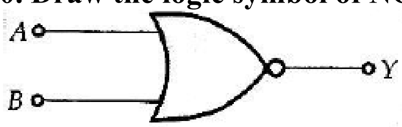
19. Draw the logic symbol of AND gate and write its truth table.



Truth Table: AND gate

A	B	y=A.B
0	0	0
0	1	0
1	0	0
1	1	1

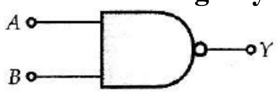
20. Draw the logic symbol of NOR gate and write its truth table.



Truth table: NOR gate

A	B	$y=\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

21. Draw the logic symbol of NAND gate and write its truth table.



Truth table: NAND gate

A	B	$y=\overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

(THREE MARKS QUESTIONS)

1. What is intrinsic semiconductor? Explain the formation of a hole in the covalent bond structure of a Ge crystal.

2. How is an n-type semiconductor formed? Name the majority charge carriers in it. Draw the energy band diagram of an n-type semiconductor.

3. How is a p-type semiconductor formed? Name the majority charge carriers in it. Draw the energy band diagram of a p-type semiconductor.

4. Distinguish between n-type and p-type semiconductors on the basis of energy band diagrams.

5. Explain the formation of the depletion region in a p-n junction. How does the width of this region change when the junction is (i) forward biased? and (ii) reverse biased?

6. What is forward bias? Draw a circuit diagram for the forward biased p-n junction and sketch the voltage-current graph for the same.

7. What is reverse bias? Draw a circuit diagram for the reverse biased p-n junction and sketch the voltage-current graph for the same.

8. With the help of a circuit diagram explain the use of Zener diode as a voltage regulator.

9. What is a transistor? Describe the various regions of a transistor.

10. Describe briefly with the help of a circuit diagram, the paths of current carriers in an npn transistor with emitter-base junction forward biased and collector-base junction reverse biased.

11. Draw a circuit diagram of a transistor amplifier in the common-emitter configuration. Briefly explain, how the input and output signals differ in phase by 180º.

12. Draw the transfer characteristic curve of a base biased transistor CE configuration. Explain how the active region of the Vo versus Vi curve is used for amplification.

13. What is a logic gate? Draw the symbol of a NOT gate and write its truth table.

14. With the help of a block diagram, briefly explain the principle of transistor oscillator.

(**FIVE MARKS QUESTIONS**)

1. What is energy band? On the basis of energy band diagrams, distinguish between metals, insulators and semiconductors.

2. What are intrinsic semiconductors? Explain the formation of a hole in an intrinsic semiconductor. Draw the energy level diagram.

3. What is extrinsic semiconductor? Distinguish between n-type and p-type semiconductors. Draw relevant energy level diagrams.

4. What is a p-n junction? Explain the formation of the depletion region in a p-n junction. How does the width of this region change when the junction is (i) forward biased? (ii) reverse biased? Explain.

5. Draw the circuit diagrams of a p-n junction diode in (i) forward bias and (ii) reverse bias. Draw the I-V characteristics for the same and discuss the resistance of the junction in both the cases.

6. With a neat circuit diagram, explain the working of a half wave rectifier employing a semiconductor diode. Draw the relevant waveforms.

7. With a neat circuit diagram, explain the working of a full wave centre-tap rectifier using junction diodes. Draw the input and output waveforms.

8. Draw the circuit arrangement for studying the input and output characteristics of an npn transistor in CE configuration. Draw these characteristics and define input resistance and output resistance.

9. With the help of a circuit diagram explain the action of a npn transistor in CE configuration as a switch. Draw the transfer characteristics and indicate the relevant regions of operation.

10. Describe with a circuit diagram the working of an amplifier using an npn transistor in CE configuration. Draw relevant waveforms and obtain an expression for the voltage gain.

MCQ

Q1. A semiconductor device is connected in a series circuit with battery and resistance. A current is found to pass through the circuit. When polarities of the battery are reversed the current drops to zero. The device may be:

[a] pn junction
[b]intrinsic semiconductor
[c] p type semiconductor
[d] n type semiconductor

Q.2 On increasing the reverse bias to a large value in pn junction diode the current:

[a] increases slowly [b] remains fixed
[c] suddenly increases [d] decreases slowly

Q.3 The electrical resistance of depletion layer is large because:

[a] it has no charge carriers
[b] it has large number of charge carriers
[c] it contains electrons as charge carriers
[d] it has holes as charge carriers

Q.4 To measure light intensity we use

[a] LED with forward bias
[b] LED with reverse bias
[c] photodiode with reverse bias
[d] photodiode with forward bias

Q.5 In forward biased pn junction the current is of the order of

[a] ampere [b] milliampere
[c] microampere [d] nanoampere

Q.6 When pn junction diode is reverse biased the flow of current across the junction is mainly due to

[a] diffusion of charges
[b] depends on nature of material
[c] drift of charges
[d] both drift and diffusion of charges

Q.7 In a common emitter circuit, the collector current is 0.9mA, base current is 100μA. The value of current gain and emitter current is

[a] 49 and 2mA [b] 9 ad 1mA
[c] 0.9 and 0.1mA [d] none of these

Q.8 The concentration of impurities in a transistor :

[a] equal for emitter, base and collector
[b] least for emitter region
[c] largest for emitter region
[d] largest for collector region

Q.9 Application of forward bias to the pn junction

[a] increases the number of donors on n side
[b] increases electric field in depletion region

[c] increases potential difference across the depletion region
[d]widens the depletion zone

Q.10 Within depletion region of the pn junction diode
[a] p side is positive and n side is negative
[b] p side is negative and n side is +ive
[c] both sides are either positive or negative
[d] both sides are neutral

Q.11 What is the conductivity of semiconductor if electron density= $5x10^{12}/cm^3$ and hole density = $8x10^{13}/cm^3$[μ_e=2.3 and μ_h=0.01 in SI units]
[a]5.634 [b] 1.968
[c]3.421 [d] 8.964

Q.12 The electrical conductivity of semiconductor increases when an electromagnetic radiation of wavelength 1125nm is incident on it. The band gap of semiconductor is
[a]0.9eV [b] 0.7eV
[c] 0.8eV [d]0.5eV

Q.13 In common base mode of transistor, the collector current is 5.488mA, the emitter current is 5.6mA. the value of current amplification factor is
[a] 48 [b] 49
[c] 50 [d] 51

Q.14 A semiconductor is doped with donor impurity is
[a] p type [b] n type
[c] npn type [d] pnp type

Q.15 IN NPN transistor, the collector current is 24mA. If 80% of the electrons reach collector, the base current in mA is
[a] 36 [b] 26
[c] 16 [d]6

Q.16 The difference in the variation of resistance with temperature in a metal and semiconductor arises essentially due to the difference in
[a] type of bonding
[b] crystal structure
[c]scattering mechanism with temperature
[d]number of charge carriers with temperature

Q.17 barrier potential of pn junction does not depend on
[a] temperature [b] forward bias
[c] reverse bias [d] diode design

Q.18 One serious drawback of semiconductors is
[a] they are costly
[b] they pollute the environment
[c] they do not last for long time
[d] they can’t withstand high voltage

Q.19 Resistance of semiconductor
[a] increases with temperature
[b] decreases with temperature

[c] remains unaffected with temperature
[d] none of these

Q.20 The dominant mechanism for motion of charge carriers in forward and reverse biased silicon pn junction are
[a] drift in both forward and reverse bias
[b]diffusion in both forward and reverse
[c] diffusion in forward and drift in reverse
[d] drift in forward and diffusion in reverse

Q.21 When NPN transistors is used as amplifier
[a] electrons move from base to emitter
[b] electrons move from emitter to base
[c] electrons move from collector to base
[d] holes move from base to emitter

Q.22 In a transistor circuit base current is increased by 50μA keeping the collector voltage fixed at 2 volts, the collector current increases by 1mA. The current gain of the transistor is
[a] 20 [b] 40
[c] 60 [d] 80

Q.23 A common emitter transistor amplifier has a current gain of 50. if the load voltage is 4kΩ and input resistance is 500Ω, the voltage gain in amplifier is
[a]160 [b] 200
[c] 300 [d] 400

Q.24 Which gate corresponds to action of parallel switches
[a] OR gate [b] AND
[c] NOR [d]NAND

Q.25 Which gate is similar to function of two series switches
[a] AND [b] OR
[c] NAND [d] NOR

Q.26 The electrical conductivity of the semiconductor increases when em radiation of wavelength shorter than 2480nm is incident on it. The band gap of the semiconductor in eV is
[a] 0.9 [b] 0.7
[c] 0.5 [d] 0.1

Q.27 If the forward voltage of semiconductor is doubled, the width of depletion layer will;
[a] become half [b] becomes $1/4^{th}$
[c] remain unchanged [d] becomes double

Q.28 A sinusoidal voltage of peak value 200V is connected to diode and resistance in series so that half wave rectification occurs. If forward resistance of the diode is negligible, the rms voltage across resistance is
[a] 200 [b] 100
[c] $100\sqrt{2}$ [d] 283

Q.29 A common emitter amplifier is designed with npn transistor with α=0.99. the input impedance is 1 Kilo ohm and load is 10 kiloohm. The voltage gain will be
[a]9.9 [b]99
[c]990 [d]9900

Q.30 The main cause of Zener breakdown is
[a]high doping [b]low doping
[c]collision ionization
[d]production of electron hole pair due to thermal excitation

Q.31 In binary number system, 11000101 represents which number on decimal system
[a] 4 [b] 401
[c] 197 [d] 204

Q.32 For common base transistor the numerical value is least for
[a] voltage gain [b] power gain
[c] resistance gain [d] current gain

Q.33 In a common emitter amplifier output resistance is 5000 ohm and input resistance is 2000 ohm. If the peak value of signal voltage is 10mV and β=50, the peak value of voltage output is
[a] $5x10^{-6}$V [b]$2.5x10^{-4}$V
[c]1.25V [d]125V

Q.34 What is the value of the voltage gain in the common emitter amplifier, where input resistance is 3 ohm and load resistance is 24 ohm. Take β=0.6
[a]8.4 [b]4.8
[c]2.4 [d]1.2

Q.35 If R_1 is the input resistance and R_2 is the output resistance of the voltage gain A in the common emitter configuration is
[a] $\alpha[R_2/R_1]$ [b]$\beta\ [R_2/R_1]$
[c]α [d]β

Q.36 A non conducting device is connected in a series circuit with battery and resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, the current drops to almost zero. The device may be
[a] p n junction diode
[b] an intrinsic semiconductor
[c] a p type semiconductor
[d] an n type semiconductor

Q.37 the temperature coefficient of resistance of the semiconductor is
[a] positive always [b] negative always
[c] zero [d] infinite

Q.38 in a p type semiconductor, the acceptor valence band is
[a]above the conduction band of the host crystal
[b] below the conduction band of the crystal
[c] above the valence band of the crystal
[d] below the valence band of the crystal

Q.39 The forbidden energy gap in an insular is
[a] > 6eV [b] < 6eV
[c]1 eV [d]4 eV

Q.40 In common base transistor amplifier, the phase difference between output voltage and input voltage is
[a] zero [b]180^0

[c] 90^0 [d] 45^0

Q.41 Transistors may not replace vacuum tubes in all uses because
[a] because transistors requires long warm ups than the vacuum tubes
[b] vacuum tubes are more resistant in shocks and vibrations than transistors
[c] vacuum tubes can handle greater power than transistors
[d] transistors use high voltages

Q.42 The base is made thin and lightly doped because
[a] about 95% of the charge carriers may cross
[b] about 100% of the charge carriers may cross
[c] the transistors can be saved from large currents
[d] none of these

Q.43 The current gain of common base npn transistor is 0.96. what is the current gain if it is used as common emitter amplifier?
[a]16 [b] 24
[c] 20 [d] 32

Q.44 In forward biased pn junction diode, the collector current is of the order of
[a] microampere [b] milliampere
[c] ampere [d] nanoampere

Q.45 The temperature coefficient of the resistance of semiconductors is always
[a] positive [b] negative
[c] zero [d] infinite

Q.46 In an insulator, the number of electrons in the valence shell in general is
[a] less than 4 [b] more than 4
[c] equal to 4 [d] none of these

1. N-type semiconductors will be obtained, when germanium is doped with. [AIIMS 2000]

Phosphorus	Arsenic
Aluminium	Both (a) or (c)

2. Which of the following logic gate is an universal gate . [AIIMS 2005]

OR	AND
NOT	NOR

3. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the. [AIEEE 2003]

Variation of scattering mechanism with temperature	Variation of the number of charge carriers with temperature
Crystal structure	Type of bon

4. The valence of an impurity added to germanium crystal in order to convert it into a P-type semi conductor is . [MP PMT 1989; CP]

6	4
5	3

5. To a germanium sample, traces of gallium are added as an impurity. The resultant sample would behave like . [AIIMS 2003]

A conductor	An N-type semiconductor
A P-type semiconductor	An insulator

6. In a PN-junction diode . [MP PET 1993]

The current in the reverse biased condition is generally very small	The reverse biased current is strongly dependent on the applied bias voltage
The current in the reverse biased condition is small but the forward biased current is independent of the bias voltage	The forward biased current is very small in comparison to reverse biased current

7. Biaxial crystal among the following is . [Pb. CET 1998]

Calcite	Selenite
Quartz	Tourmaline

8. In a PN-junction . [CBSE PMT 2002]

P and N both are at same potential	High potential at P side and low potential at N side
High potential at N side and low potential at P side	Low potential at N side and zero potential at P side

9. Boolean algebra is essentially based on . [AIIMS 1999]

Truth	Symbol
Logic	Numbers

10. The ionic bond is absent in . [J & K CET 2005]

NaCl	LiF
CsCl	H_2O

11. The energy band gap is maximum in . [AIEEE 2002]

Metals	Insulators
Superconductors	Semiconductors

12. The nature of binding for a crystal with alternate and evenly spaced positive and negative ions is . [CBSE PMT 2000]

Covalent	Dipolar
Metallic	Ionic

13. The coordination number of Cu is . [AMU 1992]

1	8
6	12

14. When Ge crystals are doped with phosphorus atom, then it becomes . [AFMC 1995;]

Insulator	N-type
P-type	Superconductor

15. PN-junction diode works as a insulator, if connected . [CPMT 1987]

To A.C.	In reverse bias
In forward bias	None of these

16. When NPN transistor is used as an amplifier . [AIEEE 2004]

Electrons move from base to collector	Electrons move from collector to base
Holes move from emitter to base	Holes move from base to emitter

17. Which of the following statements concerning the depletion zone of an unbiased PN junction is (are) true . [IIT-JEE 1995]

The width of the zone is independent of the densities of the dopants (impurities)	The electric field in the zone is produced by the ionized dopant atoms
The width of the zone is dependent on the densities of the dopants	The electric field in the zone is provided by the electrons in the conduction band and the holes in the valence band

18. In a semiconductor . [AIEEE 2002; AII]

There are no free electrons at any temperature	There are no free electrons at 0 K
The number of free electrons is more than that in a conductor	None of these

19. Which of the following is an amorphous solid . [AIIMS 2005;]

Glass	Salt
Diamond	Sugar

20. In N-type semiconductors, majority charge carriers are . [AIIMS 1999]

Holes	Neutrons
Protons	Electrons

21. In the middle of the depletion layer of a reverse-biased PN junction, the. [AIEEE 2003]

Potential is zero	Potential is maximum
Electric field is zero	Electric field is maximum

22. Electronic configuration of germanium is 2, 8, 18 and 4. To make it extrinsic semiconductor small quantity of antimony is added . [MP PET 1999]

The material obtained will be N-type germanium in which electrons and holes are equal in number	The material obtained will be N-type germanium which has more electrons than holes at room temperature

The material obtained will be P-type germanium	The material obtained will be N-type germanium which has less electrons than holes at room temperature

23. In a PN-junction diode not connected to any circuit . [IIT-JEE 1998]

The potential is the same everywhere	There is an electric field at the junction directed from the N- type side to the P- type side
The P-type is a higher potential than the N-type side	There is an electric field at the junction directed from the P-type side to the N-type side

24. A PN- junction has a thickness of the order of . [BIT 1990]

1 cm	10^{-6} m
1 mm	10^{-12} cm

25. On increasing the reverse bias to a large value in a PN-junction diode, current . [MP PMT 1994; BH]

Increases slowly	Suddenly increases
Remains fixed	Decreases slowly

26. For a crystal system, a = b = c, $\alpha = \beta = \lambda$!= 90^0, the system is . [BHU 2000]

Tetragonal system	Orthorhombic system
Cubic system	Rhombohedral system

27. Holes are charge carriers in . [IIT-JEE 1996]

Intrinsic semiconductors	P-type semiconductors
Ionic solids	Metals

28. A semiconductor dopped with a donor impurity is . [AFMC 2005]

P-type	NPN type
N-type	PNP type

29. The output of OR gate is 1. [CBSE PMT 2004]

If both inputs are zero	Only if both input are 1
If either or both inputs are 1	If either input is zero

30. A piece of semiconductor is connected in series in an electric circuit. On increasing the temperature, the current in the circuit will. [RPMT 2003]

Decrease	Increase
Remain unchanged	Stop flowing

31. When the P end of P-N junction is connected to the negative terminal of the battery and the N end to the positive terminal of the battery, then the P-N junction behaves like. [MP PET 2002]

A conductor	A super-conductor
An insulator	A semi-conductor

32. Which of the following materials is non crystalline . [CBSE PMT 1993]

Copper	Wood
Sodium chloride	Diamond

33. Energy bands in solids are a consequence of . [DCE 1999, 2000;]

Ohm's Law	Bohr's theory
Pauli's exclusion principle	Heisenberg's uncertainty principle

34. Bonding in a germanium crystal (semi- conductor) is . [CPMT 1986; MP P]

Metallic	Vander Waal's type
Ionic	Covalent

35. In P-type semiconductor, there is . [MP PMT 1989]

An excess of one electron	A missing atom
Absence of one electron	A donar level

36. A transistor is used in common emitter mode as an amplifier. Then . [IIT-JEE 1998]

The base-emitter junction is forward biased	The input signal is connected in series with the voltage applied to the base-emitter junction
The base-emitter junction is reverse biased	The input signal is connected in series with the voltage applied to bias the base collector junction

37. In extrinsic P and N-type, semiconductor materials, the ratio of the impurity atoms to the pure semiconductor atoms is about . [MP PET 2003]

1	10^{-4}
10^{-1}	10^{-7}

38. The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon P-N junctions are . [AIIMS 2000]

Drift in forward bias, diffusion in reverse bias	Diffusion in both forward and reverse bias
Diffusion in forward bias, drift in reverse bias	Drift in both forward and reverse bias

39. Zener breakdown in a semi-conductor diode occurs when . [UPSEAT 2002]

Forward current exceeds certain value	Forward bias exceeds certain value
Reverse bias exceeds certain value	Potential barrier is reduced to zero

40. A piece of copper and the other of germanium are cooled from the room temperature to 80 K, then which of the following would be a correct statement . [IIT-JEE 1988;MP]

Resistance of each increases	Resistance of copper increases while that of germanium decreases
Resistance of each decreases	Resistance of copper decreases while that of germanium increases

41. The emitter-base junction of a transistor is …… biased while the collector-base junction is ……. biased . [KCET 2004]

Reverse, forward	Forward, forward
Reverse, reverse	Forward, reverse

1) In a common base amplifier, the phase difference between the Input signal voltage and output voltage is

(a) π (b) π / 4 (c) π/2 (d) zero [AIEEE 2005]

2. In a full wave rectifier, circuit operating from 50 Hz mains frequency, the out terminal frequency in the ripple would be

(a) 25 Hz (b) 50 Hz (c)I 70.7 Hz (d)I 100 Hz

3. When npn transistor is used as an amplifier

(a) electrons move from base to collector (b) holes moves from emitter to base

(c) electrons move from collector to base (d) holes moves from base to emitter

4) For a transistor amplifier in common emitter configuration for load impedance of 1 kΩ, (h_{fe} = 50 and h_{oe} = 25 μA/V), the current gain is

a} -5.2 (b) -15.7 (c) -24.8 (d) - 48.7. [AIEEE 2004]

5) A strip of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

(a) each of these increases (b) each of these decreases

(c) copper strip increases and that of germanium decreases

(d) copper strip decreases and that of germanium increases [AIEEE 2004, 2003]

6) The menifestation of band structure in solids is due to

(a) Heisenberg's uncertainty principle (b) Pauli's exclusion principle

(c) Bohr's correspondence principle (d) Boltzmann's law [AIEEE 2004]

7) When p-n junction diode is forward biased, then

(a) the depletion region is reduced and barrier eight is increased

(b) the depletion region is widened and barrier height is reduced

(c) both the depletion region and barrier height are reduced

(d) both the depletion region and barrier height are increased [AIEEE 2004]

8) The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in

(a) type of bonding (b) crystal structure

(c) scattering mechanism with temperature (d) no. of charge carriers with temp. [AIEEE 2003]

9) In the middle of the depletion layer of a reverse biased p-n junction, the

(a) the potential is zero (b) electric field is zero

(c) potential is maximum (d) electric field is maximum [AIEEE 2003]

10) In a p-n junction, the depletion layer consists of

(a) electrons (b) protons (c) mobile ions (d) immobile ions [AIEEE 2002]

11) In forward bias, the width of otential barrier in p-n junction diode
(a) increases (b) decreases (c) remans constant
(d) first increases, then decreases [AIEEE 2002]

12) When a potential difference is applied across, the current passing through,
(a) an insulator at 0 K is zero
(b) a semiconductor at 0 K is zero
(c) a metal at 0 K is finite
(d) a p-n diode at 300 K is finite if it is reverse biased [IIT 1999]

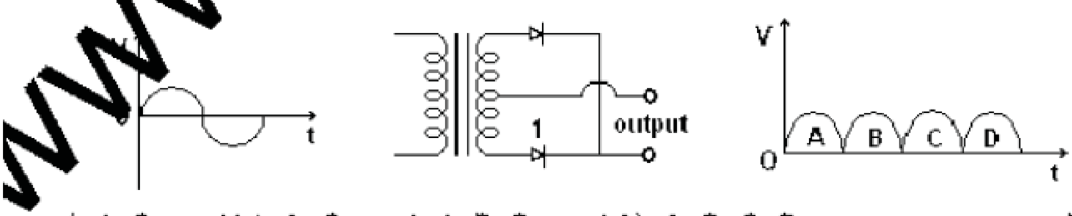
13) A transistor is used in common emitter mode as an amplifier, then
(a) the base emitter junction is forward biased
(b) the base emitter junction is reverse biased
(c) the input signal is connected in series with the voltage applied to bias the base emitter junction
(d) the input signal is connected in series with the voltage applied to bias the base connector junction [IIT 1998]

14) In a p-n junction diode not connected to any circuit
(a) the potential is the same everywhere
(b) the p-type side is at a higher potential than the n-type side
(c) there is an electric field at the junction directed from the n-type side to the p-type side
(d) there is an electric field at the junction directed from the p-type side to the n-type side
[IIT 1998]

15) Which of the following statement is not true –
(a) The resistance of intrinsic semiconductors decreases with increase of temperature
(b) Doping pure Si with trivalent impurities gives p-type semiconductors.
(c) The majority carriers in n-type semiconductors are holes
(d) A p-n junction can act as a semiconductor diode [IIT 1997]

16) Holes are charge carriers in
(a) intrinsic semiconductors (b) ionic solids
(c) p-type semiconductors (d) metals [IIT 1996]

17) A full wave rectifier circuit alongwith the output is shown in the figure. The contribution(s) from the diode 1 is (are)

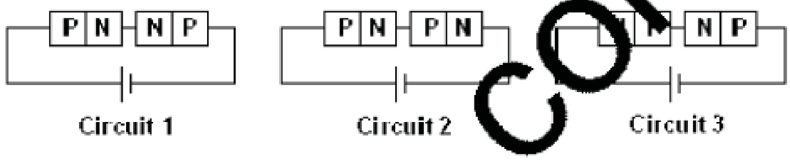


(a) C (b) A, C (c) B, D (d) A, B, C, D [IIT 1996]

18) Read the following statements carefully:
Y : The resistivity of a semiconductor decreases with increase of temperature
Z : In a conducting solid, the rate of collisions between free electrons and ions increases with increase of temperature
(a) Y is true but Z is false (b) Y is false but Z is true
(c) Both Y and Z are true (d) Y is true and Z is the correct reason for Y
[IIT 1993]

19) In an n-p-n transistor circuit, the collector current is 10 mA. If 90 % of the electrons emitted reach the collector
(a) the emitter current will be 9 mA (b) the emitter current will be 11 mA
(c) the base current will be 1 mA (d) the base current will be -1 mA [IIT 1992]

20) Two identical p-n junctions may be connected in series with a battery in three ways as shown in the figure. The potential drops across the two n-p junctions are equal in



(a) circuit 1 and circuit 2 (b) circuit 2 and circuit 3
(c) circuit 3 and circuit 1 (d) circuit 1 only [IIT 1989]

21) A piece of copper and another of germanimum are cooled from room temperature to 80° K. The resistance of
(a) each of them increases (b) each of them decreases
(c) copper increases and germanium decreases
(d) copper decreases and germanium increases [IIT 1988]

22) The impurity atoms with which pure silicon should be doped to make a p-type semiconductor are those of
(a) phosphorous (b) boron (c) antimony (c) aluminium [IIT 1988]

23) Select the correct statements from the following:
(a) A diode can be used as a rectifier.
(b) A triode cannot be used as a rectifier.
(c) The current in a diode is always proportional to the applied voltage.
(d) The linear portion of the I - V characteristic of a triode is used for amplification without distortion. [IIT 1984]

24) The plate resistance of a triode valve is 3 x 10^3 ohm and its mutual conductance is 1.5 x 10^{-3} amp / volt. The amplification factor of the triode is
(a) [illegible] x 10^5 (b) 4.5 (c) 0.45 (d) 2 x 10^6 [IIT 1983, 1981]

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	d		d	d	b	c	d	a	d	b	a,b,d	a,c	c	c	b,c	c	c	b,c	b

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
d	b,d	a,d	b																

Chapter 15:
COMMUNICATION SYSTEMS
3M
1M -1Q, 2M - 1Q

15.1 Block diagram of generalized communication system -

Communication is the act of transmission of information. Every communication system has three essential elements-transmitter, medium/channel and receiver. The block diagram shown in Fig. 15.1 depicts the general form of a communication system.

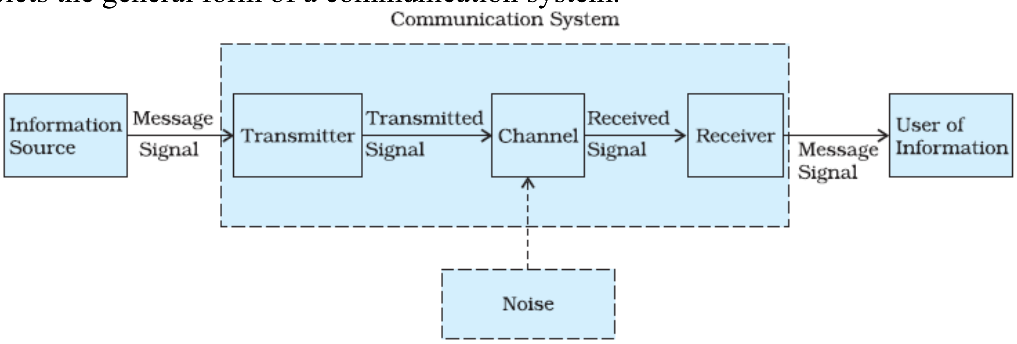


Fig. 15.1 : The general form of a communication system

In a communication system, the transmitter is located at one place, the receiver is located at some other place (far or near) separate from the transmitter and the channel is the physical medium that connects them. Depending upon the type of communication system, a channel may be in the form of wires or cables connecting the transmitter and the receiver or it may be wireless. The purpose of the transmitter is to convert the message signal produced by the source of information into a form suitable for transmission through the channel. If the output of the information source is a non-electrical signal like a voice signal, a transducer converts it to electrical form before giving it as an input to the transmitter. When a transmitted signal propagates along the channel it may get distorted due to channel imperfection. Moreover, noise adds to the transmitted signal and the receiver receives a corrupted version of the transmitted signal. The receiver has the task of operating on the received signal. It reconstructs a recognisable form of the original message signal for delivering it to the user of information.

There are two basic modes of communication: *point-to-point* and *broadcast*.

(1) In point-to-point communication mode, communication takes place over a link between a single transmitter and a receiver. Telephony is an example of such a mode of communication.

(2) In the broadcast mode, there are a large number of receivers corresponding to a single transmitter. Radio and television are examples of broadcast mode of communication.

15.2 Basic terminology used in electronic communication systems :

(a) Transducer : Any device that converts one form of energy into another can be termed as a transducer. In electronic communication systems, we usually come across devices that have either their inputs or outputs in the electrical form. An electrical transducer may be defined as a device that converts some physical variable (pressure, displacement, force, temperature, etc) into corresponding variations in the electrical signal at its output.

(b) Signal : Information converted in electrical form and suitable for transmission is called a signal. Signals can be either *analog or digital*. Analog signals are continuous variations of voltage or current. *They are essentially single-valued functions of time*. Sine wave is a fundamental analog signal. All other analog signals can be fully understood in terms of their sine wave components. Sound and picture signals in TV are analog in nature. Digital signals are those which can take only discrete stepwise values. Binary system that is extensively used in digital electronics employs just two levels of a signal. '0' corresponds to a low level and '1' corresponds to a high level of voltage/current. There are several coding schemes useful for digital communication. They employ suitable combinations of number systems such as the binary coded decimal (BCD). American Standard Code for Information Interchange (ASCII) is a universally popular digital code to represent numbers, letters and certain characters.

(c) Noise : Noise refers to the unwanted signals that tend to disturb the transmission and processing of message signals in a communication system. The source generating the noise may be located inside or outside the system.

(d) Transmitter : A transmitter processes the incoming message signal so as to make it suitable for transmission through a channel and subsequent reception.

(e) Receiver : A receiver extracts the desired message signals from the received signals at the channel output.

(f) Attenuation : The loss of strength of a signal while propagating through a medium is known as attenuation.

(g) Amplification : It is the process of *increasing the amplitude* (and consequently the strength) of a signal using an electronic circuit called the amplifier. Amplification is necessary to compensate for the attenuation of the signal in communication systems. The energy needed for additional signal strength is obtained from a DC power source. Amplification is done at a place between the source and the destination wherever signal strength becomes weaker than the required strength.

(h) Range : It is the largest distance between a source and a destination up to which the signal is received with sufficient strength.

(i) Bandwidth : Bandwidth refers to the frequency range over which an equipment operates or the portion of the spectrum occupied by the signal.

(j) Modulation : The original low frequency message/information signal cannot be transmitted to long distances. Therefore, at the transmitter, information contained in the low frequency message signal is superimposed on a high frequency wave, which acts as a carrier of the information. This process is known as modulation. There are several types of modulation, abbreviated as AM, FM and PM.

(k) Demodulation : The process of retrieval of information from the carrier wave at the receiver is termed demodulation. This is the reverse process of modulation.

(l) Repeater : A repeater is a combination of a receiver and a transmitter. A repeater, picks up the signal from the transmitter, amplifies and retransmits it to the receiver sometimes with a change in carrier frequency. Repeaters are used to extend the range of a communication system.

(j) Modem : An electronic device used for modulation and /or demodulation of electric signal is called modem.

15.3 Mention of bandwidth of signals for speech, TV and digital data ?

In a communication system, the message signal can be voice, music, picture or computer data. Each of these signals has different ranges of frequencies. The type of communication system needed for a given signal depends on the band of frequencies which is considered essential for the communication process.

(a) Bandwidth of speech : For speech signals, frequency range 300 Hz to 3100 Hz is considered adequate. Therefore speech signal requires a bandwidth of 2800 Hz (3100 Hz – 300 Hz) for commercial telephonic communication.
(b) Bandwidth of Music : To transmit music, an approximate bandwidth of 20 kHz is required because of the high frequencies produced by the musical instruments.
(c) Bandwidth of Audio signal : The audible range of frequencies extends from 20 Hz to 20 kHz.
(d) Bandwidth of video signal : Video signals for transmission of pictures require about 4.2 MHz of bandwidth. A TV signal contains both voice and picture and is usually allocated 6 MHz of bandwidth for transmission.
(e) Bandwidth of digital signal : Digital signals can be ideally reconstructed if we have superimpose fundamental frequency and all the harmonics and hence implies an infinite bandwidth. However, in practice, the contribution from higher harmonics can be neglected, thus limiting the bandwidth. As a result, received waves are a distorted version of the transmitted one. If the bandwidth is large enough to accommodate a few harmonics, the information is not lost and the rectangular signal is more or less recovered. This is so because the higher the harmonic, less is its contribution to the wave form.

15.4 Mention of bandwidth of transmission medium for coaxial cable, free space and optical fibers -

Similar to message signals, different types of transmission media offer different bandwidths. The commonly used transmission media are wire, free space and fiber optic cable.

(a) Bandwidth of cable wire : Coaxial cable is a widely used wire medium, which offers a bandwidth of approximately 750 MHz. Such cables are normally operated below 18 GHz.

(b) ***Bandwidth in free space :*** Communication through free space using radio waves takes place over a very wide range of frequencies: from a few hundreds of kHz to a few GHz.

(c) ***Bandwidth of optical fiber :*** Optical communication using fibers is performed in the frequency range of 1 THz to 1000 THz (microwaves to ultraviolet). An optical fiber can offer a transmission bandwidth in excess of 100 GHz.

15.5 Propagation of electromagnetic waves: Brief explanation of ground wave, sky wave and space wave -

In communication using radio waves, an antenna at the transmitter radiates the Electromagnetic waves (em waves), which travel through the space and reach the receiving antenna at the other end. As the em wave travels away from the transmitter, the strength of the wave keeps on decreasing. Several factors influence the propagation of em waves and the path they follow. At this point, it is also important to understand the composition of the earth's atmosphere as it plays a vital role in the propagation of em waves.

(1) Ground wave :

To radiate signals with high efficiency, the antennas should have a size comparable to the wavelength λ of the signal (at least ~ l/4). At longer wavelengths (i.e., at lower frequencies), the antennas have large physical size and they are located on or very near to the ground. In standard AM broadcast, ground based vertical towers are generally used as transmitting antennas. For such antennas, ground has a strong influence on the propagation of the signal. The mode of propagation is called surface wave propagation and the wave glides over the surface of the earth. A wave induces current in the ground over which it passes and it is attenuated as a result of absorption of energy by the earth. The attenuation of surface waves increases very rapidly with increase in frequency. The maximum range of coverage depends on the transmitted power and frequency (less than a few MHz).

Table 15.1 : Layers of ionosphere :

Name of the stratum (layer)		Approximate height over earth's surface	Exists during	Frequencies most affected
Troposphere		10 km	Day and night	VHF (up to several GHz)
D (part of stratosphere)	PARTS OF IONOSPHERE	65-75 km	Day only	Reflects LF, absorbs MF and HF to some degree
E (part of Stratosphere)		100 km	Day only	Helps surface waves, reflects HF
F_1 (Part of Mesosphere)		170-190 km	Daytime, merges with F_2 at night	Partially absorbs HF waves yet allowing them to reach F_2
F_2 (Thermosphere)		300 km at night, 250-400 km during daytime	Day and night	Efficiently reflects HF waves, particularly at night

(2) Sky Wave :

In the frequency range from a few MHz up to 30 to 40 MHz, long distance communication can be achieved by ionospheric reflection of radio waves back towards the earth. This mode of propagation is called *sky wave propagation* and is used by short wave broadcast services. The ionosphere is so called because of the presence of a large number of ions or charged particles. It extends from a height of ~ 65 Km to about 400 Km above the earth's surface. Ionisation occurs due to the absorption of the ultraviolet and other high-energy radiation coming from the sun by air molecules.

The ionosphere is further subdivided into several layers, the details of which are given in Table 15.1. The degree of ionisation varies with the height. The density of atmosphere decreases with height. At great heights the solar radiation is intense but there are few molecules to be ionised. Close to the earth, even though the molecular concentration is very high, the radiation intensity is low so that the ionisation is again low. However, at some intermediate heights, there occurs a peak of ionisation density. The ionospheric layer acts as a reflector for a certain range of frequencies (3 to 30 MHz). Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape. These phenomena are shown in the Fig. 15.2. The phenomenon of bending of em waves so that they are diverted towards the earth is similar to total internal reflection in optics.

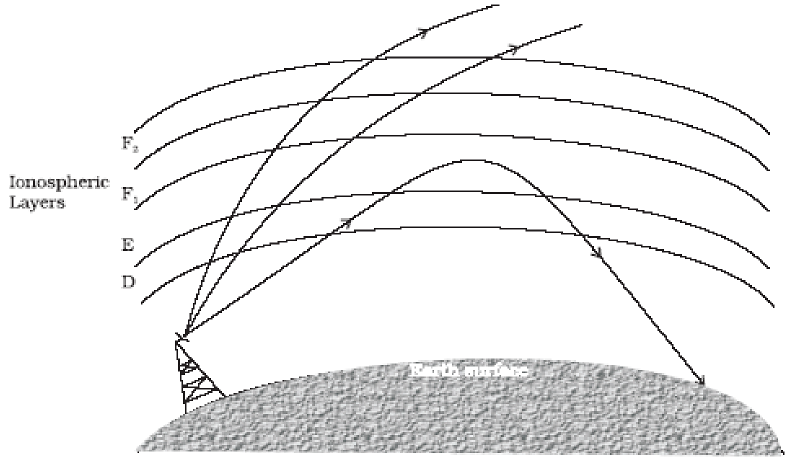


Fig. 15.2 : Sky wave propagation. The layer nomenclature

(3) Space wave :

Another mode of radio wave propagation is by *space waves*. A space wave travels in a straight line from transmitting antenna to the receiving antenna. Space waves are used for line-of-sight (LOS) communication as well as satellite communication. At frequencies above 40 MHz, communication is essentially limited to line-of-sight paths. At these frequencies, the antennas are relatively smaller and can be placed at heights of many wavelengths above the ground. Because of line-of-sight nature of propagation, direct waves get blocked at some point by the curvature of the earth as illustrated in Fig. 15.3. If the signal is to be received beyond the horizon then the receiving antenna must be high enough to intercept the line-of-sight waves.

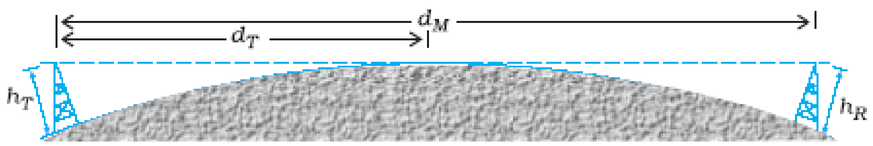


Fig. 15.3 : Line of sight communication by space waves.

Television broadcast, microwave links and satellite communication are some examples of communication systems that use space wave mode of propagation. Figure 15.4 summarises the various modes of wave propagation.

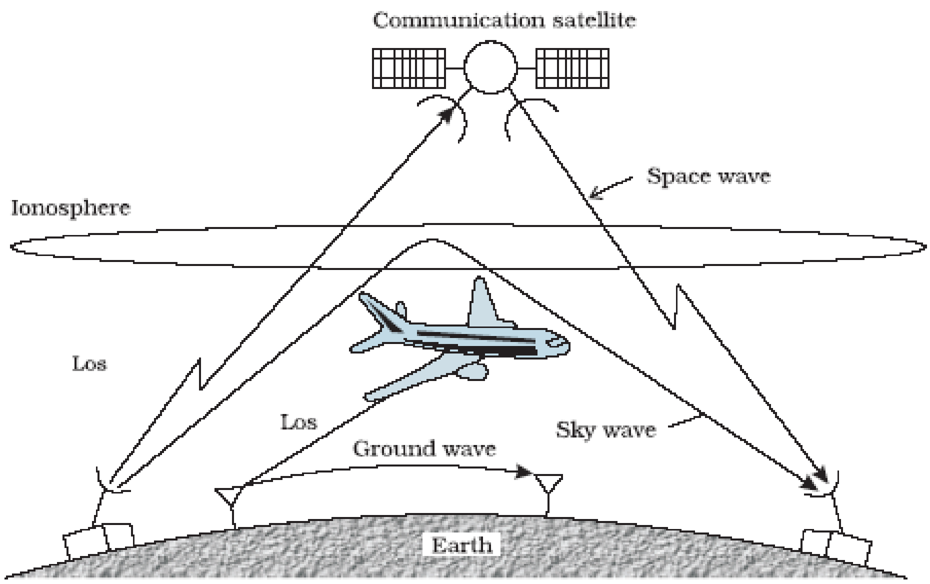


Fig. 15.4 : Various propagation modes for em waves.

15.6 Need for modulation -

Suppose we wish to transmit an electronic signal in the audio frequency (AF) range (baseband signal frequency less than 20 kHz) over a long distance directly. Let us find what factors prevent us from doing so and how we overcome these factors,

(1) Size of the antenna or aerial :

For transmitting a signal, we need an antenna or an aerial. This antenna should have a size comparable to the wavelength of the signal (at least $\lambda/4$ in dimension) so that the antenna properly senses the time variation of the signal. For an electromagnetic wave of frequency 20 kHz, the wavelength λ is 15 km. Obviously, such a long antenna is not possible to construct and operate. Hence direct transmission of such baseband signals is not practical. We can obtain transmission with reasonable antenna lengths if transmission frequency is high (for example, if n is 1 MHz, then λ is 300 m). Therefore, there is a need of *translating the information contained in our original low frequency baseband signal into high or radio frequencies before transmission*.

(2) Effective power radiated by an antenna :

A theoretical study of radiation from a linear antenna (length l) shows that the power radiated is proportional to $(l/\lambda)^2$. This implies that for the same antenna length, the power radiated increases with decreasing λ, i.e., increasing frequency. Hence, the effective power radiated by a long wavelength baseband signal would be small. For a good transmission, we need high powers and hence this also points out to *the need* of using high frequency transmission.

(3) Mixing up of signals from different transmitters :

Another important argument against transmitting baseband signals directly is more *practical* in nature. Suppose many people are talking at the same time or many transmitters are transmitting baseband information signals simultaneously. All these signals will get mixed up and there is no simple way to distinguish between them. This points out towards a possible solution by using communication at high frequencies and allotting a *band* of frequencies to each message signal for its transmission.

The above arguments suggest that there is a *need for translating the original low frequency baseband message or information signal into high frequency wave before transmission such that the translated signal continues to possess the information contained in the original signal*. In doing so, we take the help of a high frequency signal, known as the *carrier wave*, and a process known as modulation which attaches information to it.

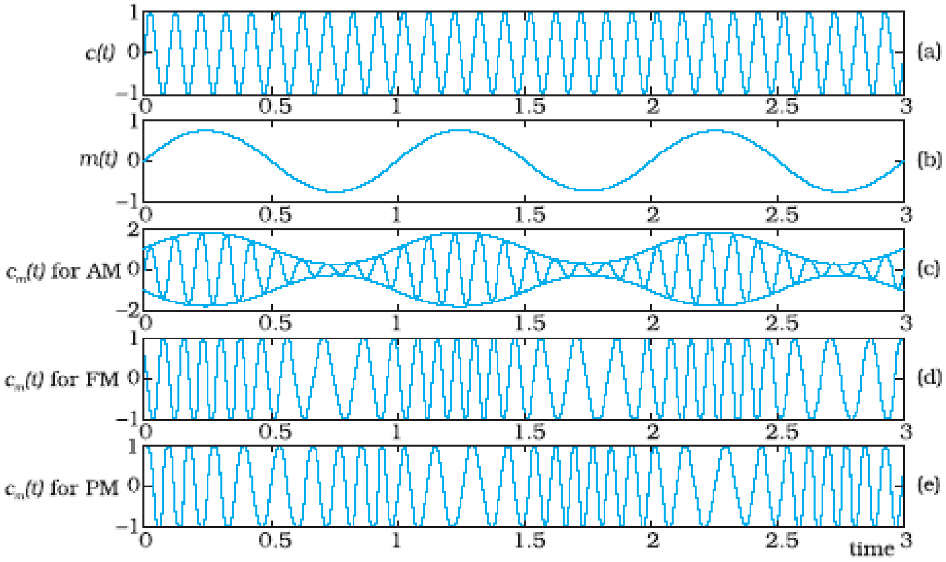


Fig. 15.5 : Modulation of a carrier wave: (a) a sinusoidal carrier wave; (b) a modulating signal; (c) amplitude modulation; (d) frequency modulation; and (e) phase modulation

15.7 Amplitude modulation: Meaning -

In amplitude modulation the amplitude of the carrier is varied in accordance with the information signal. Here we explain amplitude modulation process using a sinusoidal signal as the modulating signal.

Let $c(t) = A_c \sin \omega_c t$ represent carrier wave and $m(t) = A_m \sin \omega_m t$ represent the message or the modulating signal where $\omega_m = 2\pi f_m$ is the angular frequency of the message signal. The modulated signal $c_m(t)$ can be written as

$$c_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$$

$$= A_c \left(1 + \frac{A_m}{A_c} \sin\omega_m t\right) \sin\omega_c t \qquad \text{-------} \quad (15.3)$$

the modulated signal now contains the message signal. This can also be seen from Fig. 15.5(c).
From Eq. (15.3), we can write,

$$c_m(t) = A_c \sin\omega_c t + \mu A_c \sin\omega_m t \sin\omega_c t \qquad \text{----------} \quad (15.4)$$

Here $\mu = A_m/A_c$ is the *modulation index;* in practice, μ is kept ≤ 1 to avoid distortion.
Using the trignomatric relation $\sin A \sin B = ½ (\cos(A - B) - \cos (A + B)$,
we can write $c_m(t)$ of Eq. (15.4) as

$$c_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)\,t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t \qquad \text{-----------} \quad (15.5)$$

Here $\omega_c - \omega_m$ and $\omega_c + \omega_m$ are respectively called the lower side and upper side frequencies. The modulated signal now consists of the carrier wave of frequency ω_c plus two sinusoidal waves each with a frequency slightly different from, known as side bands. The frequency spectrum of the amplitude modulated signal is shown in Fig. 15.6.

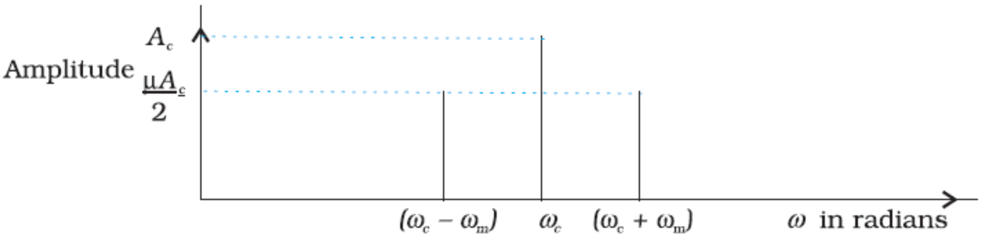


Fig. 15. 6 : A plot of amplitude versus w for an amplitude modulated signal.

As long as the broadcast frequencies (carrier waves) are sufficiently spaced out so that sidebands do not overlap, different stations can operate without interfering with each other.

15.8 Block diagram of AM transmitter and AM receiver.

(1) AM transmitter :

The modulated signal cannot be transmitted as such. The modulator is to be followed by a power amplifier which provides the necessary power and then the modulated signal is fed to an antenna of appropriate size for radiation as shown in Fig. 15.7.

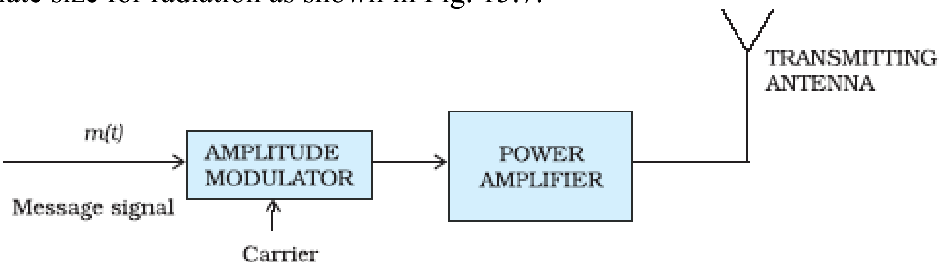


Fig. 15.7 : Block diagram of a transmitter.

(2) AM Receiver :

The transmitted message gets attenuated in propagating through the channel. The receiving antenna is therefore to be followed by an amplifier and a detector. In addition, to facilitate further processing, the carrier frequency is usually changed to a lower frequency by what is called an *intermediate frequency (IF) stage* preceding the detection. The detected signal may not be strong enough to be made use of and hence is required to be amplified. A block diagram of a typical receiver is shown in Fig. 15.8

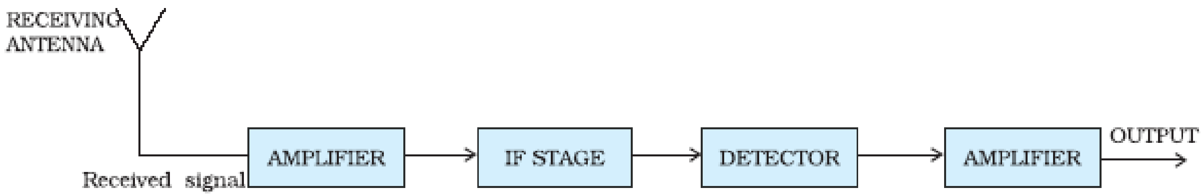


Fig. 15.8 : Block diagram of a receiver.

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IMPORTANT FORMULE :

1. Length of the antenna $L_A \geq \lambda/4$, where λ is wavelength of transmitted signal**.**

2. Power radiated bt antenna = $P_A \propto (l/\lambda)^2$ where l is length of antenna and λ is wavelength of radiated signal.

3. Line of sight distance $d_M = d_R + d_T$ where d_R is radio horizone of receiving antenna and d_T is radio horizon of transmitting antenna.

(a) $d_T = \sqrt{2Rh_T}$ and (b) $d_R = \sqrt{2Rh_R}$ where R = radius of Earth, h is height of respective antenna.

4. Modulation index of AM wave = $\mu_A = \{A_M/A_C\} = [V_m/V_c]$

5 Upper side band (USB) $= (\omega_C + \omega_M)$ **&** **Lower side band** (LSB) $= (\omega_C - \omega_M)$

6. Amplitude of USB = Amplitude of LSB $= \frac{\mu A_C}{2}$ where μ is modulation index and A_C is amplitude of carrier wave $= \frac{\mu V_c}{2}$

1 Mark Questions

1) What are the three main units of a Communication System?
Ans :
a) Transmitter
b) Transmission channel
c) Receiver.

2) What is meant by Bandwidth of transmission?
2) It is the frequency range with in which a transmission is made.

3) What is a transducer? Give an example.
3) It is a device which converts one form of energy into another. Ex: A microphone, speaker etc.

4) Why is it necessary to use satellites for long distance TV transmission?
4) TV signals being of high frequency are not reflected by ionosphere. Hence satellites are used.

5) What is the frequency range of audio waves?
5) 20Hz to 20Khz

6) What is the function of demodulator?
6) To recover the original modulating signal.

7) What is a carrier wave?
7) It is a high frequency wave which carries the information or signal.

8) What is a ground wave?
8) The radio waves propagating from one place to another following the Earth's surface are called ground waves.

9) What is a sky wave?
9) It is a mode of Communication, which uses ionosphere as a reflector for propagation.

10) Which type of communication uses discrete and binary coded version of signal?
10) Digital Communication

11) What should be the length of the dipole antenna for a carrier wave of wavelength 'λ'?
11) The size of the dipole antenna should be $1/4^{th}$ of the wavelength.

12) What is a space wave?
12) Radiowaves having high frequencies are basically called as space waves.

13) What are microwaves? What is their use?
13) Microwaves are electromagnetic waves of wavelength range 1mm to 3cm, they are used in space-communication.

14) What type of modulation is required for radio broadcast?

14) Amplitude modulation.

15) What type of modulation is required for Television broadcast?
15) Frequency modulation.

16) Which device is used for transmitting TV signals over long distances?
16) Communication satellite

17) Name any one advantage of digital signal over analog signal?
17) They are relatively Noise-free and error free.

18) What is modulation index of an AM Wave?
18) $\mu = A_m / A_c$
A_m = Amplitude of modulating wave.
A_c = Amplitude of Carrier wave.

19) What are different modes of line of communication?
19) a. Two wire transmission lines.
b. Coaxial cables
c. Optical fibers.

20) What is an digital signal?
20) It is a discontinuous and discrete signal having binary variations 1 and 0 with time.

21) What is an analog signal?
21) It's an electrical signal which varies continuously with time.

22) What is a range in a communication system?
22) It is the largest distance between the source and the destination upto which the signal is received with sufficient strength.

23) Name the device which generate Radiowaves of constant amplitude?
23) Oscillator

24) What is the frequency range for space wave propagation?
24) UHF (> 40MHz)

25) Which layer of atmosphere reflects Radio waves back to Earth?
25) Ionosphere

26) What is meant by Attenuation?
26) It refers to loss of strength of a signal during propagation of a signal.

27) What is the function of a Repeater in a Communication system?
27) It extends the range of communication.

28) What is noise in a Communication system?
28) The unwanted signal is called a noise in a communication system.

29) What is meant by Amplification of a signal?
29) It is the process of raising the strength of a signal.

30) What are the different types of Communication?
30) a. Point-to-point Communication and b. Broadcast.

31) Define line-of-sight (LOS) Communication.
31) If the signal (transmitted wave) travels the distance between the transmitter and receiver antenna in a straight line, then such a type of communication is known as LOS Communication.

32) Name the three groups into which the propagating electromagnetic waves are classified.
32) a. Ground waves.
b. Sky waves
c. Space waves

33) What is meant by phase Modulation?
33) If the phase of the carrier wave changes in accordance with the phase of the message signal, then the modulation is known as phase modulation.

Two or three marks questions

34) Give the block diagram representation of communication system?

35) Draw frequency spectrum of the amplitude modulated signal.

36) Give the block diagram of Transmission of Amplitude Modulated signal.

37) Give the block diagram of a Receiver.

38) Give the block diagram for AM signal detector.

39) Problem related to length/size of an antenna :

(40) Problem related to calculate modulation index of AM wave

(41) Problems related to calculate bandwidth of AM wave

(42)

Communication MCQ

1. Television signals on earth cannot be received at distances greater than 100 km from the transmission station. The reason behind this is that ____. [DCE 1995]

- The receiver antenna is unable to detect the signal at a distance greater than 100 km.
- The TV signals are less powerful than radio signals.
- The TV programme consists of both audio and video signals.
- The surface of earth is curved like a sphere.

2. In short wave communication waves of which of the following frequencies will be reflected back by the ionospheric layer, having electron density 10^{11} per m^3 . [AIIMS 2003]

- 2 MHz
- 12 MHz
- 10 MHz
- 18 MHz

3. Laser beams are used to measure long distances because ___ . [DCE 2002, 03]

- They are monochromatic
- They are coherent
- They are highly polarised
- They have high degree of parallelism

4. A laser beam is used for carrying out surgery because it ___ . [AIIMS 2003]

- Is highly monochromatic
- Is highly directional
- Is highly coherent
- Can be sharply focussed

5. For television broadcasting, the frequency employed is normally ____. [AMU 2002]

- 30-300 MHz
- 30-300 KHz
- 30-300 GHz
- 30-300 Hz

6. The process of superimposing signal frequency (i.e. audio wave) on the carrier wave is known as . [AIIMS 1987]

- Transmission
- Modulation
- Reception
- Detection

7. Indicate which one of the following system is digital

- Pulse position modulation
- Pulse width modulation
- Pulse code modulation
- Pulse amplitude modulation

8. An antenna is a device

- That converts electromagnetic energy into radio frequency signal
- That converts guided electromagnetic waves into free space electromagnetic waves and vice-versa
- That converts radio frequency signal into electromagnetic energy
- None of these

9. Audio signal cannot be transmitted because ___. [Kerala PMT 2005]

- The signal has more noise
- The transmitting antenna length is very small to design
- The signal cannot be amplified for distance communication
- The transmitting antenna length is very large and impracticable

10. The phenomenon by which light travels in an optical fibres is ____ . [DCE 2001]

- Reflection
- Total internal reflection
- Refraction
- Transmission

11. For sky wave propagation of a 10 MHz signal, what should be the minimum electron density in ionosphere . [AIIMS 2005]

- $\sim 1.2 * 10^{12}$ per m^3
- $\sim 10^{14}$ per m^3
- $\sim 10^{6}$ per m^3
- $\sim 10^{22}$ per m^3

12. What is the modulation index of an over modulated wave

- 1
- < 1
- Zero
- > 1

13. Through which mode of propagation, the radio waves can be sent from one place to another . [JIPMER 2003]

- Ground wave propagation
- Space wave propagation
- Sky wave propagation
- All of them

14. In an amplitude modulated wave for audio frequency of 500 cycle/second, the appropriate carrier frequency will be . [AMU 1996]

- 50 cycles/sec
- 500 cycles/sec
- 100 cycles/sec
- 50,000 cycles/sec

15. Advantage of optical fibre ___. [DCE 2005]

- High bandwidth and EM interference
- High band width, low transmission capacity and no EM interference
- Low bandwidth and EM interference
- High bandwidth, high data transmission capacity and no EM interference

16. Basically, the product modulator is

- An amplifier
- A frequency separator
- A mixer
- A phase separator

17. The maximum distance upto which TV transmission from a TV tower of height h can be received is proportional to ____. [AIIMS 2003]

- $h^{1/2}$
- $h^{3/2}$

- h
- h^2

18. Range of frequencies allotted for commercial FM radio broadcast is ____. [MNR 1997]

- 88 to 108 MHz
- 8 to 88 MHz
- 88 to 108 kHz
- 88 to 108 GHz

19. The waves used in telecommunication are

- IR
- Microwave
- UV
- Cosmic rays

20. A step index fibre has a relative refractive index of 0.88%. What is the critical angle at the corecladding interface. [Manipal 2003]

- 60°
- 45°
- 75°
- None of these

21. Consider telecommunication through optical fibres. Which of the following statements is not true ___. [AIEEE 2003]

- Optical fibres may have homogeneous core with a suitable cladding
- Optical fibres are subject to electromagnetic interference from outside
- Optical fibres can be of graded refractive index
- Optical fibres have extremely low transmission loss

22. In an FM system a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is

- 7.143
- 0.71
- 8
- 350

23. Which of the following is the disadvantage of FM over AM

- Larger band width requirement
- Higher modulation power
- Larger noise
- Low efficiency

24. In which of the following remote sensing technique is not used . [Kerala PMT 2005]

- Forest density
- Wetland mapping
- Pollution
- Medical treatment

25. In frequency modulation . [Kerala PMT 2005]

- The amplitude of modulated wave varies as frequency of carrier wave
- The amplitude of modulated wave varies as amplitude of carrier wave
- The frequency of modulated wave varies as amplitude of modulating wave
- The frequency of modulated wave varies as frequency of modulating wave

Most Likely Questions :

A. 1M Questions :
1. What is the bandwidth of Optical fibre
2. Expression for AM wave

B. 2M Questions :
1. Explain two reasons of need of modulation ?
2. Draw block diagram of AM Transmitter and Receiver