JOINT BACK-PRESSURE POWER CONTROL AND INTERFERENCE CANCELLATION IN WIRELESS MULTI-HOP NETWORKS

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ABSTRACT

Back-pressure routing and power control policies are well-appreciated for maximizing throughput in wireless multi-hop networks, where power control is used to manage interference in a way that ultimately optimizes throughput, a network-layer performance measure. In addition to transmitter power control, interference can be mitigated via selective receiver-side signal cancellation, provided that the signal to be cancelled can be reliably decoded. This paper considers joint back-pressure power control and interference cancellation, assuming that each receiver can cancel at most one interfering signal. It is shown that the joint problem is NP-hard, and a suitable convex approximation is developed and shown to yield significant gains in terms of end-to-end throughput relative to power control alone. The main methodological contribution is in terms of a fortuitous reformulation of the joint problem, which is of potentially broader interest.
Keywords: Back-pressure, power control, interference cancellation, NP-hard, convex approximation

1. INTRODUCTION

Maximizing the overall throughput of a network is a core objective that underpins most modern approaches to network design and operation. Twenty years ago, Tassiulas [4] showed that a conceptually simple control policy known as *back-pressure* enables maximal stable throughput. Back-pressure favors links and flows with high *differential backlog*, which is intuitive for congestion control; the surprise is that this seemingly greedy congestion relief strategy is optimal from the viewpoint of maximizing network throughput [4]. Over the past decade, research in cross-layer wireless networking has really picked up, driven by demand for higher data rates and companion technological advances resulting in sophisticated base stations, access points, and mobile radios with significant processing power. These have in turn enabled implementation of rather sophisticated algorithms for routing, scheduling and power control.

Back-pressure is currently an important component of modern approaches to network routing, transmission scheduling, and resource allocation, e.g., see [2, 6] and references therein. In the case of wireless networks, assuming nodes treat interference as noise and link rates are governed by the Shannon capacity formula (possibly gap-adjusted for modulation loss and coding gain), it was recently shown in [1] that the core problem to be solved at the physical layer, dubbed *Back-Pressure Power Control* (BPPC) is NP-hard. The good news in [1] is that BPPC can be affectively approximated, using a sequential convex approximation strategy.

In addition to transmitter power control, interference cancellation can be employed to mitigate interference on the receiver side, provided that the signal to be cancelled can be reliably decoded, i.e., it is received at sufficiently high signal to interference plus noise ratio (SINR) at the given receiver. Unless transmissions are carefully coordinated (e.g., spread with a large spreading factor and subject to only limited near-far effects), it is realistic to assume that at most one interferer can be cancelled at a given receiver at a time. This restriction is also appealing from a receiver complexity point of view, hence will be adopted for the sequel. Whether a receiver can cancel a particular transmission obviously depends on power control, as the latter affects the SINR at which any transmitter is heard at any receiver. Thus interference cancellation cannot be an afterthought - it has to be optimized jointly with power control across the network. Joint power-control and interference cancellation for maximizing the minimum SINR for co-channel links, has been considered in [7] using a mixed integer linear programming framework. In this paper, we consider joint the BPPC and interference cancellation problem for multi-hop networks. Not surprisingly, the joint problem is NP-hard, similar to its power-only counterpart. Interestingly, it is shown here that the joint problem can be conveniently re-formulated in a way that enables effective sequential convex approximation. The novel formulation and convex approximation methodology is shown to yield significant gains in terms of end-to-end throughput relative to power control alone.

It is worth noting that the joint problem considered here involves both continuous and discrete control variables: transmission powers and whether to cancel (and which signal) or not, at each receiver. If the control space is purely discrete and finite, it turns out that a greedy randomization policy is also optimal from a throughput point of view, as shown in [5]. This consists of drawing a random policy per scheduling slot, comparing it to the current one, and adopting the new one only if it yields better objective. In addition to formulating and approximating the joint problem of picking transmission powers and making interference cancellation decisions that maximize the differential-backlog weighted sum rate (called BPPC-IC in the sequel), we therefore also consider BPPC-RIC (for Randomized Interference Cancellation), a simpler strategy that essentially requires solving a power control (BPPC-type) problem per slot.

2. SYSTEM MODEL

Consider a wireless multi-hop network with N nodes. The topology of the network is represented by the directed graph $(\mathcal{N}, \mathcal{L})$, where $\mathcal{N} := \{1, \ldots, N\}$ and $\mathcal{L} := \{1, \ldots, L\}$ denote the set of nodes and the set of links, respectively. Each link $\ell \in \mathcal{L}$ corresponds to an ordered pair (i, j), where $i, j \in \mathcal{N}$ and $i \neq j$. Let Tx_ℓ and Rx_ℓ

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denote the transmitter and the receiver of link ℓ, i.e., when $\ell = (i, j)$, then $\text{Tx}_\ell = i$ and $\text{Rx}_\ell = j$. For simplicity of exposition, assume that node 1 is the source and node N is the destination - our results generalize easily to the case of multiple sources and destinations, see [1]. In each time slot, data can be transmitted by all nodes except the destination. Let p_ℓ denote the power transmitted on link ℓ and $G_{\ell k}$ the aggregate path loss between Rx_ℓ and Tx_k. The Signal to Interference plus Noise Ratio (SINR) at the receiver of link ℓ is

$$\gamma_\ell = \frac{G_{\ell\ell} p_\ell}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq \ell}}^{L} G_{\ell k} p_k + V_\ell}, \tag{1}$$

where V_ℓ is the background noise power, and G_{sg} models spreading / beam-forming gain. The SINR at the ℓ^{th} link receiver, when it tries to decode the transmission of the k^{th} link, is given by

$$\gamma_{\ell k} = \frac{G_{\ell k} p_k}{\frac{1}{G_{sg}} \sum_{\substack{m=1 \\ m \neq k}}^{L} G_{\ell m} p_m + V_\ell}; \quad \forall k \in \mathcal{L}_{\ell^-} \backslash \{0\}, \forall \ell \in \mathcal{L}, \tag{2}$$

where $\mathcal{L}_{\ell^-} = \{0\} \bigcup \{\mathcal{L} \backslash \{\ell\}\}$. If $\gamma_{\ell k}$ is greater than a specified threshold value T, then the ℓ^{th} link receiver can reliably decode the transmission of the k^{th} link and subsequently cancel its contribution from the received signal. We define a set of indicator random variables $\{\{c_{lk}\}_{k \in \mathcal{L}_{\ell^-}}\}_{\ell \in \mathcal{L}}$ to denote whether or not link ℓ cancels link k. For $k \neq 0$,

$$c_{\ell k} = \begin{cases} 1, & \text{if link } \ell \text{ cancels link } k \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

For $k = 0$, $c_{\ell 0}$ corresponds to no interference cancellation, i.e.

$$c_{\ell 0} = 1 \quad \text{if} \quad c_{\ell k} = 0 \; \forall \; k \in \mathcal{L}_{\ell^-} \backslash \{0\}, \; \forall \; \ell \in \mathcal{L} \tag{4}$$

The following constraint is introduced to enforce (4) as well as model that each receiver can cancel at most one interferer at a time

$$\sum_{\substack{k=0 \\ k \neq l}}^{L} c_{\ell k} = 1, \quad \forall \; \ell \in \mathcal{L} \tag{5}$$

From (3)-(5), it can be seen that for each link $\ell \in \mathcal{L}$ only one of the $\{c_{\ell k}\}_{k \in \mathcal{L}_{\ell^-}}$ can be equal to 1. Thus, the transmission rate on link ℓ is given by

$$R_\ell = \sum_{\substack{m=0 \\ m \neq \ell}}^{L} \log\left(1 + c_{\ell m} SINR_{\ell m}\right) \tag{6}$$

where $c_{\ell m} \, \epsilon \, \{0, 1\}$ as defined in (3), and $SINR_{\ell m}$ is the Signal to Interference plus Noise Ratio at the receiver of the ℓ^{th} link after cancelling the transmission on the m^{th} link, given by

$$SINR_{\ell m} = \frac{G_{\ell\ell} p_\ell}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq \ell, m}}^{L} G_{\ell k} p_k + V_\ell}. \tag{7}$$

The system is slotted in time, indexed by t. Let $W_i(t)$ denote the backlog of packets at each node i at the end of slot t. The *differential backlog* of link $\ell = (i, j)$, at the beginning of slot t, is defined as [4]

$$D_\ell(t) := \begin{cases} \max\{0, W_i(t) - W_j(t)\}, & j \neq N \\ W_i(t), & j = N. \end{cases} \tag{8}$$

The traffic flow in each link during each time slot is based on the link capacities resulting from the power allocation $\{p_\ell\}_{\ell \in \mathcal{L}}$ and cancellation coefficients $\{c_{\ell k}\}_{k \in \mathcal{L}_{\ell^-}}$ at the beginning of the slot. The powers and cancellation coefficients of each link for slot t are determined by solving an optimization problem which maximizes the differential-backlog weighted sum-rate of the wireless network [4, 5, 1]

$\mathbf{\Pi_1}$

$$\max_{\substack{\{p_\ell\}_{\ell \in \mathcal{L}} \\ \{\{c_{\ell m}\}_{m \in \mathcal{L}_{\ell^-}}\}_{\ell \in \mathcal{L}}}} \sum_{\ell=1}^{L} D_\ell(t) \sum_{\substack{m=0 \\ m \neq \ell}}^{L} \log\left(1 + c_{\ell m} SINR_{\ell m}\right)$$

$$\begin{aligned} \text{s.t.} \quad & 0 \leq p_\ell \leq P \quad \forall \ell \in \mathcal{L}, \\ & c_{\ell k} \in \{0, 1\}, \quad \forall k \in \mathcal{L}_{\ell^-}, \; \forall \ell \in \mathcal{L}, \\ & \sum_{\substack{k=0 \\ k \neq l}}^{L} c_{\ell k} = 1 \qquad \forall \ell \in \mathcal{L} \\ & \frac{G_{\ell k} p_k}{\frac{1}{G_{sg}} \sum_{\substack{m=1 \\ m \neq k}}^{L} G_{\ell m} p_m + V_\ell} \geq T c_{\ell k} \\ & \qquad \forall k \in \mathcal{L}_{\ell^-} \backslash \{0\}, \; \forall \ell \in \mathcal{L} \end{aligned} \tag{9}$$

The last set of inequalities is most interesting, and the only one whose role has not been explained so far. Each of these inequalities is active when the corresponding $c_{\ell k} = 1$, and is trivially satisfied when $c_{\ell k} = 0$. When active (i.e., link ℓ chooses to cancel link k), it ensures that the transmission of link k can be reliably decoded at the receiver of link ℓ, i.e., the SINR at which the latter hears the transmission of link k is at least T, a modulation and coding - dependent, user-specified parameter.

Unfortunately, problem $\mathbf{\Pi_1}$ is computationally intractable. The following formal claim can be established by reduction to the plain BPPC problem considered in [1]:

Claim 1 *Joint back-pressure power control and interference cancellation problem* $\mathbf{\Pi_1}$ *is NP-hard.*

Proof: For $T > \bar{T}(\mathbf{G}, P) = \max_{\ell \in \mathcal{L}, k \in \mathcal{L}_{\ell^-}} \frac{G_{\ell k} P}{V_\ell}$, there is no choice of $\{p_\ell\}_{\ell \in \mathcal{L}}$ that can support $c_{\ell k} = 1$ for some ℓ and $k \neq 0$. Therefore, the optimal value of all cancellation coefficients has to be $\{c^*_{\ell 0} = 1\}_{\ell \in \mathcal{L}}$ and $\{\{c^*_{\ell k} = 0\}_{k \in \mathcal{L}_{\ell^-} \backslash \{0\}}\}_{\ell \in \mathcal{L}}$, which is equivalent to the case of no interference cancellation. Therefore, problem $\mathbf{\Pi_1}$ contains the BPPC problem in [1] as a special case; any instance of the latter, $\Pi'(\mathbf{G}, P)$, can be mapped to an instance of $\mathbf{\Pi_1}$, namely $\mathbf{\Pi_1}(\mathbf{G}, P, T)$ with $T > \bar{T}(\mathbf{G}, P)$. The result then follows from NP-hardness of BPPC [1].

2.1. Interval relaxation

In the optimization problem $\mathbf{\Pi_1}$, the cancellation coefficients $c_{\ell k}$ are restricted to binary values. Rewriting the interference cancellation constraints for any link $\ell \in \mathcal{L}$ and for some $m, n \in \mathcal{L}_{\ell^-}$ with $m \neq n$, we have

$$\begin{aligned} & \frac{G_{\ell m} p_m}{\sum_{\substack{j=1 \\ j \neq m}}^{L} \frac{G_{\ell j}}{G_{sg}} p_j + V_\ell} \geq T c_{\ell m} \quad \Rightarrow \quad \frac{G_{\ell m} p_m}{G_{\ell n} p_n} \geq \frac{T c_{\ell m}}{G_{sg}} \\ & \frac{G_{\ell n} p_n}{\sum_{\substack{j=1 \\ j \neq n}}^{L} \frac{G_{\ell j}}{G_{sg}} p_j + V_\ell} \geq T c_{\ell n} \quad \Rightarrow \quad \frac{G_{\ell n} p_n}{G_{\ell m} p_m} \geq \frac{T c_{\ell n}}{G_{sg}} \\ & \Rightarrow \frac{T c_{\ell m}}{G_{sg}} \leq \frac{G_{\ell m} p_m}{G_{\ell n} p_n} \leq \frac{G_{sg}}{T c_{\ell n}} \quad \Rightarrow \quad \boxed{c_{\ell m} \, c_{\ell n} \leq \frac{G_{sg}^2}{T^2}} \end{aligned} \tag{10}$$

It follows that, for high enough T (which is also required for reliable cancellation, and thus reliable detection of the underlying signal of interest), at least one of the two must be small. By the same token, in fact, all except possibly one of $\{c_{\ell n}\}_{n \in \mathcal{L}_{\ell -} \setminus \{0\}}$ must be small. Thus, even if we replace the binary $\{0, 1\}$ constraints on the cancellation coefficients with $[0, 1]$ interval constraints, the feasible region for these coefficients will not extend to the product space: it will be limited close to the axes. This is illustrated in Fig. 1, and it motivates relaxing $\{\{c_{\ell k} \in \{0,1\}\}_{k \in \mathcal{L}_{\ell -}}\}_{\ell \in \mathcal{L}}$ in (9) to $\{\{c_{\ell k} \in [0,1]\}_{k \in \mathcal{L}_{\ell -}}\}_{\ell \in \mathcal{L}}$. Once we have relaxed the cancellation coefficients to lie in $[0, 1]$, the summation constraint can also be modified to $\sum_{\substack{k=0\\k\neq l}}^{L} c_{\ell k} \leq 1,\ \forall \ell \in \mathcal{L}$

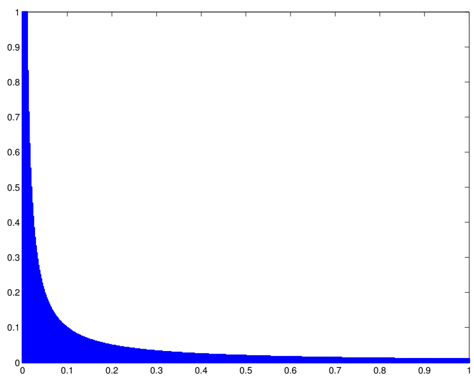

Fig. 1. **Illustration of feasible region after interval relaxation for two cancellation coefficients** $c_{\ell m}$ vs. $c_{\ell n}$, for $\frac{G_{sg}^2}{T^2} = 10^{-2}$.

3. CONVEX APPROXIMATION

The objective function in $\mathbf{\Pi_1}$ can be rewritten as

$$\max_{\substack{\{p_\ell\}_{\ell \in \mathcal{L}} \\ \{\{c_{\ell m}\}_{m \in \mathcal{L}_{\ell -}}\}_{\ell \in \mathcal{L}}}} \sum_{\ell=1}^{L} D_\ell(t) \Bigg[\sum_{\substack{m=0\\m\neq\ell}}^{L} \log \Big(G_{\ell\ell} p_\ell c_{\ell m} + \sum_{\substack{k=1\\k\neq\ell,m}}^{L} \frac{G_{\ell k}}{G_{sg}} p_k + V_\ell \Big) - \log \Big(\sum_{\substack{k=1\\k\neq\ell,m}}^{L} \frac{G_{\ell k}}{G_{sg}} p_k + V_\ell \Big) \Bigg] \quad (11)$$

Since the each term in the inner summation is a difference of two logarithmic functions of the optimization variables, the objective function is neither convex nor concave. In order to proceed, we will lower bound the link rates using the following tunable concave lower bound of $\log(1 + z)$ introduced in [3]

$$\log(1+z) \geq \alpha \log(z) + \beta \ \text{ for } \begin{cases} \alpha = \frac{z_o}{1+z_o}, \\ \beta = \log(1+z_o) - \frac{z_o}{1+z_o} \log(z_o) \end{cases} \quad (12)$$

Notice that the bound is tight at z_o; and as $z_o \to \infty$, the inequality becomes $\log(z) \leq \log(1 + z)$.

Applying (12) to $\mathbf{\Pi_1}$, the objective function can be bounded below as follows.

$$\sum_{\ell=1}^{L} D_\ell(t) \Bigg[\sum_{\substack{m=0\\m\neq\ell}}^{L} \alpha_{\ell m}^t \log\left(c_{\ell m} SINR_{\ell m}\right) + \beta_{\ell m}^t \Bigg] \quad (13)$$

The lower bound in (13) is still not concave in the variables p_ℓ and $c_{\ell k}$, since

$$\log(SINR_{\ell m}) \overset{(7)}{=} \log(G_{\ell\ell} p_\ell) - \log \left(\sum_{\substack{k=1\\k\neq\ell,m}}^{L} \left(\frac{G_{\ell k}}{G_{sg}}\right) p_k + V_\ell \right). \quad (14)$$

Introducing a logarithmic change of variables, $\tilde{p}_\ell := \log p_\ell$, $\tilde{c}_{\ell k} := \log c_{\ell k}$, and using (13), we can approximate[1] $\mathbf{\Pi_1}$ with

$\mathbf{\Pi_2}$

$$\max_{\substack{\{\tilde{p}_\ell\}_{\ell \in \mathcal{L}} \\ \{\{\tilde{c}_{\ell m}\}_{m \in \mathcal{L}_{\ell -}}\}_{\ell \in \mathcal{L}}}} \sum_{\ell=1}^{L} D_\ell(t) \Bigg[\sum_{\substack{m=0\\m\neq\ell}}^{L} \alpha_{\ell m}^t \Bigg(\tilde{G}_{\ell\ell} + \tilde{G}_{sg} + \tilde{p}_\ell - \log \left(\sum_{\substack{k=1\\k\neq\ell}}^{L} e^{\tilde{G}_{\ell k} + \tilde{p}_k} + e^{\tilde{V}_\ell} \right) + \tilde{c}_{\ell m} \Bigg) + \beta_{\ell m}^t \Bigg]$$

$$\begin{aligned} \text{s.t.} \quad & \tilde{p}_\ell \leq \tilde{P} := \log(P), \quad \forall \ell \in \mathcal{L}, \qquad (15) \\ & \tilde{c}_{\ell k} \in [\log(\varepsilon), 0], \quad k \in \mathcal{L}_{\ell -}, \forall \ell \in \mathcal{L}, \\ & \log \left(\sum_{\substack{k=0\\k\neq l}}^{L} e^{\tilde{c}_{\ell k}} \right) \leq \log(\varepsilon) \quad \forall \ell \in \mathcal{L}, \\ & \tilde{G}_{\ell k} + \tilde{G}_{sg} + \tilde{p}_k - \log \left(\sum_{\substack{k=1\\k\neq\ell}}^{L} e^{\tilde{G}_{\ell k} + \tilde{p}_k} + e^{\tilde{V}_\ell} \right) \geq \log(T) + \tilde{c}_{\ell k} \\ & \qquad \forall k \in \mathcal{L}_{\ell -}, \forall \ell \in \mathcal{L}. \end{aligned}$$

where $\tilde{G}_{\ell k} := \log\left(\frac{G_{\ell k}}{G_{sg}}\right)$, $\tilde{V}_\ell := \log(V_\ell)$ and ε is a small positive constant, which is introduced for avoiding numerical errors (since $\log(0) = -\infty$).

The objective function in $\mathbf{\Pi_2}$ is concave, since it is a sum of linear and concave functions of $\{\tilde{p}_\ell\}_{\ell \in \mathcal{L}}$ and $\{\{\tilde{c}_{\ell m}\}_{m \in \mathcal{L}_{\ell -}}\}_{\ell \in \mathcal{L}}$. The weights $D_\ell(t)$ and constants $\alpha_{\ell m}^t$ and $\beta_{\ell m}^t$ are nonnegative, cf. (8) and (12). Furthermore, all constraints in $\mathbf{\Pi_2}$ are convex with respect to optimization variables $\{\tilde{p}_\ell\}_{\ell \in \mathcal{L}}$ and $\{\{\tilde{c}_{\ell k}\}_{k \in \mathcal{L}_{\ell -}}\}_{\ell \in \mathcal{L}}$. Thus, $\mathbf{\Pi_2}$ is a convex optimization problem.

4. SUCCESSIVE APPROXIMATION ALGORITHM

The convex optimization problem $\mathbf{\Pi_2}$ can be solved using a variation of the batch successive approximation algorithm mentioned in [1]. Here, in the first iteration (n=1), we solve $\mathbf{\Pi_2}$ with $\alpha_{\ell m}^t(n) = 1$ and $\beta_{\ell m}^t(n) = 0, \forall m \in \mathcal{L}_{\ell -}, \forall \ell \in \mathcal{L}$. Then, for higher iterations, using (12), (16), the values of $\alpha_{\ell m}^t(n)$ and $\beta_{\ell m}^t(n), \forall m \in \mathcal{L}_{\ell -}\ \forall \ell \in \mathcal{L}$ are updated using the optimal values $\{\tilde{p}_\ell^*\}_{\ell \in \mathcal{L}}$, $\{\{\tilde{c}_{\ell k}^*\}_{k \in \mathcal{L}_{\ell -}}\}_{\ell \in \mathcal{L}}$ obtained from the previous iteration, in order to tighten the individual link rate lower bounds so that the bounds coincide with the link rates at $\{\tilde{p}_\ell(t)\}_{\ell \in \mathcal{L}}$ and $\{\{\tilde{c}_{\ell k}(t)\}_{k \in \mathcal{L}_{\ell -}}\}_{\ell \in \mathcal{L}}$) when the objective function value converges. The steps of the algorithm are listed below.

[1] Even though we are maximizing a lower bound to the original objective function in $\mathbf{\Pi_1}$, the solution to $\mathbf{\Pi_2}$ is only an approximation which may yield a higher value of the original objective in $\mathbf{\Pi_1}$, due to the interval relaxation introduced for $\{c_{\ell k}\}_{k \in \mathcal{L}_{\ell -}}, \forall \ell \in \mathcal{L}$.

Algorithm 1 Successive Approximation BPPC-IC:

1. ***Initialization***: *For each time slot* t*, calculate* $D_\ell(t)$*, reset iteration counter* $n = 1$*, and set* $\alpha^t_{\ell m}(n) = 1$ *and* $\beta^t_{\ell m}(n) = 0, \forall\, m \in \mathcal{L}_{\ell-},\ \forall \ell \in \mathcal{L}$.
2. **repeat:**
3. ***Optimization Step***: *Solve* $\mathbf{\Pi_2} \rightarrow \{\tilde{p}^*_\ell\}_{\ell\in\mathcal{L}}$, $\{\{\tilde{c}^*_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$.
4. ***Update step***: *Update* $\alpha^t_{\ell m}(n+1)$, $\beta^t_{\ell m}(n+1)$ *according to* (12) *for* $\forall \ell \in \mathcal{L}$

$$z_0 = \left(\frac{\left(G_{\ell\ell}\, p^*_\ell\,(t,n)\right) c^*_{\ell m}\,(t,n)}{\frac{1}{G_{sg}} \sum_{\substack{k=1\\ k\neq \ell, m}}^{L} G_{\ell k}\, p^*_k(t,n) + V_\ell} \right), \forall m \in \mathcal{L}_{\ell-}, \tag{16}$$

where $p^*_\ell\,(t,n) = e^{\tilde{p}^*_\ell\,(t,n)}$ *and* $c^*_{\ell m}\,(t,n) = e^{\tilde{c}^*_{\ell m}\,(t,n)}$, $\forall m \in \mathcal{L}_{\ell-}, \forall \ell \in \mathcal{L}$

5. $n = n + 1$
6. **until** *convergence of the objective value (within* ε*- accuracy)*
7. ***Rounding step***: $c_{\ell k} = 1$, $c_{\ell m} = 0, \forall m \neq k, m \in \mathcal{L}_{\ell-}$ *where* $k = \arg\max_{m\in\mathcal{L}_{\ell-}} \left(\frac{\left(G_{\ell\ell}\, p^*_\ell\,(t,n)\right) c^*_{\ell m}\,(t,n)}{\frac{1}{G_{sg}} \sum_{\substack{j=1\\ j\neq \ell, m}}^{L} G_{\ell j}\, p^*_j(t,n) + V_\ell} \right)$

4.1. Random Interference Cancellation Policy

As mentioned earlier, when the control space is finite, it was shown in [5] that a simple randomized policy is - surprisingly - optimal from a throughput point of view (the price paid is excess delay). This motivates comparing our joint power control and interference cancellation algorithm (called BPPC-IC in the sequel) to what we will refer to as BPPC-RIC (for Randomized Interference Cancellation), a simpler strategy that essentially requires solving a power control (BPPC-type) problem per slot. BPPC-RIC works as follows. At the start of each time slot, the receiver of each link $\ell \in \mathcal{L}$ chooses any other link at random, say $k_\ell \in \mathcal{L}_{\ell-}$ for cancellation, i.e., sets $\{c_{\ell k}\}_{k\in\{0,k_\ell\}} \in [\varepsilon, 1]$ and fixes the remaining coefficients i.e. $\{c_{\ell k}\}_{k\in\mathcal{L}_{\ell-}\setminus\{0,k_\ell\}}$, $\forall \ell \in \mathcal{L}$ to zero (ε for avoiding numerical errors). Then it solves optimization problem $\mathbf{\Pi_2}$ with the above-mentioned constraints on $c_{\ell k}$, to obtain the optimal values of $\{\tilde{p}^*_\ell(t)\}_{\ell\in\mathcal{L}}$ and $\{\{\tilde{c}^*_{\ell m}(t)\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$. The value of the objective function for the present time slot attained by $\{\tilde{p}^*_\ell(t)\}_{\ell\in\mathcal{L}}$ and $\{\{\tilde{c}^*_{\ell m}(t)\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$ is compared to the value of the objective function for the present time slot achieved by $\{\tilde{p}^{opt}_\ell(t-1)\}_{\ell\in\mathcal{L}}$ and $\{\{\tilde{c}^{opt}_{\ell m}(t-1)\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$ (power allocation and cancellation coefficients for slot $t-1$), and the pair resulting in the higher value is chosen for the present time slot. This effectively explores 2^L cancellation configurations per slot (notice that each receiver retains the freedom to choose between cancelling the randomly chosen link or not), thus being in fact in-between BPPC and BPPC-IC in terms of complexity.

5. SIMULATION RESULTS

In our simulations, we compare the performance of three algorithms: BPPC-IC, BPPC-RIC, and BPPC-noIC (that is, plain BPPC as in [1]). For BPPC-noIC, we employ the batch SA algorithm in [1]. The comparisons assess maximum stable throughput, and source and relay backlogs for each algorithm. The simulation parameters are specified in Table 1. The channel is assumed to have only propagation path-loss, i.e. $G_{\ell k} = (d_{\ell k})^{-\alpha}$, where α is the path-loss exponent. Figure 2 illustrates the 4-node and 5-node wireless network topologies used for simulation. The nodes are denoted by filled circles and the solid lines denote the bi-directional link between node-pairs.

Table 1. Simulation Parameters

Symbol	Description	Value
N	Number of nodes	4 / 5
P	Max. power per link	5 W
V	Noise variance	$10^{-12}\,W$
G_{sg}	Spreading / Beam-forming gain	128
T	SINR threshold for decoding	30 dB
ε	Tolerance parameter	0.1

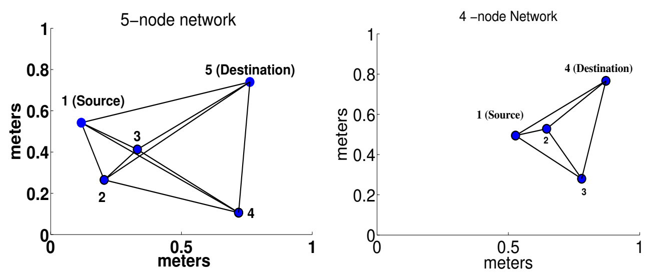

Fig. 2. **Illustration of the Network Topology**

Figures 3-4 highlight the queue-length stabilization properties for interference cancellation policies. In Figure 3, the simulation has been done for $N = 4$, and input traffic of 10 packets per slot. During the first 25 slots, the network is controlled by BPPC-noIC, while for the next 25 slots BPPC-IC is employed. It can be seen that the network is unstable under BPPC-noIC, resulting in the backlogs increasing with time. However, the BPPC-IC algorithm quickly stabilizes the network, bringing all backlogs down to stable territory. It can be seen that the average throughput increases to 10 which is the arrival rate, once BPPC-IC takes over.

Figure 4 compares the stabilization properties of BPPC-IC and BPPC-RIC policies. Here the simulation has been done for $N = 5$ and input traffic = 11 packets per slot. During the first 50 slots, the network evolves under control of BPPC-noIC, followed by BPPC-IC in one case and BPPC-RIC in the other, for the next 25 time slots. From the plots, it is clear that both BPPC-IC and BPPC-RIC stabilize the network backlogs. But the steady state average source backlog for BPPC-RIC is higher than for BPPC-IC, which means that the network is more congested under BPPC-RIC than under BPPC-IC. Furthermore, it can also be seen from Figure 4 that BPPC-IC stabilizes the network much more quickly that BPPC-RIC.

Table 2. Maximum Stable Throughput comparison

IC policy	4-node network	5-node network
BPPC-noIC	7	8
BPPC-RIC	9	12
BPPC-IC	10	14

In Table 2, the maximum stable throughput for the different algorithms in a 4-node and 5-node wireless network are shown. It should be noted that there is a significant increase in the maximum stable throughput (up to 42.8% for 4-node and 75% for 5- node case) when interference cancellation is employed at the receiver of each link. From these results, it can be inferred that interference cancellation enables the network to handle significantly more traffic.

Figure 5 compares the source-node backlog of BPPC-IC with that of BPPC-RIC for a 5-node network with input traffic of 14 packets per slot. From table 2, it can be seen that the input traffic is greater than the maximum stable throughput for BPPC-RIC. Therefore, the source backlog increases with time for BPPC-RIC. BPPC-IC, on the other hand, manages to contain the source backlog in this case. This plot corroborates the data provided in table 2, underscoring that the proposed BPPC-IC policy enables the network to handle considerably higher input traffic than BPPC-noIC or even BPPC-RIC.

In addition to higher throughput, BPPC-IC also features lower average delay relative to BPPC-RIC and BPPC-noIC. This is a consequence of Little's theorem applied to the whole network, taking into account that BPPC-IC has the lowest backlogs among the three algorithms considered. This and other issues will be explored in the journal version.

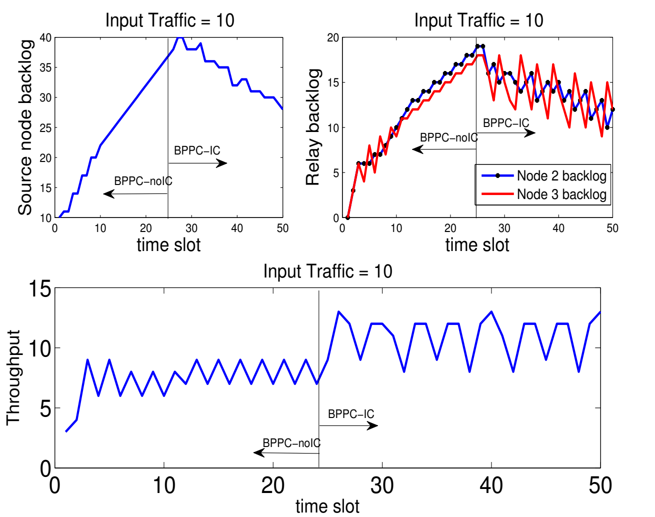

Fig. 3. **Network stability and throughput performance of BPPC-noIC and BPPC-IC policies in a 4-node network for $\lambda = 10$**

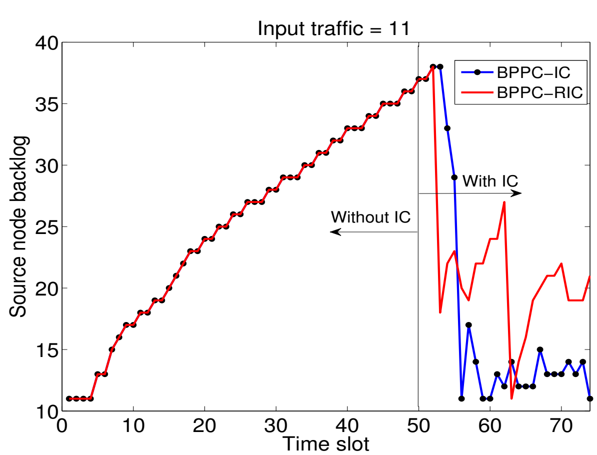

Fig. 4. **Comparison of source node backlog stability of BPPC-IC and BPPC-RIC policies in a wireless multi-hop network with 5 nodes for Input traffic = 11**

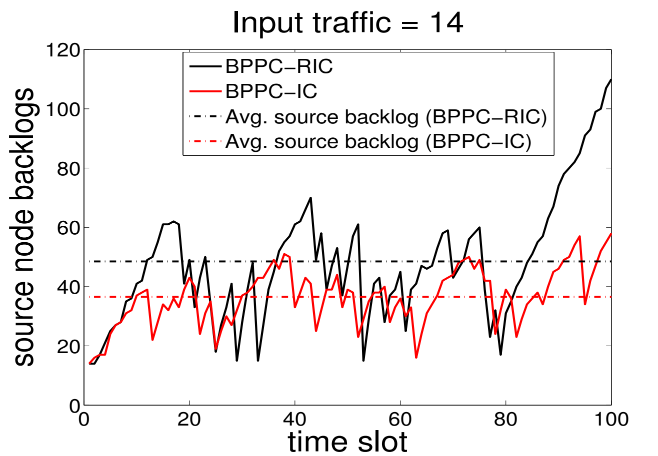

Fig. 5. **Comparison of source backlogs for BPPC-IC and BPPC-RIC policies for 5-node wireless network with input traffic 14**

6. CONCLUSION

We have considered joint back-pressure power control and interference cancellation for throughput maximization in wireless multi-hop networks. The problem is NP-hard, but we developed a formulation leading to a suitable convex approximation. The main conclusion is that joint optimization of power control and interference cancellation pays off: it offers a significant increase in the maximum stable throughput, in addition to lowering transport delay by better controlling the queueing delays in the network. The problem formulation and its approximation appears to be useful in a number of related contexts that are currently under investigation.

7. REFERENCES

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