Deterministic Binary Sequences for Modulated Wideband Converter

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Abstract—**The modulated wideband converter (MWC) is a promising spectrum blind, sub-Nyquist multi-channel sampling scheme for sparse multi-band signals. In an MWC, the input analog signal is modulated by a bank of periodic binary waveforms, low-pass filtered and then down sampled uniformly at a low rate. One important issue in the design and implementation of an MWC system is the selection of binary waveforms, which impacts the stability of sparse reconstruction. In this paper, we propose to construct the binary pattern with a circulant structure, in which each row is a random cyclic shift of a single deterministic sequence or a pair of complementary sequences. Such operators have hardware friendly structures and fast computation in recovery. They are incoherent with the FFT matrix and the corresponding sampling operators satisfy the restricted isometry property with sub-optimal bounds. Some simulation results are included to demonstrate the validity of the proposed sampling operators.**

I. INTRODUCTION

The modulated wideband converter (MWC) proposed by Mishali and Eldar [1], [2] is a multi-channel, uniform sub-Nyquist sampling system for sparse multi-band signals. It holds great potential in applications such as communications, radar and sonar. Consider an analog signal $x(t)$ whose Fourier transform $X(f)$ is bandlimited in $\left[-\frac{f_{NYQ}}{2}, \frac{f_{NYQ}}{2}\right]$ Hz. Assume that $x(t)$ has only K active disjoint frequency bands, each of which has a maximum bandwidth of B Hz. $x(t)$ is said to be a *sparse multi-band signal* if $KB \ll f_{NYQ}$. Figure 1 shows the implementation diagram of an m-channel MWC. In each channel, the input signal is first modulated by a periodic waveform $p_i(t)$, $(i = 0, 1, \cdots m-1)$, low-pass filtered by $h(t)$ and then decimated at the rate of $1/T$ to produce $y_i[n]$. For ease of presentation, we consider the basic configuration of an MWC in which $p_i(t)$ is chosen as the sign alteration waveforms with period of T [1]. Within each sampling period T, there are M intervals of length T/M each and $p_i(t)$ takes the following form [2]

$$p_i(t) = s_{ik}, \quad k\frac{T}{M} \le t \le (k+1)\frac{T}{M} \tag{1}$$

with $s_{ik} \in \{1, -1\}$. Reconstruction of $x(t)$ from $y_i[n]$ ($0 \le i \le m-1$) exploits the recently emerged compressed sensing theory [3], [4], which searches for the sparsest solution of a parameterized linear equation. Details can be found in [5].

The selection of an $m \times M$ binary pattern $\mathbf{S} = \{s_{ik}\}$ ($0 \le i \le m-1, 0 \le k \le M-1$) is crucial to the performance of an MWC. From the theoretical perspective, $\mathbf{S}$ needs to offer stable reconstruction performance. From the implementation perspective, it is desirable that $\mathbf{S}$ requires the minimal number of hardware elements with flexible choice of m and M. In [2], $\mathbf{S}$ is constructed from a full random Bernoulli operator. Although such an operator offers near optimal theoretical guarantee, it requires mM flip-flops to implement [2]. To simplify the design, [2] proposed a mixed scheme, in which the first $r < m$ rows of $\mathbf{S}$ are Bernoulli matrices, and the remaining rows
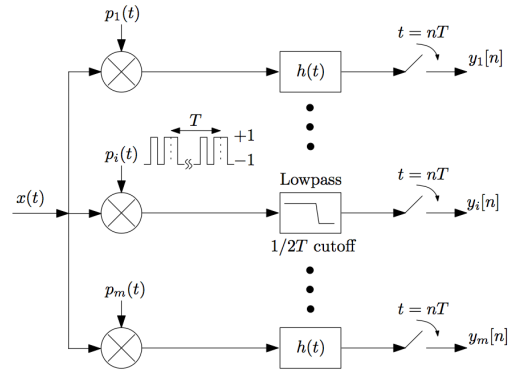

Fig. 1. Implementation diagram of the modulated wideband converter [2].

are cyclic shifts of them. Such a scheme needs only rM flip-flops. However, the theoretical performance guarantee of these operators is unknown. Besides, simulation results in [2] indicate performance degradation when r is small. In [6], deterministic operators using maximal, Gold and Kasami codes have been used. However, these codes only exist when $M = 2^\beta - 1$ ($\beta \in \mathbb{Z}^+$), which is not flexible for practical applications.

In this paper, we propose to construct $\mathbf{S}$ with a circulant structure, where each of its row is obtained by random cyclic shift of a single sequence (e.g., the m-sequence or the Legendre sequence) or a pair of cyclic complementary sequences. Due to their circulant structures, the proposed binary patterns are memory efficient with simple hardware implementation. They also offer fast calculation in reconstruction as the matrix multiplication requires only $\mathcal{O}(M \log M)$ operations. Moreover, they exist for a large choice of M. It can be shown that the corresponding sampling operator satisfies the restricted isometry property, which guarantees stable reconstruction in sparse optimization. Experimental results have shown that the proposed binary patterns can offer nearly the same performance as that of the random Bernoulli operator at much lower complexity.

The rest of the paper is organized as follows. In Section II, we briefly review mathematical formulation of the MWC system and related theory in compressed sensing. Section III presents our proposed binary patterns with circulant structure using a single sequence or a pair of complementary sequences. Their restricted isometry properties have been analyzed. Experimental results are shown in Section IV, followed by conclusions in Section V.

Notations: Throughout this paper, vectors are denoted by boldfaced lowercase letters and matrices by boldfaced uppercase characters. If their sizes are not clear from the context, subscripts are provided. For

a matrix $\mathbf{A}$, $\mathbf{A}(i,:)$ denotes its i-th row and $\mathbf{A}_{k,l}$ represent its (k,l)-th element. $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and the Hermitian transpose of $\mathbf{A}$, respectively. $\mathbf{I}$ is the identity matrix and $\mathbf{F}_M$ is an $M \times M$ FFT matrix with $\mathbf{F}_{k,l} = e^{-j\frac{2\pi kl}{M}}$. For an $M \times M$ matrix $\mathbf{A}$, $\mu(\mathbf{A})$ denotes its coherence parameter, i.e., the maximum magnitude of its elements $\mu(\mathbf{A}) = \max_{0 \le k,l \le M-1} |\mathbf{A}_{k,l}|$.

II. REVIEW

Consider an m-channel MWC system in Figure 1. Let $\mathbf{y}[n]$ denote the $m \times 1$ sampled vector

$$\mathbf{y}[n] = \begin{bmatrix} y_0[n] & y_1[n] & \cdots & y_{M-1}[n] \end{bmatrix}^T .$$

Define $\mathbf{y}(f)$ as its discrete-time Fourier transform, i.e., $\mathbf{y}(f) = \sum_{n=-\infty}^{\infty} \mathbf{y}[n]e^{-j2\pi fnT}$. Also, define $z_i(f)$ ($i = 0, \cdots, M-1$) as a slice of $X(f)$ with bandwidth of $\frac{1}{T}$

$$z_i(f) = X(f + (i - M_0)/T), \quad |f| \le \frac{1}{2T}$$

in which $M_0 = \lfloor M/2 \rfloor$. Let $\mathbf{z}(f)$ denote the $M \times 1$ vector $\mathbf{z}(f) = \begin{bmatrix} z_0(f) & z_1(f) & \cdots & z_{M-1}(f) \end{bmatrix}^T$, the input-output relation in an MWC system can be written as [2]

$$\mathbf{y}(f) = \mathbf{SF}\left(\mathbf{PDz}(f)\right), \quad |f| \le \frac{1}{2T}, \tag{2}$$

where $\mathbf{F}$ is an $M \times M$ FFT matrix, $\mathbf{P}$ is a permutation matrix and $\mathbf{D}$ is diagonal matrix which accounts for the decay of the Fourier transform of $p_i(t)$ at high frequencies. In general, (2) is an under-determined linear equation. But as $X(f)$ is a multi-band sparse signal, $\mathbf{z}(f)$ is a sparse vector with only $K \ll M$ active elements. Based on sparse reconstruction in compressed sensing theory [3], [4], $x(t)$ can be recovered from $\mathbf{y}[n]$ by first identifying the spectral support and then reconstructed using a close-form expression [5].

Note that as $\mathbf{PDz}(f)$ is also a sparse vector with K non-zero elements, we will only focus on the matrix product $\mathbf{SF}$ hereafter. Let us consider the following simplified equation

$$\mathbf{v} = \mathbf{SFu}, \tag{3}$$

in which $\mathbf{u}$ is an $M \times 1$ sparse vector with only K nonzero elements and $\mathbf{v}$ is an $m \times 1$ vector. According to the compressed sensing theory [3], [4], $\mathbf{u}$ can be reconstructed from $\mathbf{v}$ stably when the operator $\Phi = \frac{1}{\sqrt{mM}}\mathbf{SF}$ satisfies the restricted isometry property (RIP):

Definition 1 (RIP): An $m \times M$ matrix Φ with normalized columns is said to satisfy the RIP with parameters (K, δ) ($\delta \in (0,1)$) if [3], [4]

$$(1-\delta)\|\mathbf{u}\|^2 \le \|\Phi\mathbf{u}\|^2 \le (1+\delta)\|\mathbf{u}\|^2 \tag{4}$$

for all K-sparse vectors of $\mathbf{u}$.

It is well known if $\mathbf{S}$ is a full-random Bernoulli matrix, then $\Phi = \frac{1}{\sqrt{mM}}\mathbf{SF}$ satisfies the RIP when $m \ge \mathcal{O}(K \log(M/K))$ [3], [4]. However, full random matrix incurs large memory in storage and high cost in implementation. Another class of operators satisfying the RIP is the randomly subsampled unitary matrix, as presented in the following theorem [7].

Theorem 1 (RIP of a partial unitary matrix): Consider an $m \times M$ matrix $\Phi = \frac{1}{\sqrt{m}}\mathbf{R}_\Omega\mathbf{U}$, where $\frac{1}{\sqrt{m}}$ is a normalizing coefficient, $\mathbf{R}_\Omega$ is a random sampling operator which selects m samples out of M ones uniformly at random, and $\mathbf{U}$ is an $M \times M$ unitary matrix satisfying $\mathbf{U}^*\mathbf{U} = M\mathbf{I}_M$. Φ satisfies the RIP with high probability when [7]

$$M \ge \mathcal{O}\left(\mu^2(\mathbf{U})K \log^4 M\right), \tag{5}$$

in which $\mu(\mathbf{U})$ represents the maximum magnitude of the elements in $\mathbf{U}$, i.e., $\mu(\mathbf{U}) = \max_{k,l} |\mathbf{U}_{k,l}|$.

Note that the unitary property of $\mathbf{U}$ implies that $1 \le \mu(\mathbf{U}) \le \sqrt{M}$. Hence, when $\mu(\mathbf{U}) = \mathcal{O}(1)$, we can get the sub-optimal bound $M \ge \mathcal{O}(K \log^4 M)$. In the next section, we will develop deterministic binary sequnces for the MWC system based on the above Theorem.

III. BINARY PATTERNS CONSTRUCTED FROM DETERMINISTIC SEQUNCES

A. Construction from a single sequence

In this subsection, we consider $\mathbf{S}$ constructed from a partial circulant matrix with the following form

$$\mathbf{S} = \mathbf{R}_\Omega\mathbf{C} \tag{6}$$

where $\mathbf{R}_\Omega$ is a random subsampling operator, which selects m rows out of M ones uniformly at random. $\mathbf{C}$ is a circulant operator that can be expressed as

$$\mathbf{C} = \begin{bmatrix} c_0 & c_1 & \cdots & c_{M-1} \\ c_{M-1} & c_0 & \cdots & c_1 \\ \vdots & \vdots & \ddots & \vdots \\ c_1 & a_2 & \cdots & c_0 \end{bmatrix}, \tag{7}$$

in which $\mathbf{c} = \begin{bmatrix} c_0, c_1, \cdots, c_{M-1} \end{bmatrix}$ is a *deterministic* sequence. According to [2], such a sampling operator can be easily implemented in hardware with only M flip-flops.

It is well known that an $M \times M$ real-coefficient circulant matrix can be factorized into

$$\mathbf{C} = \frac{1}{M}\mathbf{F}\text{diag}(\hat{\mathbf{c}})\mathbf{F}^H, \tag{8}$$

in which $\mathbf{F}$ is the $M \times M$ FFT matrix, and the $1 \times M$ row vector $\hat{\mathbf{c}} = \begin{bmatrix} \hat{c}_0, \hat{c}_1, \cdots, \hat{c}_{M-1} \end{bmatrix}$ is the IFFT of $\mathbf{c}$, i.e., $\hat{\mathbf{c}} = \mathbf{c}\mathbf{F}^H$. Hence, the matrix product $\mathbf{SF}$ can be expressed as

$$\mathbf{SF} = \mathbf{R}_\Omega\mathbf{F}\text{diag}(\hat{\mathbf{c}}). \tag{9}$$

To make use of Theorem 1, $\frac{1}{\sqrt{M}}\mathbf{F}\text{diag}(\hat{\mathbf{c}})$ needs to be a unitary matrix, which implies that each element of $\hat{\mathbf{c}}$ has the same magnitude, i.e., $|\hat{c}_i| = \sqrt{M}$. However, the only known binary sequence with constant FFT magnitudes is $\mathbf{c} = \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix}$ or its cyclic shift. Thus, we consider binary sequences whose FFT coefficients are *nearly* flat. Two popular choices are the maximum length sequence and the Legendre sequence [8]. Specifically,

- m-**sequence**: The maximum length sequence exists for $M = 2^\beta - 1$ ($\beta \in \mathbb{Z}^+$). It can be easily implemented using β shift registers and has found wide applications in spread-spectrum communications and measurement of impulse response. If $\mathbf{c}$ is a maximum length sequence, then $|\hat{c}_i|$ can be expressed as

$$|\hat{c}_i| = \begin{cases} 1 & i = 0; \\ \sqrt{M+1} & 1 \le i \le M-1. \end{cases} \tag{10}$$

- **Legendre sequence**: A Legendre sequence $\mathbf{c}$ has length M (M prime) and is given by the Legendre symbol [8]

$$c_0 = 1,$$
$$c_i = \begin{cases} 1 & \textit{if } i \textit{ is a square (mod } M) \\ -1 & \textit{if } i \textit{ is a non-square (mod } M). \end{cases} \quad i > 1 \tag{11}$$

For such a sequence, its IFFT coefficients $\hat{c}_i$ ($0 \le i \le M-1$) take the form of [8]

$$\hat{c}_0 = 1,$$
$$\hat{c}_i = \begin{cases} 1 + c_i\sqrt{M} & \textit{if } M = 1 \text{ (mod 4)} \\ 1 + jc_i\sqrt{M} & \textit{if } M = 3 \text{ (mod 4)} \end{cases} \tag{12}$$

It is clear that both the maximum length sequence and the Legendre sequence have a (nearly) flat spectrum except for $\hat{c}_0$. By exploiting such a property, we could arrive at the following theorem:

Theorem 2: Consider an sampling operator $\Phi = \frac{1}{\sqrt{mM}}\mathbf{SF}$, in which $\mathbf{F}$ is an $M \times M$ FFT matrix and $\mathbf{S}$ takes the form of (6), where $\mathbf{c}$ is a maximum length sequence or the Legendre sequence. For all K-sparse vector $\mathbf{u} = [u_0, u_1, \cdots, u_{M-1}]$ with $u_0 = 0$, Eq. (4) holds with high probability provided that $m \geq \mathcal{O}(K \log^4 M)$.

The proof of the above theorem can be achieved by using (10), (12) and Theorem 1. Details are omitted due to lack of space. Note that when $\mathbf{S}$ is constructed from the maximum-length sequence or the Legendre sequence, Theorem 2 implies that stable reconstruction can be achieved as long as $X(f) = 0$ in $|f| < \frac{1}{2T}$. When $X(f)$ is non-zero in $|f| < \frac{1}{2T}$, we can first apply a lowpass filter with cut-off frequency of $\frac{1}{2T}$ to $x(t)$ first and then sample it at the rate of $1/T$. Combined with the samples from MWC, $x(t)$ can then be recovered.

B. Construction from a periodic complementary pair

Both the maximum-length sequence and the Legendre sequence only exist for odd M. In this section, we consider the construction of $\mathbf{S}$ when M is even. To this end, we first present the definition of periodic complementary sequences (PCS) [9]–[11].

Definition 2: For a length-M, real-valued sequence $\mathbf{c} = [c_0, c_1, \cdots, c_{M-1}]$, its periodic autocorrelation $R_{\mathbf{c}}(l)$ $(0 \leq l \leq M-1)$ is given by

$$R_{\mathbf{c}}(l) = \sum_{k=0}^{M-1} c_k \cdot c_{\text{mod }(k+l,M)}. \tag{13}$$

Let $\mathbf{a}$ and $\mathbf{b}$ be a pair of length-M bipolar sequences. They are said to form a *periodic complementary pair* (PCP) [9], [11] if

$$R_{\mathbf{a}}(l) + R_{\mathbf{b}}(l) = 0, \quad 1 \leq l \leq M-1. \tag{14}$$

$\mathbf{a}$ (or $\mathbf{b}$) is called as a *periodic complementary sequence* (PCS).

It is known that periodic complementary sequences exist for $M = 2^{\kappa_1} 10^{\kappa_2} 26^{\kappa_3}$, $M = 2^{\kappa_1} 34^{\kappa_2}$ or $M = 2^{\kappa_1} 50^{\kappa_2}$ with κ_i $(1 \leq i \leq 3)$ being non-negative integers [11]. It is worth mentioning that a periodic complementary sequence is also *nearly flat* in the FFT domain. To see this, let $\mathbf{a}$ and $\mathbf{b}$ be a PCP and define $\hat{\mathbf{a}} = \mathbf{aF}$ and $\hat{\mathbf{b}} = \mathbf{bF}$. From (14), it can be shown that [9]

$$|\hat{a}_k|^2 + |\hat{b}_k|^2 = 2M, \quad 0 \leq k \leq M-1, \tag{15}$$

in which $\hat{a}_k$ and $\hat{b}_k$ represent the k-th element of $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, respectively. Therefore,

$$|\hat{a}_k| < \sqrt{2M} \quad \text{and} \quad |\hat{b}_k| < \sqrt{2M}, \quad 0 \leq k \leq M-1. \tag{16}$$

In Theorem 3, we will use this property to derive the coherence bound.

We now move on to consider the construction of $\mathbf{S}$ using two circulant cores. Let $\mathbf{a}$ and $\mathbf{b}$ be a PCP of length-$M/2$ and define $\mathbf{A}$ and $\mathbf{B}$ as two $\frac{M}{2} \times \frac{M}{2}$ circulant matrices whose first rows are $\mathbf{a}$ and $\mathbf{b}$, respectively. Eq. (14) implies that an $M \times M$ operator $\mathbf{G}$ given below is a binary orthogonal matrix [12]:

$$\mathbf{G} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & -\mathbf{A}^T \end{bmatrix}. \tag{17}$$

Based on (17), we propose the following binary pattern $\mathbf{S}$:

$$\mathbf{S} = \mathbf{R}_\Omega \mathbf{G}\tilde{\mathbf{P}}, \tag{18}$$

in which $\mathbf{R}_\Omega$ is the same as that in (6), $\mathbf{G}$ is given by (17) and $\tilde{\mathbf{P}}$ is a permutation matrix so that for a vector $\mathbf{c} = [c_0, c_1, , \cdots, c_{M-1}]$,

$$[c_0, \cdots, c_{M/2-1}, c_{M/2}, \cdots c_{M-1}]\tilde{\mathbf{P}}$$
$$= [c_0, c_{M/2}, c_1, c_{M/2+1}, \cdots, c_{M/2-1}, c_{M-1}],$$

i.e., it interleaves the first $M/2$ elements and the last $M/2$ elements of $\mathbf{c}$. The following Lemma presents some properties of the product matrix $\tilde{\mathbf{G}} = \mathbf{G}\tilde{\mathbf{P}}$:

Lemma 1: Consider $\tilde{\mathbf{G}} = \mathbf{G}\tilde{\mathbf{P}}$, in which $\mathbf{G}$ and $\tilde{\mathbf{P}}$ are the same as in (18). $\tilde{\mathbf{G}}$ has the following properties:

- $\tilde{\mathbf{G}}$ is an orthogonal matrix satisfying $\tilde{\mathbf{G}}\tilde{\mathbf{G}} = M\mathbf{I}_M$.
- $\tilde{\mathbf{G}}$ has a circulant structure. Specifically, $\tilde{\mathbf{G}}(k,:)$ and $\tilde{\mathbf{G}}(k+M/2,:)$ $(1 \leq k \leq M/2-1)$ are respectively, the cyclic shift of $\tilde{\mathbf{G}}(0,:)$ and $\tilde{\mathbf{G}}(M/2,:)$ to the right by displacement of $2k$, i.e., the following relations hold

$$\tilde{\mathbf{G}}_{k,l} = \tilde{\mathbf{G}}_{0,\text{ mod }(2k+l,M)} \tag{19}$$
$$\tilde{\mathbf{G}}_{k+M/2,l} = \tilde{\mathbf{G}}_{M/2,\text{ mod }(2k+l,M)}. \tag{20}$$

- Each row of $\tilde{\mathbf{G}}$ is a periodic complementary sequence.

Sketch of the proof: The orthogonal property of $\tilde{\mathbf{G}}$ is straightforward due to the orthogonal property of $\mathbf{G}$ and $\tilde{\mathbf{P}}$. The circulant structure of $\tilde{\mathbf{G}}$ can be obtained from the definitions of $\mathbf{G}$ and $\tilde{\mathbf{P}}$. To prove that each row of $\tilde{\mathbf{G}}$ is a PCS, we need the following two facts [11]: (*i*) If $\mathbf{a}$ and $\mathbf{b}$ form a PCP, then their individual cyclic shifts by any displacement l will also produce a PCP; and (*ii*) If $\mathbf{a}$ and $\mathbf{b}$ form a PCP with length of $M/2$, by interleaving them, one can get a new PCS with length of M.

By exploiting Lemma 1, eq.(16) and Theorem 1, the following theorem can be derived:

Theorem 3: Consider an $m \times M$ matrix $\Phi = \frac{1}{\sqrt{mM}}\mathbf{SF}$, in which $\mathbf{S}$ is given by (18) and $\mathbf{F}$ is the $M \times M$ FFT matrix. Φ satisfies the RIP with high probability when $m \geq \mathcal{O}(K \log^4 M)$.

Detailed proof of Lemma 1 and Theorem 3 will be given in the journal version of this paper. Note that due to the structure of $\mathbf{G}$ in (17), only M flip-flops are required to implement $\mathbf{S}$ in (18). Besides, unlike the m-sequence and the Legendre sequence, there is no additional processing of the signal $X(f)$ in $|f| < \frac{1}{2T}$ when $\mathbf{S}$ is constructed from a PCP.

IV. SIMULATIONS RESULTS

Extensive simulations have been carried out to evaluate the performance of the proposed binary patterns. Due to lack of space, we only present the results using the Legendre sequence. The experimental setup is very similar to that in [2]. Specifically, the signal $x(t)$ has $f_{NYQ} = 10$ GHz with 3 pairs of active bands (i.e, $K = 6$), each of width $B = 50$ MHz, constructed as follows

$$x(t) = \sum_{i=1}^{3} \sqrt{E_i B}\,\text{sinc}(B(t - \tau_i)) \cos(2\pi f_i(t - \tau_i)), \tag{21}$$

with $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. The energy coefficients are $E_i = \{1, 2, 3\}$ and the time offsets are $\tau_i = \{0.4, 0.7, 0.2\}$. The frequency components f_i is selected uinformly at random from $[f_{NYQ}/2, f_{NYQ}/2]$. In [2], M is selected as 195. Here, we choose $M = 197$, the smallest prime number greater than 195 so that the Legendre sequence can be used. Just as in [2], we assume that $x(t)$ is corrupted by white Gaussian noise and 500 test signals have been evaluated. The reconstruction algorithm is based on that proposed in [5].

We first present the performance of Legendre sequence-based sampling operators for different number of channels with m ranging

from 20 to 100, and different input signal to noise ratio (SNR), ranging from -20 dB to 30 dB. For comparison purposes, the results of full-random binary pattern (i.e., when $\mathbf{S}$ is a Bernoulli matrix) are also included, as shown in Figure 2. One can observe that the proposed sampling operator using the Legendre sequence offers very similar performance to that of the full binary sampling operator at much lower implementation cost. Next, we compare our proposed sampling operators with the mixed scheme proposed in [2]. Specifically, in the mixed scheme, the first r rows are full random Bernoulli operators. Then, the i-th row ($r \leq i \leq m - 1$) is five cyclic shifts (to the right) of the $(i - r)$-th row. This mixed scheme requires rM flip-flops, while our proposed sampling operator needs only M ones. Figure 3 presents the reconstruction performance of different binary patterns with $m = 49$ and $M = 197$. As can be seen, the proposed Legendre sequence-based sampling operator provides slightly better reconstruction performance than the full-random sampling operator when the SNR is below 0 dB. On the other hand, the mixed scheme is inferior to the full random sampling operator. In fact, substantial performance loss can be observed when r is small (i.e., $r = 4$). These simulation results demonstrate the effectiveness of using deterministic sequences for an MWC system.
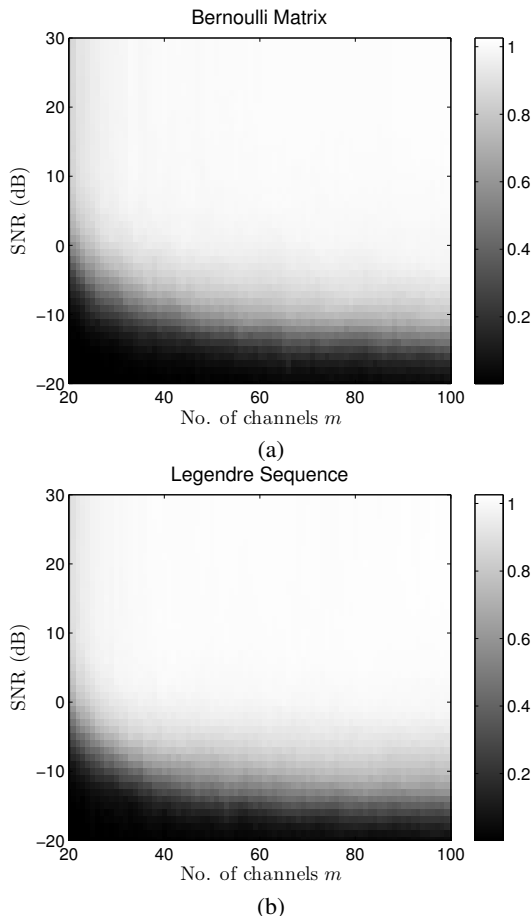

Fig. 3. Successful recovery rate using different 49×197 binary patterns under different input SNR.

sparse signals. These include the maximum-length sequence, the Legendre sequence and periodic complementary sequences, all of which have nearly flat spectrum in the (I)FFT domain. The corresponding binary operator $\mathbf{S}$ features hardware friendly implementation, fast computation and near-optimal performance guarantees. Simulation results show that despite their simplicity, the proposed sampling operators can offer very similar performance as that of the full random sampling operators, which imply they are promising in practical applications of the MWC system.

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(a)

(b)

Fig. 2. Probabilities of successful support set recovery for different number of channels m and different SNR levels. (a) Results when $\mathbf{S}$ is a full-random Bernoulli operator. (b) Results when $\mathbf{S}$ is partial circulant matrix in (6) with $\mathbf{c}$ being the Legendre sequence.

V. CONCLUSIONS

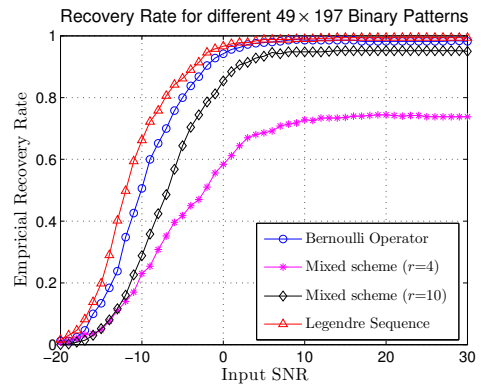In this paper, we have proposed to use deterministic sequences for modulated wideband converter in sub-Nyquist sampling of spectrally