

SPECTRAL SEPARATION COEFFICIENTS FOR DIGITAL GNSS RECEIVERS

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ABSTRACT

The extreme weakness of a GNSS (Global Navigation Satellite System) signal makes it vulnerable to almost every kind of interferences, that can be radically different in terms of time and frequency characteristics. For this reason the development of a consistent theory allowing comparative analysis was needed and the concepts of effective C/N_0 and SSCs (Spectral Separation Coefficients) were introduced as reliable measures of the interfering degradations. However these parameters were defined only in the analog domain, not considering specific features due to digital synthesis. In this article an alternative derivation for the analog case and the extension to digital devices are provided. The analysis is particularly focused on the acquisition block, the first element of a GNSS receiver that provides a roughly estimated code delay and Doppler shift. The innovative approach presented in the paper is based on the fact that effective C/N_0 and SSCs are interpreted in terms of ROCs (Receiver Operative Characteristics) showing how the system performance strictly depends on these parameters.

1. INTRODUCTION

One of the main problems connected to GNSS (Global Navigation Satellite System) is the evaluation of the impact of different interferences on the receiving device. The extreme weakness of a GNSS signal makes it vulnerable to different kinds of interfering, like spurious and out-of-band emissions that can be originated by telecommunication systems, either operating in adjacent bands or working at frequencies relatively far from the GPS ones. Since several types of interference, potentially widely different in terms of time and frequency characteristics, can affect GNSS receivers, the development of a consistent theory, allowing comparative analysis, was needed. In addition the acquisition and the tracking processes within the GNSS receiver modify the shape of the interfering, either mitigating or amplifying its impact and a reliable measure of the interfering degradation should account these interactions.

Different parameters have been investigated in order to quantify the effect of interference on the signal quality, and in particular a quantity called “effective C/N_0 ” was introduced to reflect the effect of interference at the input of the receiver, avoiding receiver-specific details such as integration time and the use of coherent or non-coherent processing. Furthermore a parameter called spectral separation coefficient (SSC) was

introduced [1, 2] to distinguish the effects of the interference spectral shape from effects due to interfering power. These parameters were first introduced by J.W. Betz in [1, 2] and then widely accepted as reliable and effective measures of interference degradation. In particular, both Galileo Signal Task Force and ESA adopted them to investigate mutual system interference between GPS and Galileo signals.

The performance of signal acquisition, carrier tracking and data demodulation essentially depends on the SNIR (Signal to Noise and Interference Ratio) at the output of each correlator in a receiver. Two different but related post-correlation quantities, the coherent output SNIR and the non-coherent SNIR, can be defined. Coherent output SNIR is defined under the hypothesis of knowing the phase of the received signal that can thus be perfectly aligned with the local signal replica. The coherent output SNIR is an important indicator of the received signal quality and we will show that, for gaussian interference, the ROCs (Receiver Operative Characteristics) are essentially determined by this parameter. Non-coherent output SNIR is defined under the assumption that the phase of the received signal is unknown and a non-coherent correlation is employed. This implies that the receiver signal is multiplied by two orthogonal sinusoids at the same frequency and the signals obtained at the in-phase and quadrature branches are squared and summed providing a correlation independent from the initial phase. The non-coherent output SNIR is evaluated over this correlation.

Although the coherent and non-coherent output SNIR are distinct quantities, [1] shows that they can both be determined from the effective C/N_0 . In absence of interference the effective C/N_0 corresponds to the traditional C/N_0 at the receiver input, whereas, when non-white interference is present, it can be interpreted as the carrier to noise density ratio caused by an equivalent white noise that would yield to the same output SNIRs.

It is important to highlight that in [1] the concepts of SNIR and therefore of the SSCs were not directly related to the receiver functional blocks. In addition, such parameters were derived in the analog domain without taking into account the specific features of digital receivers (like as example the sampling rate). The innovative contribution of this paper can be then summarized into two points:

- the paper provides the definition and the analysis of the effective C/N_0 and SSCs for digital receivers. The final equation will be easier to compute with respect to what

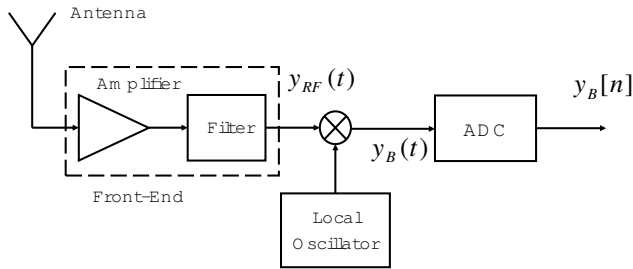


Figure 1: First stage of a GNSS receiver

reported in [1]

- the paper explains and analyzes the meaning and the effects of effective C/N_0 , SNIR and SSCs considering the impact of such parameters on the acquisition block (which is the first functional block of a digital GNSS receiver). It has been proved that the acquisition performance directly depends on SNIR and so on SSCs.

The paper is organized as follows: in Section 2 the basic signal model is reported for both digital and analog devices, introducing the basic notation used in the paper; Section 3 reviews Betz's theory using a different derivation with respect to [1]: the correlation operation is interpreted as an equivalent filtering and the analog output coherent SNIR is evaluated using linear system properties. In Sections 4 and 5 the effective C/N_0 and the SSCs definition are extended to digital devices and related to the ROCs as indicators of system performance. Some simulations support the theoretical analysis in 6 and finally Section 7 concludes the paper.

2. SIGNAL MODEL

According to [7], the signal obtained by demodulating the output of the front-end (see Figure 1) is given by

$$y_B(t) = x_B(t) + \eta_B(t) + i_B(t) \\ = s(t - \tau_0^a) \cos(2\pi(f_0^a + f_D^a)t + \theta) + \eta_B(t) + i_B(t)$$

where:

- $s(t) = Ac(t)d(t)$ is the GNSS signal composed by the PRN sequence $c(t)$ and the navigation message $d(t)$. A is the amplitude of $s(t)$ since both $c(t)$ and $d(t)$ are BPSK signals. In this analysis the filter used to recover $y_B(t)$ is supposed to have a flat frequency response over its band and therefore neglected. In [1] the effect of transmission and reception filters is taken into account but the results do not essentially change. In the following the navigation message $d(t)$ will be considered constant over the interval used for the acquisition processing.
- τ_0^a is the GNSS signal delay; f_0^a and f_D^a are respectively the analog local and Doppler frequencies; θ is a random phase introduced by the communication channel;
- $\eta_B(t)$ is the noise contribution with flat spectral density $N_0/2$;
- $i_B(t)$ is the interfering signal with non-flat spectral density.

The noise and the interfering random processes are supposed independent. The frequency f_0^a can be either close to zero or not [5][6], according to the adopted demodulation scheme.

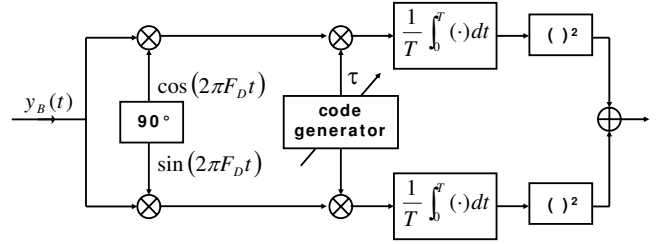


Figure 2: The basic analog non-coherent acquisition scheme

In the analog receiver the signal $y_B(t)$ directly enters the acquisition block and it is processed in order to find a rough estimation of the Doppler frequency and of the code delay. In a digital receiver the $y_B(t)$ is sampled at the frequency $f_s = \frac{1}{T_s}$, obtaining:

$$y_B[n] = y_B(nT_s) = x_B[n] + \eta_B[n] + i_B[n] \\ = s[n - \tau_0] \cos(2\pi(f_0 + f_D)n + \theta) + \eta_B[n] + i_B[n]$$

where $x_B[n]$, $\eta_B[n]$ and $i_B[n]$ are the sampled versions of the useful signal, the noise and the interfering. $\tau_0 = \tau_0^a/T_s$ is the code delay expressed in terms of the sampling interval T_s , $f_0 = f_0^a T_s$ and $f_D = f_D^a T_s$ are the digital local and Doppler frequencies. The noise $\eta_B[n]$ is in general a band-pass random process with flat spectral density $\frac{1}{2}N_0 f_s = \frac{N_0}{2T_s}$. The factor f_s is due to the sampling operation supposed to be a multiplication by an ideal Dirac pulse train. The variance of $\eta_B[n]$ should be evaluated by multiplying the spectral density by the noise band expressed in terms of numerical frequencies. In the following the sample index will be indicated with n and the digital frequency with f_d .

3. ANALOG SSCS

According to [1] the interfering entering the acquisition block should be a zero mean, wide sense stationary, gaussian random process. These hypotheses guarantee that the correlator outputs, before the squaring operation are still gaussian and therefore the false alarm and detection probabilities have the same theoretical expressions with or without interference. In the rest of the paper these properties will be assumed and a more detailed interpretation will be provided in Section 5 for the digital case. In Figure 2 the basic scheme of an analog non-coherent acquisition block is presented: the input signal $y_B(t)$ is multiplied by two orthogonal sinusoids for different values of the frequency F_D that accounts for both local and Doppler frequencies. The signal is then multiplied by a local code replica delayed of τ and integrated over T , the integration interval. The outputs of the in-phase and quadrature ways are squared and summed providing the non-coherent correlation. The multiplication by the local code with variable delay and the successive integration can be interpreted as an equivalent filtering whose impulse response is given by $h_c(t) = \frac{1}{T}c(-t)$, where $c(t)$ is the local code replica of length T . $h_c(t)$ behaves like a low-pass filter and this consideration allows to use the linear system properties in order to evaluate the coherent output SNIR.

When the frequency F_D does not match the sum of the local and Doppler frequencies or the delay under test is not correct, the signal is almost completely removed, due to the PRN

code properties, so only the case of perfect alignment is considered. For this reason $F_D = f_0^a + f_D^a$ is assumed; this condition is quite unrealistic since the local and Doppler frequencies are generally recovered with an error within the width of the Doppler bin, that is the step used to explore all the possible signal frequencies. However it has been proved [4] that this frequency uncertainty can be modeled as an additional loss that reduces the output SNIRs and that can be analyzed separately.

When the condition $F_D = f_0^a + f_D^a$ is achieved the signals on the two branches of the acquisition, after the sinusoids multiplication, assume the following expressions:

$$y_I(t) = \frac{1}{2}s(t - \tau_0^a) \cos \theta + \frac{1}{2}\eta_I(t) + \frac{1}{2}i_I(t) \quad (1)$$

$$y_Q(t) = -\frac{1}{2}s(t - \tau_0^a) \sin \theta + \frac{1}{2}\eta_Q(t) + \frac{1}{2}i_Q(t) \quad (2)$$

In expressions (1) and (2) the high frequency components of the signal have been neglected since they will be eliminated by the code equivalent filter. $\eta_I(t)$ and $\eta_Q(t)$ are obtained demodulating the noise components and it is possible to show that they are independent and with spectral density N_0 . Since $\eta_I(t)$ and $\eta_Q(t)$ have a flat spectrum within the bands of the GNSS receivers and of the equivalent code filter, they can be treated as white gaussian noise. $i_I(t)$ and $i_Q(t)$ are two independent not-white gaussian random processes with spectral density $C_l G_l(f)$ where C_l is the power of the two signals and $G_l(f)$ is their normalized power spectral density (PSD). In the following the factor 1/2 will be ignored without changing the analysis results, since it affects both signal and noise components. From Equations (1) and (2) and using the equivalent filter representation it is possible to evaluate the outputs of the correlators before the squaring operations:

$$\lambda_I(t, \theta) = y_I(t) * h_c(t) = y_I(t) * \frac{1}{T}c(-t)$$

$$\lambda_Q(t, \theta) = y_Q(t) * h_c(t) = y_Q(t) * \frac{1}{T}c(-t)$$

Finally the coherent output SNIR is defined as

$$\rho_c = \max_{\theta} \frac{|E[\lambda_I(\tau_0^a, \theta)]|^2}{\text{Var}\{\lambda_I(\tau_0^a, \theta)\}} = \max_{\theta} \frac{|E[\lambda_Q(\tau_0^a, \theta)]|^2}{\text{Var}\{\lambda_Q(\tau_0^a, \theta)\}} \quad (3)$$

By means of some algebra, considering the useful signal $s(t)$ as a deterministic process and using the linear system properties, the following expression for ρ_c is obtained:

$$\rho_c = \frac{\frac{C}{N_0} T \left[\int_{-\beta_r/2}^{\beta_r/2} G_s(f) df \right]^2}{\int_{-\beta_r/2}^{\beta_r/2} G_s(f) df + \frac{C_l}{N_0} \int_{-\beta_r/2}^{\beta_r/2} G_l(f) G_s(f) df}$$

where β_r is the equivalent two-sided band of the receiver, $C = A^2$ is the power of the useful received signal $s(t)$ and $G_s(f)$ is the normalized code PSD given by

$$G_s(f) = T |H_c(f)|^2 = \frac{1}{T} |C(f)|^2$$

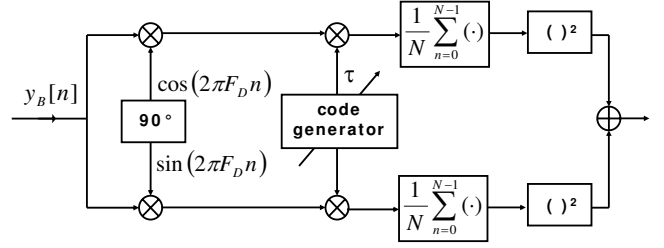


Figure 3: The basic digital non-coherent acquisition scheme

with $H_c(f)$ and $C(f)$ Fourier Transforms of $h_c(t)$ and $c(t)$ respectively. The equivalent band of the acquisition block is given by

$$B_{\text{acq}} = \frac{1}{T \int_{-\beta_r/2}^{\beta_r/2} G_s(f) df}$$

and the effective C/N_0 is defined as

$$\left(\frac{C}{N_0} \right)_{\text{eff}} = \rho_c B_{\text{acq}} = \frac{\frac{C}{N_0} \int_{-\beta_r/2}^{\beta_r/2} G_s(f) df}{\int_{-\beta_r/2}^{\beta_r/2} G_s(f) df + \frac{C_l}{N_0} \int_{-\beta_r/2}^{\beta_r/2} G_l(f) G_s(f) df} \quad (4)$$

From (4) it is clear that the impact of the interference is proportional to the power independent factor

$$k_{ls} = \int_{-\beta_r/2}^{\beta_r/2} G_l(f) G_s(f) df$$

Such factor is called Spectral Separation Coefficient and it accounts the interaction of the interfering spectrum with the acquisition device, providing a quantitative measure of the interfering impact.

4. DIGITAL SCCS

When a digital GNSS receiver is considered, all the classical analog operations are replaced by their numerical equivalent. Considering Figure 3, it is easy to notice that all the operations are performed between digital signals and the analog integrations have been replaced by summations over N samples. Also in the case of digital acquisition the code integration can be interpreted as a digital filtering with an equivalent impulse response $h_c[n] = \frac{1}{N}c[-n]$, where $c[n]$ is the digital version of the PRN code.

Proceeding in the same way exposed in Section 3 it is possible to show that, when the condition $F_D = f_0 + f_D$ is achieved, the signals before the code correlation assume the following form:

$$y_I[n] = \frac{1}{2}s[n - \tau_0] \cos \theta + \frac{1}{2}\eta_I[n] + \frac{1}{2}i_I[n]$$

$$y_Q[n] = -\frac{1}{2}s[n - \tau_0] \sin \theta + \frac{1}{2}\eta_Q[n] + \frac{1}{2}i_Q[n]$$

where $s[n]$ is the digitalized version of the received signal, $\eta_I[n]$ and $\eta_Q[n]$ are two independent gaussian processes

with spectral density N_0 over the digital receiver band $\beta_r^d = \beta_r T_s$, and $i_I[n]$ and $i_Q[n]$ two independent non-white random signals with power C_I and normalized spectral density $G_I(e^{j2\pi f_d})$. $y_I[n]$ and $y_Q[n]$ are then processed by the equivalent digital filter having impulse response $h_c[n]$, leading to

$$\lambda_I[t, \theta] = y_I[n] * h_c[n] = y_I[n] * \frac{1}{T} c[-n]$$

$$\lambda_Q[n, \theta] = y_Q[n] * h_c[n] = y_Q[n] * \frac{1}{T} c[-n]$$

And finally, in the same way of (3), a digital coherent output SNIR can be defined

$$\rho_c^d = \max_{\theta} \frac{|E[\lambda_I[\tau_0, \theta]]|^2}{\text{Var}\{\lambda_I[\tau_0, \theta]\}} = \max_{\theta} \frac{|E[\lambda_Q[\tau_0, \theta]]|^2}{\text{Var}\{\lambda_Q[\tau_0, \theta]\}}$$

and doing some algebra it results

$$\rho_c^d = \frac{\frac{C}{N_0} N T_s \left[\int_{-\beta_r^d/2}^{\beta_r^d/2} G_s(e^{j2\pi f_d}) df_d \right]^2}{\int_{-\beta_r^d/2}^{\beta_r^d/2} G_s(e^{j2\pi f_d}) df_d + \frac{C_I}{N_0} T_s k_{Is}^d}$$

where $C = A^2$ is the power of the useful received signal $s[n]$ and $G_s(e^{j2\pi f_d})$ is the normalized digital code PSD given by $G_s(e^{j2\pi f_d}) = N |H_c(e^{j2\pi f_d})|^2 = \frac{1}{N} |C(e^{j2\pi f_d})|^2$ with $H_c(e^{j2\pi f_d})$ and $C(e^{j2\pi f_d})$ DTFTs (Discrete Time Fourier Transforms) of $h_c[n]$ and $c[n]$ respectively. k_{Is}^d is the digital SSC that is given by

$$k_{Is}^d = \int_{-\beta_r^d/2}^{\beta_r^d/2} G_I(e^{j2\pi f_d}) G_s(e^{j2\pi f_d}) df_d \quad (5)$$

For the digital SSCs it is possible to use the Parseval equality to express Equation (5) avoiding the integral. In fact we have

$$k_{Is}^d = \sum_{n=-N}^{N-1} R_I[n] R_s[n] \quad (6)$$

where $R_I[n] = \frac{1}{C_I} E[\sum_{k=-\infty}^{\infty} i_I[k] i_I[k-n]]$ and $R_s[n] = \frac{1}{N} \sum_{k=-\infty}^{\infty} c[k] c[k-n]$ are the normalized autocorrelations of the interference components and the code. The summation in (6) is performed only on $[-N; N-1]$ since $R_s[n]$ is non-zero only on this interval.

The effective C/N_0 becomes

$$\left(\frac{C}{N_0} \right)_{eff}^d = \frac{\frac{C}{N_0} \int_{-\beta_r^d/2}^{\beta_r^d/2} G_s(e^{j2\pi f_d}) df_d}{\int_{-\beta_r^d/2}^{\beta_r^d/2} G_s(e^{j2\pi f_d}) df_d + \frac{C_I}{N_0} k_{Is}^d}$$

5. ROC ANALYSIS AND SSCS INTERPRETATION

The efficiency of an acquisition block is measured by the ROC (Receiver Operative Characteristics), curves reporting the false alarm versus the detection probability of the system. The false alarm and the detection probabilities measure the capability of the system of correctly finding the GNSS signal coming from the satellite SV_i . The presence of an interference impacts the ROC reducing the detection probability for a fixed value of false alarm. In this section we highlight how

the SSCs and the coherent output SNIR are directly linked to the ROCs.

The acquisition block tests different values of code delay and Doppler frequency producing a random variable, often indicated with cell, for each pair of these parameters and forming the search space. The detection of the satellite SV_i depends on the value of the search space in the "correct" cell, that is the one that matches both code delay and Doppler frequency: only if this value passes a fixed threshold V_i the exact detection is obtained. It can be shown [3] that if the in-phase and quadrature components, before the squaring blocks, are independent gaussian random variables with variance σ_{out}^2 , then the false alarm and detection probabilities assume the following expressions:

$$P_{fa}(V_i) = \exp\left\{-\frac{V_i^2}{2\sigma_{out}^2}\right\} \quad (7)$$

$$P_{det}(V_i) = \int_{V_i}^{+\infty} \frac{z}{\sigma_{out}^2} \exp\left\{-\frac{z^2 + \alpha^2}{2\sigma_{out}^2}\right\} I_0\left(\frac{z\alpha}{\sigma_{out}^2}\right) dz \quad (8)$$

with $\alpha = \sqrt{\mu_I^2 + \mu_Q^2}$. μ_I and μ_Q are the means of the random variables on the in-phase and quadrature ways in case of perfect delay/frequency alignment. In case of missed alignment the gaussian variables are supposed to be zero mean. $I_0(\cdot)$ is the modified Bessel function of zero order.

The Betz's hypotheses reported in Section 3 guarantee that the random variables before the squaring are gaussian with zero mean when the code delay and the Doppler frequency are not matched. Furthermore the variance σ_{out}^2 can be expressed as:

$$\sigma_{out}^2 = \frac{N_0}{N} f_s \int_{-\beta_r^d/2}^{\beta_r^d/2} G_s(e^{j2\pi f_d}) df_d + \frac{C_I}{N} k_{Is}^d \quad (9)$$

and

$$\begin{aligned} \mu_I &= \sigma_{out} \sqrt{\rho_c^d} \cos \theta \\ \mu_Q &= -\sigma_{out} \sqrt{\rho_c^d} \sin \theta \\ \alpha &= \sigma_{out} \sqrt{\rho_c^d} \end{aligned} \quad (10)$$

Equations (9) and (10) prove that the ROCs in presence of interference are completely determined by the knowledge of the SSCs and of the output coherent SNIR.

Equations (7) and (8) refer to the acquisition block of Figure 3, however more complex systems could be employed, for example by introducing non-coherent averaging or multi-threshold detection methods. Also in these cases it is possible to show that the false alarm and detection probabilities depend only on σ_{out}^2 and α , that are strictly related to the output coherent SNIR and to the SSCs by (9) and (10), that result still valid: in this sense the SSCs are a system independent measure of the interfering impact.

6. SIMULATION RESULTS

The presented analysis has been supported by simulations. An acquisition system like the one represented in Figure 3 has been implemented and both false alarm and detection probabilities have been evaluated using error count techniques. The system has been fed with the useful signal, white

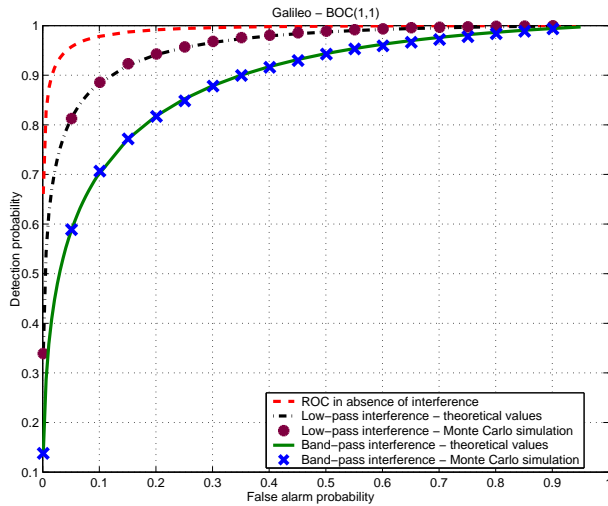


Figure 4: ROC curves for the BOC(1,1) Galileo signal

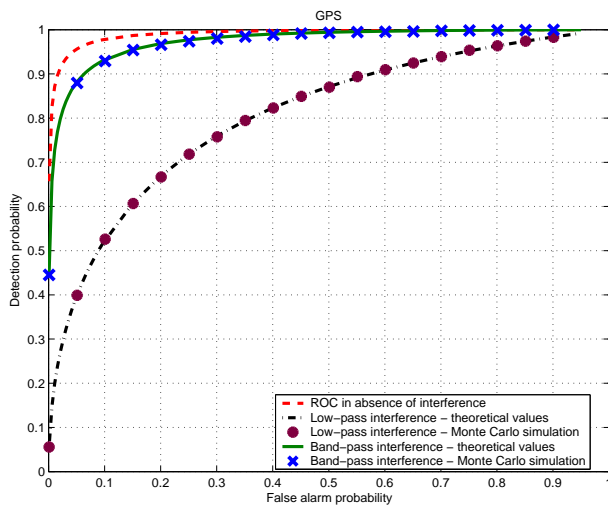


Figure 5: ROC curves for the GPS signal

noise and different types of interference. The narrow band interference has been simulated filtering white gaussian noise. A base-band model has been used since the demodulation and the Doppler frequency removal produce base-band signals. For this reason the notation “low-pass” interference indicates a signal whose central frequency was originally close to the GNSS signal carrier and that has assumed a spectrum concentrated around the zero frequency after the demodulation and the Doppler removal. The simulations have been carried out for different kinds of interference and for both GPS and Galileo systems, always leading to results in agreement with the theoretical model. In Figures 4 and 5 the analysis of the impact of a band-pass and a low-pass interference for the Galileo and GPS signal acquisition has been reported; the simulation parameters are reported in table 1 and the SSC values are listed in table 2. As expected the GPS signal is more sensitive to interferences with spectra concentrated around the GNSS carrier. This is due to the spectral shape of the GPS signal that has a main lobe at the frequency carrier: in this case the SSC is greater than the one of the

Table 1: Simulation parameters

C/N_0 Galileo	30 dB-Hz
C/N_0 GPS	36 dB-Hz
samples per chip	4
sampling frequency	$f_s = 4.092$ MHz
Low-pass interference cut-off frequency	$f_c = 0.125 f_s$
Band-pass interference frequency interval	$[0.125 f_s; 0.375 f_s]$
Interference to noise ratio $C_I/(N_0 f_s)$	0 dB

Table 2: SSCs values, pure number

	GPS	Galileo
Low-pass interference	3.198	0.617
Band-pass interference	0.337	1.661

BOC(1,1) that presents a zero at those frequencies and the ROCs worsen. On the contrary the Galileo signal is more fragile respect to interference centered on its side lobes.

7. CONCLUSIONS

In this paper the theory relative to the SSCs has been presented and extended for digital GNSS receivers. These parameters are essential for the determination of the system performance and can be used as reliable measure of the interfering impact over the acquisition block. Simulations prove the consistency of the developed theory.

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