View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by ZENODO

LATTICE FACTORIZATION AND DESIGN OF PERFECT RECONSTRUCTION FILTER BANKS WITH ANY LENGTH YIELDING LINEAR PHASE

Zhiming Xu and Anamitra Makur

School of Electrical and Electronic Engineering
Nanyang Technological University, Singapore 639798
email: zhmXU@pmail.ntu.edu.sg, eamakur@ntu.edu.sg

ABSTRACT

This paper introduces the lattice factorizations and designs of a large class of critically sampled linear phase perfect reconstruction filter banks. We deal with FIR filter banks with real-valued coefficients in which all filters have the same *arbitrary* length and symmetry center. Refined existence conditions on the length are given first. Lattice structures are developed for *both* even and odd channel filter banks. Compared to most existing design methods, the proposed approach can offer better trade off between performance and filter length. Finally, several design and application examples are presented to validate our approach.

1. INTRODUCTION

There has been enormous research works in the area of filter banks (FB) [1, 2] which find many applications in image/audio/video processing and communication systems. Of extreme importance is the capability to design a filter bank for some desired applications. In image and video processing, linear phase (LP) property of filters is always crucial. Moreover, simple symmetric extension methods can be employed for LP filters to accurately handle the boundaries of finite length signals, such as images. For practical purposes, only FIR, causal, LP and perfect reconstruction (PR) filter banks with real-valued coefficients are considered in this paper.

Consider the polyphase form of an M-channel critically sampled linear phase perfect reconstruction filter bank (LPPRFB) shown in Fig. 1. Suppose all the filters have the same *arbitrary* length $L = KM + \beta$ $(0 \le \beta < M)$ and the same symmetry center. Previous research works on such FBs have been focused on the necessary conditions and factorizations of the polyphase matrix. The restrictions on the filter symmetry polarities and length were studied thoroughly in [3]. However, most design methods [4, 5, 6] concentrated only on the constrained case $\beta = 0$, i.e., filter length with multiple of M. As far as the case $\beta > 0$ is concerned, the cosine-modulated FBs with arbitrary length were studied in [7]. Another design method without constrained cosine-modulated structure was discussed in the restrictive even channel paraunitary FBs [3]. In [8], the FBs with arbitrary filter length were also studied based on minimization of mean square error. However, their design methods cannot structurally enforce LP and PR properties into FBs. Thus the designed FBs based on their methods have neither LP nor PR property in general and are source dependent.

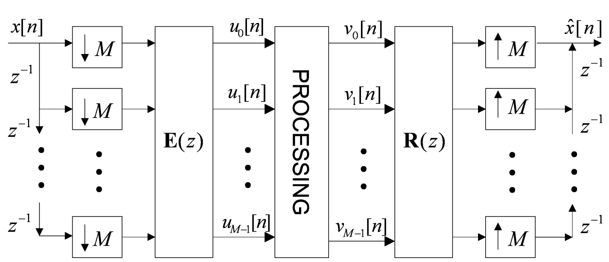


Figure 1: Polyphase form of an M-channel LPPRFB

In this paper, we study the more general class of LPPRFBs with arbitrary but equal filter length. First, the refined existence conditions for such class of FBs are given. Lattice factorizations and designs for *both* even and odd channel cases are developed. Different from [8], our method can structurally enforce LP and PR properties into FBs and is source independent. Finally, several design examples are presented to validate the proposed lattice structures.

The main motivation for this work is to design more flexible FBs which may give more possible choices for a given desired application. In most traditional works [4, 5, 6], the filter length must be multiple of M, which greatly limits the possible design of LPPRFBs. For example, for 8-channel LPPRFBs, if the length is constrained to be less than 24 by the constraints of complexity, there are only two possible choices in conventional designs. The restriction becomes more severe for large M. However, this restriction is from the traditional FB design methods, *not* from LPPRFBs, which can be seen from existence conditions in section 2. Our proposed design method can provide more possible LPPRFBs satisfying given constraint length and the length increment among FBs can be made as small as possible. Continuing the above example, the proposed design method can give 8 possible LPPRFBs compared to only 2 choices in traditional method. Our design method can offer better trade off between filter length and performance than traditional methods. To our knowledge, this is the most general LPPRFB with the same length to date.

Notations: Bold-faced quantities denote matrices and vectors. $\mathbf{I}_M$, $\mathbf{J}_M$ and $\mathbf{0}_M$ denote the identity matrix, reversal matrix and null matrix, all with size $M \times M$. For FIR FBs, the polyphase matrix can be written as $\mathbf{E}(z) = \sum_{i=0}^{K-1} \mathbf{e}[i] z^{-i}$, where $\mathbf{e}[K-1] \ne \mathbf{0}$. $K-1$ is defined as the order of the polyphase matrix, i.e., the FIR FB. It is related to the maximum possible filter length L of the analysis filters by $L = KM$. In addition, $\mathbf{W}_{2m}$ and $\mathbf{W}_{2m+1}$ are $2m \times 2m$ and $(2m+1) \times (2m+1)$ butterfly-like matrices, respectively, as follows.

$$\mathbf{W}_{2m} = \begin{bmatrix} \mathbf{I}_m & \mathbf{I}_m \\ \mathbf{I}_m & -\mathbf{I}_m \end{bmatrix}, \quad \mathbf{W}_{2m+1} = \begin{bmatrix} \mathbf{I}_m & \mathbf{0} & \mathbf{I}_m \\ \mathbf{0} & \sqrt{2} & \mathbf{0} \\ \mathbf{I}_m & \mathbf{0} & -\mathbf{I}_m \end{bmatrix}$$

2. EXISTENCE CONDITIONS AND GENERAL LATTICE STRUCTURES

In [3], some necessary conditions for LPPRFBs with length $L = K_i M + \beta$ were derived. For the class of LPPRFBs with equal filter length, i.e., $K_i = K$, some further refined existence conditions can be obtained and listed in Table 1. Note that even channel LPPRFBs with the same length can only have even length. Thus, the case with even M and odd β does not exist.

Without loss of generality, we can always arrange the M channel linear phase filters in such an order that the first n_s filters are symmetric, while the other n_a are antisymmetric filters. The associated analysis polyphase matrix $\mathbf{E}(z)$ should satisfy the LP condition [3],

$$\mathbf{E}(z) = z^{-(K-1)} \mathbf{D}_M \mathbf{E}(z^{-1}) \hat{\mathbf{J}}_M(z) \tag{1}$$

where

$$\hat{\mathbf{J}}_M(z) = \begin{bmatrix} z^{-1}\mathbf{J}_\beta & \mathbf{0}_{\beta \times (M-\beta)} \\ \mathbf{0}_{(M-\beta)\times\beta} & \mathbf{J}_{M-\beta} \end{bmatrix}$$

Table 1: Existence conditions for M-channel critically sampled LP-PRFB with length $L = KM + \beta$

	Symmetry Polarity Condition	Order K
M even, β even	$\frac{M}{2}$ S and $\frac{M}{2}$ A	Arbitrary
M odd, β even	$\frac{M+1}{2}$ S and $\frac{M-1}{2}$ A	odd
M odd, β odd	$\frac{M+1}{2}$ S and $\frac{M-1}{2}$ A	even

and $\mathbf{D}_M = \text{diag}(\mathbf{I}_{n_s}, -\mathbf{I}_{n_a})$.

Similar to [3, 4, 5, 6], a lattice factorization for the analysis polyphase matrix $\mathbf{E}(z)$ of LPPRFB can be formulated in the following form,

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0(z) \quad (2)$$

where the starting block $\mathbf{E}_0(z)$ with order N_0 and length $N_0M + \beta$ (with order $N_0 - 1$ and length N_0M if $\beta = 0$) has both LP and PR properties, and each block $\mathbf{G}_i(z)$ with order N_1 can propagate both LP and PR properties and increase filter length by N_1M . Such a cascade form would finally generate a LPPRFB with filter length $L = (KN_1 - N_1 + N_0)M + \beta$. The simplified LPPR propagating blocks in [6] can be used here for $\mathbf{G}_i(z)$. For even channel case, i.e., $M = 2m$, $\mathbf{G}_i(z)$ has the following form,

$$\begin{aligned}\mathbf{G}_i(z) &= \frac{1}{2}\mathbf{\Phi}_i\mathbf{W}_{2m}\mathbf{\Lambda}(z)\mathbf{W}_{2m} \quad (3)\\ &= \frac{1}{2}\begin{bmatrix}\mathbf{U}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_m\end{bmatrix}\mathbf{W}_{2m}\begin{bmatrix}\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & z^{-1}\mathbf{I}_m\end{bmatrix}\mathbf{W}_{2m}\end{aligned}$$

However, for odd channel case, i.e., $M = 2m+1$, $\mathbf{G}_i(z)$ has the following form,

$$\begin{aligned}\mathbf{G}_i(z) &= \frac{1}{4}\mathbf{\Phi}_{i,0}\mathbf{W}_{2m+1}\mathbf{\Lambda}_0^o(z)\mathbf{W}_{2m+1}\mathbf{\Phi}_{i,1}\mathbf{W}_{2m+1}\mathbf{\Lambda}_1^o(z)\mathbf{W}_{2m+1} \quad (4)\\ &= \frac{1}{4}\begin{bmatrix}\mathbf{U}_{i,0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_m\end{bmatrix}\mathbf{W}_{2m+1}\begin{bmatrix}\mathbf{I}_{m+1} & \mathbf{0}\\ \mathbf{0} & z^{-1}\mathbf{I}_m\end{bmatrix}\mathbf{W}_{2m+1}\\ &\times\begin{bmatrix}\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_{i,1}\end{bmatrix}\mathbf{W}_{2m+1}\begin{bmatrix}\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & z^{-1}\mathbf{I}_{m+1}\end{bmatrix}\mathbf{W}_{2m+1}\end{aligned}$$

The difference between our factorization and previous ones is the starting block $\mathbf{E}_0(z)$. Contrary to [4, 5, 6], $\mathbf{E}_0(z)$ cannot be made order zero if $\beta \neq 0$, i.e., constant matrix, which is treated as a trivial case because it would impose multiple of $(M - \beta)$ zero filter coefficients at fixed positions. Thus, $\mathbf{E}_0(z)$ has at least order one for the case of $\beta \neq 0$, i.e., $N_0 \geq 1$.

3. EVEN CHANNEL LPPRFB

Consider an M-channel LPPRFB ($M = 2m$) with filter length $L = KM + \beta$ $(0 \leq \beta < M)$. From the existence conditions in section 2, we know that β must be even, i.e., $\beta = 2l, (0 \leq l < m)$. For the even channel case, the LPPR propagating block $\mathbf{G}_i(z)$ has order one, i.e., $N_1 = 1$. Thus, the minimal order of initial block $\mathbf{E}_0(z)$ can be one if $\beta \neq 0$, i.e., $N_0 = 1$, thus length $M + \beta$. By the linear phase condition (1), $\mathbf{E}_0(z)$ can be written in the following form,

$$\mathbf{E}_0^e(z) = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{S}_{00} + z^{-1}\mathbf{S}_{00}\mathbf{J}_{2l} & \mathbf{S}_{01} & \mathbf{S}_{01}\mathbf{J}_{m-l}\\ \mathbf{A}_{00} - z^{-1}\mathbf{A}_{00}\mathbf{J}_{2l} & \mathbf{A}_{01} & -\mathbf{A}_{01}\mathbf{J}_{m-l}\end{bmatrix} \quad (5)$$

where matrices $\mathbf{S}_{00}$ and $\mathbf{A}_{00}$ have the same size $m \times 2l$, $\mathbf{S}_{01}$ and $\mathbf{A}_{01}$ have the same size $m \times (m - l)$. The corresponding synthesis LPPRFB has a starting block,

$$\mathbf{R}_0^e(z) = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{T}_{00} + z\mathbf{J}_{2l}\mathbf{T}_{00} & \mathbf{W}_{00} - z\mathbf{J}_{2l}\mathbf{W}_{00}\\ \mathbf{T}_{01} & \mathbf{W}_{01}\\ \mathbf{J}_{m-l}\mathbf{T}_{01} & -\mathbf{J}_{m-l}\mathbf{W}_{01}\end{bmatrix} \quad (6)$$

where matrices $\mathbf{T}_{00}$ and $\mathbf{W}_{00}$ have the same size $2l \times m$, $\mathbf{T}_{01}$ and $\mathbf{W}_{01}$ have the same size $(m - l) \times m$. Then, with the PR condition $\mathbf{R}_0^e(z)\mathbf{E}_0^e(z) = \mathbf{I}_M$, the following matrix equations can be established.

$$\mathbf{T}_{00}\mathbf{S}_{00} + \mathbf{J}_l\mathbf{T}_{00}\mathbf{S}_{00}\mathbf{J}_l = \mathbf{I}_{2l} = \mathbf{W}_{00}\mathbf{A}_{00} + \mathbf{J}_l\mathbf{W}_{00}\mathbf{A}_{00}\mathbf{J}_l \quad (7)$$

$$\mathbf{T}_{01}\mathbf{S}_{01} = \mathbf{I}_{m-l} = \mathbf{W}_{01}\mathbf{A}_{01} \quad (8)$$

$$\mathbf{T}_{00}\mathbf{S}_{01} = \mathbf{0}_{2l\times(m-l)} = \mathbf{W}_{00}\mathbf{A}_{01} \quad (9)$$

$$\mathbf{T}_{01}\mathbf{S}_{00} = \mathbf{0}_{(m-l)\times 2l} = \mathbf{W}_{01}\mathbf{A}_{00} \quad (10)$$

With these equations, a rank condition on the matrices $\mathbf{S}_{00}$ and $\mathbf{S}_{01}$ can be found, which is a key to the lattice factorization.

Theorem 1. *For the class of LPPRFBs and its starting block $\mathbf{E}_0^e(z)$ stated above, the matrix $\mathbf{S}_{01}$ has full column rank and $\mathbf{S}_{00}$ has rank l, i.e., rank$(\mathbf{S}_{01}) = m - l$ and rank$(\mathbf{S}_{00}) = l$.*

Proof. From (8), we know that $m - l = \text{rank}(\mathbf{T}_{01}\mathbf{S}_{01}) \leq \text{rank}(\mathbf{S}_{01})$. In addition, $\text{rank}(\mathbf{S}_{01}) \leq \min\{m, m-l\} = m - l$. Thus, the matrix $\mathbf{S}_{01}$ has full rank, i.e., $\text{rank}(\mathbf{S}_{01}) = m - l$. The matrix $\mathbf{T}_{01}$ also has full rank by similar derivation. From (7) and the rank inequality for matrix in [9], we can obtain that $2l = \text{rank}(\mathbf{T}_{00}\mathbf{S}_{00} + \mathbf{J}_l\mathbf{T}_{00}\mathbf{S}_{00}\mathbf{J}_l) \leq 2\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00})$, i.e., $\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) \geq l$. Define matrix $\mathbf{T} = [\mathbf{T}_{00}^T, \mathbf{T}_{01}^T]^T$ and $\mathbf{S} = [\mathbf{S}_{00}, \mathbf{S}_{01}]$, then from Eq. (8)-(10), we know,

$$\mathbf{TS} = \begin{bmatrix}\mathbf{T}_{00}\mathbf{S}_{00} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{m-l}\end{bmatrix}$$

which means $\text{rank}(\mathbf{TS}) = \text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) + m - l \leq \text{rank}(\mathbf{T}) \leq \min\{m + l, m\} = m$, i.e., $\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) \leq l$. Then, we get $\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) = l$. Finally, from (10) and Sylvester rank inequality in [9], $0 = \text{rank}(\mathbf{T}_{01}\mathbf{S}_{00}) \geq \text{rank}(\mathbf{T}_{01}) + \text{rank}(\mathbf{S}_{00}) - m \geq \text{rank}(\mathbf{S}_{00}) - l$, i.e., $\text{rank}(\mathbf{S}_{00}) \leq l$. However, we know that $\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) = l \leq \text{rank}(\mathbf{S}_{00})$. Therefore, $\text{rank}(\mathbf{S}_{00}) = l$. This finishes the proof. □

From the above analysis on the rank of matrix $\mathbf{S}_{00}$, we propose a parameterized form for matrix $\mathbf{S}_{00}$ to help factorization,

$$\mathbf{S}_{00} = [\mathbf{U}_{00}\mathbf{\Gamma}_p \;\; \mathbf{U}_{00}\mathbf{\Gamma}_m] \quad (11)$$

where $\mathbf{U}_{00}$ has size $m \times l$ and $\mathbf{\Gamma}_p = (\mathbf{\Gamma}_1 + \mathbf{\Gamma}_2)/2$ and $\mathbf{\Gamma}_m = (\mathbf{\Gamma}_1 - \mathbf{\Gamma}_2)\mathbf{J}_l/2$, where $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ are two arbitrary square *invertible* matrices with size $l \times l$. Apply similar parameterized form for $\mathbf{A}_{00}$ with matrix $\mathbf{V}_{00}$ with the same size as $\mathbf{U}_{00}$ and replace the matrices $\mathbf{S}_{01}$ and $\mathbf{A}_{01}$ with $\mathbf{U}_{01}$ and $\mathbf{V}_{01}$. By such parameterization, it can be shown that $\mathbf{E}_0^e(z)$ in (5) can be written in (12) and factorized in the form (13) shown on the top of next page, where $\mathbf{\Phi}_0^e$ can be further factorized in the form (14) and matrices $\mathbf{U}_0 = [\mathbf{U}_{00}, \mathbf{U}_{01}]$, $\mathbf{V}_0 = [\mathbf{V}_{00}, \mathbf{V}_{01}]$. We also explicitly show the inverse $\mathbf{\Gamma}_e^{-1}$ in (15),

$$\mathbf{\Phi}_0 = \begin{bmatrix}\mathbf{U}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_0\end{bmatrix}\begin{bmatrix}\mathbf{I}_m & \mathbf{I}_m\\ \mathbf{I}_m & \mathbf{I}_m\end{bmatrix}\begin{bmatrix}\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_m\end{bmatrix} \quad (14)$$

$$\mathbf{\Gamma}_e^{-1} = \begin{bmatrix}\hat{\mathbf{\Gamma}}_p & \hat{\mathbf{\Gamma}}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{2(m-l)}\\ \mathbf{J}_l\hat{\mathbf{\Gamma}}_m\mathbf{J}_l & \mathbf{J}_l\hat{\mathbf{\Gamma}}_p\mathbf{J}_l & \mathbf{0}\end{bmatrix} \quad (15)$$

where $\hat{\mathbf{\Gamma}}_p = (\mathbf{\Gamma}_1^{-1} + \mathbf{\Gamma}_2^{-1})/2$ and $\hat{\mathbf{\Gamma}}_m = (\mathbf{\Gamma}_1^{-1} - \mathbf{\Gamma}_2^{-1})\mathbf{J}_l/2$. It can be shown easily that such factorization can ensure PR property of FB as long as $\mathbf{U}_0$ and $\mathbf{V}_0$ are square invertible matrices. Note that a similar factorization proposed in [3] is only applicable to PU system, where $\mathbf{U}_0$, $\mathbf{V}_0$, $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ are constrained to be square orthogonal matrices. On the contrary, the proposed factorization extends it to the more general class of PR systems.

The proposed factorization is not only more general than before, but also good in terms of implementation delays. The following theorem states that the proposed lattice structure employs the fewest number of delays in its implementation.

$$\mathbf{E}_0^e(z) = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{U}_{00}\mathbf{\Gamma}_p + z^{-1}\mathbf{U}_{00}\mathbf{\Gamma}_m\mathbf{J}_l & \mathbf{U}_{00}\mathbf{\Gamma}_m + z^{-1}\mathbf{U}_{00}\mathbf{\Gamma}_p\mathbf{J}_l & \mathbf{U}_{01} & \mathbf{U}_{01}\mathbf{J}_{m-l} \\ \mathbf{V}_{00}\mathbf{\Gamma}_p - z^{-1}\mathbf{V}_{00}\mathbf{\Gamma}_m\mathbf{J}_l & \mathbf{V}_{00}\mathbf{\Gamma}_m - z^{-1}\mathbf{V}_{00}\mathbf{\Gamma}_p\mathbf{J}_l & \mathbf{V}_{01} & -\mathbf{V}_{01}\mathbf{J}_{m-l} \end{bmatrix} \quad (12)$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{U}_{00} & \mathbf{U}_{01} & \mathbf{U}_{01}\mathbf{J}_{m-l} & z^{-1}\mathbf{U}_{00}\mathbf{J}_l \\ \mathbf{V}_{00} & \mathbf{V}_{01} & -\mathbf{V}_{01}\mathbf{J}_{m-l} & -z^{-1}\mathbf{V}_{00}\mathbf{J}_l \end{bmatrix}\begin{bmatrix} \mathbf{\Gamma}_p & \mathbf{\Gamma}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{2m-2l} \\ \mathbf{J}_l\mathbf{\Gamma}_m\mathbf{J}_l & \mathbf{J}_l\mathbf{\Gamma}_p\mathbf{J}_l & \mathbf{0} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{U}_{00} & \mathbf{U}_{01} & \mathbf{U}_{01}\mathbf{J}_{m-l} & \mathbf{U}_{00}\mathbf{J}_l \\ \mathbf{V}_{00} & \mathbf{V}_{01} & -\mathbf{V}_{01}\mathbf{J}_{m-l} & -\mathbf{V}_{00}\mathbf{J}_l \end{bmatrix}\begin{bmatrix} \mathbf{I}_{2m-l} & \mathbf{0} \\ \mathbf{0} & z^{-1}\mathbf{I}_l \end{bmatrix}\begin{bmatrix} \mathbf{\Gamma}_p & \mathbf{\Gamma}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{2m-2l} \\ \mathbf{J}_l\mathbf{\Gamma}_m\mathbf{J}_l & \mathbf{J}_l\mathbf{\Gamma}_p\mathbf{J}_l & \mathbf{0} \end{bmatrix} = \frac{1}{\sqrt{2}}\mathbf{\Phi}_0^e\,\mathbf{\Lambda}_e(z)\,\mathbf{\Gamma}_e \quad (13)$$

Theorem 2. *The factorization in (2) is minimal, where its factors are given in (3) and (13).*

Proof. The degree of a causal rational system is defined as the minimum number of delays required for its implementation [1]. A structure is said to be minimal if the number of delays used is equal to the degree of the transfer function. For the class of FBs described above, it can be proven that [10],

$$\deg(\mathbf{E}(z)) = \deg(|\mathbf{E}(z)|)$$

By the LP condition in (1), we have

$$\deg(|\mathbf{E}(z)|) = \deg\left(z^{-M(K-1)}|\mathbf{D}| \times |\mathbf{E}(z^{-1})| \times |\hat{\mathbf{J}}_M(z)|\right) \quad (16)$$

For even channel case, the number of symmetric filters n_s is equal to the number of anti-symmetric filters n_a, thus $|\mathbf{D}| = 1$. From [3], we know that $|\hat{\mathbf{J}}_M(z)| = (-1)^{\beta+M/2}z^{-\beta}$. Then, from (16) and the equality $\deg(\mathbf{E}(z^{-1})) = -\deg(|\mathbf{E}(z)|) = -\deg(\mathbf{E}(z))$, we obtain,

$$\deg(\mathbf{E}(z)) = M(K-1) + \beta - \deg(\mathbf{E}(z))$$

which means $\deg(\mathbf{E}(z)) = M(K-1)/2 + \beta/2$. In our factorization, there are $(K-1)$ order one building blocks according to (3), in which each block uses $M/2$ delays seen in (3). The initial block $\mathbf{E}_0^e(z)$ employs $\beta/2 = l$ delays seen in (13). Therefore, the total number of delays in use is $M(K-1)/2 + \beta/2$, which is just the degree of the transfer function $\mathbf{E}(z)$. This finishes the proof. □

4. ODD CHANNEL LPPRFB

From the existence conditions in Table 1, there are two possible cases for odd channel LPPRFBs with $M = 2m+1$. One case is even β with odd K, the other is odd β with even K. Because the LPPR propagating block we employed for odd channel LPPRFB has order two, the minimal order of initial block $\mathbf{E}_0(z)$ for even β case ($\beta \neq 0$) can be order one and length $M+\beta$, whereas it should be order two and length $2M+\beta$ for odd β case ($\beta \neq 0$).

4.1 β is even

We show the even β case in detail here. For the even $\beta = 2l, (0 \leq l \leq m)$, from the linear phase condition (1), the starting block $\mathbf{E}_0(z)$ can be written in the following form,

$$\mathbf{E}_0^o(z) = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{S}_{00} + z^{-1}\mathbf{S}_{00}\mathbf{J}_{2l} & \mathbf{S}_{01} & \mathbf{q} & \mathbf{S}_{01}\mathbf{J}_{m-l} \\ \mathbf{A}_{00} - z^{-1}\mathbf{A}_{00}\mathbf{J}_{2l} & \mathbf{A}_{01} & \mathbf{0} & -\mathbf{A}_{01}\mathbf{J}_{m-l} \end{bmatrix}$$

where matrices $\mathbf{S}_{00}$, $\mathbf{A}_{00}$, $\mathbf{S}_{01}$ and $\mathbf{A}_{01}$ have size $(m+1)\times 2l, m\times 2l, (m+1)\times(m-l)$ and $m\times(m-l)$, respectively. $\mathbf{q}$ and $\mathbf{0}$ are column vectors with size $m+1$ and m, respectively. The corresponding synthesis LPPRFB has a starting block,

$$\mathbf{R}_0^o(z) = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{T}_{00} + z\mathbf{J}_{2l}\mathbf{T}_{00} & \mathbf{W}_{00} - z\mathbf{J}_{2l}\mathbf{W}_{00} \\ \mathbf{T}_{01} & \mathbf{W}_{01} \\ \mathbf{r}^T & \mathbf{0}^T \\ \mathbf{J}_{m-l}\mathbf{T}_{01} & -\mathbf{J}_{m-l}\mathbf{W}_{01} \end{bmatrix}$$

where matrices $\mathbf{T}_{00}$, $\mathbf{W}_{00}$, $\mathbf{T}_{01}$ and $\mathbf{W}_{01}$ have size $2l\times(m+1), 2l\times m, (m-l)\times(m+1)$ and $(m-l)\times m$, respectively. The row vectors $\mathbf{r}^T$ and $\mathbf{0}^T$ have size $m+1$ and m, respectively. Similar to even channel case, a set of matrix equations can be established for the PR condition. Below is half of them.

$$\mathbf{T}_{00}\mathbf{S}_{00} + \mathbf{J}_{2l}\mathbf{T}_{00}\mathbf{S}_{00}\mathbf{J}_{2l} = \mathbf{I}_{2l} \quad (17)$$

$$\mathbf{T}_{01}\mathbf{S}_{01} = \mathbf{I}_{m-l} \quad (18)$$

$$\mathbf{T}_{00}\mathbf{S}_{01} = \mathbf{0}, \mathbf{T}_{01}\mathbf{S}_{00} = \mathbf{0} \quad (19)$$

$$\mathbf{T}_{00}\mathbf{q} = \mathbf{0}, \mathbf{T}_{01}\mathbf{q} = \mathbf{0}, \mathbf{r}^T\mathbf{S}_{00} = \mathbf{0}, \mathbf{r}^T\mathbf{S}_{01} = \mathbf{0} \quad (20)$$

$$\mathbf{r}^T\mathbf{q} = 2 \quad (21)$$

From these equations, a rank condition on the matrices $\mathbf{S}_{00}$ and $\mathbf{S}_{01}$ can also be found.

Theorem 3. *For the class of LPPRFBs and its starting block $\mathbf{E}_0^o(z)$ stated above, rank$(\mathbf{S}_{01}) = m-l$, rank$(\mathbf{S}_{00}) = l$ and $\mathbf{q}$ is a nonzero vector satisfying (20) and (21).*

Proof. From (18), we know that $m-l = \text{rank}(\mathbf{T}_{01}\mathbf{S}_{01}) \leq \text{rank}(\mathbf{S}_{01})$. In addition, $\text{rank}(\mathbf{S}_{01}) \leq \min\{m+1, m-l\} = m-l$. Thus, the matrix $\mathbf{S}_{01}$ has full rank, i.e., $\text{rank}(\mathbf{S}_{01}) = m-l$. The matrix $\mathbf{T}_{01}$ also has full rank by similar derivation. From (17) and the rank inequality for matrix in [9], we can obtain that $2l = \text{rank}(\mathbf{T}_{00}\mathbf{S}_{00} + \mathbf{J}_l\mathbf{T}_{00}\mathbf{S}_{00}\mathbf{J}_l) \leq 2\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00})$, i.e., $\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) \geq l$. Define matrices $\mathbf{T}_a = [\mathbf{T}_{00}^T, \mathbf{T}_{01}^T, \mathbf{r}^T]^T$ and $\mathbf{S}_a = [\mathbf{S}_{00}, \mathbf{S}_{01}, \mathbf{q}]$, then from Eq. (18)-(21), we can see,

$$\mathbf{T}_a\mathbf{S}_a = \begin{bmatrix} \mathbf{T}_{00}\mathbf{S}_{00} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{m-l} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 2 \end{bmatrix}$$

which means $\text{rank}(\mathbf{TS}) = \text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) + m - l + 1 \leq \text{rank}(\mathbf{S}) \leq \min\{m+l, m+l+1\} = m+1$, i.e., $\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) \leq l$. Combined with above inequality, we get $\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) = l$. Similarly, define matrices $\mathbf{T}_b = [\mathbf{T}_{01}^T, \mathbf{r}^T]^T$ and $\mathbf{S}_b = [\mathbf{S}_{01}, \mathbf{q}]$, then it can be shown easily that $\mathbf{T}_b\mathbf{S}_b = \text{diag}(\mathbf{I}_{m-l}, 2)$, i.e., $\text{rank}(\mathbf{T}_b) = m-l+1$. However, from $\mathbf{T}_b\mathbf{S}_{00} = \mathbf{0}$ and Sylvester rank inequality in [9], we can obtain that $0 = \text{rank}(\mathbf{T}_b\mathbf{S}_{00}) \geq \text{rank}(\mathbf{T}_b) + \text{rank}(\mathbf{S}_{00}) - (m+1)$ from which the inequality $\text{rank}(\mathbf{S}_{00} \leq (m+1) - \text{rank}(\mathbf{T}_b) = (m+1) - (m+1-l) = l$ can be established. Finally, from $\text{rank}(\mathbf{T}_{00}\mathbf{S}_{00}) = l \leq \text{rank}(\mathbf{S}_{00})$, we know that $\text{rank}(\mathbf{S}_{00}) = l$. This finishes the proof. □

From the rank of matrix $\mathbf{S}_{00}$, we also propose a parameterized form similar to the even channel case for $\mathbf{S}_{00} = [\mathbf{U}_{00}\mathbf{\Gamma}_p, \mathbf{U}_{00}\mathbf{\Gamma}_m]$, where $\mathbf{U}_{00}$ has size $(m+1)\times l$, and $\mathbf{\Gamma}_p$, $\mathbf{\Gamma}_m$ are defined as the same way in even M case. Apply similar parameterized form for $\mathbf{A}_{00}$ with matrices $\mathbf{V}_{00}$ with the size $m\times l$. Finally, replace the matrices $\mathbf{S}_{01}$ and $\mathbf{A}_{01}$ with $\mathbf{U}_{01}$ and $\mathbf{V}_{01}$. It can be shown that $\mathbf{E}_0^o(z)$ can be written in the form (22) and factorized in the form (23) shown on the top of next page, where $\mathbf{\Phi}_0^o$ can be further factorized in the form (24) and matrices $\mathbf{U}_0 = [\mathbf{U}_{00}, \mathbf{U}_{01}, \mathbf{q}]$ and $\mathbf{V}_0 = [\mathbf{V}_{00}, \mathbf{V}_{01}]$. We also explicitly show the inverse $\mathbf{\Gamma}_o^{-1}$ in (25),

$$\mathbf{\Phi}_0 = \begin{bmatrix} \mathbf{U}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_0 \end{bmatrix}\begin{bmatrix} \mathbf{I}_m & \mathbf{0} & \mathbf{I}_m \\ \mathbf{0} & \sqrt{2} & \mathbf{0} \\ \mathbf{I}_m & \mathbf{0} & \mathbf{I}_m \end{bmatrix}\begin{bmatrix} \mathbf{I}_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_m \end{bmatrix} \quad (24)$$

$$
\begin{aligned}
\mathbf{E}_0^o(z) &= \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{U}_{00}\boldsymbol{\Gamma}_p + z^{-1}\mathbf{U}_{00}\boldsymbol{\Gamma}_m\mathbf{J}_l & \mathbf{U}_{00}\boldsymbol{\Gamma}_m + z^{-1}\mathbf{U}_{00}\boldsymbol{\Gamma}_p\mathbf{J}_l & \mathbf{U}_{01} & \mathbf{q} & \mathbf{U}_{01}\mathbf{J}_{m-l} \\ \mathbf{V}_{00}\boldsymbol{\Gamma}_p - z^{-1}\mathbf{V}_{00}\boldsymbol{\Gamma}_m\mathbf{J}_l & \mathbf{V}_{00}\boldsymbol{\Gamma}_m - z^{-1}\mathbf{V}_{00}\boldsymbol{\Gamma}_p\mathbf{J}_l & \mathbf{V}_{01} & \mathbf{0} & -\mathbf{V}_{01}\mathbf{J}_{m-l} \end{bmatrix} \quad (22) \\
&= \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{U}_{00} & \mathbf{U}_{01} & \mathbf{q} & \mathbf{U}_{01}\mathbf{J}_{m-l} & z^{-1}\mathbf{U}_{00}\mathbf{J}_l \\ \mathbf{V}_{00} & \mathbf{V}_{01} & \mathbf{0} & -\mathbf{V}_{01}\mathbf{J}_{m-l} & -z^{-1}\mathbf{V}_{00}\mathbf{J}_l \end{bmatrix} \begin{bmatrix} \boldsymbol{\Gamma}_p & \boldsymbol{\Gamma}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{2m-2l+1} \\ \mathbf{J}_l\boldsymbol{\Gamma}_m\mathbf{J}_l & \mathbf{J}_l\boldsymbol{\Gamma}_p\mathbf{J}_l & \mathbf{0} \end{bmatrix} \\
&= \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{U}_{00} & \mathbf{U}_{01} & \mathbf{q} & \mathbf{U}_{01}\mathbf{J}_{m-l} & \mathbf{U}_{00}\mathbf{J}_l \\ \mathbf{V}_{00} & \mathbf{V}_{01} & & \mathbf{0}-\mathbf{V}_{01}\mathbf{J}_{m-l} & \mathbf{V}_{00}\mathbf{J}_l \end{bmatrix} \begin{bmatrix} \mathbf{I}_{2m-l+1} & \mathbf{0} \\ \mathbf{0} & z^{-1}\mathbf{I}_l \end{bmatrix} \begin{bmatrix} \boldsymbol{\Gamma}_p & \boldsymbol{\Gamma}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{2m-2l+1} \\ \mathbf{J}_l\boldsymbol{\Gamma}_m\mathbf{J}_l & \mathbf{J}_l\boldsymbol{\Gamma}_p\mathbf{J}_l & \mathbf{0} \end{bmatrix} = \frac{1}{\sqrt{2}}\boldsymbol{\Phi}_0^o\,\boldsymbol{\Lambda}_o(z)\,\boldsymbol{\Gamma}_o \quad (23)
\end{aligned}
$$

$$
\boldsymbol{\Gamma}_o^{-1} = \begin{bmatrix} \hat{\boldsymbol{\Gamma}}_p & \hat{\boldsymbol{\Gamma}}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{2(m-l)+1} \\ \mathbf{J}_l\hat{\boldsymbol{\Gamma}}_m\mathbf{J}_l & \mathbf{J}_l\hat{\boldsymbol{\Gamma}}_p\mathbf{J}_l & \mathbf{0} \end{bmatrix} \quad (25)
$$

where $\hat{\boldsymbol{\Gamma}}_p$ and $\hat{\boldsymbol{\Gamma}}_m$ are the same ones as in the even channel case. It can be shown easily that PR property is ensured as long as $\mathbf{U}_0$ and $\mathbf{V}_0$ are invertible. Note that this is a *new* result, not reported in the literature before. This new lattice factorization is also good in terms of implementation delays. The following theorem states that the proposed lattice structure employs the fewest number of delays in its implementation.

Theorem 4. *The factorization in (2) is minimal, where its factors are given in (4) and (23).*

Proof. This result is similar to the proof of Theorem 2. For the case of odd M and even β, $|\hat{\mathbf{J}}_M(z)| = (-1)^{(M-1)/2}z^{-\beta}$. Thus,

$$
\begin{aligned}
\deg(\mathbf{E}(z)) &= \deg\left(z^{-M(K-1)}|\mathbf{D}| \times |\mathbf{E}(z^{-1})| \times |\hat{\mathbf{J}}_M(z)|\right) \\
&= M(K-1) + \beta - \deg(\mathbf{E}(z))
\end{aligned}
$$

which leads to $\deg(\mathbf{E}(z)) = M(K-1)/2 + \beta/2$, same as the even channel case. In this case, from the existence conditions in Table 1, we know that K should be odd. In our factorization, there are $(K-1)/2$ order-2 building blocks where each employs M delays seen in (4) ($\Lambda_0^o(z)$ has $M/2$ and $\Lambda_1^o(z)$ has $(M+1)/2$ delays). The initial block $\mathbf{E}_0^o(z)$ employs $\beta/2 = l$ delays seen in (23). Therefore, the total number of delays in use is $M(K-1)/2 + \beta/2$, which is just the degree of the transfer function $\mathbf{E}(z)$. This finishes the proof. □

4.2 β is odd

For the odd $\beta = 2l+1, (0 \le l < m)$, the starting block $\mathbf{E}_0(z)$ has minimal order two. From the LP condition (1) and PR condition, we can also derive a set of matrix equations similar to the previous ones, but they are more complex than before. Due to the space limitation, the specific equations are omitted here. Different from the previous two cases, it is very involved to obtain efficient rank conditions and complete solutions to these equations due to the higher order of $\mathbf{E}_0(z)$. Although the complete solutions cannot be established, we can at least find a simple but meaningful solution, thus obtain a factorization for this case in the following form.

$$
\mathbf{E}_0(z) = \frac{1}{\sqrt{2}}\boldsymbol{\Phi}_0\boldsymbol{\Lambda}(z)\boldsymbol{\Gamma} \quad (26)
$$

where various matrices are,

$$
\boldsymbol{\Phi}_0 = \operatorname{diag}(\mathbf{U}_0, \mathbf{V}_0)\mathbf{W}_{2m+1}\operatorname{diag}(\mathbf{I}_{m+1}, \mathbf{J}_m)
$$

$$
\boldsymbol{\Lambda}(z) = \begin{bmatrix} \mathbf{I}_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & z^{-1}\mathbf{I}_{m-l+1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & z^{-2}\mathbf{I}_l \end{bmatrix}
$$

$$
\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_p & \mathbf{0} & \boldsymbol{\Gamma}_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\Gamma}_s & \boldsymbol{\Gamma}_t \\ \mathbf{0} & 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J}_{m-l}\boldsymbol{\Gamma}_t\mathbf{J}_{m-l} & \mathbf{J}_{m-l}\boldsymbol{\Gamma}_s\mathbf{J}_{m-l} \\ \mathbf{J}_l\boldsymbol{\Gamma}_m\mathbf{J}_l & \mathbf{J}_l\boldsymbol{\Gamma}_p\mathbf{J}_l & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}
$$

where $\mathbf{U}_0 = [\mathbf{U}_{00}, \mathbf{U}_{01}, \mathbf{q}]$, $\mathbf{V}_0 = [\mathbf{V}_{00}, \mathbf{V}_{01}]$ are square matrices with size $(m+1)\times(m+1)$ and $m \times m$, respectively, and $\boldsymbol{\Gamma}_p$ and $\boldsymbol{\Gamma}_m$ are same as before, $\boldsymbol{\Gamma}_s = (\boldsymbol{\Gamma}_3 + \boldsymbol{\Gamma}_4)/2$ and $\boldsymbol{\Gamma}_t = (\boldsymbol{\Gamma}_3 + \boldsymbol{\Gamma}_4)\mathbf{J}_{m-l}/2$, where $\boldsymbol{\Gamma}_3$ and $\boldsymbol{\Gamma}_4$ are two arbitrary square invertible matrices with size $(m-l)\times(m-l)$. We also explicitly show the inverse $\boldsymbol{\Gamma}^{-1}$ as follows,

$$
\boldsymbol{\Gamma}^{-1} = \begin{bmatrix} \hat{\boldsymbol{\Gamma}}_p & \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\boldsymbol{\Gamma}}_m \\ \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} & \mathbf{0} \\ \mathbf{J}_l\hat{\boldsymbol{\Gamma}}_m\mathbf{J}_l & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J}_l\hat{\boldsymbol{\Gamma}}_p\mathbf{J}_l \\ \mathbf{0} & \hat{\boldsymbol{\Gamma}}_s & \mathbf{0} & \hat{\boldsymbol{\Gamma}}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{m-l}\hat{\boldsymbol{\Gamma}}_t\mathbf{J}_{m-l} & \mathbf{0} & \mathbf{J}_{m-l}\hat{\boldsymbol{\Gamma}}_s\mathbf{J}_{m-l} & \mathbf{0} \end{bmatrix}
$$

where $\hat{\boldsymbol{\Gamma}}_s = (\boldsymbol{\Gamma}_3^{-1} + \boldsymbol{\Gamma}_4^{-1})/2$ and $\hat{\boldsymbol{\Gamma}}_t = (\boldsymbol{\Gamma}_3^{-1} - \boldsymbol{\Gamma}_4^{-1})\mathbf{J}_{m-l}/2$. The PR property can be guaranteed if matrices $\mathbf{U}_0$ and $\mathbf{V}_0$ are invertible.

5. DESIGN EXAMPLES

In this section, several LPPRFBs were constructed by using the proposed lattice factorization methods. We apply these methods in designing LPPRFBs for two different cases studied above. For all methods, the invertible matrices are decomposed by SVD through orthogonal and diagonal matrices. The invertibility is ensured as long as the diagonal elements are nonzero.

One objective function for optimization is minimization of the stopband attenuation and passband error for ideal filter shape, which is a classical one in FB design and optimization. Denote the passband of the ith filter $H_i(z)$ as $[\omega_{i,L}, \omega_{i,H}]$ and the transition bandwidth as ε. The cost function to be optimized is $C = C_{\text{passband}} + C_{\text{stopband}}$, where

$$
C_{\text{stopband}} = \sum_{i=0}^{P-1}\left(\int_0^{\omega_{i,L}-\varepsilon}|H_i(e^{j\omega})|^2 \mathrm{d}\omega + \int_{\omega_{i,H}+\varepsilon}^{\pi}|H_i(e^{j\omega})|^2 \mathrm{d}\omega\right)
$$

$$
C_{\text{passband}} = \sum_{i=0}^{P-1}\int_{\omega_{i,L}+\varepsilon}^{\omega_{i,H}-\varepsilon}(|H_i(e^{j\omega})| - 1)^2 \mathrm{d}\omega
$$

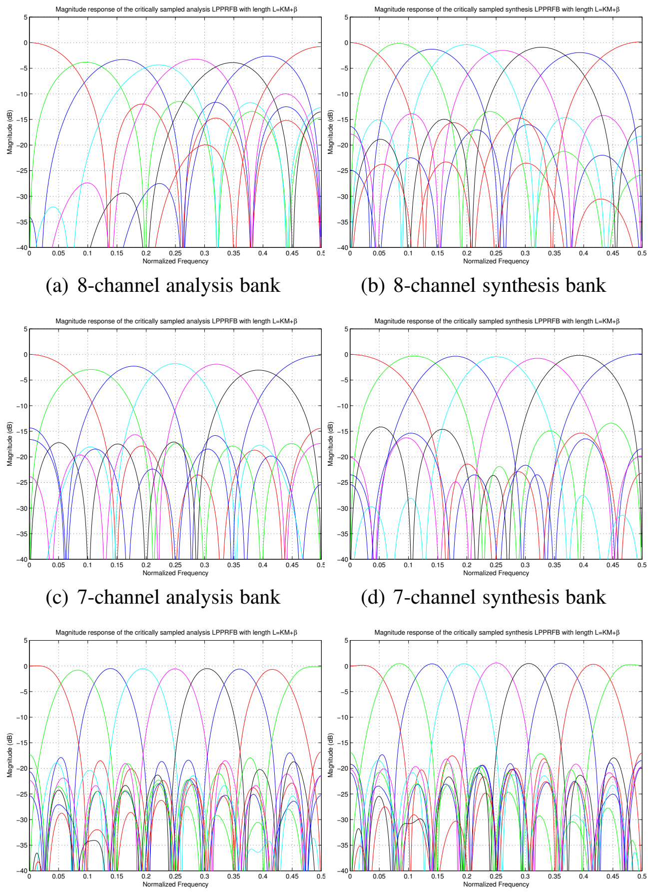
ε is selected to be 0.1 in the following example which used this objective function. Another optimization criterion which is more related to efficient image compression is the generalized coding gain for LPPRFBs [11, 5], which for 1-D source is,

$$
C_{\text{coding gain}} = 10\log_{10}\frac{\sigma_x^2}{\left(\prod_{i=0}^{M-1}\sigma_i^2\|f_i\|^2\right)^{1/M}} \quad (27)
$$

where σ_x^2 is the input signal variance, σ_i^2 is the variance of the ith subband signal and $\|f_i\|^2$ is the norm of the ith synthesis filter impulse response. It may also be easily extended to 2-D sources like real images. We consider AR(1) process with $\rho = 0.95$, as well as some images, as our sources.

The first design example is an 8-channel LPPRFB with filter length 10, i.e., $K = 1, \beta = 2$, optimized for coding gain. The analysis bank for AR(1) case is shown in Fig. 2(a). The corresponding synthesis bank is shown in in Fig. 2(b). The coding gain is shown in Table 2. Compared to the restrictive paraunitary case [3], our proposed general PR solution can avoid the inevitable zero filter coefficient at fixed positions [3], thus obtaining better performance.

The second example is a 7-channel LPPRFB with filter length 11, i.e., $K=1, \beta=4$, optimized for ideal filter shape. The analysis bank is shown in Fig. 2(c) and the corresponding synthesis bank is shown in in Fig. 2(d). The last design example is a 9-channel LPPRFB with length 41, i.e., $K=4, \beta=5$, optimized for ideal filter shape. The analysis and synthesis banks are shown in Fig. 2(e) and 2(f), respectively.



(a) 8-channel analysis bank (b) 8-channel synthesis bank

(c) 7-channel analysis bank (d) 7-channel synthesis bank

(e) 9-channel analysis bank (f) 9-channel synthesis bank

Figure 2: Design examples of critically sampled LPPRFBs

Table 2: Comparisons of coding gain (dB) for 8-channel critically sampled LPPRFBs with different properties

LPPRFBs → Sources ↓	8×16 LOT [12]	8×10 GLBT	8×12 GLBT	8×14 GLBT	8×16 GLBT [5]
AR(1)	9.22	9.12	9.38	9.52	9.62
Lena	16.05	15.96	16.11	16.21	16.23
Cameraman	12.74	12.51	12.67	12.74	12.77
Goldhill	11.77	11.55	11.69	11.77	11.81

Here we also show the comparison of coding gains for different LPPRFBs. Following the traditional transform coding terminology, the LPPUFB with 8 channels and length 16 and LPPRFBs are also called LOT and GLBT, respectively. We can see that our proposed lattice structure can offer more flexible tradeoff for coding gain and the filter length of LPPRFB. We plan to include some image coding results in the final version besides this if space permits.

6. CONCLUSION

In this paper, we have presented new lattice structures and design methods for a large class of LPPRFBs, where all FIR filters have the same arbitrary length $L=KM+\beta,(0\leq\beta<M)$ and symmetry center. The refined existence conditions for this class of FBs are established. Lattice structures are developed for both even and odd channel LPPRFBs. Compared to the existing works [4, 5, 6], our structures are more general than theirs and cover them as special cases. In addition, our result is also more general than the restrictive LP paraunitary FB studied in [3] or constrained cosine-modulated FB reported in [7]. Different from [8] whose design can only give near PR FB with arbitrary length, the proposed design method can structurally enforce the LP and PR properties into our lattice structures. To our knowledge, this is the most general LPPRFB with the same length to date. Furthermore, minimality of the proposed structure is proved which can guarantee the minimal number of delays used in implementation. Finally, several design and application examples are presented to validate the proposed novel lattice structures and to show the better trade off provided by the our design methods.

REFERENCES

[1] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Upper Saddle River, NJ: Prentice Hall, 1993.

[2] G. Strang and T. Q. Nguyen, *Wavelets and Filter Banks*, Wellesley, MA: Wellesley-Cambridge Press, 1996.

[3] T. D. Tran and T. Q. Nguyen, "On *M*-channel linear-phase FIR filter banks and application in image compression", *IEEE Trans. Signal Processing*, vol. 45, pp. 2175 - 2187, Sep. 1997.

[4] R. L. de Queiroz, T. Q. Nguyen and K. R. Rao, "GenLOT: generalized linear-phase lapped orthogonal transform", *IEEE Trans. Signal Processing*, vol. 44, pp. 497 - 507, Mar. 1996.

[5] T. D. Tran, R. L. de Queiroz and T. Q. Nguyen, "Linear-phase perfec reconstruction filter banks: lattice structure, design, and application in image coding", *IEEE Trans. Signal Processing*, vol. 48, pp. 133 - 147, Jan. 2000.

[6] L. Gan and K.-K. Ma, "A simplified lattice factorization for linear-phase perfect reconstruction filter bank", *IEEE Signal Processing Letter*, vol. 8, pp. 207-209, Jul. 2001.

[7] T. Q. Nguyen and R. D. Koilpillai, "The theory and design of arbitrary-length cosine-modulated filter banks and wavelets, satisfying perfect reconstruction", *IEEE Trans. Signal Processing*, vol. 44, pp. 473 - 483, Mar. 1996.

[8] A. Hjorungnes, H. Coeard and T. Ramstad, "Minimum mean suqare error FIR filter banks with arbitrary filter lengths", *Proc. IEEE Int. Conf. Image Processing*, pp. 961-964, 1999.

[9] F. R. Gantmacher, *The Theory of Matrices*, vol. 1, New York: Chelsea Publishing Company, 1977.

[10] P. P. Vaidyanathan and T. Chen, "Role of anticausal inverses in multirate filter banks - Part I: System-theoretical fundamentals", *IEEE Trans. Signal Processing*, vol. 43, pp. 1090 - 1102, May 1995.

[11] J. Katto and Y. Yasuda, "Performance evaluation of subband coding and optimization of its filter coefficients", *Proc. SPIE Visual Commun. Image Processing*, pp. 95 - 106, 1991.

[12] H. S. Malvar, *Signal Processing with Lapped Transforms*, Norwood, MA: Artech House, 1992.